A new method for determining resonance poles in finite volume

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In Collaboration with:

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Resonance

- Quasi-bound; Finite lifetime; Barrier tunneling
- Theoretically, S-matrix pole in the complex energy plane ($E_0 - i\,\Gamma/2$)
- Experimentally, a peak in the cross section

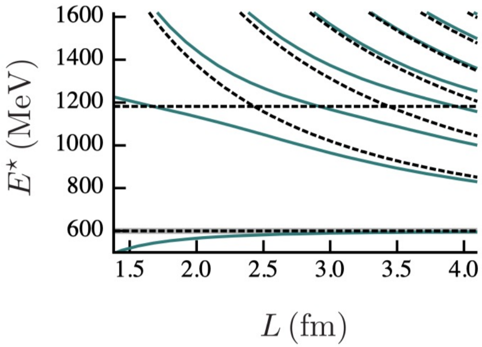
Background: Resonance in finite volume (FV)

- Standard procedure in lattice QCD R. Briceño, J. Dudek, R. Young, RMP 90, 025001 (2018)

lattice observables → GEVP → energy spectrum

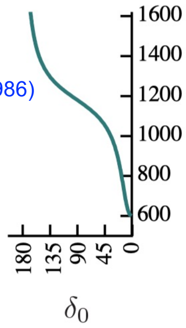
phase-shifts → analytical continuation → resonance poles

Finite volume



Lüscher formula

M. Lüscher, Comms. Math. Phys. 105, 153 (1986)



Infinite volume

- Lüscher formula is difficult to apply successfully in nuclear lattice EFT
  - the errors of the energy levels are larger than their separations
    B.-N. Lu, T. Lähde, D. Lee, U.-G. Meißner , PLB 760, 309-313 (2016)

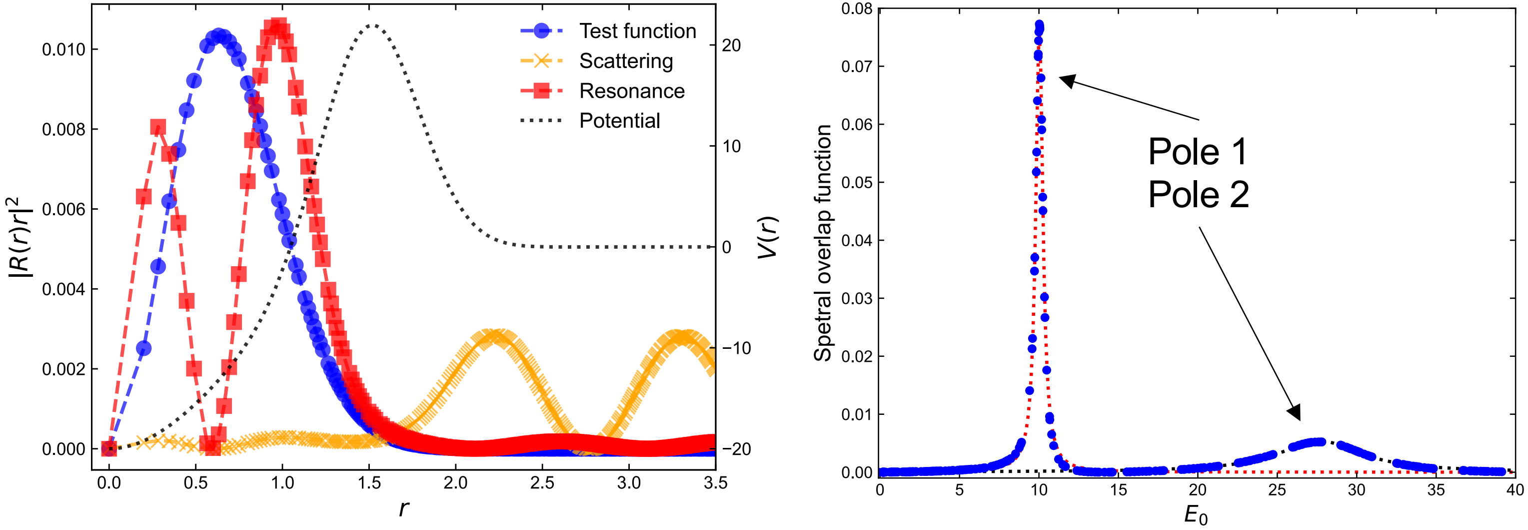
➢ see D. Lee's talk on NLEFT's review (29.08)

