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Application of chiral two- and three-baryon interactions to light hypernuclei

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11th Workshop on Chiral Dynamics, Ruhr-Universität Bochum, August 26-30, 2024

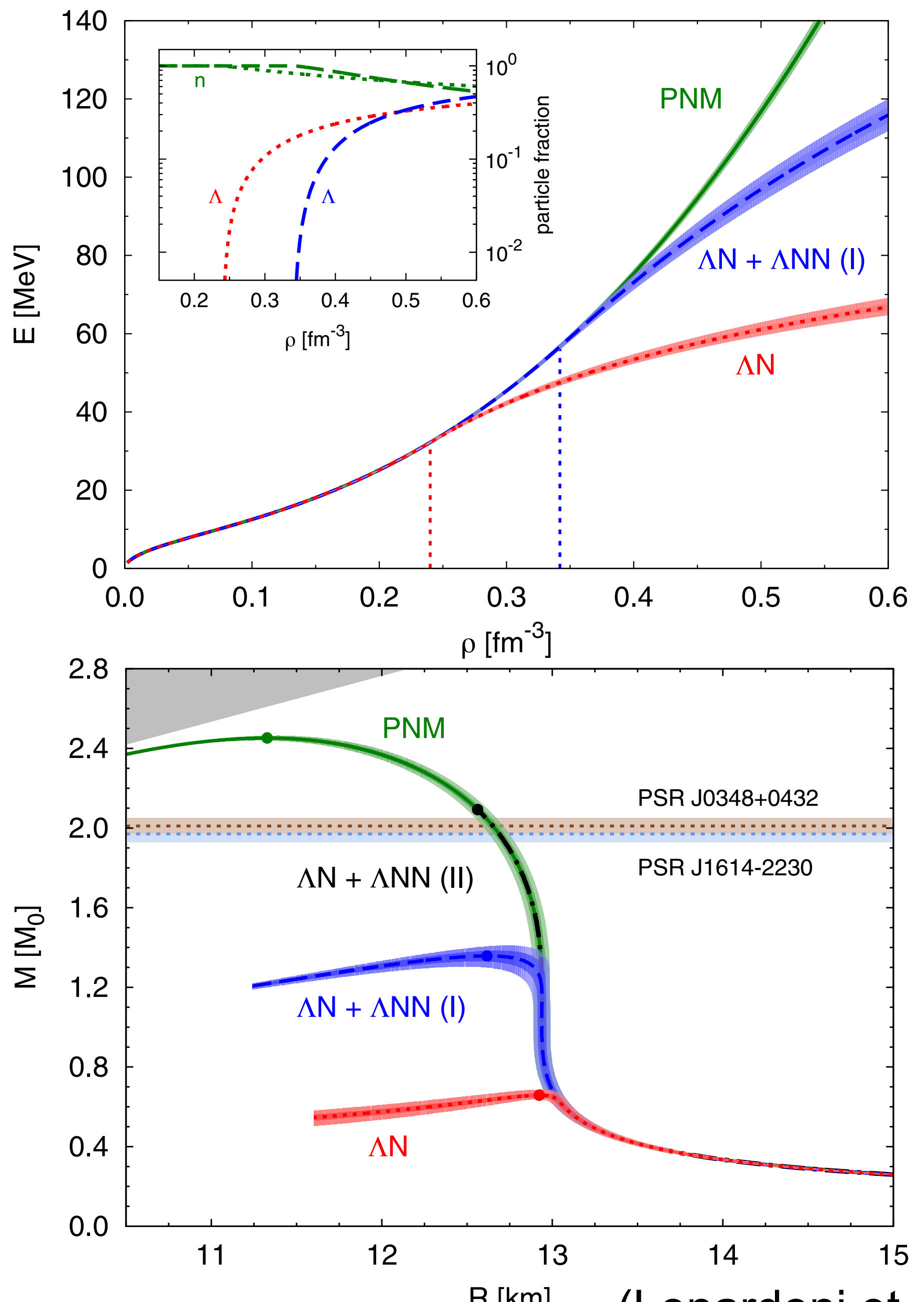
- Motivation
- YN interactions
- SRG evolution of (hyper-)nuclear interactions
- Determination of CSB contact interactions and Λn scattering length
- Application to $A = 7$ and 8 hypernuclei
- Uncertainty of Λ separation energies and size of chiral 3BF contributions
- Chiral YNN interactions
- Conclusions & Outlook

in collaboration with Johann Haidenbauer, **Hoai Le**, Ulf Meißner

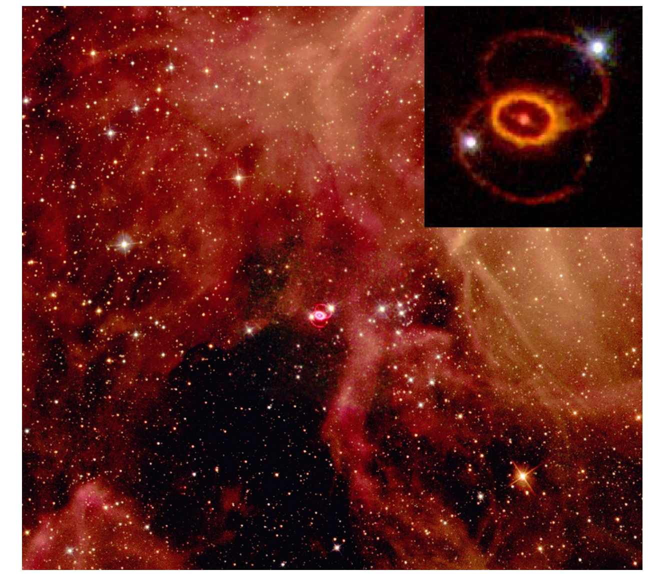
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

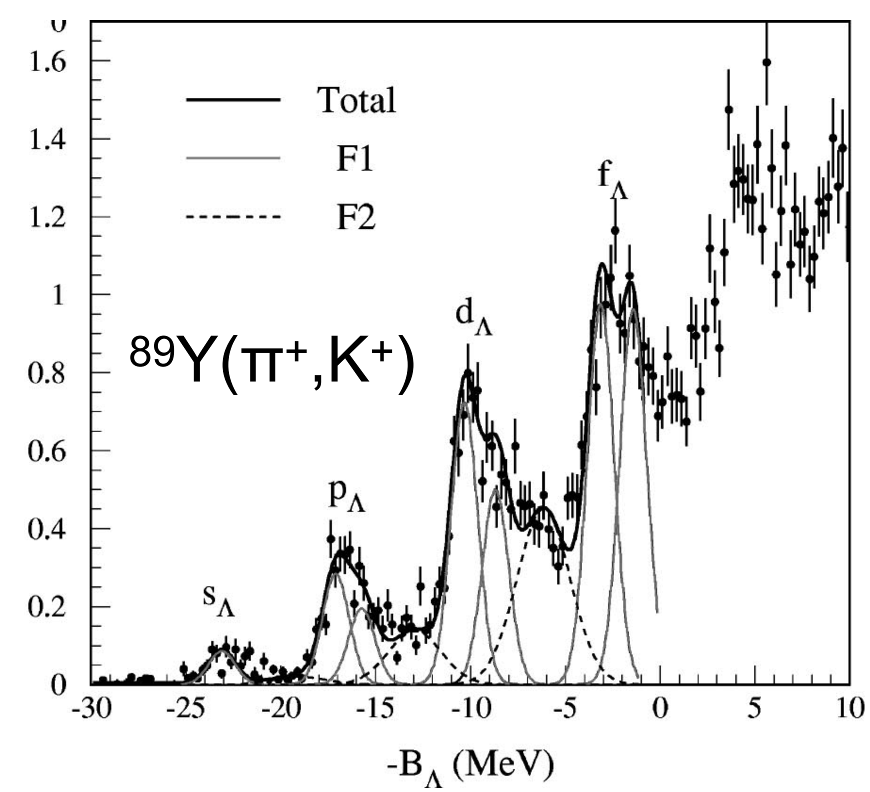
- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- ***flavor dependence of baryon-baryon interactions***



(Lonardoni et al. (2015))



(SN1987a, Wikipedia)



(Hotchi et al. (2001))

Hypernuclei

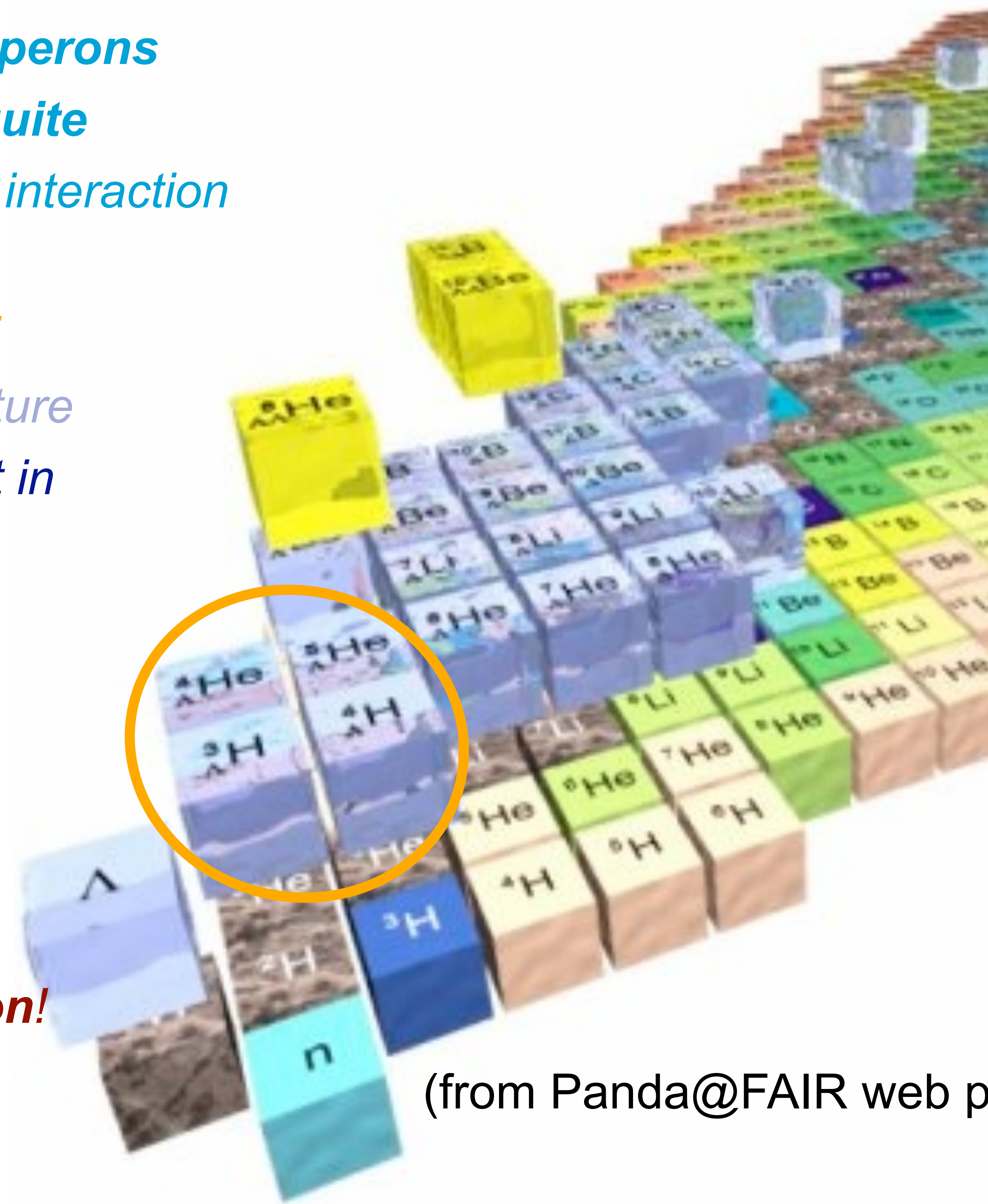
Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively: **core nucleus + hyperons***
  - *„separation energies“ are **quite** independent from NN(+3N) interaction*

- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in **several spin states***

- ***non-trivial constraints** on the YN interaction even from lightest ones*

- *size of **YNN** interactions? need to include **Λ-Σ conversion**!*



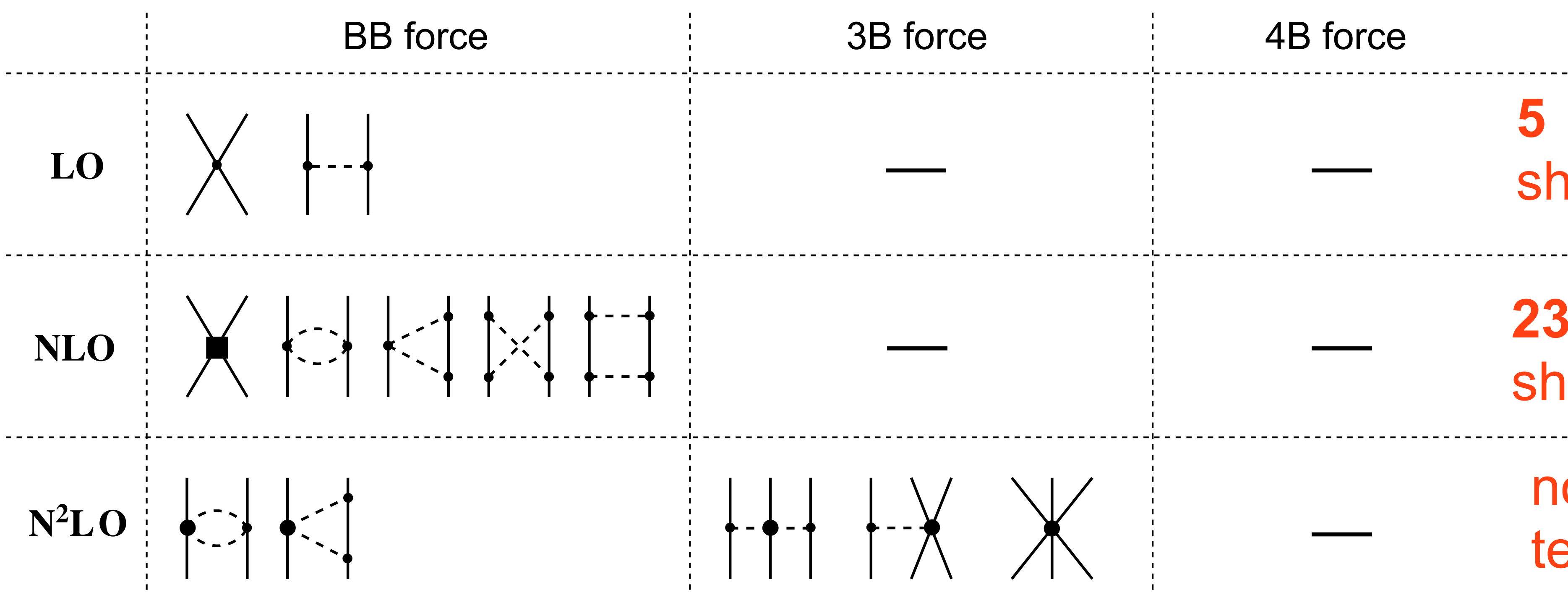
(from Panda@FAIR web page)

Chiral NN & YN interactions

EFT based approaches

Chiral EFT implements **chiral symmetry of QCD**

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N²LO (for YN)



5 NN/YN short range parameters

23 NN/YN short range parameters

no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

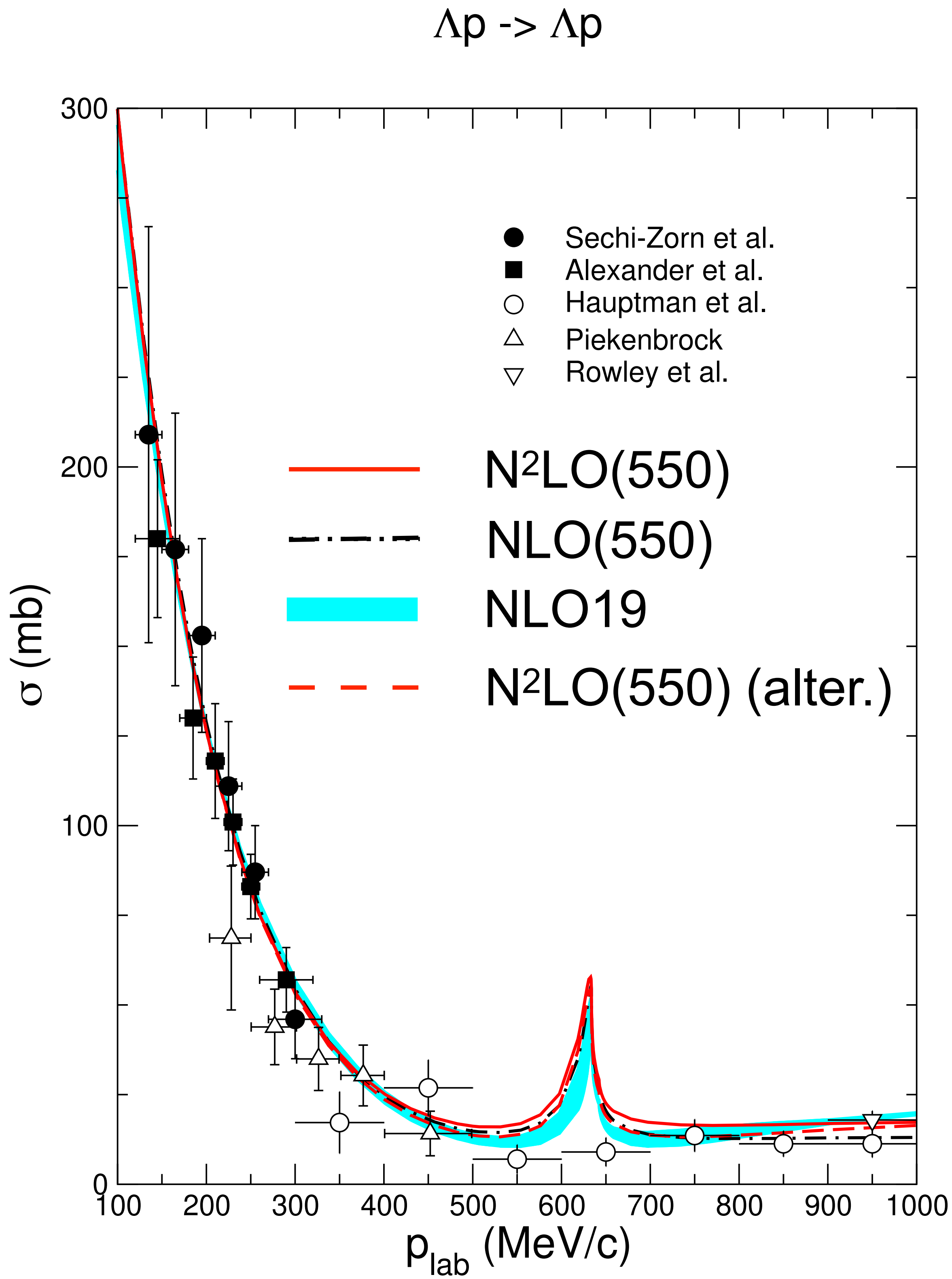
Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

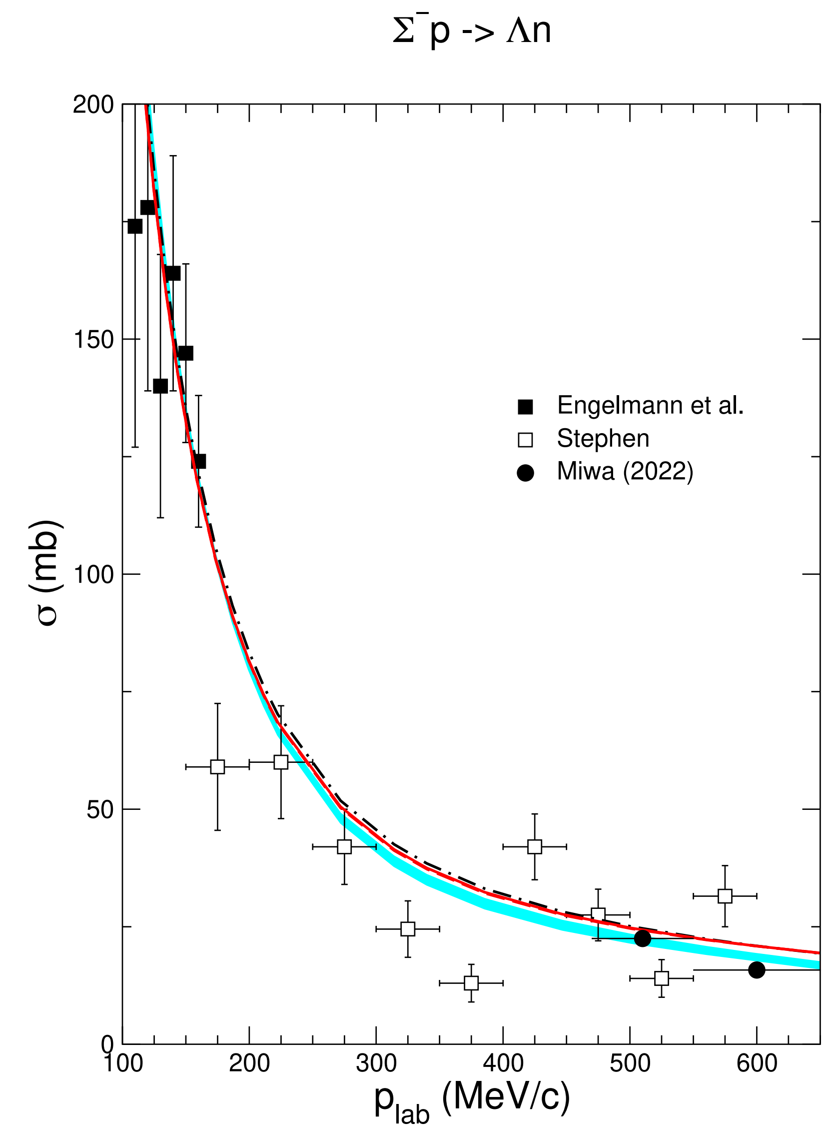
$\Lambda - \Sigma$ **conversion** is explicitly included (3BFs only in N²LO)

