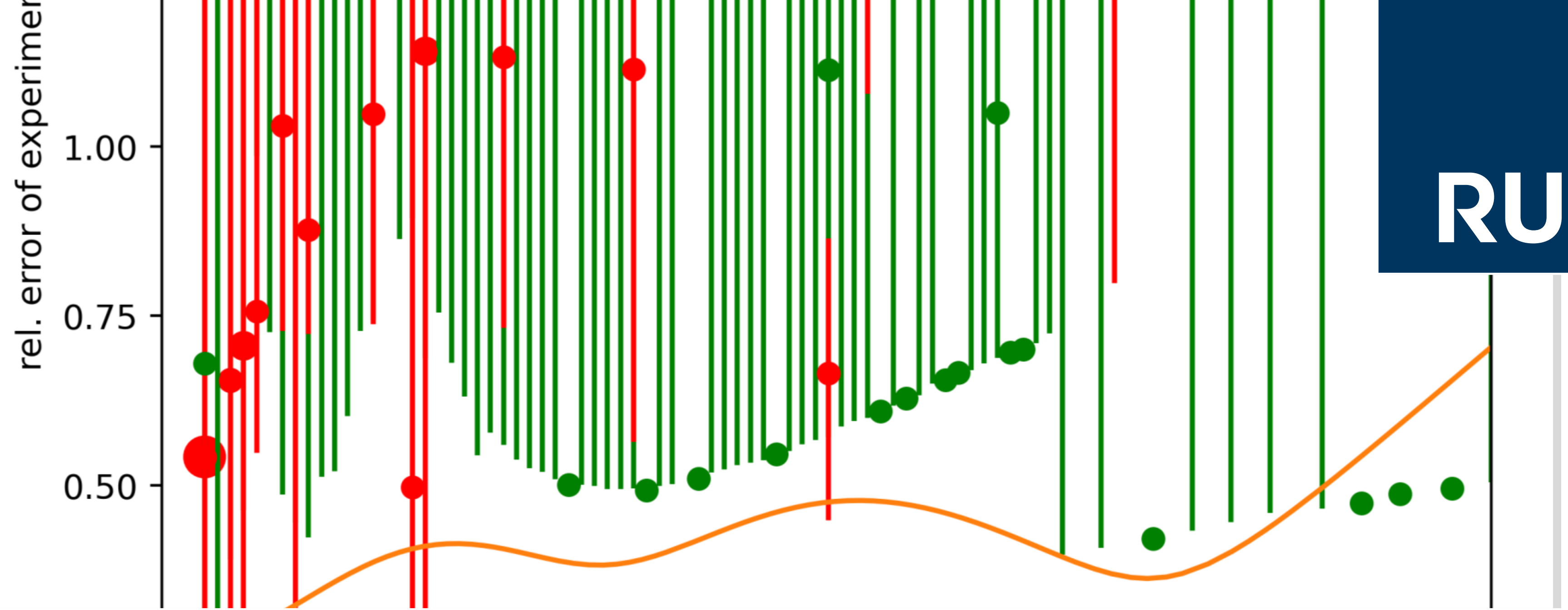

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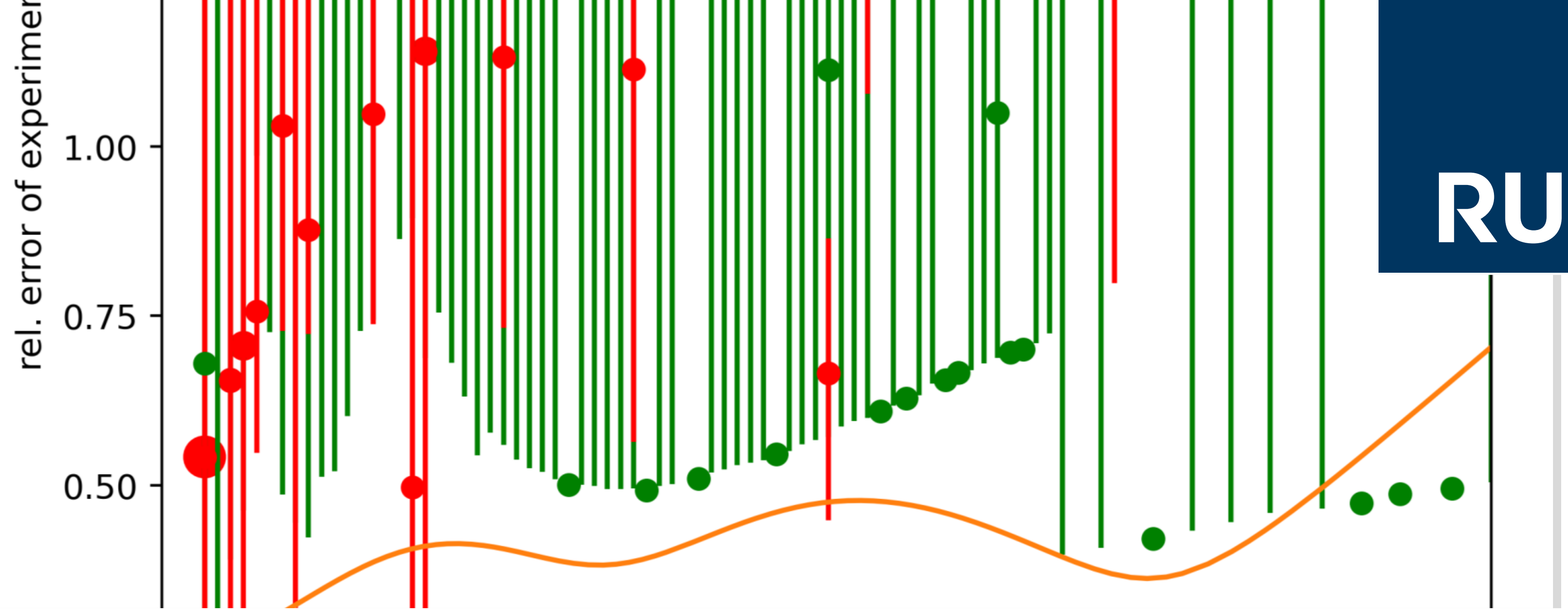
EXPLICIT ESTIMATION OF TRUNCATION UNCERTAINTIES IN CHIRAL EFT

Chiral Dynamics 2024, WG 3, Few-Body physics

Sven Heihoff, Evgeny Epelbaum, Arseniy Filin

contact: sven.heihoff@rub.de



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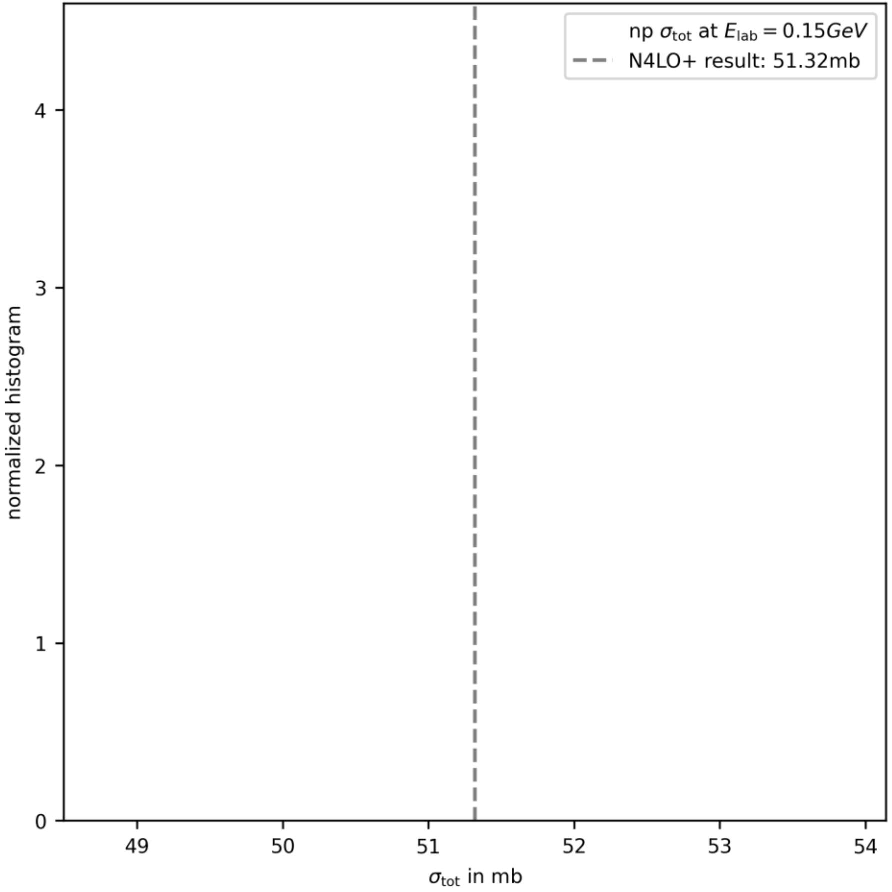
Outline

- **Aim**: Calculate some Nucleon-Nucleon scattering observable $\boldsymbol{X}$

1.) What is the theory and how to go from theory to $\boldsymbol{X}$?



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Outline

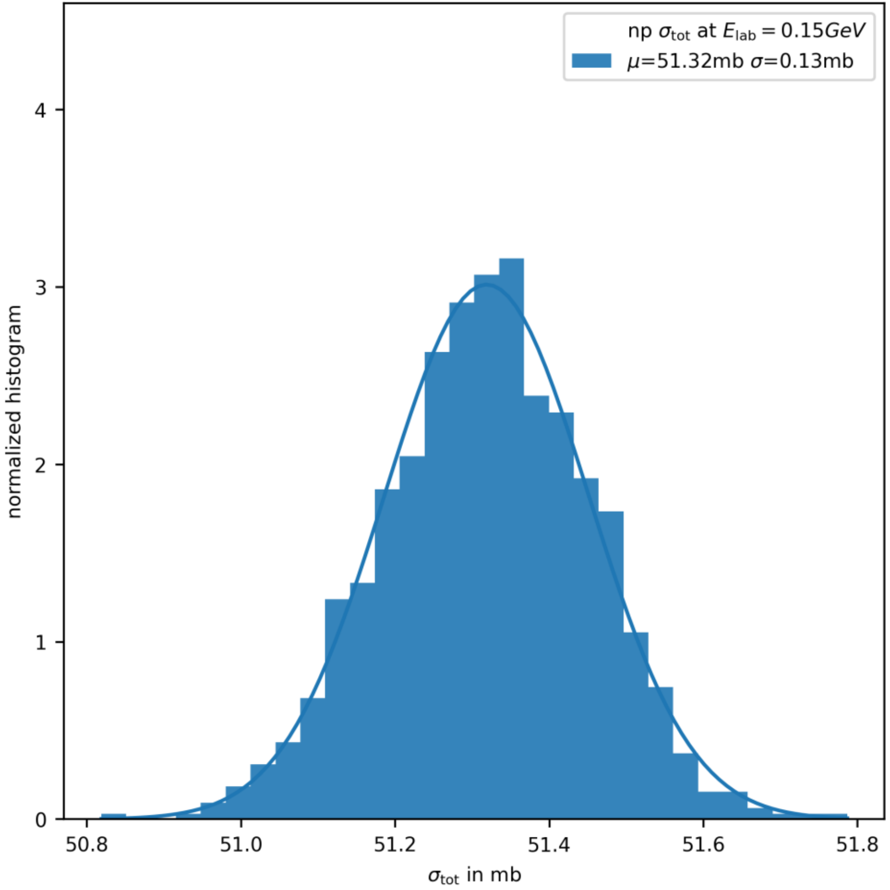
- **Aim**: Calculate some Nucleon-Nucleon scattering observable $\boldsymbol{X}$

1.) What is the theory and how to go from theory to $\boldsymbol{X}$?

