Recent progresses in few-nucleon structure and dynamics in chiral effective field theory

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Outline

- The plethora of χEFT potentials
  - The adopted χEFT potentials
- Available electro-weak currents
- The Hyperspherical Harmonics *ab-initio* method
- Selected results
  - $A = 2$: muon capture on deuteron
  - $A = 3$: the *pd* radiative capture
  - $A = 4$: – the ^{4}He monopole form factor
    – the $^3\mathrm{He}(\vec{n}, p)\,^3\mathrm{H}$ parity-conserving asymmetry
- Conclusions and outlook

The χEFT *NN* interaction models

R. Machleidt and F. Sammarruca — *Progress in Particle and Nuclear Physics 137 (2024) 104117*

Table 3.1
Chiral NN potentials published during the past 20 years.

Year	Authors	Name	Order(s)	Δ's[a]	Locality	Cutoff(s)	Max. T_{lab}[b]	χ^2/datum[c]	Ref(s).
Set 1 (Early Birds):									
2003	Entem, Machleidt	Idaho	N^3LO	No	Nonlocal	500 MeV	300 MeV	1.3	[39]
2005	Epelbaum et al.	Bochum	N^3LO	No	Nonlocal	450, 600 MeV	300 MeV	14.5, 2.3	[83]
Set 2 (Göteborg/Oak Ridge):									
2013	Ekström et al.	$NNLO_{opt}$	NNLO	No	Nonlocal	500 MeV	290 MeV	9.5	[86]
2015	Ekström et al.	$NNLO_{sat}$	NNLO	No	Nonlocal	450 MeV	35 MeV	39.0[d], 42.7[e]	[93]
2018	Ekström et al.	ΔNNLO	NNLO	Yes	Nonlocal	450 MeV	200 MeV	25.1[f]	[94]
2020	Jiang, Ekström et al.	$\Delta NNLO_{GO}$	NNLO	Yes	Nonlocal	394, 450 MeV	200 MeV	32.6[f], 29.6[f]	[99]
Set 3 (Configuration space):									
2014	Gezerlis et al.		LO-NNLO	No	Local	1.0–1.2 fm	250 MeV	12.2[g]	[87]
2015	Piarulli et al.		NNLO/N^3LO[h]	Yes	Min. nonloc.[i]	0.8–1.2 fm	300 MeV	1.35(2)	[88]
2016	Piarulli et al.	Norfolk, NV2	LO-N(3)LO[h]	Yes	Local	0.8–1.2 fm	200 MeV	1.40[j]	[89]
2023	Saha et al.	Idaho	LO-N^3LO	No	Local	1.0–1.2 fm	200 MeV	1.45[k]	[100]
2023	Somasundaram et al.	LO-N^3LO_{LA}	LO-N^3LO	No	Max. local[l]	0.6–0.9 fm	400 MeV		[101]
Set 4 (Latest high accuracy and high precision at N^4LO):									
2015	Epelbaum et al.	LENPIC	LO-N^4LO	No	Semilocal[m]	0.8–1.2 fm	300 MeV		[84,85]
2017	Entem et al.	Idaho	LO-N^4LO	No	Nonlocal	450–550 MeV	300 MeV	1.15[n]	[95]
2018	Reinert et al.	LENPIC	LO-N^4LO^+	No	Semilocal[m]	350–550 MeV	300 MeV	1.03[p]	[92]
2021	Nosyk et al.	Idaho	LO-NNLO	Yes	Nonlocal	394, 450 MeV	200 MeV	3.71[q]	[63]

- **Plethora of models** $\rightarrow$ 15
- **Mainly non-local** ($\rightarrow$ more difficult when charged scattering systems)

R. Machleidt and F. Sammarruca, Prog. Part. Nucl. Phys. **137**, 104117 (2024)

The adopted χEFT nuclear interactions

Two-nucleon interactions:

