Studies of $A > 3$ few-nucleon systems within next-to-leading order Pionless Effective Field Theory ($\not\pi$EFT)

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Nuclear interaction behind the looking glass

Nuclear interaction :

Two-body
(*NN* scattering, $^2\mathrm{H}$)

Three-body
(*Nd* scattering, $^3\mathrm{H}$, $^3\mathrm{He}$, ...)

Bound state properties
($^4\mathrm{He}$, ...)

$\longrightarrow$
Precise few-body methods
$\longrightarrow$

Few-body $A > 3$ continuum
(scattering, reactions, ...)

Nuclear few-body continuum

Analyzing power A_y in low-energy N-d and p-$^3\mathrm{He}$ elastic scattering

(A. Margaryan et al. Phys. Rev. C 93 (2016) 054001; L. Girlanda, Phys. Rev. C 99 (2019) 054003)

Isoscalar monopole resonance of $^4\mathrm{He}$ (the first $^4\mathrm{He}$ excited state)

(S. Bacca et al., Phys. Rev. Lett. 110 (2013) 042503; S. Kegel et al. Phys. Rev. Lett. 130 (2023) 152502)

→ discrepancy between experimental and theoretically predicted monopole transition form factor

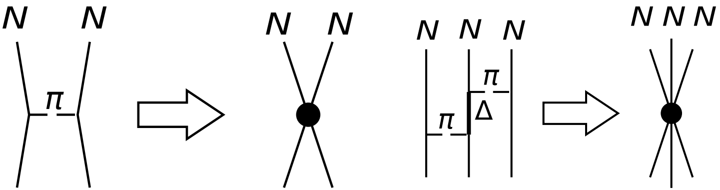
Splitting between $^2P_{3/2}$ and $^2P_{1/2}$ partial waves in $^4\mathrm{He} + \mathrm{n}$

(R. Lazauskas, Phys. Rev C 97 (2018) 044002; A. M. Shirokov et al.,Phys. Rev. C 98 (2018) 044624)

Big Bang Nucleosynthesis reactions

→ astrophysical S-factors of $d(d,p)^3\mathrm{H}$ and $d(d,n)^3\mathrm{He}$ reactions at very low energies (100 keV)

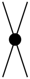
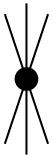
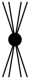
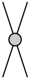
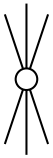
$\not\pi$EFT - basic idea



Baryonic EFT :

→ no pionic degrees of freedom

$\not\pi$EFT

LO			$\delta(\mathbf{r}_{12}),\ \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})$
NLO			$\overleftarrow{\nabla}^2_{\mathbf{r}_{12}}\delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12})\overrightarrow{\nabla}^2_{\mathbf{r}_{12}},\ \delta(\mathbf{r}_{12})\delta(\mathbf{r}_{23})\delta(\mathbf{r}_{34})$
N^2LO			$S-D$ tensor ($T=0$), momentum dep. 3-body
N^3LO	...		$(\nabla_{\mathbf{r}_1}\cdot\nabla_{\mathbf{r}_2})\delta(\mathbf{r}_{12})$, LS, tensor ($T=1$), more 4-body ?

(H.-W. Hammer, Sebastian König, and U. van Kolck, Rev. Mod. Phys. 92 (2020) 025004)

$\not\pi$EFT

- breakdown scale $M \sim m_\pi$, estimate of typical momentum $Q(^4\mathrm{He}) \approx 115\mathrm{MeV}$
- nuclear pionless EFT has large truncation error at LO
  $\rightarrow$ however, it seems to works well in few-body physics