SMS NLO/N²LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



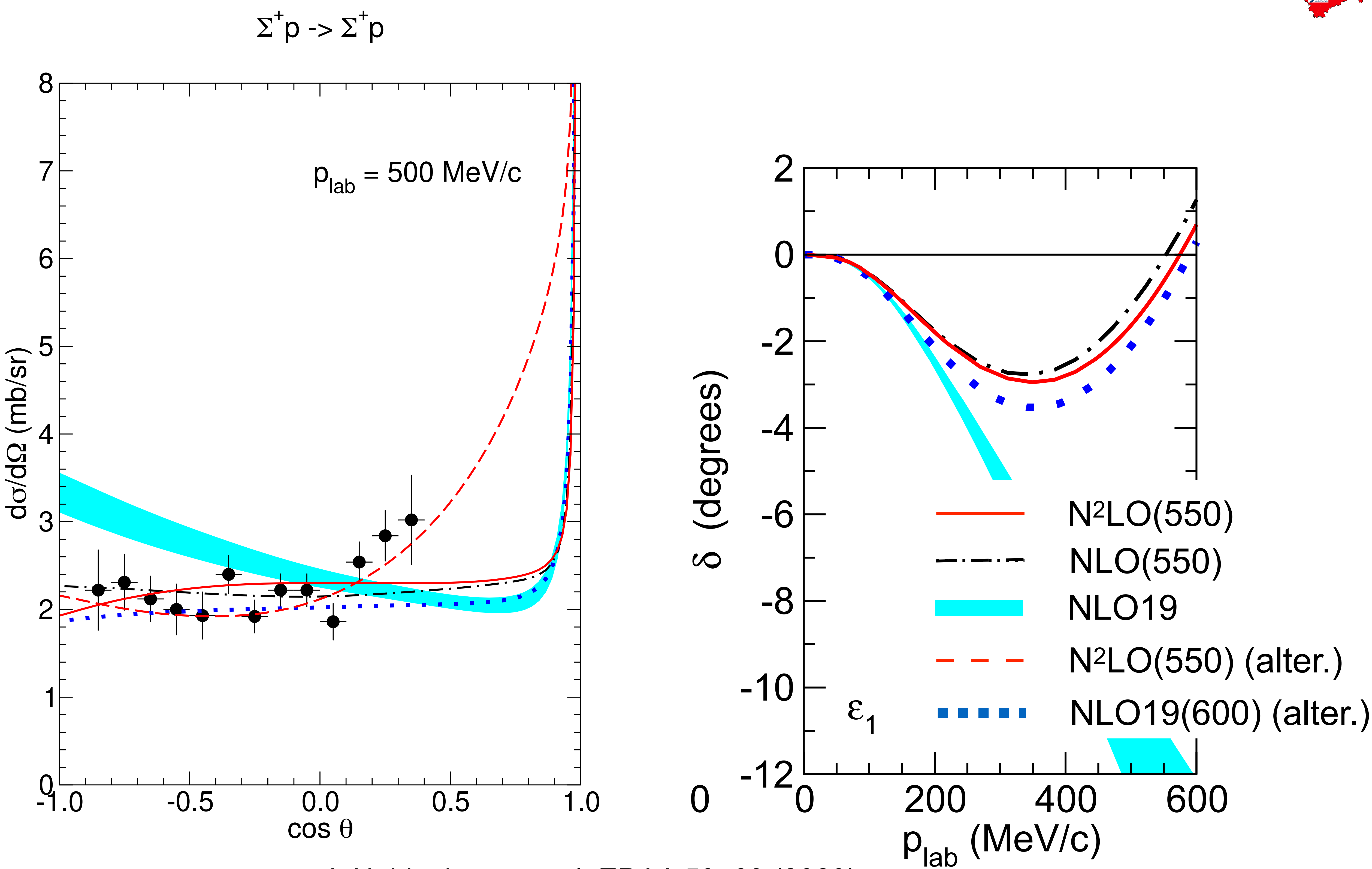
- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)



J. Haidenbauer et al. EPJ A 59, 63 (2023).

SMS NLO/N²LO interaction

new data (Miwa(2022)) at higher energies provides **new constraints**!



J. Haidenbauer et al. EPJ A 59, 63 (2023).

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches
(NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

$$\frac{dH_s}{ds} = [\,\underbrace{[T\,,\,H(s)]}_{\equiv \eta(s)}\,,\,H(s)\,] \qquad H(s) = T + V(s)$$

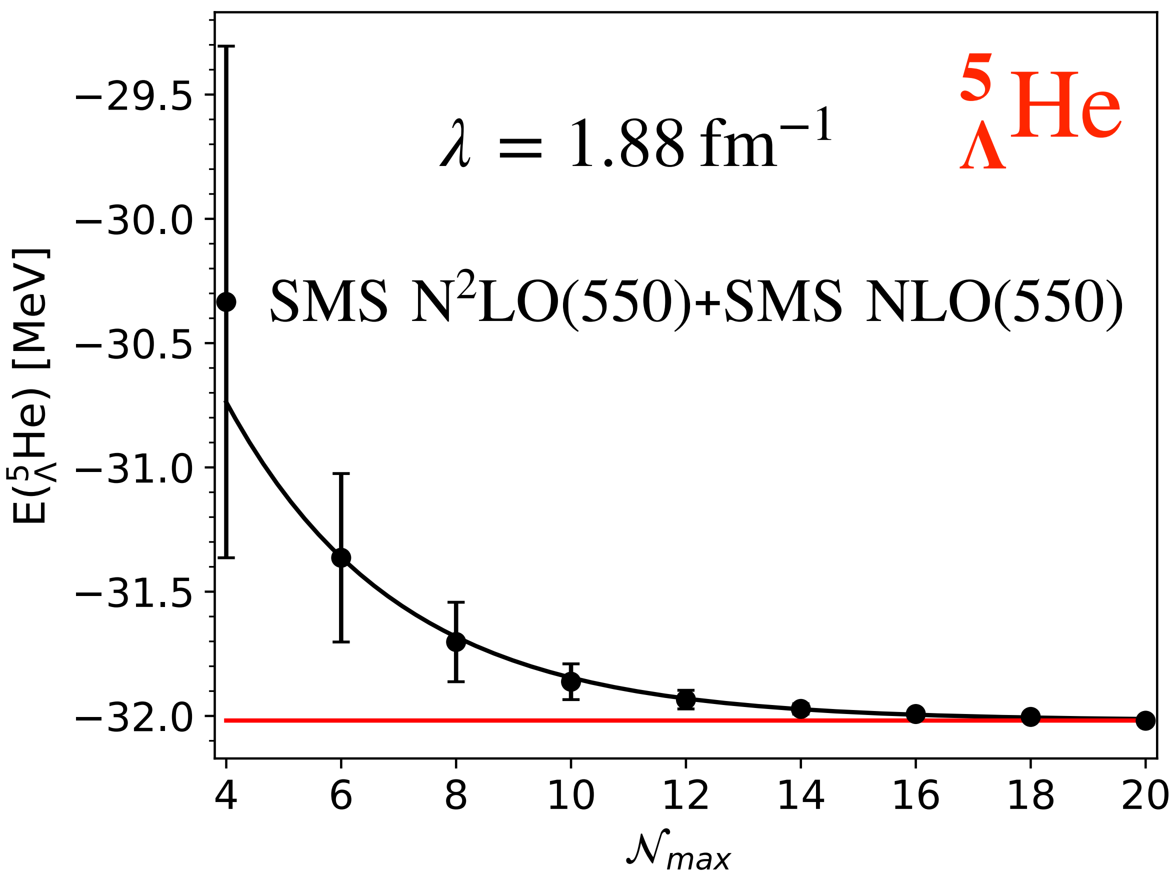
this choice of generator drives *V(s)* into a diagonal form in momentum space

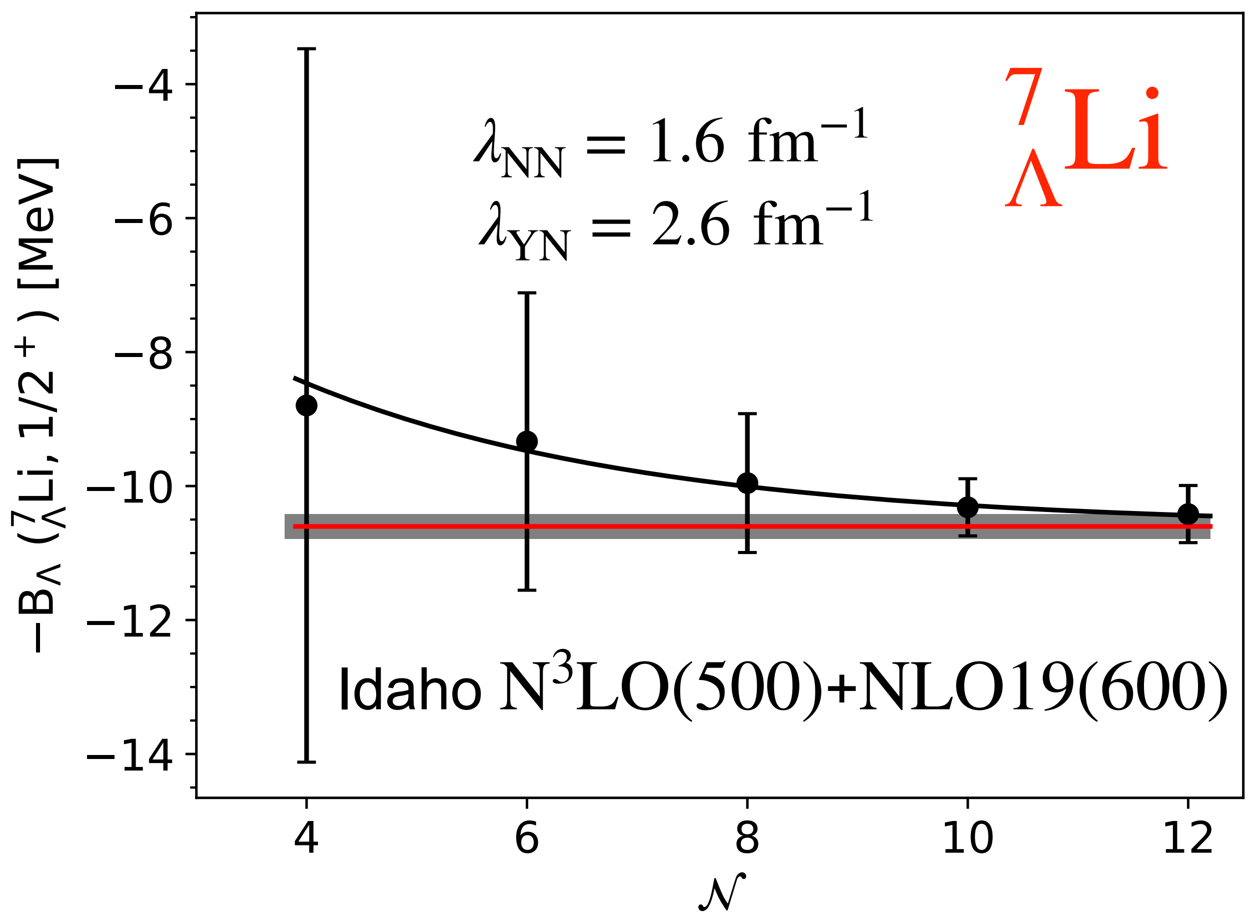
- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

