JÜLICH Forschungszentrum

NRW-FAIR

Hypernuclear 3BF within the NCSM

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Workshop on "Three-Nucleon Interactions and Nuclear Dynamics", Bochum, Germany

- Motivation
- YN & YY interactions
- J-NCSM & SRG evolution of (hyper-)nuclear interactions
- Uncertainty of Λ separation energies & chiral YNN interactions
- Chiral YNN forces
- Application of YNN forces to light hypernuclei
- Conclusions & Outlook

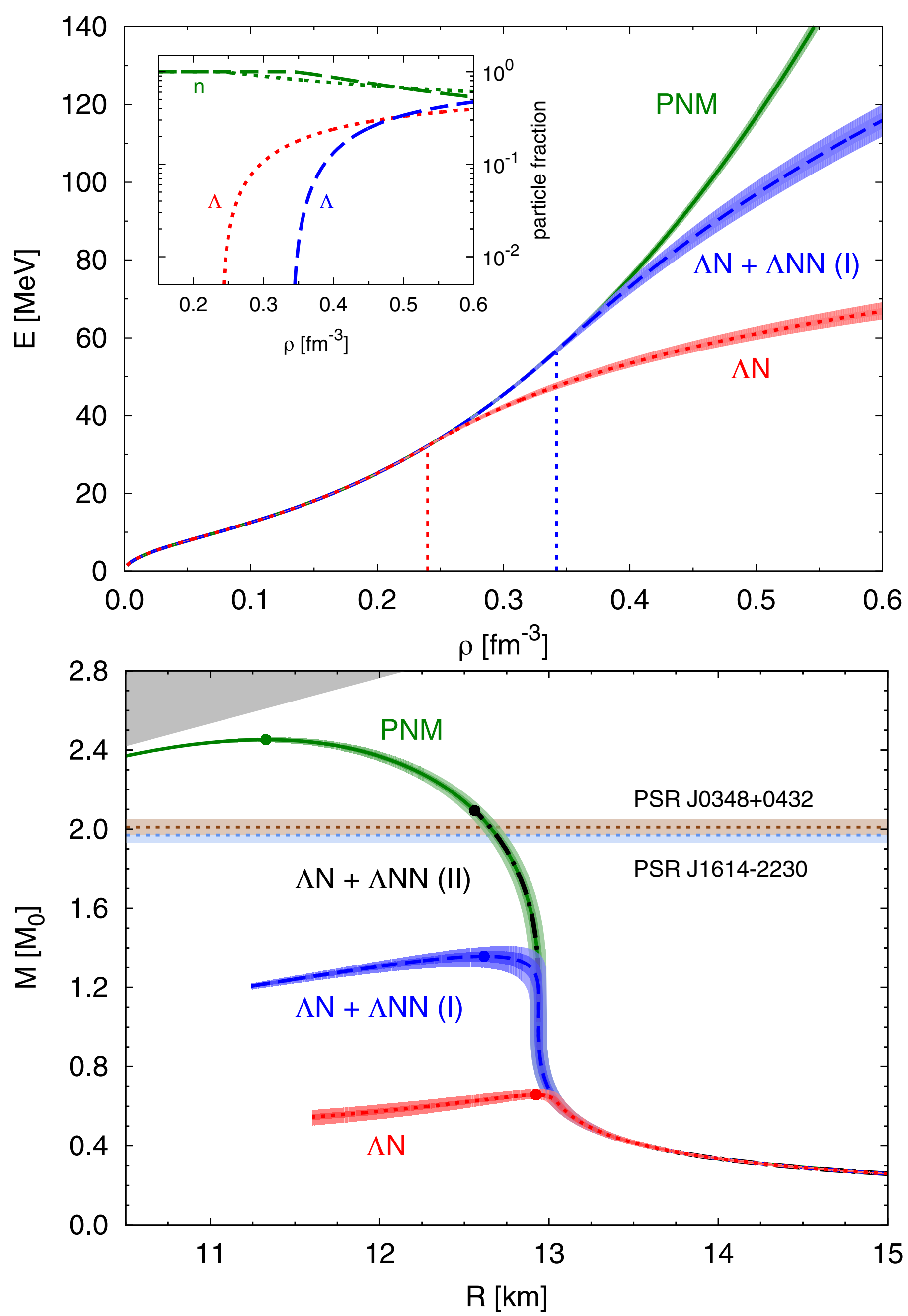
in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

December 11th, 2025

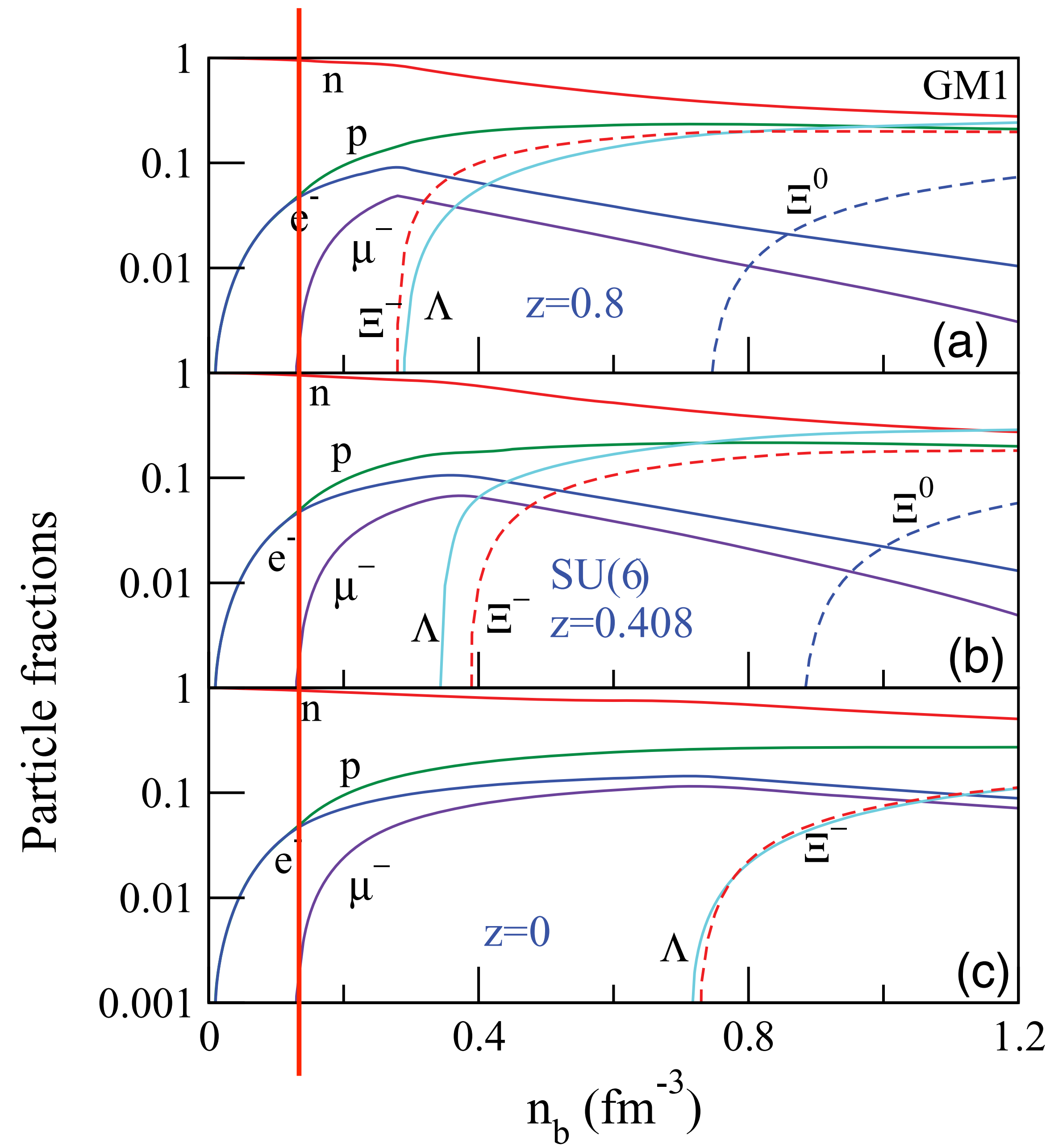
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- ***flavor dependence of baryon-baryon interactions***



(Lonardoni et al. (2015))

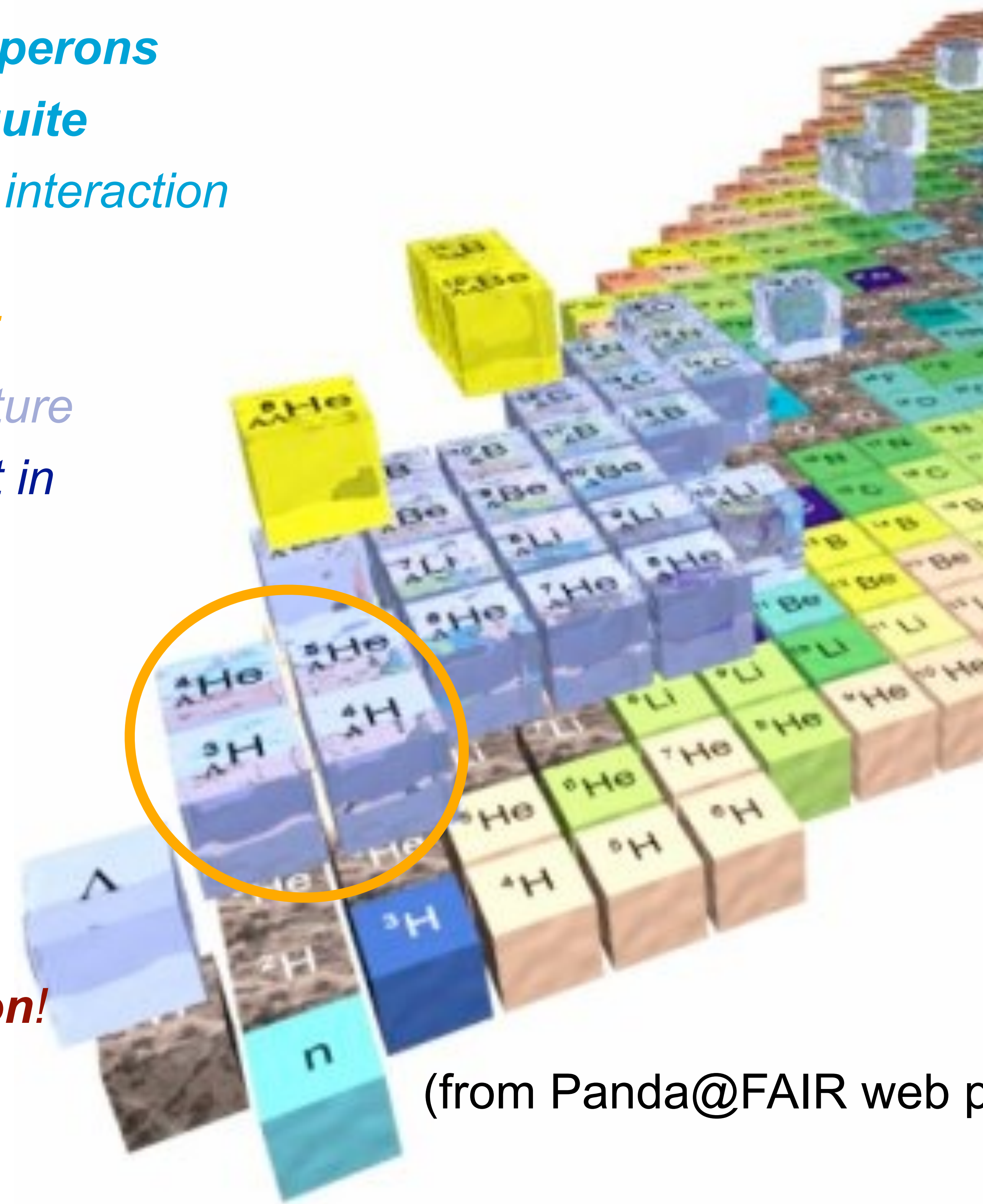


(Weissenborn et al. 2012)

Hypernuclei

Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively: **core nucleus + hyperons***
  - *„separation energies“ are **quite** independent from NN(+3N) interaction*
- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in **several spin states***
- ***non-trivial constraints** on the YN interaction even from lightest ones*
- *size of **YNN** interactions? need to include **Λ-Σ conversion**!*