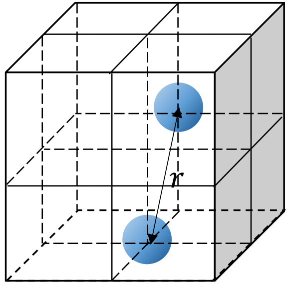
➢ see L. Bovermann's talk on beta decay in NLEFT (27.08)

Persistent state method: intuitive explanation

- A resonance pole implies the existence of long-lived states that are compact in size

- $|\langle\psi_{\text{test}}|E\rangle|^2$ (spectral overlap function) peaked around resonance energy E_0 for compact $\psi_{\text{test}}(r)$





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Persistent state method: two realizations

■ Spectral function and survival amplitude

The QM survival amplitude of a resonant state is given as

$$f(t) = \langle\psi| e^{-iHt} |\psi\rangle$$

In the Fock–Krylov method (i.e., the resonant state expressed as expansion of the scattering states), thus the survival amplitude is

N. S Krylov, V. A. Fock, JETP 17, 93 (1947)

$$f(t) = \int dE \, |a(E)|^2 e^{-iEt} \qquad |\psi\rangle = \int dE \, a(E) \, |E\rangle$$

One can proceed to evaluate f(t) if the spectral function is known. For Breit-Wigner parameterization,

$$a(E) \propto \frac{1}{E - E_0 + i\Gamma/2} \qquad f(t) \propto e^{-iE_0 t - \Gamma t/2}$$

■ Ansatz: simple persistent state approximating the resonant state

- Persistent state: compact spatially (e.g., Gaussian) and decaying slowly

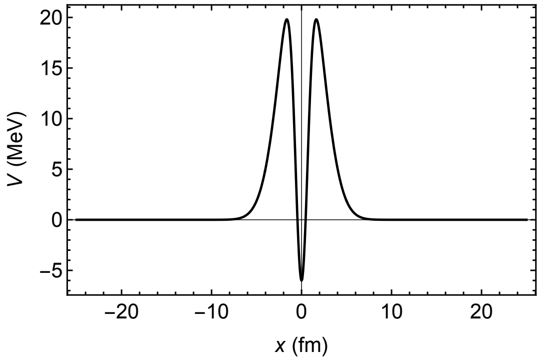
input: $\langle\psi_{\text{persis.}}|E\rangle$ → fitting function: $a(E)$ → output: E_0 Γ

input: $\langle\psi_{\text{persis.}}|e^{-iHt}|\psi_{\text{persis.}}\rangle$ → fitting function: $f(t)$ → output: E_0 Γ

Benchmark calculation

Toy model: two spinless particles (1D)

- $V(x) = 30\, e^{-\left(\frac{x}{3\text{ fm}}\right)^2} - 36\, e^{-\left(\frac{x}{1\text{ fm}}\right)^2}$ (MeV), $\mu = 938.92$ (MeV)
- Single resonance pole in even parity
  $E_0 = 10.55$ (MeV) $\Gamma = 1.45$ (MeV) (from solving the L−S equation)

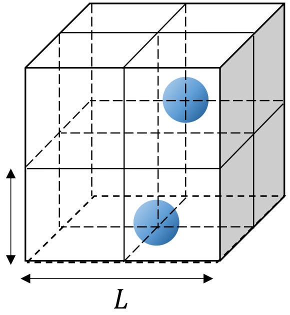


Finite volume continuum calculation

- Solving the eigenvalue problem by the plane-wave based discrete variable representation S. König, FB 61, 20 (2020)
- Infinite volume extrapolation ($L \to \infty$) for $E_0(L)$ and $\Gamma(L)$

Finite volume Lattice calculation

- Non-perturbative calculation using the Lanczos algorithm
- Infinite volume and continuum extrapolation ($L \to \infty$ and $a \to 0$) for $E_0(L, a)$ and $\Gamma(L, a)$

