Three-body analysis of $T_{cc}^{+}(3875)$

Sebastian M. Dawid

with the honorable
F. Romero-López & S. Sharpe

SUMMARY

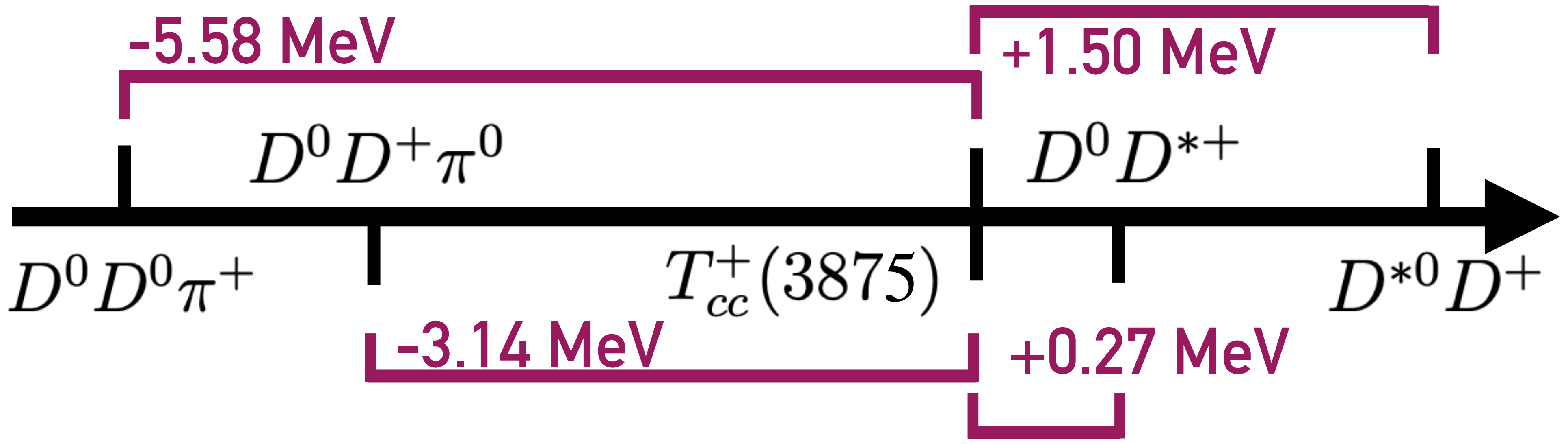
1) We lay out a strategy for a rigorous determination of T_{cc} and related systems from Lattice QCD
2) We discuss resolution of the "left-hand cut problem" both in the finite volume and in the continuum
3) We generalize and solve relativistic EFT three-body equations and apply them to existing lattice data

UNIVERSITY *of* WASHINGTON

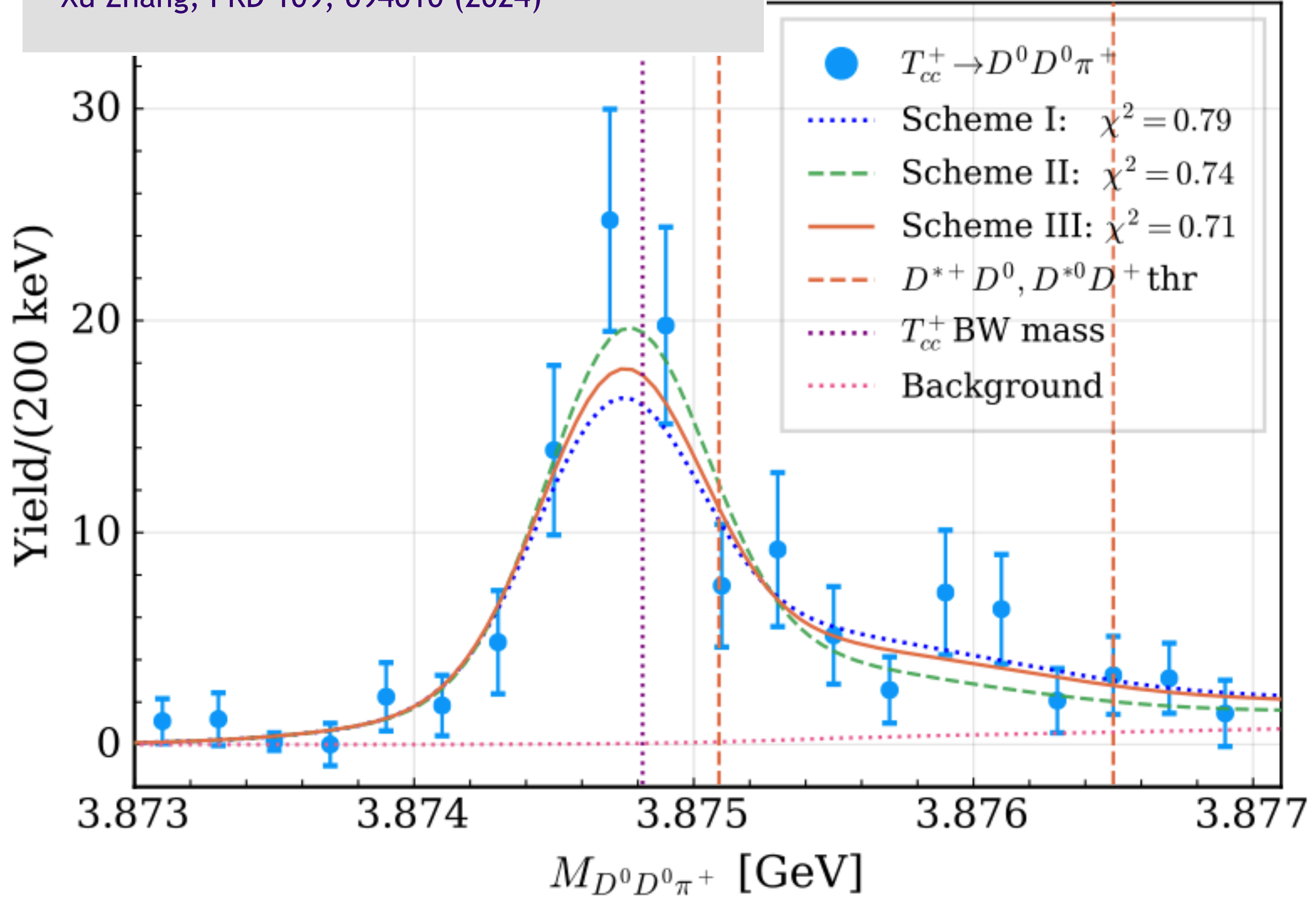
Chiral Dynamics 2024, Bochum, 27.08.2024

Infinite Volume

Three-body effects strongly impact properties of the tetraquark due to the proximity of the DDπ thresholds.

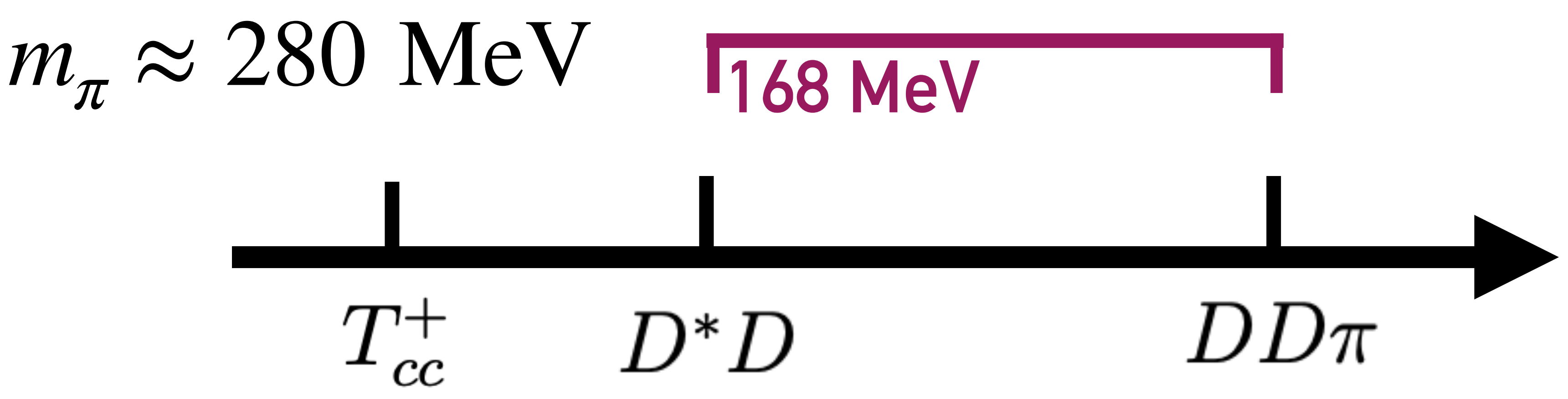


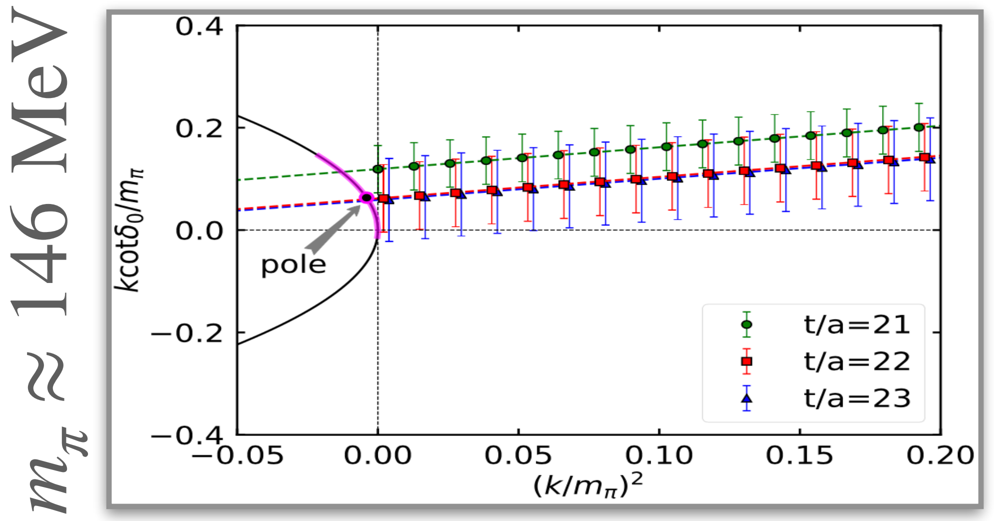
Meng-Ling Du et al., PRD 105, 014024 (2022)
Xu Zhang, PRD 109, 094010 (2024)



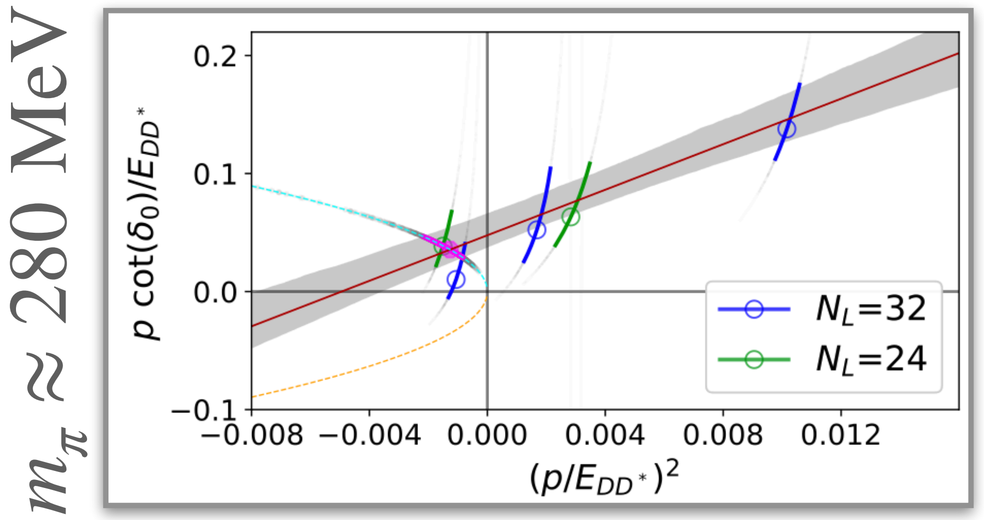
Finite Volume

For heavy pion, thresholds are inverted but three-body effects still play an important role

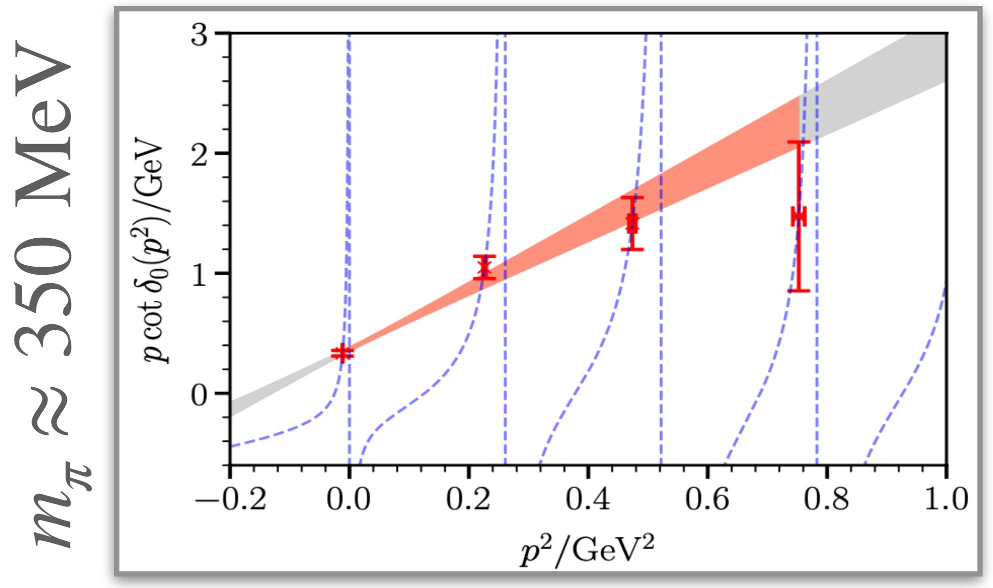




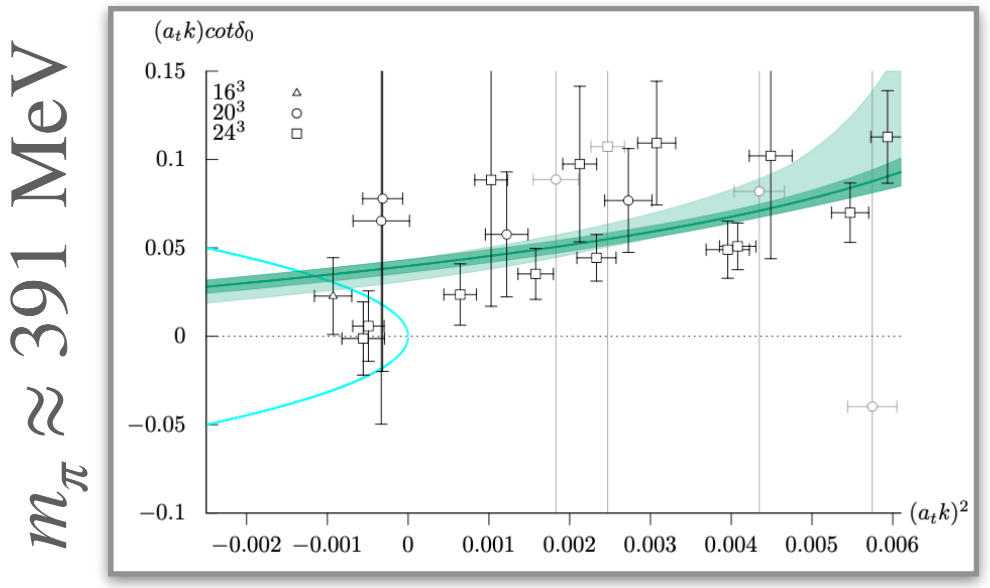
Lyu et al., PRL 131, 161901 (2023)