2.) For a falsifiable theory, distribution crucial:

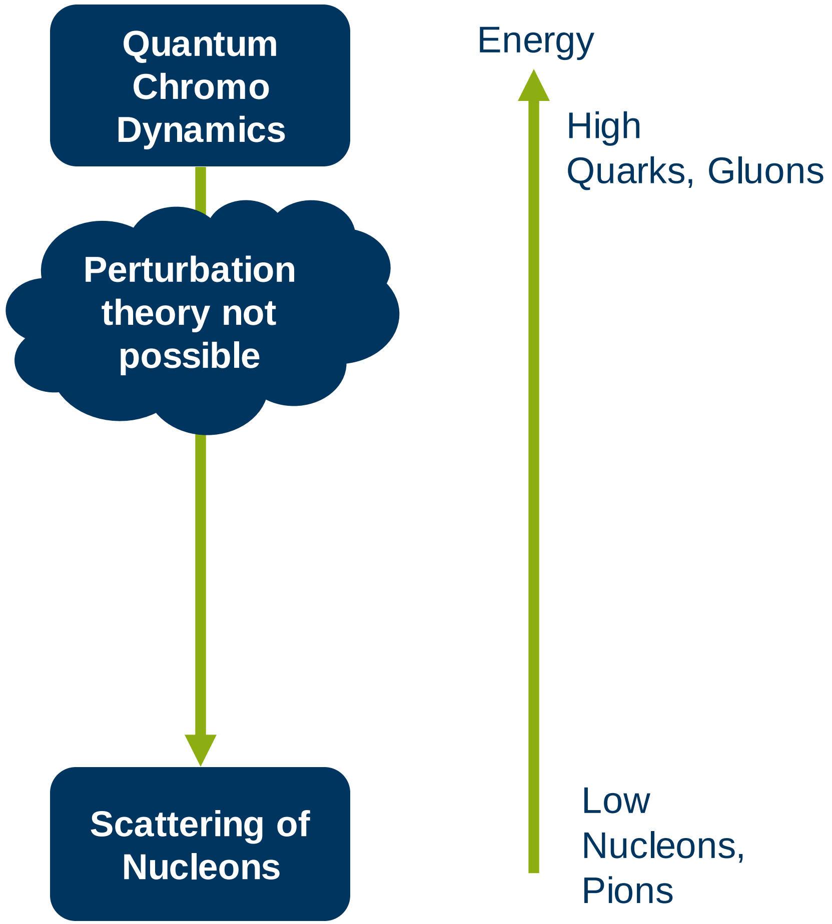
$$p\left(X^{(k)} \mid \text{theory}\right)$$

3.) Results – What can we learn from $p\left(X^{(k)} \mid \text{theory}\right)$?



What is chiral EFT?

- QCD is best description of strong force so far
- Has not been solved at level of nucleons



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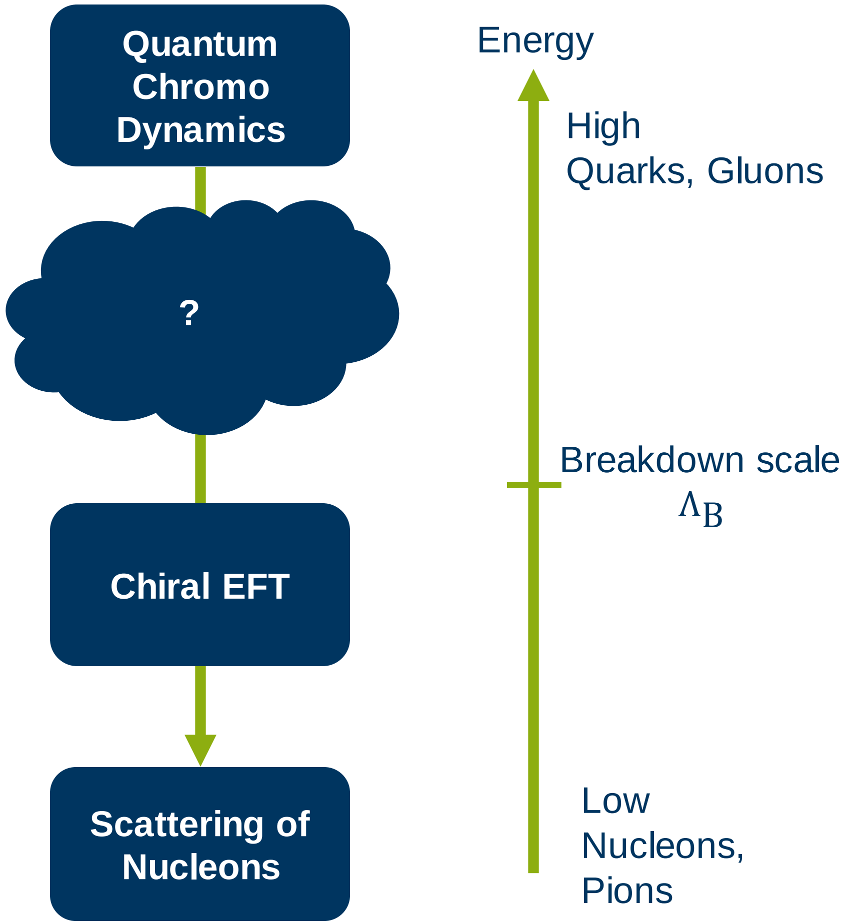
- Degrees of freedom: **Nucleons and pions**
- Start with most general Lagrangian ← a **lot** of terms!



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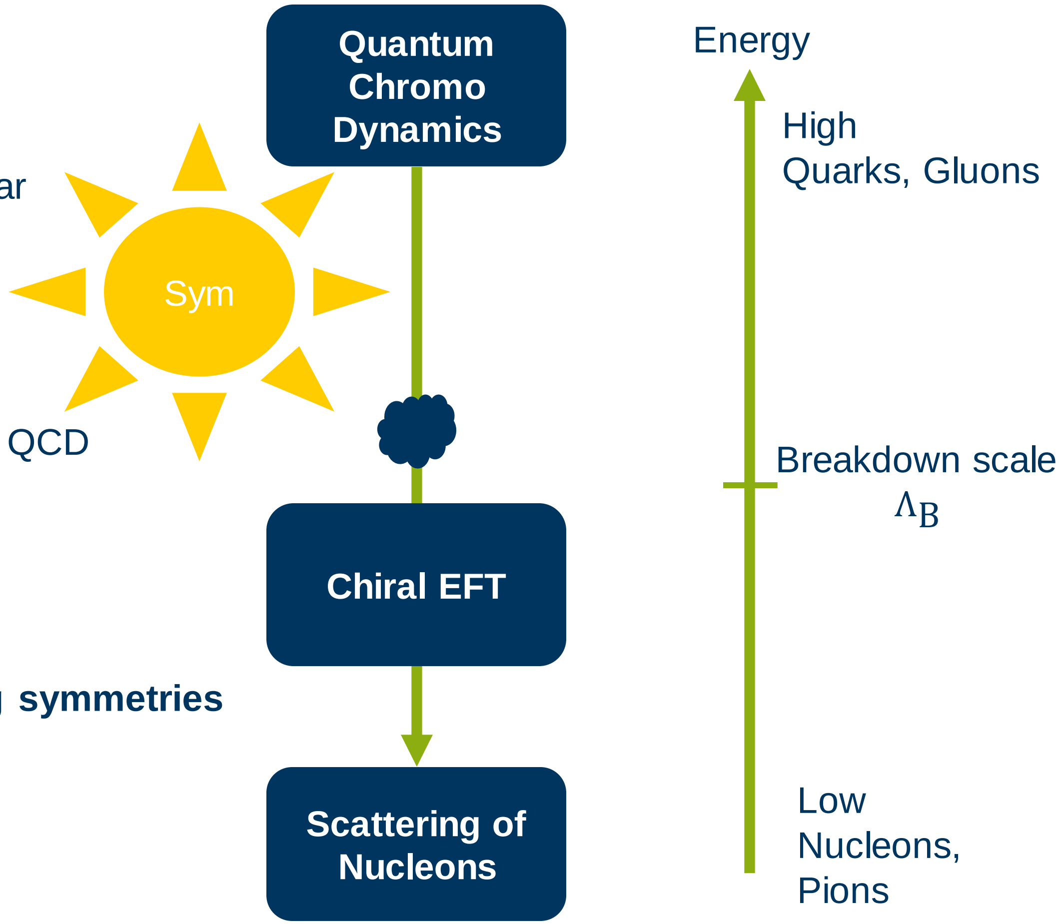

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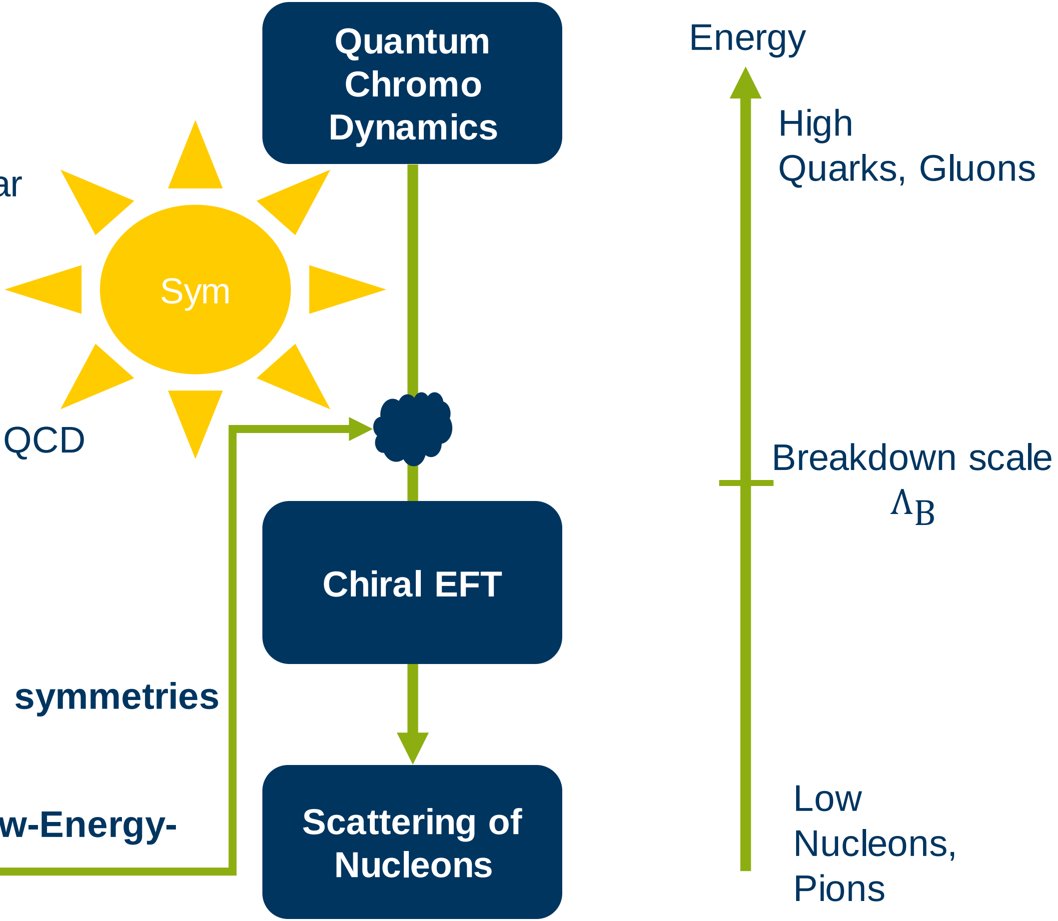
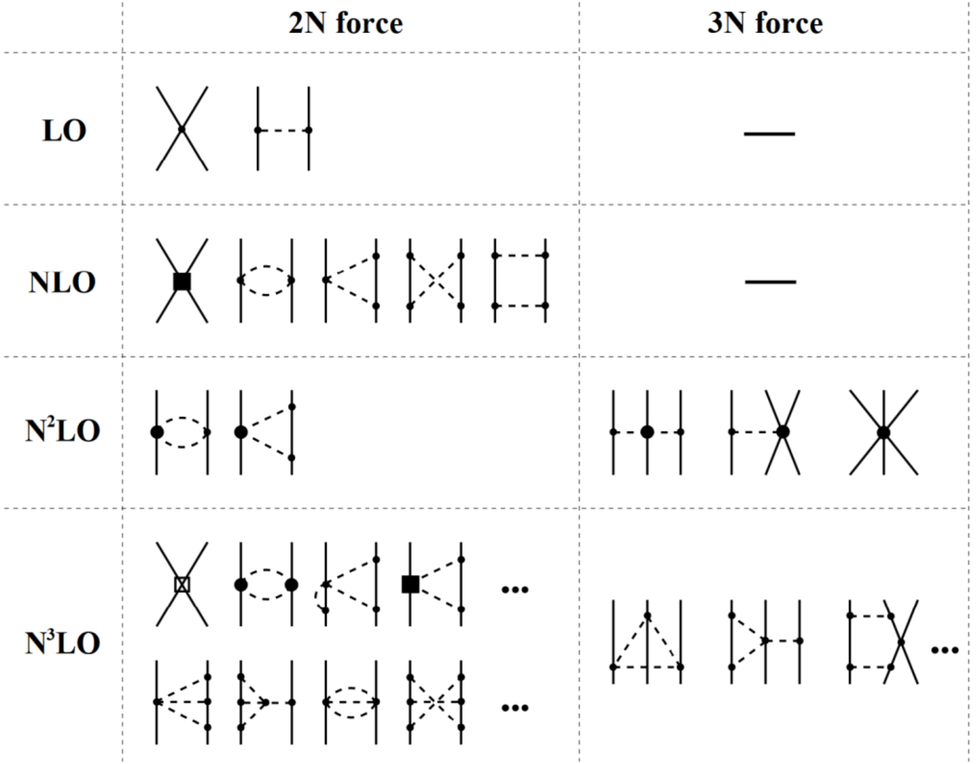
- Start with most general Lagrangian, fulfilling **symmetries**
- Chiral EFT comes with a-priori unknown **Low-Energy-Constants (LECs)**