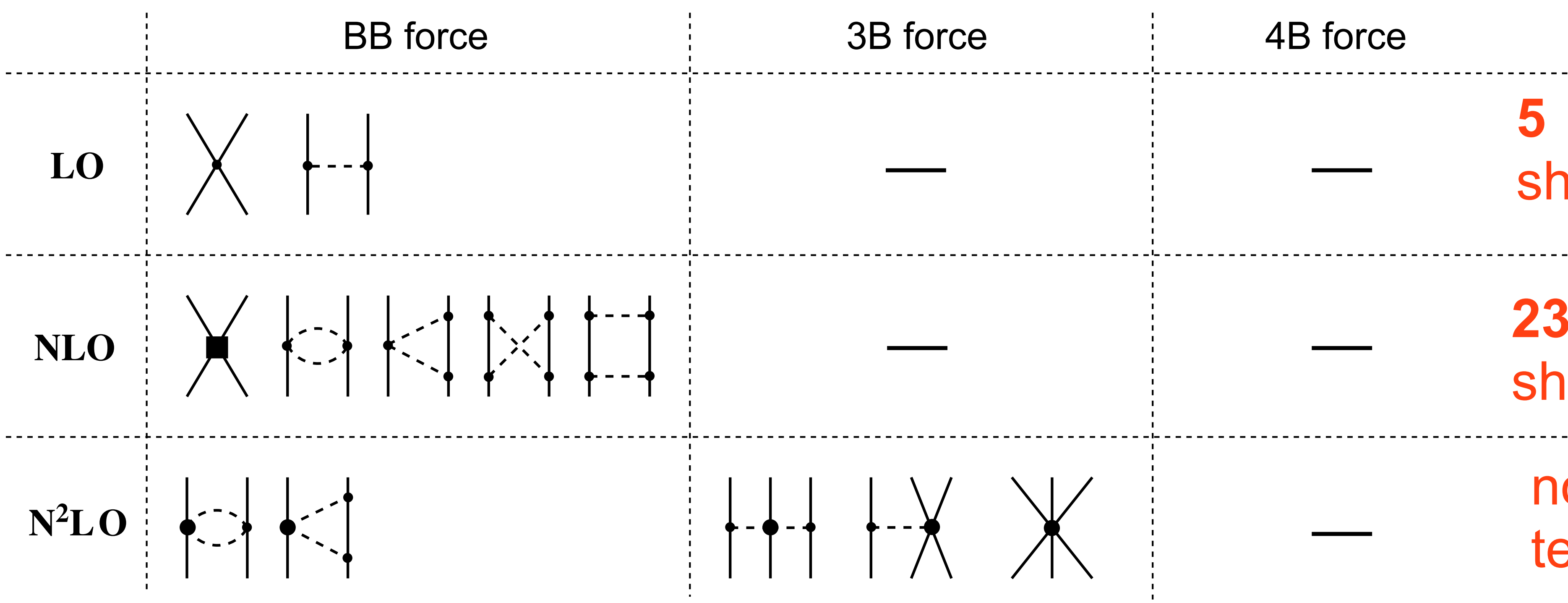
(from Panda@FAIR web page)

Chiral NN & YN interactions

EFT based approaches

Chiral EFT implements **chiral symmetry of QCD**

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N²LO (for YN)



5 NN/YN short range parameters

23 NN/YN short range parameters

no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

$\Lambda - \Sigma$ **conversion** is explicitly included (3BFs starting from N²LO)

SMS NLO/N²LO interaction

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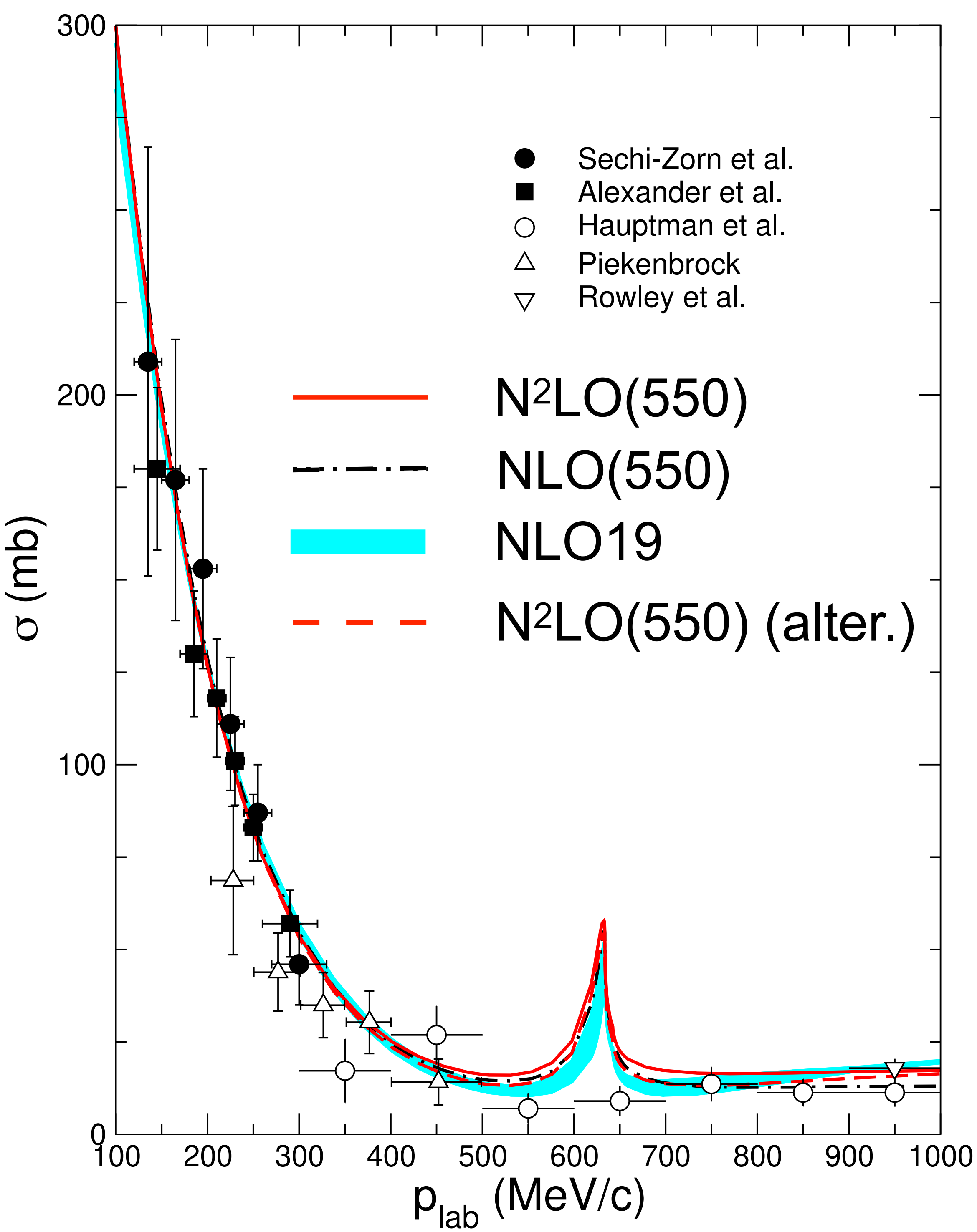
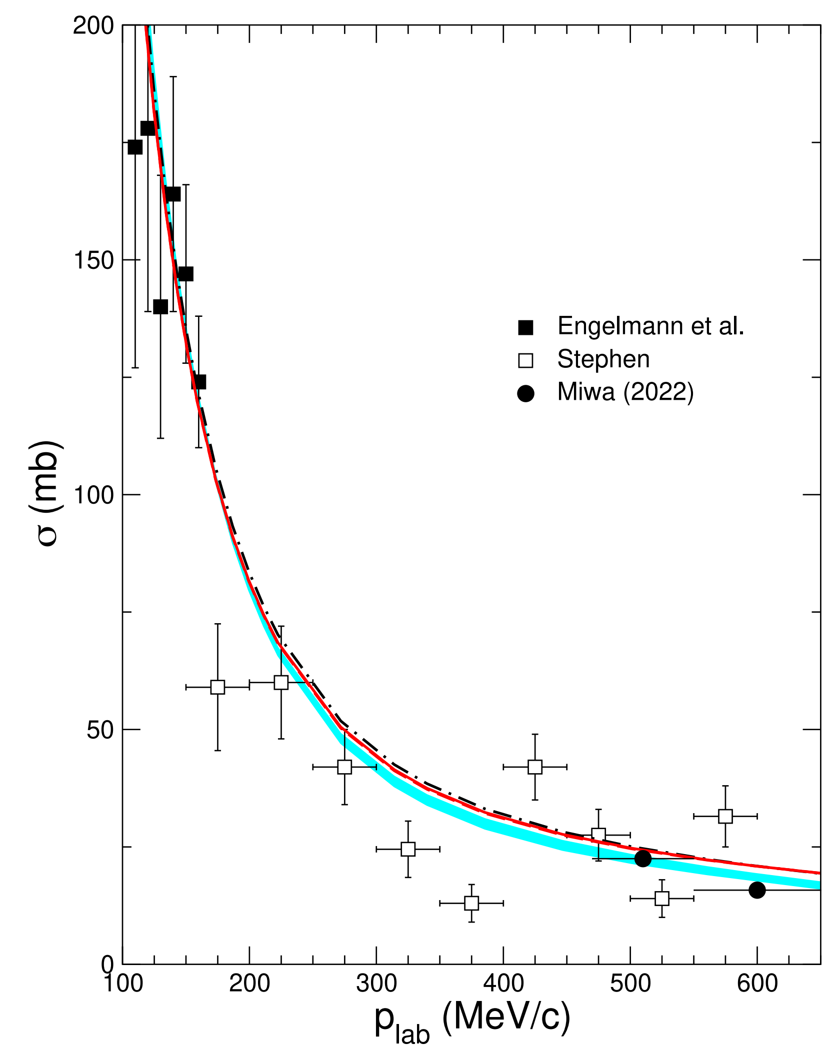
Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)

Λp -> Λp



- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)