Padmanath, Prelovsek, PRL 129, 032002 (2022)
Collins et al., PRD 109 (2024) 9, 094509



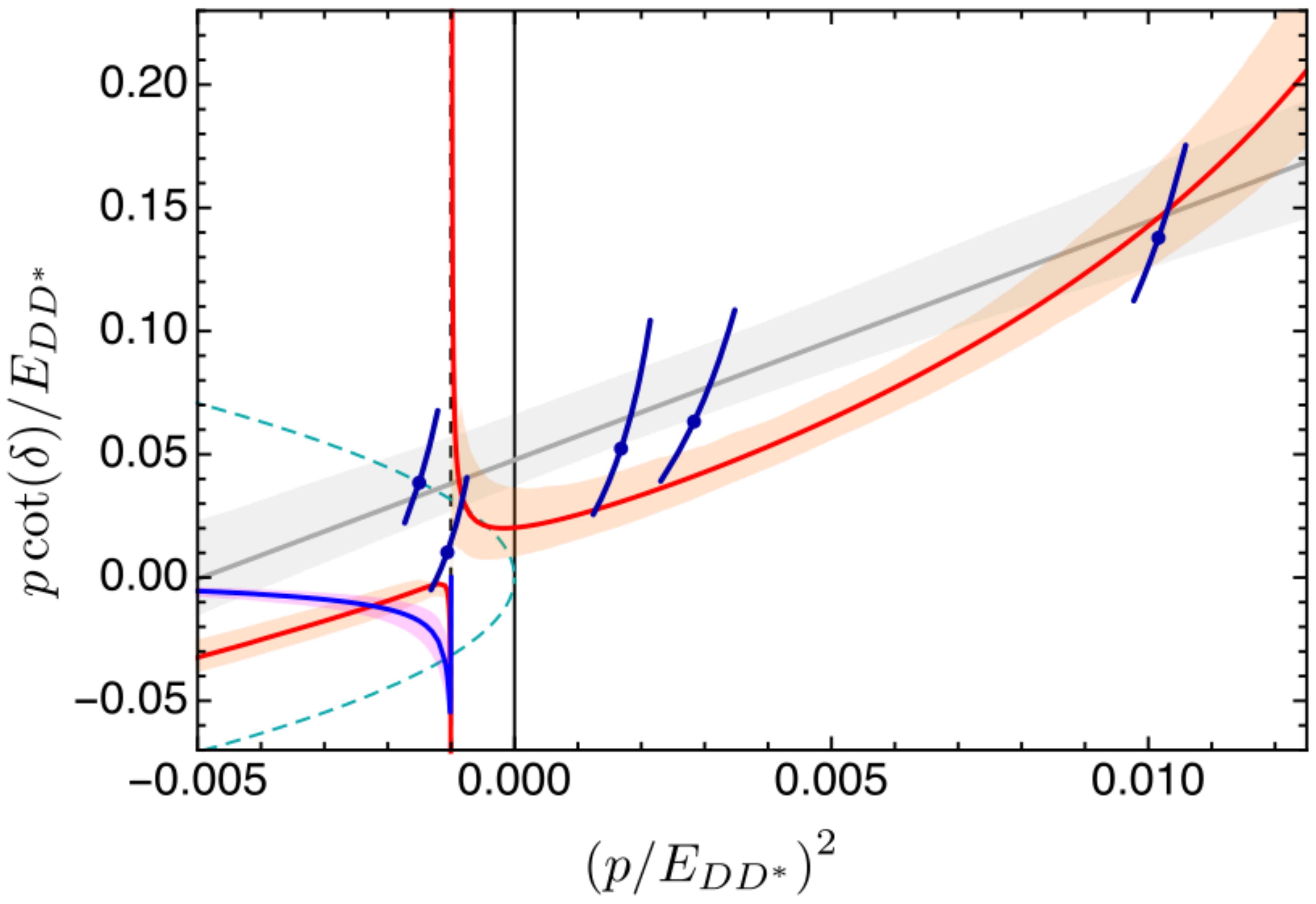
Chen et al. PLB 833, 137391 (2022)



Whyte, Wilson, Thomas, arXiv:2405.15741

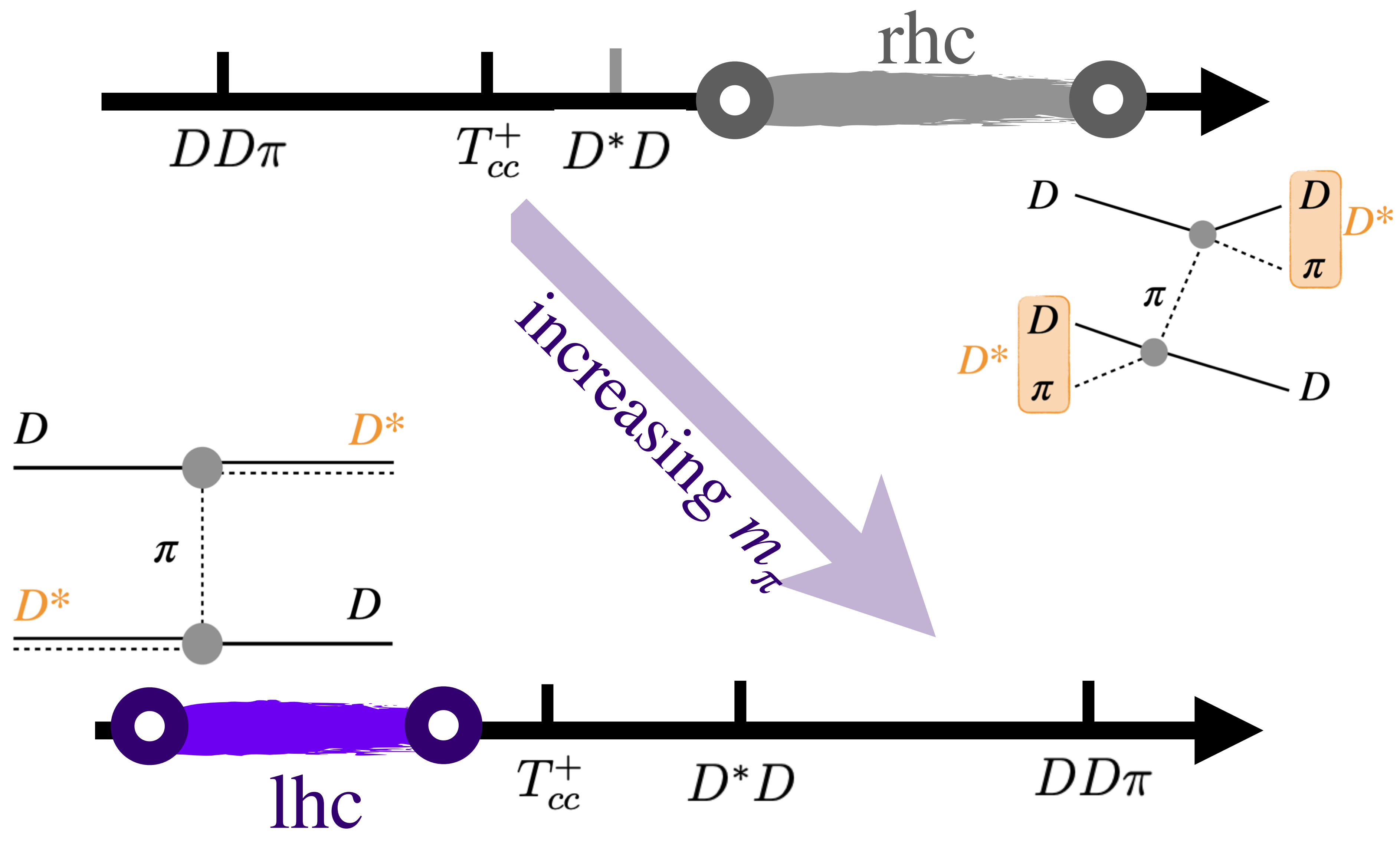
The left-hand cut problem

Role of the left-hand cut contributions on pole extractions from lattice data...
Meng-Lin Du et al., PRL 131, 131903 (2023)



Presence of the left-hand cut:
a) invalidates the Lüscher formalism
b) invalidates the effective-range expansion

At the pion mass increases, the right-hand cut of the physical pion exchange travels below the threshold

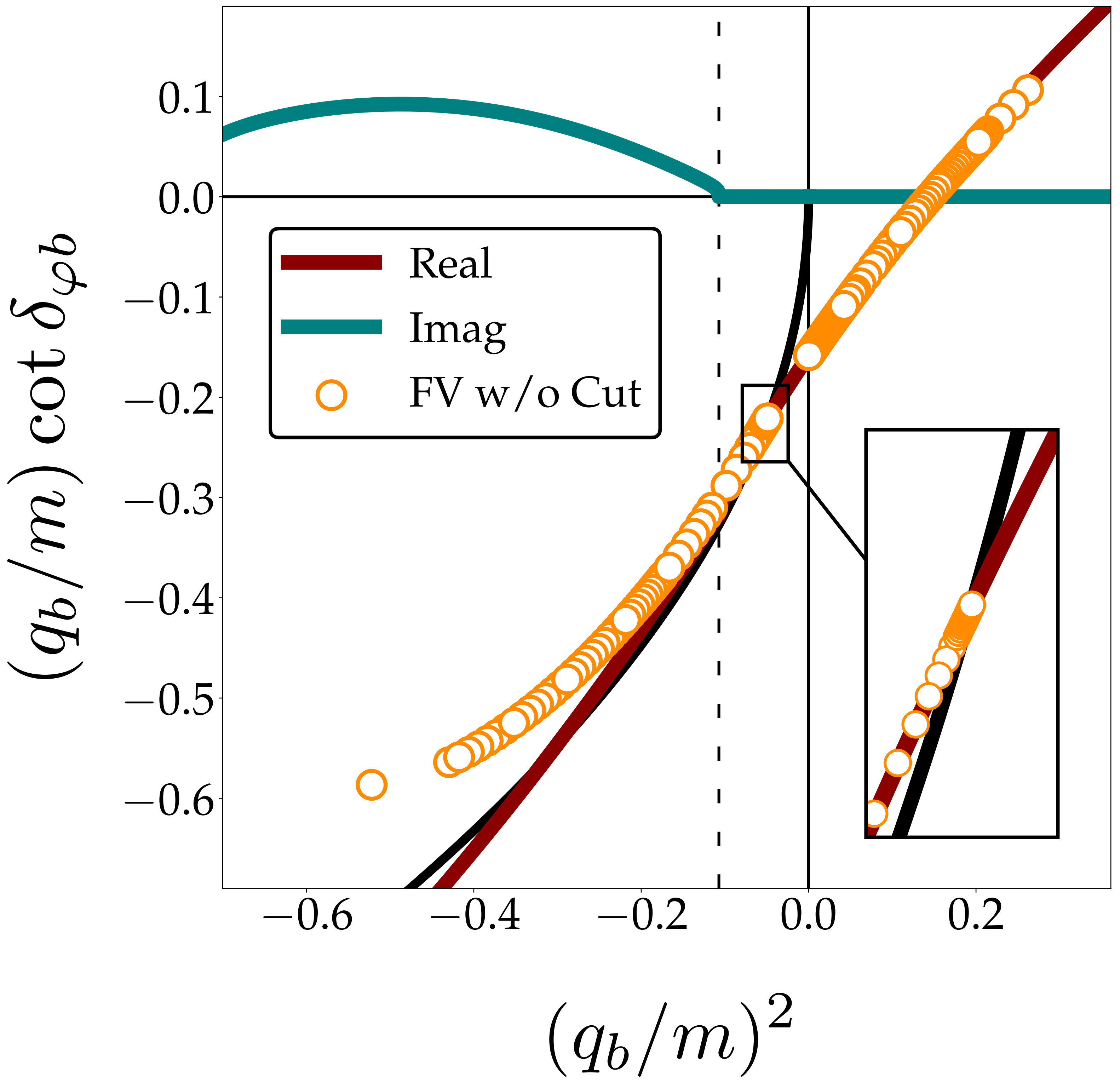


Incorporating $DD\pi$ effects and left-hand cuts in lattice QCD studies of T_{cc}
Hansen, Romero-López, Sharpe, arXiv:2401.06609

Raposo, Hansen, arXiv:2311.18793
Lu Meng et al., Phys.Rev.D 109, L071506 (2024)
Bubna et al. JHEP 05 (2024)

Breakdown of the Lüscher formalism

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, PRD 108 (2023) 3, 034016
Numerical exploration of three relativistic particles...
Romero-Lopez et al. JHEP 10 (2019) 007

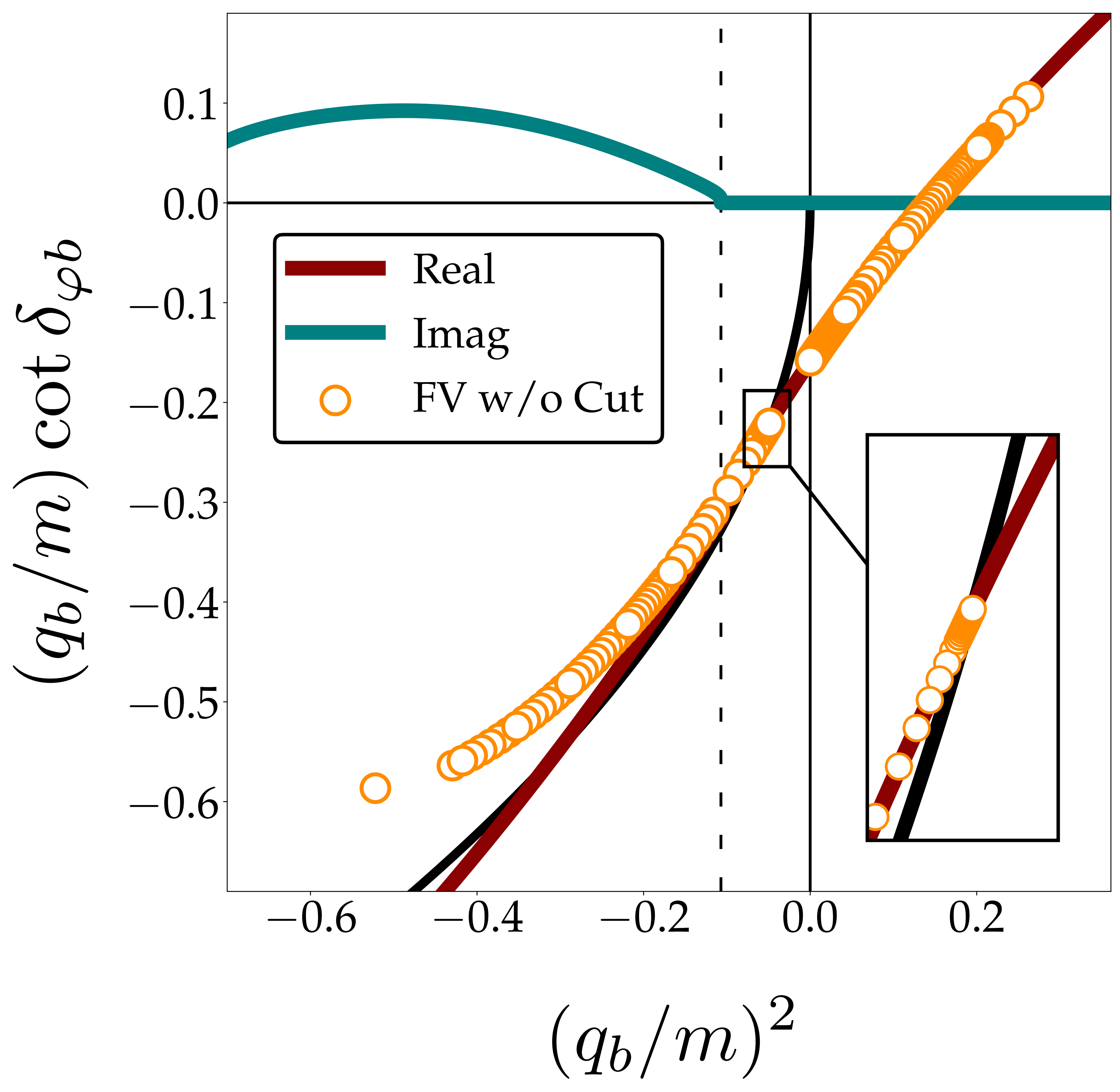


$$\lim_{\sigma',\sigma\to m_{D^*}^2} \mathcal{M}_{DD\pi} = \frac{g}{\sigma' - m_{D^*}^2}\mathcal{M}_{DD^*}\frac{g}{\sigma' - m_{D^*}^2}$$