Regularization/Renormalization

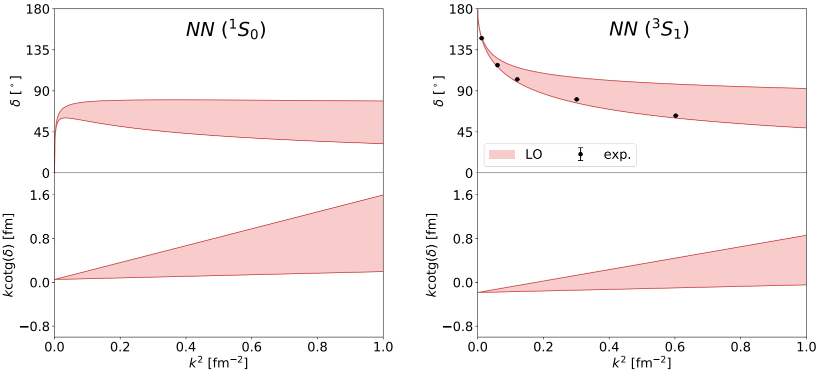
$$C\ \delta(\mathbf{r}_{ij}) \rightarrow C(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \mathrm{e}^{\frac{-\lambda^2 r_{ij}^2}{4}}$$

$$D\ \delta(\mathbf{r}_{ij})\delta(\mathbf{r}_{jk}) \rightarrow D(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^6 \mathrm{e}^{\frac{-\lambda^2 (r_{ij}^2+r_{jk}^2)}{4}}$$

- $C(\lambda), D(\lambda)$ are low energy constants (LECs) tuned to reproduce two-body resp. three-body observables for each λ
- required (RG invariance for $\lambda >> M$)
  $\rightarrow$ all observable will become λ independent when $\lambda \rightarrow \infty$

  $O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \dots$

LO $\pi\!\!\!/$EFT



Leading order (LO) :

(exp. constraints)

$a^0_{NN}\ (a^0_{nn}) = -18.95(40)$ fm

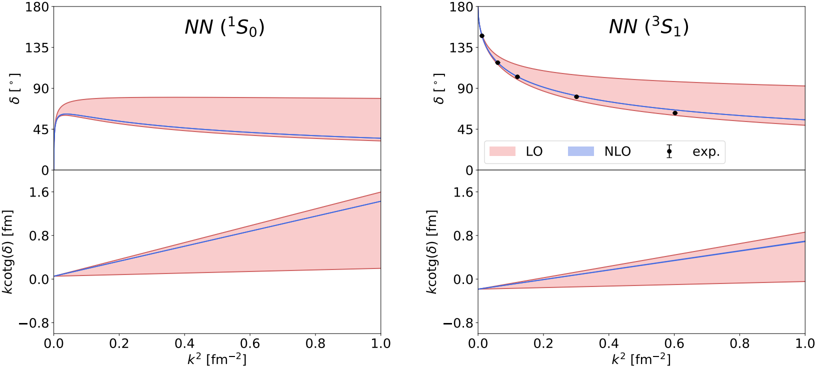
$a^1_{NN}\ (a^1_{np}) = 5.419(7)$ fm

$B(^3\mathrm{H}) = 8.482$ MeV

Effective range expansion :

$$k\mathrm{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

NLO $\not\pi$EFT



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$B(^3\mathrm{H}) = 8.482$ MeV

Next-to-leading order (NLO) :

(exp. constraints)

$r^0_{NN}\ (r^0_{nn}) = \ 2.75(11)$ fm

$r^1_{NN}\ (r^1_{np}) = \ 1.753(8)$ fm

$B(^4\mathrm{He}) = 28.296$ MeV

NLO $\not\pi$EFT

Where we stand ?

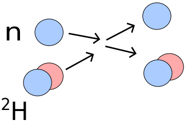
- NLO $\not\pi$EFT using 6 experimental constraints
  (a, r) of $NN(^1S_0)$ and $NN(^3S_1)$, $B(^3\mathrm{H})$, $B(^4\mathrm{He})$
- perturbative NLO using potentials
  (easily extended to 2, 3, 4, 5, 6, ...-body systems)

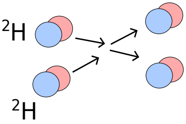
What do we want to study ?

