

Scalar and tensor charmonium resonances from lattice QCD

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Chiral Dynamics, Bochum
26th August 2024

based on work:

PRL Editors' choice: arXiv: [2309.14070](https://arxiv.org/abs/2309.14070) (7 pages)

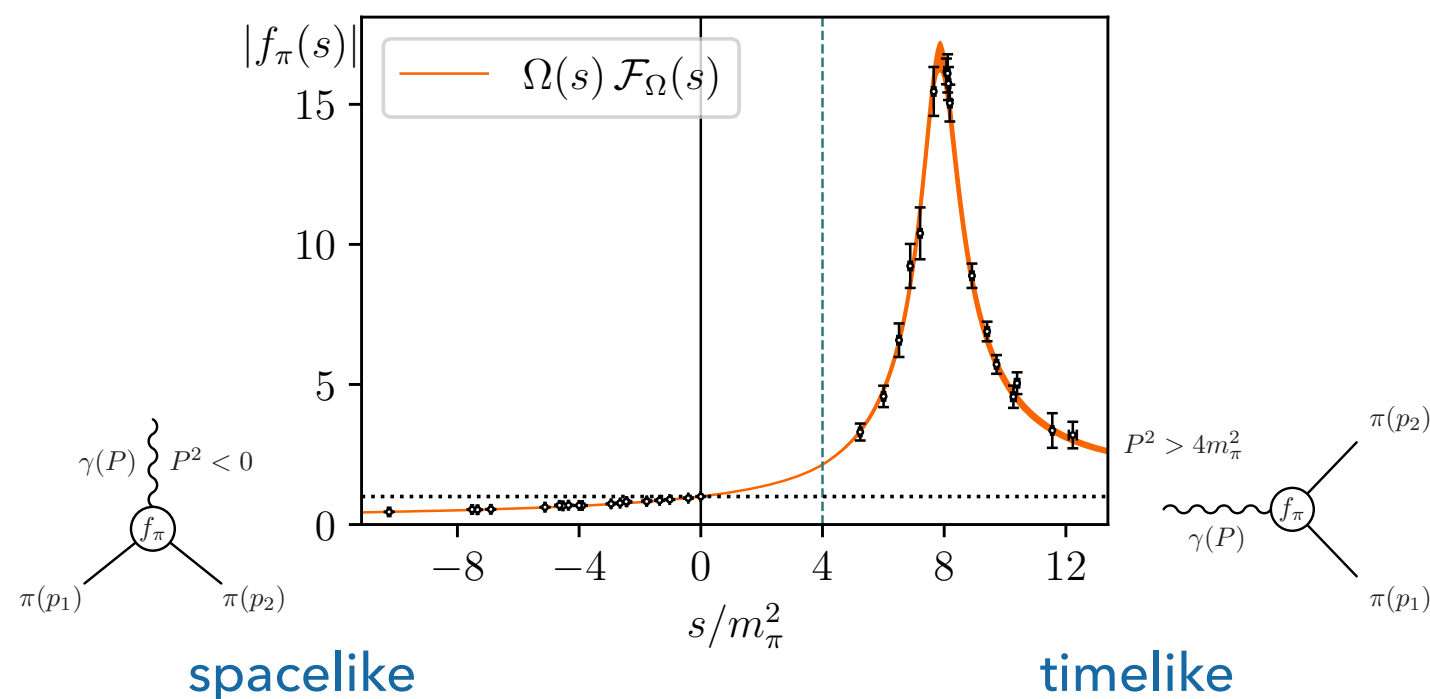
PRD Editors' choice: arXiv: [2309.14071](https://arxiv.org/abs/2309.14071) (55 pages)



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THE ROYAL SOCIETY

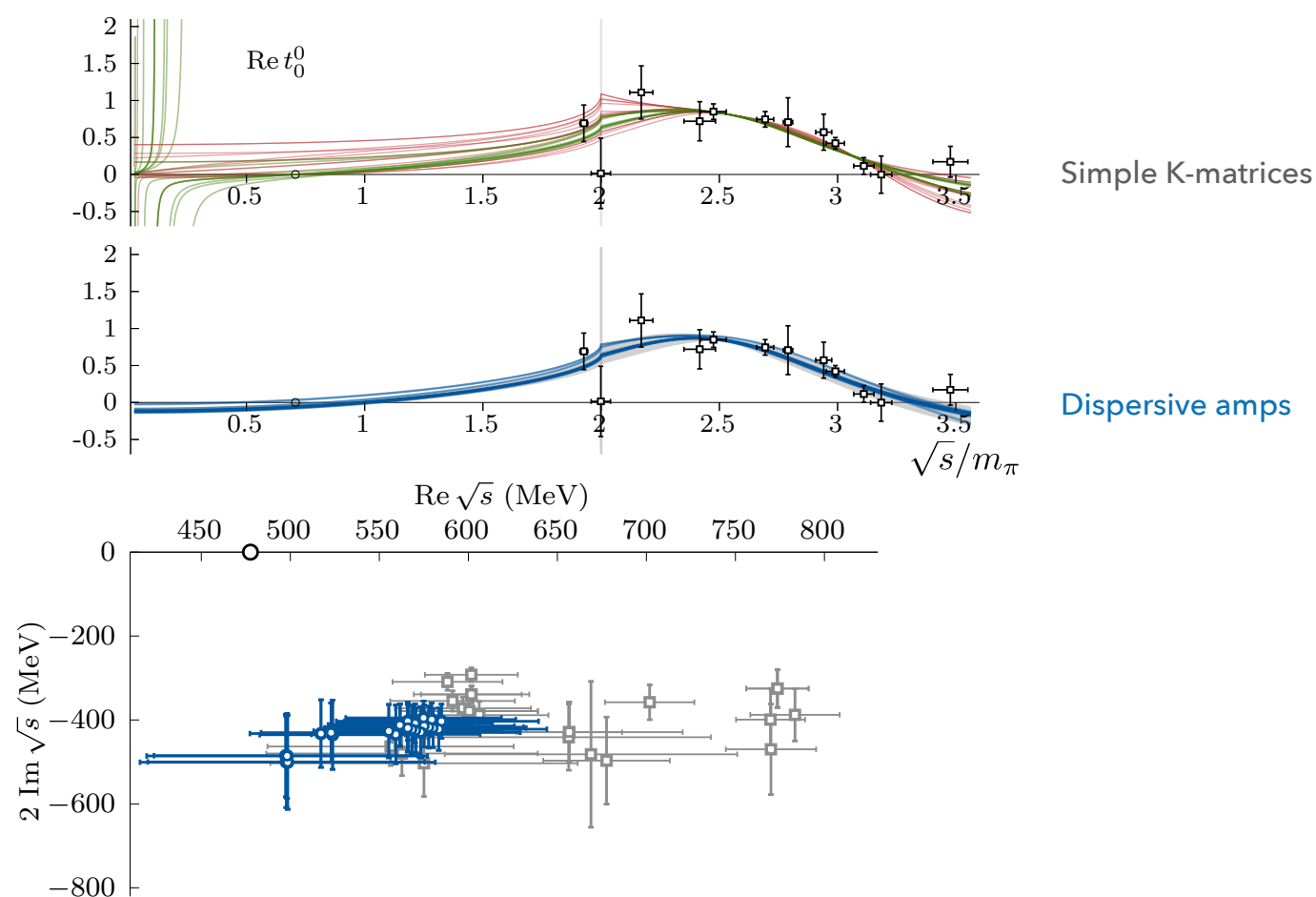


Timelike meson form-factors

Ortega-Gama, Dudek, Edwards

[arXiv:2407.20617](https://arxiv.org/abs/2407.20617)

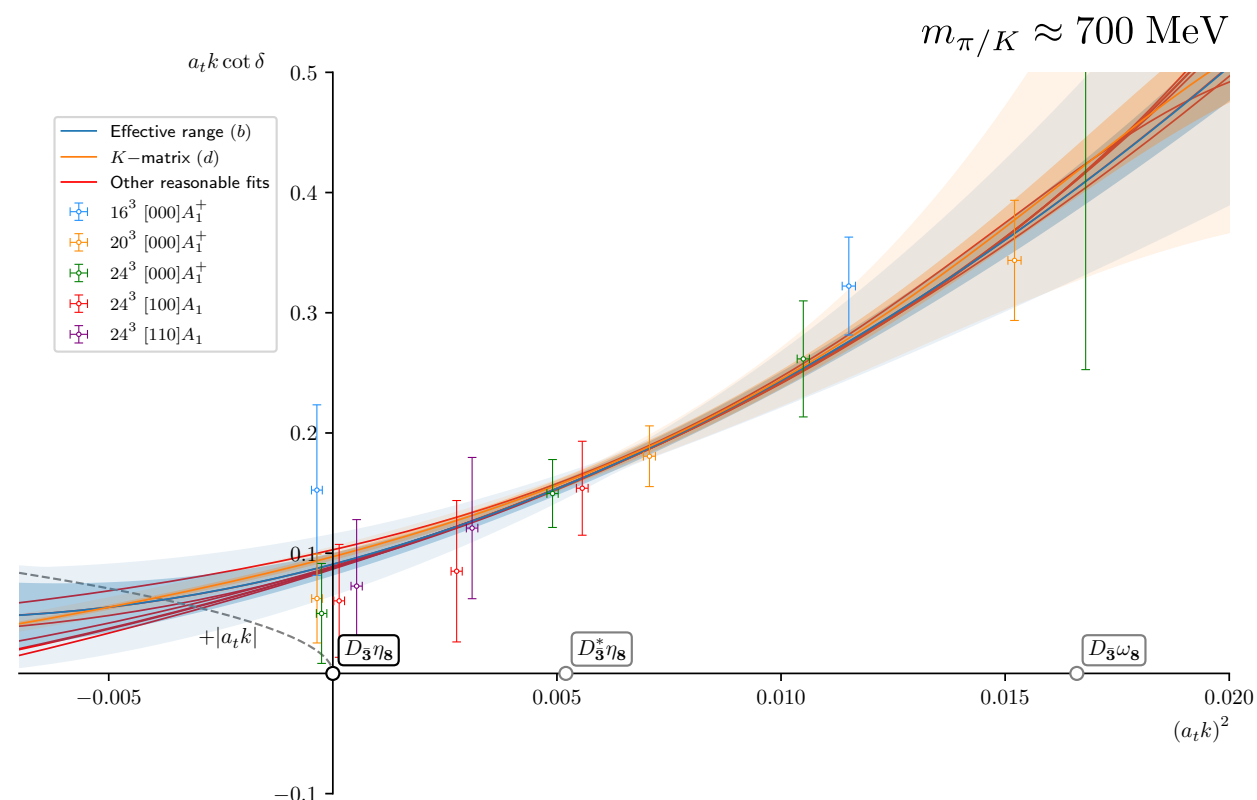
- Lellouch-Lüscher analysis to extract infinite volume scattering amplitude
- Extended to coupled-channel region (KKbar)

 $\pi\pi$ dispersive + light quark mass-dependence

Rodas, Dudek, Edwards

[arXiv:2303.10701](https://arxiv.org/abs/2303.10701) & [arXiv:2304.03762](https://arxiv.org/abs/2304.03762)

- pins down σ -pole position
- Adler zero arises naturally
- interesting interplay with bound-state σ and Adler zeros when considering the light-quark mass dependence

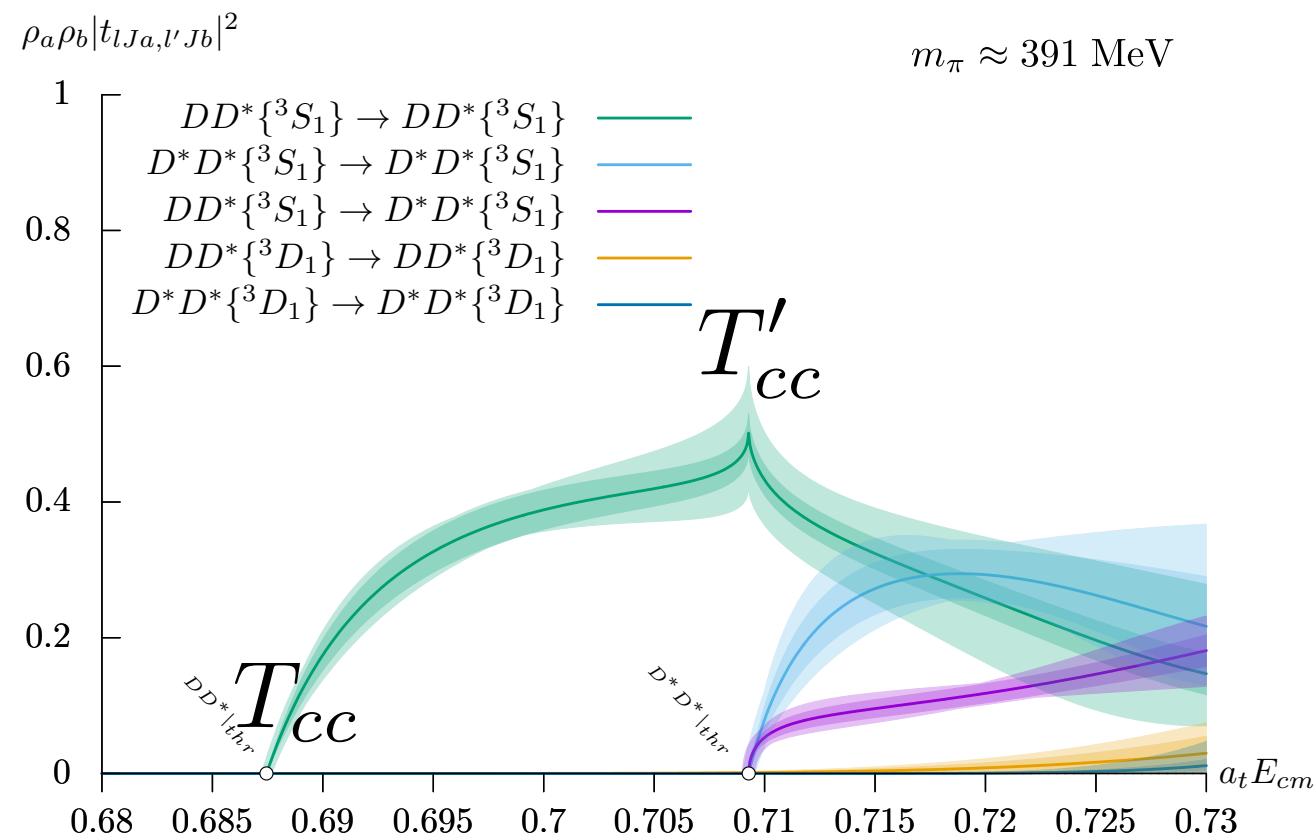


$D\pi/DK$ scattering with SU(3) flavour symmetry

Yeo, Thomas, Wilson

[arXiv:2403.10498](https://arxiv.org/abs/2403.10498)

- S-wave interactions in flavour SU(3)
- 3bar, 6, 15bar
- Virtual bound state sextet pole
- Also deeply bound 3bar state, similar to $Ds_0(2317)$, much greater binding

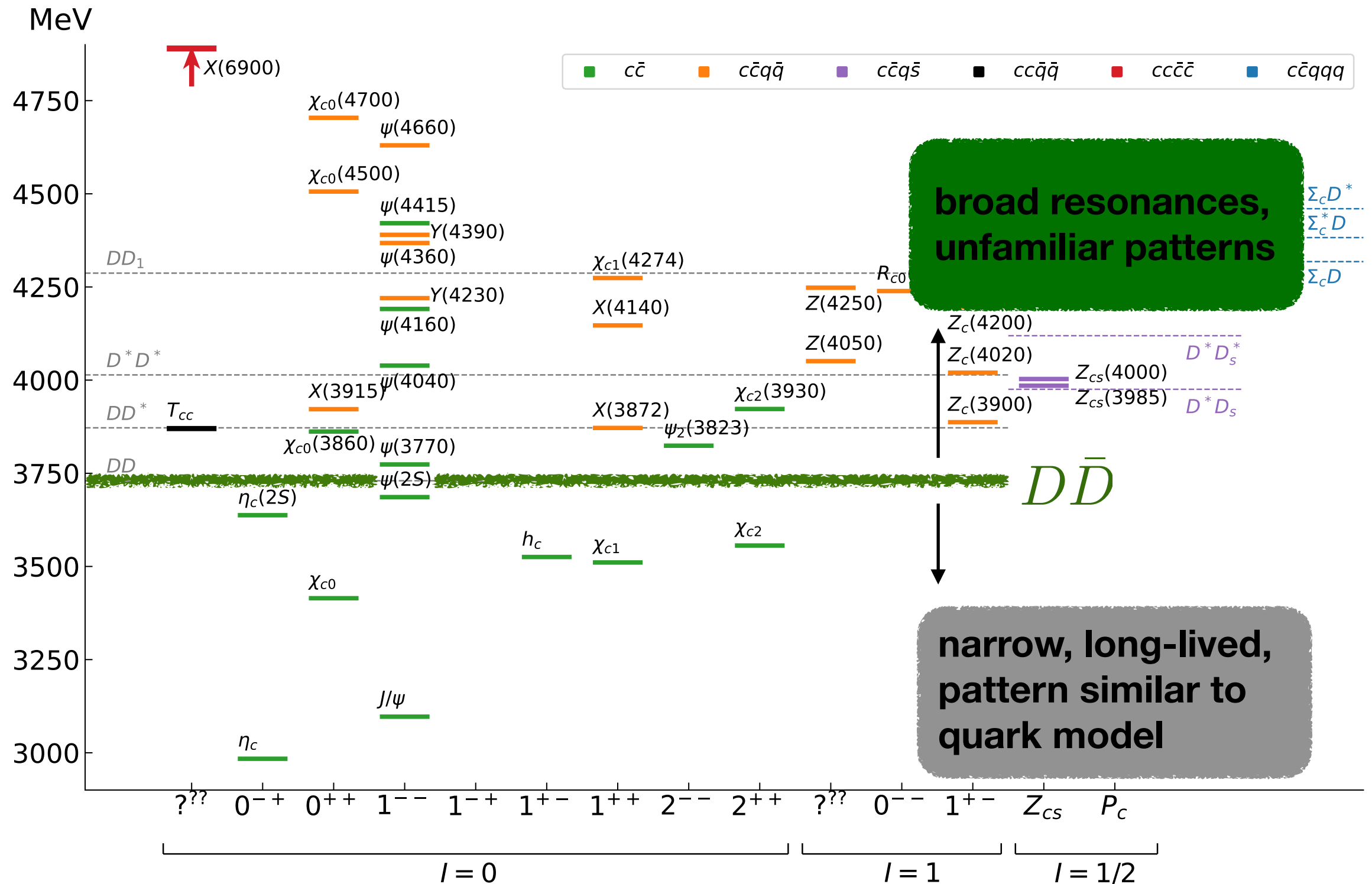


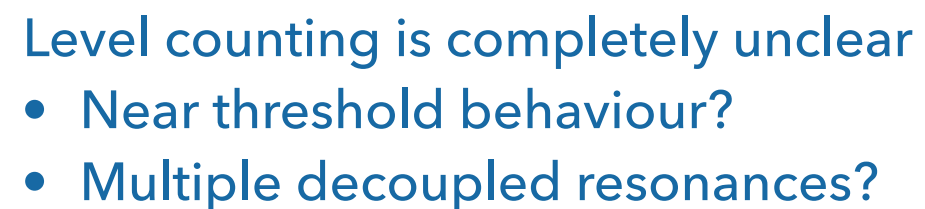
$DD^*-D^*D^*$ coupled channel

Whyte, Wilson, Thomas

[arXiv:2405.15741](https://arxiv.org/abs/2405.15741)

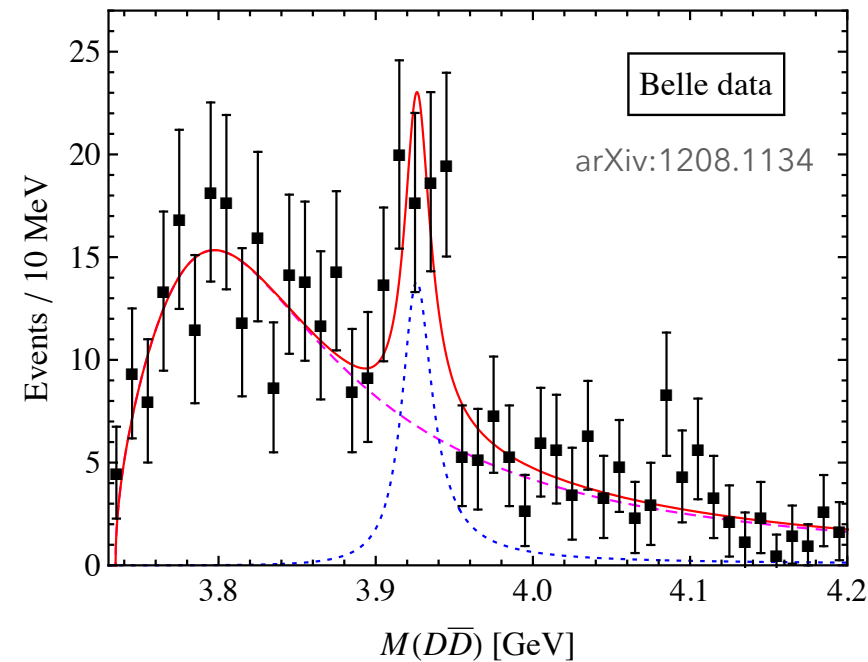
- S and D-wave in $J^P=1^+$
- virtual bound state below DD^* and resonance below D^*D^*
- (neglecting left cuts)



