Anthropic Considerations for Big Bang Nucleosynthesis

Chiral Dynamics 2024

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Motivation

- Fundamental constants: show up in every discipline of science
- We know them to precisions given units of parts per 10^9 [1]

permeability of free space	μ_0	$4\pi \times 10^{-7}$ N A^{-2} = 12.566 370 614 ... $\times 10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 352 5664(17)$\times 10^{-3}$ = 1/137.035 999 139(31)†	0.23, 0.23
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 3227(19)$\times 10^{-15}$ m	0.68
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e \alpha^{-1}$	3.861 592 6764(18)$\times 10^{-13}$ m	0.45
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	5.670 367(13)$\times 10^{-8}$ W m^{-2} K^{-4}	2300
Fermi coupling constant**	$G_F/(\hbar c)^3$	1.166 378 7(6)$\times 10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\hat{\theta}(M_Z)$ ($\overline{\rm MS}$)	0.231 22(4)††	1.7×10^5
$W^\pm$ boson mass	m_W	80.379(12) GeV/c^2	1.5×10^5

- Some theories predict changes in these constants over cosmological time scales

Are fundamental constants really constant?[2]

- How can we test this? $\Rightarrow$ Laboratory: Big Bang Nucleosynthesis (BBN)[3]

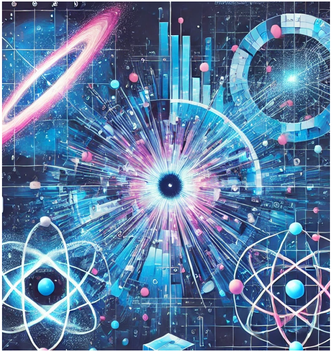
[1] PDG: Workman et al., 2022, [2] Dirac, 1973 and many others, [3] Olive, Steigman, and Walker, 2000; Iocco et al., 2009; Cyburt et al., 2016; Pitrou et al., 2018a

This talk

In this work: studied BBN under variation of

- the electromagnetic coupling constant α [1]
  - ☞ also using results from Halo EFT calculations[2]
- the Higgs vacuum expectation value (VEV) v [3]

Goal: find a bound on these variations through comparing calculations with experimental values for light element abundances



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[1] Meißner, Metsch, HM 2023; Bergström, Iguri, Rubenstein, 1999; Nollett, Lopez, 2002; Dent,Stern,Wetterich, 2007; Coc et al., 2007;

[2] Meißner, Metsch , HM 2024; Hammer, Ji, Phillips, 2017; [3] Meißner, HM 2024; Burns et al., 2024

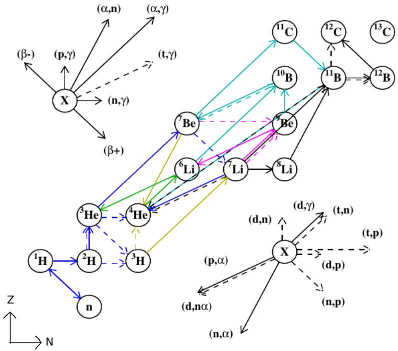
Introducing BBN – Evolution of Abundances

- abundance $Y_i = n_i / n_b$, with n_i density of nucleus i and n_b total baryon density
- Need to solve system of rate equations

$$\dot{Y}_i \supset -Y_i \Gamma_{i \to \dots} + Y_j \Gamma_{j \to i + \dots} + Y_k Y_l \Gamma_{kl \to ij} - Y_i Y_j \Gamma_{ij \to kl}$$

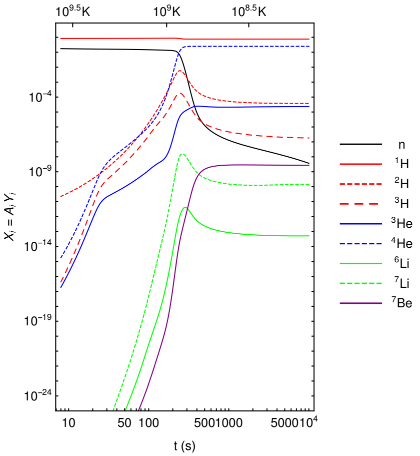
- Used five different codes[1] to get an estimate of systematical errors

[1] PRIMAT: Pitrou et al., 2018b, AlterBBN: Arbey et al., 2020, PArthENoPE: Gariazzo et al., 2022, NUC123: Kawano, 1992 and PRyMordial: Burns, Tait, and Valli, 2023