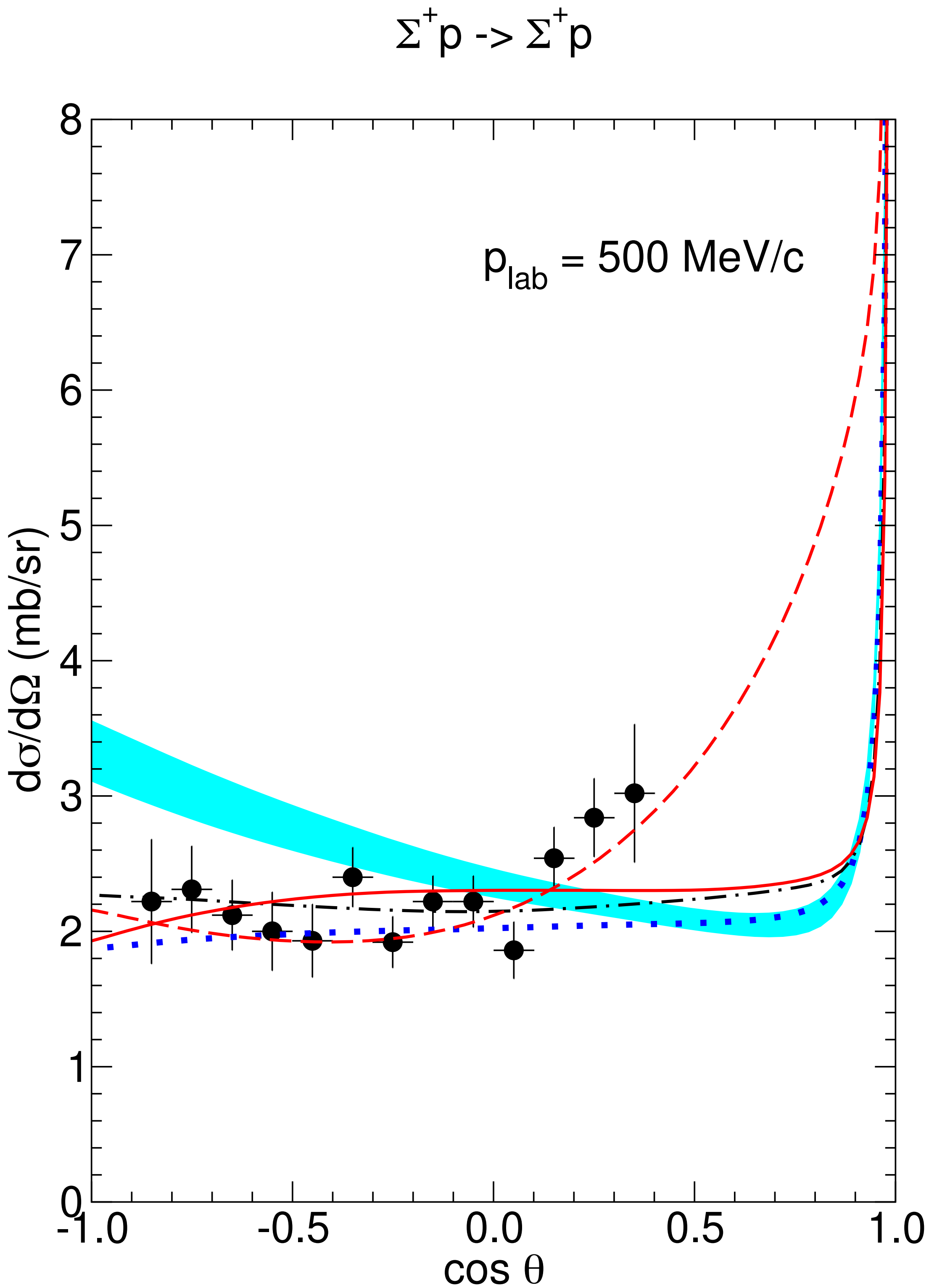
Σ⁻p -> Λn

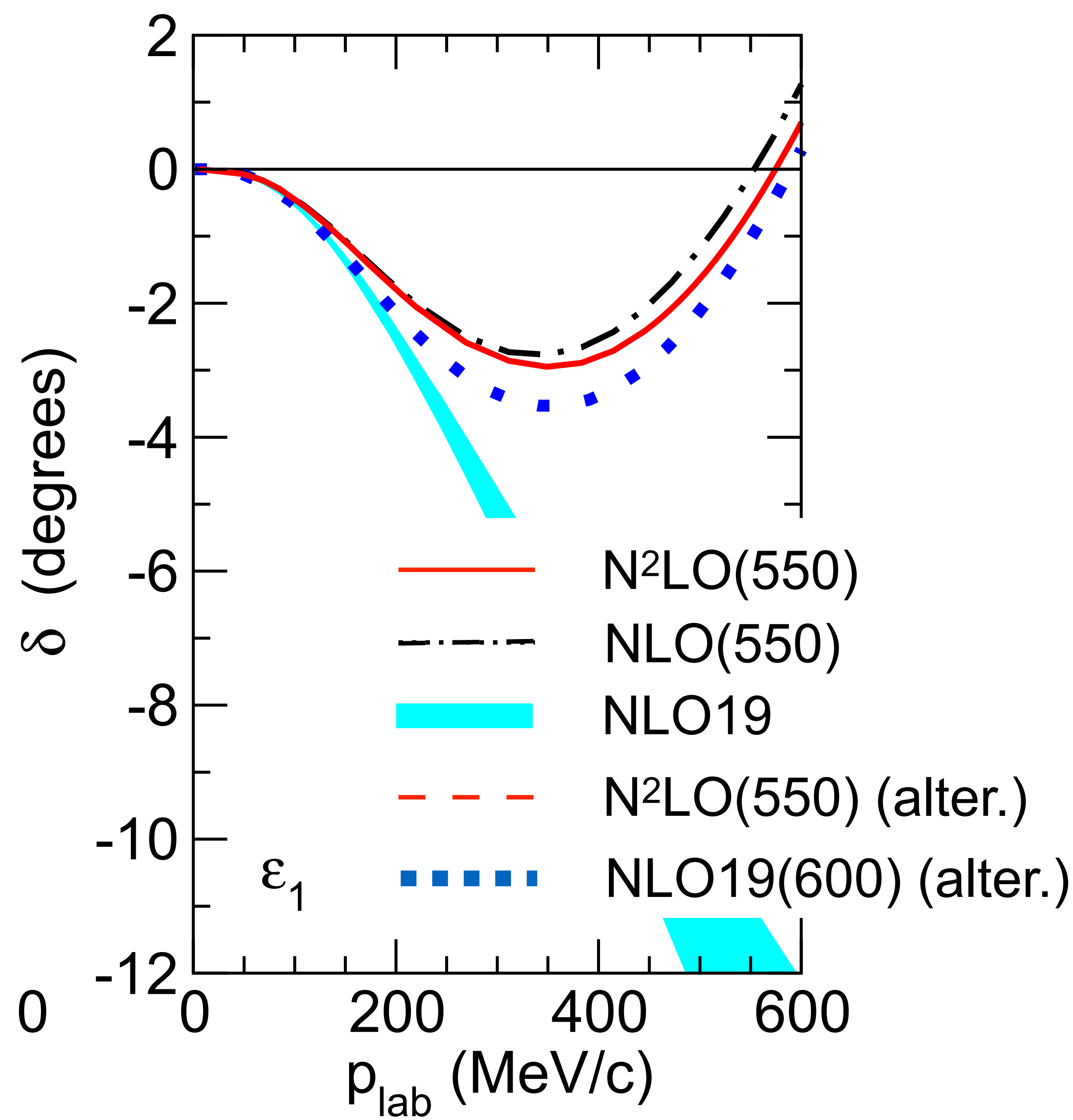


J. Haidenbauer et al. EPJ A 59, 63 (2023)

SMS NLO/N²LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!





J. Haidenbauer et al. EPJ A 59, 63 (2023)

Tools

Need reliable predictions for hypernuclei to further constrain interactions

Faddeev-Yakubovsky (FY) equations for $A = 3$ and 4 (momentum space)

- long distance tails of wave functions can be well represented
- uses Jacobi coordinates separating off CM motion
- chiral interactions can be directly used
- hugh linear eigenvalue problem (dimension 10^9x10^9) even for A=4 systems
- is feasible only for $A \leq 4$

(see AN, Glöckle, Kamada, 2002))

Jacobi-no core shell model (J-NCSM) for $A \geq 4$ (HO space)

- smaller dimensions allow to tackle p-shell nuclei
- exact antisymmetrization of wave functions can be prepared
- uses Jacobi coordinates separating off CM motion
- chiral interactions require similarity renormalization group (SRG) evolution
- long distance wave functions require large HO model spaces

(see Liebig et al., 2016; Le et al., 2020 & 2021)

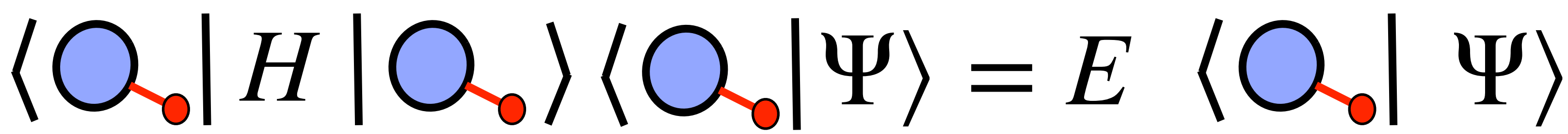
Jacobi-NCSM

Solve the Schrödinger equation using **HO states**

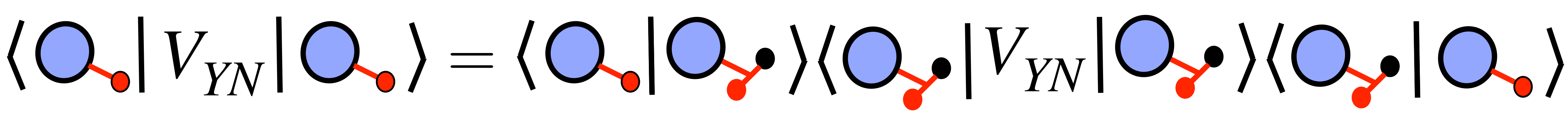
Two ingredients are necessary:

- **cfp** — antisymmetrized states for nucleons
- **transition coefficients** to separate off NN, YN, 3N and YNN

Schrödinger equation



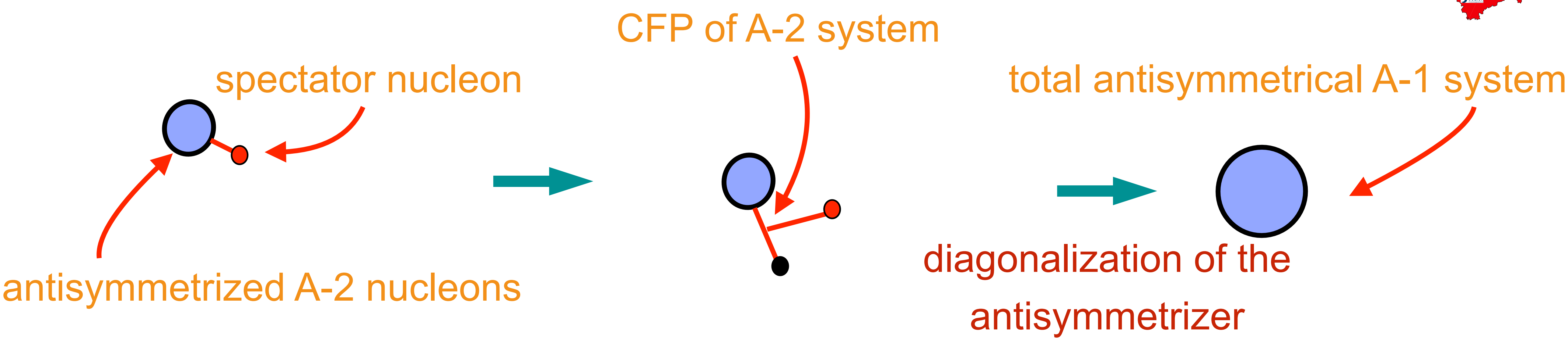
e.g. for YN interaction

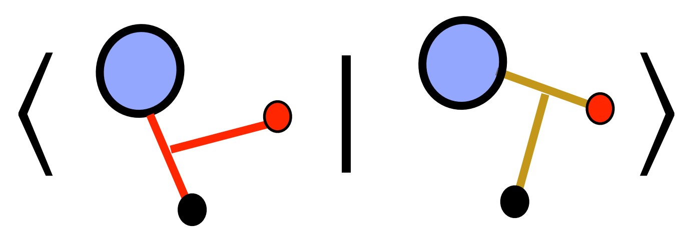


Application of to NN, YN, 3N and YNN interactions require the representation of particle transitions. (see Liebig et al. EPJ A 52,103 (2016), Le et al. EPJ A 56, 301 (2020) for combinatorical factors see Le et al. EPJ A 57, 217 (2021))

Jacobi-NCSM — CFP

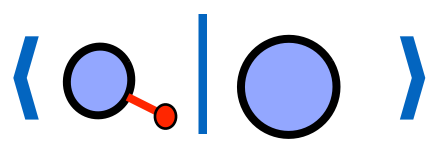
First, generate **antisymmetrized states** for the A-1 nucleon system





antisymmetrizer is equivalent to coordinate trafo
expression in terms of Talmi-Moshinsky brackets

(Navrátil et al. PRC 61,044001(2000))

The CFP coefficients  are obtained by diagonalization of the antisymmetrizer.

HO states guarantee:

- complete separation of antisymmetrized and other states
- **independence** of HO **length/frequency**

These coefficients will be openly accessible as **HDF5** data files

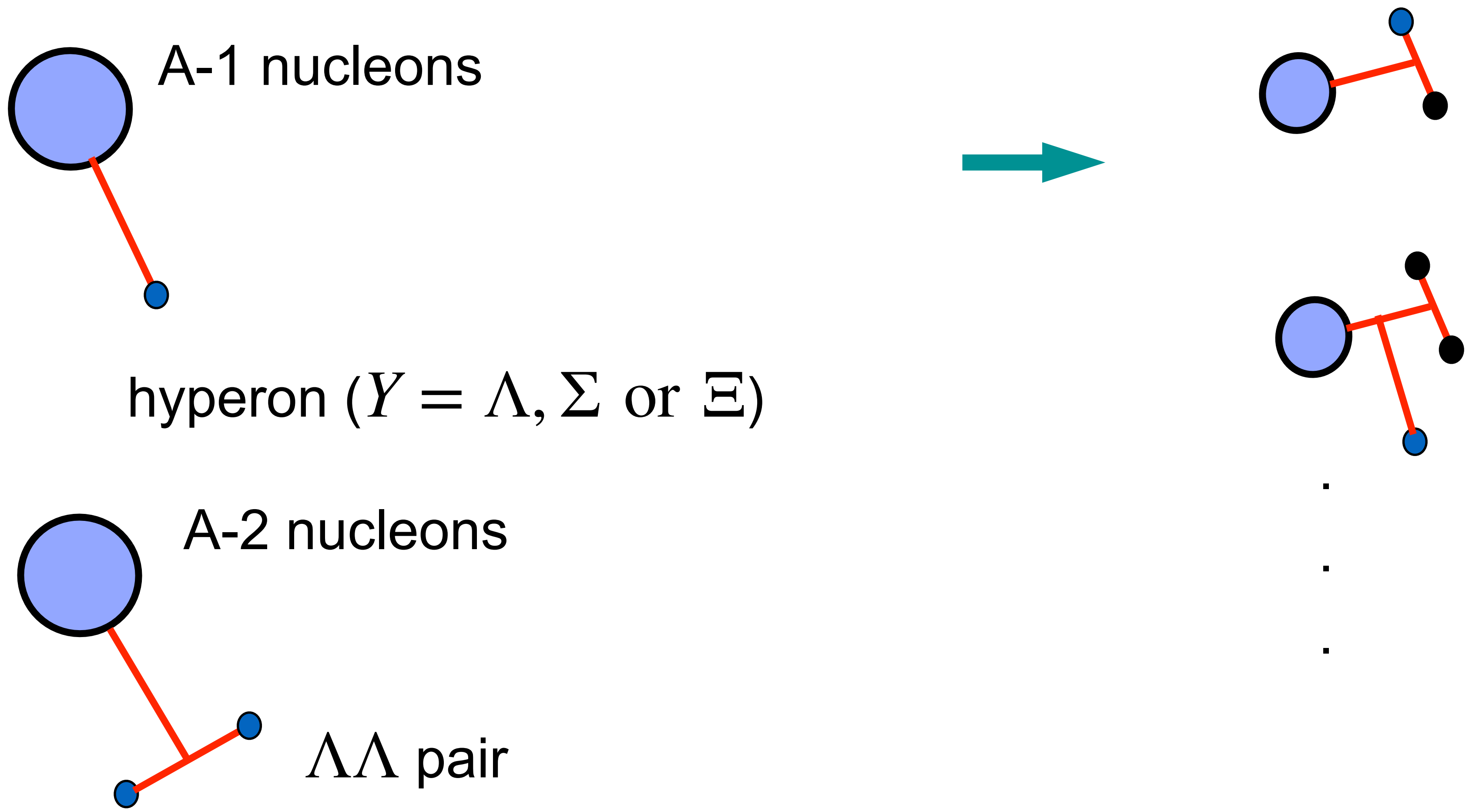
(download server is in preparation *(please contact me when interested!)*)

(Liebig et al. EPJ A 52,103 (2016))

Jacobi-NCSM states for $S = -1$

A-body hypernuclei state (no antisymmetrization with respect to nucleons required)

Third, rearrange baryons for the application of interactions, …



Again HO states guarantee the independence of HO length/frequency.