(Bogner et al., 2007)

J-NCSM convergence

SRG evolution improves convergence





$$E\left({}^{5}_{\Lambda}\text{He}\right) = -32.018 \pm 0.001\ \text{MeV}$$

$$E_\Lambda\left({}^{7}_{\Lambda}\text{Li}\right) = 10.6 \pm 0.2\ \text{MeV}$$

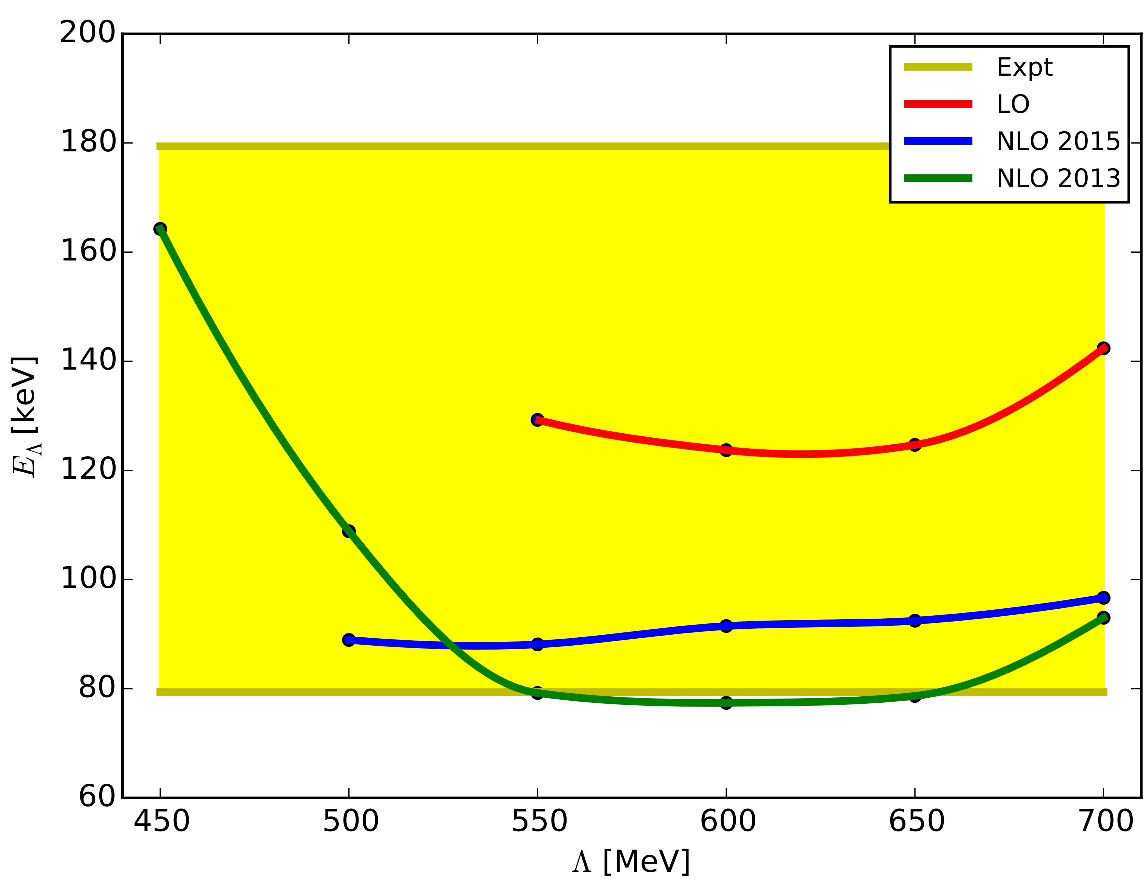
- for light nuclei and hypernuclei, the numerical uncertainty is negligible.
- for p-shell nuclei/hypernuclei, the uncertainty is visible
- extrapolation of separation energy can reduce uncertainty of this quantity

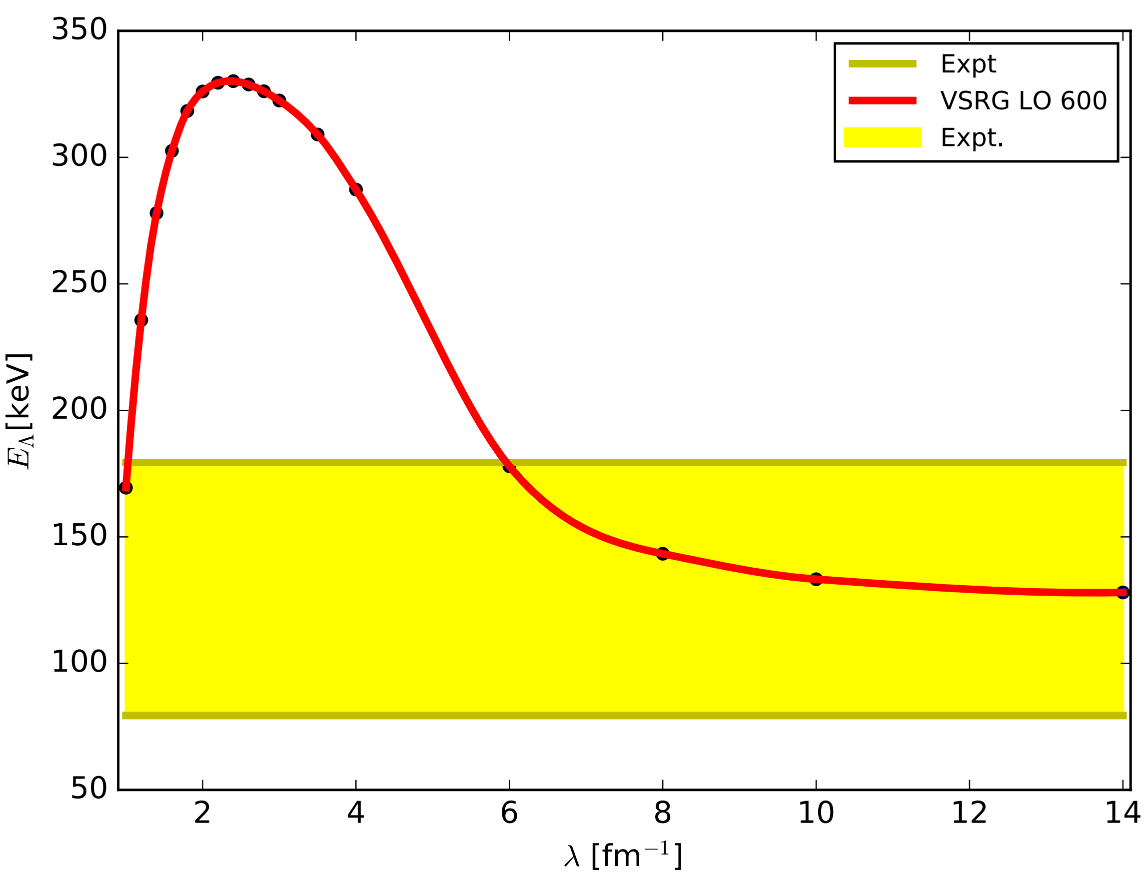
Induced 3BF …

SRG parameter dependence is significant when NN and YN interactions are evolved

→ missing 3N and YNN interactions

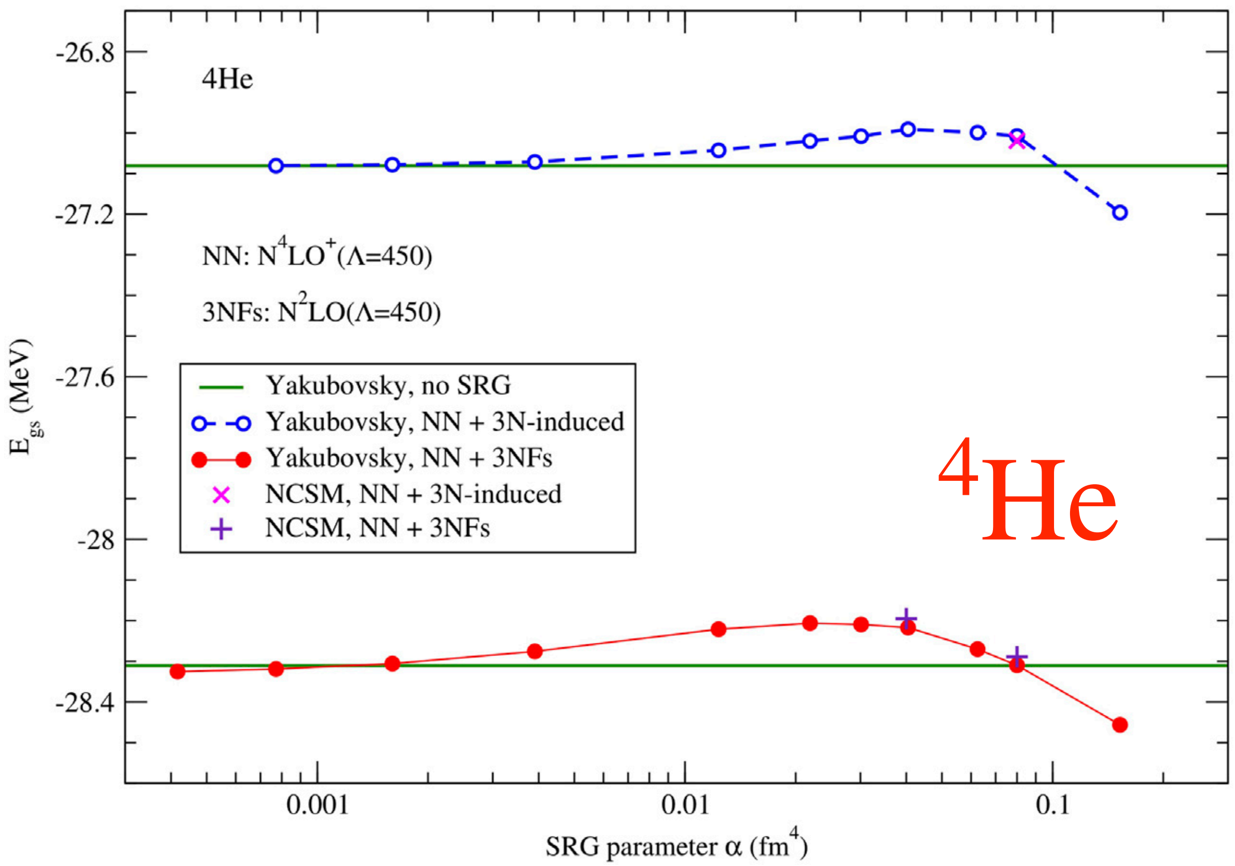
- 3NF is comparable to chiral 3NF
- YNN is larger than chiral YNN