: Taken from Pitrou et al., 2018a

Introducing BBN – The Timescales



: produced by PRIMAT

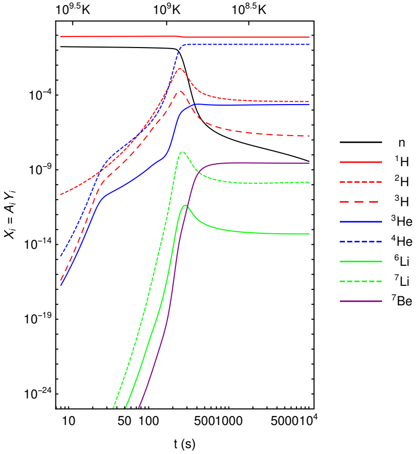
- $t \lesssim 1\,\text{s}$

  Weak $n \leftrightarrow p$ reactions

  ☞ number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference

  ☞ at 1 s or $T \approx 1\,\text{MeV}$: freeze-out and free neutron decay

Introducing BBN – The Timescales



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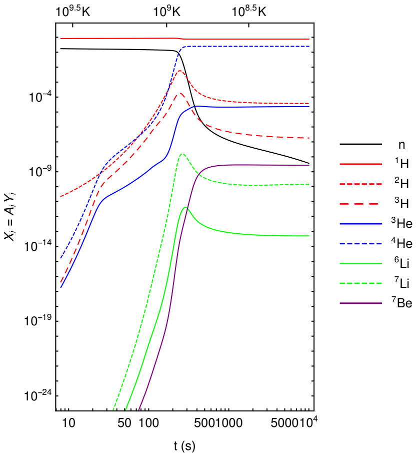
  - ☞ at 1 s or $T \approx 1\,\mathrm{MeV}$: freeze-out and free neutron decay
- $t = 1\,\mathrm{min}$

  Deuterium bottleneck: $n + p \to d + \gamma$ efficient

Introducing BBN – The Timescales



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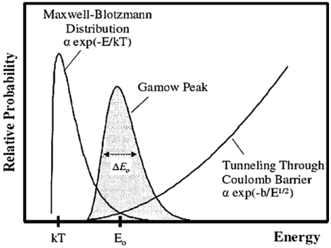
  Weak $n \leftrightarrow p$ reactions
  - number density ratio $\frac{n_n}{n_p} = e^{-Q_n/T}$, Q_n: mass difference
  - at 1 s or $T \approx 1\,\mathrm{MeV}$: freeze-out and free neutron decay
- $t = 1\,\mathrm{min}$

  Deuterium bottleneck: $n + p \to d + \gamma$ efficient
- $t \lesssim 3\,\mathrm{min}$

  Fusion of light elements (up to $^7\mathrm{Be}$)

Variation of α – What to consider

- Nuclear reaction rates: Coulomb barrier $\rightarrow$ energy-dependent penetration factor in cross section[1]
- Radiative capture



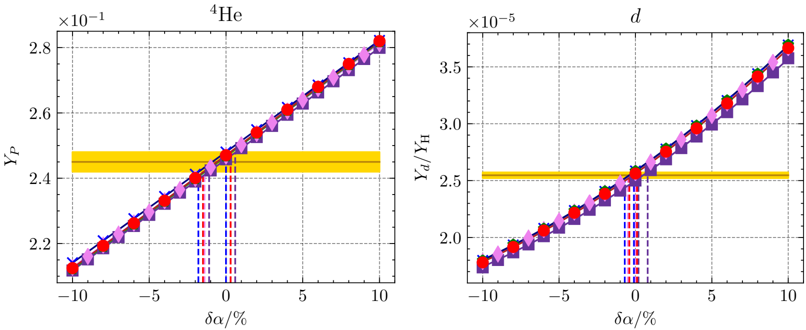
- $n \leftrightarrow p$ and β-decay rates: final (initial) state interactions between charged particles
- Indirect effects: binding energies[2] and Q_n (QED contribution)[3]

$$\Delta Q_n = Q_n^{\mathrm{QED}} \cdot \delta\alpha = -0.58(16)\,\mathrm{MeV} \cdot \delta\alpha$$

[1] Blatt and Weisskopf, 1979; [2] Elhatisari et al., 2024; [3] Gasser, Leutwyler, and Rusetsky, 2021