Transition coefficients are also accessible as **HDF5** data files to anyone interested.

(Le, Haidenbauer, Meißner, AN, 2020 & 2021)

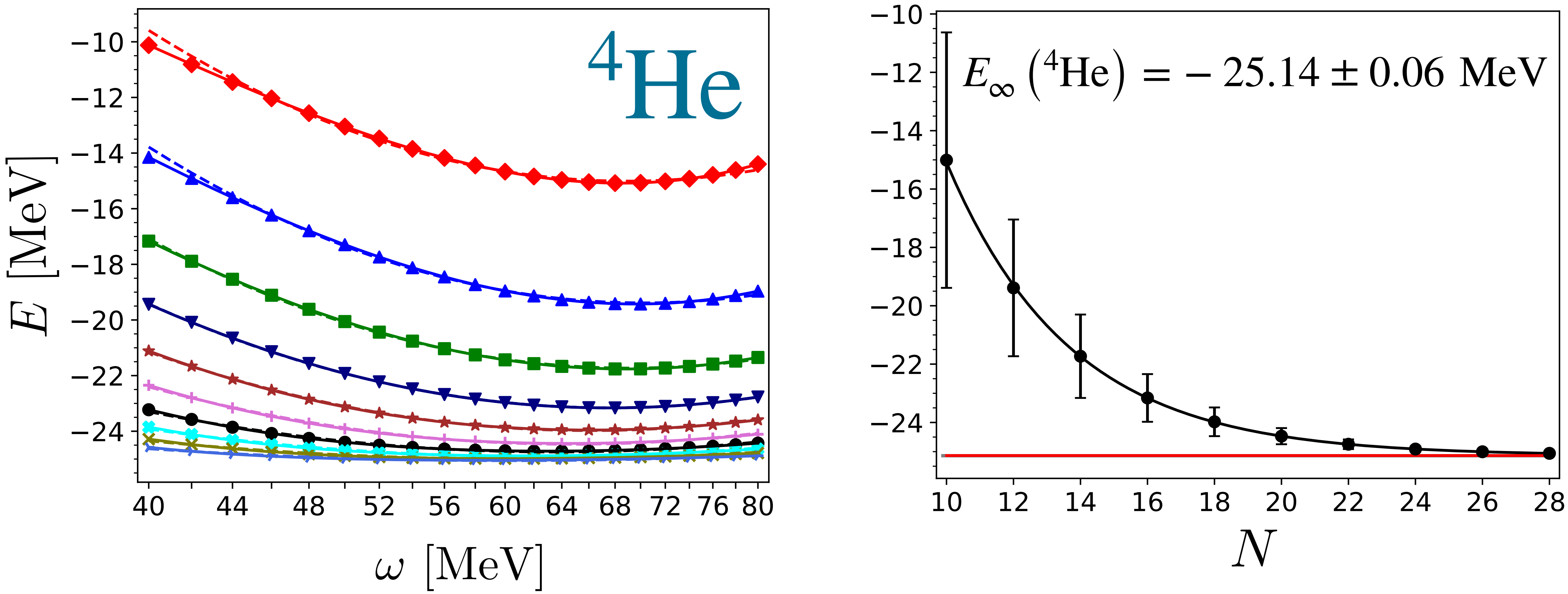
Converged results feasible for **"soft"** interactions.

Convergence for Jacobi-NCSM

Simple example: ^{4}He with SMS $N^2LO(550)$

observed dependence on ω and N

$$E(\omega) = E_N + \kappa\left(\log(\omega) - \log(\omega_{opt})\right)^2 \longrightarrow E_N = E_\infty + A\, e^{-bN}$$



Conservative uncertainty estimate: difference of $E_{N_{max}}$ and E_∞

Numerical uncertainties for light nuclei are small.

For p-shell, numerical uncertainty is more sizable due to smaller N_{max} and smaller separation energies.

(Liebig et al. EPJ A 52,103 (2016))

In future: neural networks for extrapolation (see Wolfgruber et al. PRC 110,014327 (2024))

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

(Bogner et al. PRC 75,061001 (2007))

$$\frac{dH_s}{ds} = \Big[\underbrace{\big[T \,, H(s) \big]}_{\equiv \eta(s)} , H(s) \Big] \qquad H(s) = T + V(s)$$

this choice of generator drives *V(s)* into a diagonal form in momentum space

- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s} \right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

SRG interactions

The evolution naturally separated in 2- and 3-body,… parts.

$$\frac{dV_{ij}(s)}{ds} = \left[\left[T_{ij}, V_{ij}(s)\right], T_{ij} + V_{ij}(s)\right]$$

easily done — we use momentum space

(Bogner et al. PRC 75,061001 (2007))

$$\frac{dV_{ijk}(s)}{ds} = \left[\left[T_{ij}, V_{jj}(s)\right], V_{ki}(s) + V_{jk}(s) + V_{ijk}(s)\right] + \left[\left[T_{jk}, V_{jk}(s)\right], V_{ki}(s) + V_{ij}(s) + V_{ijk}(s)\right]$$

$$+ \left[\left[T_{ki}, V_{ki}(s)\right], V_{ij}(s) + V_{jk}(s) + V_{ijk}(s)\right]$$

more involved but implemented

$$+ \left[\left[T_{ij} + T_k, V_{ijk}(s)\right], T_{ij} + T_k + V_{ij}(s) + V_{ij}(s) + V_{ki}(s) + V_{ijk}(s)\right]$$

(Hebeler PRC 85,021002(R) (2012))

4-body SRG-induced interactions small (see later), not necessary for hypernuclei

For 3N forces: induced interactions are of similar size a chiral 3N forces

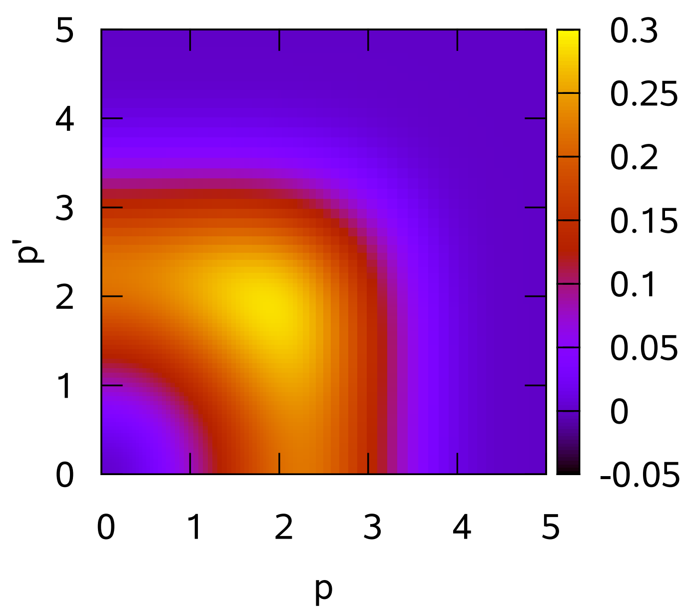
For ΛNN: SRG induced-3BFs are large, probably much larger than chiral ones!

(see also Wirth et al. (2016))

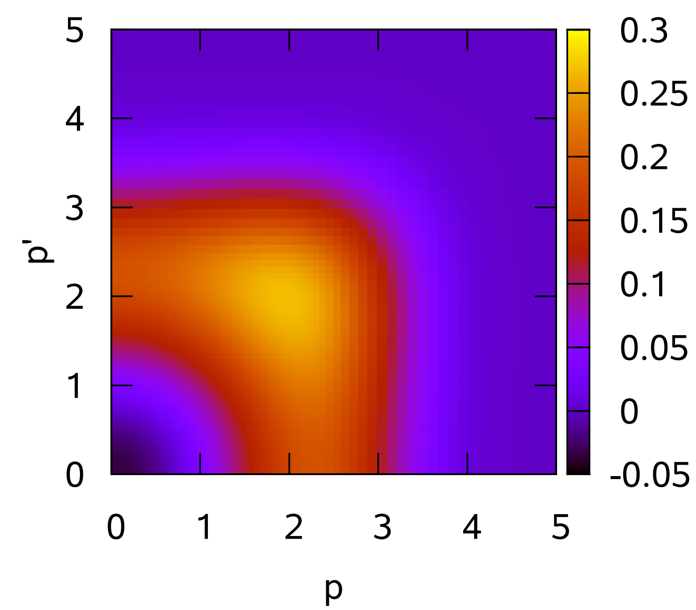
SRG interactions (YN)