$\mathcal{K}_2, \mathcal{K}_3$ —Int. eqs.→ $\mathcal{M}_3$ —LSZ→ $\mathcal{M}_{\varphi b}$ ←2-body QC— $E_3 \to E_{\varphi b}$ ←3-body QC— $\mathcal{K}_2, \mathcal{K}_3$

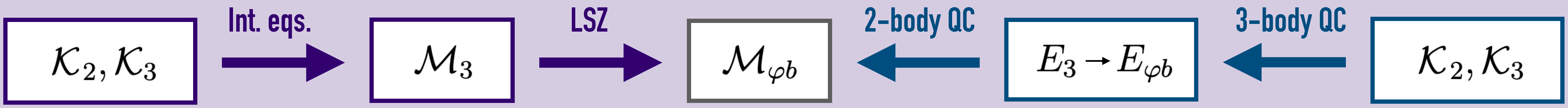
Breakdown of the Lüscher formalism

Analytic continuation of the relativistic three-body amplitudes
Dawid, Islam, Briceño, PRD 108 (2023) 3, 034016
Numerical exploration of three relativistic particles...
Romero-Lopez et al. JHEP 10 (2019) 007



STRATEGY

1. Apply the three-body quantization condition to states with T_{cc} quantum numbers (regardless of the pion mass)
2. Extract the DDπ-relevant two-and three-body K matrices
3. Solve the integral equations relating these objects to the continuum DDπ scattering amplitude
4. Employ the LSZ reduction formula to obtain the DD* amplitude that accounts for the pion exchanges

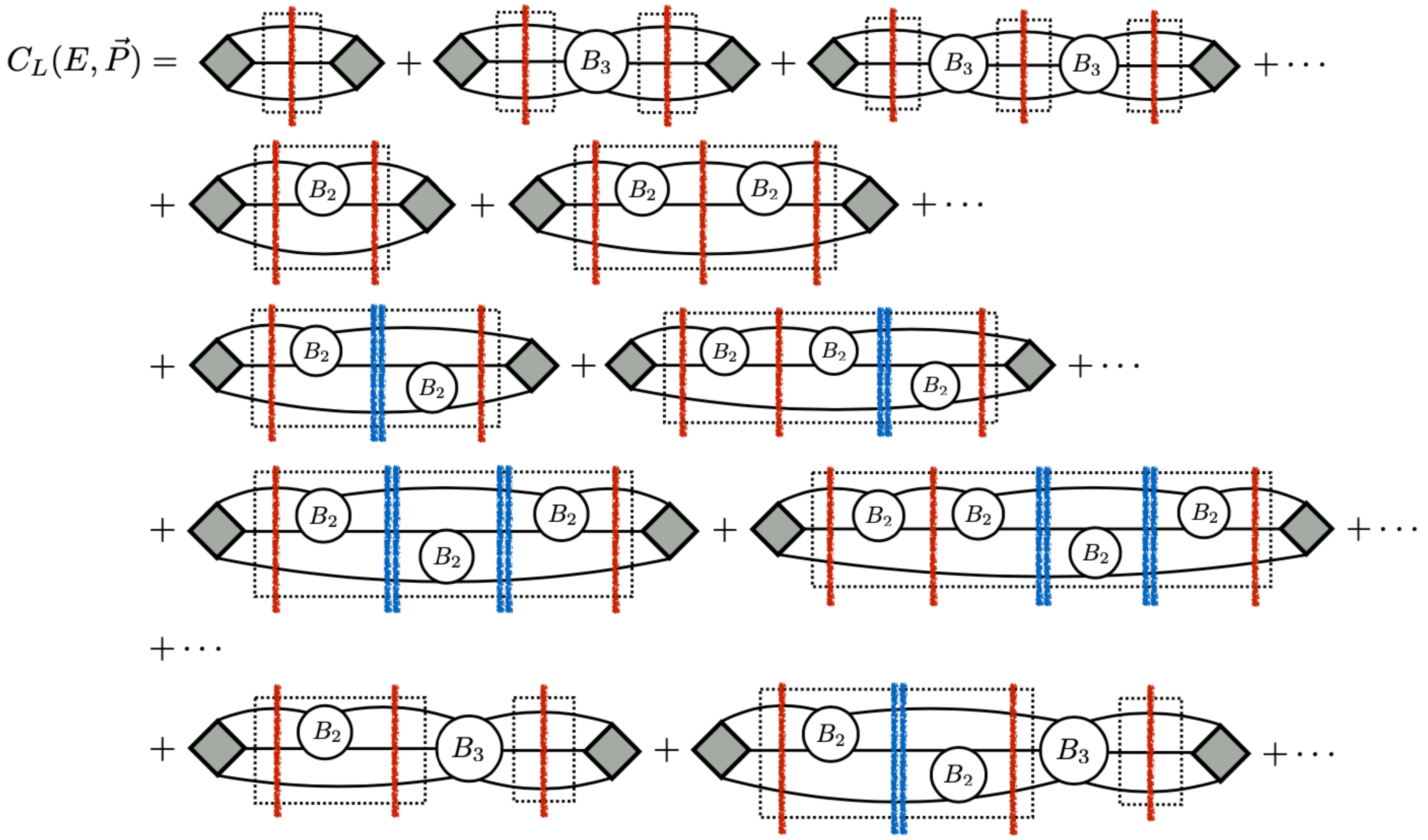


REFT finite-volume quantization

Relativistic three-particle quantization condition for non-degenerate scalars
Three-particle finite-volume formalism for $\pi^+\pi^+K^+$ and related systems
Blanton, Sharpe, PRD 103 (2021) 5, 054503 and PRD 104 (2021) 3, 034509

Lattice QCD and three-particle decays of resonances
Hansen, Sharpe, Ann. Rev. Nucl. Part. Sci. 69 (2019) 65-107

Incorporating DDπ effects and left-hand cuts in lattice QCD studies of T_{cc}^+
Hansen, Romero-López, Sharpe, arXiv:2401.06609



$$\begin{array}{cc} (D\pi)D & (DD)\pi \end{array}$$
$$\begin{pmatrix} \mathcal{K}_3^{(11)} & \mathcal{K}_3^{(12)} \\ \mathcal{K}_3^{(21)} & \mathcal{K}_3^{(22)} \end{pmatrix} \begin{array}{c} (D\pi)D \\ (DD)\pi \end{array}$$

Generalization to the relevant isospin

$$\mathbf{\frac{1}{2} \otimes \frac{1}{2} \otimes 1 = 0 \oplus 1 \oplus 1 \oplus 2}$$

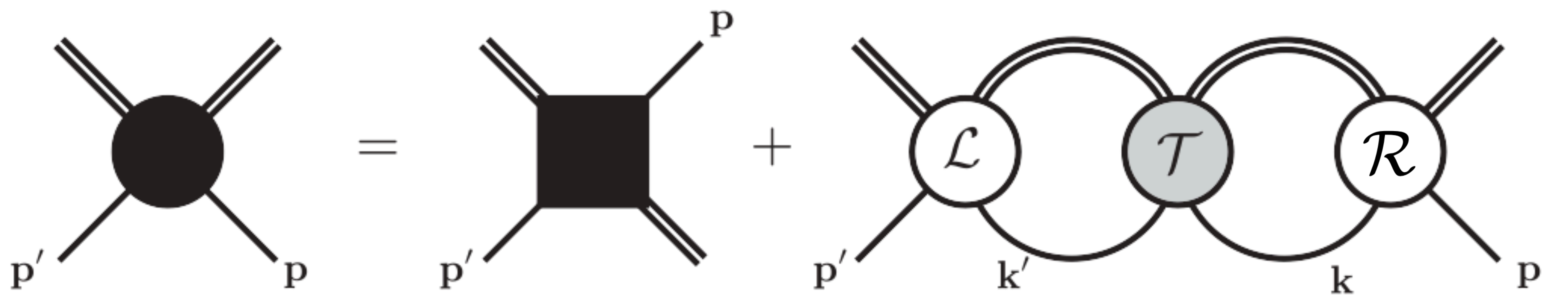
$$\det_{k,\ell,m} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3(E^\star) \boldsymbol{F}_3(E, \boldsymbol{P}, L) \right] = 0$$

$$\prod_{I \in \{0,1,2\}} \det_{k,\ell,m,f} \left[\mathbb{1} - \boldsymbol{\mathcal{K}}_3^I(E^\star) \boldsymbol{F}_3^I(E, \boldsymbol{P}, L) \right] = 0$$

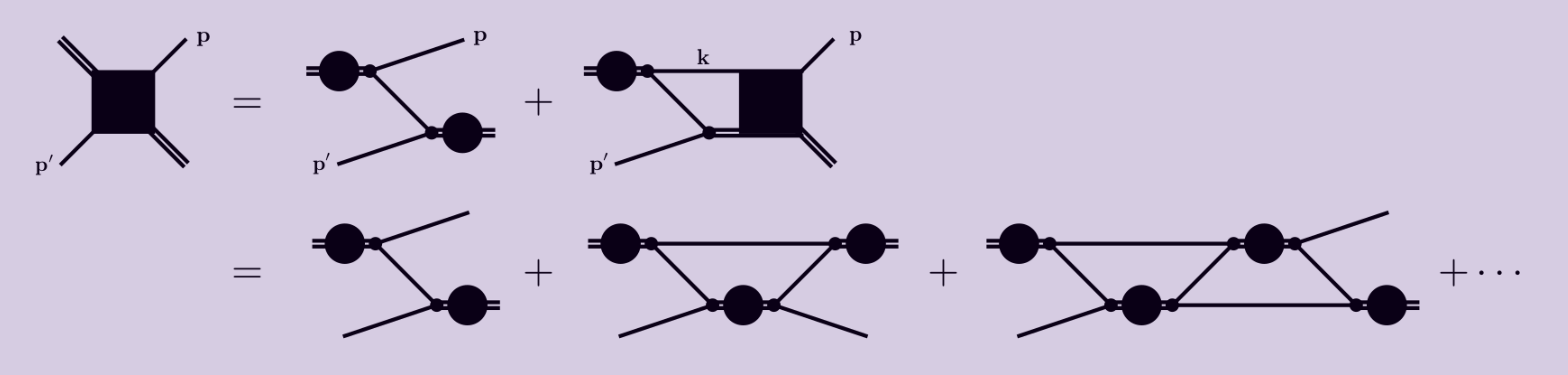
REFT three-body integral equations

diagrams by A. Jackura from PRD 100 (2019) 3, 034508

$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{3,\mathrm{df}}$$



One−particle exchanges



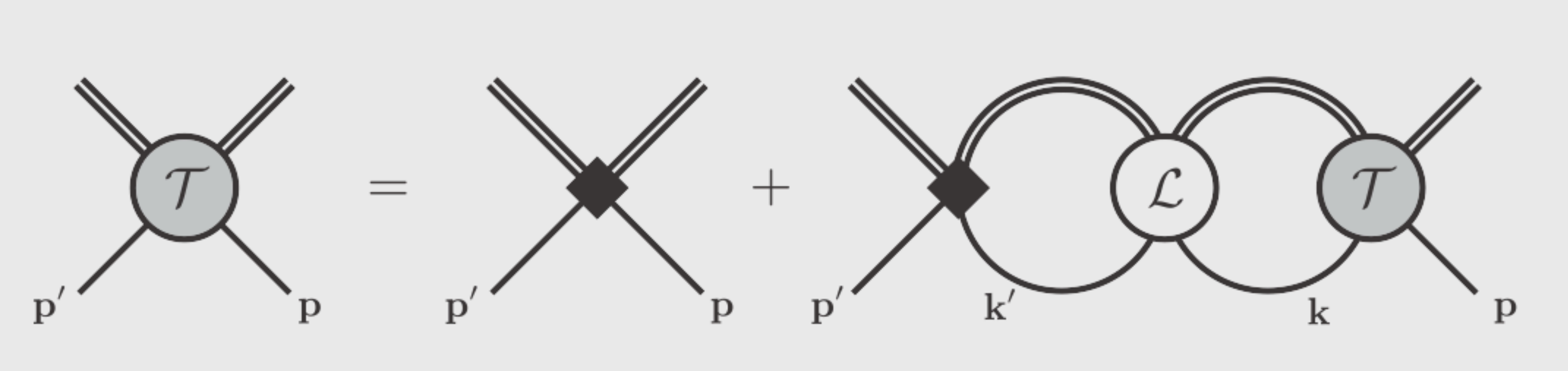
Diffractive Production and Rescattering of Three Particle Systems
Brayshaw, Phys.Rev.D 18 (1978) 2638

Expressing the three-particle finite-volume spectrum in terms of the three-to-three scattering amplitude
Hansen, Sharpe, Phys.Rev.D 92 (2015) 11, 114509

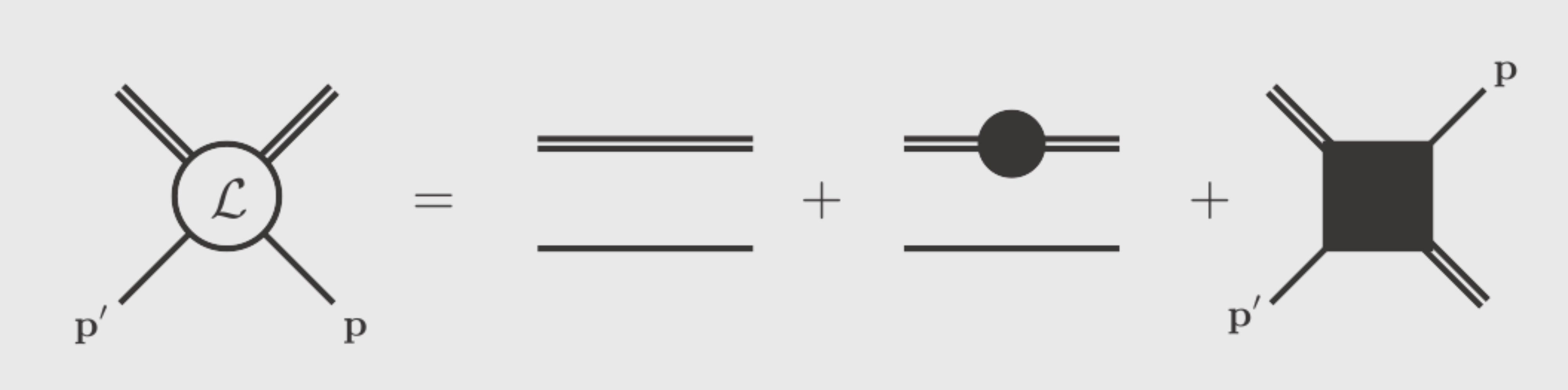
Three-body scattering: Ladders and Resonances
Mikhasenko, Wunderlich, Jackura, et al., *JHEP* 08 (2019) 080