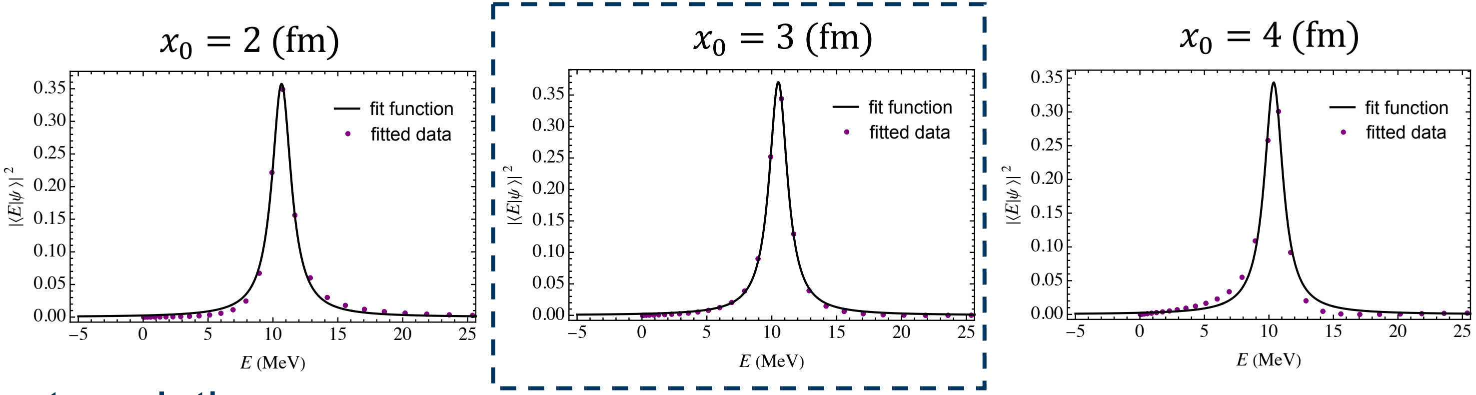


$$\psi(x + n\,L) = \psi(x)$$

Spectral overlap function in FV continuum

■ Breit-Wigner fit

- Gaussian test state:
  $\psi(x) = e^{-(x/x_0)^2}$
- Search for narrowest (~best fitting) peak → persistent state



■ Infinite volume extrapolation

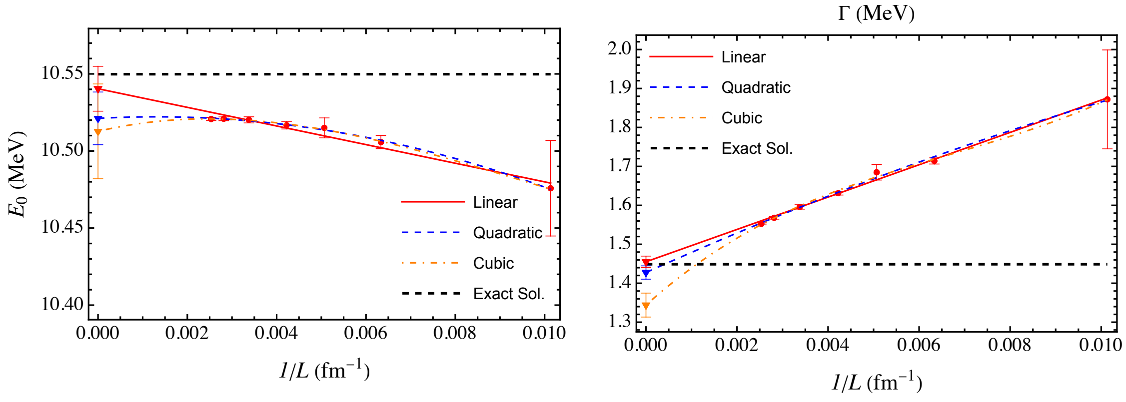
- Akaike information criterion (AIC) suggests quadratic fit in $1/L$

$$\mathrm{AIC} = 2k - 2\ln(\hat{L})$$

H. Akaike, IEEE TAC 19, 716 (1974)

result:
$E_0 = 10.52 \pm 0.02$ (MeV)
$\Gamma = 1.43 \pm 0.02$ (MeV)

exact sol.:
$E_0 = 10.55$ (MeV)
$\Gamma = 1.45$ (MeV)