- convergence of all $\not\pi$EFT NLO predictions with λ
- comparison with experimental results

$\longrightarrow$ **no more $A \leq 5$ nuclear bound states** to test the theory $\longrightarrow$ **few-body scattering**

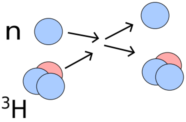
$\rightarrow$ perturbative NLO $\not\pi$EFT predictions at 4- and 5-body level

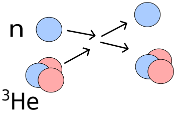




Few-Body scattering

(Phys. Rev. C 107 (2023) 064001; Phys. Lett. B 844 (2023) 138078)





Universality

Universal fermionic relations (STM, Petrov, Deltuva, ...)
Atom-Dimer scattering

$$\frac{a_{ad}}{a_{aa}} = 1.1791 + 0.553\frac{r_{aa}}{a_{aa}}; \;\; \frac{r_{ad}}{a_{aa}} = -0.038 + 1.04\frac{r_{aa}}{a_{aa}}$$

Dimer-Dimer scattering

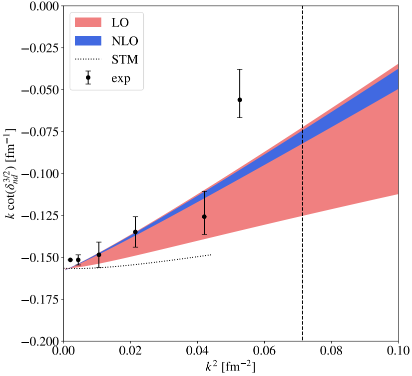
$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105\frac{r_{aa}}{a_{aa}}; \;\; \frac{r_{dd}}{a_{aa}} = 0.133 + 0.51\frac{r_{aa}}{a_{aa}}$$

These results are reproduced for spin-saturated systems:

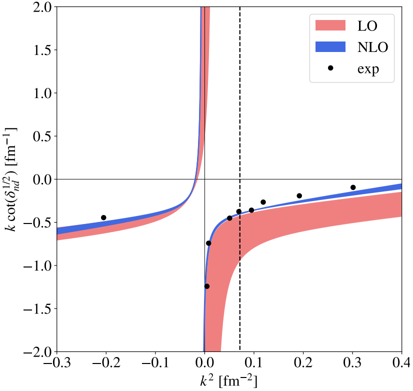
- Neutron-Deuteron $S = 3/2$ scattering
- Deuteron-Deuteron $S = 2$ scattering

$n + d$ $(S = 3/2, T = 1/2)$ and $(S = 1/2, T = 1/2)$ scattering

$(\mathbf{S} = 3/2, \mathbf{T} = 1/2)$



$(\mathbf{S} = 1/2, \mathbf{T} = 1/2)$



$n+{}^3\mathrm{H}$ and $n+{}^3\mathrm{He}$ scattering

- four different 4-body channels $(S=0, T=1)$, $(S=0, T=0)$, $(S=1, T=1)$, and $(S=1, T=0)$
- no isospin breaking terms, our approach does not distinguish between different 4-body T_z

For $n+{}^3\mathrm{H}\ (T_z=-1)$ **:**

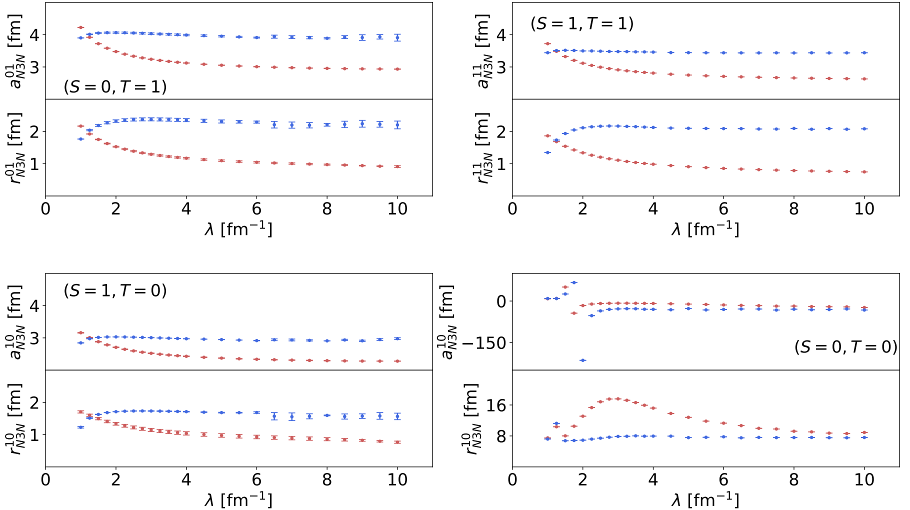
$$S=0 \longrightarrow (S=0, T=1)$$
$$S=1 \longrightarrow (S=1, T=1)$$