- Entem and Machleidt 2003: N3LO, $\Lambda = 500, 600$ MeV [N3LO500/600]
- Entem, Machleidt, and Nosyk 2017: LO-N4LO, $\Lambda = 450, 500, 600$ MeV [EMN450/500/550]
- Piarulli *et al.*: N3LO, $(R_\mathrm{S}, R_\mathrm{L}) = (0.7/0.8, 1.0/1.2)$ fm [NV I/II a/b]

Name	DOF	O_χ	$(R_\mathrm{S}, R_\mathrm{L})$ or Λ	E range	Space
NVIa	π, N, Δ	N3LO	(0.8,1.2) fm	0–125 MeV	r
NVIb	π, N, Δ	N3LO	(0.7,1.0) fm	0–125 MeV	r
NVIIa	π, N, Δ	N3LO	(0.8,1.2) fm	0–200 MeV	r
NVIIb	π, N, Δ	N3LO	(0.7,1.0) fm	0–200 MeV	r
EMN450	π, N	N3LO	450 MeV	0–300 MeV	p
EMN500	π, N	N3LO	500 MeV	0–300 MeV	p
EMN550	π, N	N3LO	550 MeV	0–300 MeV	p

A. Gnech, L.E. Marcucci, and M. Viviani, Phys. Rev. C **109**, 035502 (2024)

Three-nucleon interactions:

- N2LO-local, Λ consistent with N3LO and EMN potentials
- N2LO with Δ's, $(R_\mathrm{S}, R_\mathrm{L})$ consistent with NV potentials

$c_D - c_E$ from $B(A = 3)$ and GT^exp of ^{3}H β-decay

The electro-weak nuclear current operator

Two groups (Bochum & Jlab-Pisa) with different expressions and power counting

Bochum group

Oper.	LO (Q^{-3})	NLO (Q^{-2})	N2LO (Q^{-1})	N3LO (Q^{0})
$\rho(A)$	—	—	1b(NR)	—
$\mathbf{j}(A)$	1b(NR)	—	OPE OPE-Δ^* [1b(RC)]	CT(d_R) OPE
$\rho(V)$	1b(NR)	—	[1b(RC)]	[OPE(RC)]
$\mathbf{j}(V)$	—	—	1b(NR) OPE	OPE-Δ^* [1b(RC)]

Jlab-Pisa

Oper.	LO (Q^{-3})	NLO (Q^{-2})	N2LO (Q^{-1})	N3LO (Q^{0})	N4LO (Q^{1})
$\rho(A)$	—	1b(NR)	OPE	—	TPE^A CT†
$\mathbf{j}(A)$	1b(NR)	—	1b(RC) OPE-Δ^*	CT(d_R) OPE	TPE OPE(sub)
$\rho(V)$	1b(NR)	—	1b(RC)	OPE(RC)	TPE
$\mathbf{j}(V)$	—	1b(NR)	OPE	1b(RC) OPE-Δ^*	TPE OPE(d_2^V, d_3^V, d_2^S) CT(d_1^V, d_1^S)

A. Gnech, L.E. Marcucci, and M. Viviani, Phys. Rev. C **109**, 035502 (2024)

→ Jlab-Pisa model

- **Axial opertors**: A. Baroni *et al.*, Phys. Rev. C **93**, 015501 (2016); Phys. Rev. C **98**, 044003 (2018)
- **EM/vector operators**: S. Pastore *et al.*, Phys. Rev. C **80**, 034004 (2009); M. Piarulli *et al.*, Phys. Rev. C **87**, 014006 (2013); A. Gnech and R. Schiavilla, Phys. Rev. C **106**, 044001 (2022)

The Hyperspherical Harmonics (HH) method

Bound states

$$\Psi^{JJ_z} = \sum_\mu c_\mu \Psi_\mu$$

- $\Psi_\mu \to$ known functions (spin-isospin HH functions)
- Rayleigh-Ritz var. principle: $\delta_c \langle \Psi^{JJ_z} | H - E | \Psi^{JJ_z} \rangle = 0$
  $\Rightarrow$ Solve for E and c_μ