Equivalence of three-particle scattering formalisms
Jackura, Dawid, Fernandez-Ramirez, et al., PRD 100 (2019) 3, 034508

Short−range amplitude



External−state rescatterings



Generalizing to DDπ

$J^P = 1^+$

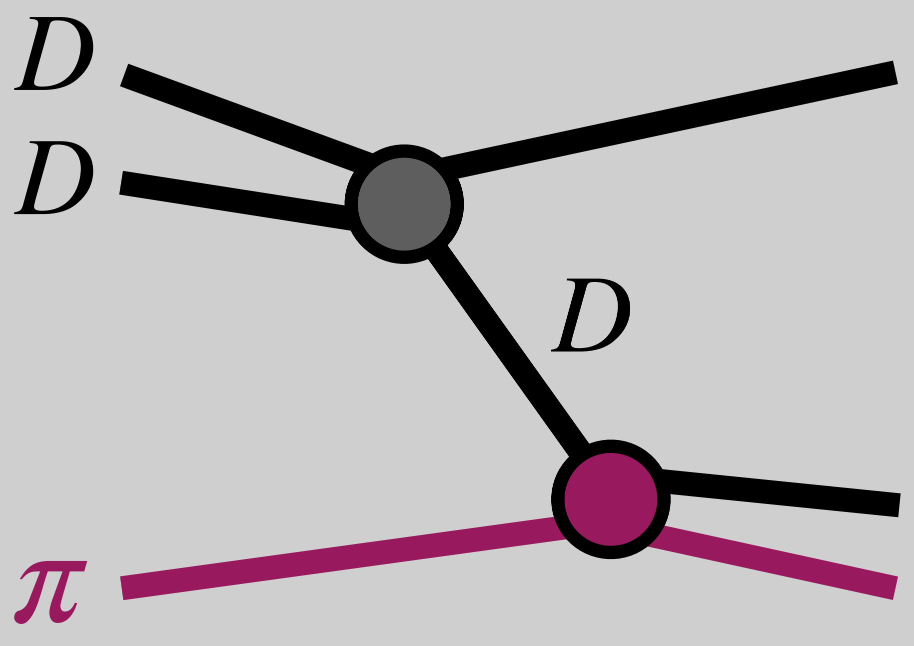
Dawid, Romero-López, Sharpe, in preparation

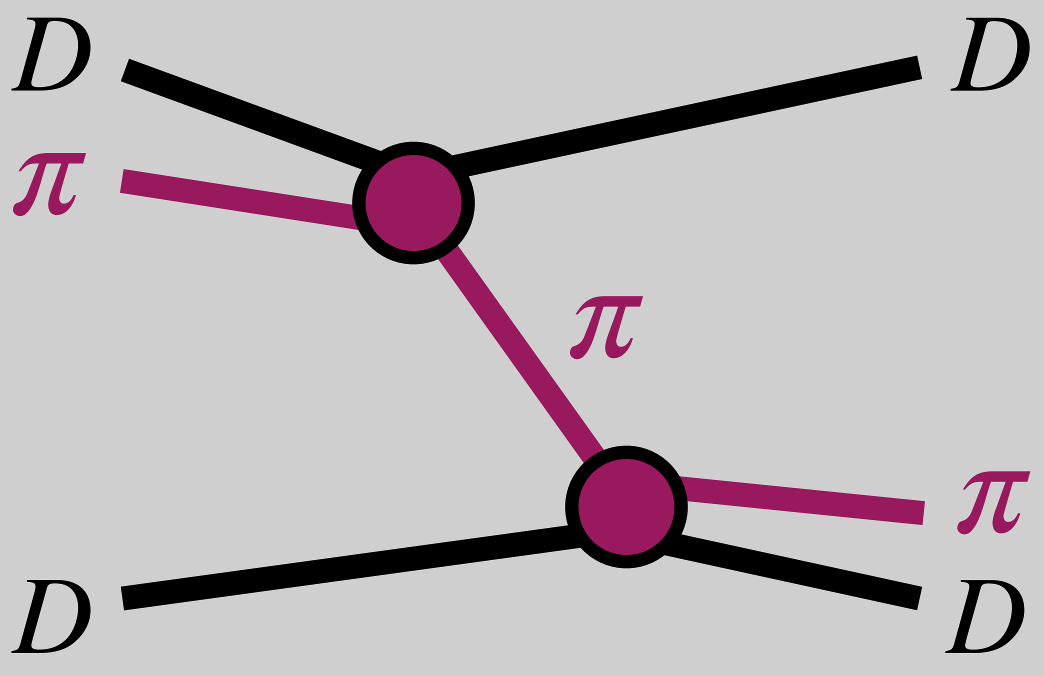
$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \mathcal{M}_2 G \mathcal{D}$$

The amplitude becomes a matrix describing coupled-channel scattering between pairs and spectators of different angular momenta (PW mixing allowed)

$$\begin{array}{ccccc}
(D\pi)D & (D\pi)D & (D\pi)D & (DD)\pi & \\
\end{array}$$
$$\begin{bmatrix}
\mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^3P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1) \\
\mathcal{M}_3({}^3S_1|{}^1P_1) & \mathcal{M}_3({}^3S_1|{}^3S_1) & \mathcal{M}_3({}^3S_1|{}^3D_1) & \mathcal{M}_3({}^3S_1|{}^1P_1) \\
\mathcal{M}_3({}^3D_1|{}^1P_1) & \mathcal{M}_3({}^3D_1|{}^3S_1) & \mathcal{M}_3({}^3D_1|{}^3D_1) & \mathcal{M}_3({}^3D_1|{}^1P_1) \\
\mathcal{M}_3({}^1P_1|{}^1P_1) & \mathcal{M}_3({}^1P_1|{}^3S_1) & \mathcal{M}_3({}^1P_1|{}^3D_1) & \mathcal{M}_3({}^1P_1|{}^1P_1)
\end{bmatrix}
\begin{array}{c} (D\pi)D \\ (D\pi)D \\ (D\pi)D \\ (DD)\pi \end{array}$$

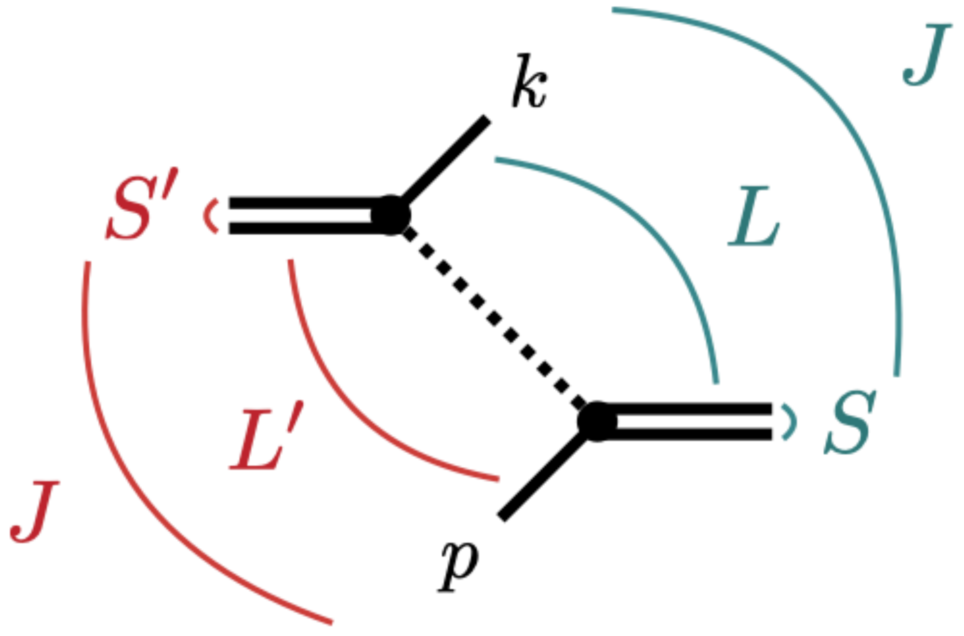
Driving processes





$$\propto -\frac{4\pi\, Y^*_{s'\lambda'}(\hat{\boldsymbol{p}}^\star)\, Y_{s\lambda}(\hat{\boldsymbol{k}}^\star)}{(P-p-k)^2 - m^2 + i\epsilon}$$

$$G({}^{2S'+1}L'_J|{}^{2S+1}L_J) =$$



$$\propto \frac{1}{4pk} \log\left(\frac{1+z(p,k;E)}{1-z(p,k;E)}\right)$$

Partial-wave projection of the one-particle exchange in three-body scattering amplitudes
Jackura, Briceño, PRD 109, 096030 (2024)

Including three-body forces $J^P = 1^+$

Implementing the three-particle quantization condition for ππK and related systems
Blanton, Romero-López, Sharpe, JHEP 02 (2022) 098

Dawid, Romero-López, Sharpe, in preparation

$$\mathcal{T} = \mathcal{K}_3 - \mathcal{K}_3\, \rho\, \mathcal{L}\, \mathcal{T}$$

Matrix-integral equation governed by the symmetric three-body K matrix and two-body rescatterings.

$$\mathcal{K}_3^{(ij)}(p,k) = \sum_a \mathcal{K}_{L,a}^{(i)}(p)\, \mathcal{K}_{R,a}^{(j)}(k)$$

$$\mathcal{T} = \mathcal{K}_L^T\, [1 + \mathcal{I}]^{-1}\, \mathcal{K}_R$$

Solution of another integral equation is unnecessary for certain models of the three-body K matrix

Threshold expansion

$$\mathcal{K}_3 = \mathcal{K}_3^{\text{iso},0} + \mathcal{K}_3^{\text{iso},1}\Delta + \mathcal{K}_3^B \Delta_2^S + \mathcal{K}_3^E t'_{22}$$

$$\Delta = \frac{s - (2m_D + m_\pi)^2}{(2m_D + m_\pi)^2} \qquad t'_{22} = \frac{(p_2 - p'_2)^2}{(2m_D + m_\pi)^2}$$

The last term contributes, for instance,

$$\mathcal{K}_3({}^3S_1|{}^3S_1) = \frac{2}{27}\mathcal{K}_3^E\, q_p^\star\, q_k^\star\, (\gamma_p + 2)(\gamma_k + 2)$$

Relative two−body momentum in a pair

Boost to pair's rest frame

Fixing the scattering parameters

Dawid, Romero-López, Sharpe, in preparation

$$q_b^{2s+1} \cot \delta_s^{(n)} = -\frac{1}{a_s^{(n)}} + \frac{1}{2} r_s^{(n)} q_b^2$$

Scattering parameters

$$a_S^{D\pi},\ r_S^{D\pi},\ a_P^{D\pi},\ r_P^{D\pi},\ a_S^{DD},\ \mathcal{K}_3^E$$

Mohler et al., PRD 87, 034501 (2012)
Becirevic, Sanfilippo, PLB 721 (2013) 94-100
Moir et al. (HadSpec), JHEP 10 (2016) 011 (2016)
Gayer et al. (HadSpec), JHEP 07 (2021) 123
Yan et al., arXiv: 2404.13479 (2024)

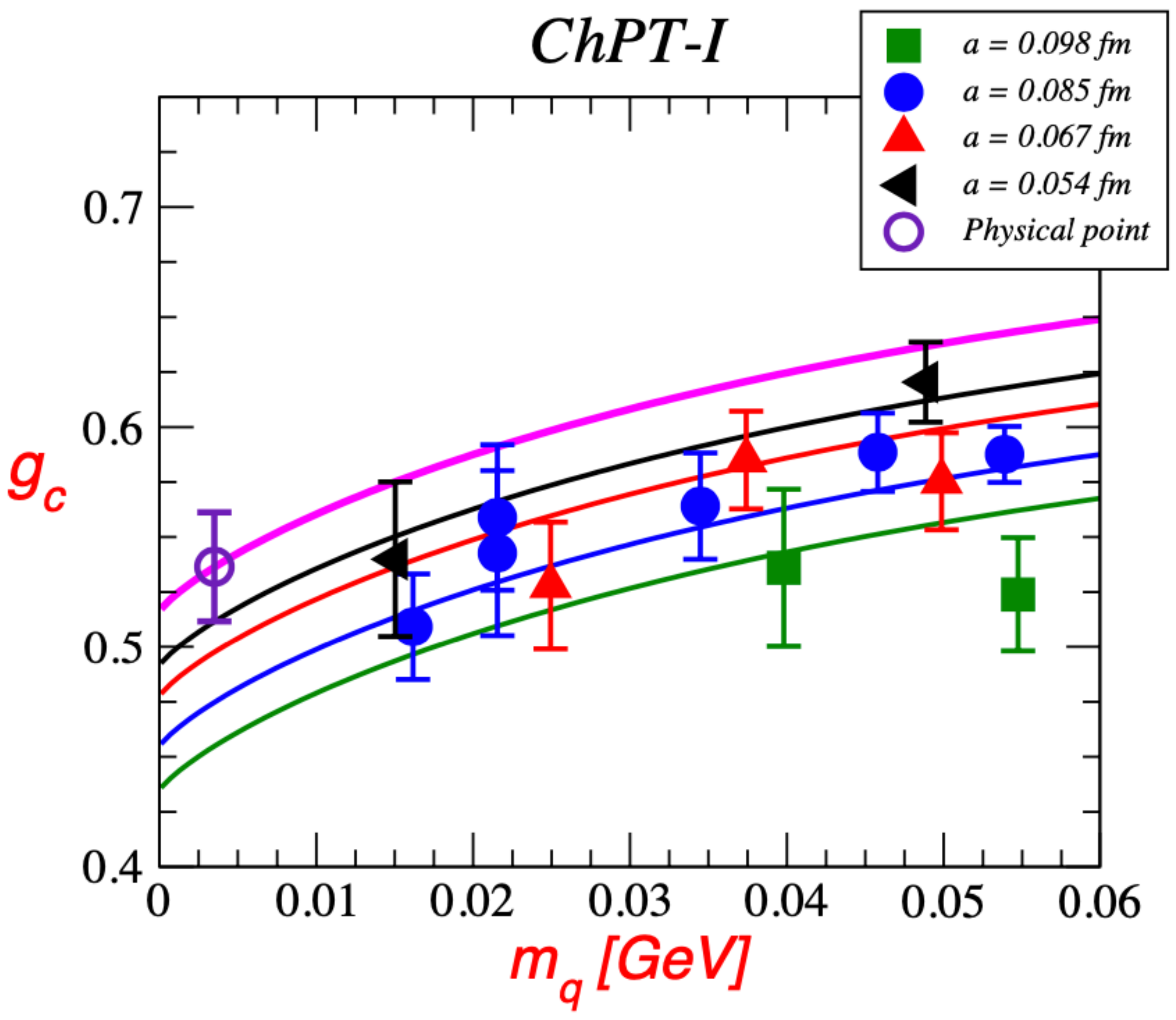
$m_\pi \approx 280$ MeV
$m_D \approx 1927$ MeV
$m_{D^*} \approx 2049$ MeV

$\kappa = m_\pi / m_D \approx 0.145$
$\kappa_{\text{phys}} \approx 0.073$

Padmanath, Prelovsek, PRL 129, 032002 (2022)

Heavy−light meson ChPT

$$\mathcal{L} = \frac{f_\pi^2}{8} \partial^\mu \Sigma_{ab} \partial_\mu \Sigma^\dagger_{ab} + \lambda_0 [\hat{m}\Sigma + \hat{m}\Sigma^\dagger]_{aa}$$
$$- \text{Tr}[\bar{H}_a i v_\mu D^\mu_{ba} H_b] + g\, \text{Tr}[\bar{H}_a H_b \not{A}_{ba} \gamma_5] + \ldots$$