Experimental constraints

- PDG[1]: reliable measurements for ^{4}He, d and ^{7}Li (But: Lithium problem[2])



- 5 codes give similar results
- Only α-variation of $|\delta\alpha| < 1.8\%$ is consistent with experiment

[1] Workman et al., 2022; [2] Fields, 2011

Halo Effective Field Theory (EFT)

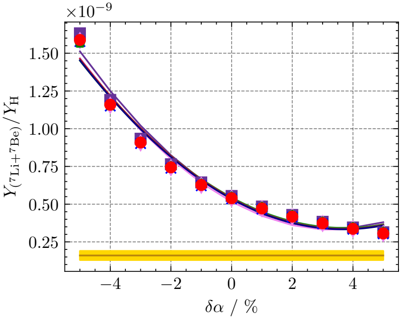
Biggest source of uncertainty: reaction rates and cross sections

$\Rightarrow$ Need theoretical predictions

- So far: only pionless EFT for $n + p \to d + \gamma$ [1]
- Now: include Halo EFT[2] rates for
  - $n + {}^7\mathrm{Li} \to {}^8\mathrm{Li} + \gamma$ [3]
  - $p + {}^7\mathrm{Be} \to {}^8\mathrm{B} + \gamma$ [4]
  - ${}^3\mathrm{H} + {}^4\mathrm{He} \to {}^7\mathrm{Li} + \gamma$ and ${}^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma$ [5]

[1] Rupak, 2000; [2] review: Hammer, Ji, Phillips, 2017; [3] Fernando, Higa, Rupak 2012; Higa, Premarathna, Rupak, 2021; [4] Higa, Premarathna, Rupak, 2022;

[5] Higa, Rupak, Vaghani, 2018; Premarathna, Rupak, 2020



: Meißner, Metsch, HM 2024: in print (EPJA)

${}^7\mathrm{Li} + {}^7\mathrm{Be}$ abundance diverges?

Higgs VEV Variation – What to consider

- QCD scale $\Lambda_{\mathrm{QCD}} \propto (1+\delta v)^{0.25}$[1]
- Fermi constant $G_F \propto (1+\delta v)^{-2}$
- Change of electron and quark masses $\Rightarrow M_\pi$ through Gell-Mann-Oakes-Renner relation

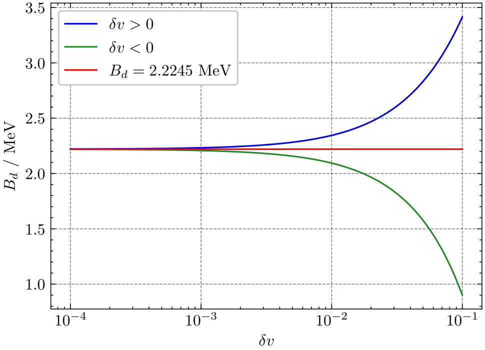
→ Q_n (QCD part)[2]

→ Deuteron binding energy (right)

→ nucleon mass and axial-vector coupling (from Lattice QCD or ChPT)

Remember Ulf-G. Meißner's talk?

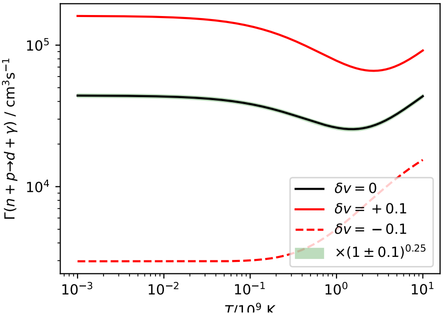
→ nucleon-nucleon scattering parameters (low energy theorems)[3]



[1] Burns et al., 2024, [2] Gasser, Leutwyler, and Rusetsky, 2021, [3] Baru et al., 2015, 2016