Scattering states

$$\Psi_{LSJ} = \Psi^{LSJ}_{core} + \Psi^{LSJ}_{asym}$$

- $\Psi^{LSJ}_{core} = \sum_\mu c_\mu \Psi_\mu$
- $\Psi^{LSJ}_{asym} \propto \Omega^R_{LS} + \sum_{L'S'} R_{LL',SS'} \Omega^I_{L'S'}$
- Kohn var. principle:
  $[R_{LL',SS'}] = R_{LL',SS'} - \langle \Psi_{L'S'J} | H - E | \Psi_{LSJ} \rangle$
  $\Rightarrow$ Solve for c_μ and $R_{LL',SS'} \to$
  phase-shifts and mixing angles

Strength and weakness

- very accurate
- both r- and p-space
- both bound and scattering states

- at present limited to $A = 6$
- in prospective $A = 8$
- not much more ...

L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

SELECTED RESULTS

- $A = 2$: muon capture on deuteron

  A. Gnech, L.E. Marcucci, and M. Viviani, Phys. Rev. C **109**, 035502 (2024)

Bayesian analysis for $\mu^- + d \to n + n + \nu_\mu$

$\Gamma^{1/2}$&$\Gamma^{3/2}$ calculation with a **robust theoretical uncertainty evaluation** $\to$

- LECs in the nuclear electroweak currents

$$
\begin{aligned}
g_A(q^2) &= g_A\left(1 - \frac{1}{6}{r_A}^2 q^2\right) \\
d_R &= -\frac{M_N}{4\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6} \\
\sigma^2_{\mathrm{LECs}} &= \left(\frac{\partial\Gamma}{\partial r_A^2}\right)^2 \sigma^2(r_A^2) + \left(\frac{\partial\Gamma}{\partial d_R}\right)^2 \sigma^2(d_R)
\end{aligned}
$$

- Chiral expansion truncation for current and interaction
  $\to$ Bayesian analysis as in J A. Melendez *et al.*, Phys. Rev. C **100**, 044001 (2019)
  $\to$ $\sigma^{C(I)}_{k=3(4)}$ = standard deviation of the truncation error for C/I at the $k = 3/4$-th order
- Dependence on the nuclear interaction model $\to$ various interaction models

Model	$\Gamma^{1/2}$	$\sigma^C_{k=3}$[BPC]	$\sigma^C_{k=3}$[JPPC]	$\sigma^I_{k=4}$	$\sigma_{\rm LECs}$	$\Gamma^{3/2}$
NVIa	393.5	5.1	1.1	n.a.	3.9	13.3
NVIb	393.7	5.1	1.1	n.a.	3.9	13.5
NVIIa	392.5	5.1	1.1	n.a.	3.9	13.3
NVIIb	392.6	5.1	1.1	n.a.	3.9	13.3
EMN450	396.0	6.4	2.2	0.3	3.9	13.7
EMN500	397.3	6.0	2.2	0.3	3.9	13.8
EMN550	397.0	6.2	2.1	0.4	3.9	13.8

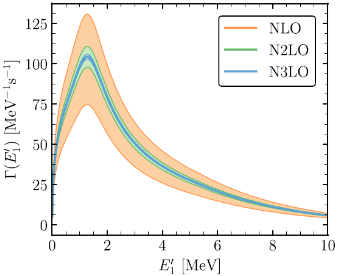
All the uncertainties are reported at 68% CL

Absolute and **relative** contributions to the estimated total theoretical error

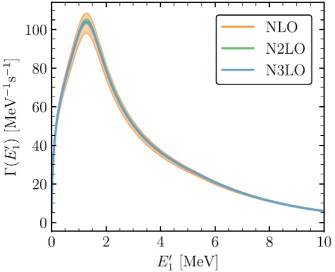
W.I. Jay and E.T. Neil, Phys. Rev. D **103**, 114502 (2021)

Uncertainty source	BPC	JPPC
r_A^2	5.6(30.8%)	5.4(75.2%)
Other current LECs	negligible	
χEFT truncation - currents	8.1(65.2%)	2.4(14.6%)
χEFT truncation - interactions	0.5(0.3%)	0.5(0.7%)
Model dependence	1.9(3.7%)	1.9(9.5%)