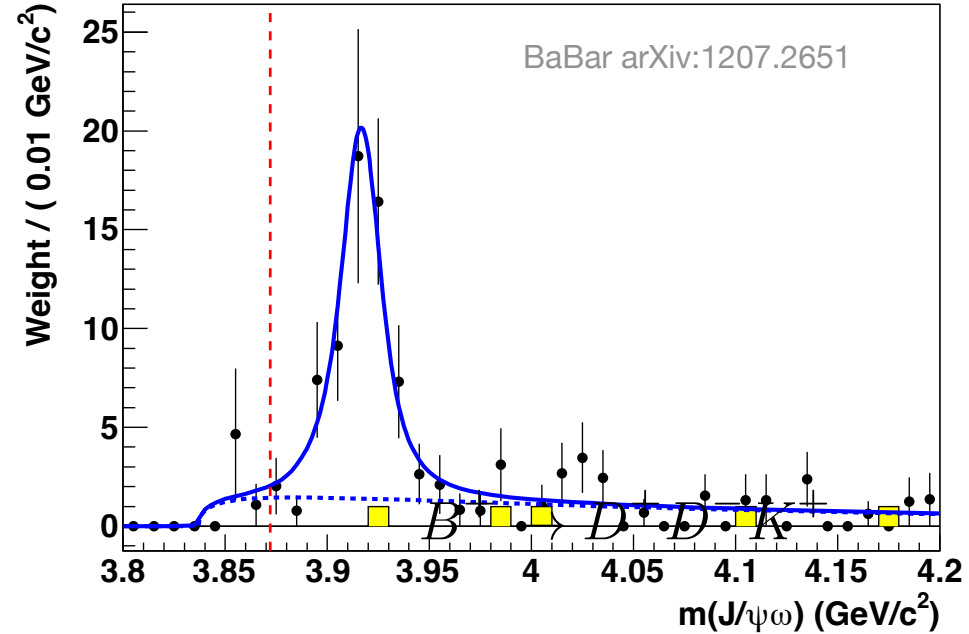


Probably one
resonance

$$\gamma\gamma \rightarrow D\bar{D}$$



$$J/\psi\omega$$



Just a few examples
Many many more
(References in the longer paper)

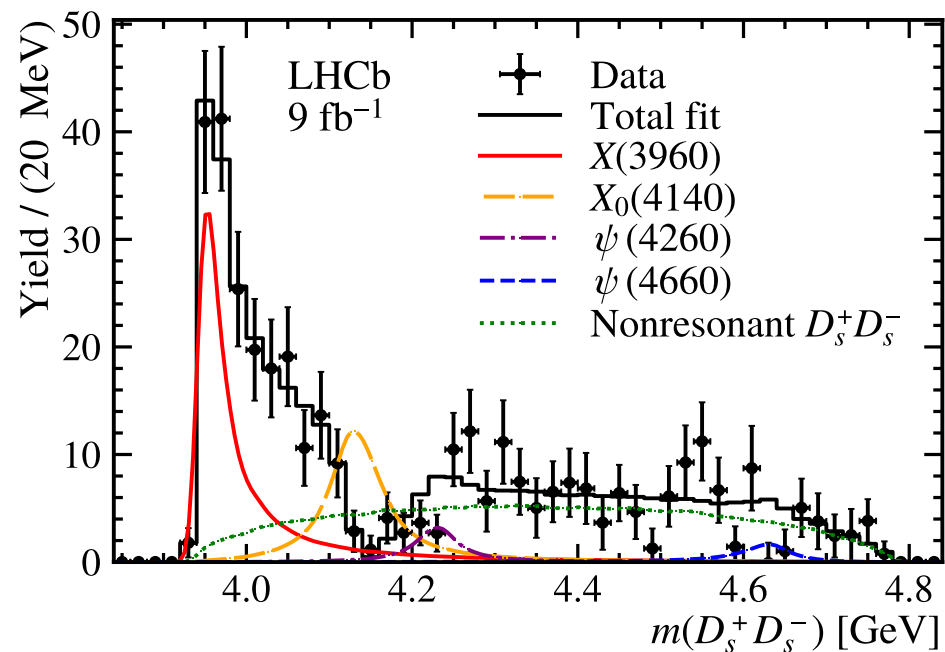
$$m = (3919.4 \pm 2.2 \pm 1.6) \text{ MeV}$$

$$\Gamma = (13 \pm 6 \pm 3) \text{ MeV}$$

$$J^P = 0^+$$

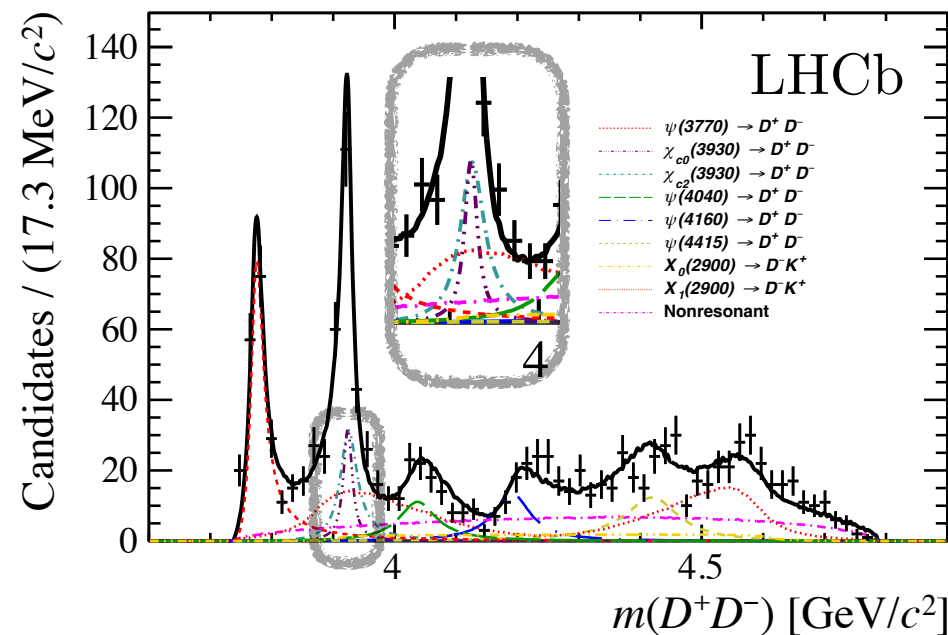
$$B^+ \rightarrow D_s^+ D_s^- K^+$$

arXiv:2210.15153
LHCb



$$B^+ \rightarrow D^+ D^- K^+$$

arXiv:2009.00026

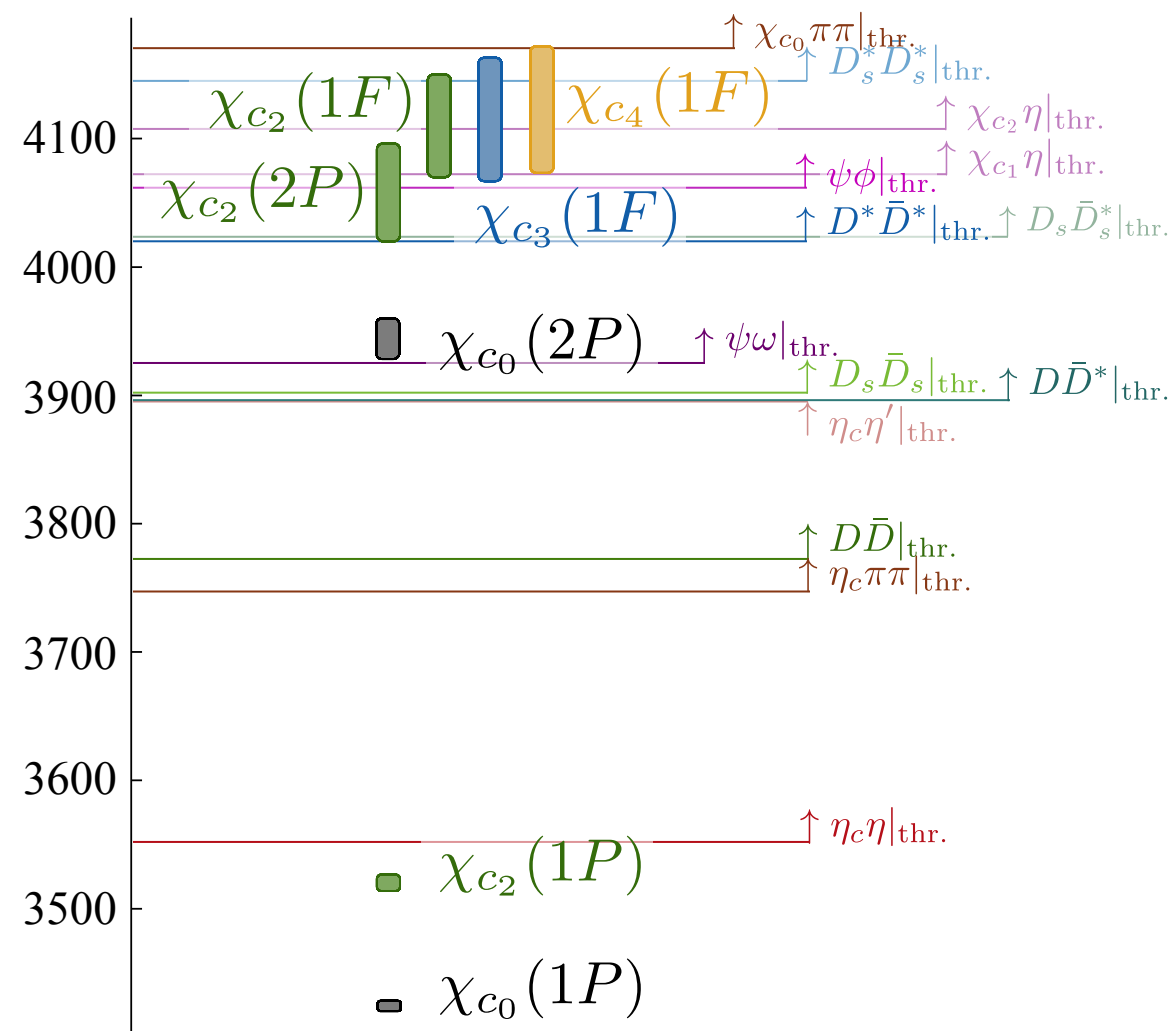


overlapping 0^{++} and 2^{++}
resonances around 3925 MeV

no need for a low 0^{++} resonance

Previously:

E_{cm}/MeV



spectra from qqbar operators only,
Liu et al JHEP 1207 (2012) 126

“HadSpec” lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

$L/a_s = 16, 20, 24$

$m_\pi = 391 \text{ MeV}$

rest and moving frames

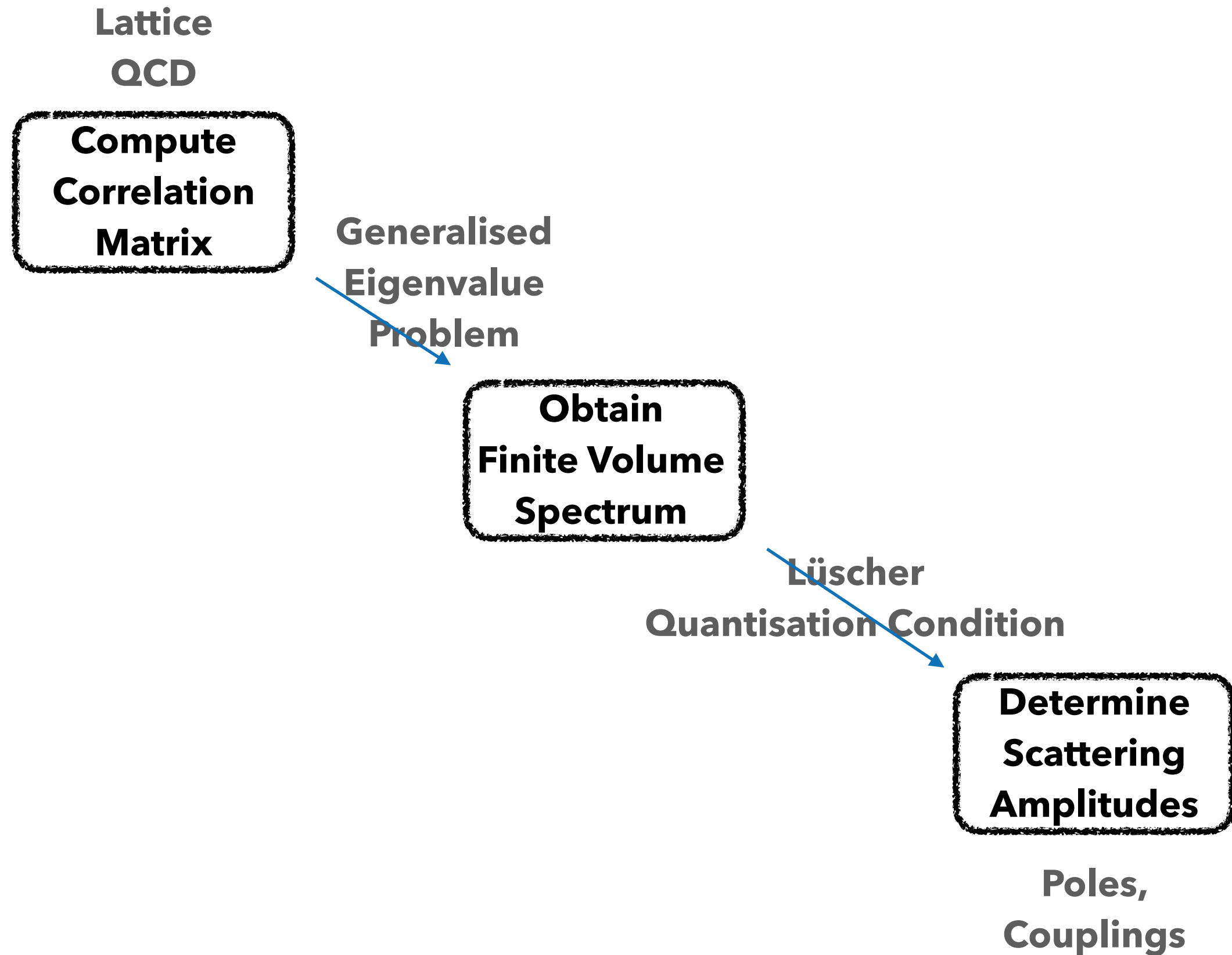
$N_f = 2+1$ flavours

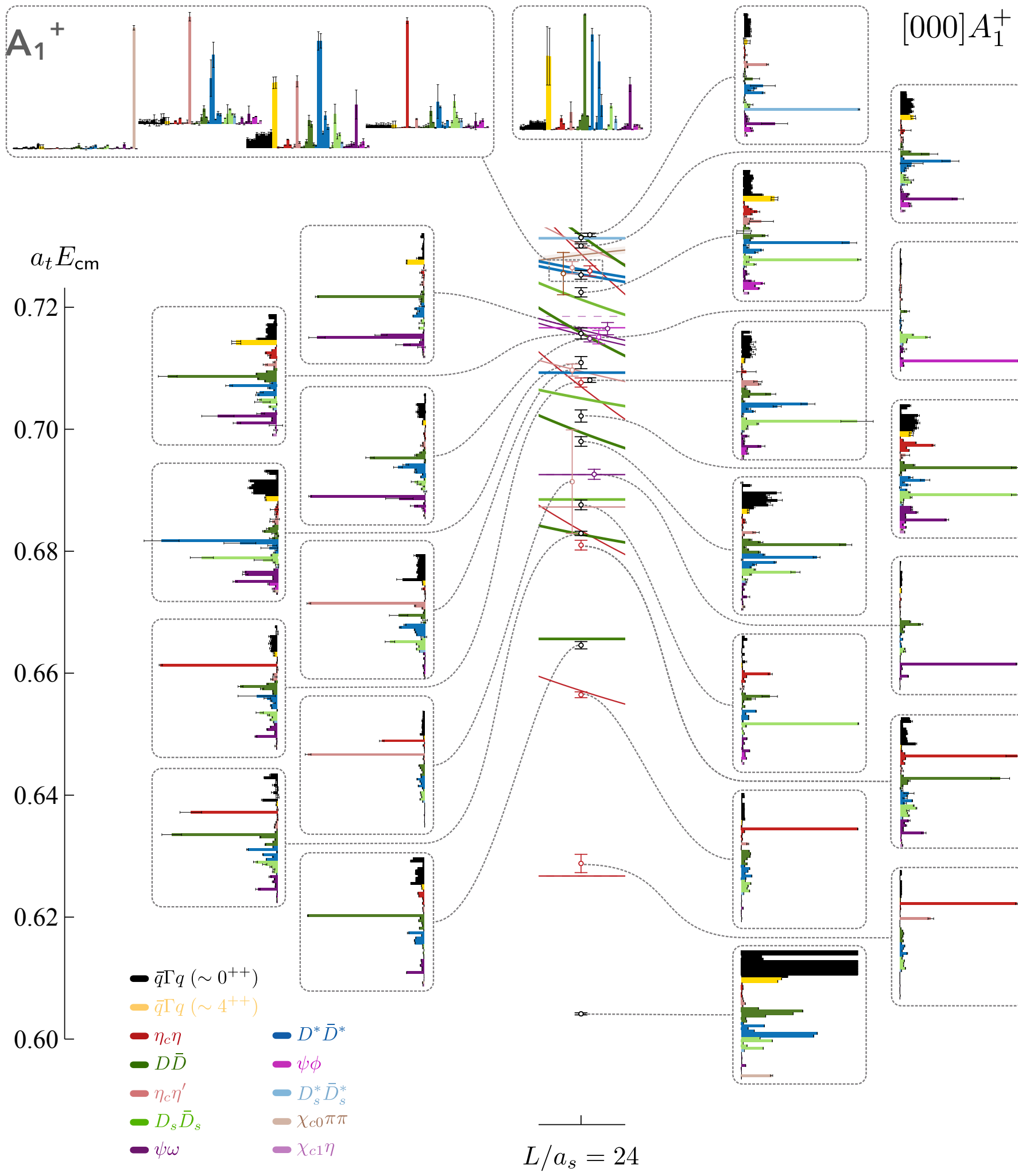
all light+strange annihilations included
no charm annihilation

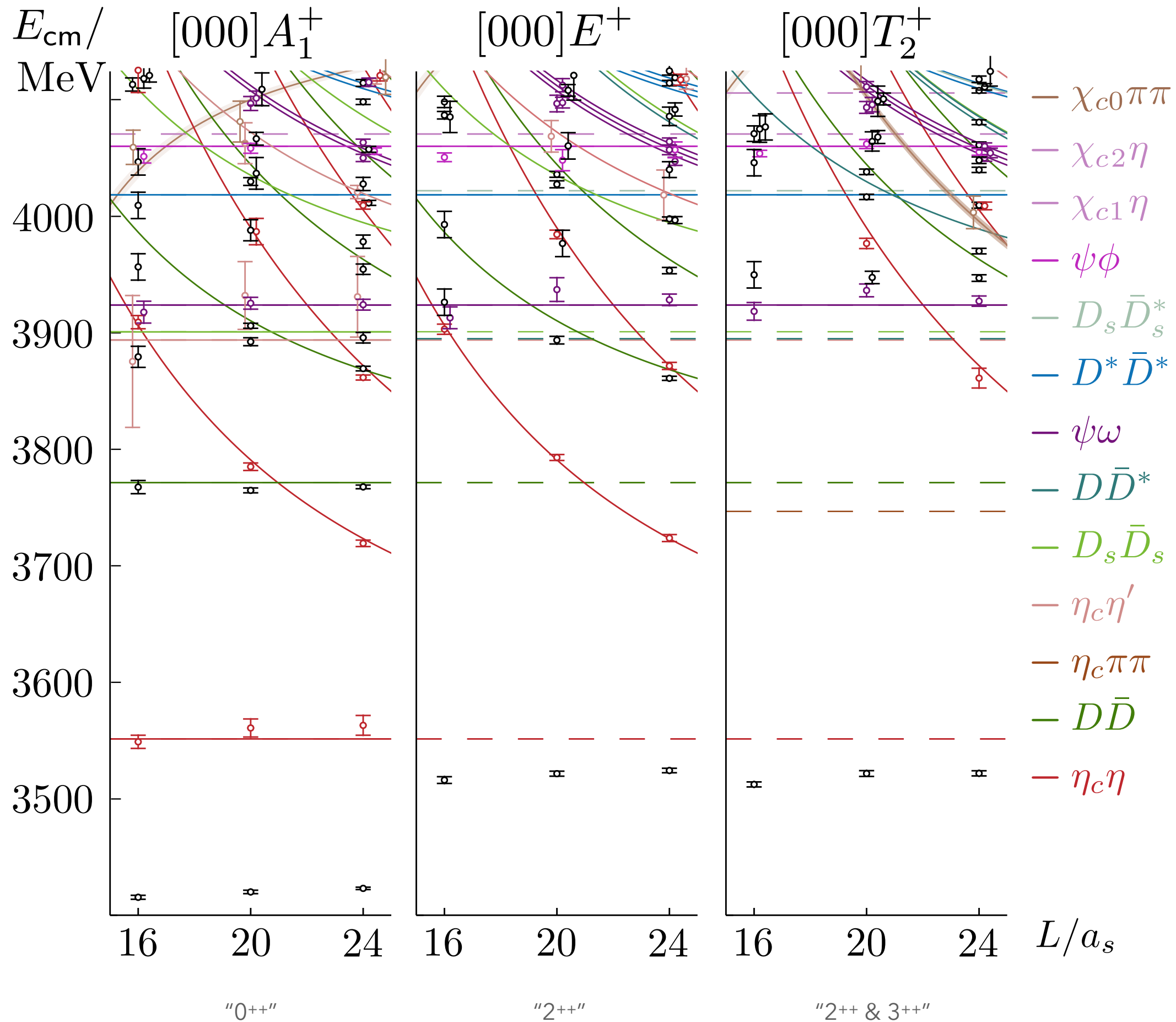
using *distillation* (Peardon et al 2009)
many channels, many wick contractions

This study: Meson-meson + qqbar ops

- compute a large correlation matrix
- solve generalised eigenvalue problem to extract energies