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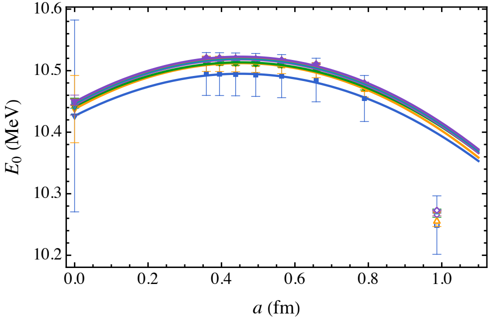
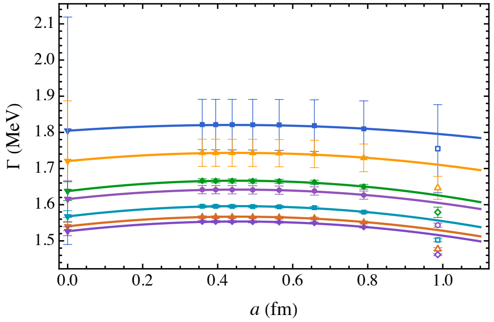
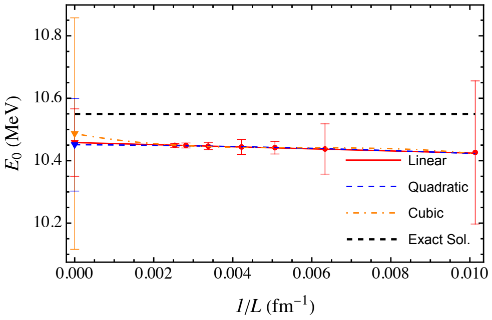
Spectral overlap function on lattice

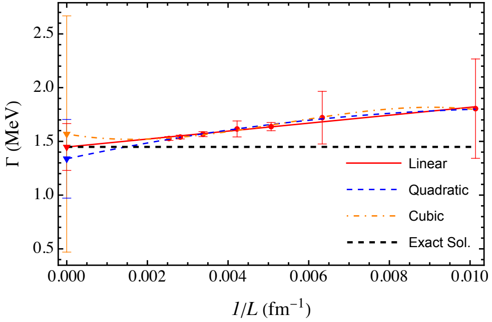
- Breit-Wigner fit $E_0(L,a)\ \Gamma(L,a)$
- Continuum extrapolation $E_0(L,a\to 0)\ \Gamma(L,a\to 0)$





- Infinite volume extrapolation $E_0(L\to\infty,a\to 0)\ \Gamma(L\to\infty,a\to 0)$





- AIC suggests linear fit:
  $E_0 = 10.46 \pm 0.11$ (MeV)
  $\Gamma = 1.45 \pm 0.22$ (MeV)