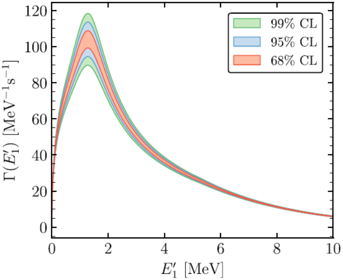
EMN550@N3LO, JPPC current @ NLO–N3LO



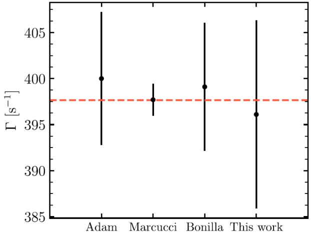
EMN550@NLO–N3LO, JPPC current order fixed



Recommended differential capture rate

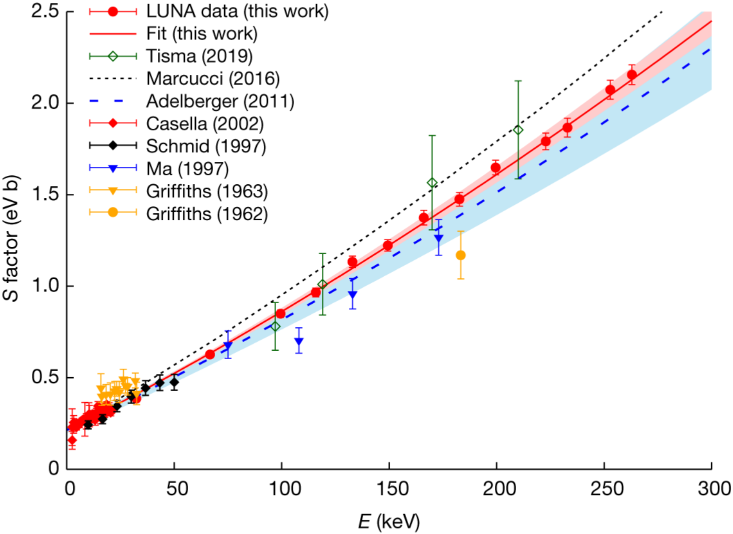


$$\Gamma_{\rm th}(\rm BPC) = (395 \pm 10)\,s^{-1} \quad (68\%CL)$$
$$\Gamma_{\rm th}(\rm JPPC) = (395 \pm 6)\,s^{-1} \quad (68\%CL)$$



SELECTED RESULTS

- $A = 3$: the *pd* radiative capture



V. Mossa *et al.* (LUNA Collab.), Nature **587**, 210 (2020)

AV18/UIX phenomenological approach $\rightarrow \sim 10$**% discrepancy**

Theoretical error? $\Rightarrow \chi$EFT

Preliminary results (work in progress)

Local potentials NV2+3 I/II a/b* and associated currents

E_{cm} [keV]	AV18/UIX	χEFT-NV Full	χEFT-NV Siegert	Exp. [LUNA]
262.9	2.37	2.35(2)	2.33(3)	2.156(58)
252.9	2.28	2.26(2)	2.24(3)	2.073(53)
232.9	2.09	2.09(2)	2.04(3)	1.866(52)

Further work:

- check numerics and convergences

NV2+3 I/a*, $E_{cm} = 252.9$ keV, MC=800k config., QRN= 50k

N_{Lag}	LO(MC)	Full(MC)	Sieg.(MC)	Sieg.(QRN)
26	1.96	2.26	2.19	2.18
28	1.95	2.25	2.21	2.21
30	1.96	2.26	2.23	2.23
32	1.96	2.26	2.24	2.24
$K_{\max}^{(1-3)}$	LO(MC)	Full(MC)	Sieg.(MC)	Sieg.(QRN)
60	1.95	2.25	2.09	2.11
80	1.95	2.25	2.21	2.22
90	1.96	2.26	2.23	2.24
100	1.96	2.26	2.24	2.24

- study other energies (I am afraid the discrepancy will remain)
- apply other potentials $\rightarrow$ EMN LO-N4LO $\Lambda = 450, 500, 550$ MeV