Power counting

- In principle infinitely many terms in Lagrangian
- Expansion parameter: $Q \in \left(\frac{p}{\Lambda_B}, \frac{M_\pi}{\Lambda_B} \right) \sim \frac{1}{3}$
- Each diagram is assigned to a power of Q



E. Epelbaum, Nuclear Forces from Chiral Effective Field Theory: A Primer (2010)

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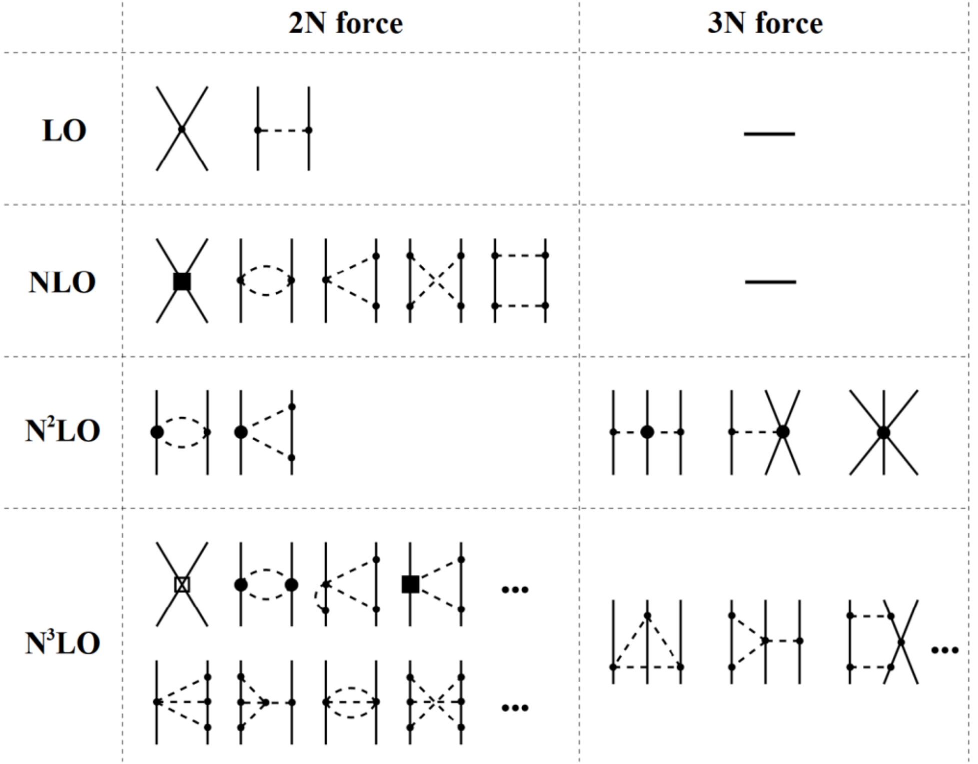
- Each diagram is assigned to a power of Q
  → Hierarchy of diagrams by their importance
  → Finite order → finite set of diagrams
  → Allows for systematic improvement of theory



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- Lagrangian looks like:

$L = A_{\text{LO}} + A_{\text{NLO}} + A_{N^2LO} + \cdots$ **with** $A_{\text{LO}} \propto Q^0$ $A_{\text{NLO}} \propto Q^2, A_{N^2LO} \propto Q^3$ **thus** $A_{\text{LO}} > A_{\text{NLO}} > A_{N^2LO}$

The final step: How to get observables?

- Finite set of diagrams can be summed into a QM potential $V = V(p, p')$
- Lippmann-Schwinger-equation:

$$T = V + VGT \Rightarrow (1 - VG)T = V$$

- With G as free propagator and T as transition matrix (directly connected to observables)
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- What about LECs? Fitted to experimental data! ← **inverse problem**
- 2N potential in chiral EFT leads to high-precision description of 2N data

P. Reinert, et al., Eur. Phys. J. A 54, 86 (2018)

Nice theory, but

“a physicist who has no errorbars is also just a religious person.”

Three important sources of uncertainties

- Uncertainties of the fitting protocol of the LECs
  - Numerical approximations (especially in EM contributions)
  - Fitting algorithm
  - Physical constants
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In this talk

- **Aim**: probability distribution $\boldsymbol{p}\left(\boldsymbol{X}^{(\boldsymbol{k})}\right)$ of observable X at some order k of Chiral EFT.

Bayesian estimation of truncation uncertainties

- Assumption: Expansion is the same for observable $X^{(k)}$ as for potential

$$X^{(k)} = X_{\text{ref}} \cdot \left(c_0 + \sum_{i=2}^{k} c_i \cdot Q^i + \Delta_k \right)$$

- With expansion coefficients $c_i = \frac{X^{(i)}}{X_{\text{ref}} \cdot Q^i}$ and trunc. uncertainty $X_{\text{ref}}\Delta_k = X_{\text{ref}} \sum_{i=k+1}^{\infty} c_i \cdot Q^i$

J. A. Melendez, S. Wesolowski, and R. J. Furnstahl, Phys. Rev. C 96, 024003 (2017), arXiv:1704.03308

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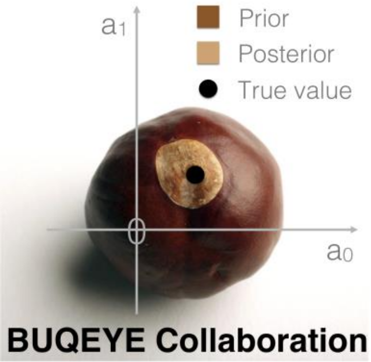
- Bayes theorem yields: $\mathrm{p}\left(\{c_{j\epsilon[k+1,\infty]}\} \mid \{c_{i\epsilon[0,k]}\}\right)$

  → Obtain a probability distribution $p\left(\Delta_k^h \middle| \{c_i\}\right)$ for $\Delta_k^h = \sum_{i=k+1}^{h} c_i \cdot Q^i$



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Issues of Bayesian approach

- Chiral EFT results at $E_{\text{lab}} = 150\ \text{MeV}$ and cutoff $\Lambda = 450\text{MeV}$

$$\sigma_{\text{tot}} = 52.15_{\text{LO}} - 2.94_{\text{NLO}} + 1.25_{\text{N2LO}} + 0.34_{\text{N3LO}} + 0.44_{\text{N4LO}} + 0.07_{\text{N4LO+}}$$

- Final result with Bayesian error: $\sigma_{\text{tot}}^{\text{N4LO+}} = 51.32 \pm 0.19\ \text{mb}$

For algorithm see: E. Epelbaum et al., Eur. Phys. J. A 56, 92 (2020), arXiv:1907.03608

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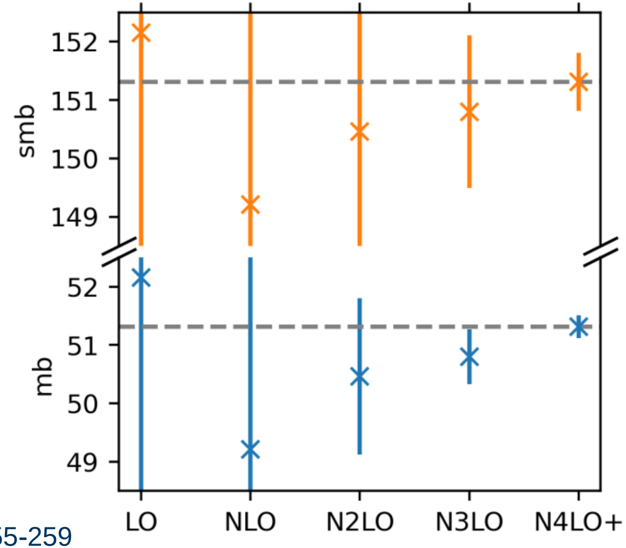

- What if, we measure in smb? $x\ \mathrm{mb} = (x + 100)\ \mathrm{smb}$
  - $\sigma_{\mathrm{tot}}^{\mathrm{N4LO+}} = 151.32 \pm 0.49\ \mathrm{smb}$

  → Bayesian uncertainty dependent on absolute scale!

- Experiment: $51.02 \pm 0.3\ \mathrm{mb}$ P.W. Lisowski et al.,Phys. Rev. Lett. 49 (1982), 255-259

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Expansion of potential really also valid for observables?

Expansion of potential really also valid for observables?

Including next-higher order at level of nuclear potential!

Method

- How to obtain $p\left(X^{(k)}\right)$?

  1.) Include contact diagrams of next higher order $\mathrm{k}+1$ to potential

  2.) Set LECs $\vec{a}^{(k+1)}$ to natural values

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- Mathematically: Explicitly integrating out the diagrams of order $\mathrm{k}+1$

$$p(X^{(k)}) = \int \mathrm{d}\,\vec{a}^{(k+1)} \cdot \delta(X^{(k+1)}\left(\vec{\mathrm{a}}^{(i\,\in\,[0,\mathrm{k}+1])}\right) - X^{(k)}) \cdot p(\vec{\mathrm{a}}^{(i\,\in\,[0,\mathrm{k}+1])})$$

- Uncertainty: width of $p(X^{(k)})$

What is natural for the LECs?