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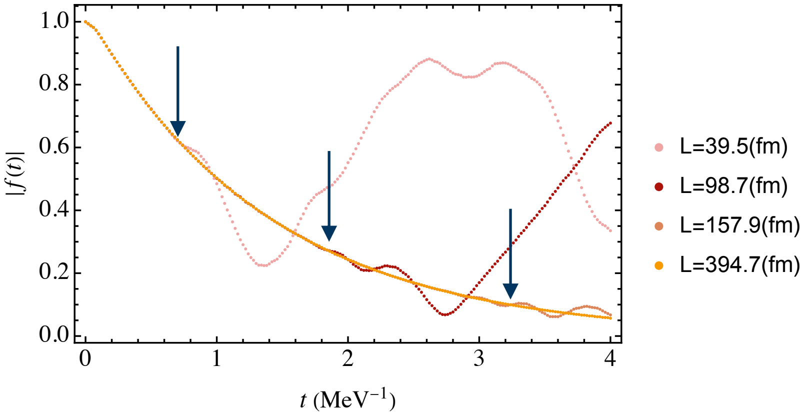
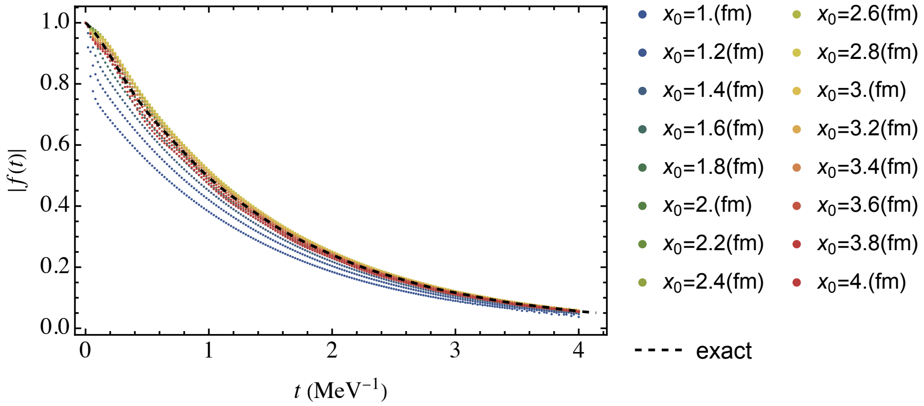
Survival amplitude in FV continuum

■ Non-obvious finite volume effect



- Same survival amplitudes (~10^{-3} differences) before the wave packet hits the boundary
- No needs for volume extrapolation

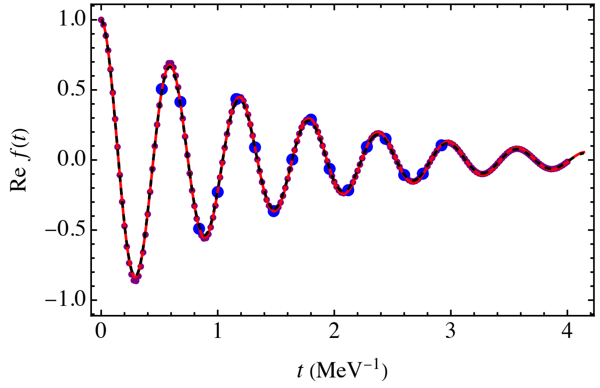
■ Varying Gaussian test states

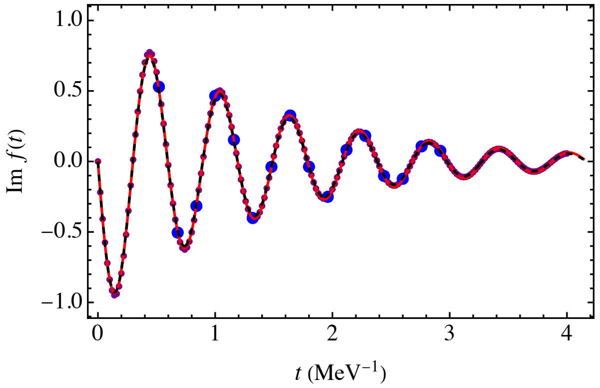


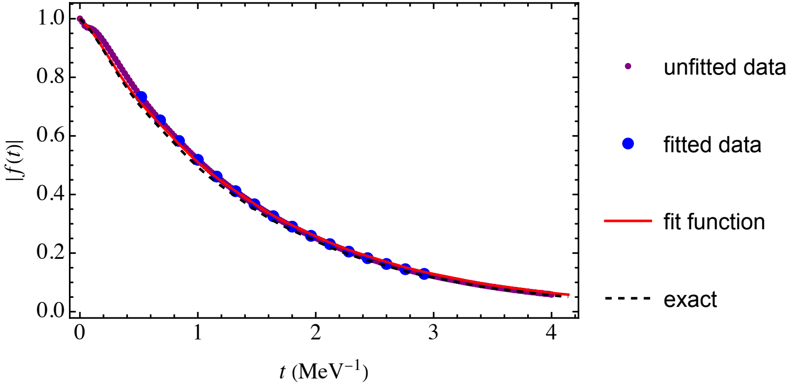
- Persistent states indeed close to the exact resonant state (Breit-Wigner parameterization of $a(E)$)
- Persistent state: x_0 = 2.6 (fm)