SELECTED RESULTS

- $A = 4$: the ^{4}He monopole form factor

  M. Viviani *et al.*, Few-Body Syst. **65**, 74 (2024)

$^4\mathrm{He}(e,e'p)^3\mathrm{H}$ & $^4\mathrm{He}(e,e'n)^3\mathrm{He}$

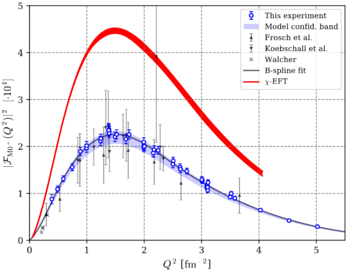
At energies below the $d+d$ threshold

$$|F_M(q)|^2 = \frac{d\sigma}{d\Omega} \Big/ \left[Z^2 4\pi \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \right] = \frac{1}{16\pi} \sum_{\gamma=1,2} \int_0^\infty dE_\gamma \; 8 p_\gamma \mu_\gamma |C_0^{000}(q, E_\gamma)|^2$$

$E_\gamma = p/n$ energy
$\mu_\gamma = p + {}^3\mathrm{H}/n + {}^3\mathrm{He}$ reduced mass
$p_\gamma = p + {}^3\mathrm{H}/n + {}^3\mathrm{He}$ relative momentum
$q =$ momentum transferred
$C_0^{000}(q, E_\gamma)$ defined as $(0_0^+ \to 0_1^+)$

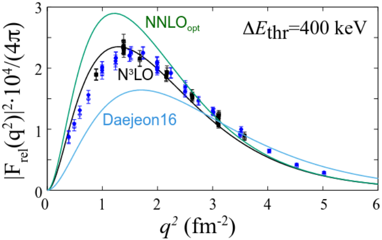
$$C_0^{000}(q, E_\gamma) = \frac{1}{\sqrt{4\pi}} \langle \Psi^{1+3}_{J=L=S=0} | \hat{\rho}(\boldsymbol{q}) | \Psi({}^4\mathrm{He}) \rangle$$

Triggered by S. Kegel *et al.*, Phys. Rev. Lett. **130**, 152502 (2023)

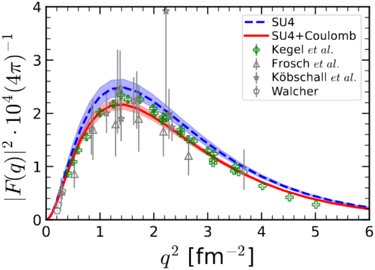


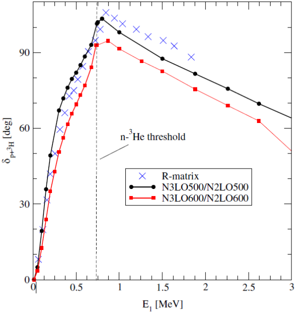
However

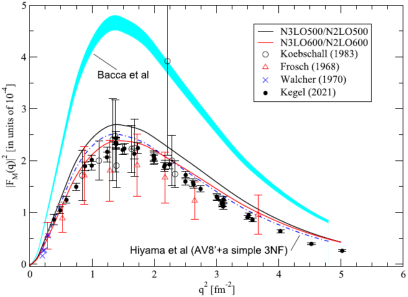
N. Michel *et al.*, Phys. Rev. Lett. **131**, 242502 (2023)

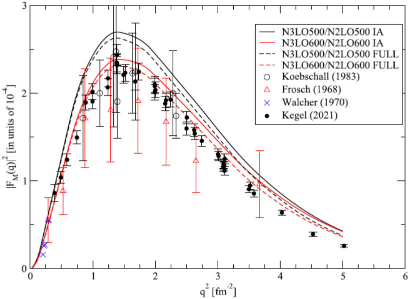


U.-G. Meißner *et al.*, Phys. Rev. Lett. **132**, 062501 (2024)