- Naturalness: Every physical parameter should be in the order of 1
  - Compared to a reasonable physical scale
  - No proof, but generally accepted idea in physics
  - Here: Encoded in probability distribution for LEC values $p(a_j^{(i)})$

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- Can we infer $p(\vec{a}^{(k+1)})$ from the LECs fitted so far?
  - Assume: $p(a_{\mathrm{j}}^{(i)})$ is independent of the order i and is the same for all elements of $\vec{a}^{(i)}$
  - Assume: $p\left(a_{\mathrm{j}}^{(i)}\right) \sim \mathrm{N}(0, \mathrm{w})$ follows Gaussian with mean 0 and standard deviation w
  - Bayes theorem:

$$p(w \mid \vec{\mathrm{a}}^{(i \in [0,\mathrm{k}])}) \propto p\left(\vec{\mathrm{a}}^{(i \in [0,\mathrm{k}])} \mid w\right) \cdot p(w)$$

Naturalness of known LECs

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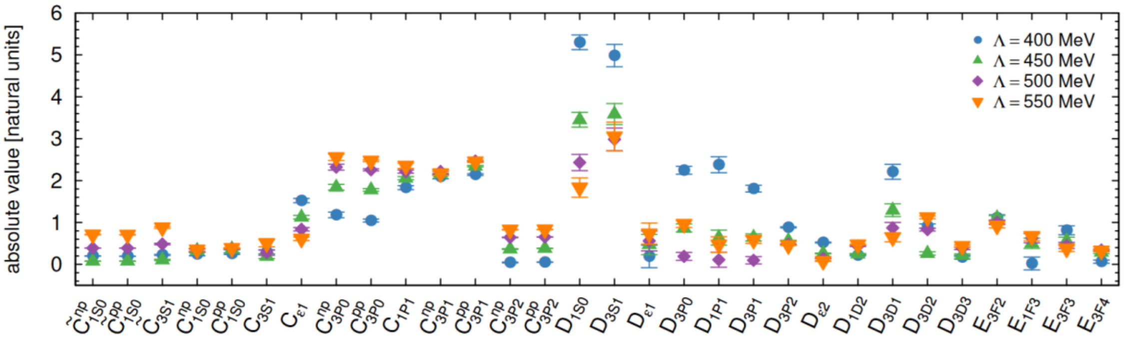
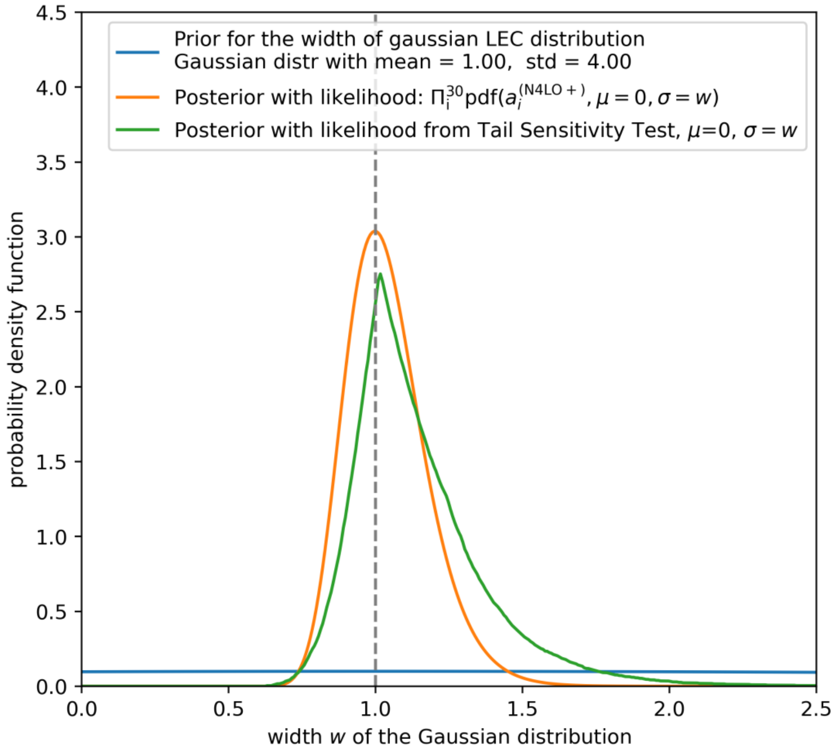
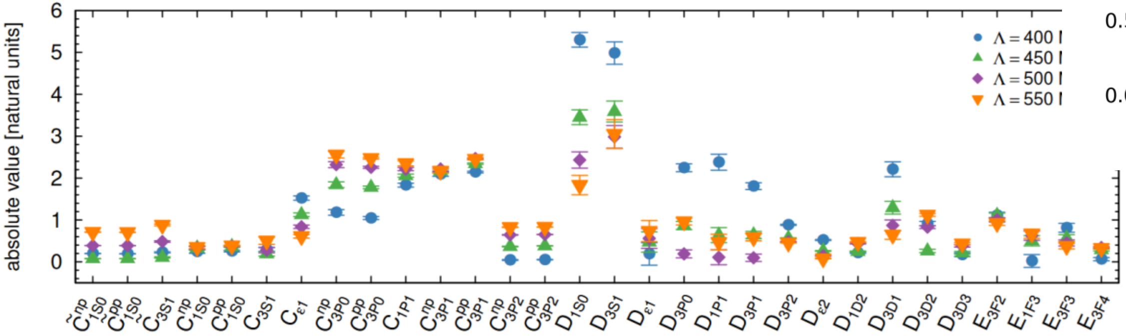


Epelbaum et al., Front. in Phys. 8, 98 (2020), arXiv:1911.11875

Naturalness of known LECs

- $p(w \mid \vec{a}^{(i \in [0,k])}) \propto p\left(\vec{a}^{(i \in [0,k])} \mid w\right) \cdot p(w)$
- What exactly is $p\left(\vec{a}^{(i \in [0,k])} \mid w\right)$?
- If assumptions are fulfilled, the **LECs** are indeed drawn from a **Gaussian distribution with $\sigma \sim 1$**





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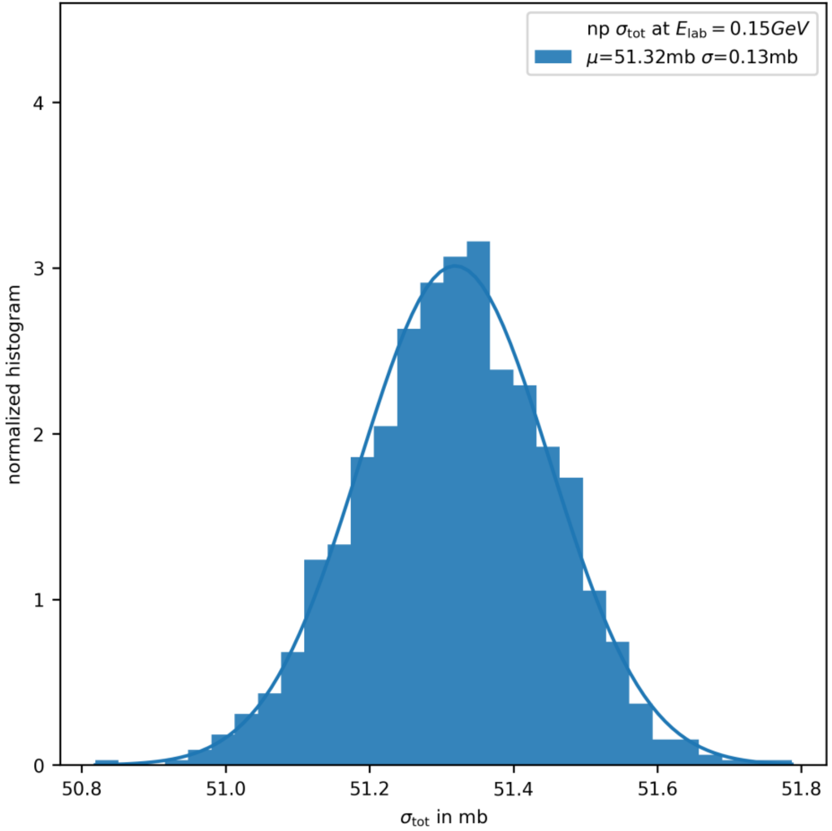
What we thought we do

Calculate $X^{(k)}$ and add randomly higher order contributions of natural size → Truncation error

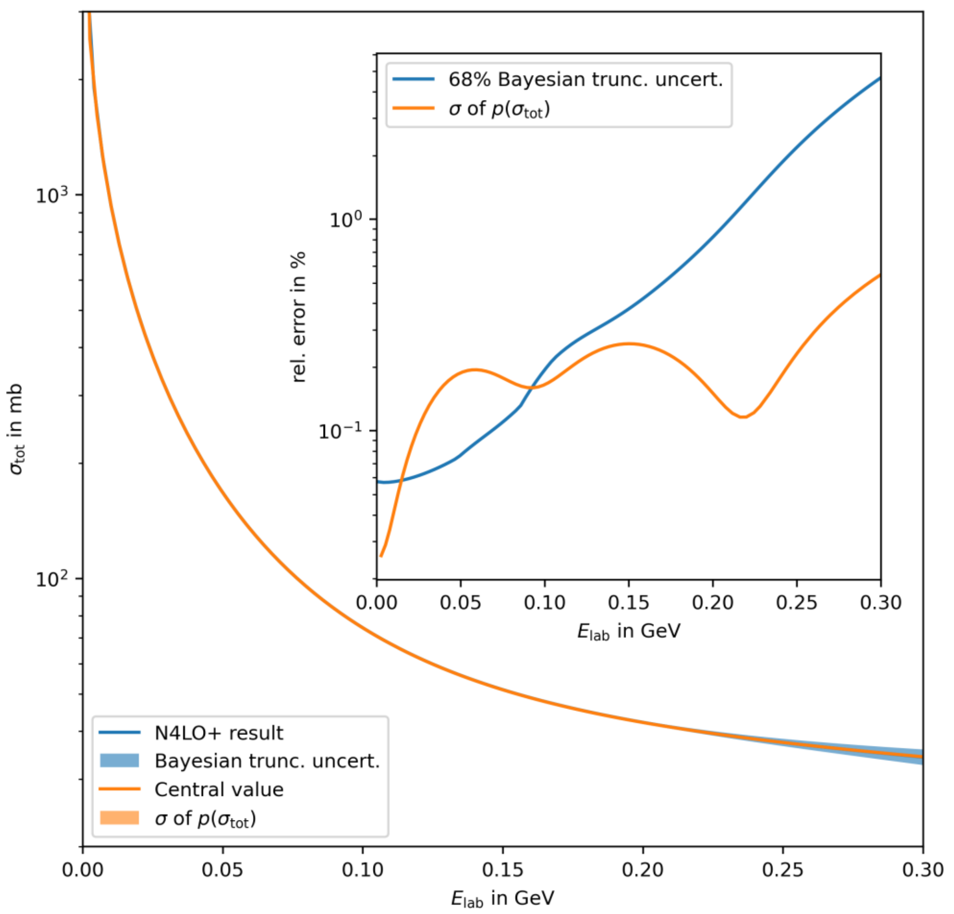
Results

PRELIMINARY!

$p\left(\sigma_{\text{tot}}^{(N4LO+)}\right)$ at 150MeV

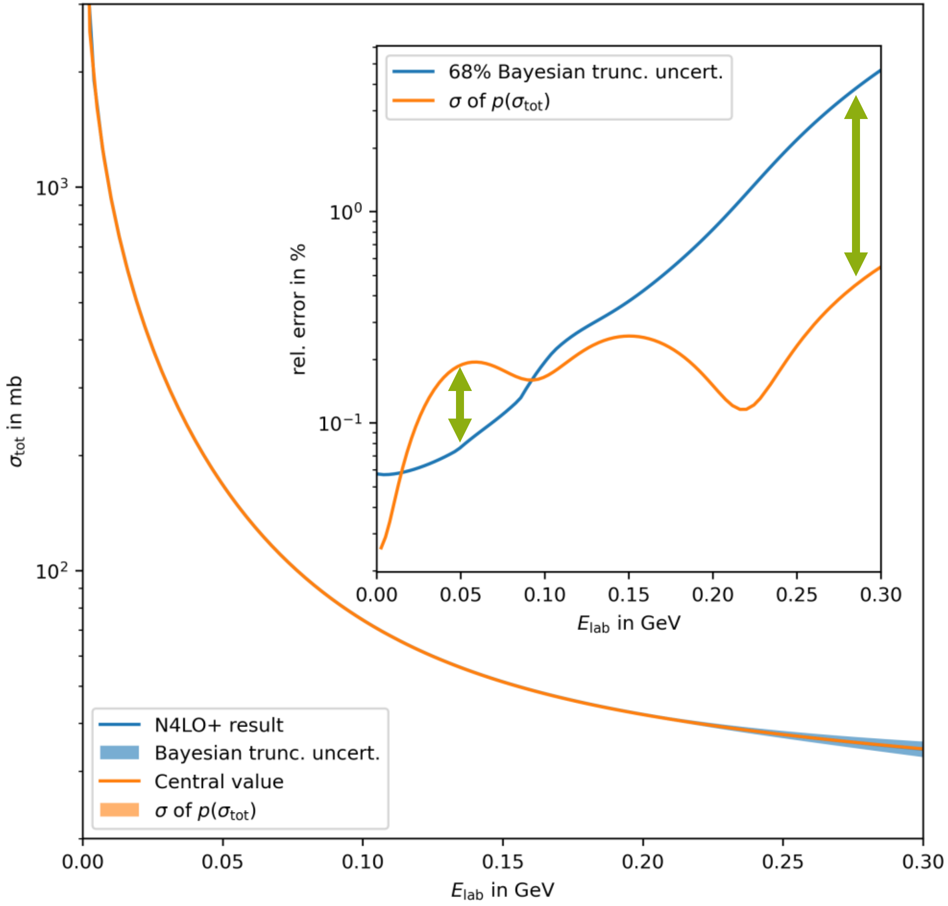


$$p\left(\sigma_{\text{tot}}^{(N4LO+)}\right)$$



$$p\left(\sigma_{\text{tot}}^{(N4LO+)}\right)$$

- Bayesian model underestimates for low-energy regime and overestimates for high-energy regime compared to uncertainty here