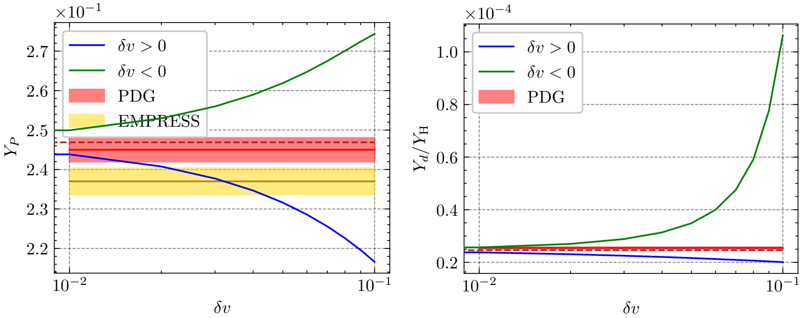
$n + p \rightarrow d + \gamma$

- $n + p \rightarrow d + \gamma$ rate[1] depends heavily on
  - deuteron binding energy
  - nucleon mass
  - nucleon-nucleon scattering parameters
- v-dependence much stronger than expected from $(1 + \delta v)^{0.25}$ [2]



[1] Rupak, 2000; [2] Burns et al., 2024

Experimental constraints



: PDG: Workman et al., 2022 ; EMPRESS: Matsumoto et al., 2022

- found more stringent 2σ-bound from deuterium abundance:

$$-0.5\% \leq \delta v \leq -0.1\%$$

To summarize...

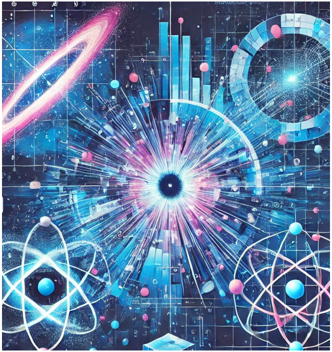
- simulated Big Bang Nucleosynthesis with 5 different codes as laboratory
- considered variation of fundamental physical constants and found
  - for the fine-structure constant (1σ)

$$\boxed{|\delta\alpha| < 1.8\%}$$

  - for the Higgs VEV (2σ)

$$\boxed{-0.5\% \leq \delta v \leq -0.1\%}$$

  to be consistent with measurements
- Now: Are they really constant?



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Outlook

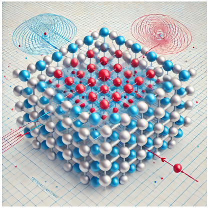
- Combined analysis of α- and v- or α- and quark mass variations
- Quantitative and detailed error estimations
- Main source of uncertainty: reaction cross sections and rates

$\Rightarrow$ need more theoretical predictions

✔ Halo EFT

☞ new: Nuclear Lattice Effective Field Theory Remember Dean Lee's talk?

- $\rightarrow$ contributions to nuclear binding energies (already used for α-variation)
- $\rightarrow$ *ab initio* calculation of scattering parameters and rates: deuteron-deuteron reactions in the making
- $\rightarrow$ can directly vary fundamental parameters: no need for approximation



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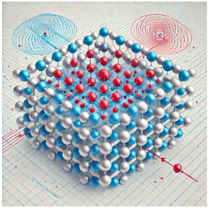

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Thank you for your attention!

Nuclear Reaction Rates – Coulomb Barrier

$$\boxed{\Gamma_{ab\to cd}(T) = N_A \langle \sigma v \rangle \propto \int_0^\infty \mathrm{d}E \, \sigma_{ab\to cd}(E) \cdot E \cdot e^{-\frac{E}{k_B T}}}, \quad E = \frac{1}{2}\mu_{ab} v^2$$

(1) Coulomb Barrier

Cross section is proportional to penetration factor [Blatt and Weisskopf, 1979]

$$\boxed{\sigma \propto v_0 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}},$$

with Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E},$$

and Gamow-energy

$$E_G = 2\mu_{ab} c^2 \pi^2 Z_a^2 Z_b^2 \alpha^2, \quad \mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

Nuclear Reaction Rates – Radiative Capture

(2) Radiative capture reactions

- Coupling $\propto e \Rightarrow$ Cross section $\sigma \propto \alpha \propto e^2$
- External capture processes [Christy and Duck, 1961]: parameterized in $f(\delta\alpha)$ [Nollett and Lopez, 2002]
- Assume dipole dominance
- For some reactions: Halo EFT cross sections $\Rightarrow$ work in progress

α-dependence of cross section ($q_\gamma = 1$ for radiative capture, zero else)