Good agreement with N. Michel *et al.*, Phys. Rev. Lett. **131**, 242502 (2023)
U.-G. Meißner *et al.*, Phys. Rev. Lett. **132**, 062501 (2024)

SELECTED RESULTS

- $A = 4$: the ^{3}He$(\vec{n}, p)\,{}^3$H parity-conserving asymmetry

  M. Viviani *et al.*, arXiv: 2405.10258 (2024)

A_{PC} in $^3\mathrm{He}(\vec{n}, p)^3\mathrm{H}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{u}} \left(1 + A_{\mathrm{PV}} \hat{s}_n \cdot \hat{k}_p + A_{\mathrm{PC}} (\hat{s}_n \times \hat{k}_n) \cdot \hat{k}_p\right)$$

$A_{\mathrm{PC}} \rightarrow k_p$ in the plane defined by $\hat{s}_n \times \hat{k}_n$ and $\hat{k}_n$
$\rightarrow$ sensitive to the interaction in P-waves
$A_{\mathrm{PV}} \rightarrow k_p$ in the plane defined by $\hat{s}_n$ and $\hat{k}_n$

$$A_{\mathrm{PC}} = (-42.4 \pm 6.0(\mathrm{stat}) \pm 0.23(\mathrm{sys})) \times 10^{-8} = \frac{\beta}{\lambda_n} \Rightarrow$$

$$\boxed{\beta = (-1.97 \pm 0.28(\mathrm{stat}) \pm 0.02(\mathrm{sys})) \times 10^{-6}\,\mathrm{\AA}}$$

HH method + N3LO500/600 or NVIa/b with/without TNI

Interaction	$E_{\rm R}(0^-)$ [MeV]	$\Gamma(0^-)$ [MeV]	${}^0t^{2,1}_{11,11}$ [fm]	$\beta \times 10^6$ [Å]
N3LO500	0.16	0.41	20.7	− 4.83
N3LO600	0.24	0.51	16.9	− 2.68
NVIa	0.31	0.53	17.3	− 2.61
NVIb	0.30	0.54	13.0	− 0.43
N3LO500/N2LO500	0.06	0.26	30.1	−10.04
N3LO500/N2LO500*	0.14	0.41	18.5	− 2.68
N3LO600/N2LO600	0.09	0.30	25.9	− 5.28
NVIa/3N	0.04	0.36	35.4	−12.17
NVIb/3N	0.12	0.40	23.9	− 5.15
Experimental	0.44	0.84		−1.97 ±0.28 (stat) ±0.12 (sys)

${}^0t^{2,1}_{11,11} = \lim_{q_2 \to 0} |{}^0T^{2,1}_{11,11}|/q_2$, $({}^JT^{\gamma,\gamma'}_{LS,L'S'})$

- tiny differences in $E_{\rm R}(0^-)$ $\Rightarrow$ large changes in ${}^0t^{2,1}_{11,11}$
- large differences since leading contributions cancel out
- $A_{\rm PC} > 0$ without ${}^0T^{2,1}_{11,11}$

$\Rightarrow$ critical dependence on ${}^0T^{2,1}_{11,11}$ $\Rightarrow$ on *NN* interaction in *P*-waves and TNI

Conclusions and outlook

Conclusions

- Recent results for selected $A = 2, 3, 4$ observables in χEFT
- Moving forward with the most recent χEFT potentials/currents
- Some interesting "puzzles" remain ($p + d \to {}^3\mathrm{He} + \gamma$, ${}^3\mathrm{He}(\vec{n}, p)^3\mathrm{H}$)

Outlook

- Try to solve the "puzzles" (obvious!)
- Extend our studies to the largest variety of χEFT potentials/currents as possible
- Extend our studies to $A = 5, 6$ nuclear systems

A. Gnech *et al.*, Phys. Rev. C **102**, 014001 (2020) & Phys. Rev. C **104**, 035501 (2021)

Thank you for your attention!

In collaboration with

- A. Kievsky and M. Viviani (INFN-Pisa)
- L. Girlanda (Univ. del Salento)
- A. Gnech, R. Schiavilla (JLab-ODU)