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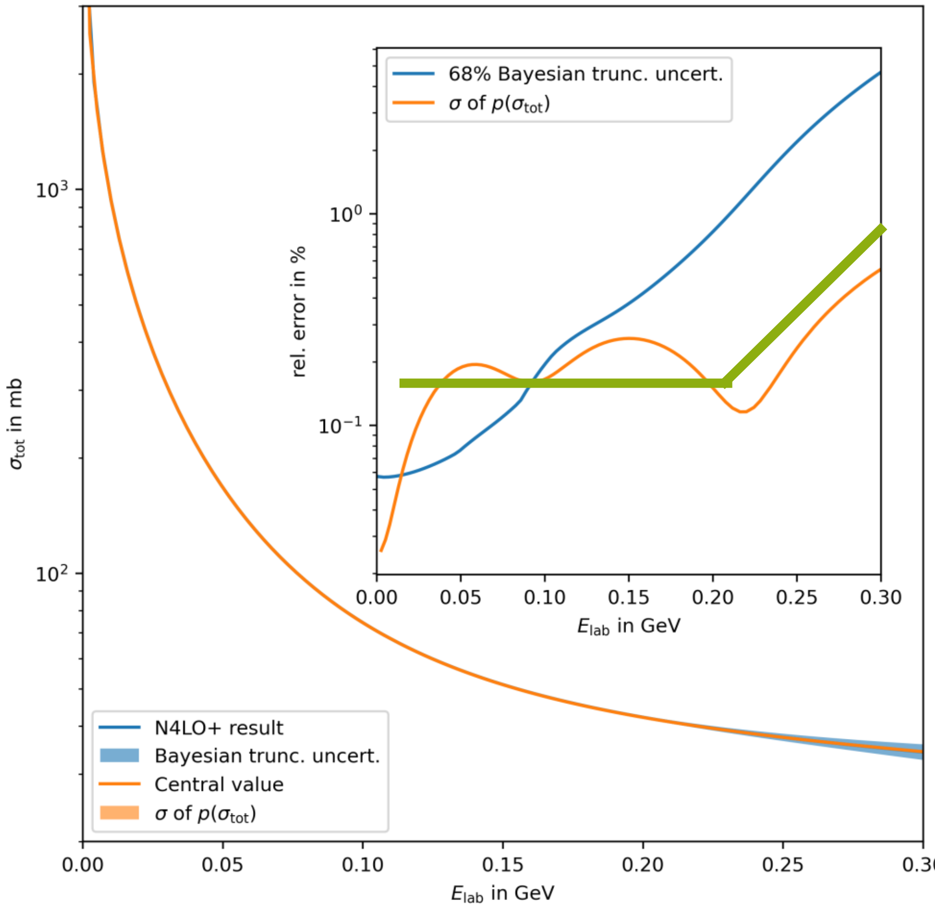
- Double expansion in momentum and pion mass

  → Two energy regimes



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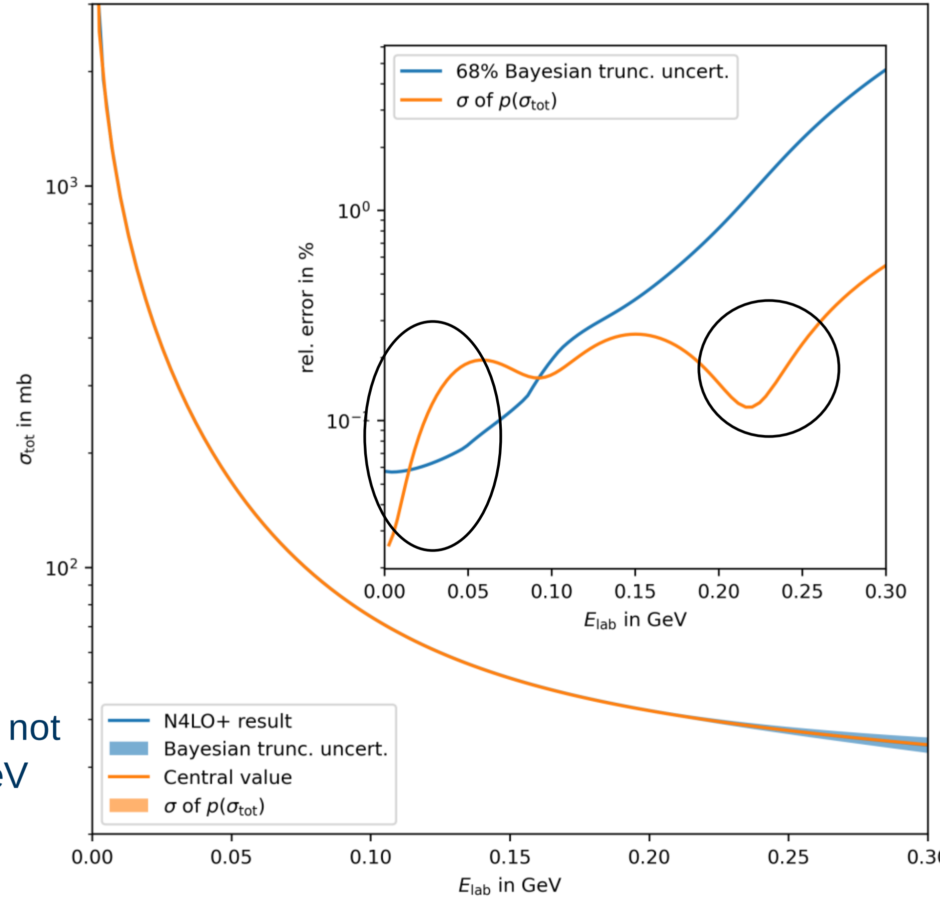
- Why the dips?
  - Number of experiments and their precision ist not equally distributed between 0MeV and 280MeV



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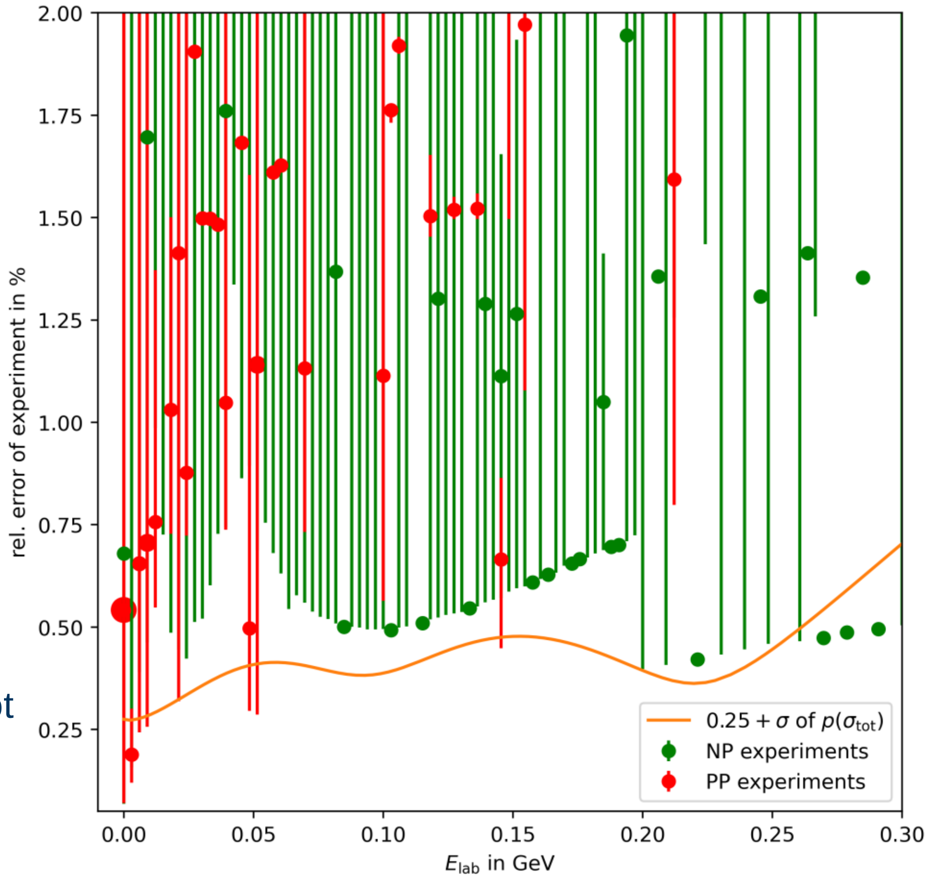


$p(X^{(k)})$ is actually not only truncation uncertainty

Theory

- Uncertainty of next-higher order contact forces

Data

- Stat. and sys. uncertainties
- Distribution of data among observables and energy ranges

Beliefs

- Natural LECs
- No inclusion of theory's precision predicition in fit
- Fit protocol
- …

$$p(X^{(k)} \mid \ldots)$$

Conclusion

- We have obtained the **combined uncertainty** of our chiral EFT model
  - Including truncation uncertainty of contact forces, as well as uncertainty from the data
  - Using our prior beliefs

Conclusion and Outlook

- We have obtained the **combined uncertainty** of our chiral EFT model
  - Including truncation uncertainty of contact forces, as well as uncertainty from the data
  - Using our prior beliefs

- Investigate different chiral orders, different cutoffs

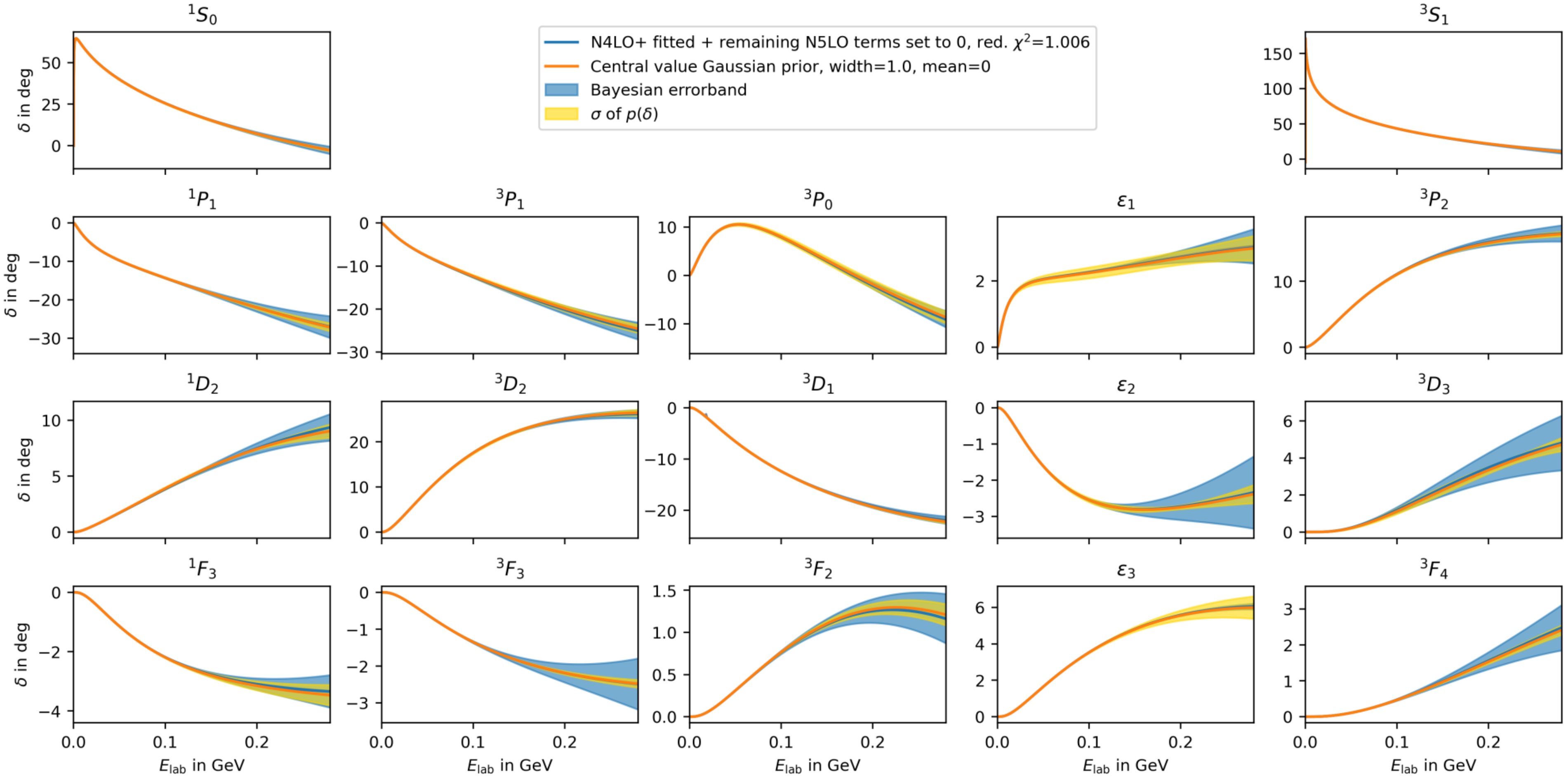
- So far, pion-exchange is used in calculation, but its truncation uncertainty is not yet estimated

- How to **disentangle statistical and truncation** uncertainty?