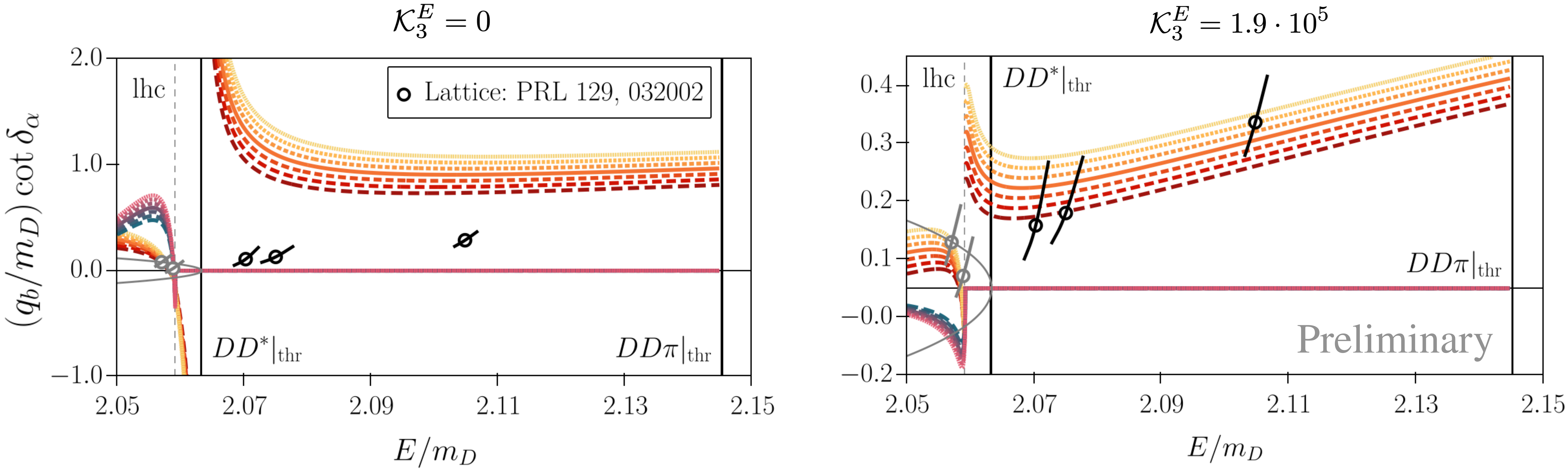


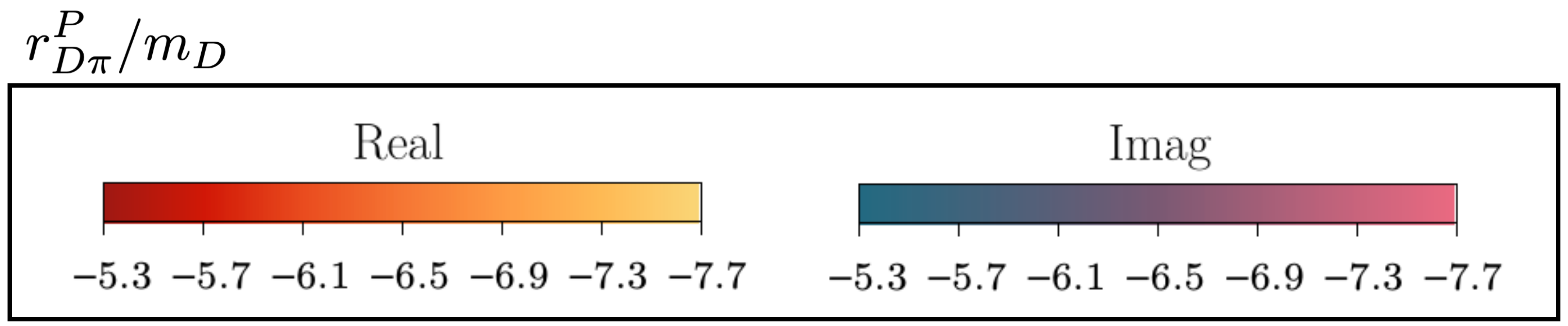
Partial-wave mixing amplitude

Blatt−Biederharn parametrization

$$\begin{bmatrix} \mathcal{M}_{DD^*}(^3S_1|^3S_1) & \mathcal{M}_{DD^*}(^3S_1|^3D_1) \\ \mathcal{M}_{DD^*}(^3D_1|^3S_1) & \mathcal{M}_{DD^*}(^3D_1|^3D_1) \end{bmatrix}$$

$$q_b^{-\ell'} \left[\mathcal{K}_{DD^*}^{-1}\right]_{\ell',\ell} q_b^{-\ell} = \begin{pmatrix} \cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon) \\ q_b^2 \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b \cot(\delta_\alpha) & 0 \\ 0 & q_b^5 \cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & q_b^2 \sin(\epsilon) \\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$





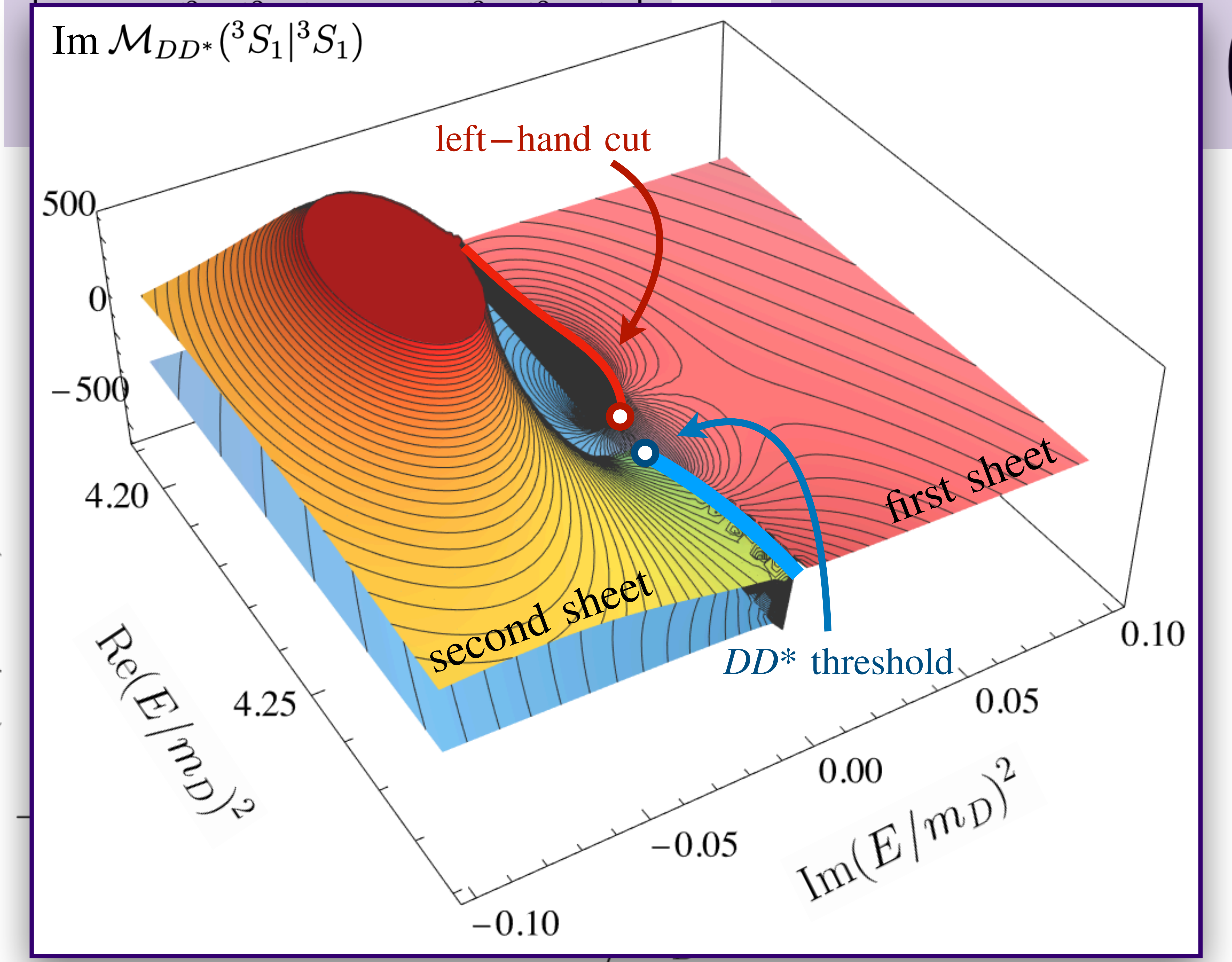
Observations:

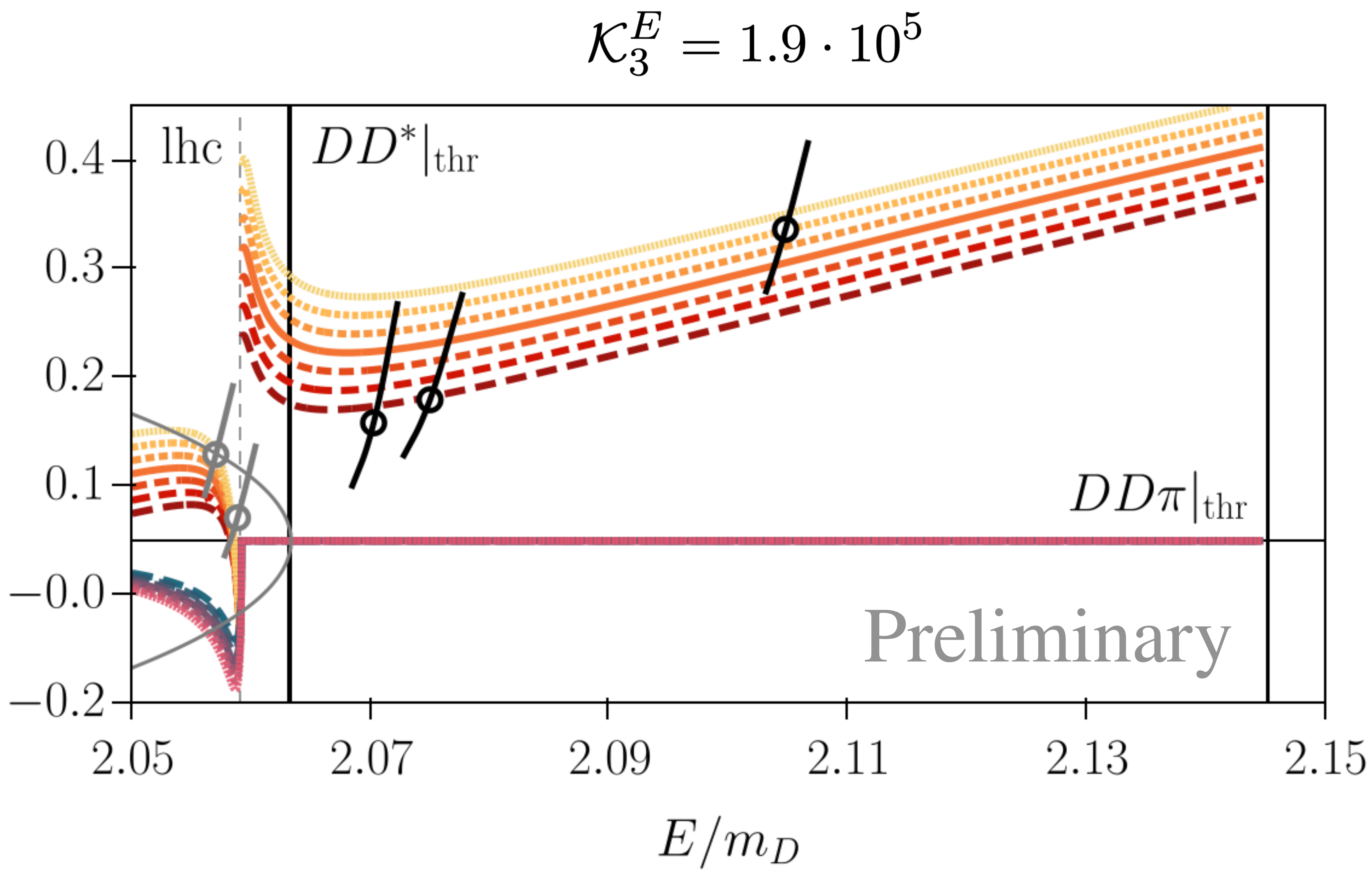
(a) we find a sub-threshold complex pole (agreement with the NR EFT analysis)
(b) simple model of three-body forces is enough to describe data
(c) partial-wave mixing is small (not shown here)
(d) Dπ S-wave scattering is (almost) negligible (not shown here)

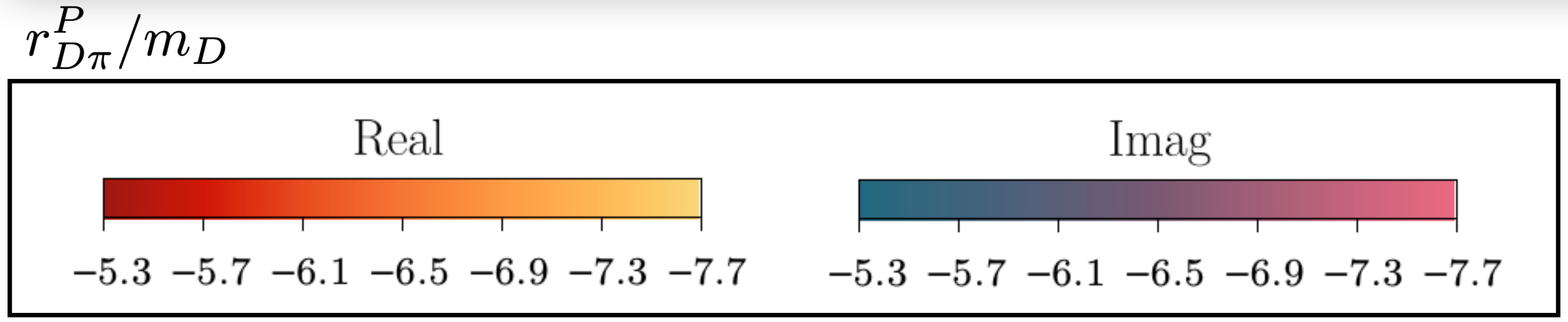
Partial-wave mixing amplitude

Blatt−Biederharn parametrization

$$\begin{pmatrix}\cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon)\\ q_b^2\sin(\epsilon) & \cos(\epsilon)\end{pmatrix}\begin{pmatrix} q_b\cot(\delta_\alpha) & 0\\ 0 & q_b^5\cot(\delta_\beta)\end{pmatrix}\begin{pmatrix}\cos(\epsilon) & q_b^2\sin(\epsilon)\\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon)\end{pmatrix}$$







Observations:

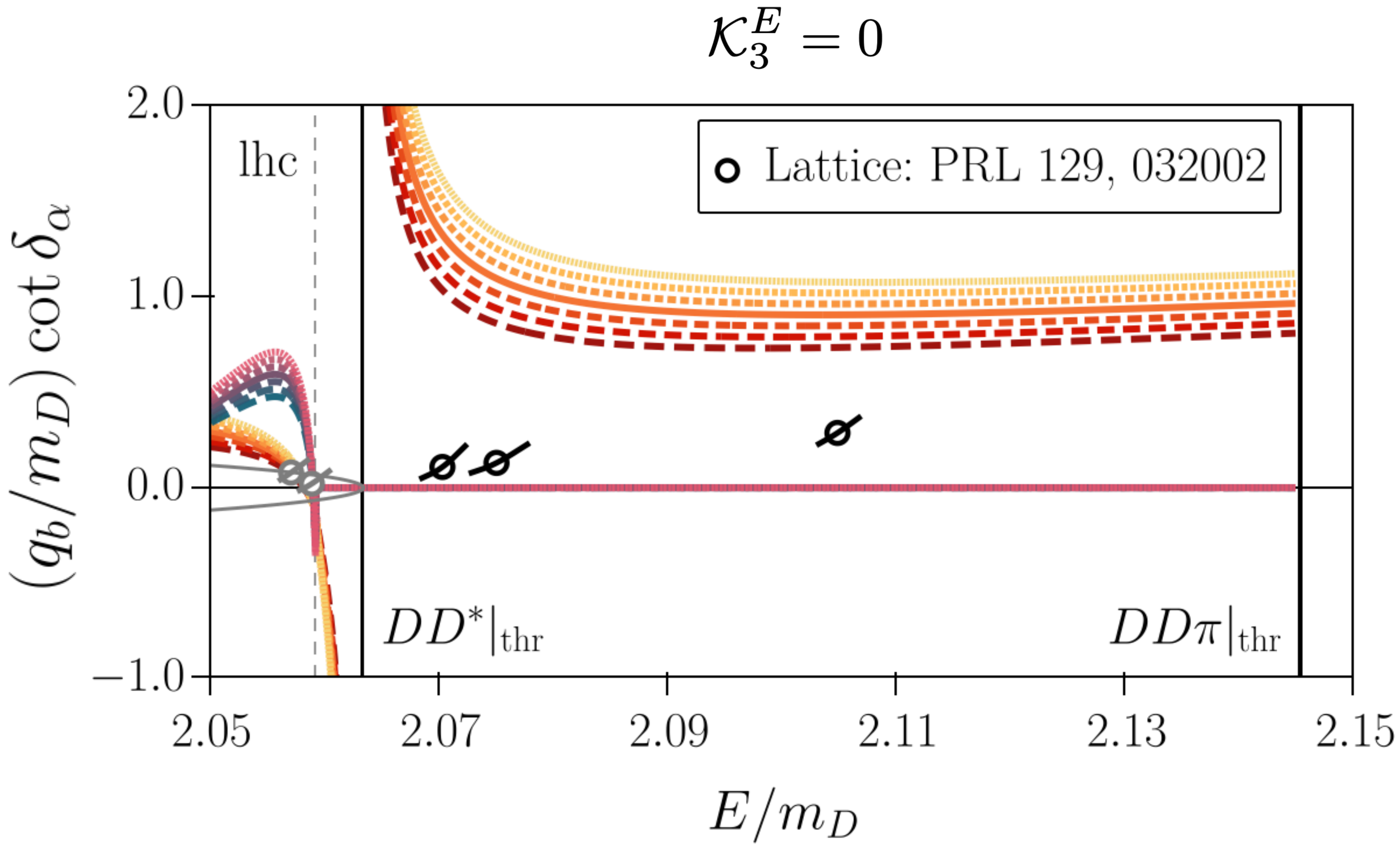
(a) we find a sub-threshold complex pole (agreement with the NR EFT analysis)
(b) simple model of three-body forces is enough to describe data
(c) partial-wave mixing is small (not shown here)
(d) Dπ S-wave scattering is (almost) negligible (not shown here)

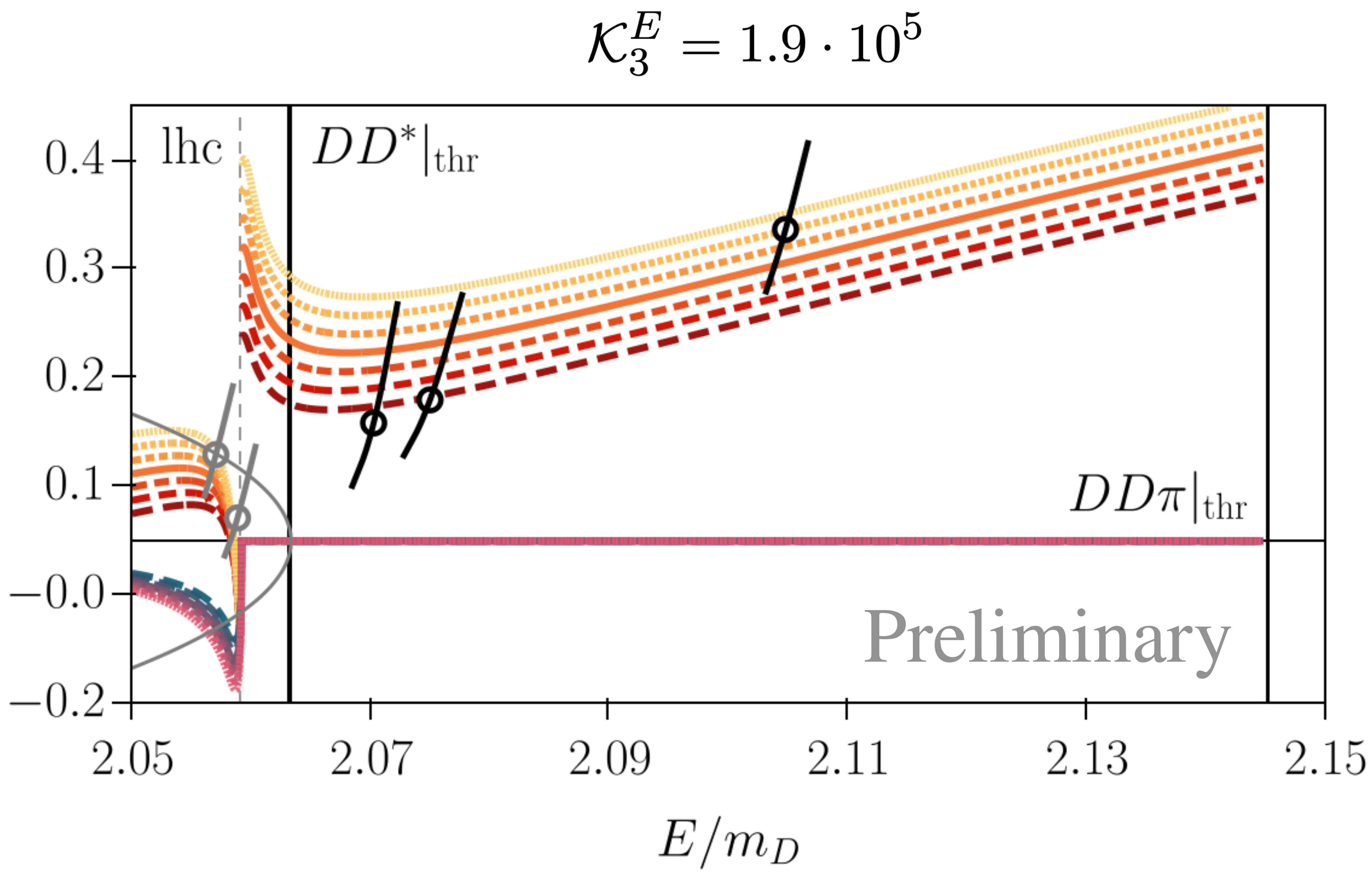
Partial-wave mixing amplitude

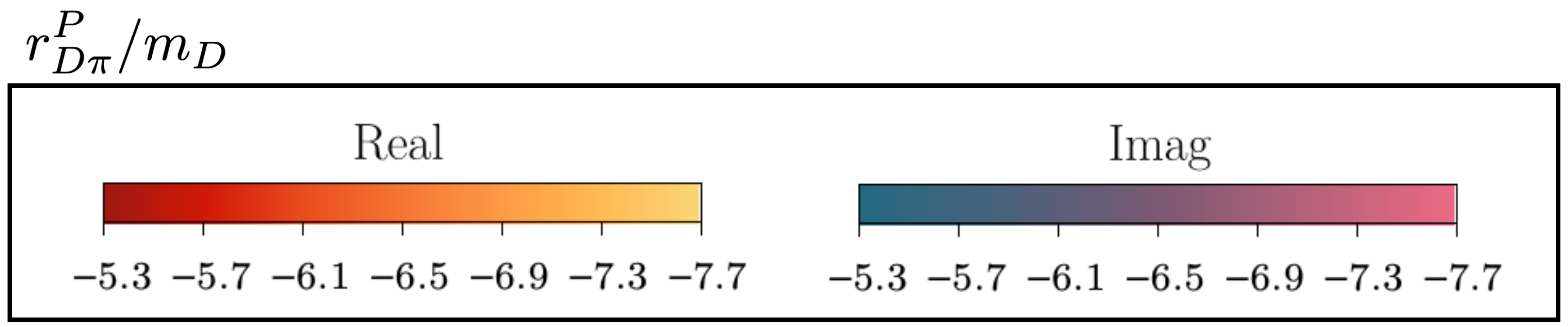
Blatt–Biederharn parametrization

$$\begin{bmatrix} \mathcal{M}_{DD^*}({}^3S_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3S_1|{}^3D_1) \\ \mathcal{M}_{DD^*}({}^3D_1|{}^3S_1) & \mathcal{M}_{DD^*}({}^3D_1|{}^3D_1) \end{bmatrix}$$

$$q_b^{-\ell'}\left[\mathcal{K}^{-1}_{DD^*}\right]_{\ell',\ell} q_b^{-\ell} = \begin{pmatrix} \cos(\epsilon) & -\frac{1}{q_b^2}\sin(\epsilon) \\ q_b^2\sin(\epsilon) & \cos(\epsilon) \end{pmatrix} \begin{pmatrix} q_b\cot(\delta_\alpha) & 0 \\ 0 & q_b^5\cot(\delta_\beta) \end{pmatrix} \begin{pmatrix} \cos(\epsilon) & q_b^2\sin(\epsilon) \\ -\frac{1}{q_b^2}\sin(\epsilon) & \cos(\epsilon) \end{pmatrix}$$







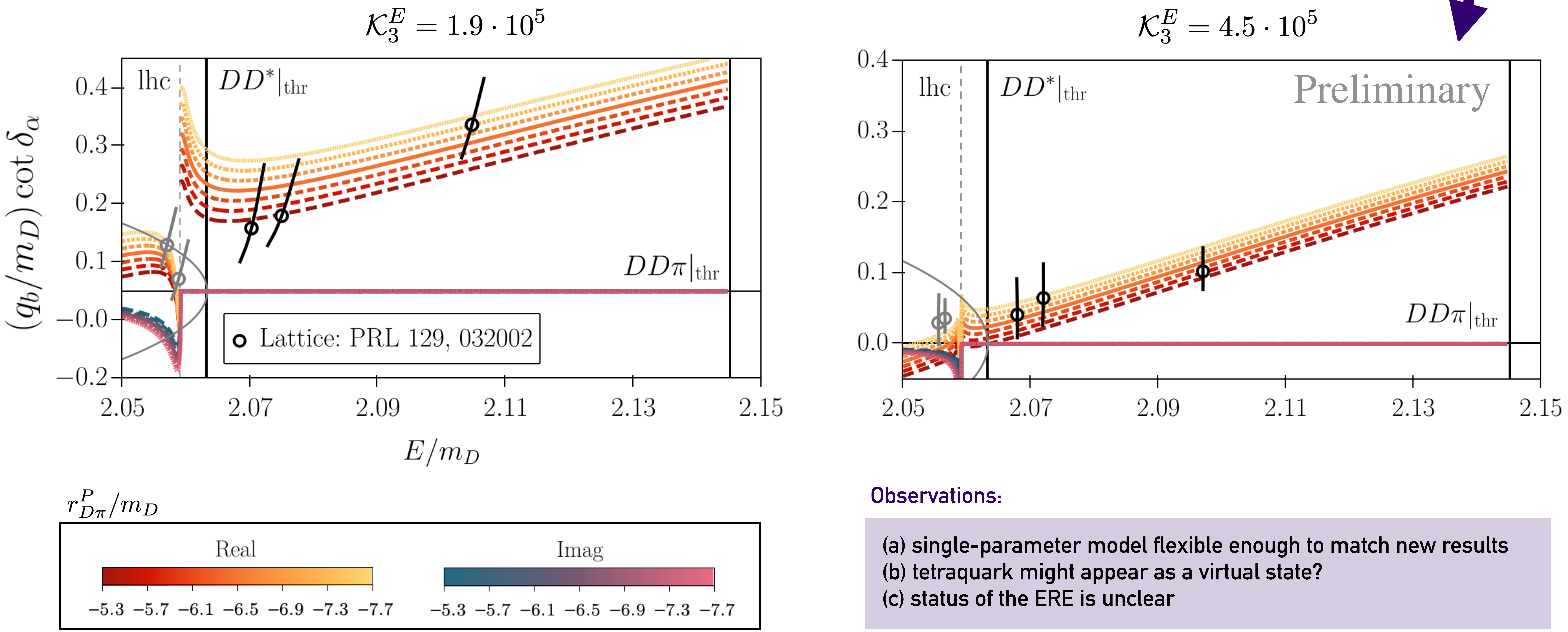
Observations:

(a) we find a sub-threshold complex pole (agreement with the NR EFT analysis)
(b) simple model of three-body forces is enough to describe data
(c) partial-wave mixing is small (not shown here)
(d) Dπ S-wave scattering is (almost) negligible (not shown here)

The recently updated lattice data

Ortiz-Pacheco et al. arXiv:2312.13441

Preliminary (unpublished) data set presented at the Lattice 24 conference talk:
Ivan Vujmilovic "T_{cc} via plane-wave approach and including diquark-antidiquark operators"



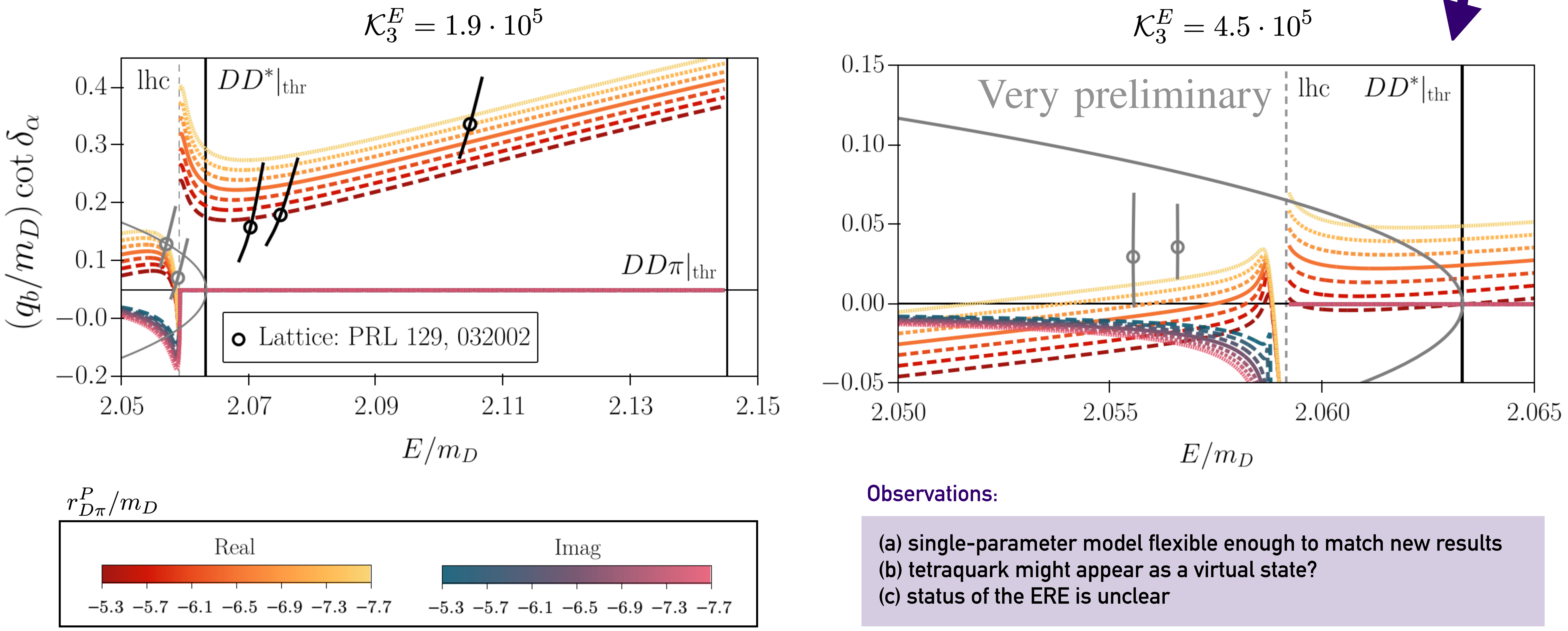
Observations:

(a) single-parameter model flexible enough to match new results
(b) tetraquark might appear as a virtual state?
(c) status of the ERE is unclear

The recently updated lattice data

Ortiz-Pacheco et al. arXiv:2312.13441

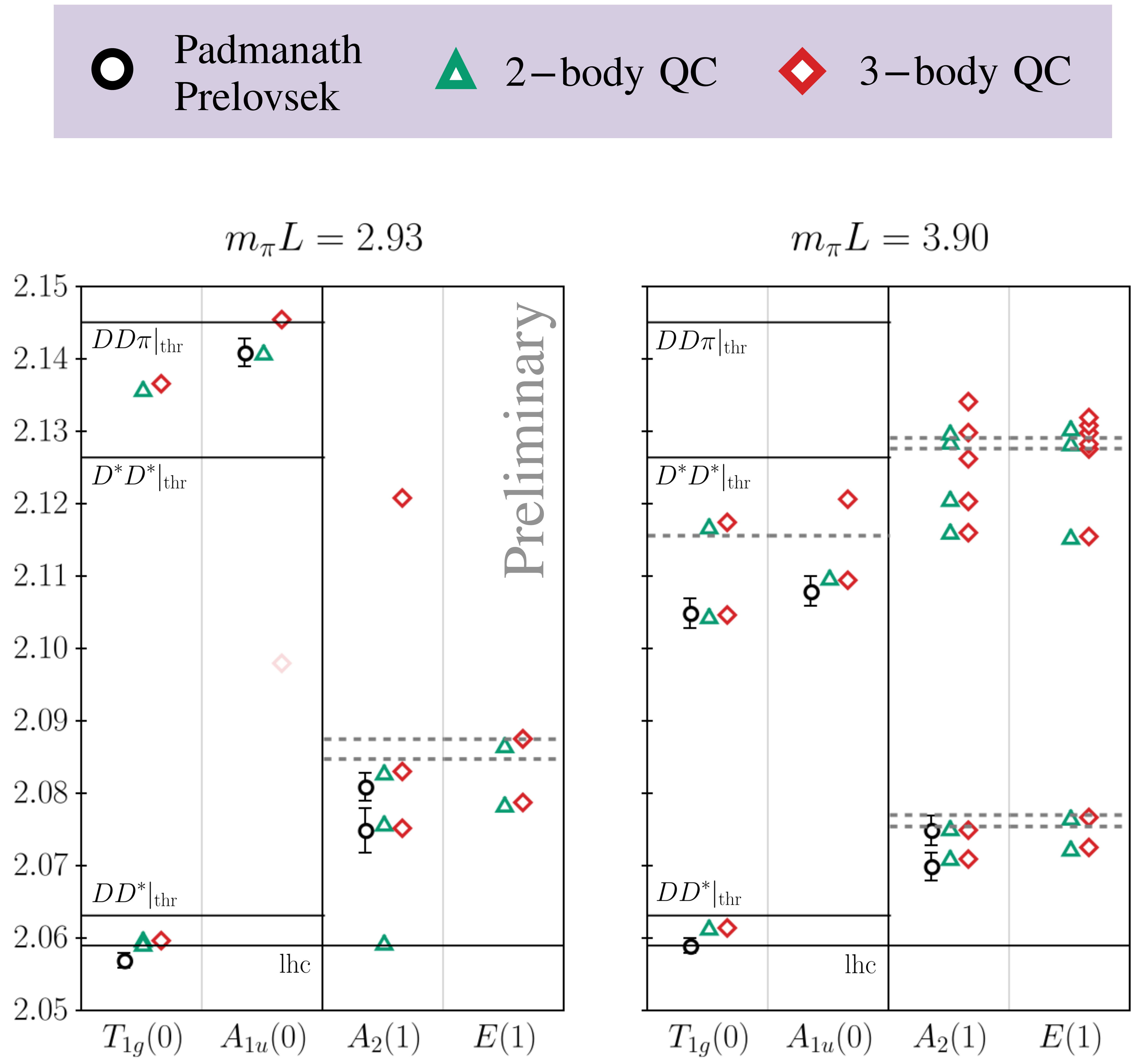
Preliminary (unpublished) data set presented at the Lattice 24 conference talk:
Ivan Vujmilovic "T_{cc} via plane-wave approach and including diquark-antidiquark operators"



Observations:

(a) single-parameter model flexible enough to match new results
(b) tetraquark might appear as a virtual state?
(c) status of the ERE is unclear

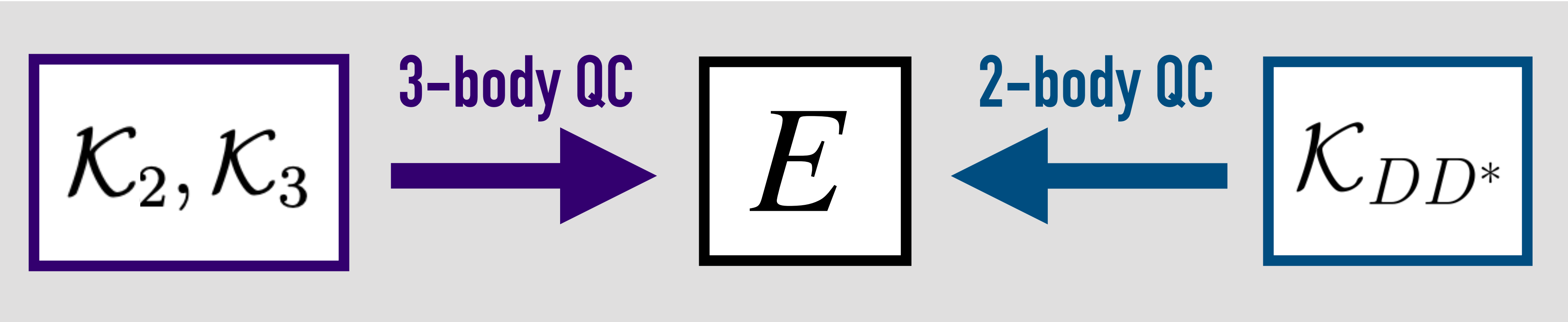
Comparing the finite-volume spectra



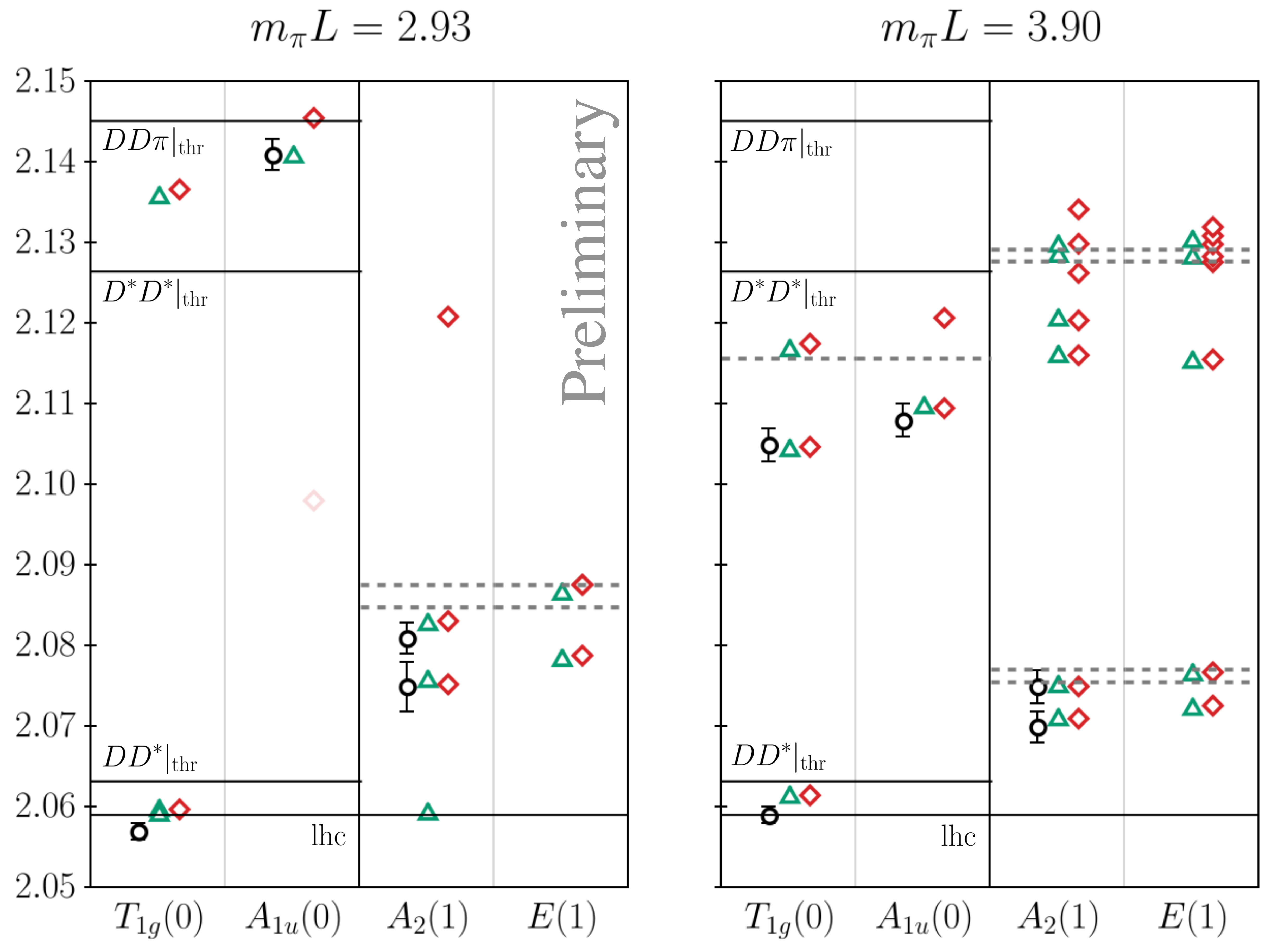
Further observations

- two-body QC breaks down near the lhc for too small lattice volumes
- attractive interaction in J=0 consistent with the lattice; (other K_3 terms needed)