Λp-Λp matrix element for the $^1S_0$ depending on incoming and outgoing momenta

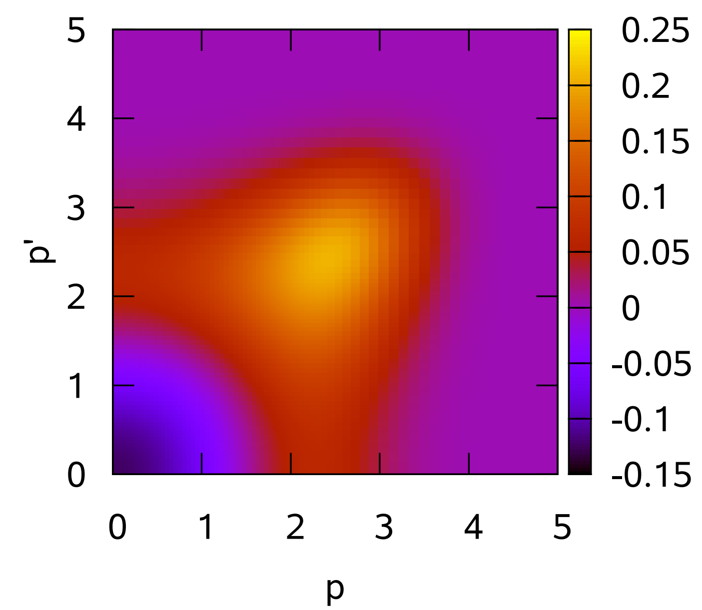
λ=10 fm^{-1}



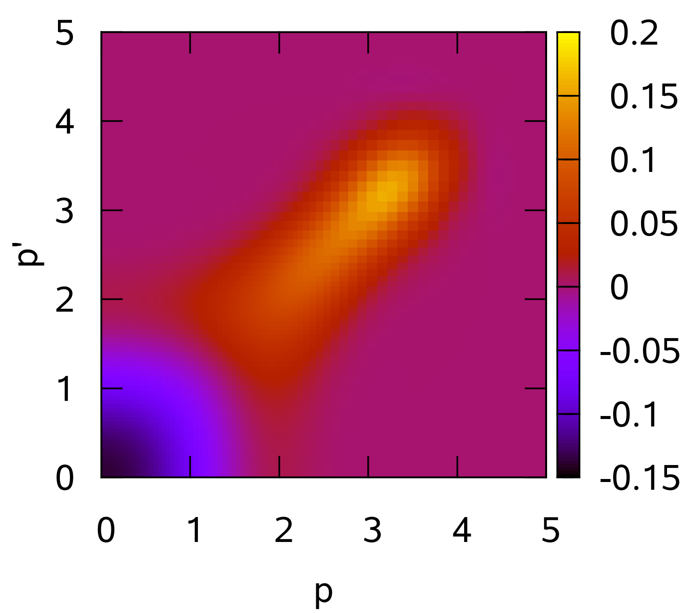
λ=5 fm^{-1}



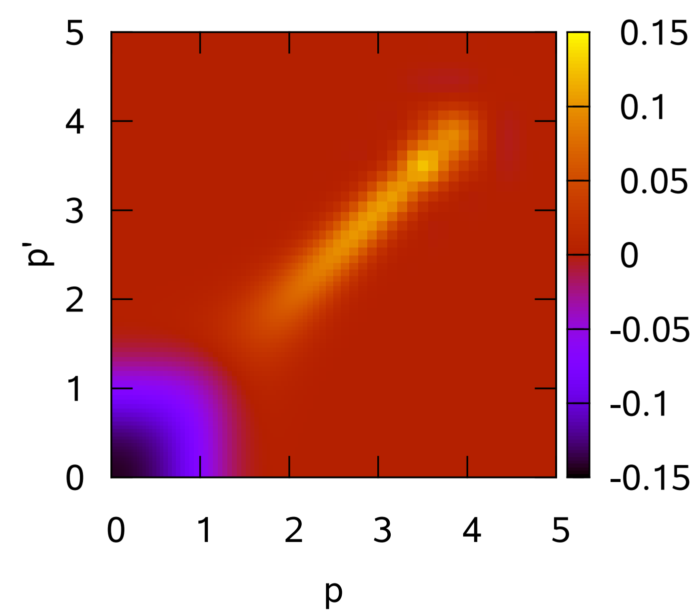
λ=3 fm^{-1}

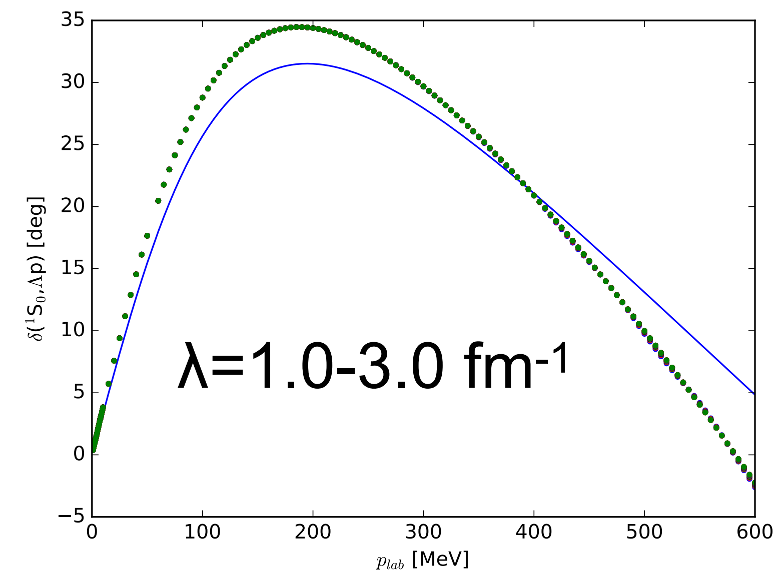


λ=2 fm^{-1}



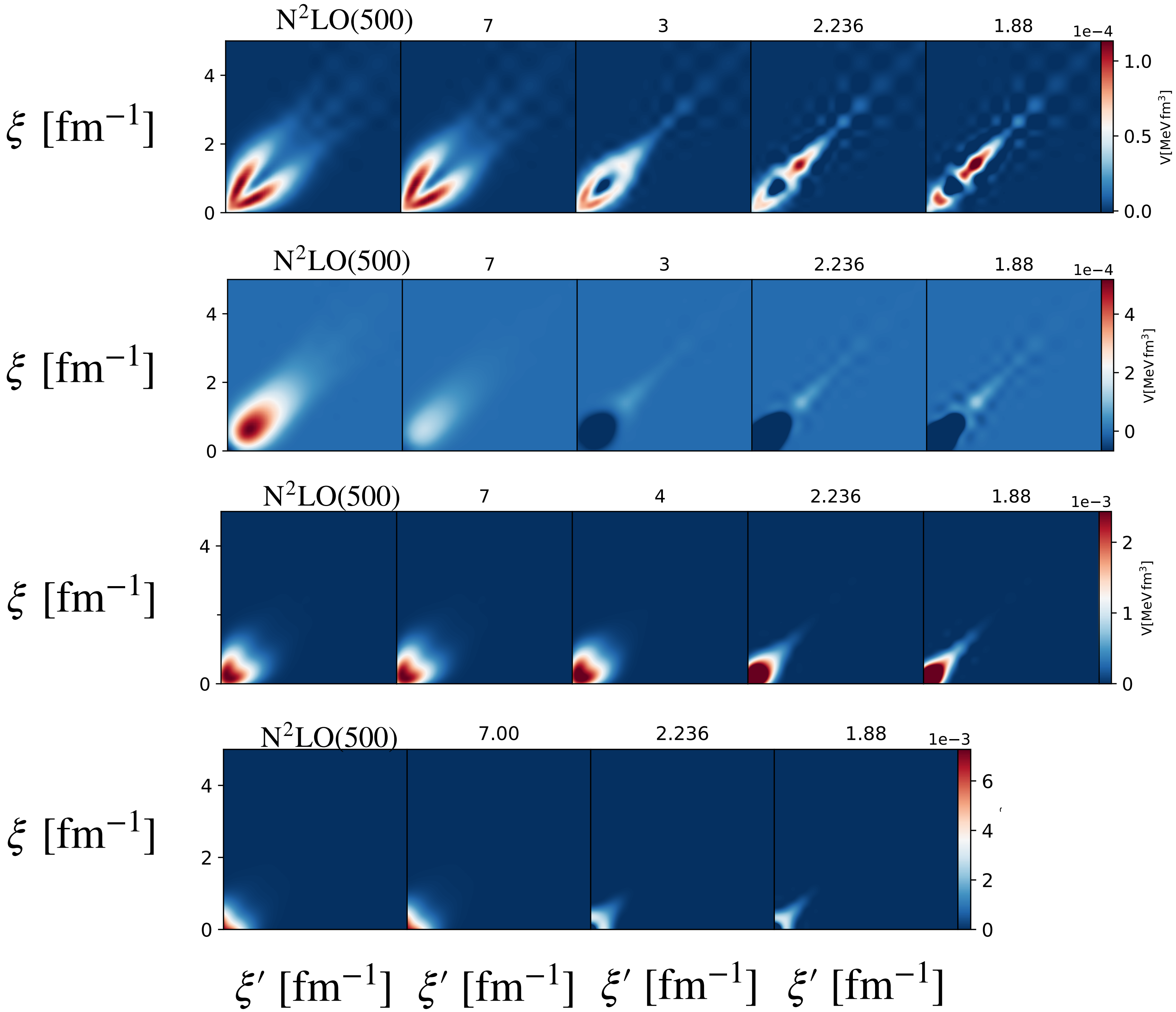
λ=1.5 fm^{-1}





SC97f compared to SRG of EFT-NLO-600

SRG 3N



$$J^\pi, T = \frac{9}{2}^+, \frac{1}{2}$$

$$J^\pi, T = \frac{7}{2}^+, \frac{1}{2}$$

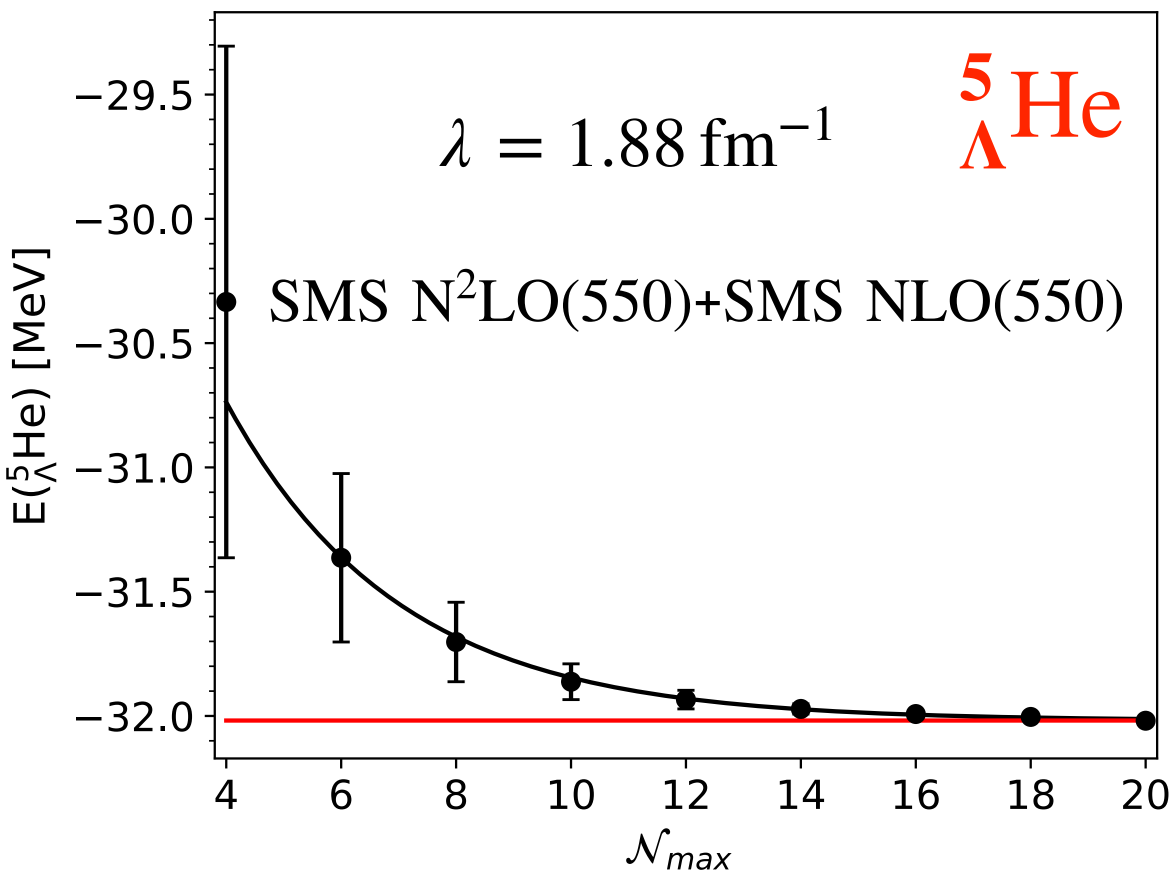
$$J^\pi, T = \frac{5}{2}^+, \frac{1}{2}$$

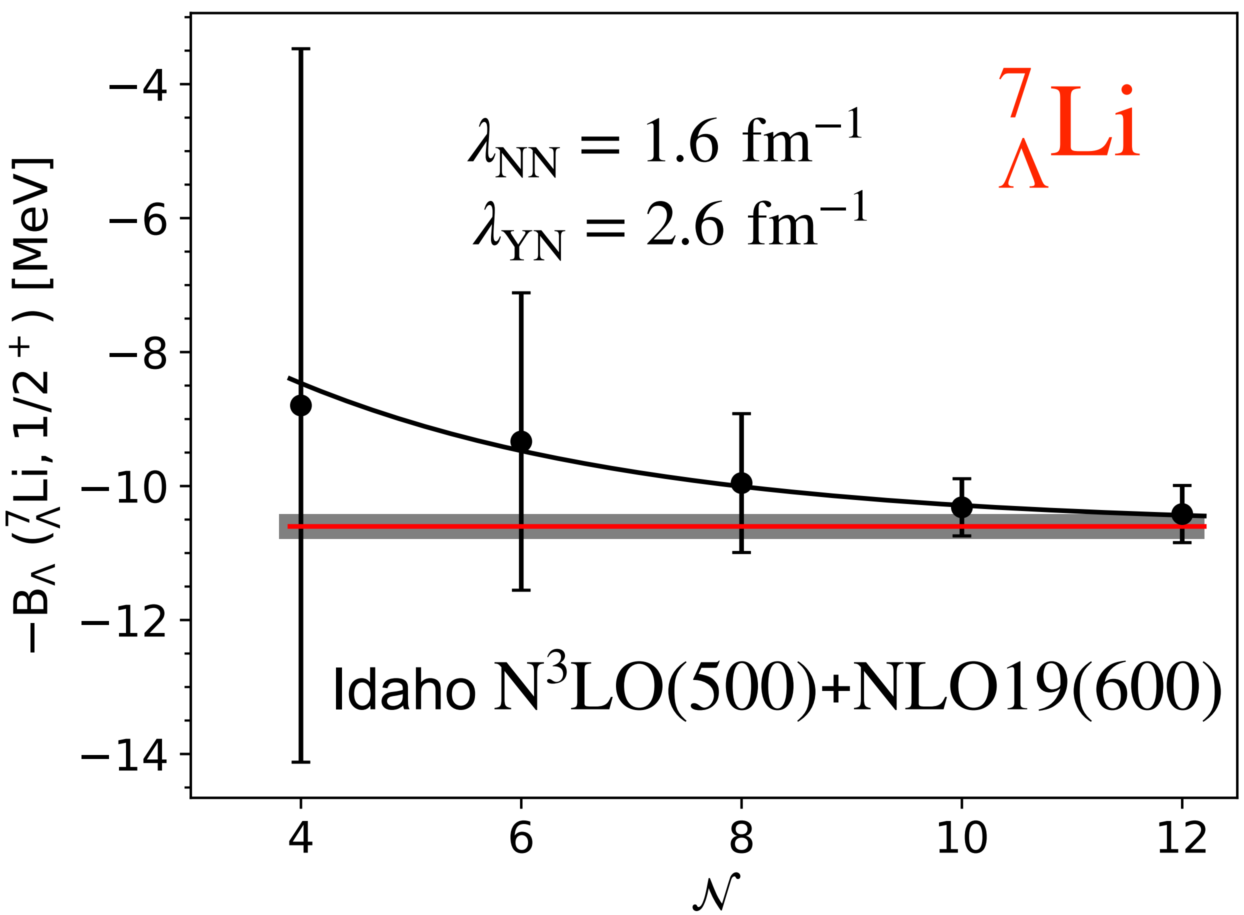
$$J^\pi, T = \frac{1}{2}^+, \frac{1}{2}$$

$$\xi, \xi' = p^2 + \frac{3}{4} q^2$$

J-NCSM convergence

SRG evolution improves convergence





$$E\left({}^5_\Lambda\text{He}\right) = -32.018 \pm 0.001\ \text{MeV} \qquad E_\Lambda\left({}^7_\Lambda\text{Li}\right) = 10.6 \pm 0.2\ \text{MeV}$$