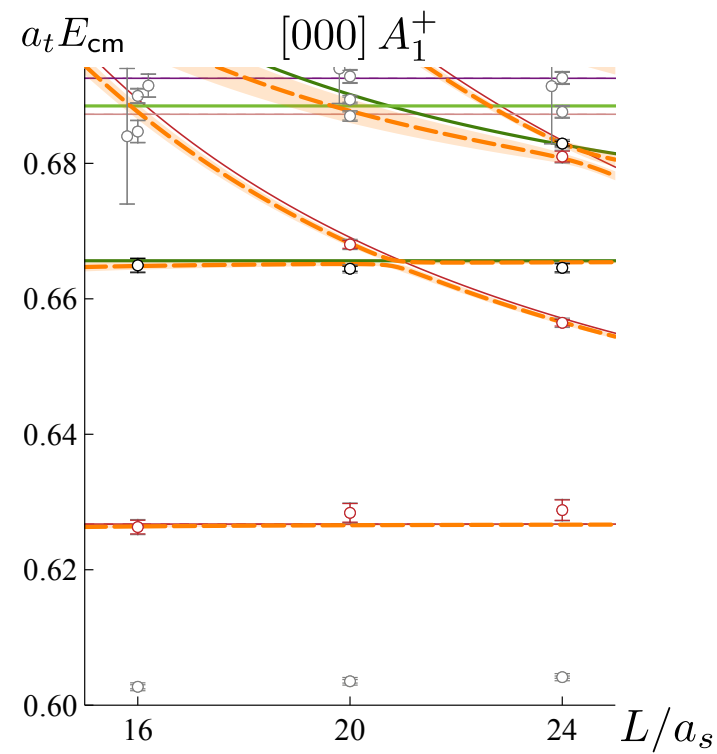
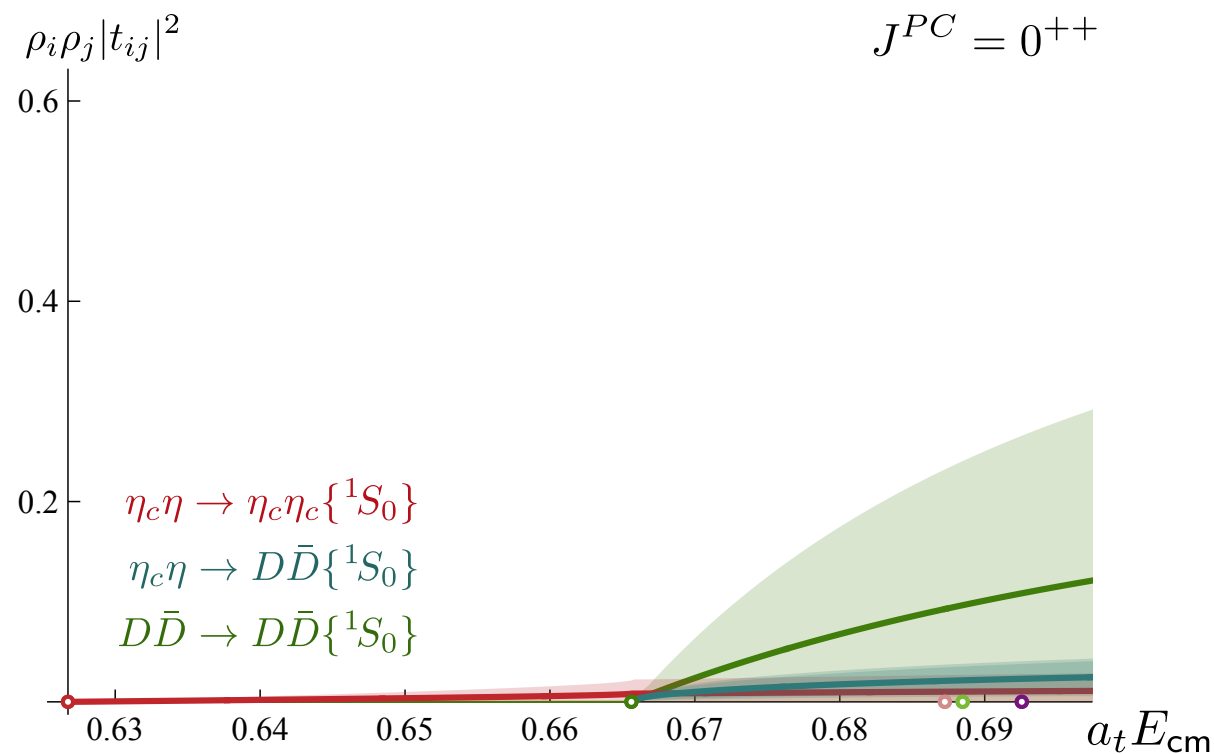
$$S = \mathbf{1} + 2i\rho^{\frac{1}{2}} \cdot t \cdot \rho^{\frac{1}{2}}$$

$$t^{-1} = K^{-1} + I$$

$$\text{Im}I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\mathbf{1} + i\rho \cdot t (\mathbf{1} + i\mathcal{M}(L))] = 0$$

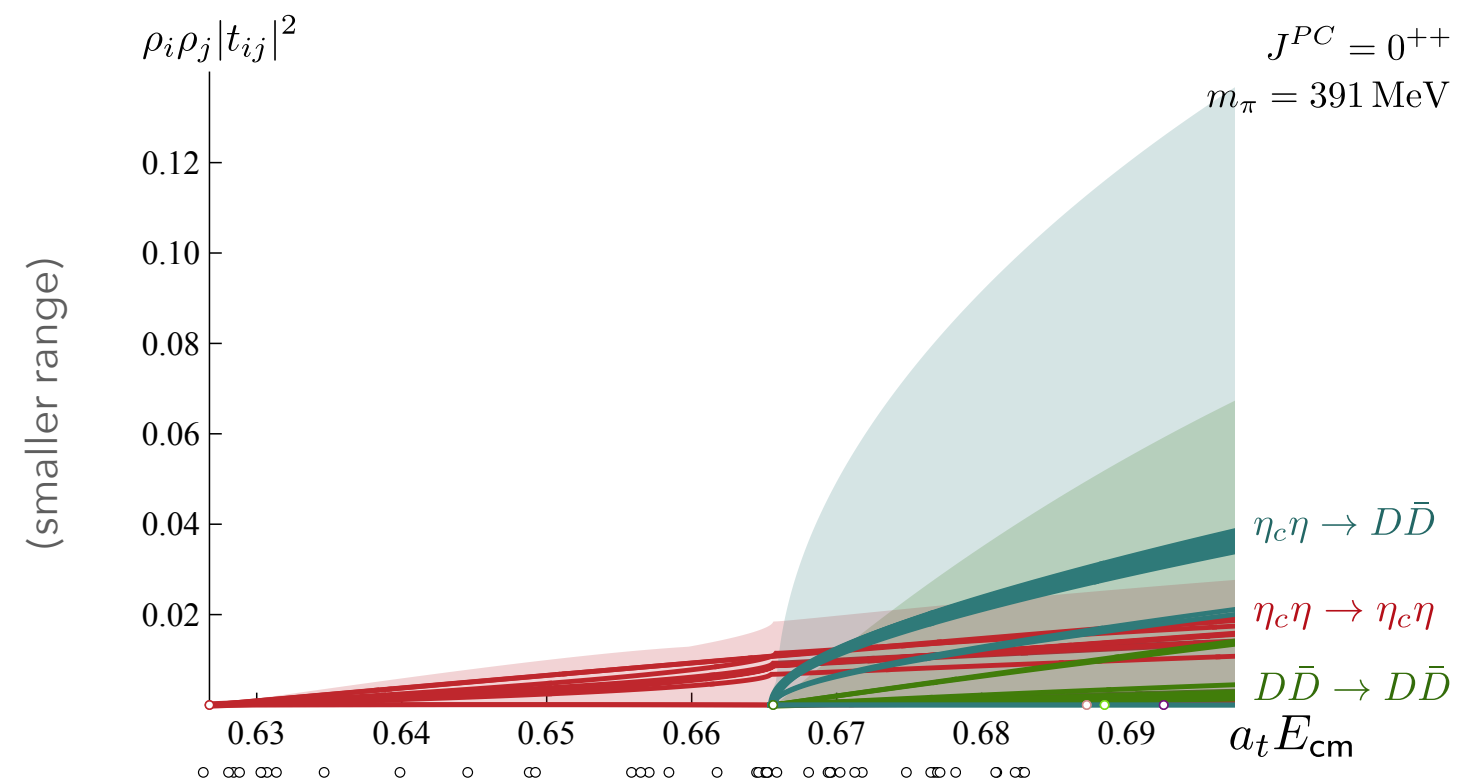
$$K = \begin{bmatrix} \gamma_{\eta_c\eta \rightarrow \eta_c\eta} & \gamma_{\eta_c\eta \rightarrow D\bar{D}} \\ \gamma_{\eta_c\eta \rightarrow D\bar{D}} & \gamma_{D\bar{D} \rightarrow D\bar{D}} \end{bmatrix}$$



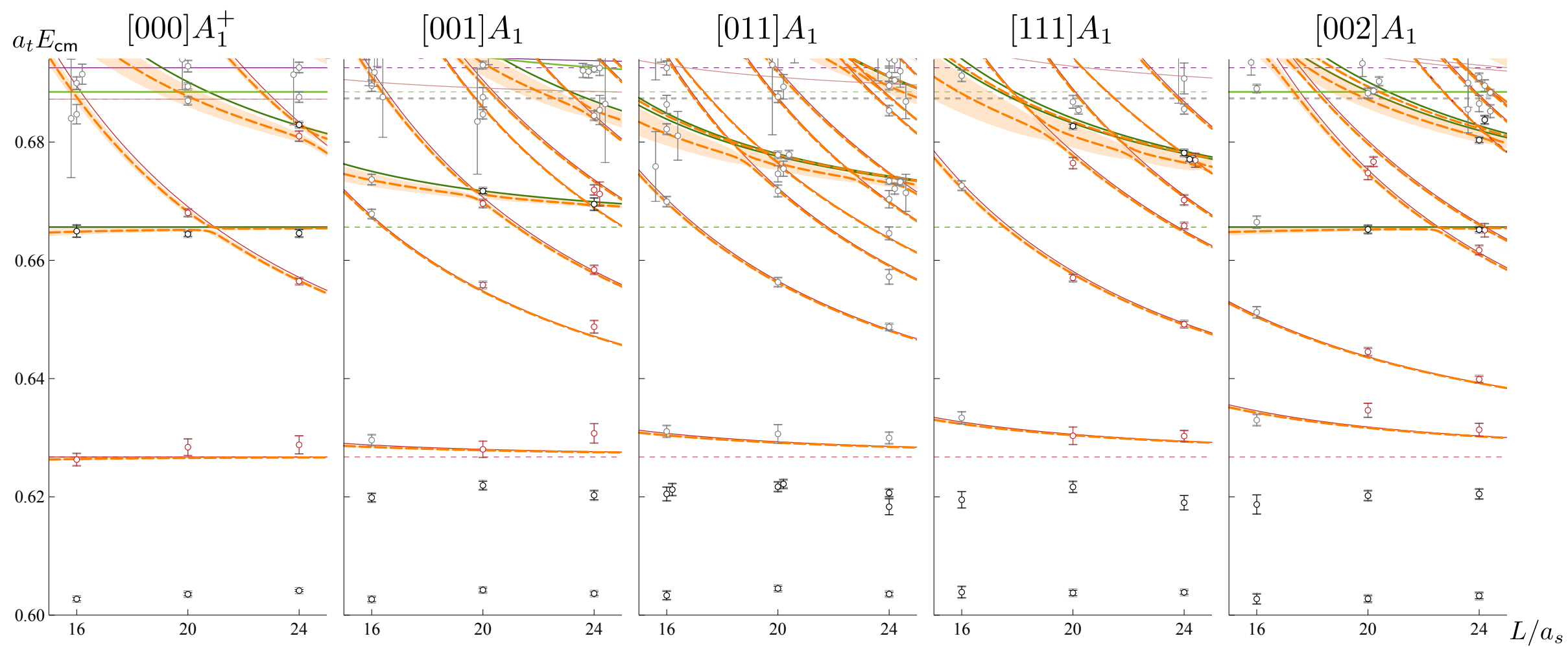
using rest-frame only

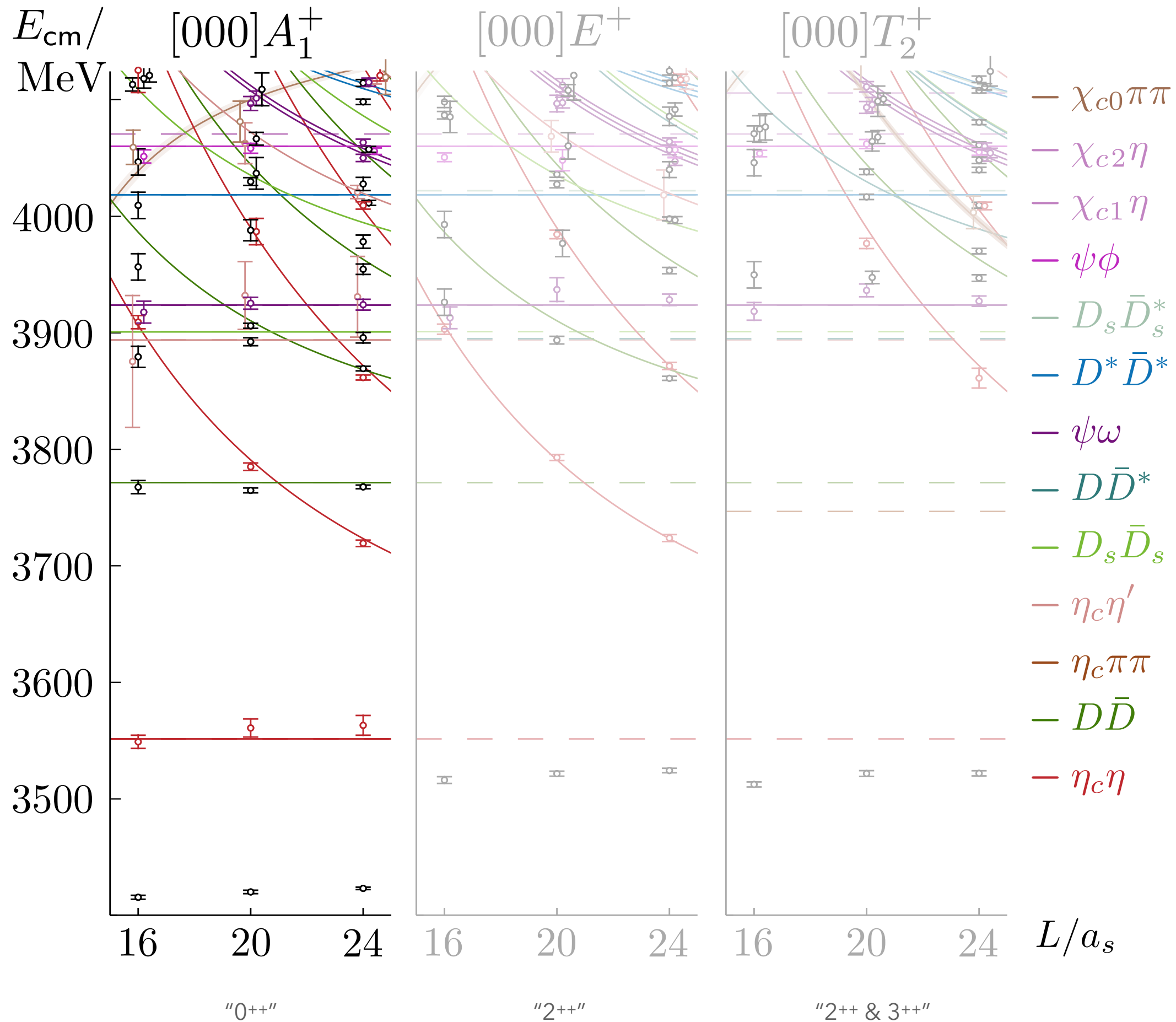
$$\begin{aligned} \gamma_{\eta_c\eta \rightarrow \eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta \rightarrow D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D} \rightarrow D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \quad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

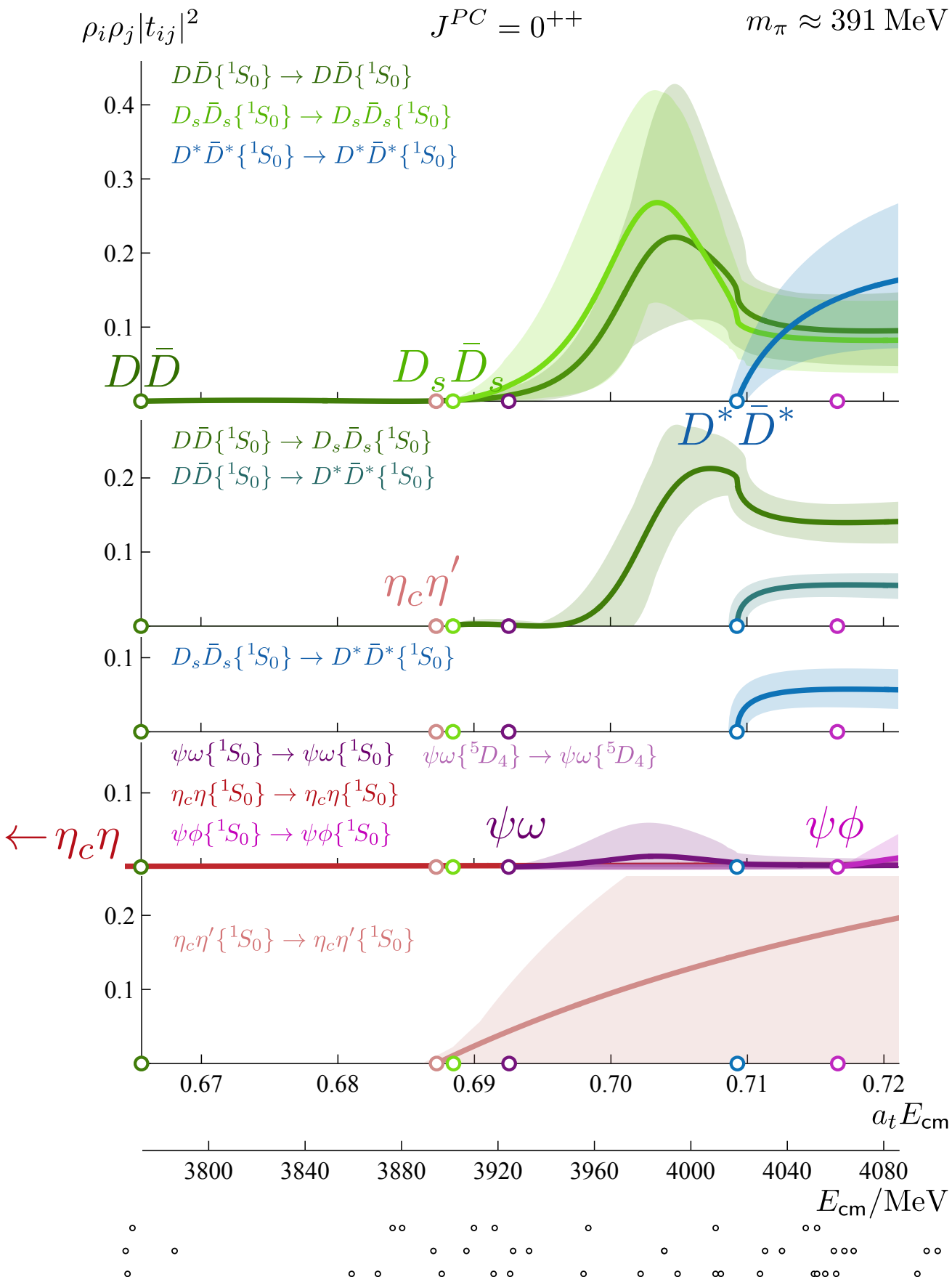
$$\chi^2/N_{\text{dof}} = \frac{5.65}{10-3} = 0.81$$



using zero and non-zero total momentum







three channels open close together:

$$\eta_c\eta', D_s\bar{D}_s, \psi\omega$$

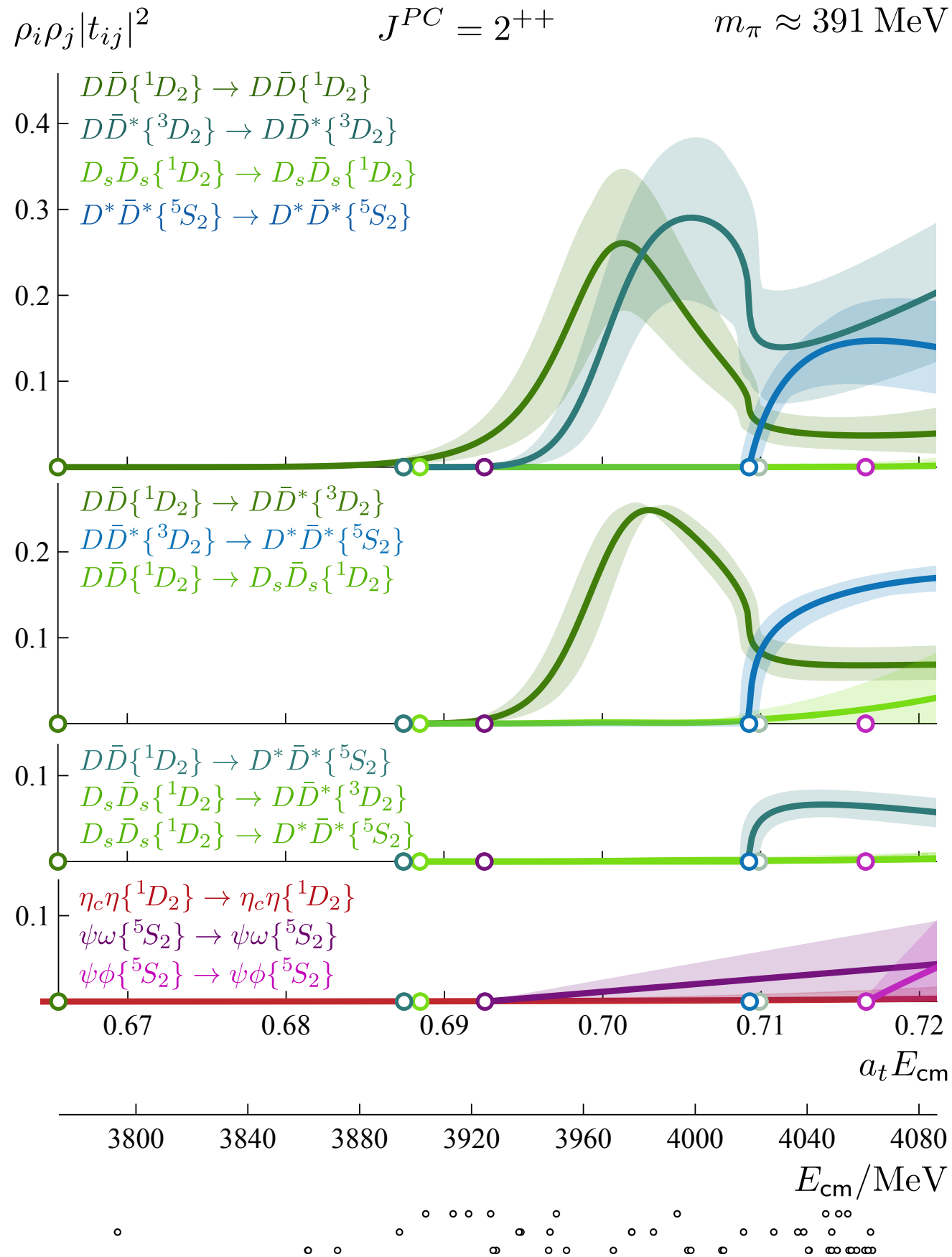
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