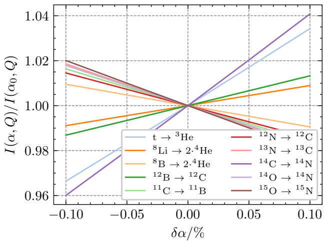
$$\sigma(\alpha, E) \propto \left(\frac{\sqrt{E_G^{\rm in}/E}}{e^{\sqrt{E_G^{\rm in}/E}} - 1} \right) \cdot \left(\frac{\sqrt{E_G^{\rm out}/(E+Q)}}{e^{\sqrt{E_G^{\rm out}/(E+Q)}} - 1} \right) \cdot (\alpha f(\delta\alpha))^{q_\gamma}$$

$$Q = m_a + m_b - m_c - m_d$$

Weak Rates – Fermi Function

β-decay rate (assume $|M_{fi}|^2$ to be p-independent) [Segrè, 1964]:

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\mathrm{max}}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2}\right)^2 F(Z, \alpha, p_e) p_e^2 \, \mathrm{d}p_e}_{= I(\alpha, Q)},$$



$$p_{e,\mathrm{max}} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}, \, W \approx M_a - M_b = Q$$

Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z, \alpha, \epsilon_e) \approx \frac{\pm 2\pi\nu}{1 - \exp(\mp 2\pi\nu)}, \quad \nu \equiv \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

Then:

$$\lambda(\alpha) = \lambda(\alpha_0) \frac{I(\alpha, Q)}{I(\alpha_0, Q)}$$

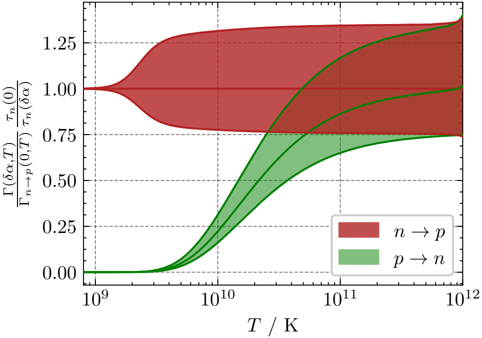
$n \leftrightarrow p$ Rates

Free neutron decay: lifetime

$$\tau_n(\alpha) = \tau_n(\alpha_0)\frac{I(\alpha_0, Q)}{I(\alpha, Q)}$$

But: Ignored Fermi-Dirac distribution of neutrino and electron

$\Rightarrow$ temperature dependence in α-variation for high temperatures



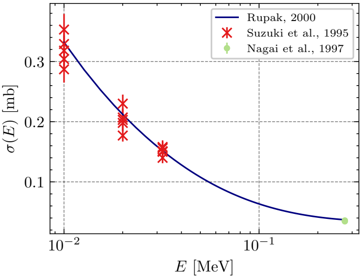
Nuclear Reaction Rates – $n + p \rightarrow d + \gamma$

Some corrections due to α variation are energy-dependent
$\Rightarrow$ need reaction cross section!

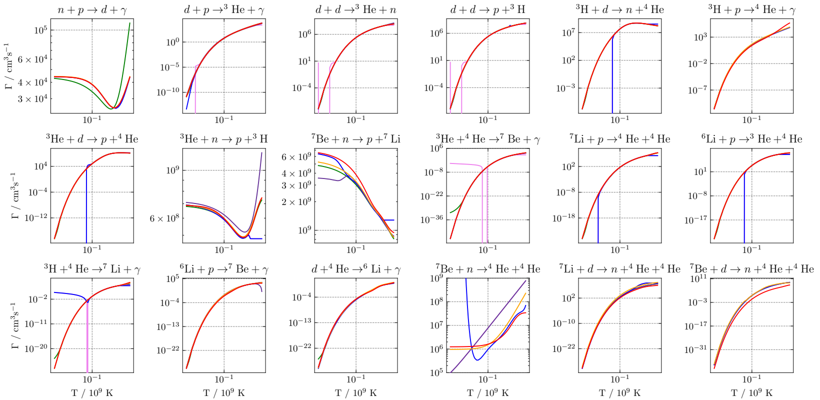
For $n + p \rightarrow d + \gamma$:

- Pionless EFT (N^4LO) approach by Rupak, 2000
- $\sigma(n + p \rightarrow d + \gamma)$ depends linearly on α

Other reaction cross section need to be parameterized by fitting to data EXFOR database



Nuclear Reaction Rates – Leading Reactions



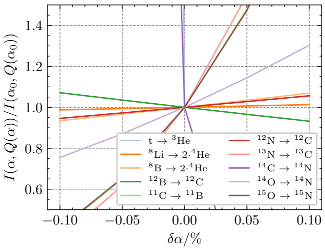
This work ; PRIMAT ; AlterBBN ; PArthENoPE ; NUC123 ; NACRE II ;