Survival amplitude in FV continuum

- The fit $f(t) \propto e^{-iE_0 t - \Gamma t/2}$







result:

$E_0 = 10.54 \pm 0.01(\mathrm{MeV})$

$\Gamma = 1.38 \pm 0.01(\mathrm{MeV})$

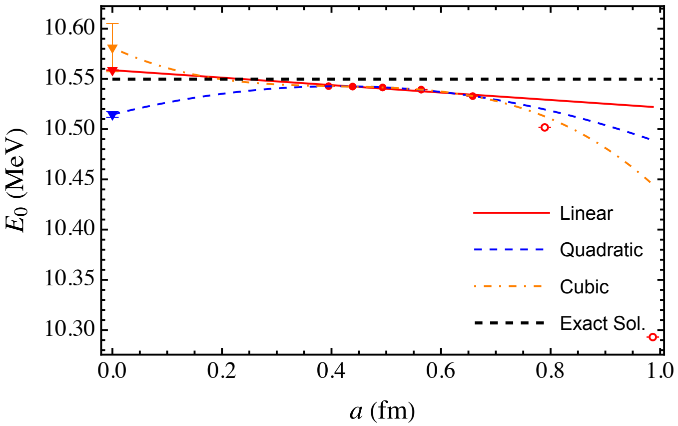
exact sol.:

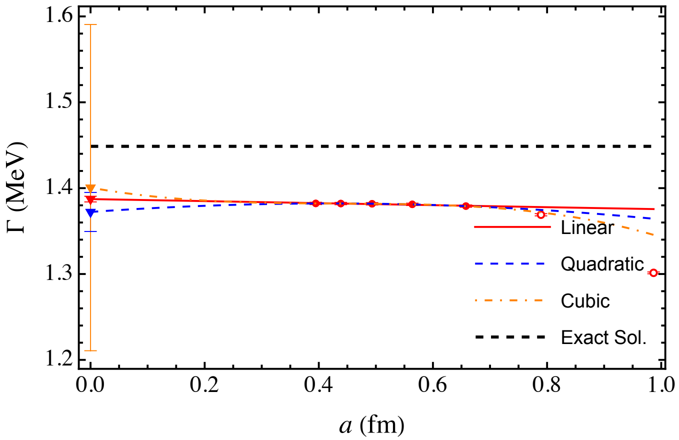
$E_0 = 10.55\ (\mathrm{MeV})$

$\Gamma = 1.45\ (\mathrm{MeV})$

Survival amplitude in FV lattice

- The fit $f(t) \propto e^{-iE_0 t - \Gamma t/2}$
- Continuum extrapolation





- AIC suggests linear fit
- result:

$$E_0 = 10.56 \pm 0.01(\text{MeV})$$
$$\Gamma = 1.39 \pm 0.01(\text{MeV})$$

Summary and outlook

- Persistent state method is proposed and benchmarked using a two-body model

		spectral overlap function		survival amplitude	
	exact	**continuum**	**lattice**	**continuum**	**lattice**
E_0 **(MeV)**	10.55	10.52 ± 0.02	10.46 ± 0.11	10.54 ± 0.01	10.56 ± 0.01
Γ **(MeV)**	1.45	1.43 ± 0.02	1.45 ± 0.22	1.38 ± 0.01	1.39 ± 0.01

- □ Multiple poles
  - ■ By spectral function (3D, >2 poles)
  - □ By survival amplitude
- □ Three-body system
- □ Monte Carlo calculation for a realistic nuclear system (by A. Sarkar et.al.)

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Summary and outlook

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		spectral overlap function		survival amplitude	
	exact	continuum	lattice	continuum	lattice
E_0 (MeV)	10.55	10.52 ± 0.02	10.46 ± 0.11	10.54 ± 0.01	10.56 ± 0.01
Γ (MeV)	1.45	1.43 ± 0.02	1.45 ± 0.22	1.38 ± 0.01	1.39 ± 0.01

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Thank you!