For $n+{}^3\mathrm{He}\ (T_z=\ 0)$ **:**

$$S=0 \longrightarrow (S=0, T=0)+(S=0, T=1) \quad {}^4\mathrm{He}(0_2^+)\ \text{resonance}$$
$$S=1 \longrightarrow (S=1, T=0)+(S=1, T=1)$$

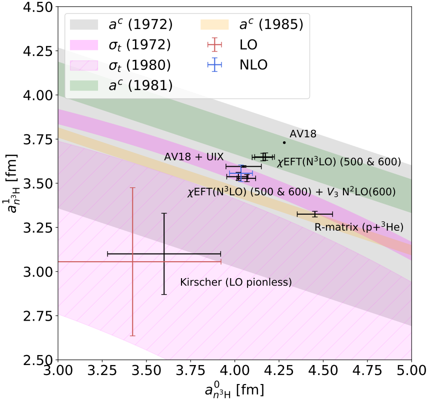
- for $n+{}^3\mathrm{He}$ scattering we must include two different isospin channels

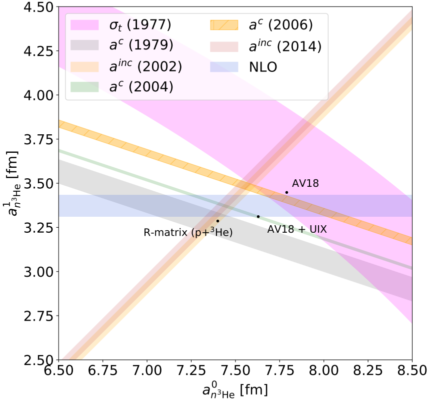
$n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering



- four-body force needed only in $(S = 0, T = 0)$ channel

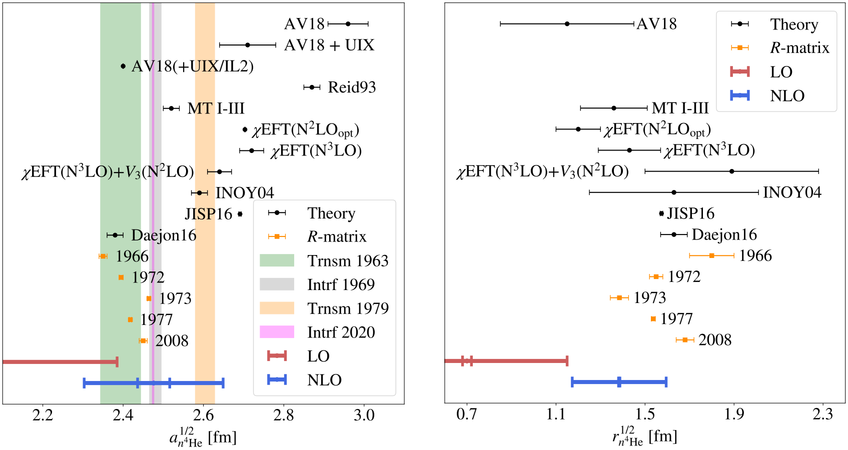
Experiment & Theory : $n+{}^3\mathrm{H}$ and $n+{}^3\mathrm{He}$ scattering lengths





(Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002)

$n + {}^4$He scattering



For references to all theoretical results, see (Phys. Lett. B 844 (2023) 138078).