(PRyMordial uses the PRIMAT rates)

Indirect Effects – Binding energies [Meißner and Metsch, 2022]

Coulomb interaction between protons in nucleus
$\Rightarrow$ Electromagnetic contribution to binding energy [Elhatisari et al., 2024]
Change in Q-value:

$$\Delta Q = \delta\alpha \left(-\sum_i B_C^i + \sum_j B_C^j \right)$$



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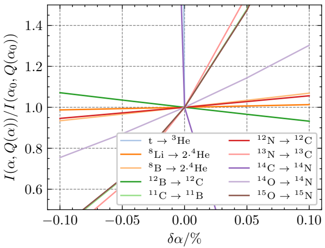


Nuclear reaction cross sections ($p_\gamma = 3, q_\gamma = 1$ for radiative capture, $p_\gamma = 1/2, q_\gamma = 0$ else)

$$\sigma(E,\alpha) \propto \underbrace{(E + Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\rm in}(\alpha)/E}}{\exp\left(\sqrt{E_G^{\rm in}(\alpha)/E}\right) - 1} \frac{\sqrt{E_G^{\rm out}(\alpha)/(E + Q(\alpha))}}{\exp\left(\sqrt{E_G^{\rm out}(\alpha)/(E + Q(\alpha))}\right) - 1}$$

Indirect Effects – Neutron-proton mass difference

$Q_n = m_n - m_p$ has QED contribution [Gasser, Leutwyler, and Rusetsky, 2021]:

$$\Rightarrow \Delta Q_n = Q_n^{\mathrm{QED}} \cdot \delta\alpha = -0.58(16)\,\mathrm{MeV} \cdot \delta\alpha$$

Affects

- weak $n \leftrightarrow p$ rates
- Q-values of β-decays
- $m_N = (m_n + m_p)/2$ appearing in $n + p \to d + \gamma$ cross section? $\to$ neglect α-dependence!

Results

Baryon-to-photon ratio $\eta = 6.14 \times 10^{-10}$; neutron lifetime $\tau_n(\alpha_0) = 879.4\,\mathrm{s}$ [PDG]

Parameter fit

$$\frac{Y(\alpha) - Y(\alpha_0)}{Y(\alpha_0)} = a \cdot \frac{\Delta\alpha}{\alpha_0} + b \cdot \left(\frac{\Delta\alpha}{\alpha_0}\right)^2$$

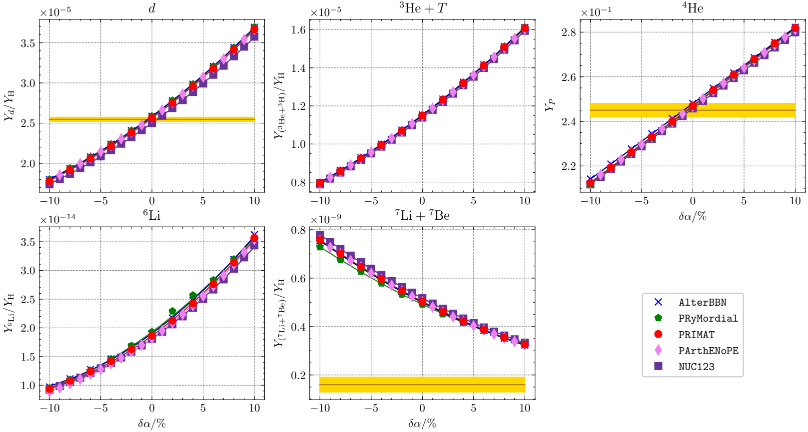
Main results see Meißner, Metsch, and Meyer, 2023:

- For most elements: change in nuclear reaction rates biggest effect.
- $^4\mathrm{He}$ indeed very sensitive to ΔQ_n.
- Lithium Problem

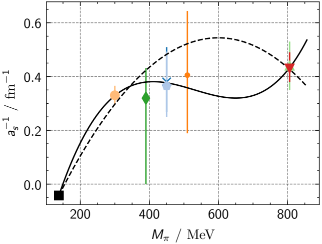
Differences to existing literature:

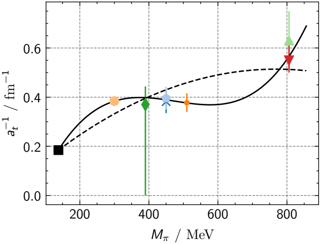
- Updated experimental values for masses, physical constants etc., more recent calculation of Q_n^{QED}
- Different reaction rates due to parameterization of cross section.
- Calculating the corrections exactly or using temperature-dependent approximations.