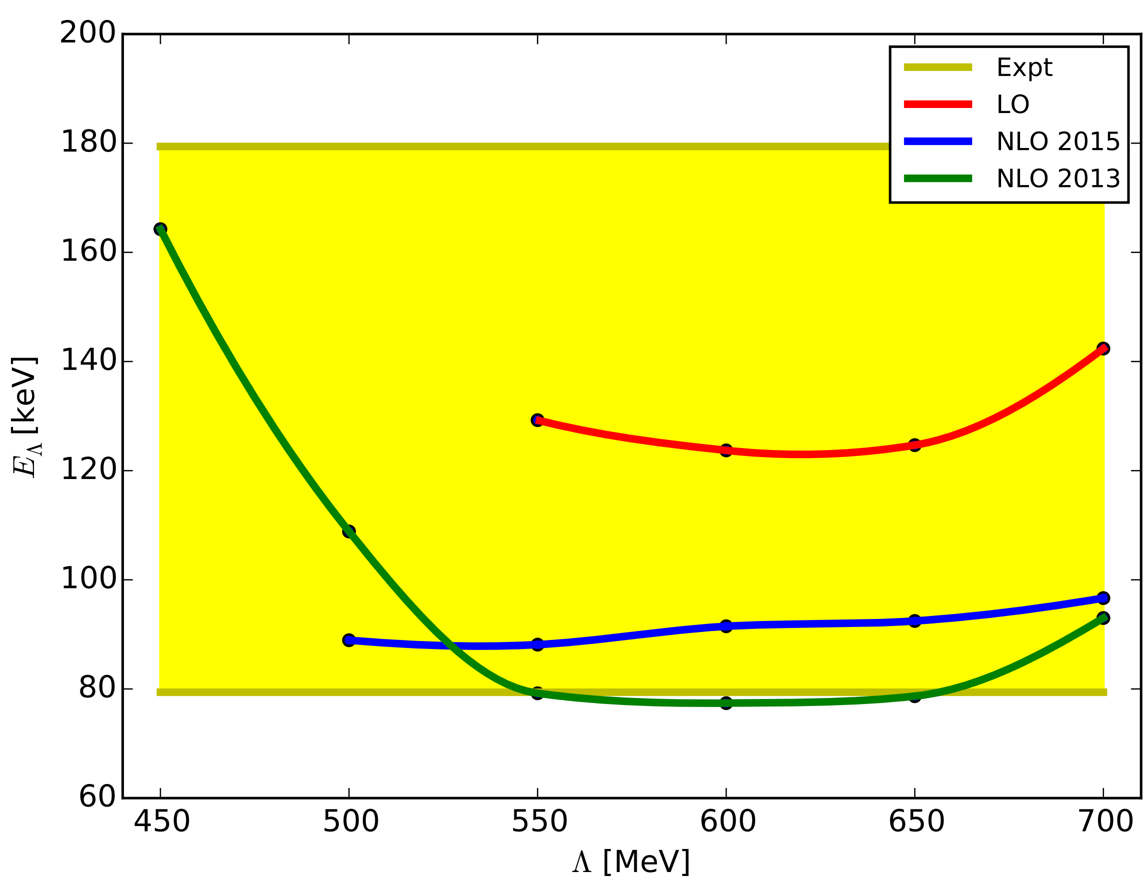
- for light nuclei and hypernuclei, the numerical uncertainty is negligible.
- for p-shell nuclei/hypernuclei, the uncertainty is visible
- extrapolation of separation energy can reduce uncertainty of this quantity

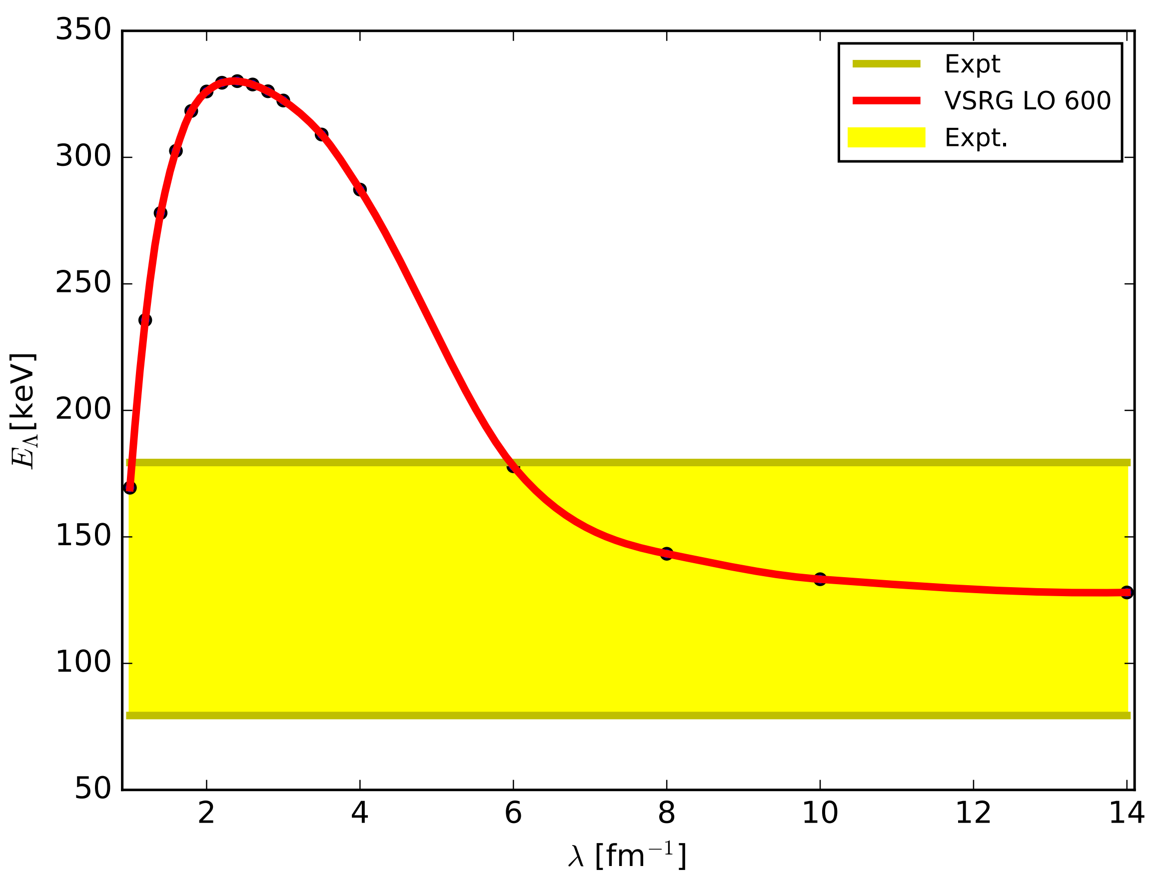
Induced 3BF …

SRG parameter dependence is significant when NN and YN interactions are evolved

→ missing 3N and YNN interactions

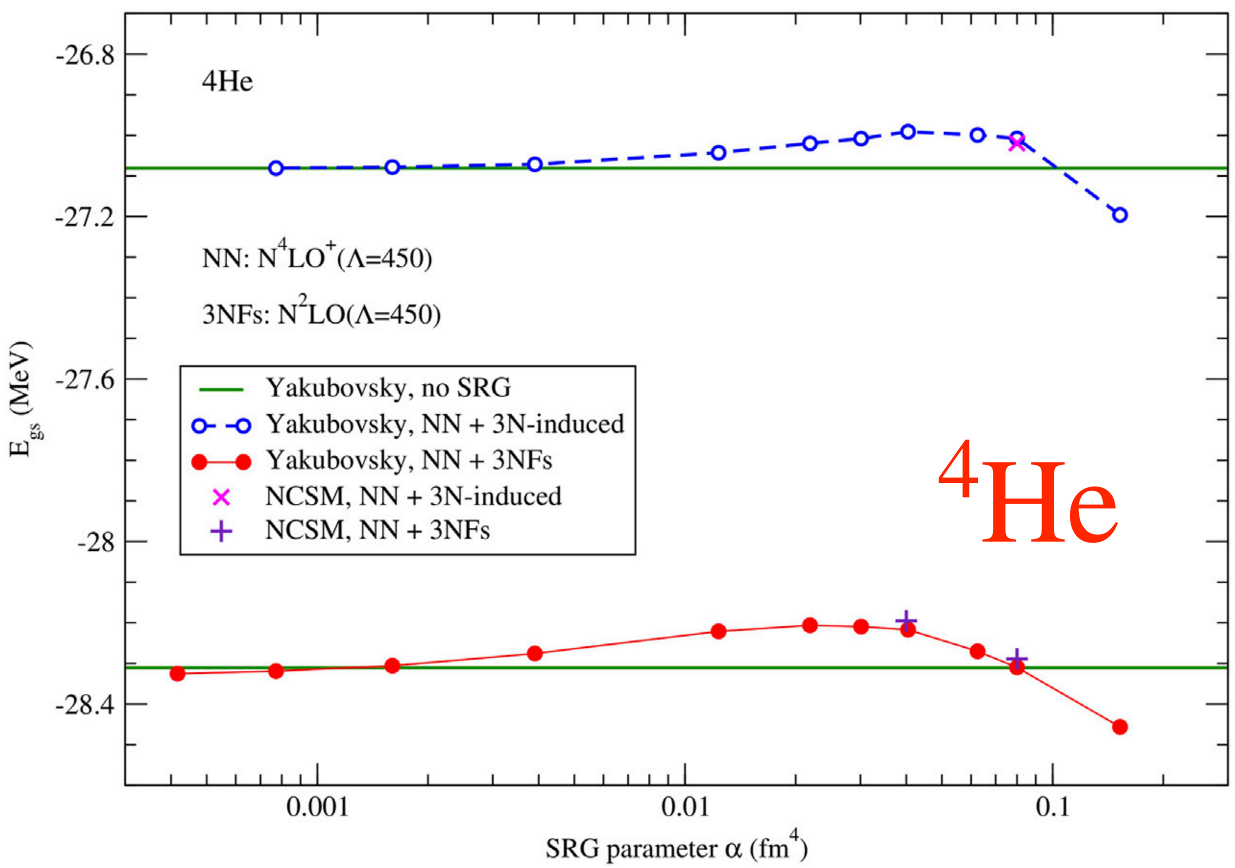
- 3NF is comparable to chiral 3NF
- YNN is larger than chiral YNN





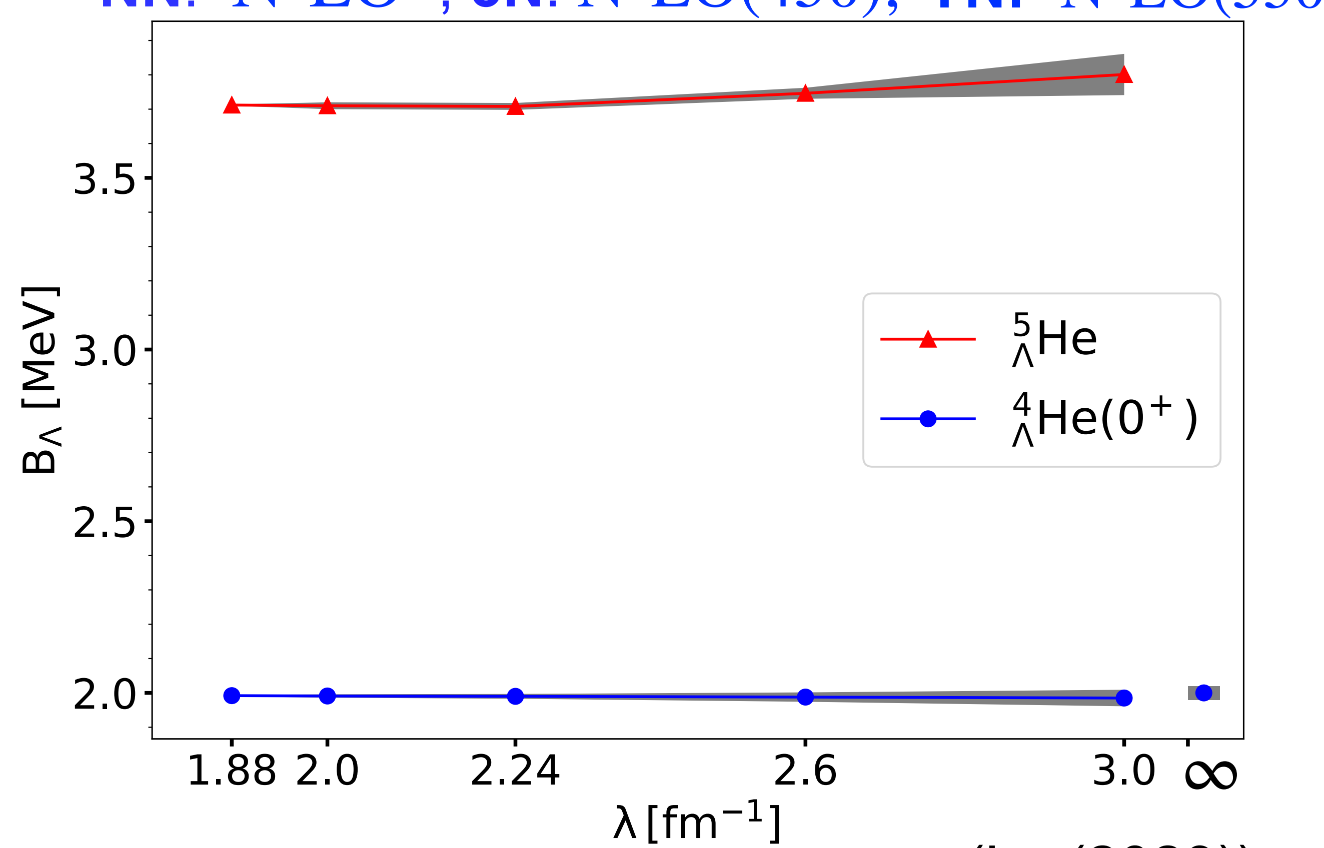
SRG dependence of results

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100-200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)