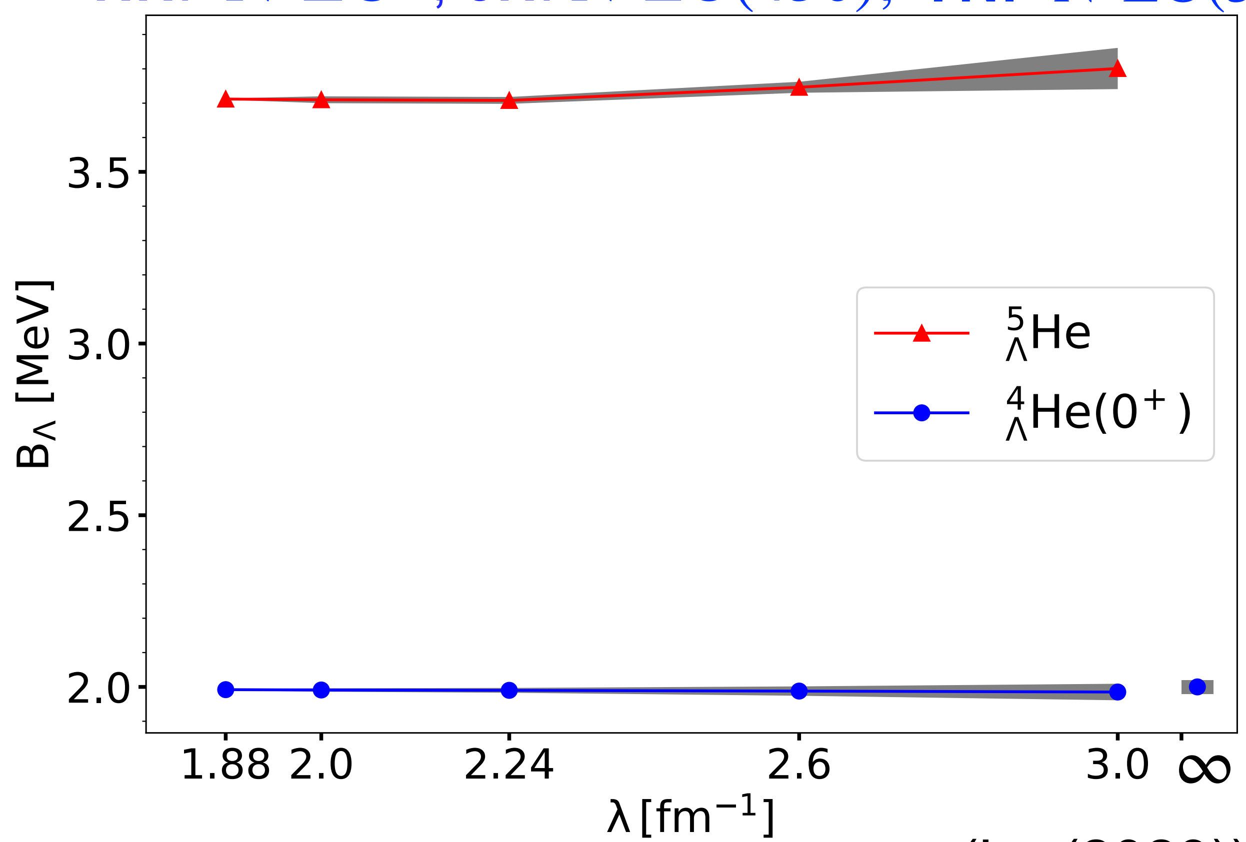
SRG dependence of results

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100-200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)

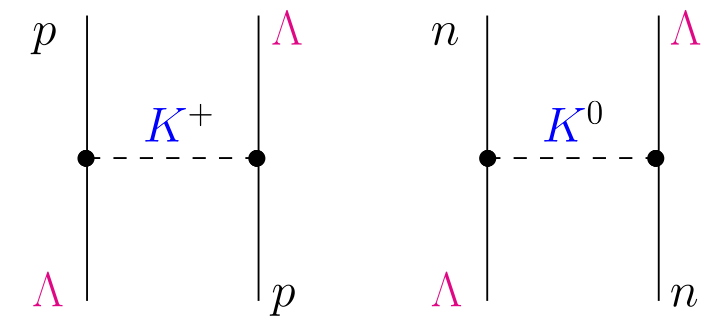


(Le (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

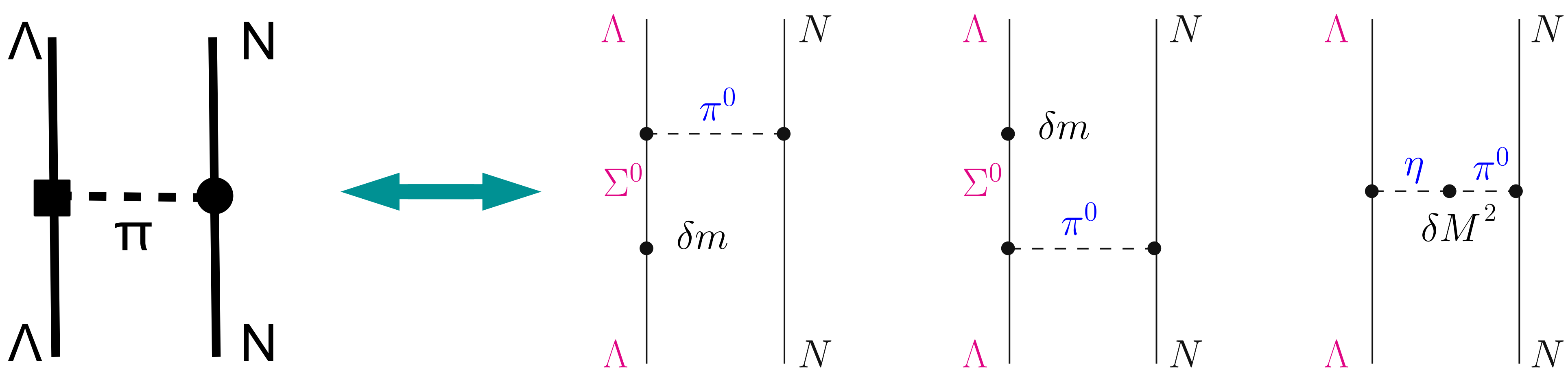
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - — very small due to the small relative difference of kaon masses

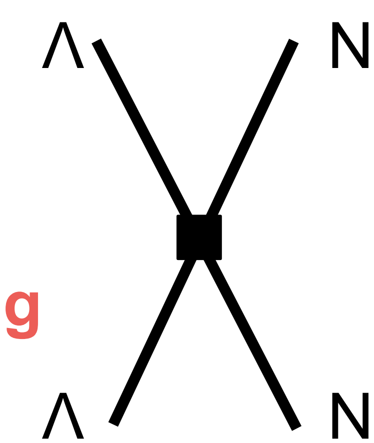


- **subleading but most important**
  — effective CSB ΛΛπ coupling constant (Dalitz, van Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$

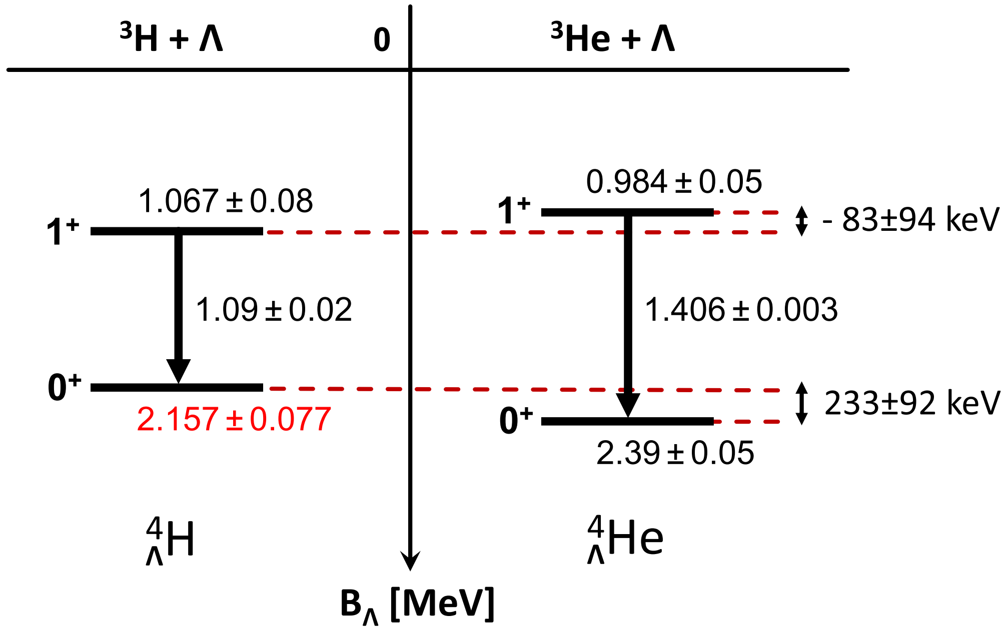


- **so far less considered but necessary for proper renormalization**
  — CSB contact interactions (for singlet and triplet)



Aim: determine the two unknown CSB LECs and predict Λn scattering

Fit of contact interactions



(Schulz et al.,2016; Yamamoto, 2015)

- Adjust the two CSB contact interactions to one main scenario (**CSB1**)

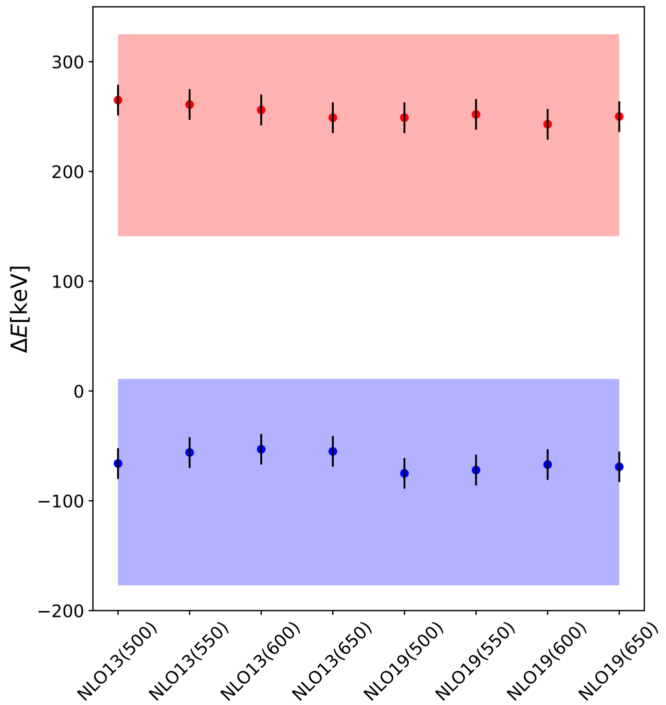
Λ	NLO13		NLO19	
	C_s^{CSB}	C_t^{CSB}	C_s^{CSB}	C_t^{CSB}
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