K-matrix pole terms become necessary to obtain a good quality of fit



7-channels, mixture of S and D

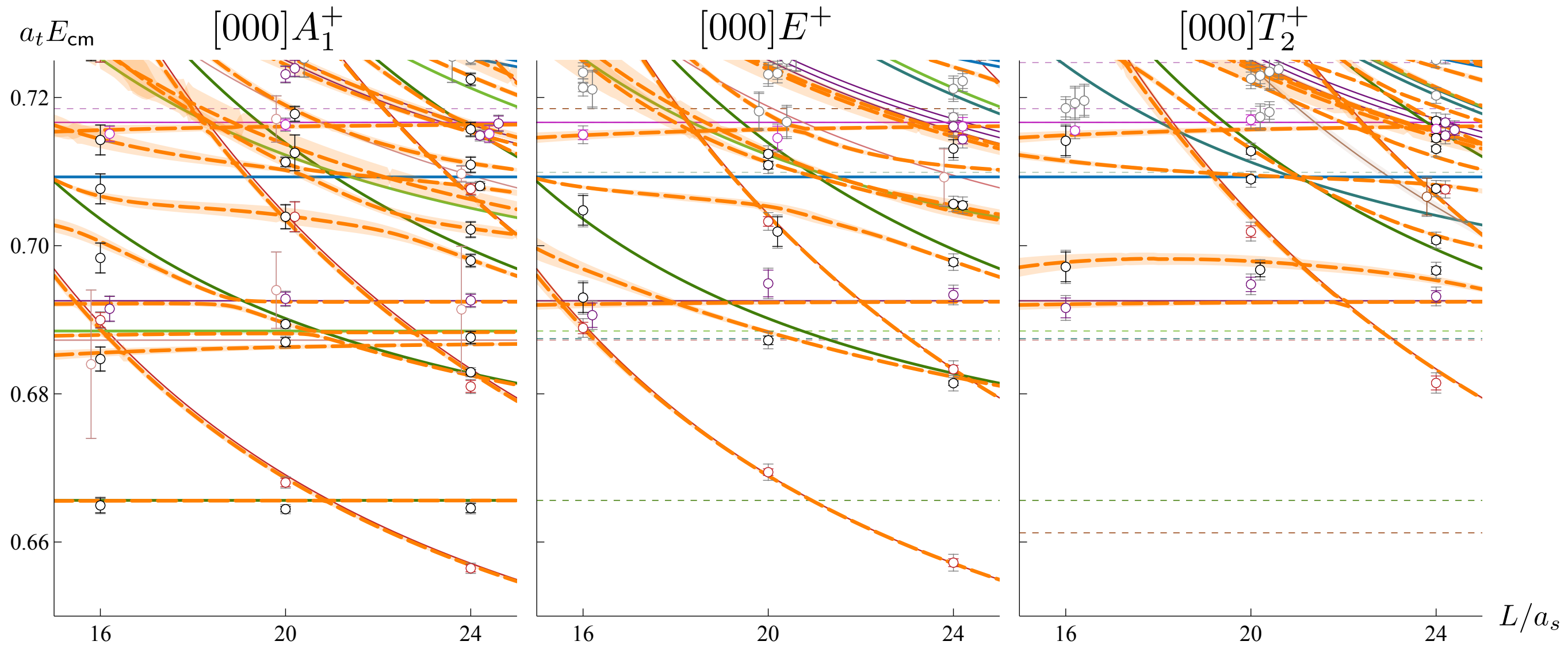
$$DD\bar{D}, D_s\bar{D}_s\{^1D_2\} \quad DD\bar{D}^*\{^3D_2\} \quad D^*\bar{D}^*\{^5S_2\}$$

$$\eta_c\eta\{^1D_2\} \quad \psi\omega, \psi\phi\{^5S_2\}$$

peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

DD* is large -
similar phase space to DsDs



$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \mathbf{t} (\mathbf{1} + i\mathcal{M}(L))] = 0$$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

$$\sqrt{s_{\text{pole}}} = m - \frac{i}{2}\Gamma$$

Sign Im k_i

$$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[+])$$

$$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[-])$$

**Physical scattering at
real energies**

**Common pole influences
both amplitudes**

Branch point

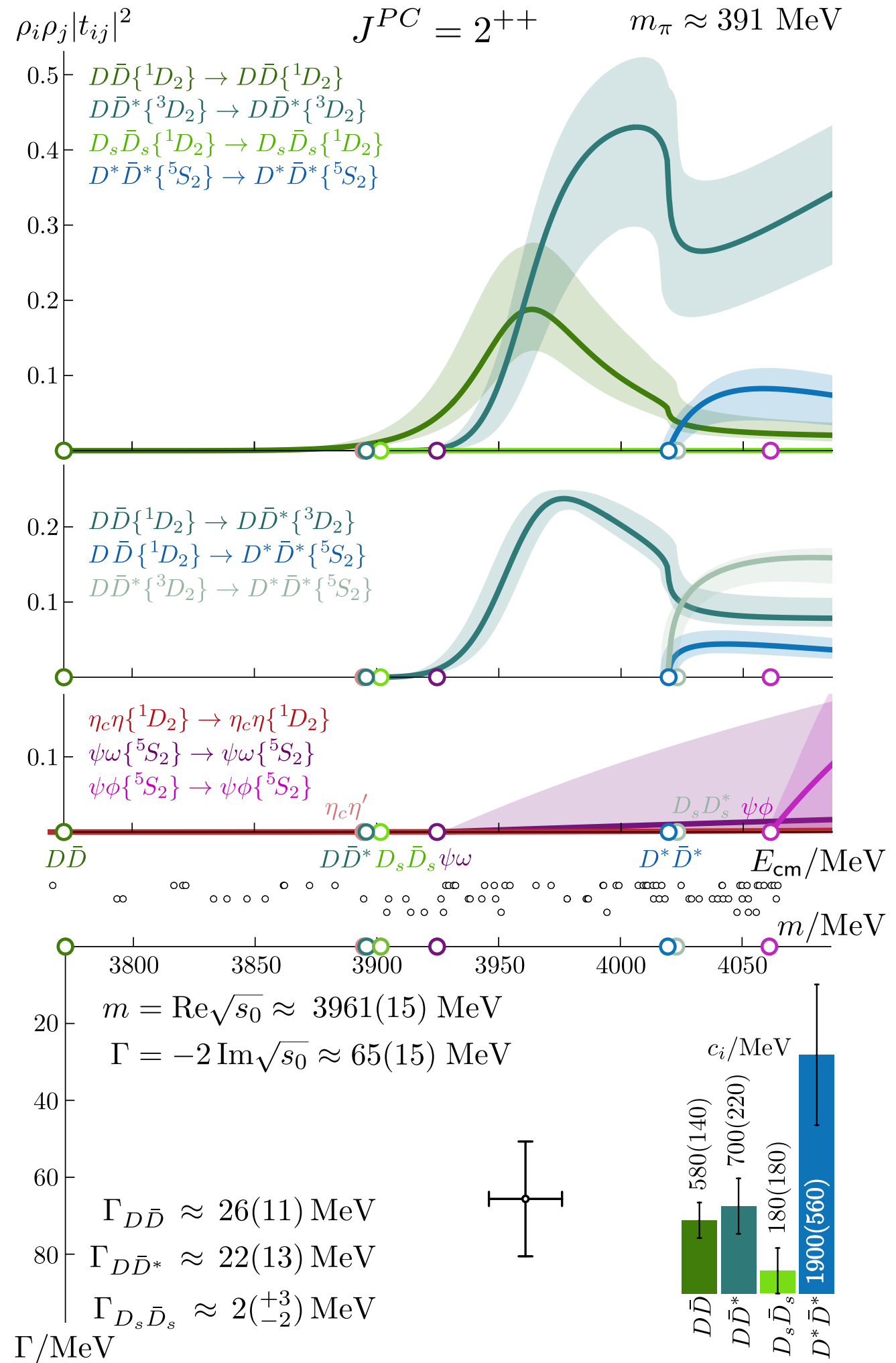
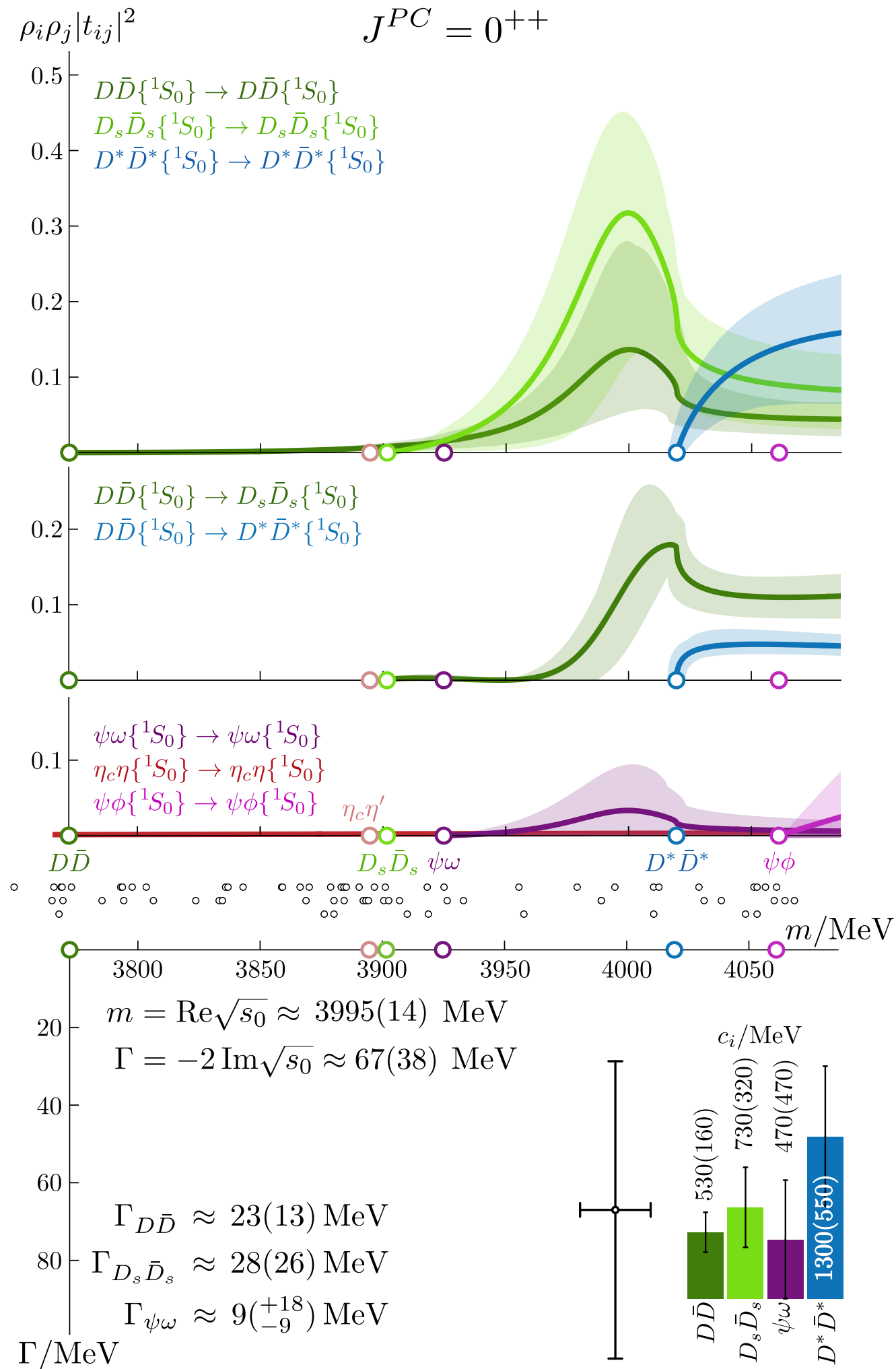
$$D^*\bar{D}^*|_{\text{thr.}}$$

$$E_{\text{cm}} = \text{Re}\sqrt{s}$$

$$\Gamma = 2\text{Im}\sqrt{s}$$

$$D\bar{D} \rightarrow D_s\bar{D}_s$$

$$D_s\bar{D}_s \rightarrow D_s\bar{D}_s$$

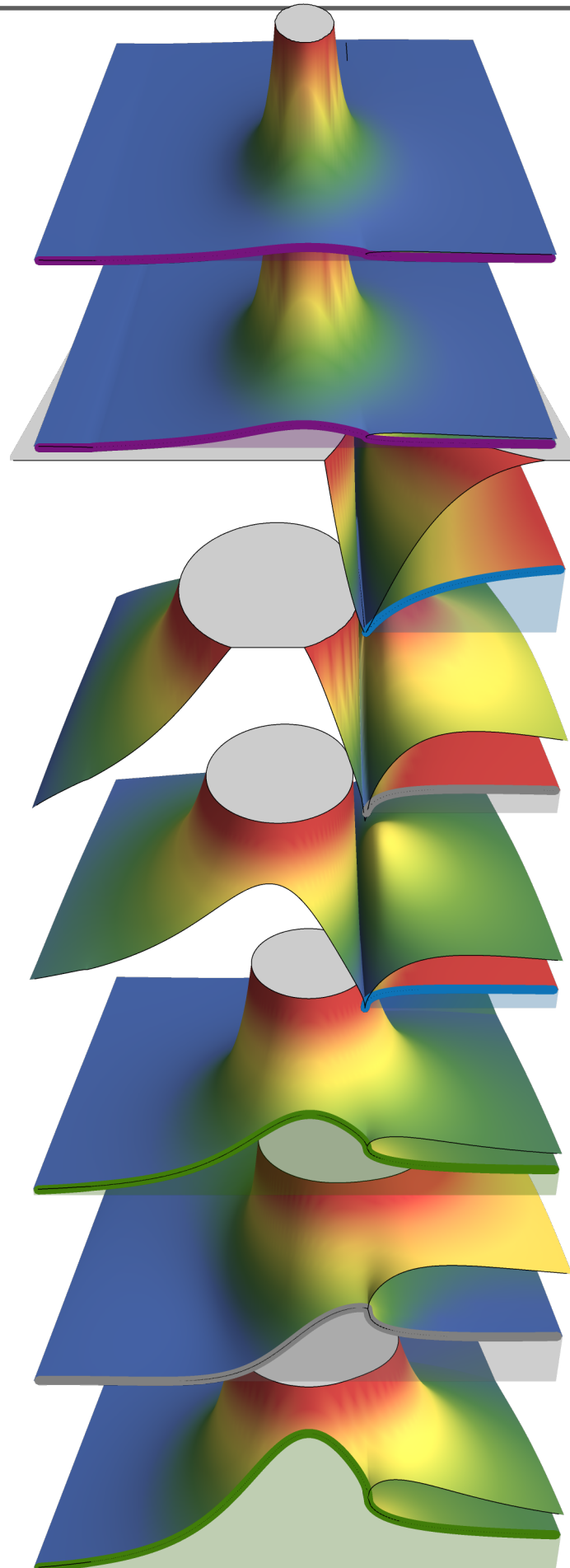


Scalar and tensor charmonium

- at $m_\pi=391$ MeV, one scalar and one tensor pole is found.
- The **level counting** is not obviously different from the **quark model**
- large **coupled-channel** effects in OZI **connected D-meson channels**
- OZI **disconnected** channels look **small everywhere**
- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture
- some amplitudes are **very different** to the simple **Breit-Wigners** often used in experimental analyses
- a clear, as yet unobserved, 3^{++} resonance is present in $D\bar{D}^*$ & a bound state in 2^{-+}
- we **do not find** a **near-threshold $D\bar{D}^*$** state (between 3700 and 3860 MeV)
- these methods can also be applied to the $X(3872)$ 1^{++} channel

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

one resonance pole
— many different amplitudes



$$J/\psi\omega \rightarrow J/\psi\omega$$

$$D\bar{D} \rightarrow J/\psi\omega$$

$$D^*\bar{D}^* \rightarrow D^*\bar{D}^*$$

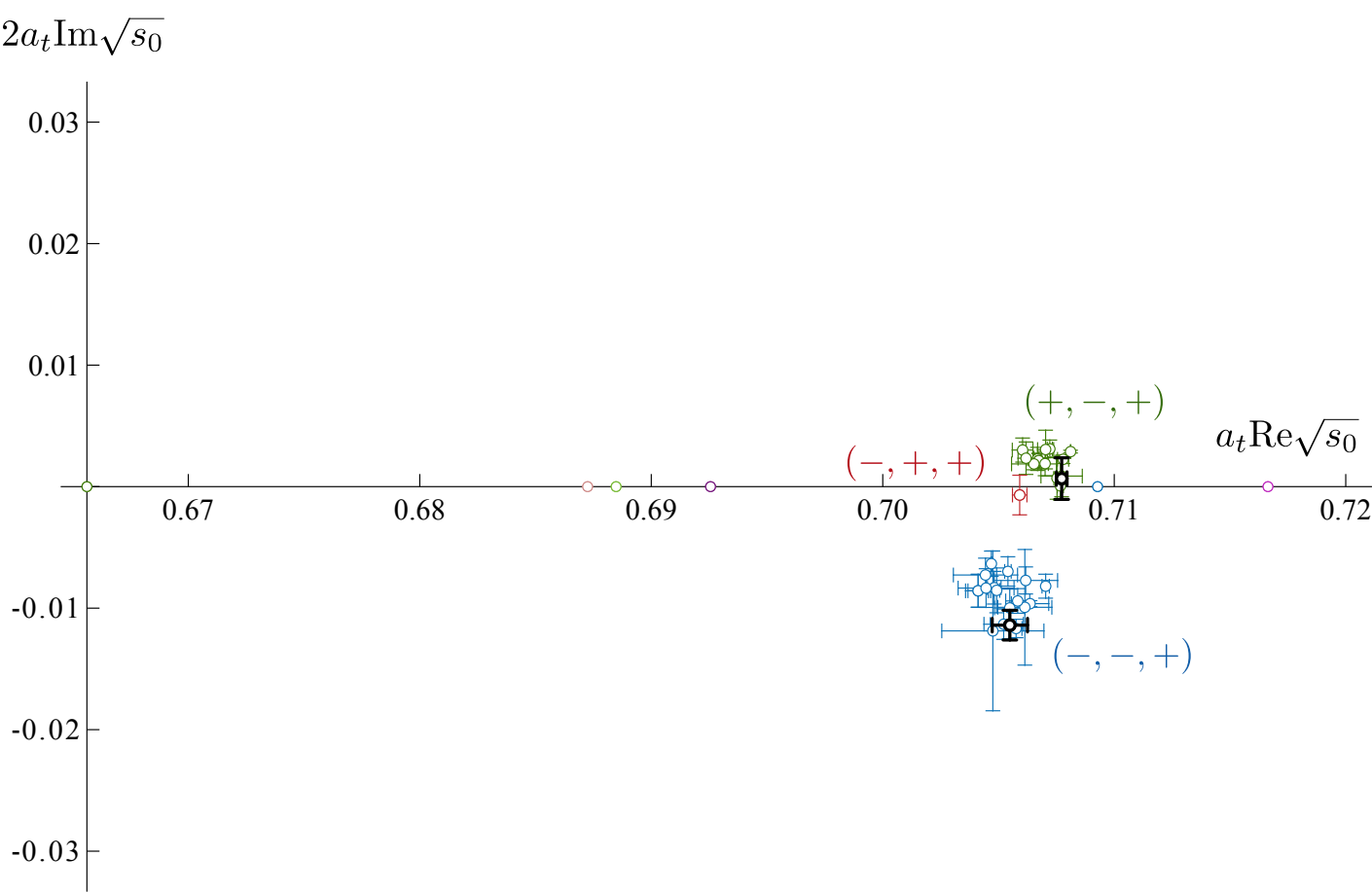
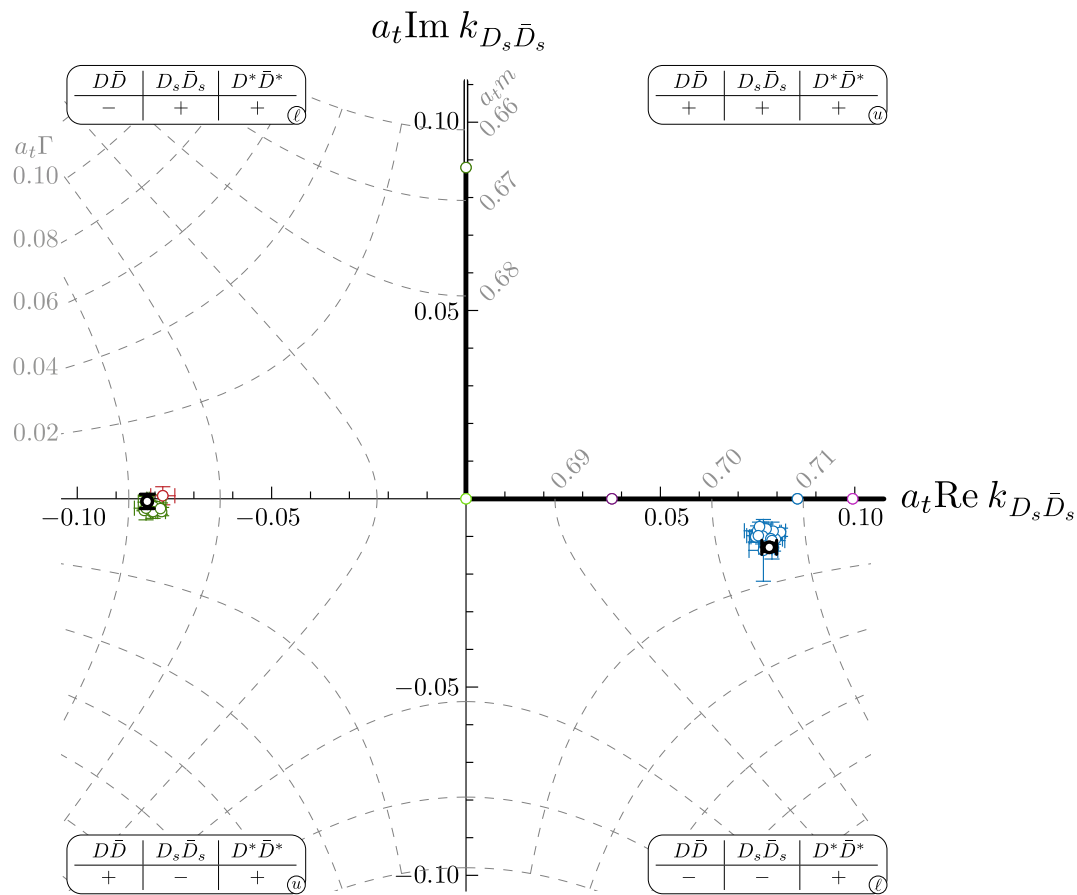
$$D_s\bar{D}_s \rightarrow D^*\bar{D}^*$$