Summary & Outlook

- constructed $\not\pi$EFT potential up to NLO
- tests on calculations of 2-, 3-, 4-, and 5-body elastic scattering

Next steps :

- addition of nonperturbative Coulomb interaction (ongoing)
- higher $\not\pi$EFT orders (ongoing work on $\mathrm{N^2LO}$)
- scattering in higher partial waves
- inelastic scattering, nuclear reactions
- $^4\mathrm{He}(0^+)$ resonance, binding of $^6\mathrm{Li}$ and $^6\mathrm{He}$ at higher orders

$\not\pi$EFT potential at LO and NLO

Leading order potential (3 LECs) :

$$V_\lambda^{(\mathrm{LO})} = \sum_{i<j} \left[C_0^{(0)}(\lambda) P_{ij}^{T=1,S=0} + C_1^{(0)}(\lambda) P_{ij}^{T=0,S=1} \right] e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ D_0^{(0)}(\lambda) \sum_{i<j<k} \mathcal{Q}_{ijk}^{T=1/2,S=1/2} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{jk}^2)}$$

Next-to-leading order potential (6 LECs) :

$$V_\lambda^{(\mathrm{NLO})} = \sum_{i<j} \left[C_0^{(1)}(\lambda) P_{ij}^{T=1,S=0} + C_1^{(1)}(\lambda) P_{ij}^{T=0,S=1} \right] e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ \sum_{i<j} \left[C_2^{(1)}(\lambda) P_{ij}^{T=1,S=0} + C_3^{(1)}(\lambda) P_{ij}^{T=0,S=1} \right] (\mathbf{k}^2 + \mathbf{q}^2) e^{-\frac{\lambda^2}{4}\mathbf{r}_{ij}^2}$$

$$+ D_0^{(1)}(\lambda) \sum_{i<j<k} \mathcal{Q}_{ijk}^{T=1/2,S=1/2} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+.\mathbf{r}_{jk}^2)}$$

$$+ E_0^{(1)}(\lambda) \sum_{i<j<k<l} \mathcal{Q}_{ijkl}^{T=0,S=0} e^{-\frac{\lambda^2}{4}(\mathbf{r}_{ij}^2+\mathbf{r}_{ik}^2+\mathbf{r}_{il}^2+\mathbf{r}_{jk}^2+\mathbf{r}_{jl}^2+\mathbf{r}_{kl}^2)}$$

BERW formula

→ assumption of short-range potential with range $R << b_{\mathrm{HO}} = \sqrt{\frac{1}{\mu\omega}}$

Bush formula

$$-\sqrt{4\mu\omega}\,\frac{\Gamma\left(3/4-\epsilon_n/2\omega\right)}{\Gamma\left(1/4-\epsilon_n/2\omega\right)} = k\ \mathrm{cotg}(\delta), \qquad k=\sqrt{2\mu\epsilon_n}$$

(A. Suzuki, Phys. Rev. A 80 (2009) 033601, T. Bush Found. of Phys. 28 (1998) 4)

LO $\not\pi$EFT calculations:

$$H(\omega) = T_k + V_\lambda^{(\mathrm{LO})} + V_{\mathrm{HO}}(\omega) \longrightarrow H(\omega)\psi_n = \epsilon_n\psi_n \overset{\mathrm{BERW}}{\overbrace{\longrightarrow}} k\ \mathrm{cotg}(\delta)$$

NLO $\not\pi$EFT calculations:

$$\epsilon_n^{(\mathrm{NLO})} = \epsilon_n + \langle\psi_n|\,V_\lambda^{(\mathrm{NLO})}\,|\psi_n\rangle \overset{\mathrm{BERW}}{\overbrace{\longrightarrow}} k\ \mathrm{cotg}(\delta^{(\mathrm{NLO})})$$

Stochastic Variational Method

(K. Varga et al., NPA571 (1994) 447, K. Varga, Y. Suzuki, PRC52 (1995) 2885)

$$H\psi = E\psi, \qquad \psi = \sum_{i=0}^{N} c_i \varphi^i$$

Basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\varphi^i_{SM_S TM_T}(\mathbf{x}, A_i) = \mathcal{A}\{G_{A_i}(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_{A_i}(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A_i\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, A_i symmetric positive definite matrix of $\frac{N(N-1)}{2}$ real parameters, spin χ_{SM_S} and isospin η_{TM_T} parts

optimization of variational basis in a **random trial and error procedure**

$n + {}^4\mathrm{He}$ scattering