The values of the LECs are in 10^4 GeV^{-2}

- Size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\mathrm{GeV}^{-2} \propto 6 \cdot 10^{-3} \cdot 10^4\,\mathrm{GeV}^{-2}$$

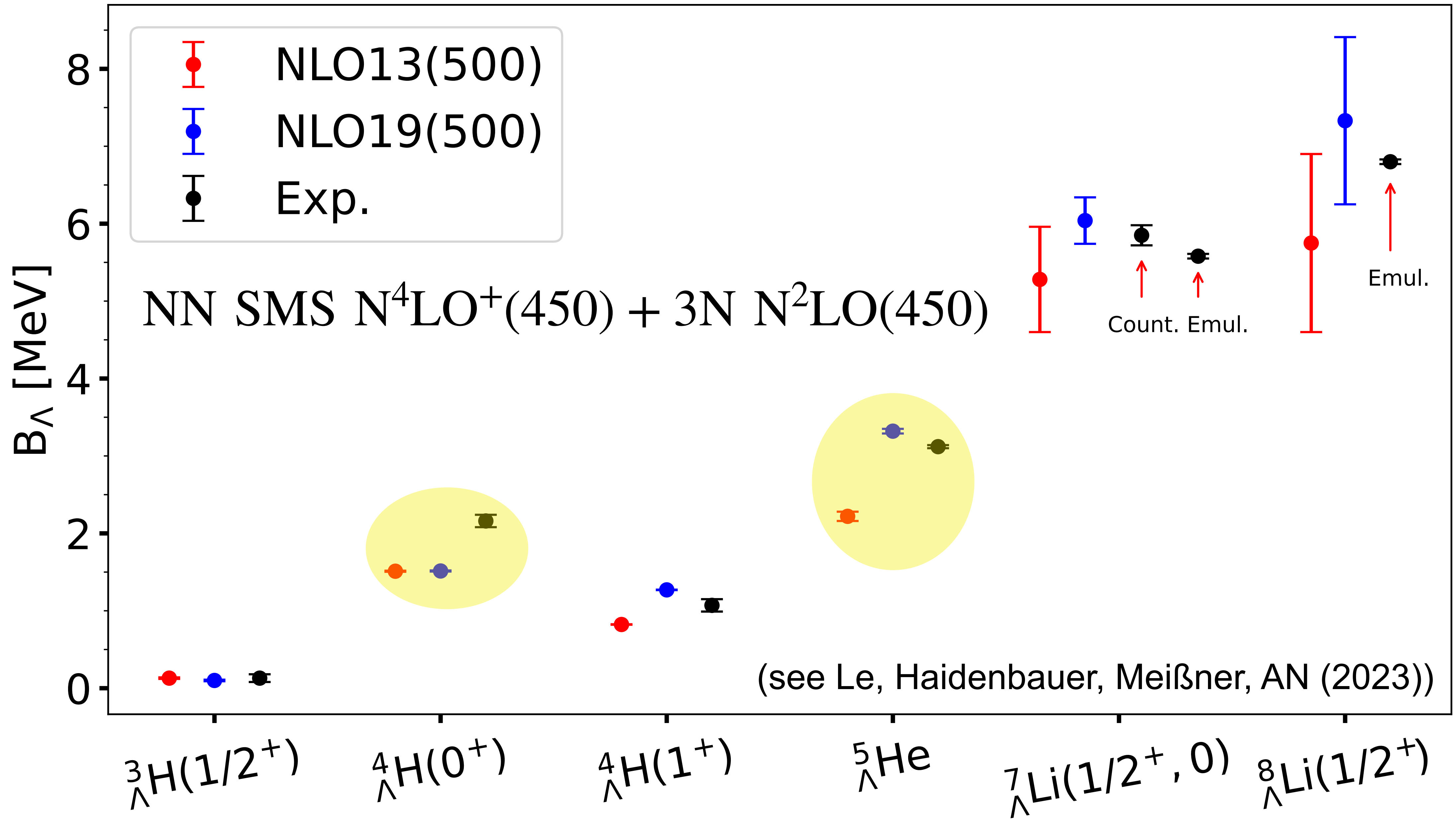
- Problem: large experimental uncertainty of experiment
- here only **fit to central values** to test theoretical uncertainties



(see Haidenbauer, Meißner, AN (2021))

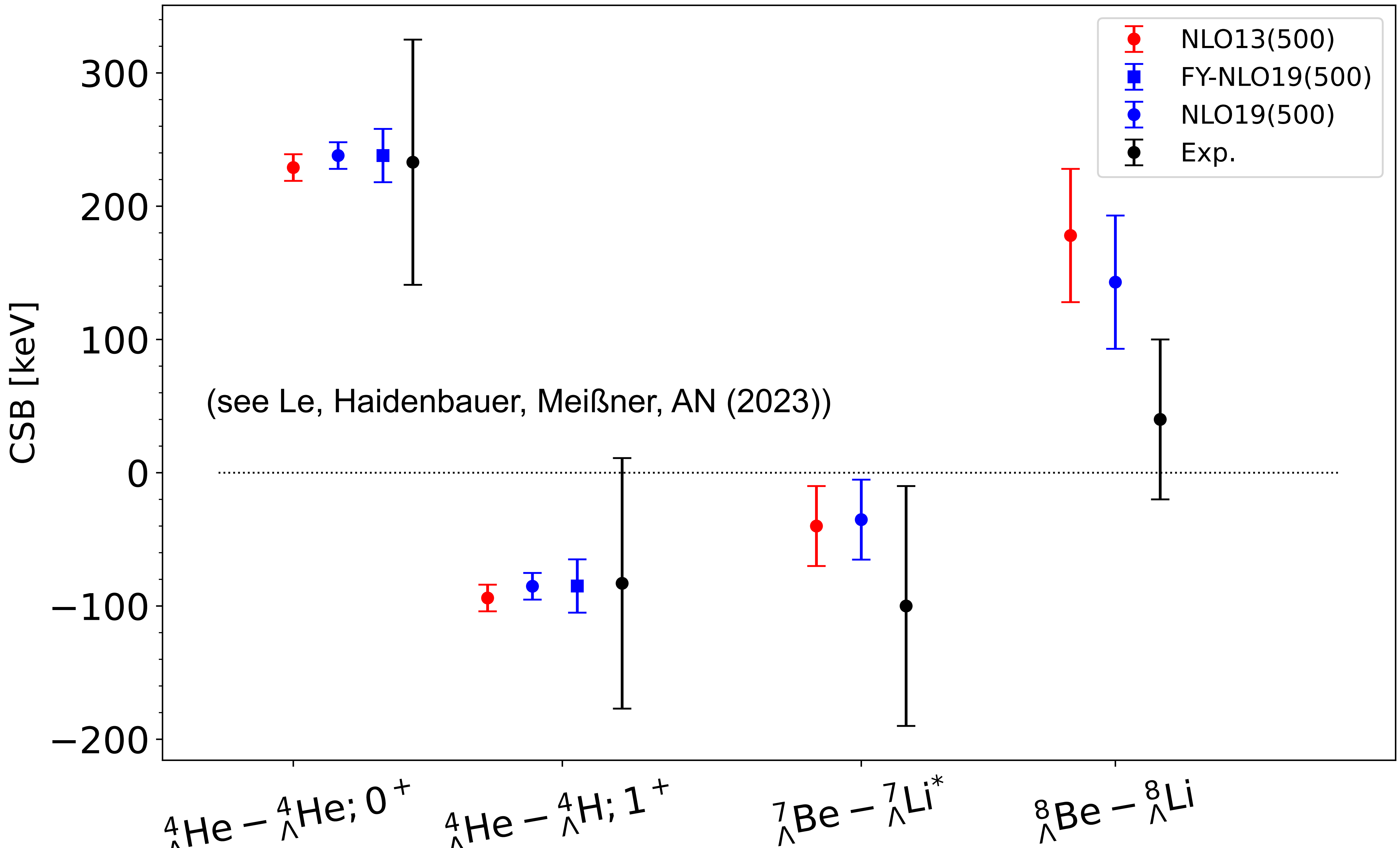
Application to $A = 7$ and 8

- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^4_\Lambda\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^5_\Lambda\mathrm{He}$ and heavier hypernuclei
  — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



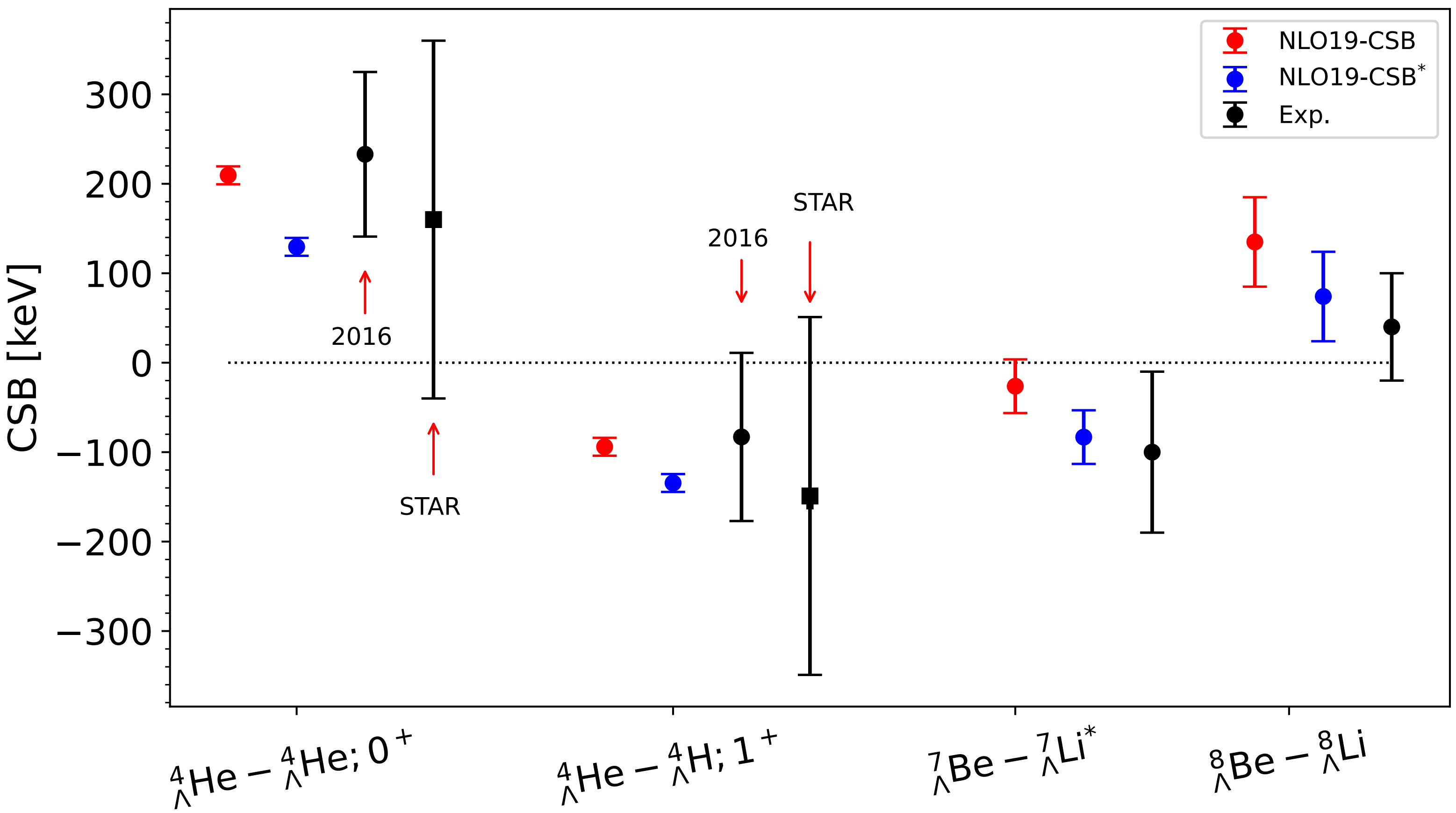
Application to $A = 7$ and 8

- CSB of singlet and triplet states interferes differently
- CSB still not fixed — experimental uncertainty is large
- scenario studied here is only **marginally consistent** with CSB in $A = 8$