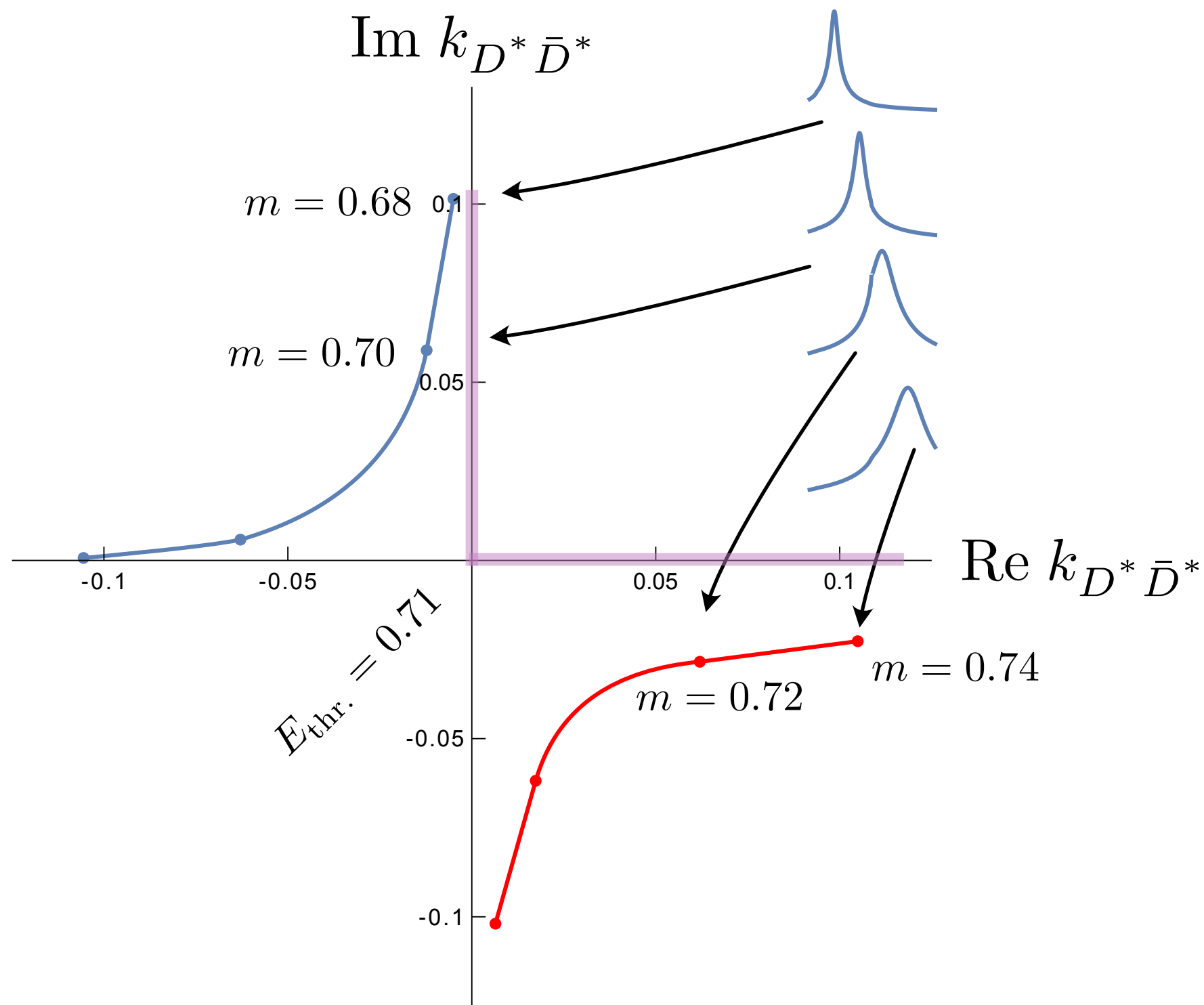
$$D\bar{D} \rightarrow D^*\bar{D}^*$$

$$D\bar{D} \rightarrow D\bar{D}$$

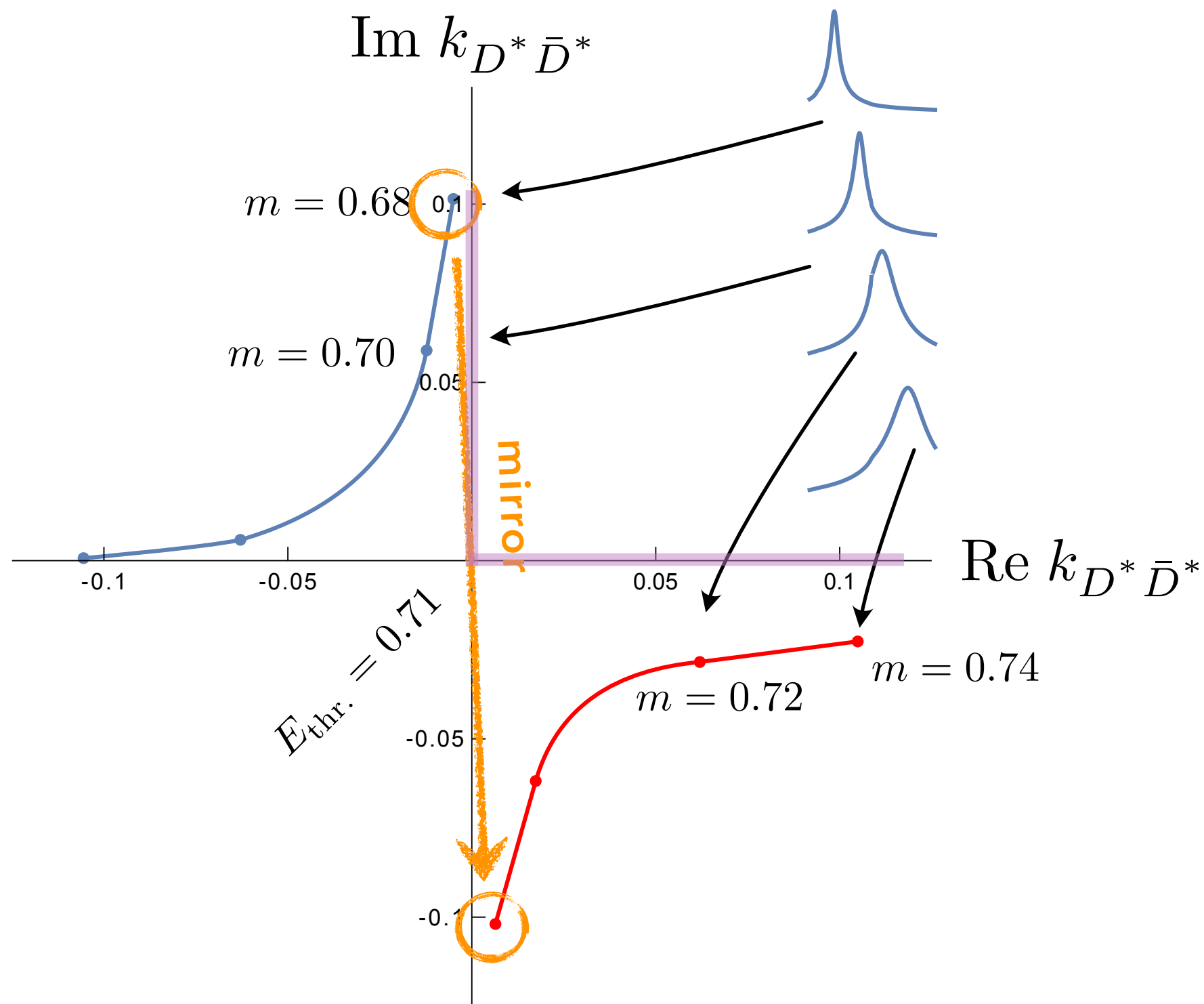
$$D\bar{D} \rightarrow D_s\bar{D}_s$$

$$D_s\bar{D}_s \rightarrow D_s\bar{D}_s$$

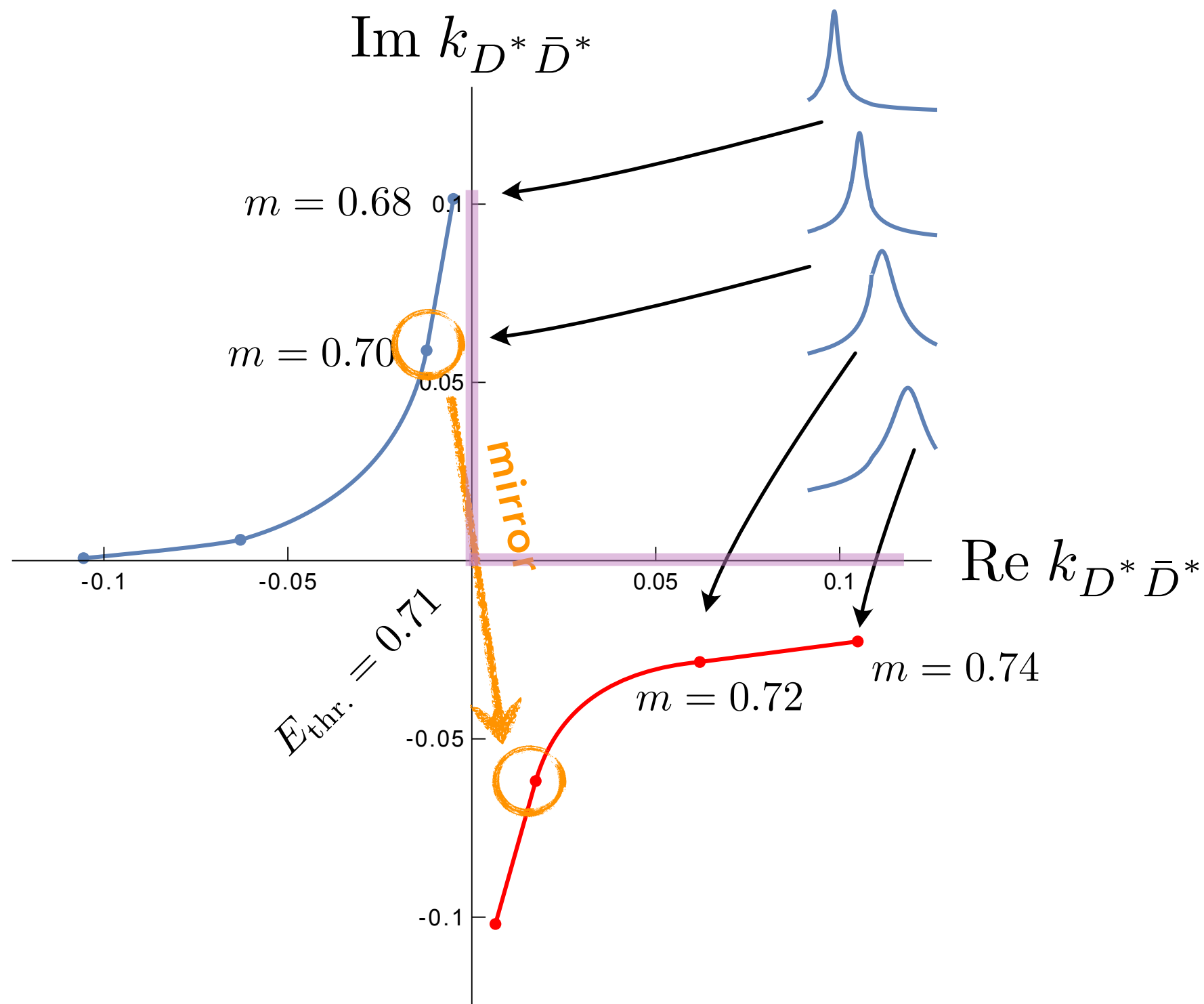




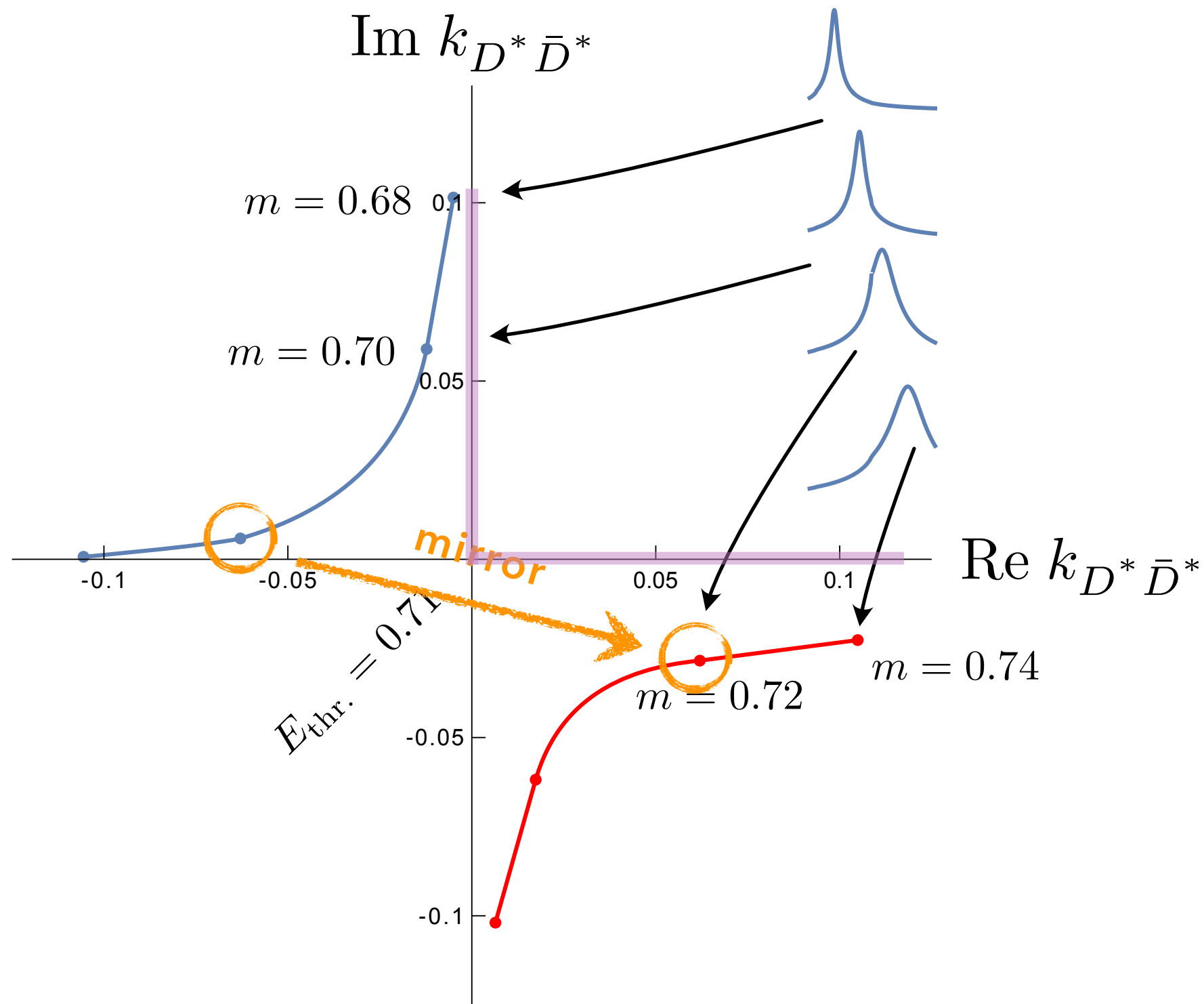
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^* \bar{D}^*}^2 \rho_{D^* \bar{D}^*}}$$



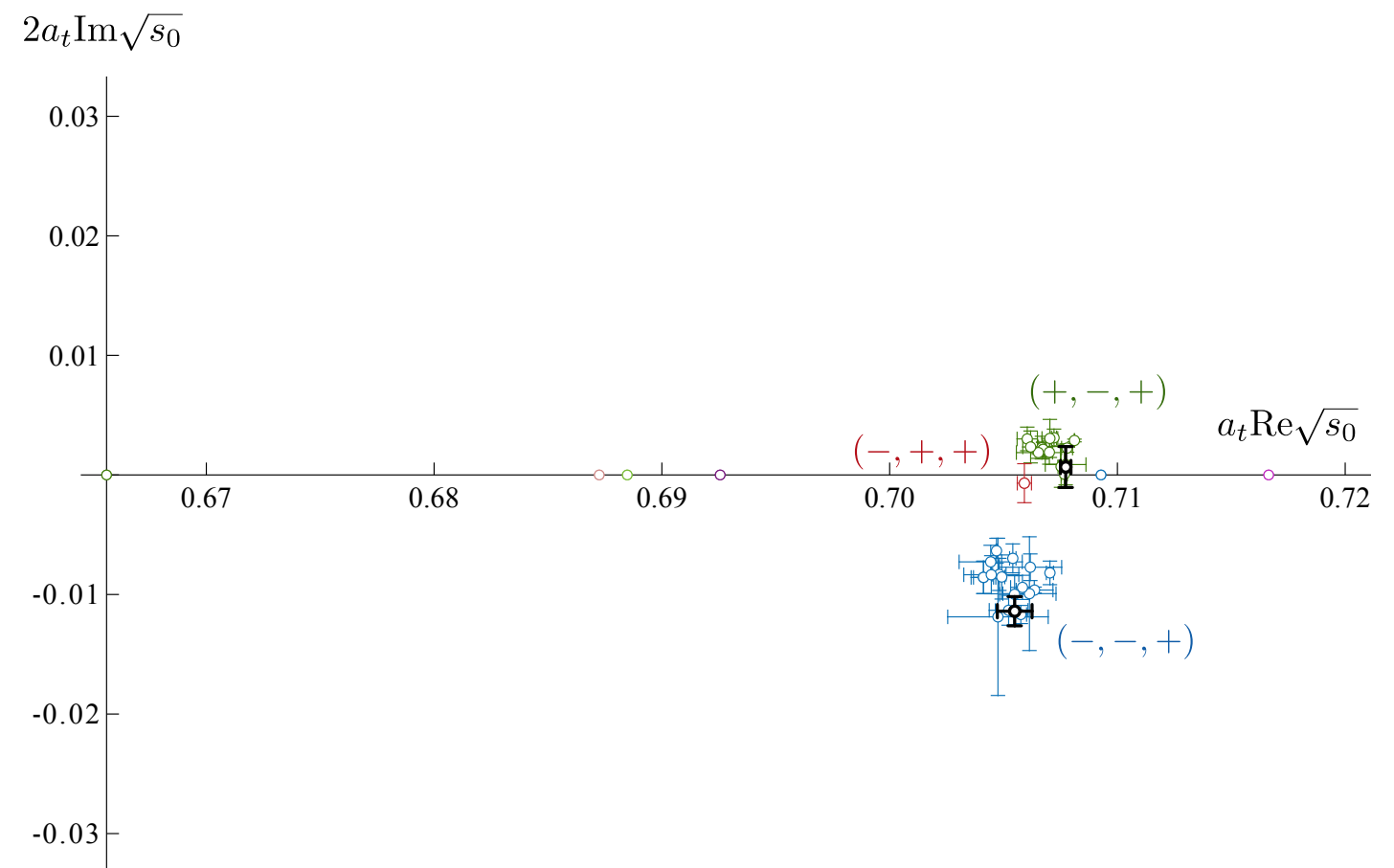
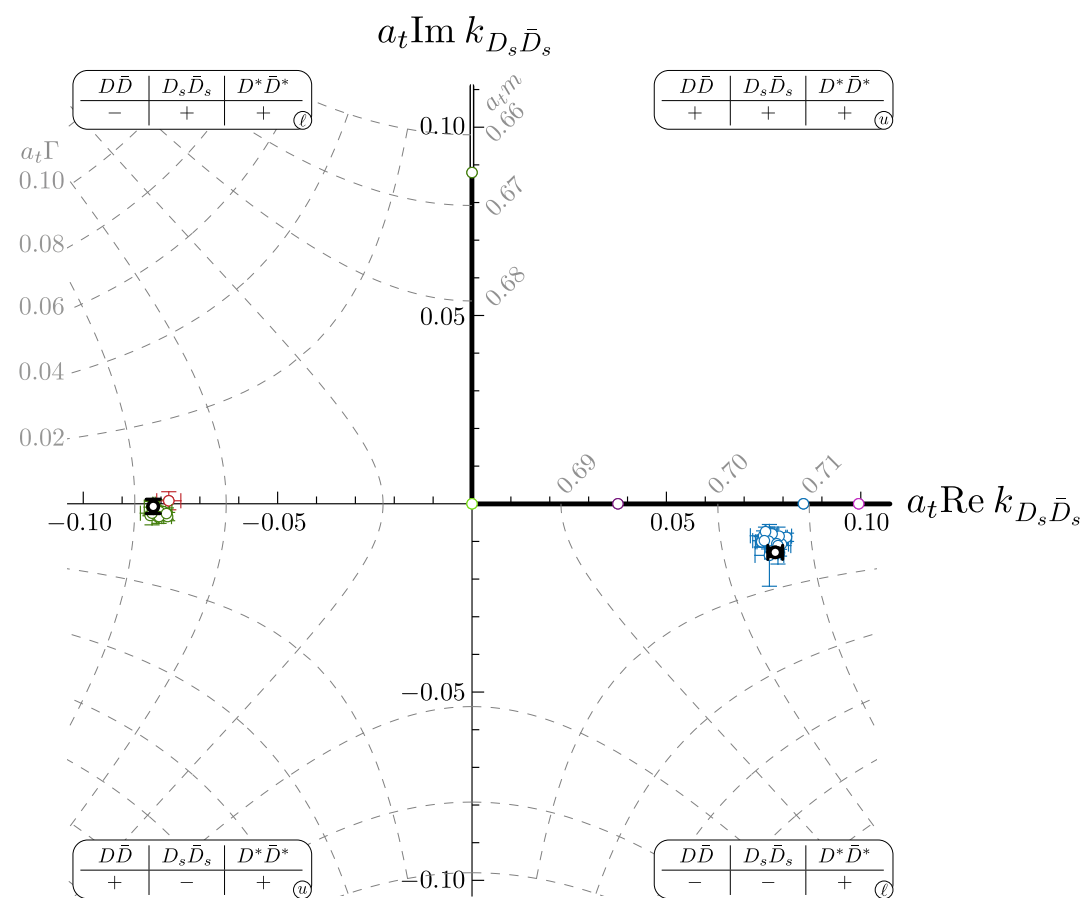
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$