(Le (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

Uncertainty analysis to $A = 3$ to 5

Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Results for **different orders** enable uncertainty estimate:

Ansatz for the order by order convergence:

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \ \text{LO, exp., max, …})$$

Bayesian analysis of the uncertainty following Melendez et al. 2017,2019

Extracting c_k for $k \leq K$ **from calculations**

→ **probability distributions** for c_k

→ $$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$$

Uncertainty due to missing higher orders is more relevant than numerical uncertainty! (for light nuclei)

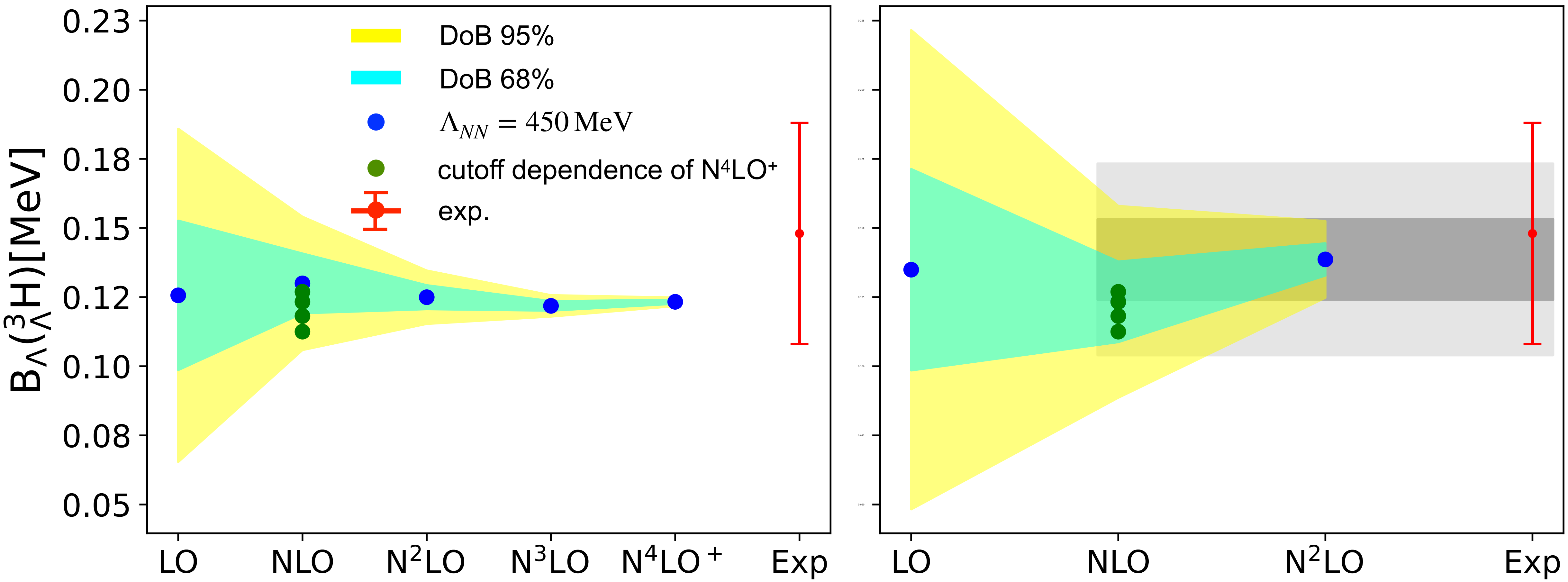
Application to ${}^{3}_{\Lambda}\mathrm{H}$

- Q, ν_0 and τ_0 are chosen using all available data (NN and YN convergence)
- uncertainties are extracted using c_k for NN or YN convergence
- use c_k of individual hypernuclei

→ individual uncertainties for NN and YN convergence for each separation energy

consistent with experimental data

cutoff dependence always at least NLO (YNN missing!)

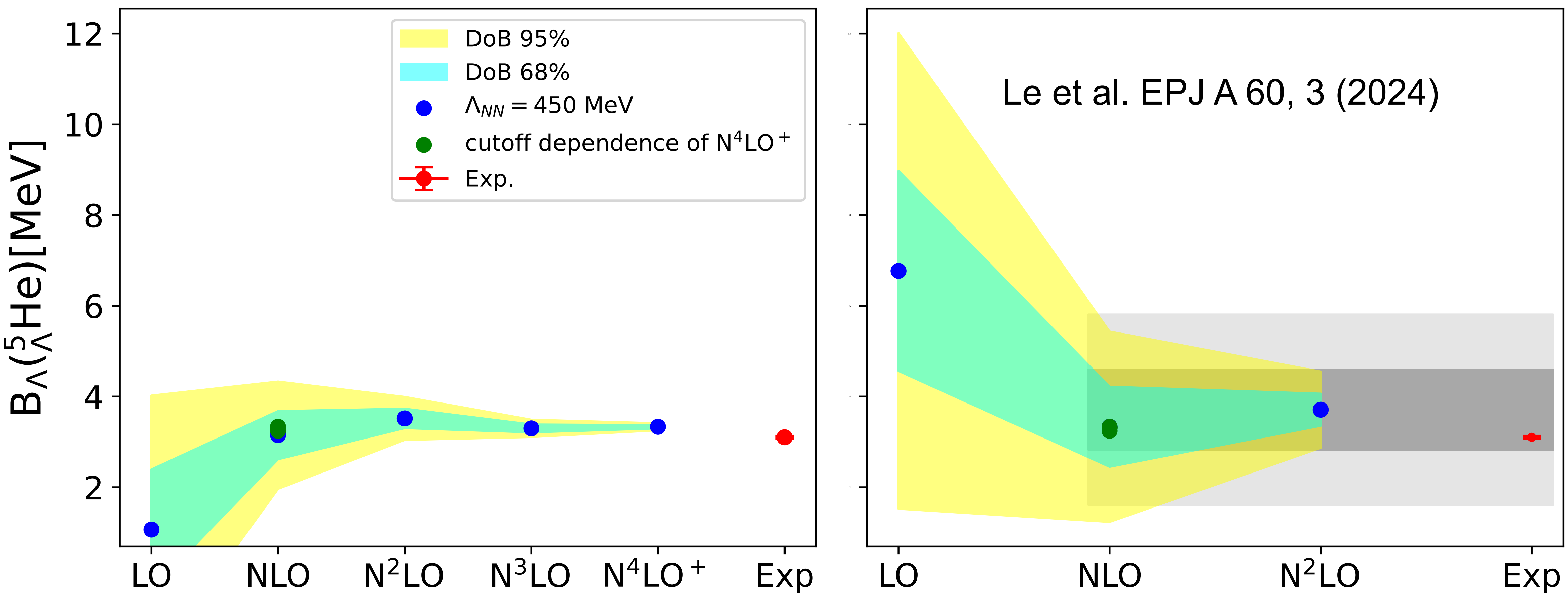


Application to $^{5}_{\Lambda}\mathrm{He}$ and summary

- **without YNN**: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
$^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
$^{4}_{\Lambda}\mathrm{He}\,(0^+)$	0.157	0.239
$^{4}_{\Lambda}\mathrm{He}\,(1^+)$	0.114	0.214
$^{5}_{\Lambda}\mathrm{He}$	0.529	0.881

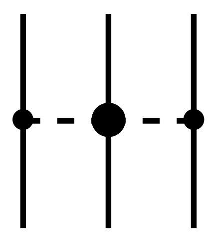
→ at the same time: estimate of YNN !



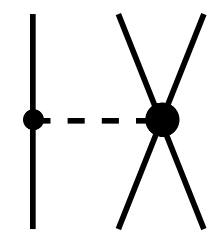
YNN (ΛNN) interactions

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

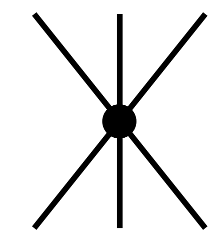
ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN
(up to 10)



2 LECs in ΛNN
(up to 14)

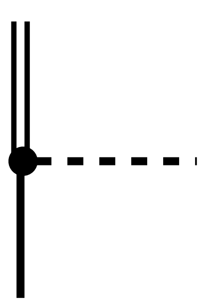


3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

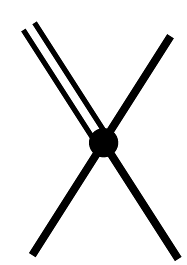
only few data → need to keep the **# of LECs** small

Decuplet baryons ($\Sigma^* \ldots$) might enhance YNN partly to NLO
(Petschauer et al., NPA 957, 347 (2017))

By decuplet saturation all LECs can be related to the following
leading octet-decuplet transitions (Petschauer et al. Front. Phys. 8,12 (2020))



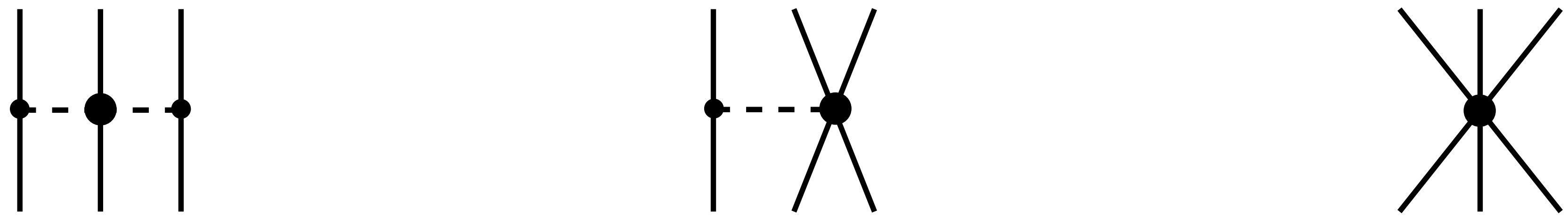
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) interactions

Decuplet saturation relates all LECs to G_1 and G_2

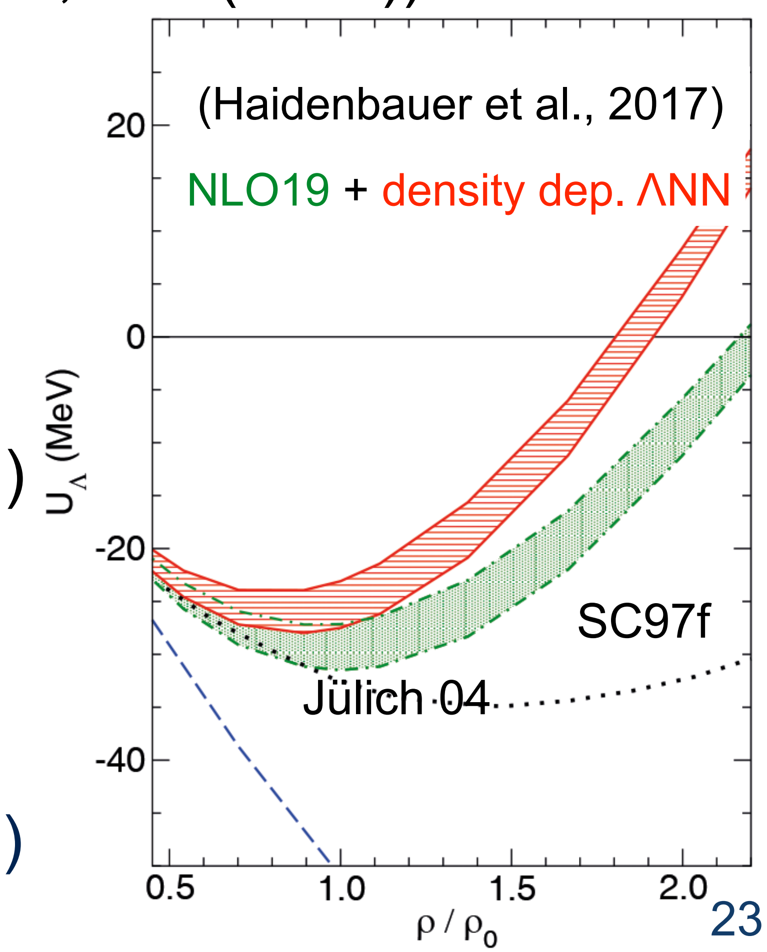


	$\propto C^2$	$\propto CG_1, CG_2$	$\propto (G_1)^2, (G_2)^2, G_1G_2$
For ΛNN:	$\propto C^2$	$\propto C(G_1 + 3G_2)$	$\propto (G_1 + 3G_2)^2$ 1 LEC

→ density dependent BB interactions (Petschauer et al., NPA 957, 347 (2017))

→ application to nuclear matter (Haidenbauer et al., EPJ A 53, 121 (2017))

neutron stars (Logoteta et al., EJA 55, 207 (2019))



- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of 2π and 1π exchange

(fixes sign of $G_1 + 3G_2$!)