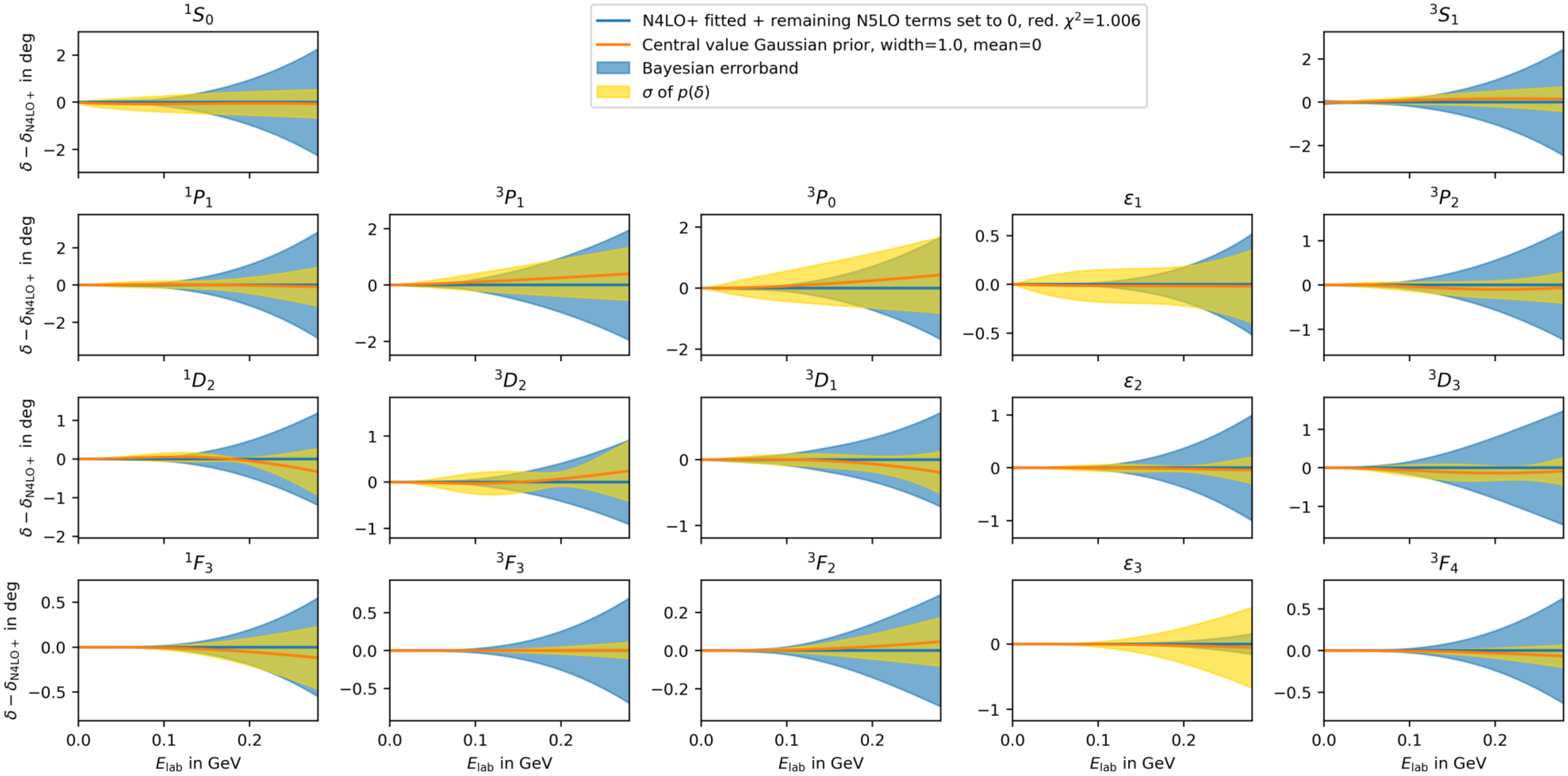
- How to propagate these uncertainties towards **few- and many-body** calculations?

Back up slides
More results, more preliminary!

Back up slides
Phaseshifts



$^1S_0$
$^3S_1$
N4LO+ fitted + remaining N5LO terms set to 0, red. χ^2=1.006
Central value Gaussian prior, width=1.0, mean=0
Bayesian errorband
σ of $p(\delta)$
$^1P_1$
$^3P_1$
$^3P_0$
ε_1
$^3P_2$
$^1D_2$
$^3D_2$
$^3D_1$
ε_2
$^3D_3$
$^1F_3$
$^3F_3$
$^3F_2$
ε_3
$^3F_4$
δ in deg
E_{lab} in GeV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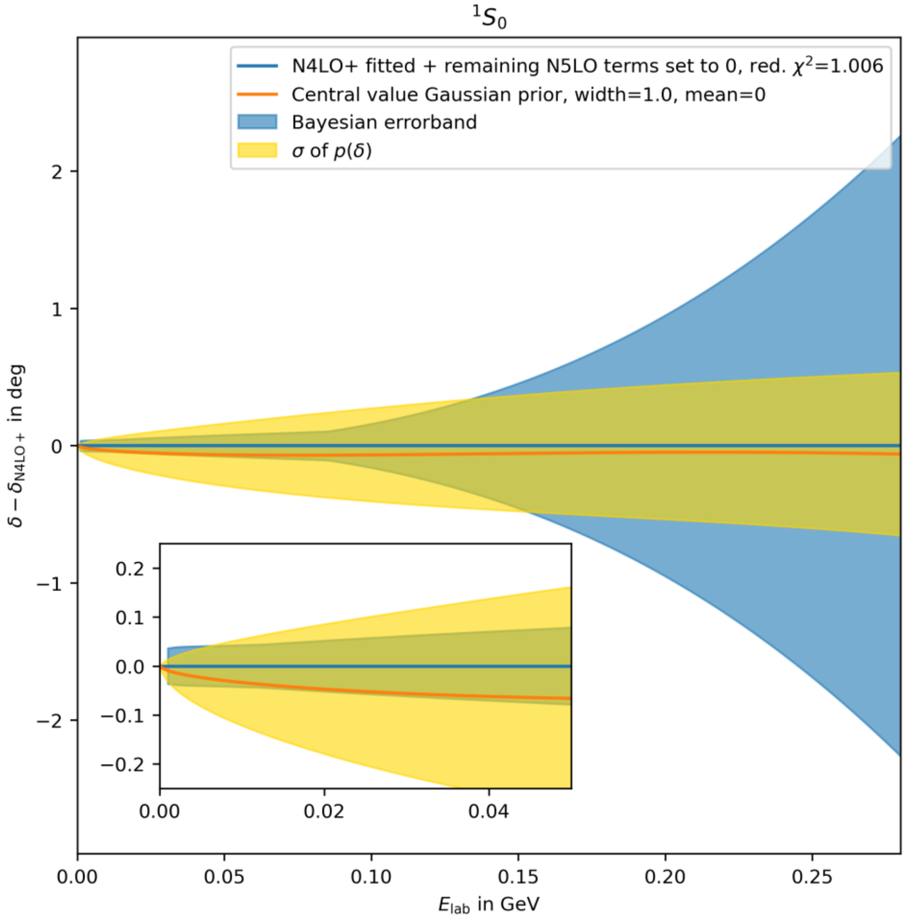
$^3D_2$
$^3D_1$
ε_2
$^3D_3$
$^1F_3$
$^3F_3$
$^3F_2$
ε_3
$^3F_4$
$\delta - \delta_{N4LO+}$ in deg
E_{lab} in GeV

1S0

- Larger uncertainty for lower energies
- Smaller uncertainty for higher energies