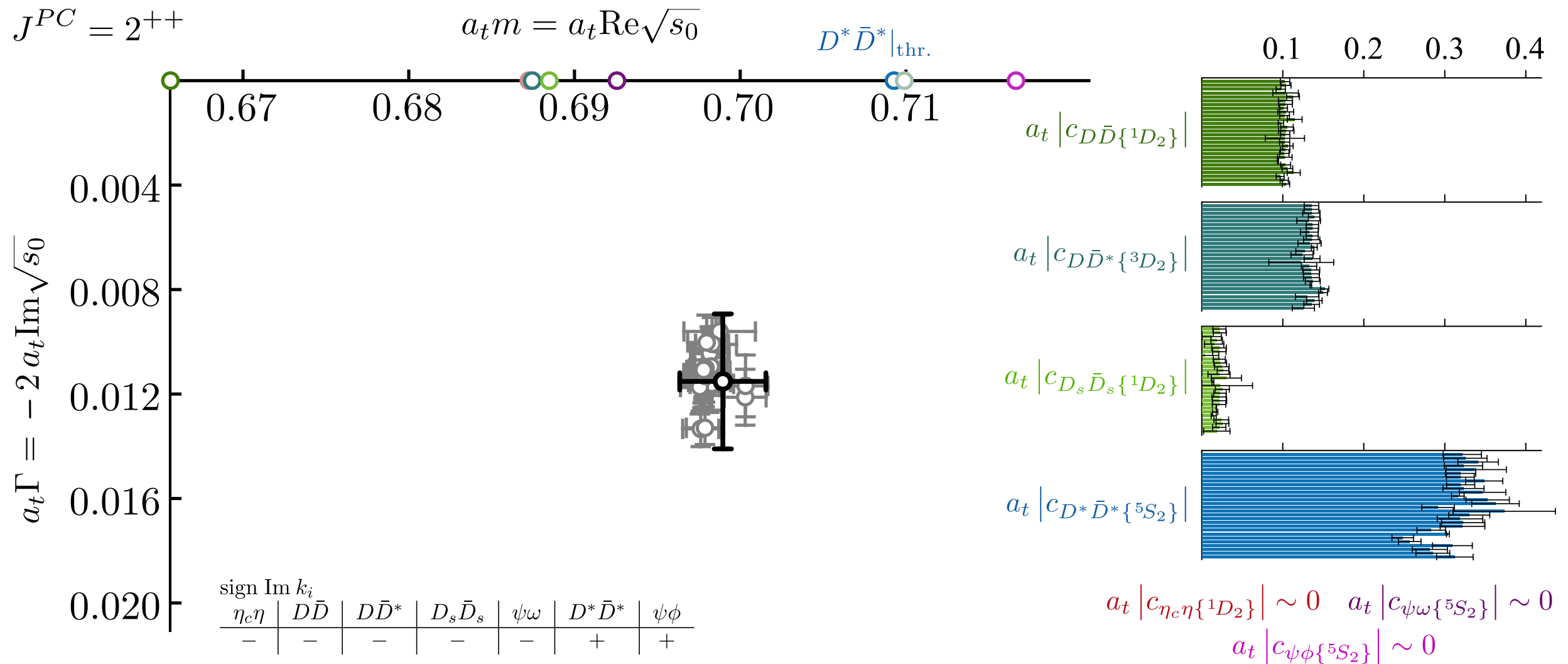


$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^* \bar{D}^*}^2 \rho_{D^* \bar{D}^*}}$$



the “green” cluster of poles are just mirror poles

- amplitude is **dominated by a single resonance pole** in this energy region

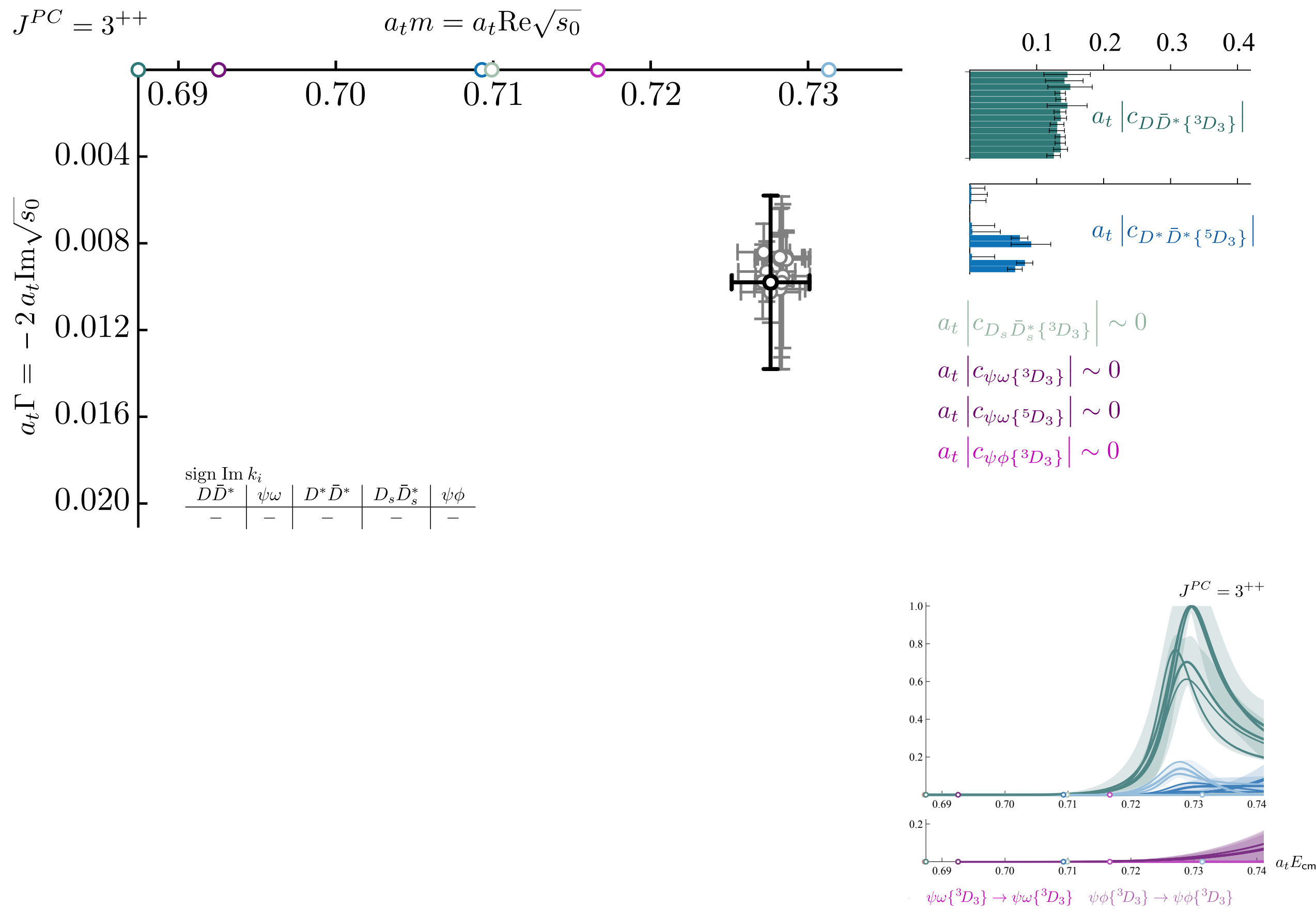


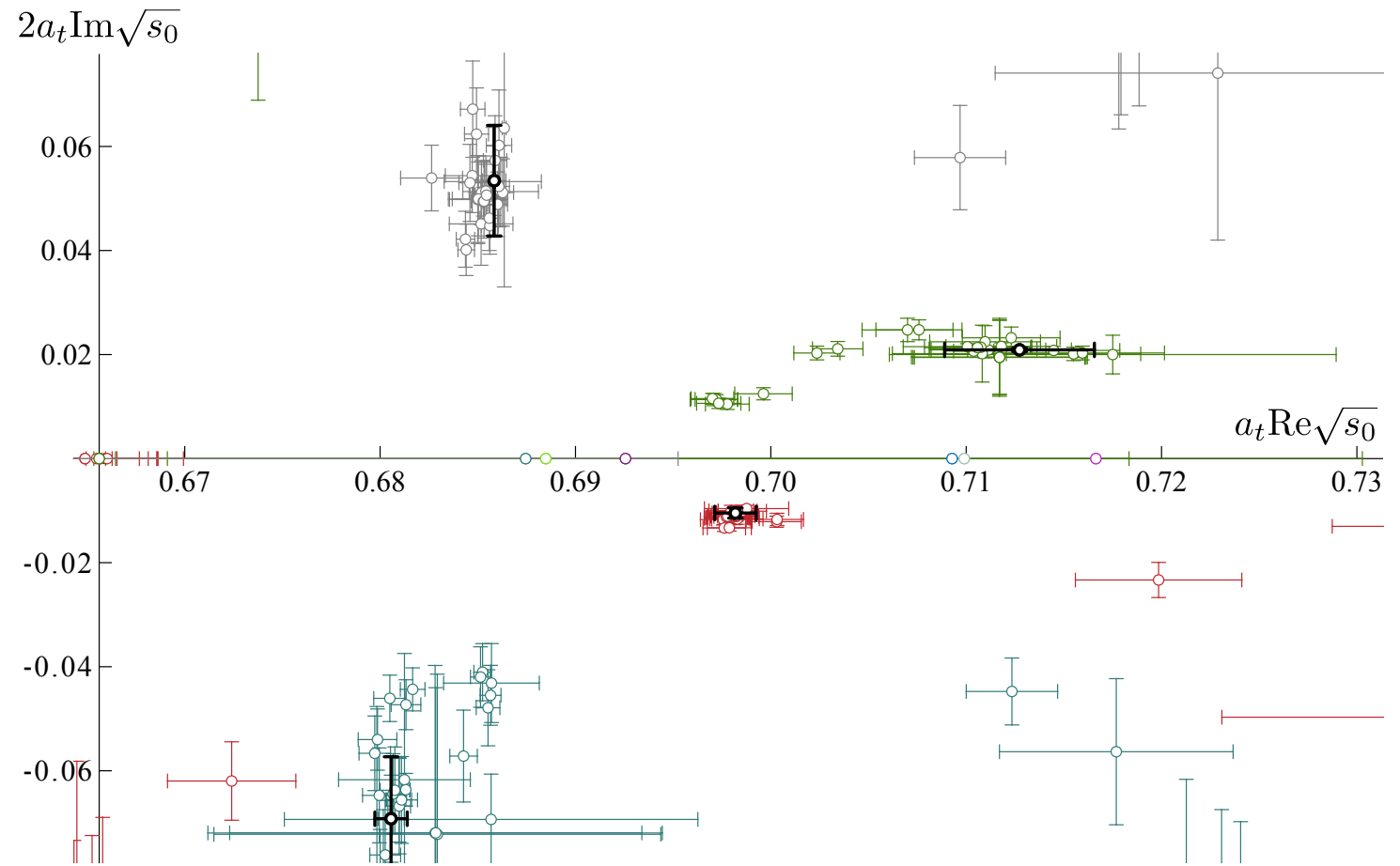
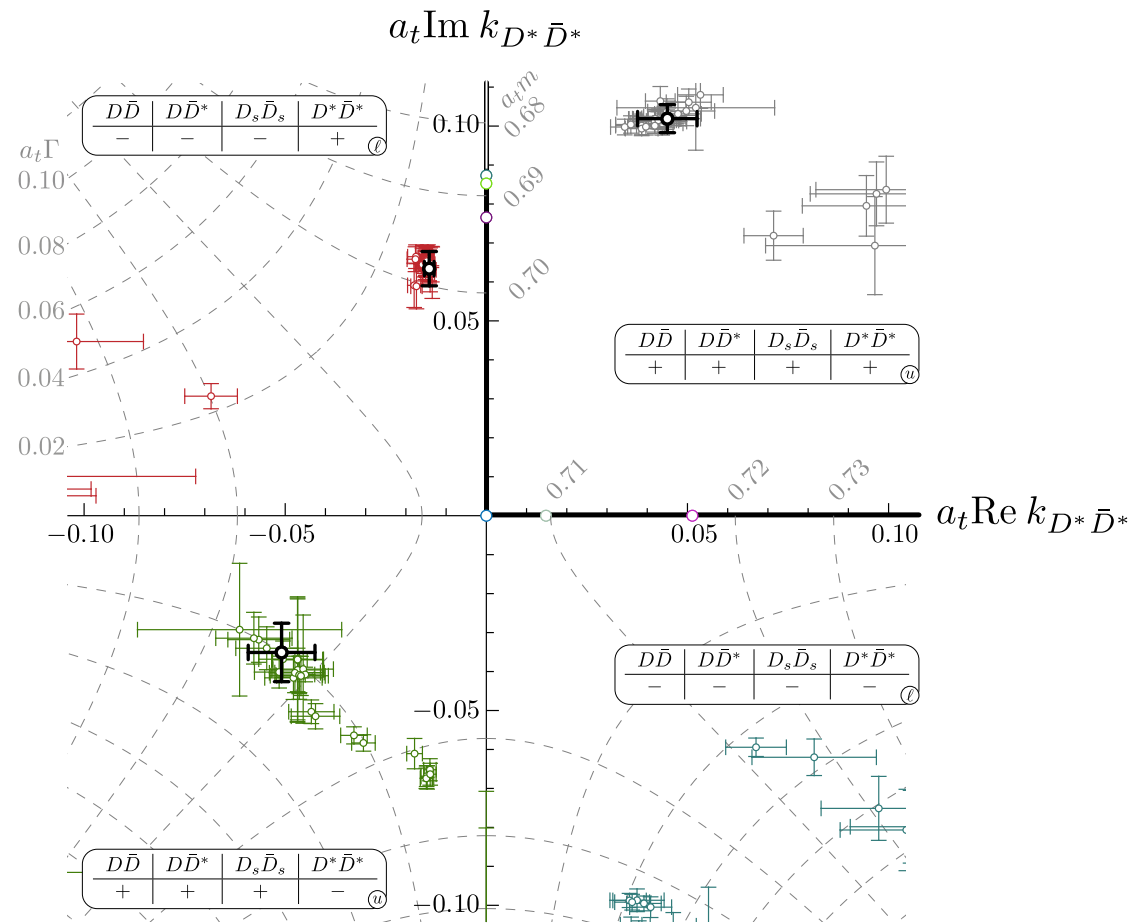
additional poles were found

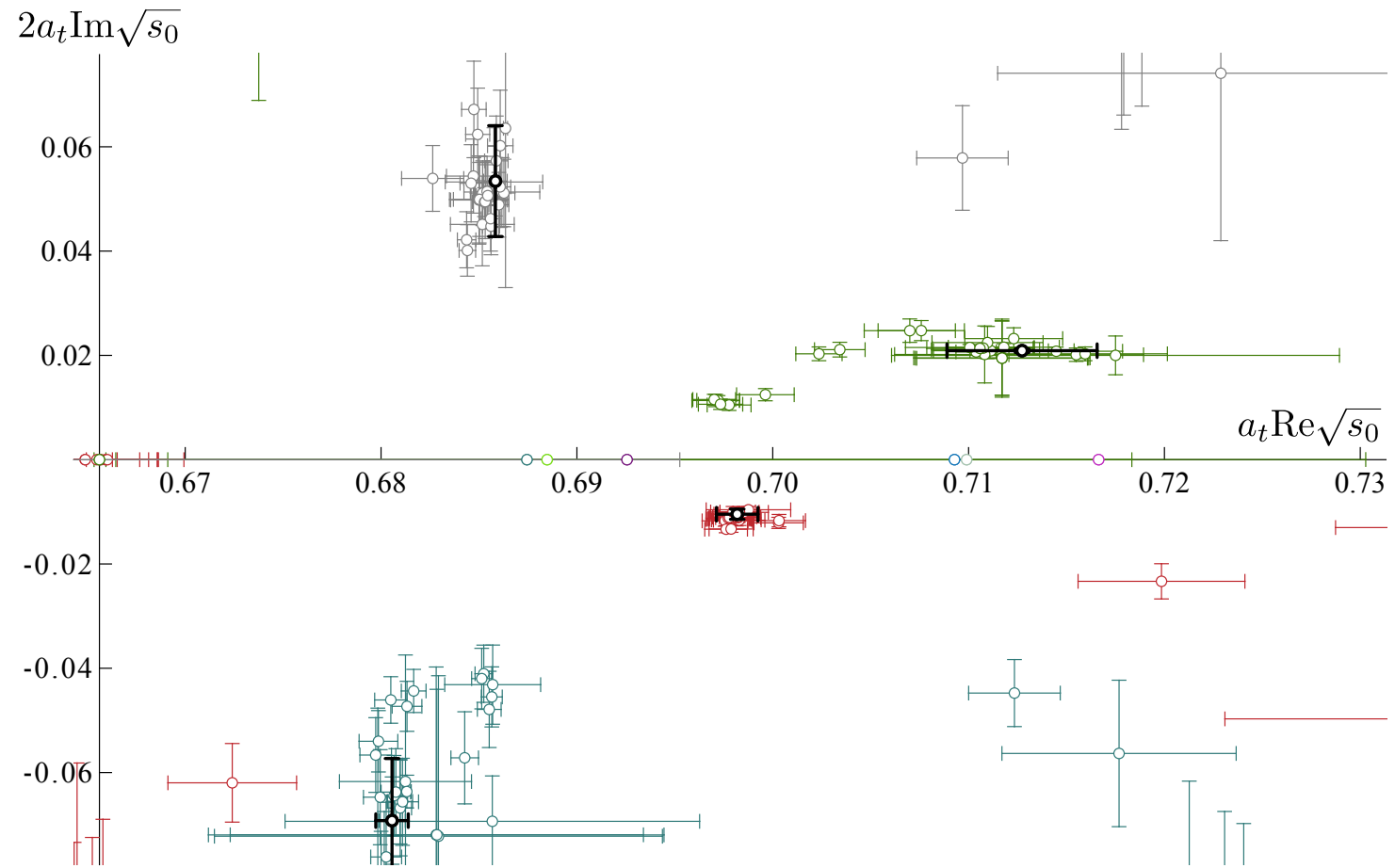
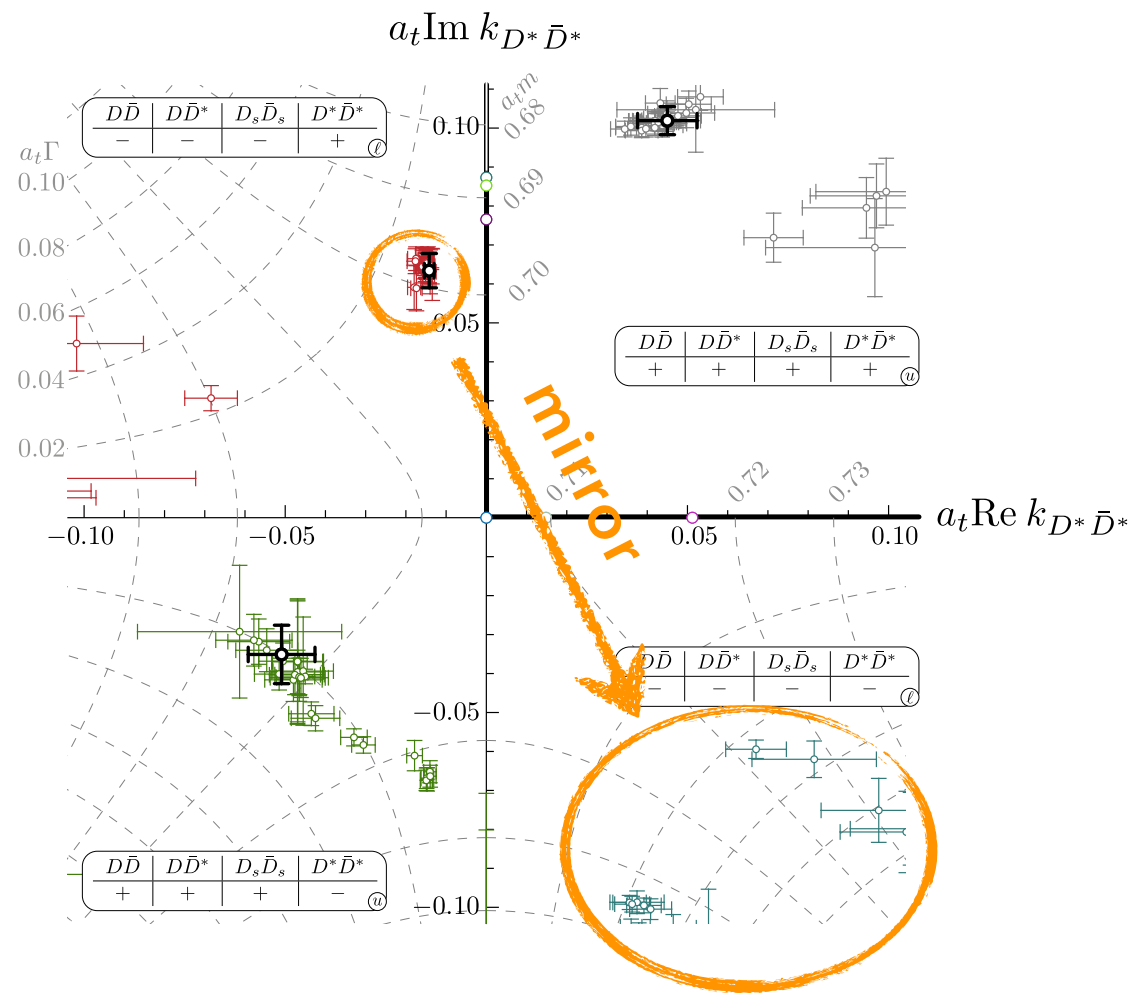
- don't appear to be important