Recently: successful benchmark of matrix elements (Hoai Le et al. EPJ A 61,21 (2025))

and first direct application to light hypernuclei including Σ's (Hoai Le et al. PRL 134, 072502 (2025))

YNN (ΛNN) interactions

Recalculate 2π, 1π and contact terms of ΛNN using old **non-local** regularization

to benchmark to Kohno et al. (use fixed constant $G_1 = G_2 = \frac{1}{4f_\pi^{\ 2}}$, $G_1 + 3G_2 = +\frac{1}{f_\pi^{\ 2}}$)

→ ΛNN matrix elements agree ✔

Comparison of separation energies $(\mathrm{SMS\ N^4LO^+(550)/N^2LO + NLO19})$:

	w/o YNN	$w/\ 2\pi$	$w/\ 2\pi/1\pi$	$w/\ 2\pi/1\pi/\mathrm{ct}$
${}^3_\Lambda\mathrm{H}$ w/o ΣNN	0.080	0.151	0.215	0.208
${}^3_\Lambda\mathrm{H}$		0.241	0.564	0.549
${}^4_\Lambda\mathrm{He}(0^+)$	1.432	2.412		
${}^4_\Lambda\mathrm{He}(1^+)$	1.164	2.623		
${}^5_\Lambda\mathrm{He}$	3.174	7.139		

Large contribution to all light hypernuclei (larger than estimate!)

— consistent description requires larger cancelation of 2π and 1π part

— contact terms neglible for ${}^3_\Lambda\mathrm{H}$

YNN (ΛNN) interactions

for the application:
apply locally regularized YNN **including subtractions**

Here test results $(\mathrm{SMS\ N^4LO^+(550)/N^2LO + SMS\ NLO(550)})$:

(use fixed constant $G_1 = G_2 = \dfrac{1}{4f_\pi^{\,2}}$, $G_1 + 3G_2 = +\dfrac{1}{f_\pi^{\,2}}$)

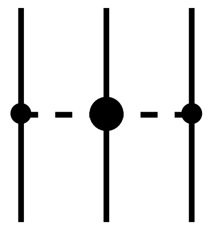
	w/o YNN	$w/\ 2\pi$	$w/\ 2\pi/1\pi$	$w/\ 2\pi/1\pi/\mathrm{ct}$
$^3_\Lambda\mathrm{H}$ w/o subtr	0.107	0.149		
$^3_\Lambda\mathrm{H}$ only subtr		0.086		
$^3_\Lambda\mathrm{H}$ ΛNN compl		0.124		
$^3_\Lambda\mathrm{H}$		0.159	0.238	
$^4_\Lambda\mathrm{He}(0^+)$	1.969	2.333		
$^4_\Lambda\mathrm{He}(1^+)$	1.063	1.367		
$^5_\Lambda\mathrm{He}$	3.247	4.294		

SMS regularization leads to much more **natural results.**

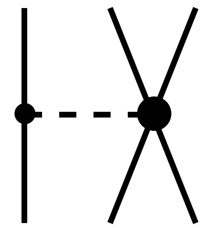
Consistent regularization of NN/3N and YN/YNN forces?

YNN (ΛNN) interactions in practice

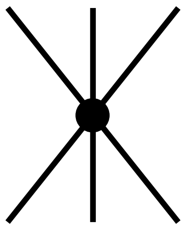
Decuplet approximation in YNN



$\propto C^2$



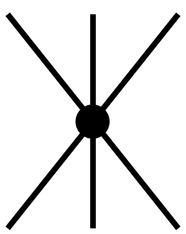
$\propto CG_1, CG_2$



$\propto (G_1)^2, (G_2)^2, G_1 G_2$

is **not** sufficient to fix spin dependence

→ + ΛNN contact terms **without decuplet constraints**



ΛNN $\propto C_1', C_2', C_3'$

***ad hoc* choice:** alter C_2:

$$C_1' = C_3' = \frac{(G_1 + 3G_2)^2}{72\Delta}$$

$$C_2' = 0$$

→

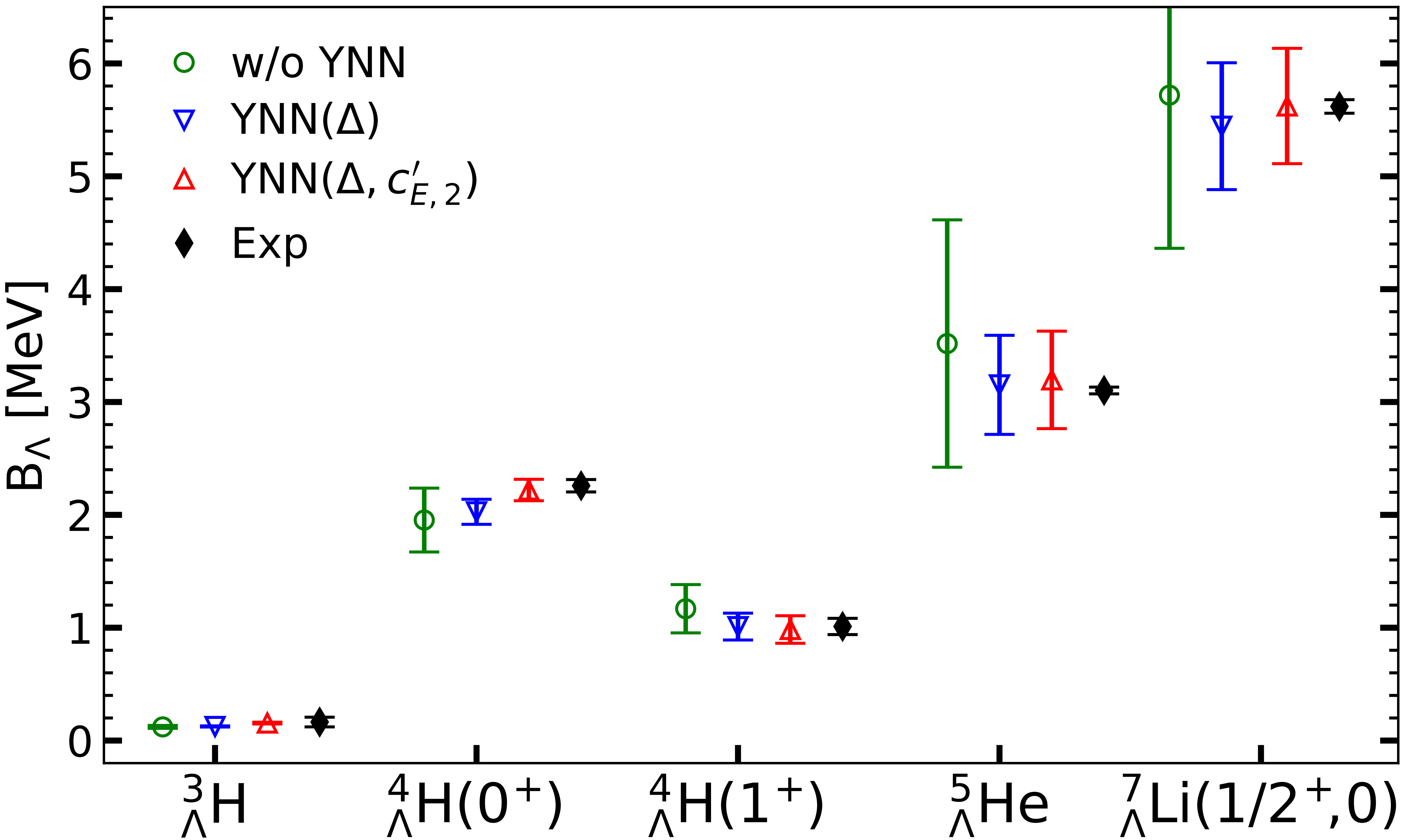
$$V_{\Lambda\mathrm{NN}} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

$$C_2' = G_3$$

C_2' introduces a spin dependent interaction in the most relevant particle channel

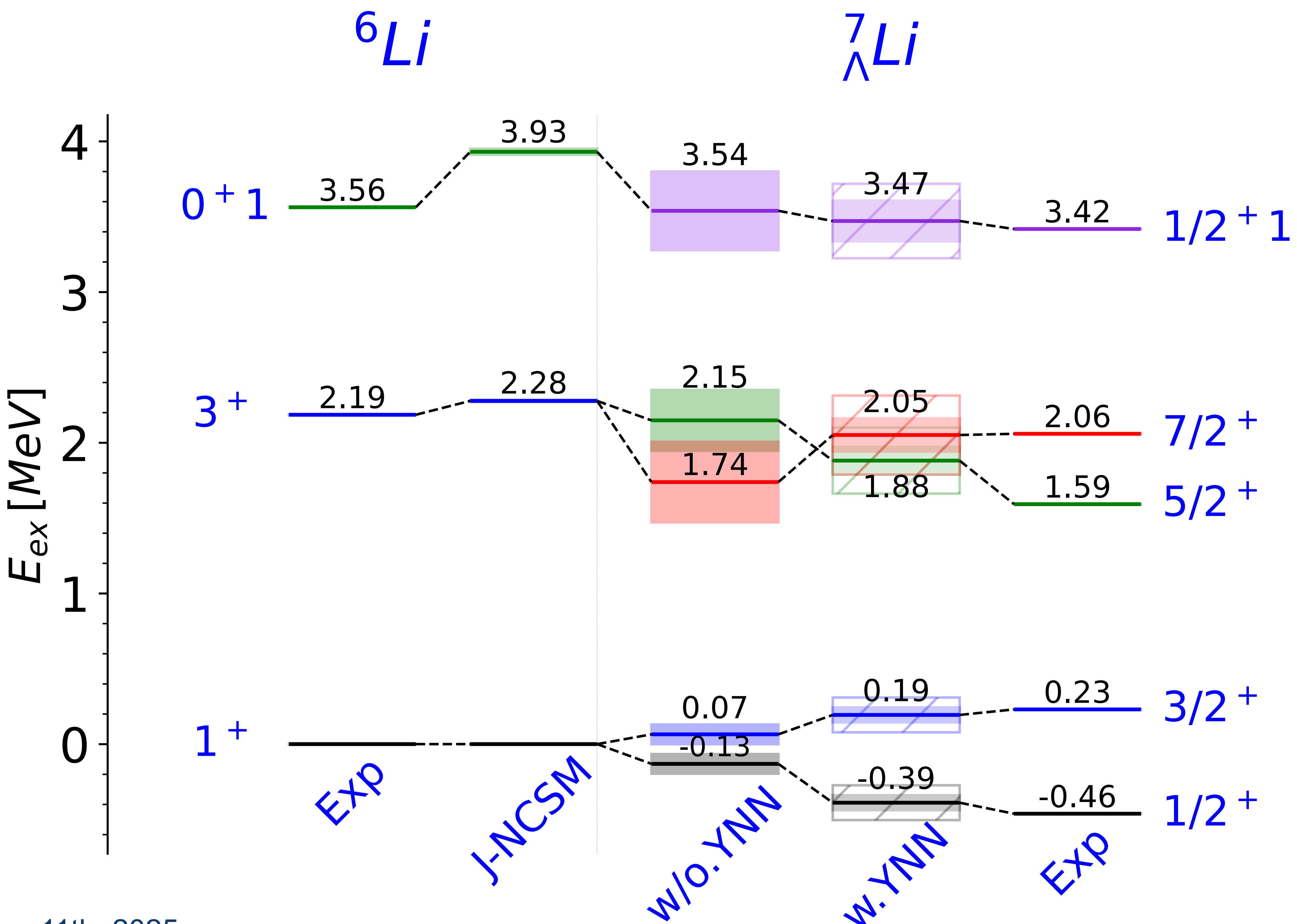
YNN fit

- Fit to 0^+ and 1^+ state of ${}^4_\Lambda\mathrm{He}$ and/or ${}^5_\Lambda\mathrm{He}$
- spin-dependence in A=4 not well explained by decuplet saturation
- C_2' term improves 0^+ of ${}^4_\Lambda\mathrm{He}$ and $1/2^+$ of ${}^7_\Lambda\mathrm{Li}$
- agreement generally much better than $\mathrm{N^2LO}$ uncertainty



YNN prediction for $^7_\Lambda\mathrm{Li}$

- good agreement
- C_2' term included, but not very important (not shown)
- higher states have significant uncertainty



Conclusions & Outlook

- **YN interactions not well understood**
  - *scarce YN data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - $^1S_0$ ΛN *scattering length &* $^3_\Lambda\mathrm{H}$
  - CSB of ΛN scattering & $^4_\Lambda\mathrm{He}$ / $^4_\Lambda\mathrm{H}$
- **SMS YN interactions up to $\mathrm{N^2LO}$**
  - *order LO, NLO and N^2LO allow uncertainty quantification*
  - *have a **non-unique** determination of contact interactions (more data necessary)*
- **Chiral 3BF**
  - *choice for regularization matters*
  - *decuplet saturation alone does not improve spin dependence*
  - *spin-dependent ΛNN leads to further improvement*

  - *study cutoff dependence / application to more p-shell hypernuclei*
  - *extension to Λd scattering: probably more insight for higher densities*
  - *extension Λd/Λpp correlations: info on different spin/isospin states*