New STAR data for $A = 4$ CSB

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

(see Le, Haidenbauer, Meißner, AN (2023))

Uncertainty analysis to $A = 3$ to 5

Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Results for **different orders** enable uncertainty estimate:

Ansatz for the order by order convergence:

$$X_K = X_{ref} \sum_{k=0}^{K} c_k\, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, …})$$

Bayesian analysis of the uncertainty following Melendez et al. 2017,2019

Extracting c_k for $k \leq K$ **from calculations**

→ **probability distributions** for c_k

→ $$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k\, Q^k$$

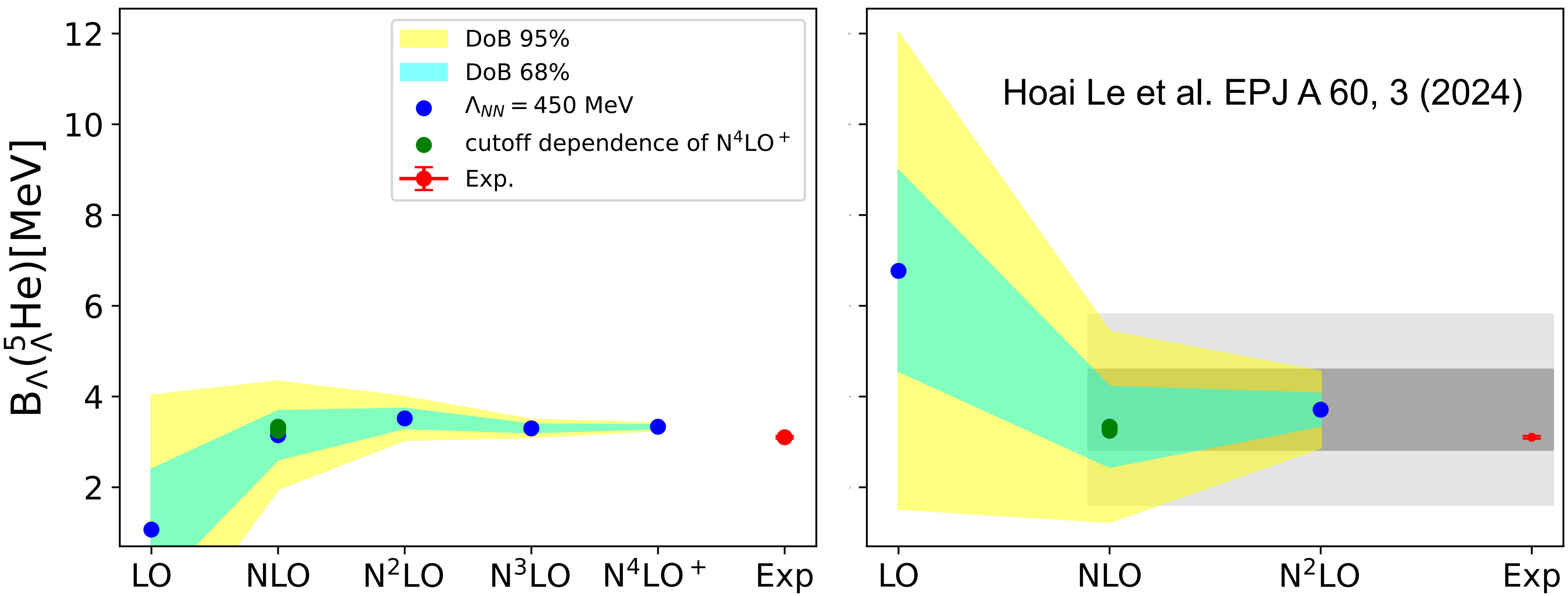
Uncertainty due to missing higher orders is more relevant than numerical uncertainty! (for light nuclei)

Application to ${}^{5}_{\Lambda}\mathrm{He}$ and summary

- **without YNN**: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^+)$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^+)$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881

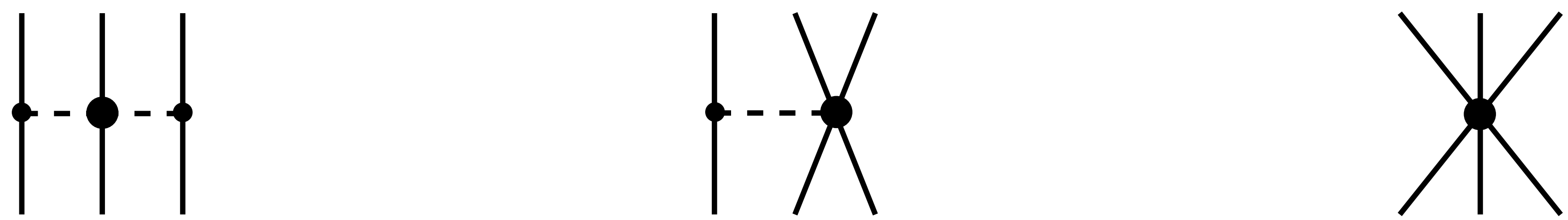
→ at the same time: estimate of YNN !



YNN (ΛNN) interactions

Leading 3BF with the usual topologies (see Petschauer et al., 2016 & 2017)

ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN
(up to 10)

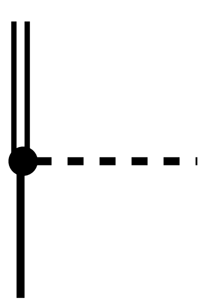
2 LECs in ΛNN
(up to 14)

3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

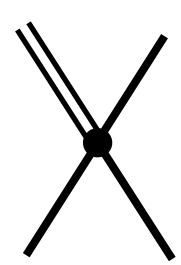
only few data → need to keep the **# of LECs** small

Decuplet baryons ($\Sigma^*\ldots$) might enhance YNN partly to NLO (see Petschauer et al., 2017)

By decuplet saturation all LECs can be related to the following leading octet-decuplet transitions (Petschauer et al. , 2020)



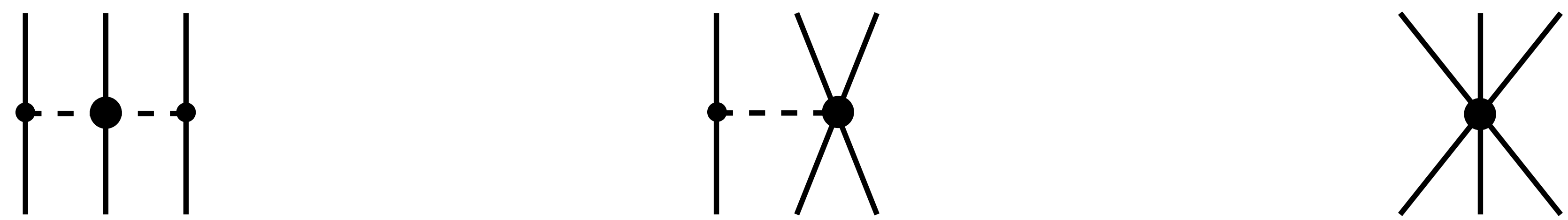
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) interactions

Decuplet saturation relates all LECs to G_1 and G_2

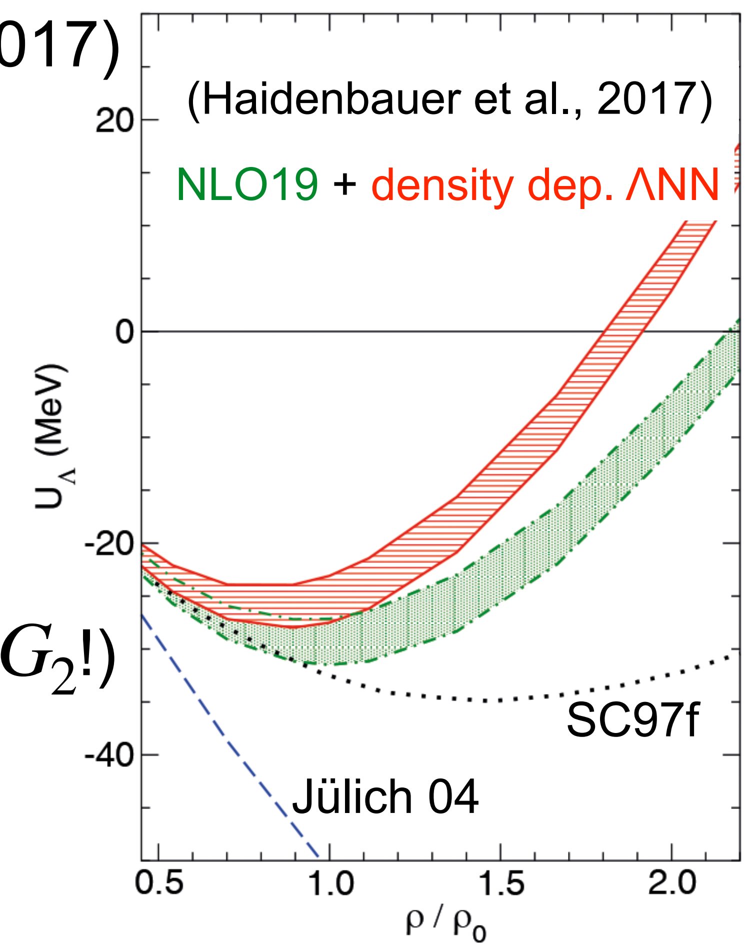


	$\propto C^2$	$\propto CG_1, CG_2$	$\propto (G_1)^2, (G_2)^2, G_1G_2$
For ΛNN:	$\propto C^2$	$\propto C(G_1 + 3G_2)$	$\propto (G_1 + 3G_2)^2$ 1 LEC

→ density dependent BB interactions (Petschauer et al., 2017)

→ application to nuclear matter (Haidenbauer et al., 2017)

neutron stars (Logoteta et al., 2019)

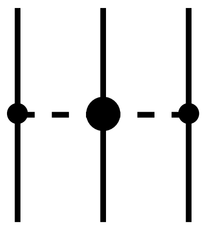


- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of 2π and 1π exchange (fixes sign of $G_1 + 3G_2$!)