Results


d
$\times 10^{-5}$
Y_d/Y_H
$^3\mathrm{He} + T$
$Y_{(^3\mathrm{He}+^3\mathrm{H})}/Y_\mathrm{H}$
$^4\mathrm{He}$
$\times 10^{-1}$
Y_P
$\delta\alpha/\%$
$^6\mathrm{Li}$
$\times 10^{-14}$
$Y_{^6\mathrm{Li}}/Y_\mathrm{H}$
$^7\mathrm{Li} + {}^7\mathrm{Be}$
$\times 10^{-9}$
$Y_{(^7\mathrm{Li}+^7\mathrm{Be})}/Y_\mathrm{H}$
$\delta\alpha/\%$
$\delta\alpha/\%$
AlterBBN
PRyMordial
PRIMAT
PArthENoPE
NUC123

Quark mass dependence of scattering parameters





Measurement of Primordial Abundances

Deuterium *d*:

- Almost completely destroyed in stars
- Observe high red-shift, low-metallicity systems

Helium-4 $^{4}\mathrm{He}$:

- Recombination lines of He and H in metal-poor extra-galactic HII regions
- Metal Production in stars positively correlated to stellar $^{4}\mathrm{He}$ contribution
  $\rightarrow$ Primordial abundance found by extrapolation to zero metallicity

Lithium-7 $^{7}\mathrm{Li}$:

- Observe stars in the galactic halo with very low metallicities
- $^{7}\mathrm{Li}$ dominant over $^{6}\mathrm{Li}$
- Lithium problem[1]: theoretical prediction three times higher

[1] LithiumProblem

Temperature-Dependent Approximation

Charged particle reactions

- Define $S(E) = \sigma(E) E e^{\sqrt{E_G^{\mathrm{in}}/E}}$ and assume $S \approx \mathrm{const.}$
- Reaction rate

$$\Gamma = \int \mathrm{d}E \, \frac{S(E)}{E} e^{-\sqrt{E_G^{\mathrm{in}}/E}} E e^{E/(k_B T)}$$

- E at maximum of integrand

$$E \to \bar{E}_c = \left(\frac{k_B T}{2}\right)^{\frac{2}{3}} (E_G^{\mathrm{in}})^{\frac{1}{3}}.$$

Neutron induced reactions

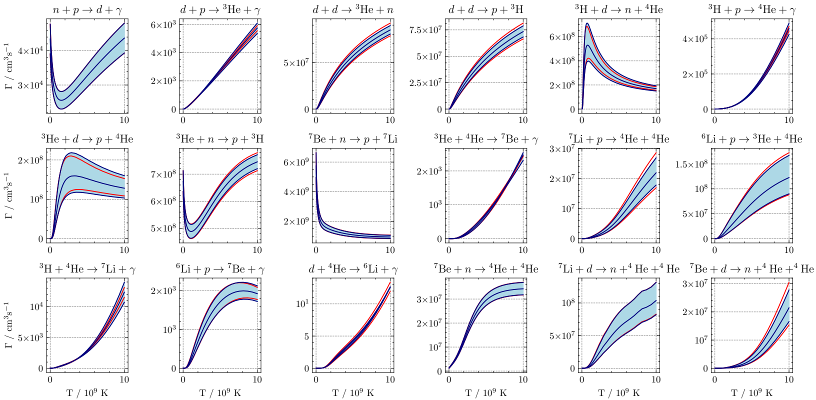
- Define $R(E) = \sigma(E)\sqrt{E}$ and assume $R \approx \mathrm{const.}$
- Reaction rate

$$\Gamma = \int \mathrm{d}E \, \frac{R(E)}{\sqrt{E}} E e^{E/(k_B T)}$$

- E at maximum of integrand

$$E \to \bar{E}_\gamma = \frac{1}{2} k_B T$$

Reaction Rates for Approximation



Reaction rates for $\delta\alpha = 0, \pm 10\%$ calculated exactly (blue) and with temperature-dependent approximation (red)