"coupling-ratio" phenomena seen in K-matrix pole parameters

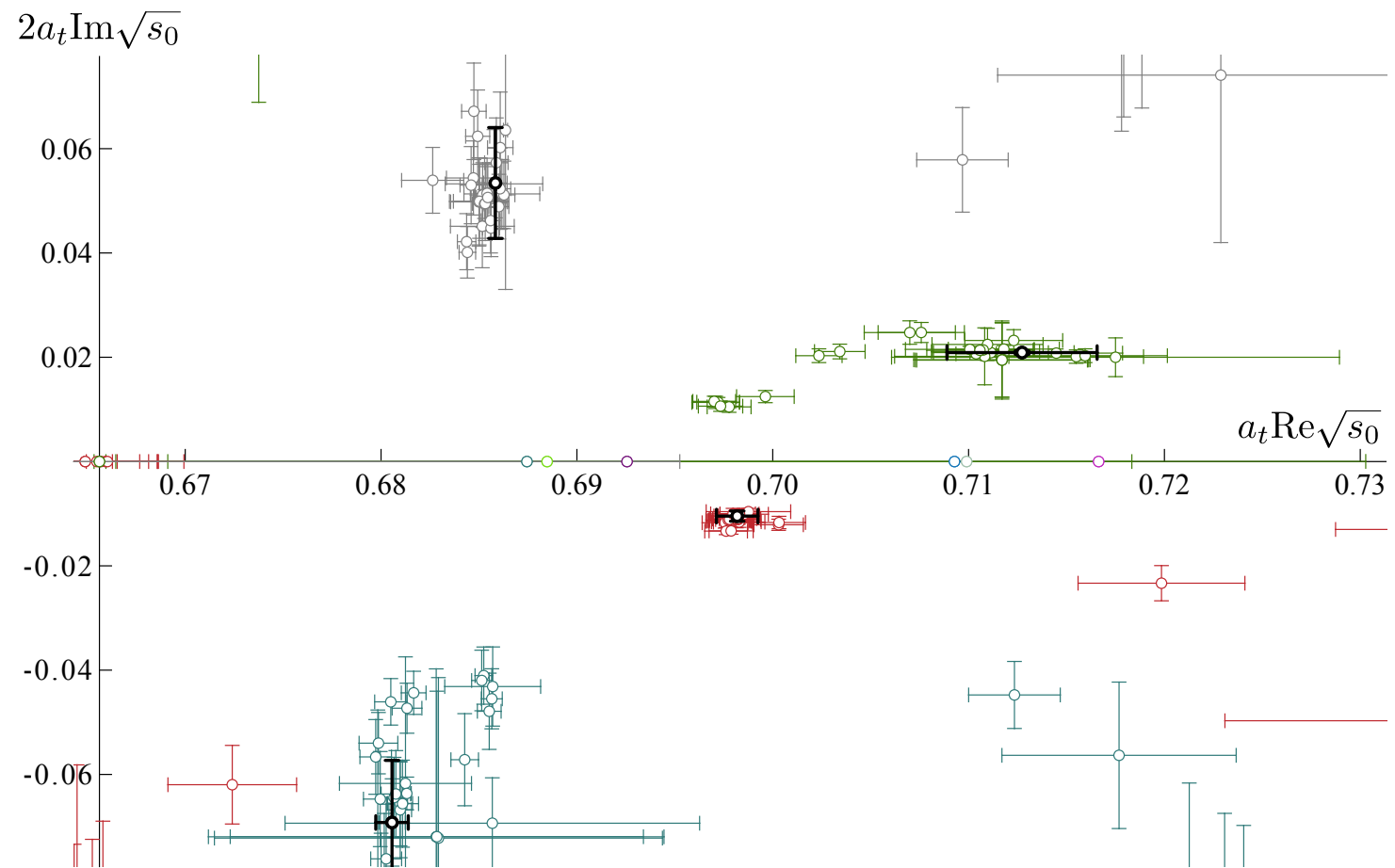
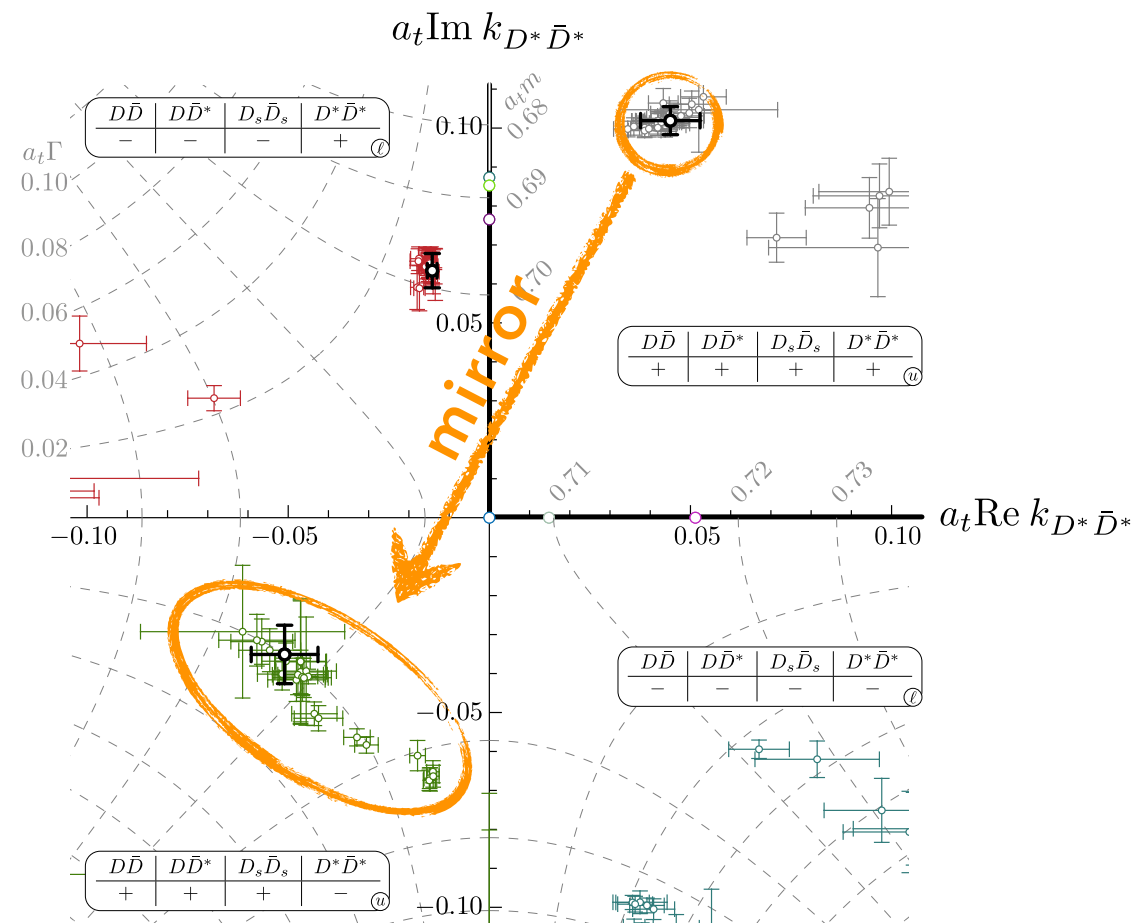
- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined





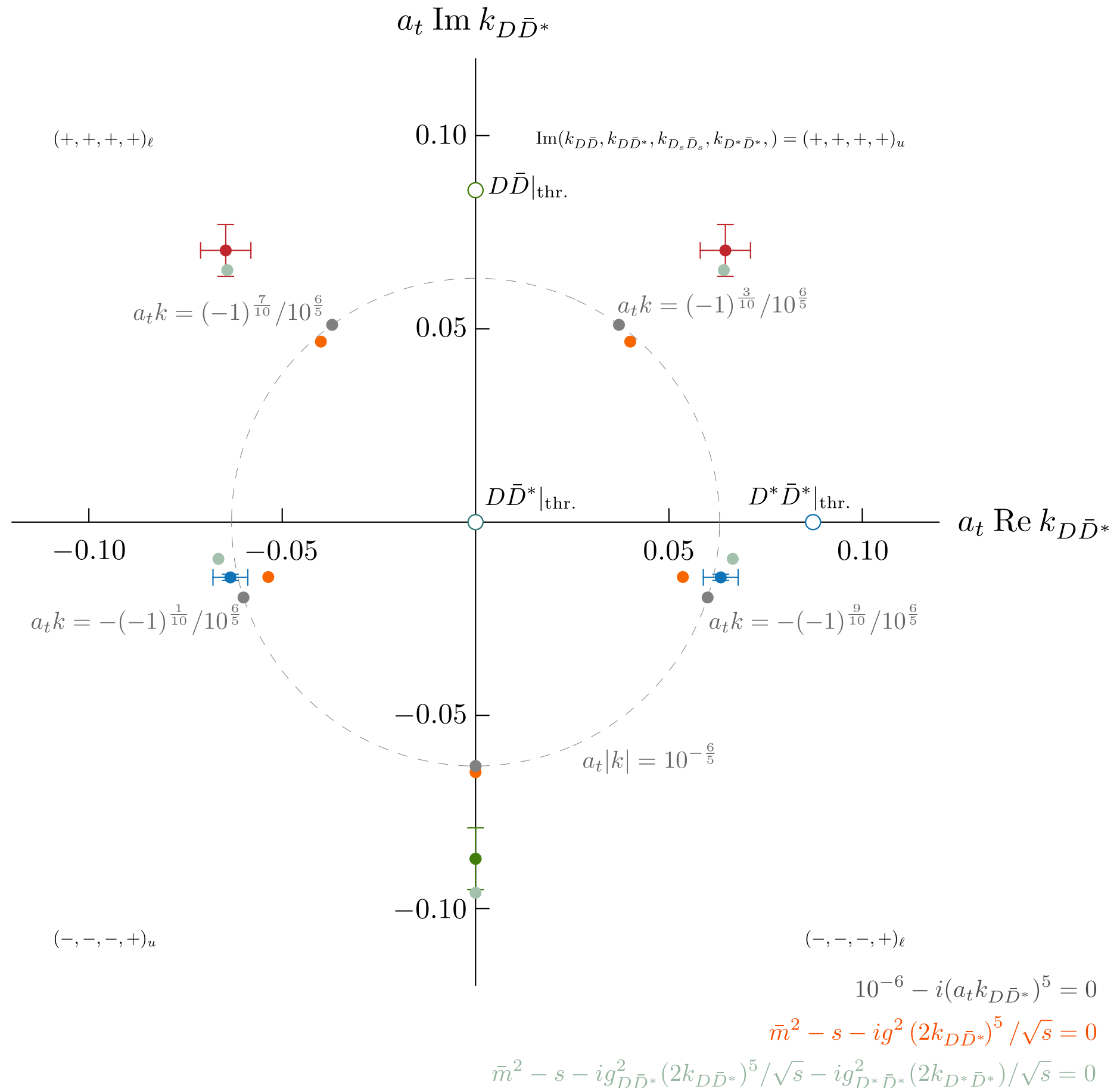


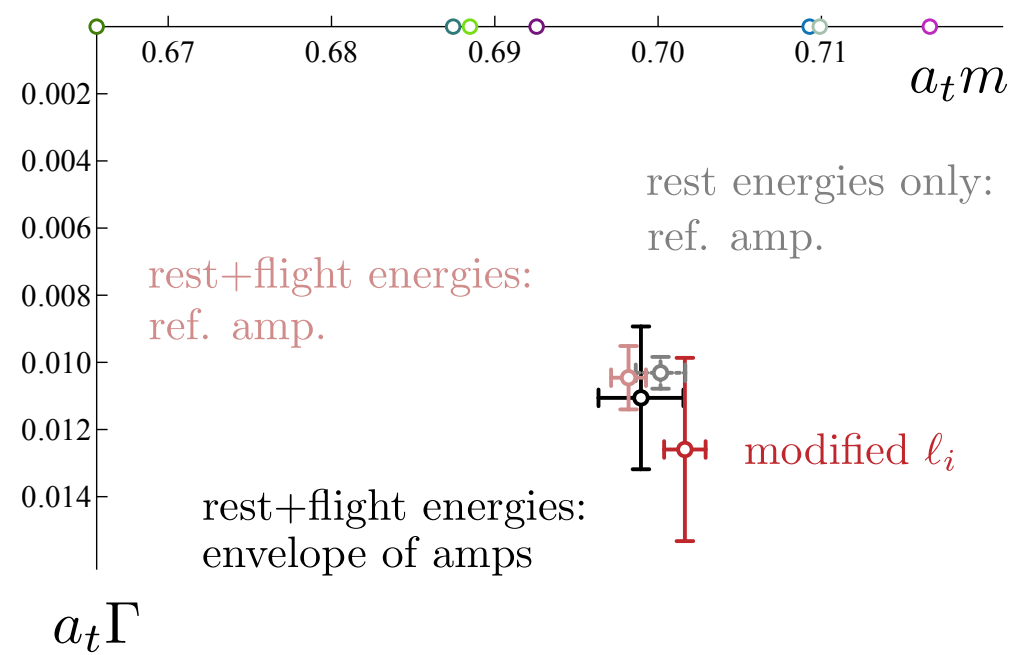
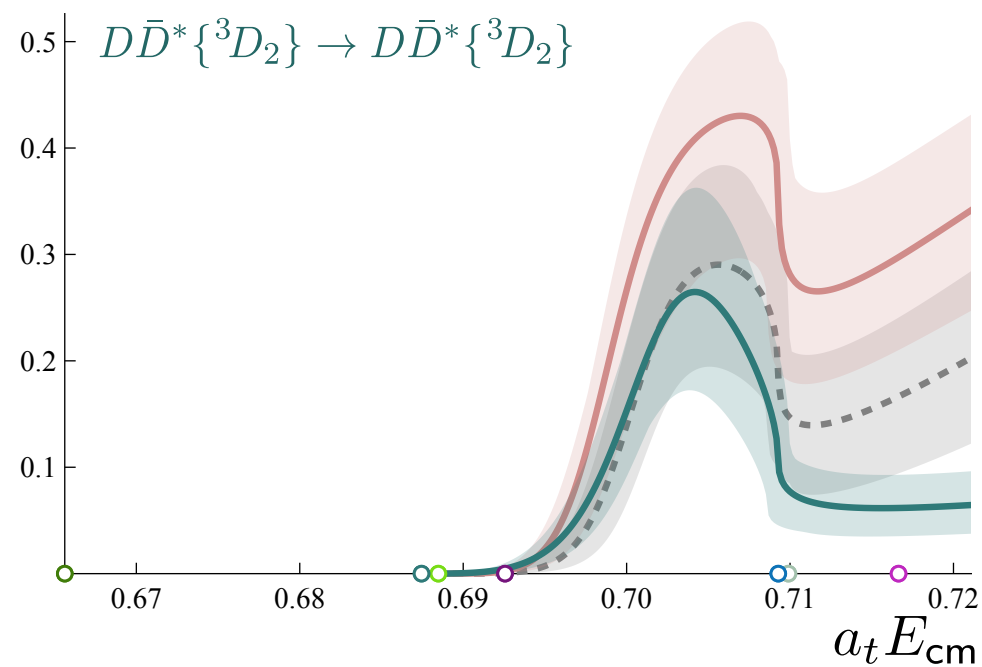
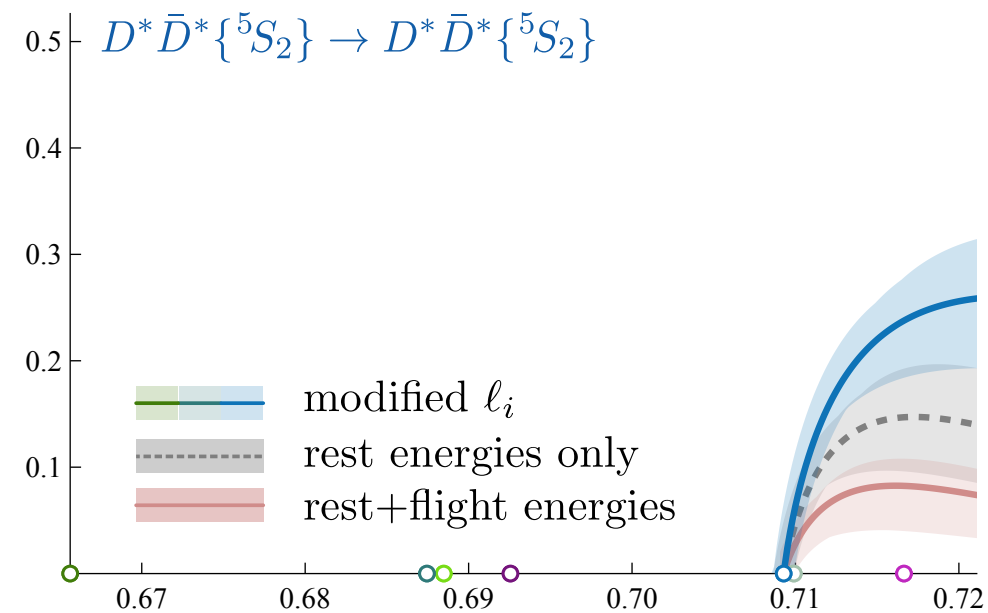
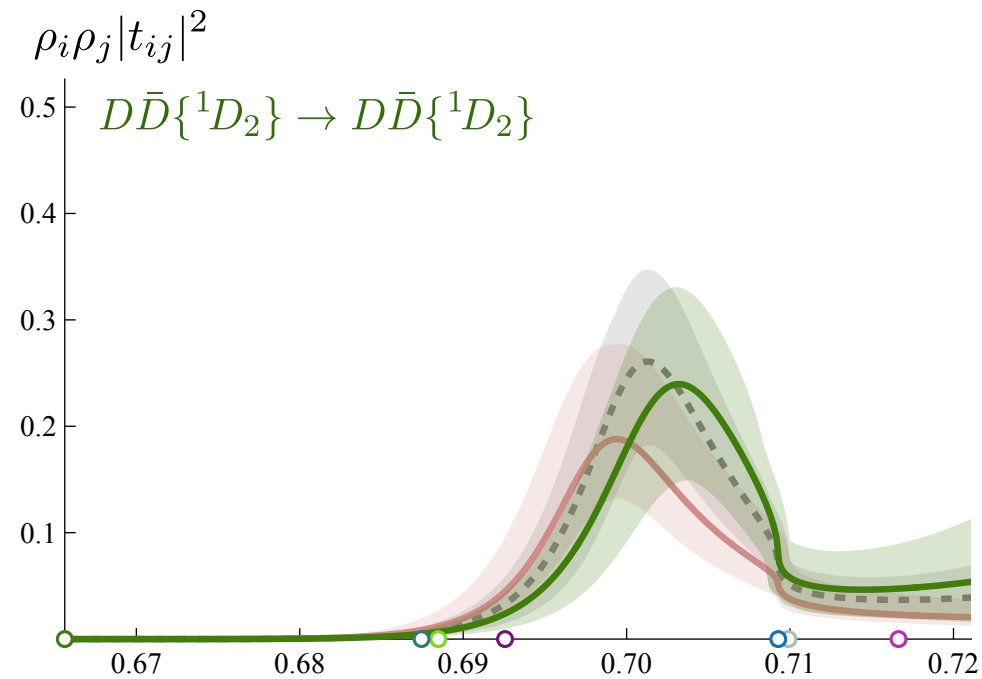
mirror pole - similar to a Flatté