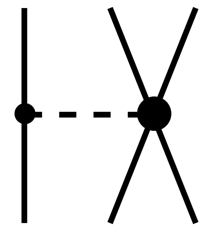
Recently: successful benchmark of matrix elements:
Hoai Le et al. arXiv:2407.02064v1

YNN (ΛNN) interactions in practice

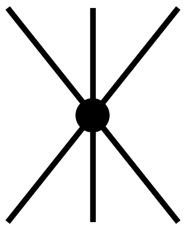
Decuplet approximation in YNN



$\propto C^2$



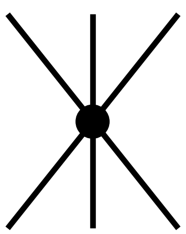
$\propto CG_1, CG_2$



$\propto (G_1)^2, (G_2)^2, G_1 G_2$

is not sufficient to fix spin dependence

→ + ΛNN contact terms **without decuplet constraints**



ΛNN $\propto C_1', C_2', C_3'$

***ad hoc* choice:** alter C_2:

$$C_1' = C_3' = \frac{(G_1 + 3G_2)^2}{72\Delta}$$

$$C_2' = 0$$

→

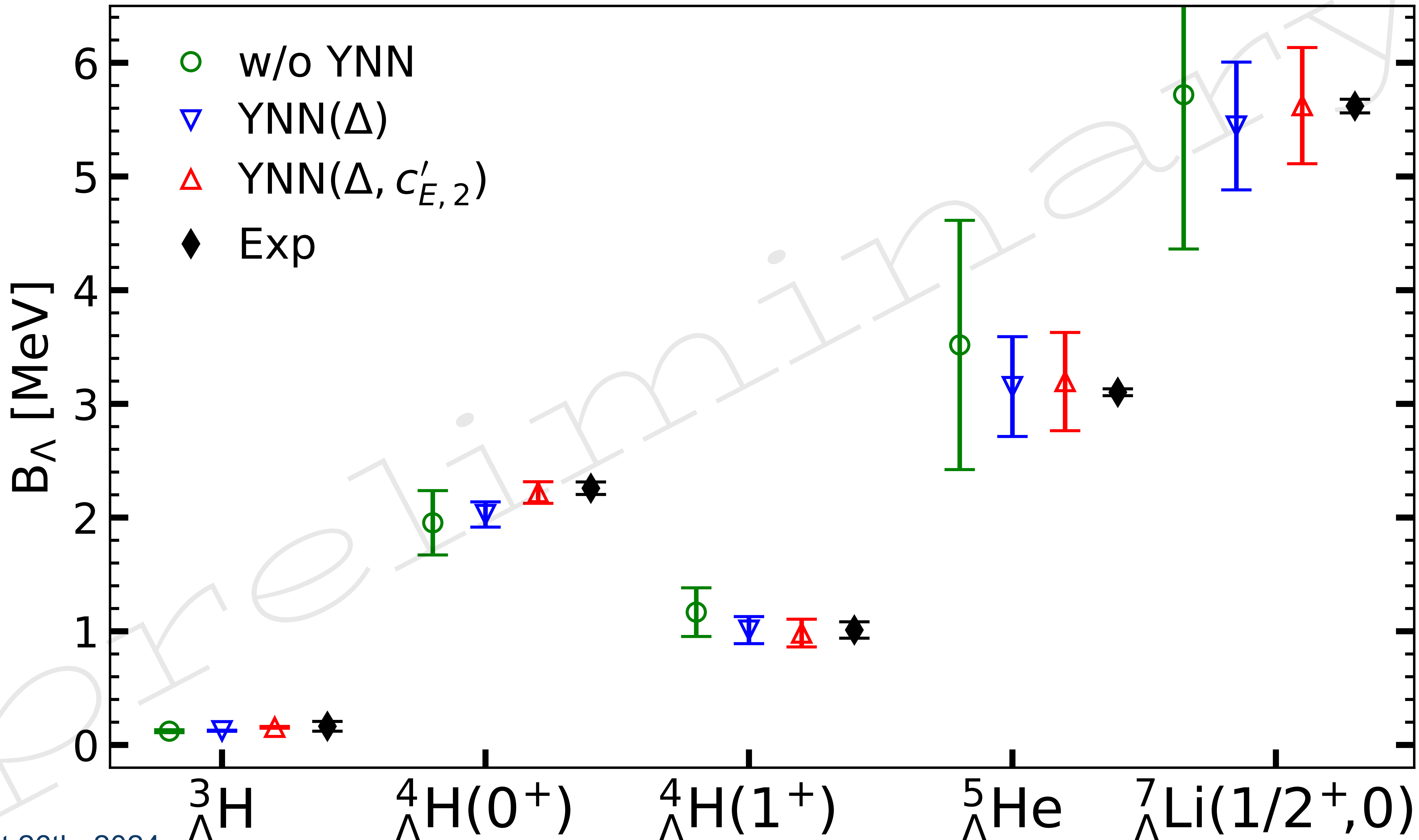
$$V_{\Lambda NN} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

$$C_2' = G_3$$

C_2' introduces a spin dependent interaction in the most relevant particle channel

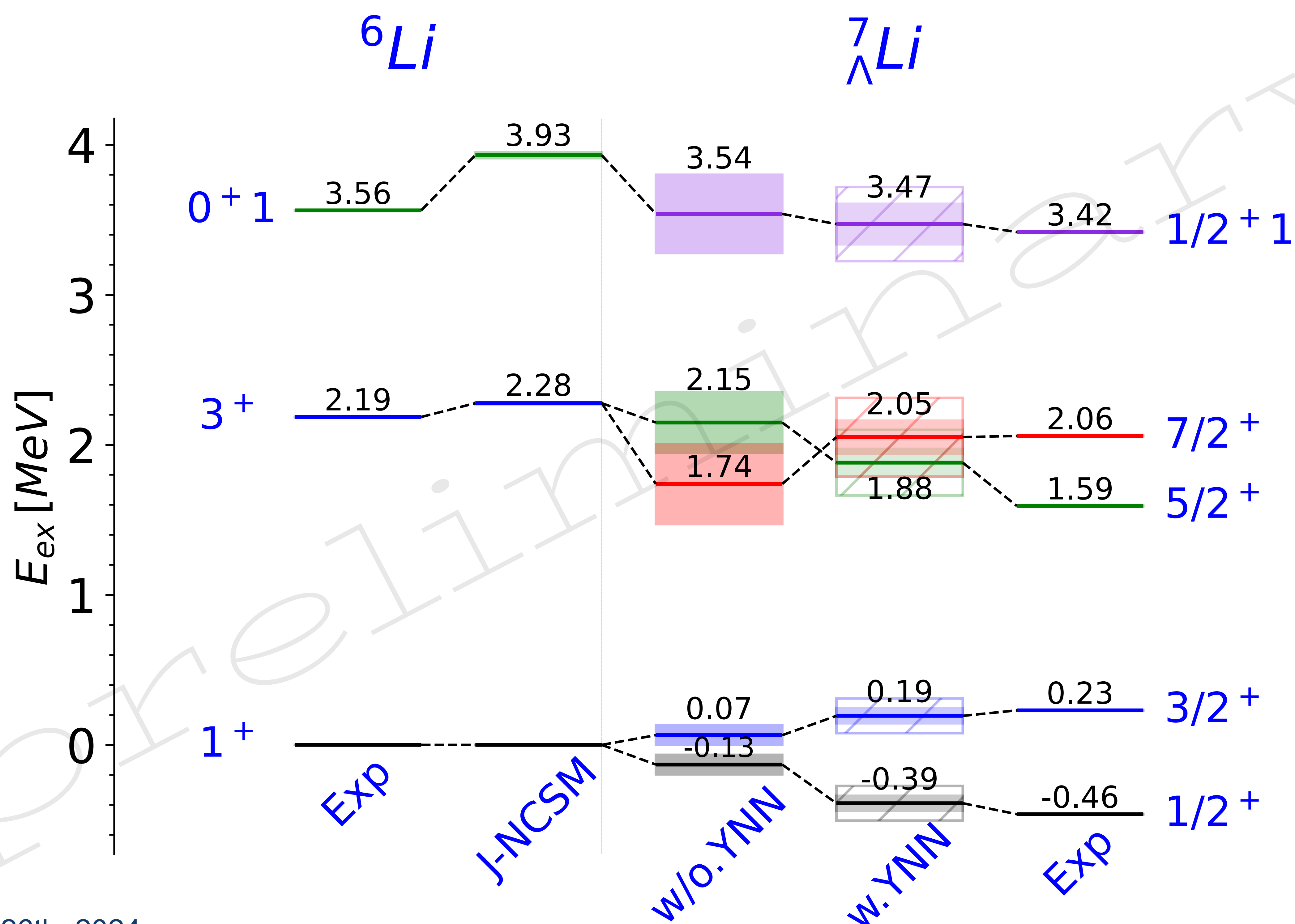
YNN fit

- Fit to 0^+ and 1^+ state of ${}^4_\Lambda\mathrm{He}$ and/or ${}^5_\Lambda\mathrm{He}$
- spin-dependence in A=4 not well explained by decuplet saturation
- C_2' term improves 0^+ of ${}^4_\Lambda\mathrm{He}$ and $1/2^+$ of ${}^7_\Lambda\mathrm{Li}$
- agreement generally much better than $\mathrm{N^2LO}$ uncertainty



YNN prediction for ${}^{7}_{\Lambda}\mathrm{Li}$

- good agreement
- C_2' term included, but not very important (not shown)
- higher states have significant uncertainty



Conclusions & Outlook

- **YN interactions not well understood**
  - *scarce YN data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - *CSB of ΛN scattering &* $^{4}_{\Lambda}\mathrm{He}\ /\ ^{4}_{\Lambda}\mathrm{H}$
  - $^{3}_{\Lambda}\mathrm{H}$ *is used to constrain the spin dependence*
  - *new experiments & analyses planned at J-PARC, MAMI, J-Lab, FAIR,…*
- **New SMS YN interactions**
  - *give an accurate description low energy YN data*
  - *order LO, NLO and N²LO allow uncertainty quantification*
  - *have a **non-unique** determination of contact interactions (data necessary)*
- **Chiral 3BF** need to be included
  - *chiral 3BFs are now available — non-local and SMS regularization*
  - *but: decuplet saturation alone does not improve spin dependence*
  - *spin-dependent $\Lambda\mathrm{NN}$ leads to further improvement*
  - *however: uncertainty estimate in N²LO of incomplete N²LO YNN force?*
  - *study cutoff dependence / application to more p-shell hypernuclei*