Compared to Bayesian estimation

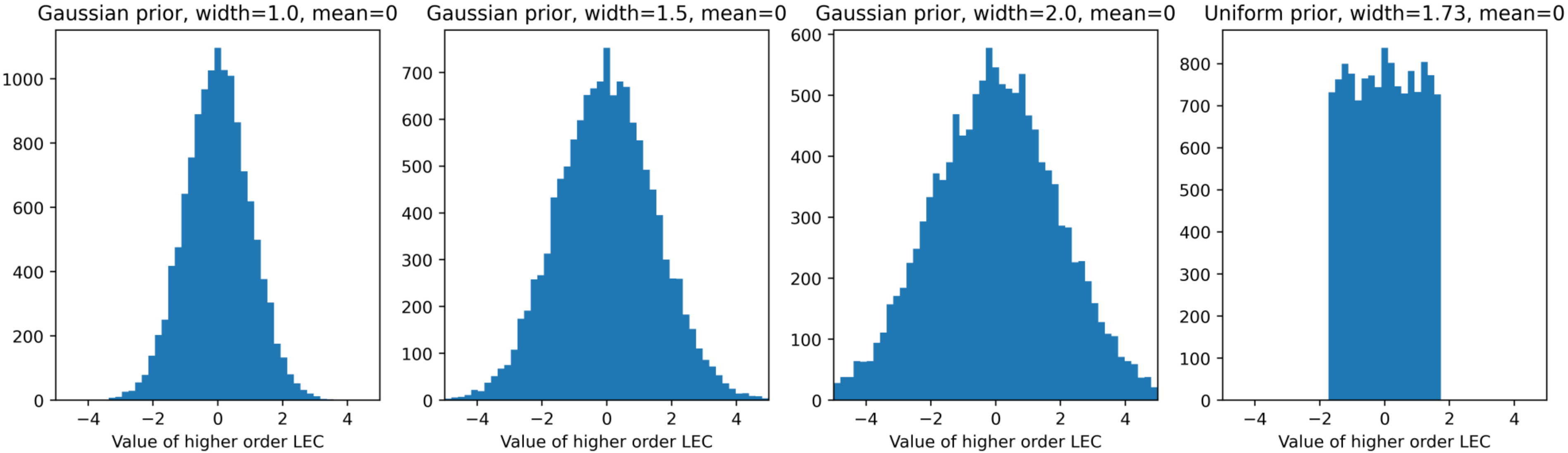


Back up slides

Using different distribution for higher-order LECs

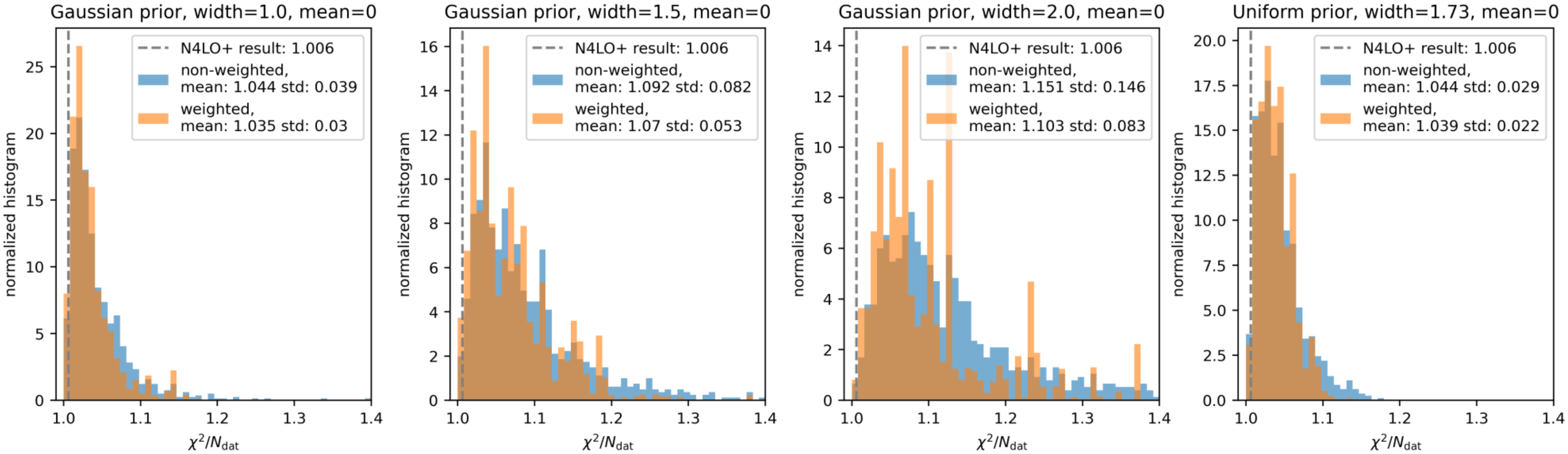
Histograms for higher-order LECs

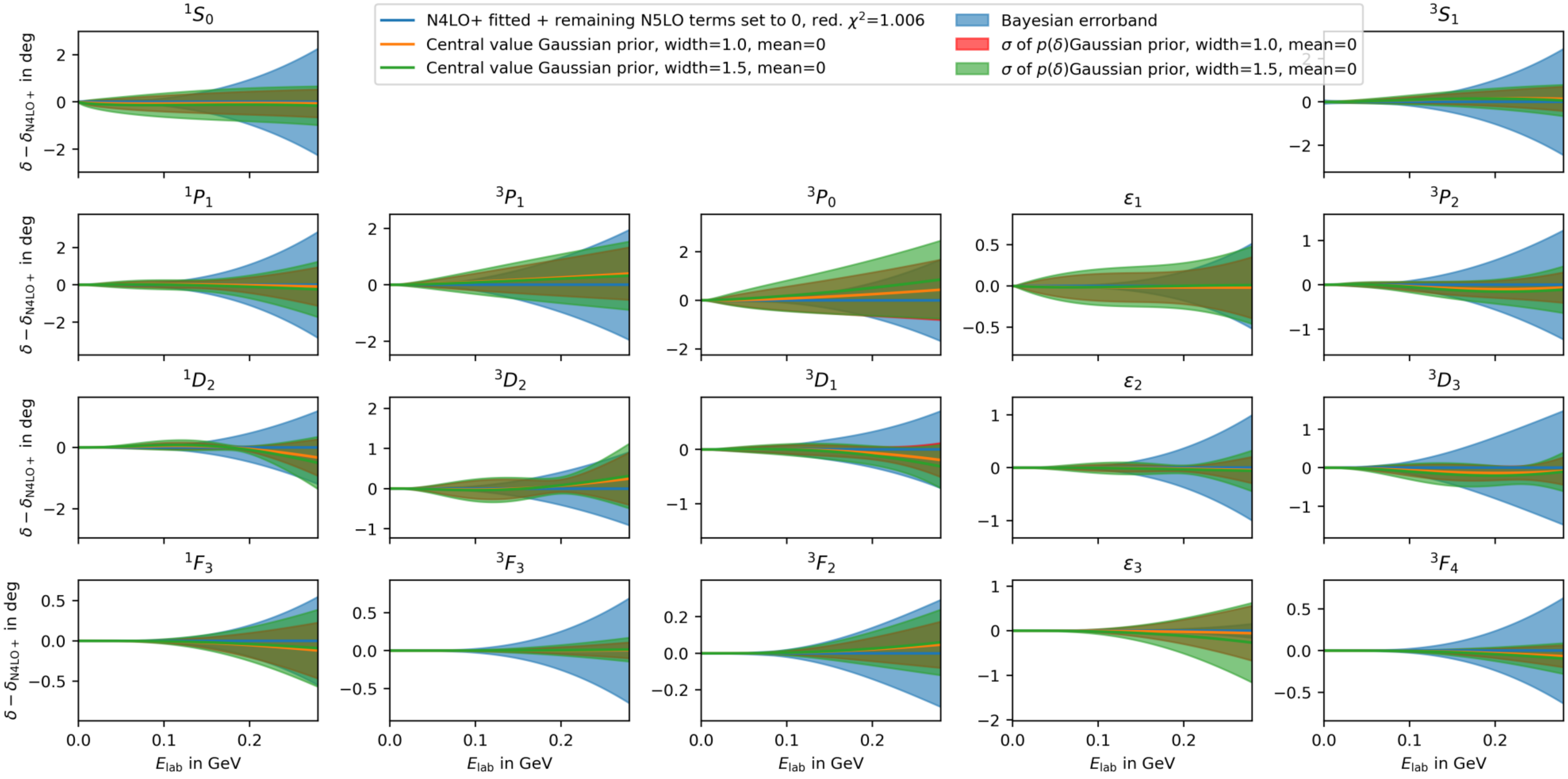
- Investigated 4 different distribution for higher-order LECs in total
- For each distribution 1000 fits have been conducted



Histograms for reduced chisq

- Entem-Machleidt-Nosyk N4LO+ best χ^2/N_{dat} (0-300MeV): ~ 1.2 E. Epelbaum et al., Handbook of Nuclear Physics 2022
- Fits without 2π exchange ~ 1.9 (deviates ~45σ from 1!)
- Successful PWA (χ^2/N_{dat} ~ 1) and model uncertainty are two different things!

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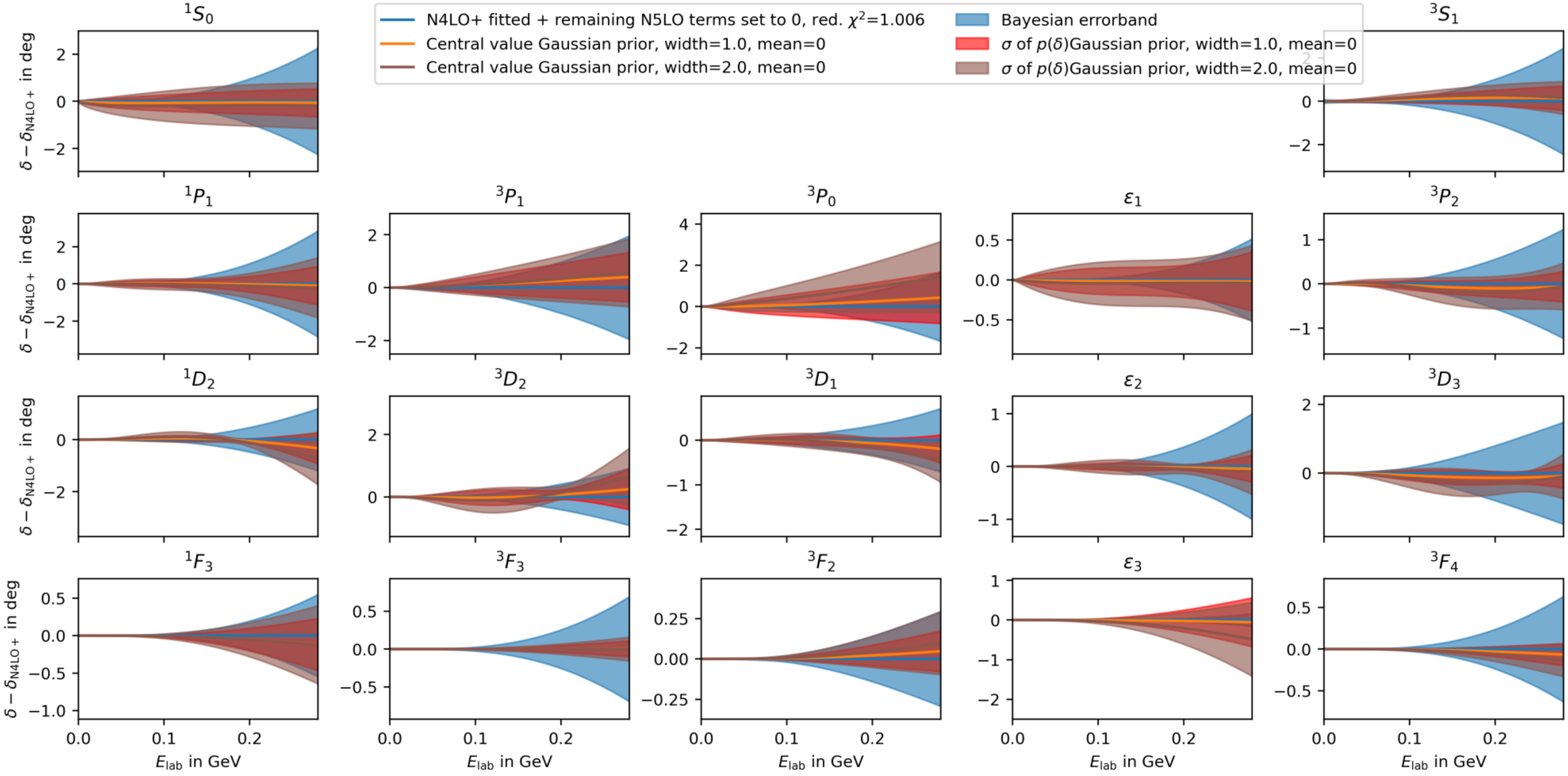
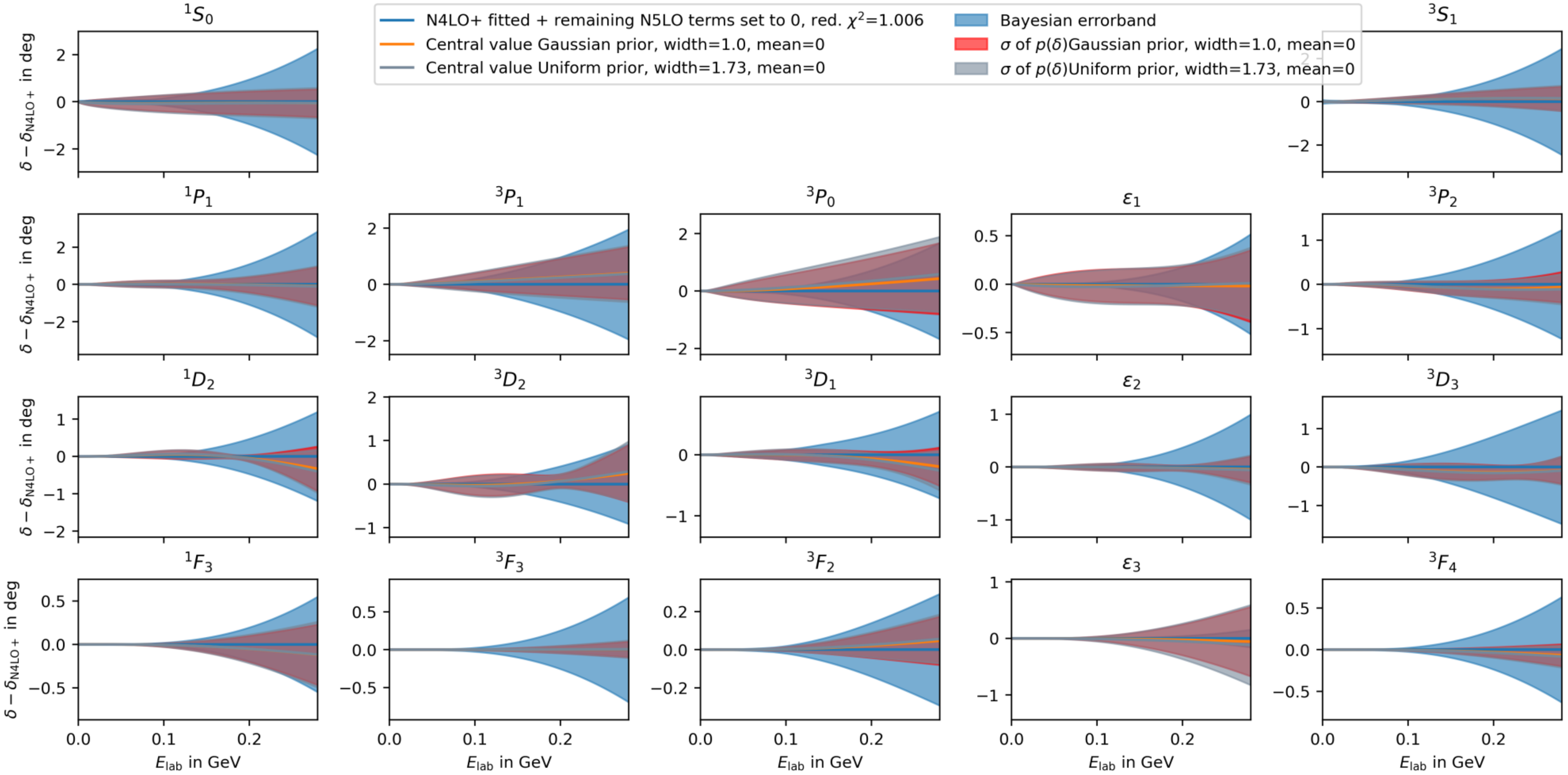
¹D₂
³D₂
³D₁
ε₂
³D₃
¹F₃
³F₃
³F₂
ε₃
³F₄
δ − δ_N4LO+ in deg
E_lab in GeV