"green" pole is a mirror of the physical sheet pole

physical sheet pole arises because of the large $g_{D\bar{D}^*}$



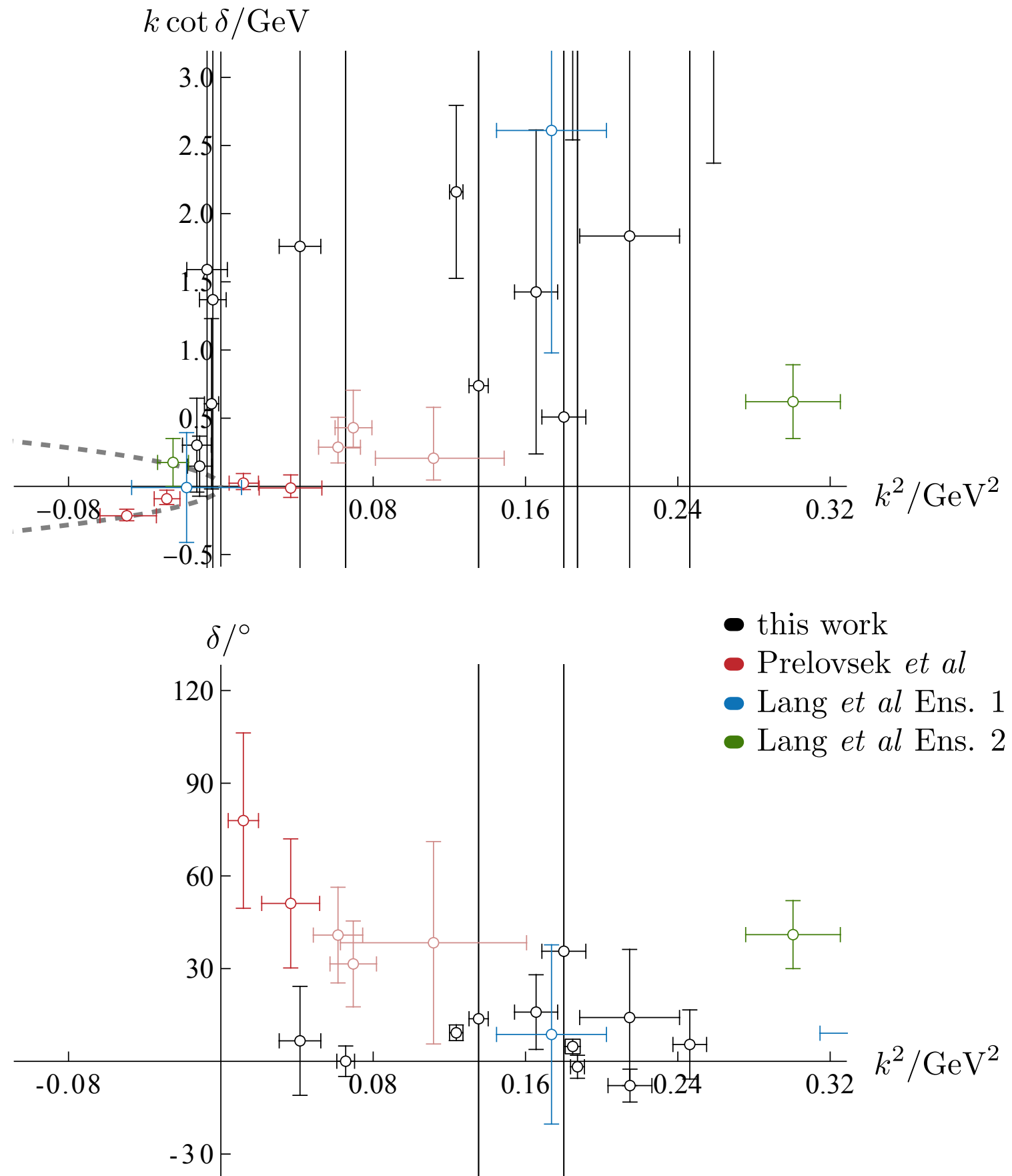


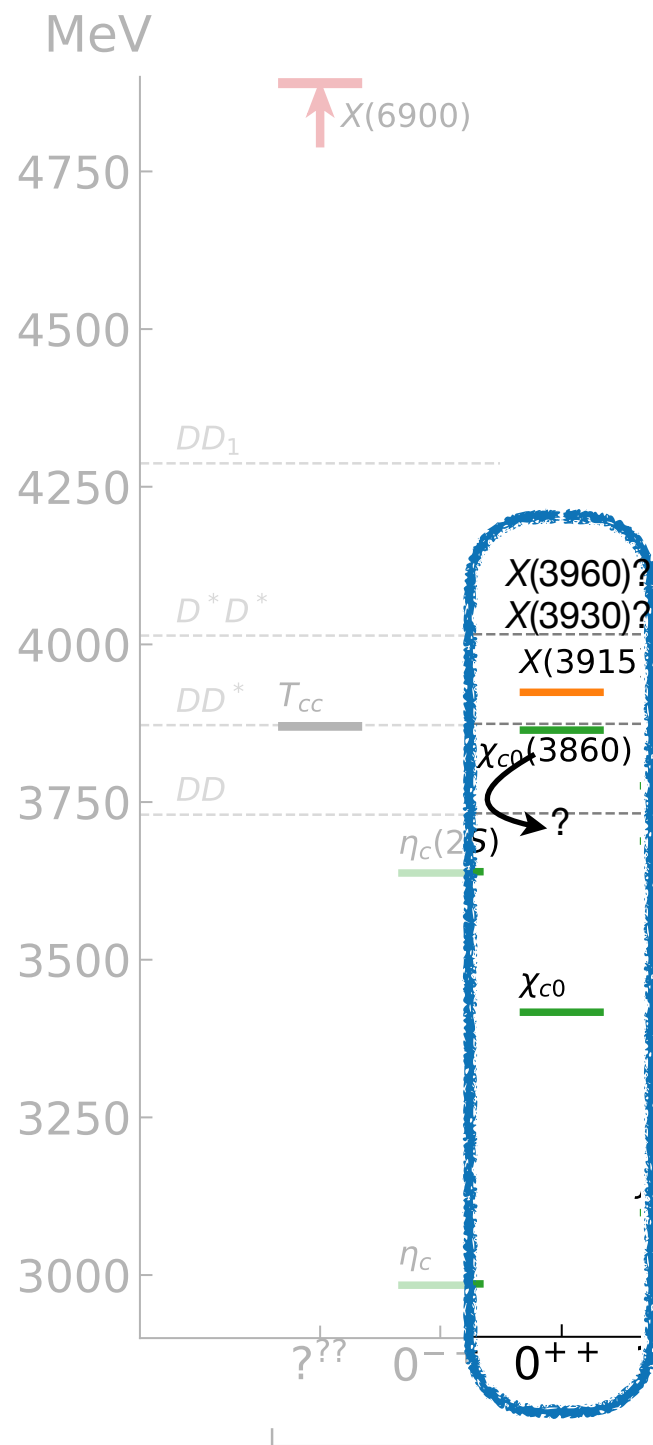
- different physical sheet pole
- no obvious nearby (+,+,+,-) sheet pole (there are some with $a_t E > 0.74$)

Results from Prelovsek, Padmanath et al, suggest effects at DDbar and DsDsbar thresholds

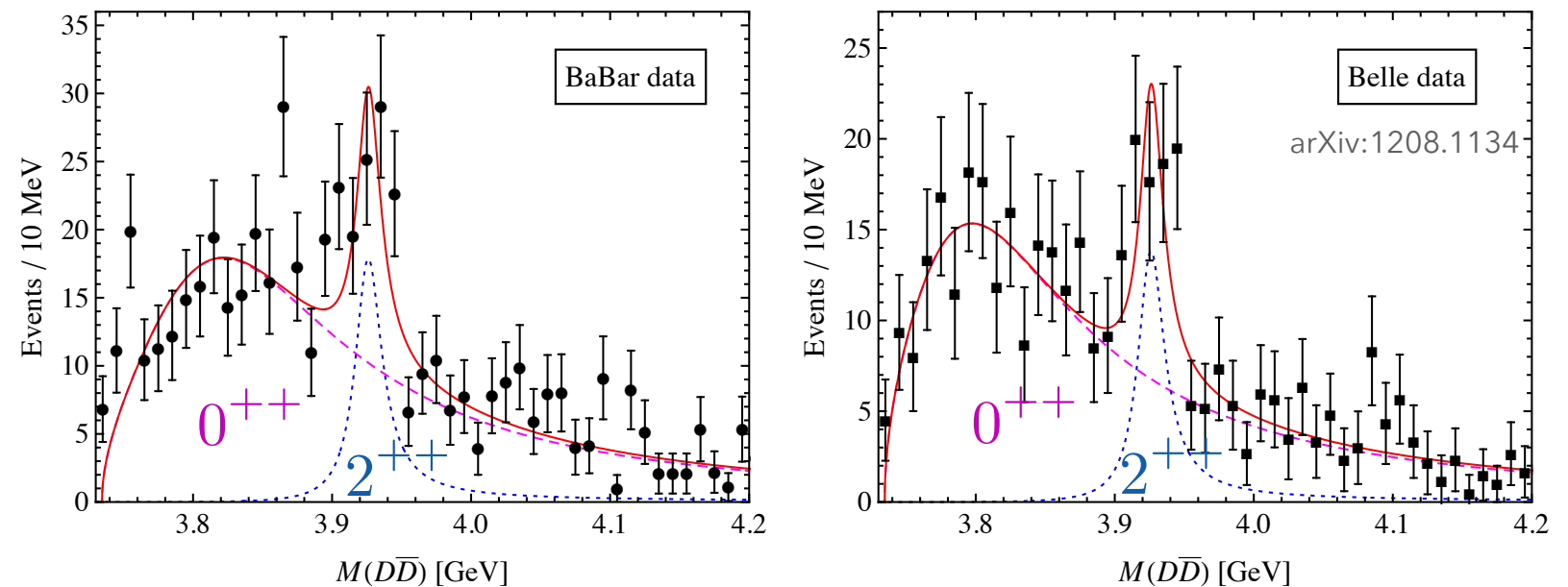
- pion mass ~ 280 MeV
- light quark heavier than physical, strange quark lighter than physical

hard to justify such a large change due to the light quark mass (no one-pion-exchange term)

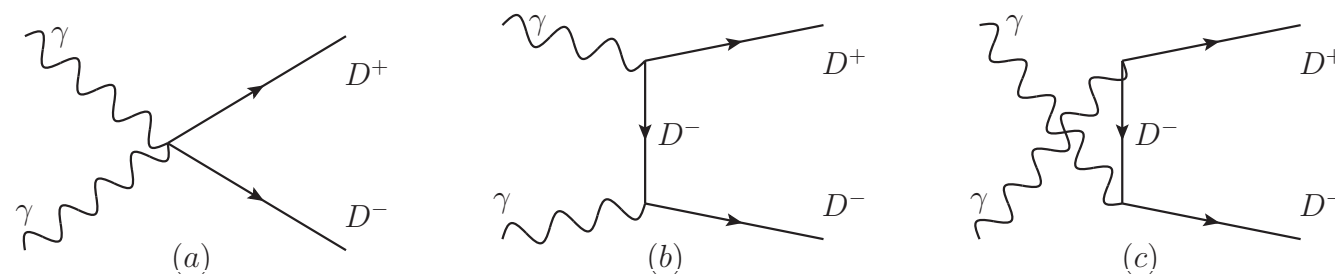




- BaBar, Belle - resonance around 3860 MeV $\gamma\gamma \rightarrow D\bar{D}$



- Guo & Meissner (2012)
 $m = 3840$ MeV, $\Gamma = 220$ MeV
- Wang et al (2021), Daneika et al (2022):
Complications from Born exchanges lead to a lower state around 3700 MeV



arXiv:2010.15431

no state around 3840-3860 MeV (?)

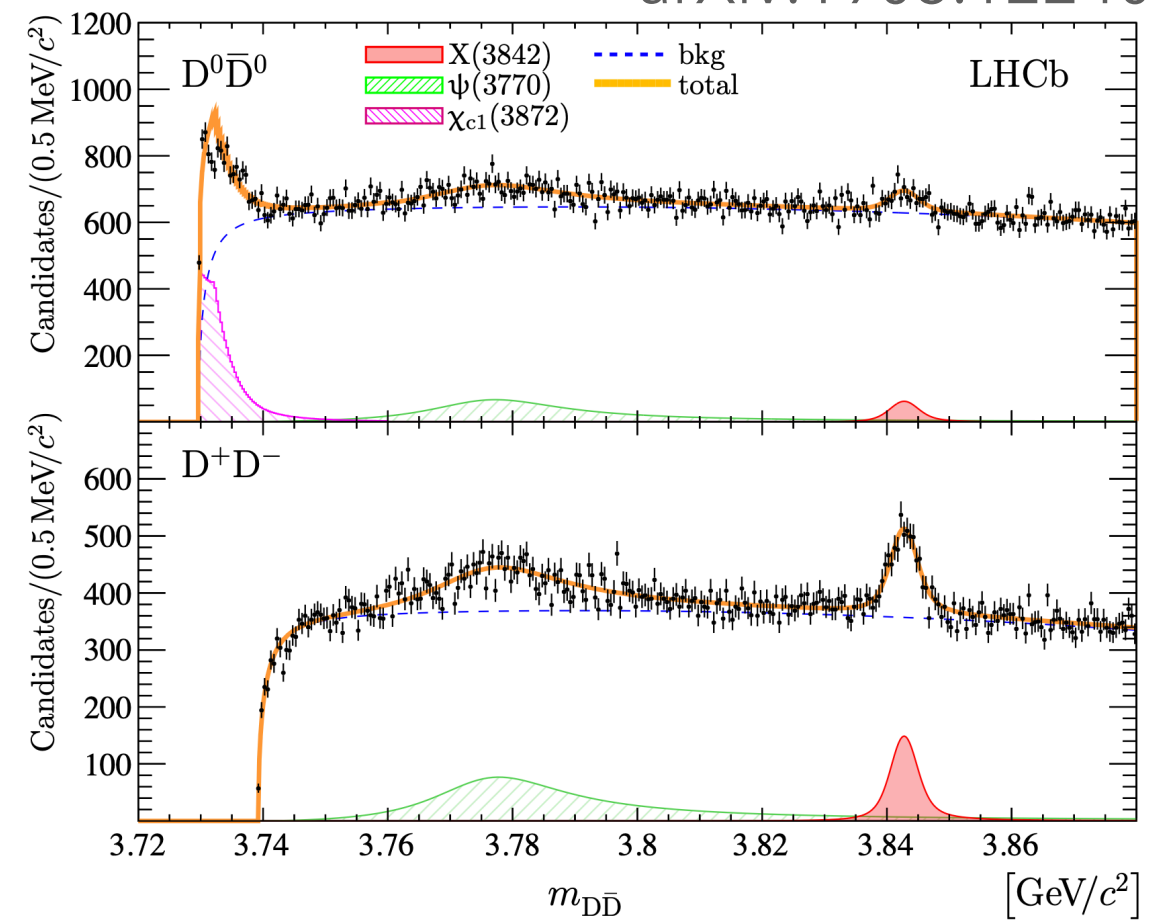
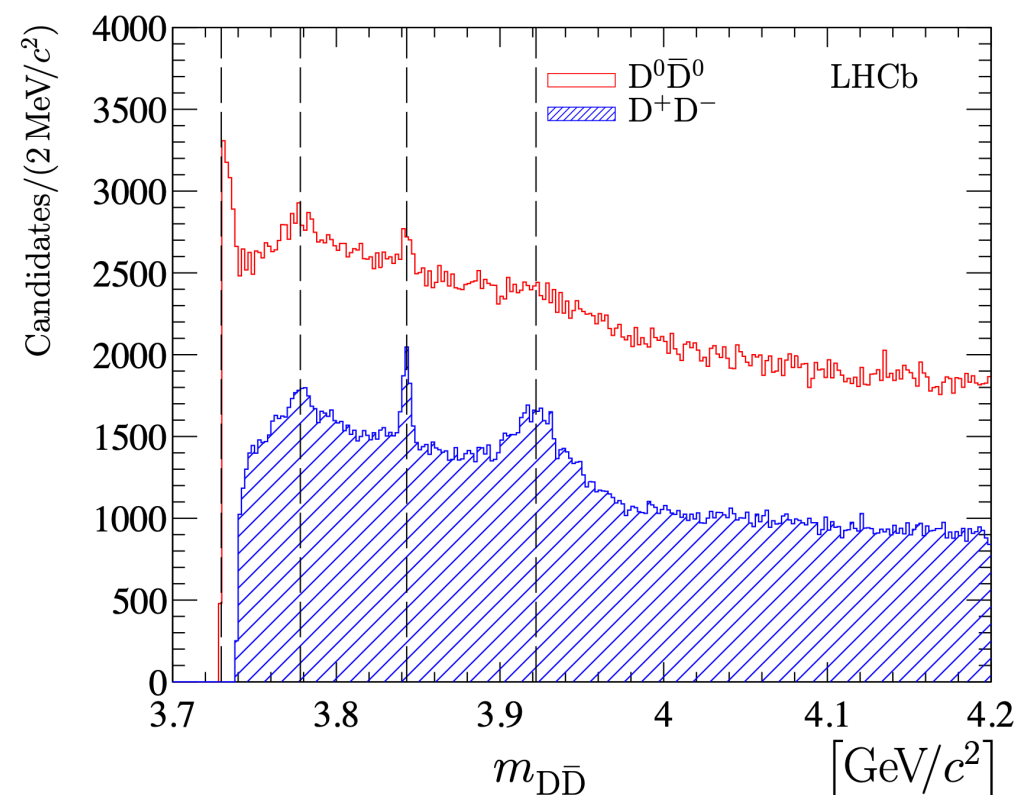
Many models with meson-meson components find strong effects in S-wave DDbar

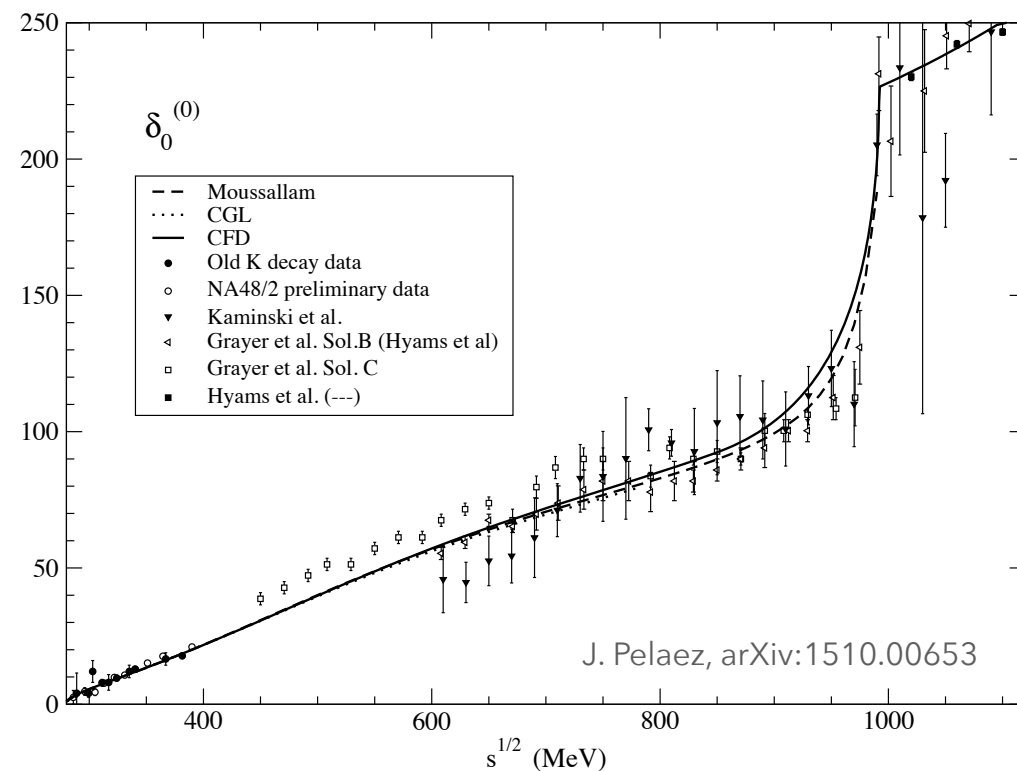
Several suggestions of a near-threshold state in DDbar scattering

- $\gamma\gamma$ to DDbar (BaBar, Belle)
- near threshold structure partly due to Born/t-channel photon exchange
- see e.g. Guo & Meißner 2012, Wang et al 2021, Deineka et al 2022

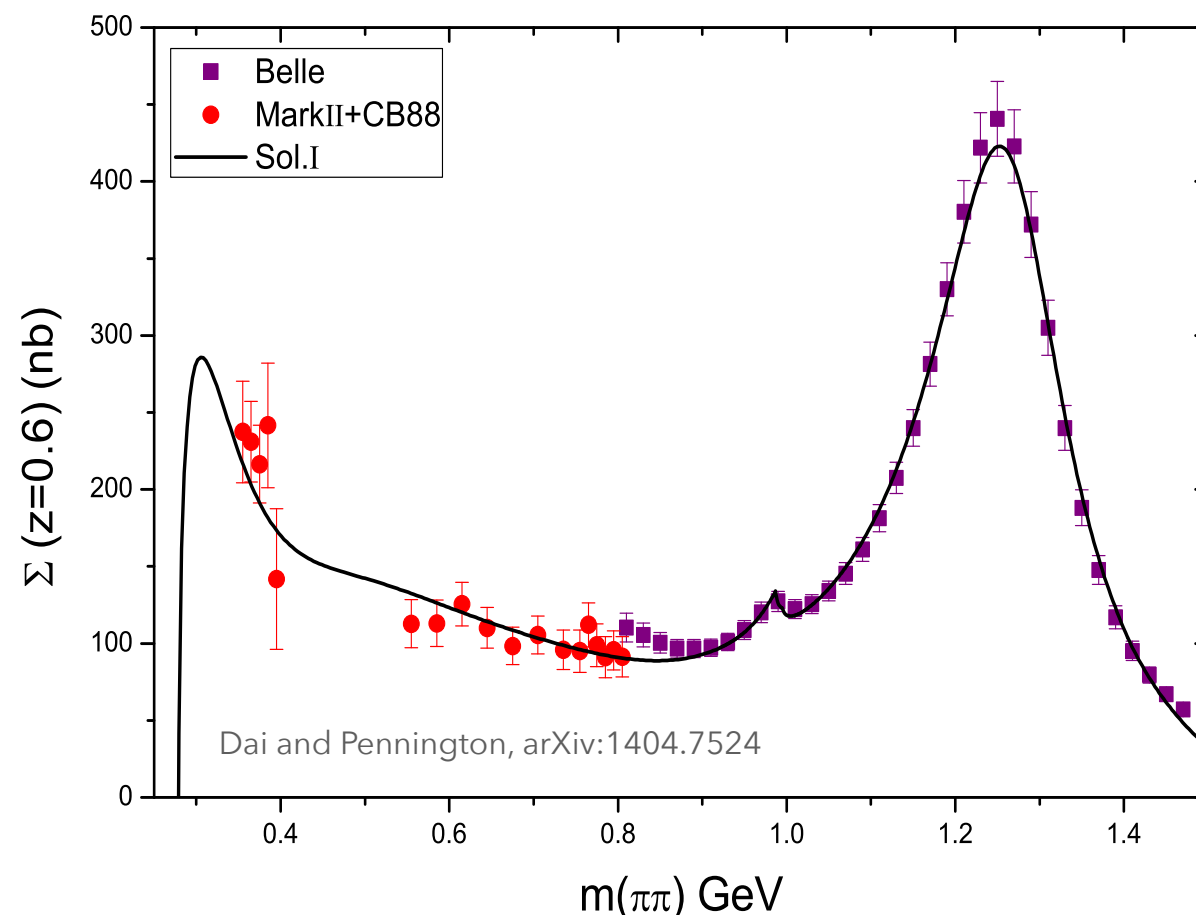
Recent LHCb analyses find a peak at DDbar threshold but attribute this to “feed-down” from X(3872) decays

arXiv:1903.12240

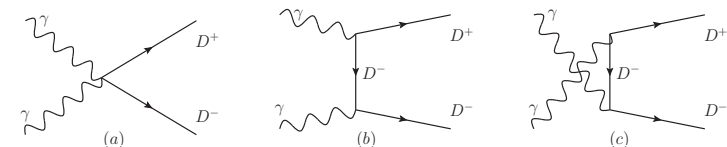




$$\pi\pi \rightarrow \pi\pi \quad (S - \text{wave})$$



$$\gamma\gamma \rightarrow \pi\pi$$



extra structure at threshold,
not linked to a resonance
or bound state