Comparing the finite-volume spectra

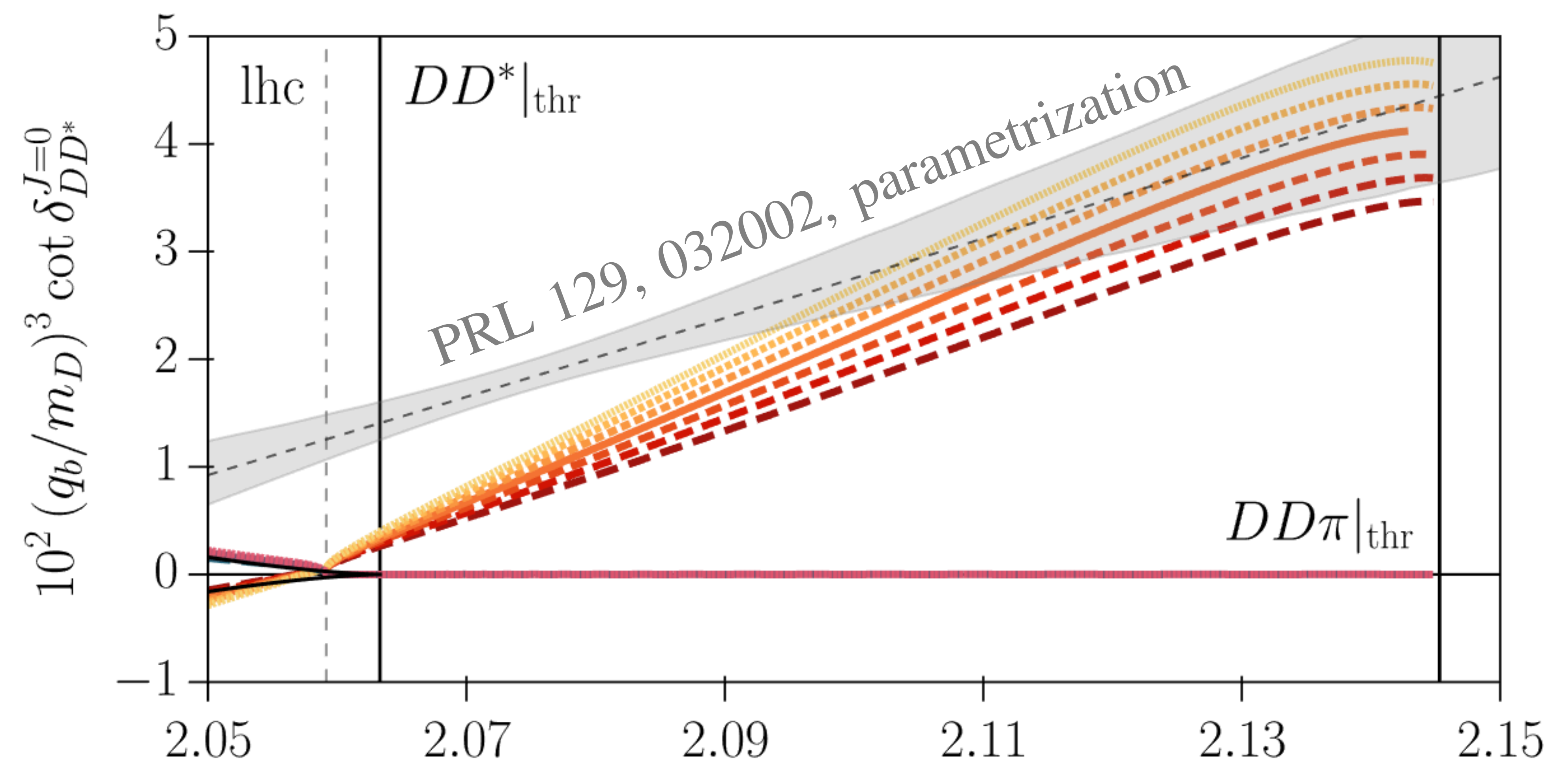


○ Padmanath Prelovsek △ 2−body QC ◇ 3−body QC



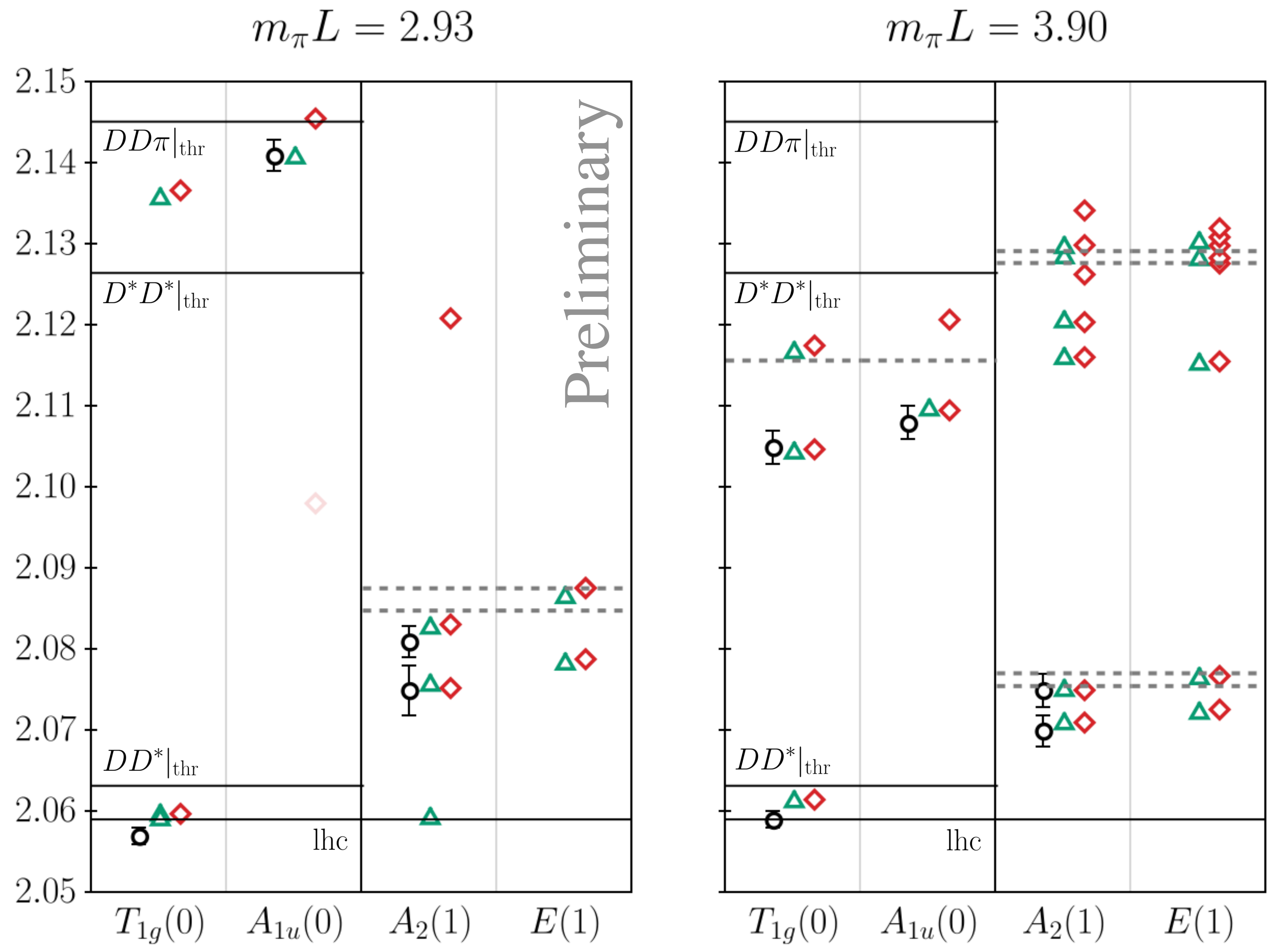
The $J^P = 0^-$ amplitude

$$\begin{bmatrix} \mathcal{M}_3({}^1S_0|{}^1S_0) & \mathcal{M}_3({}^1S_0|{}^3P_0) \\ \mathcal{M}_3({}^3P_0|{}^1S_0) & \mathcal{M}_3({}^3P_0|{}^3P_0) \end{bmatrix}$$



Comparing the finite-volume spectra

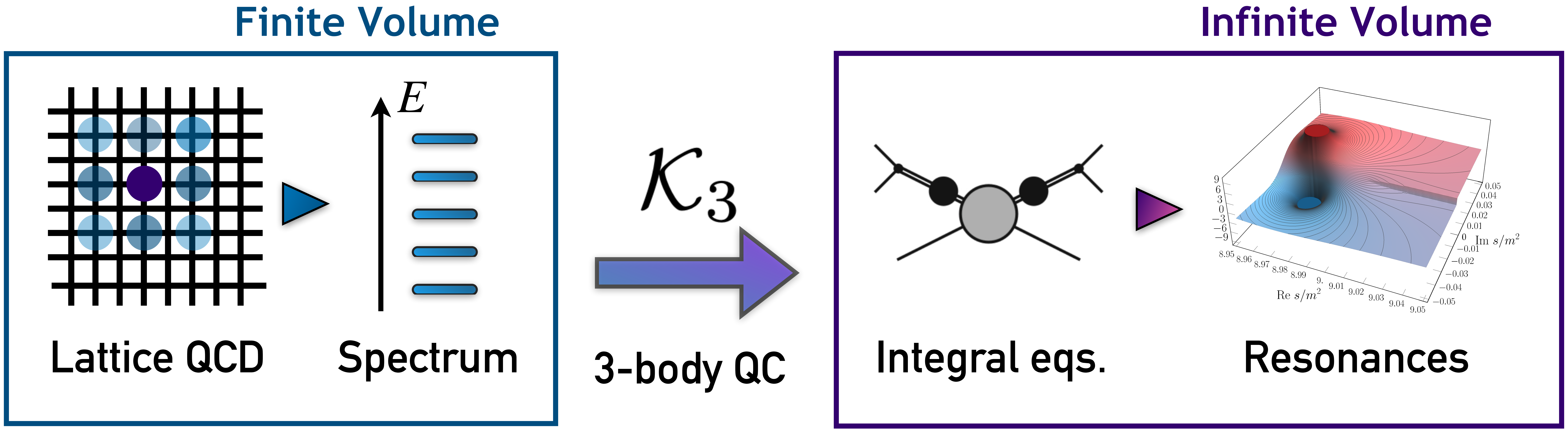
Padmanath Prelovsek | 2−body QC | 3−body QC



Further observations

- two-body QC breaks down near the lhc for too small lattice volumes
- attractive interaction in J=0 consistent with the lattice; (other K_3 terms needed)

Summary



Towards the tetraquark from Lattice QCD

- [x] resolution of the left-hand cut problem
- [x] generalization of the three-body equations
- [x] comparison with the existing lattice results
- [x] model of T_{cc} = initial condition for LQCD studies

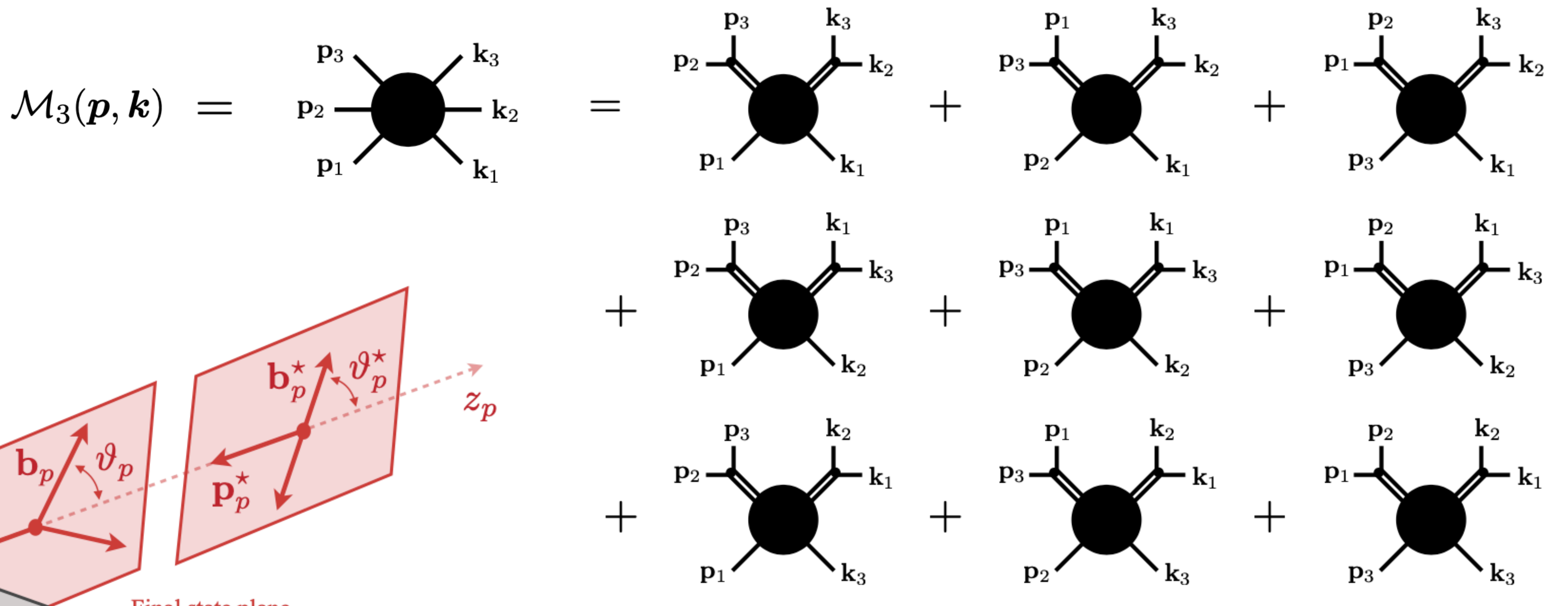
Next steps

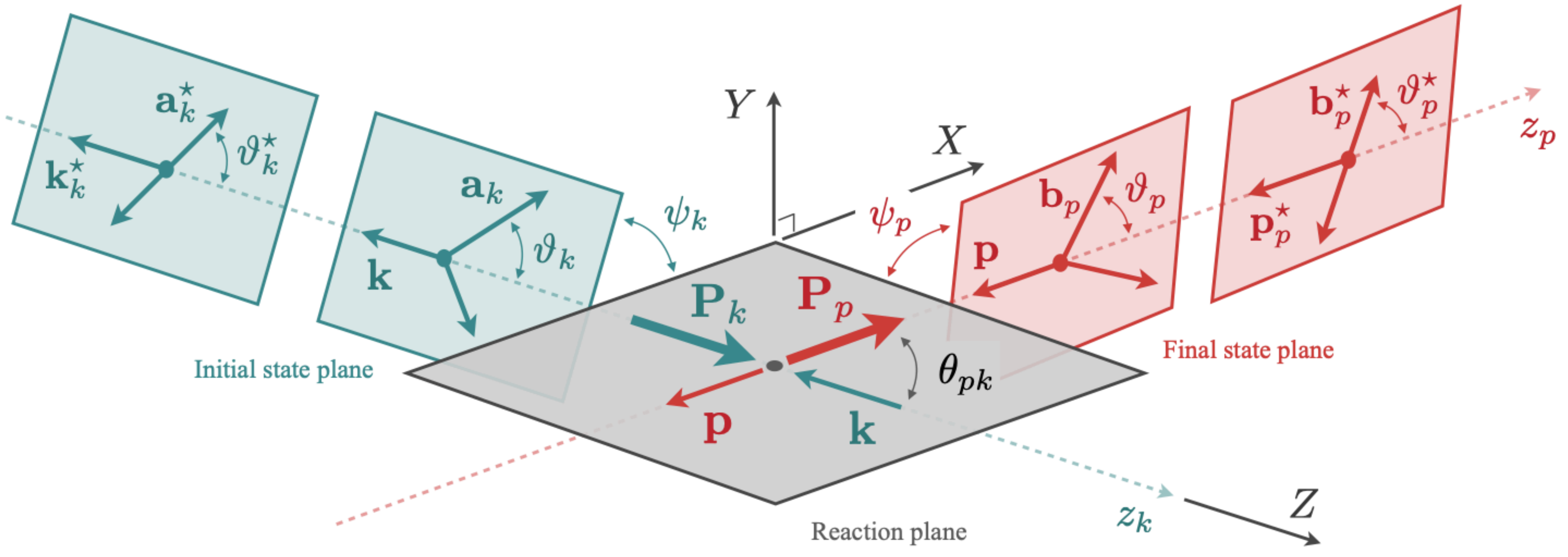
- [ ] Systematics of the K matrices
- [ ] Systematic application to lattice data
- [ ] Three-body computation of T_{cc}
- [ ] Formalism for the Roper resonance

THANK YOU

Pair-spectator amplitude

Illustrations by A. Jackura (arXiv:2208.10587, arXiv:2312.00625)



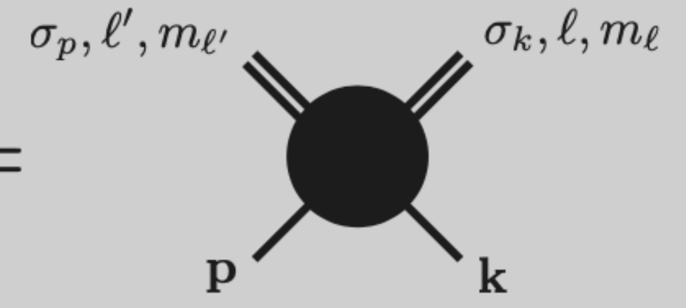


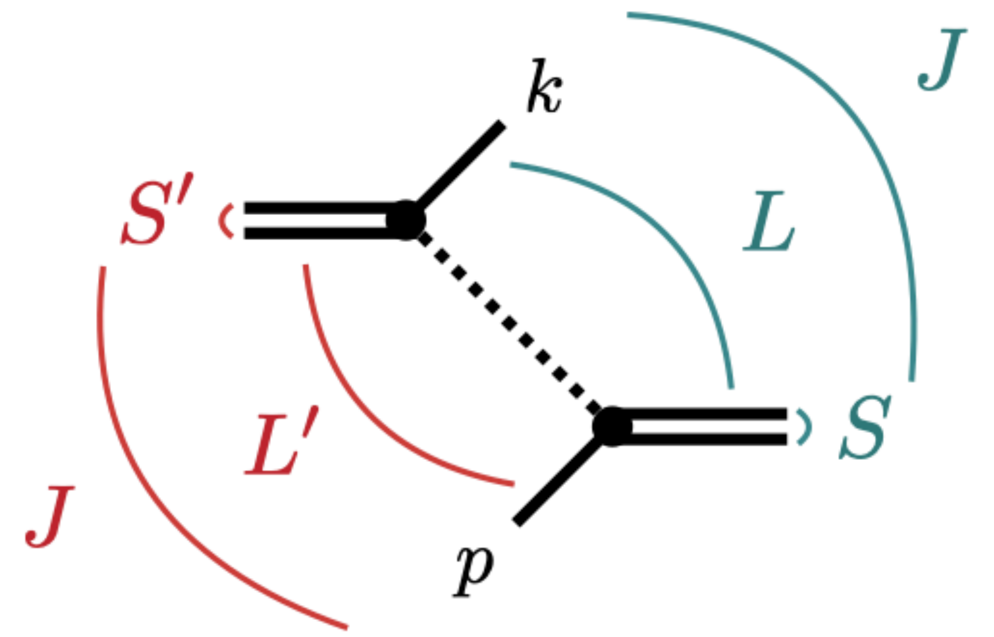
$$\mathcal{M}_3(\boldsymbol{p};\boldsymbol{k}) = \sum_{i,j}\sum_{J}\sum_{L'_i,S'_i}\sum_{L_j,S_j}\mathcal{M}^{(u,u)J}_{3,L'_iS'_i;L_jS_j}(p_i,k_j;E)\sum_{M}\mathcal{Z}^{JM*}_{L'_iS'_i}(\boldsymbol{P}_{p_i},\boldsymbol{q}^\star_{p_i})\,\mathcal{Z}^{JM}_{L_jS_j}(\boldsymbol{P}_{k_j},\boldsymbol{q}^\star_{k_j})$$

$$\mathcal{Z}^{JM}_{L_jS_j}(\boldsymbol{P}_{k_j},\boldsymbol{q}^\star_{k_j}) = \sqrt{4\pi}\sqrt{\frac{2L_j+1}{2J+1}}\sum_{m_j}(Jm_j|L_j0S_jm_j)\,D^J_{Mm_j}(\boldsymbol{P}_{k_j})\,Y^*_{S_jm_j}(\hat{\boldsymbol{q}}_{k_j})$$

Pair−spectator amplitude

$$\mathcal{M}^{(u,u)}_{3;\ell'm'_\ell;\ell m_\ell}(\boldsymbol{p},\boldsymbol{k}) =$$





On-shell three-body elastic amplitude depends on eight kinematical variables:
- two angles defining orientation of the initial-state pair
- two angles defining orientation of the final-state pair
- one angle defining orientation between spactators (pairs)
- invariant mass of the initial and final pair
- total energy