N4LO+ fitted + remaining N5LO terms set to 0, red. χ²=1.006
Central value Gaussian prior, width=1.0, mean=0
Central value Gaussian prior, width=2.0, mean=0
Bayesian errorband
σ of p(δ)Gaussian prior, width=1.0, mean=0
σ of p(δ)Gaussian prior, width=2.0, mean=0
¹S₀
³S₁
¹P₁
³P₁
³P₀
ε₁
³P₂
¹D₂
³D₂
³D₁
ε₂
³D₃
¹F₃
³F₃
³F₂
ε₃
³F₄
δ − δ_N4LO+ in deg
E_lab in GeV



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Back up slides
Naturalness of LECs

Our general approach

- Using power counting, nuclear forces from chiral Lagrangian at some fixed chiral order are derived

- Forces are regulated, with cutoff regulator as function of external nuclear momenta and cutoff Λ
  - Regulator must leave all symmetries intact
  - Infinite number of UV-divergences, if $\Lambda \to \infty$, from iterated potential, but at finite chiral order not enough counter terms available → finite cutoff!
  - Demanding no deeply bound states

  → $\Lambda \sim \Lambda_B$ and Weinberg power counting → LECs ~ 1

- Fitting the bare(!) coefficients of the short-range operators (LECs), leaving the cutoff finite

  → Implicit renormalization of LECs (Renormalized LECs are actually unknown for pionful theory)

Epelbaum et al., Front. in Phys. 8, 98 (2020), arXiv:1911.11875

What are natural values?

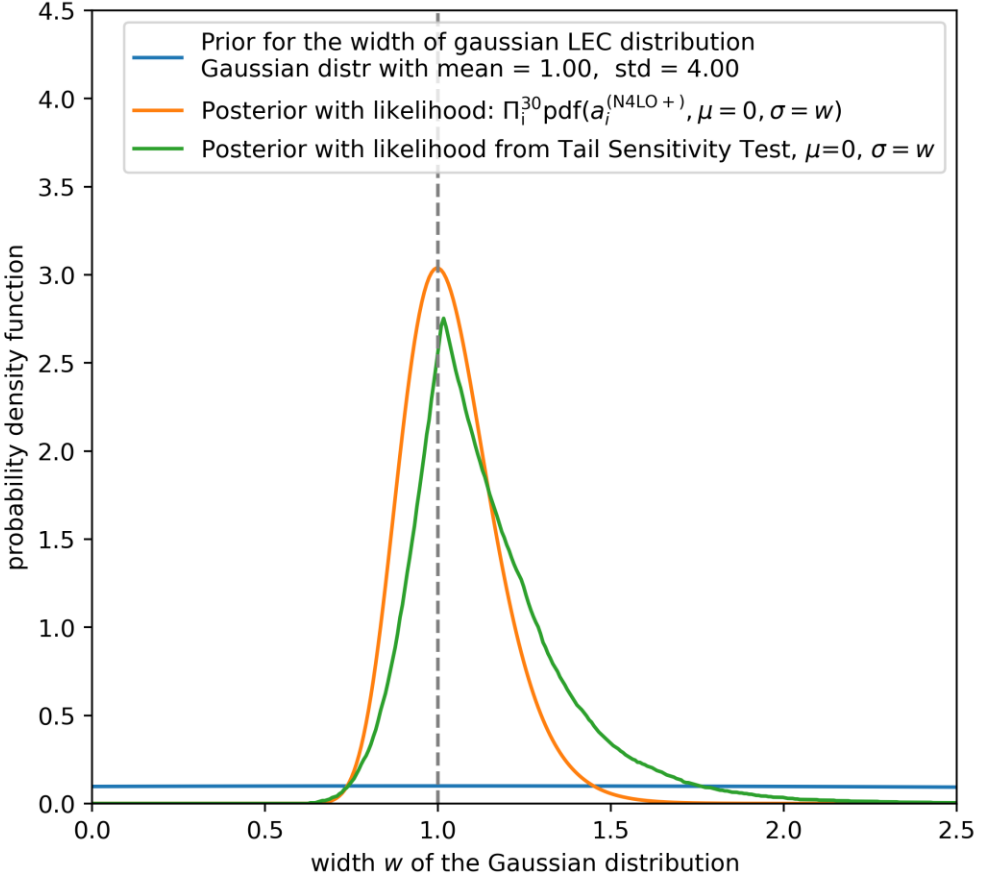
- $p(w \mid \vec{a}^{(i \in [0,k])}) \propto p\left(\vec{a}^{(i \in [0,k])} \mid w\right) \cdot p(w)$

- What is $p\left(\vec{a}^{(i \in [0,k])} \mid w\right)$?

  - Product of probabilities of every element of $\vec{a}^{(i \in [0,k])}$, that it comes from $\mathrm{N}(0, \mathrm{w})$?

  - Probability, that this set of LECs is drawn from $\mathrm{N}(0, \mathrm{w})$? Tail-sensitive test S. Aldor-Noiman et al., Am. Stat. 67, 249 (2013)

→ If assumptions are fulfilled, the LECs are indeed drawn from a distribution with $w \sim 1$



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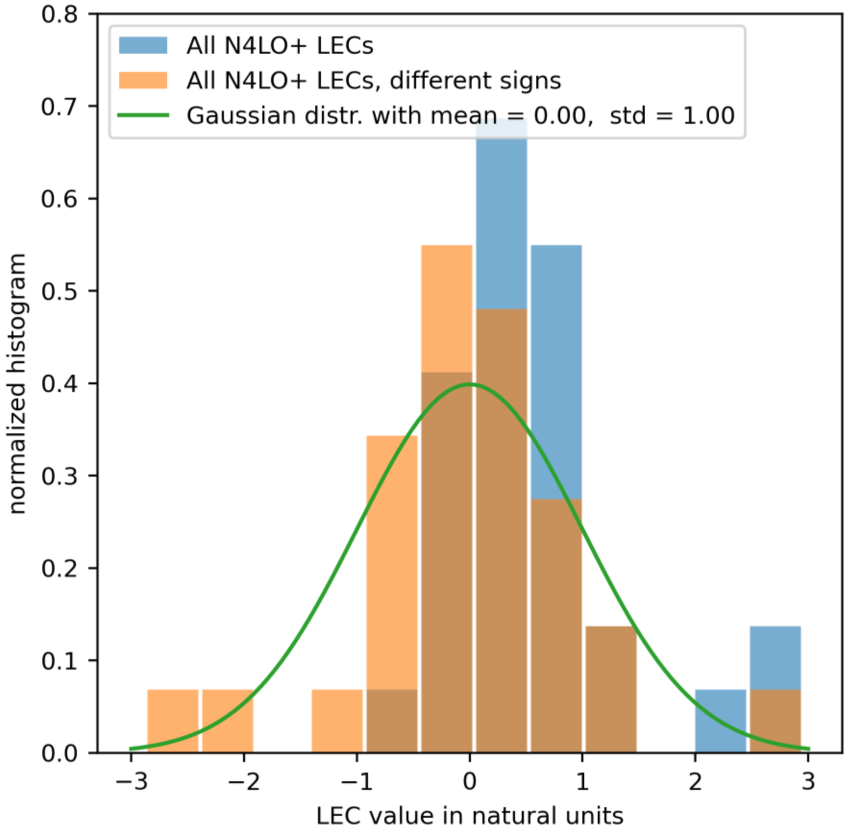
Histogram of LECs

- "Sign problem" of LECs in Chiral EFT
  - Sign of contact forces is just convention

  → Signs of LECs can be changed without any effects for theory or observables

- What are "natural units" of LECs?
  - LECs of order Q^{2n} are given in units of $10^4 \mathrm{GeV}^{-2-2n}$



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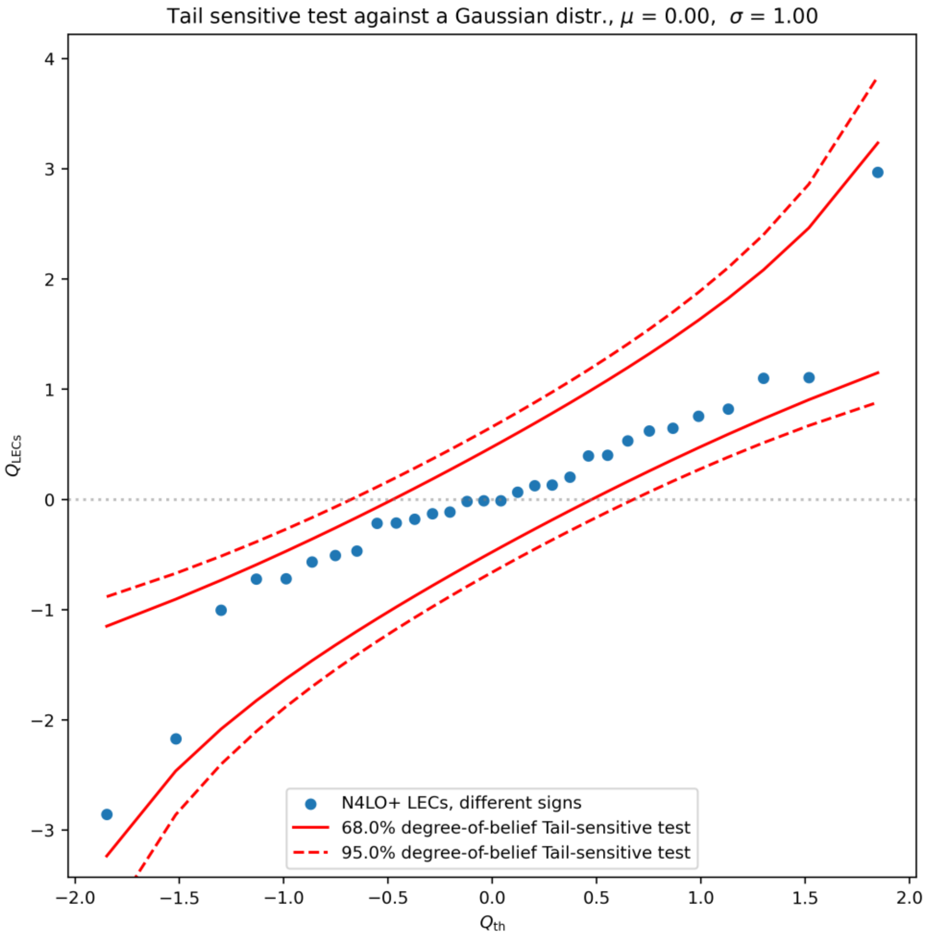

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