11th International Workshop on Chiral Dynamics

26—30.8, 2024 @ Bochum

Nucleon-nucleon interaction in manifestly Lorentz-invariant ChEFT

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In collaboration with:

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26-08-2024

OUTLINE

- Introduction

- Theoretical framework

- Results and discussion

- Summary

Nuclear forces — Weinberg's seminal work

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 14 August 1990 PLB251(1990)288-292

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON–PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 2 April 1991 NPB363(1991)3-18

- **Self-consistently** include many-body forces

$$V = V_{2N} + V_{3N} + V_{4N} + \cdots$$

- **Systematically improve** order by order (heavy baryon ChPT)

$$V_{iN} = V_{iN}^{\mathrm{LO}} + V_{iN}^{\mathrm{NLO}} + V_{iN}^{\mathrm{NNLO}} + \cdots$$

- Scattering amplitude: **Schrödinger / Lippmann-Schwinger Eq.**

$$\left[\left(\sum_{i=1}^{A} -\frac{\nabla_i^2}{2m_N}\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right] |\Psi\rangle = E|\Psi\rangle$$

- ☐ Provide **a systematic and solid theoretical approach** to study the few-nucleon scattering

Renormalization issue of chiral NF

- Iteration of the chiral NN potential within LSE

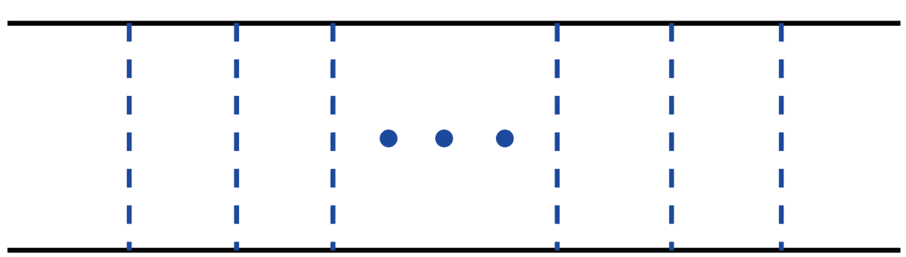
$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int \frac{d^3k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

➡ UV divergencies cannot be absorbed by contact terms!

- Leading order NN potential

$$V_{\mathrm{LO}} = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{4 f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\boldsymbol{q}^2 + m_\pi^2}$$

- Iterated one-pion exchange potential (ladder diagrams)



M. Savage, arXiv:nucl-th/9804034

$k \to \infty$ → Spin-triplet

Logarithmic Divergence

$\sim (Q m_N)^n$

cannot be absorbed by C_S, C_T

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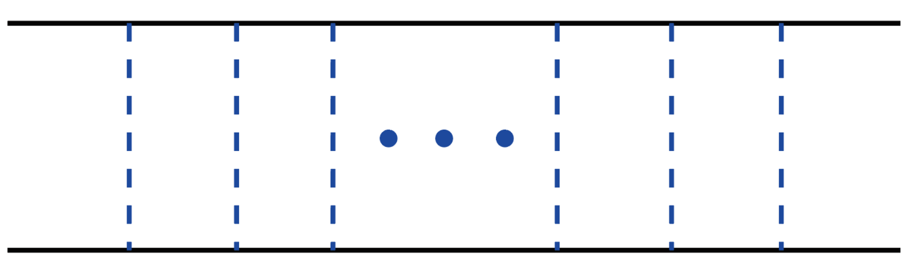

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WPC is inconsistent with renormalization, even at LO!

Deal with the renormalization issue

- Possible solutions (still controversial...)
  - **Keep cutoff lower than hard scale:** $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
    - ✓ WPC is consistent — *G.P. Lepage, nucl-th/9706029; E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*
    - ✓ Achieve great successes — *A.M. Gasparyan, E. Epelbaum PRC105(2022)024001; 107 (2023) 044002,…*

      E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773

      R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1
  - **Kaplan, Savage, and Wise (KSW) power counting**
    - ✓ Treat the exchange of pions perturbatively — *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*
    - ✓ Fail to converge in certain spin-triplet channels — *S. Fleming, et al., Nucl.Phys. A677 (2000) 313*
      - Deepen examine: only lowest spin-triplet partial waves — *D.B. Kaplan, PRC102(2020)034004*
  - **Modified WPC with renormalization group invariance (RGI)**
    - ✓ Rearrange the higher order contact terms to the lower chiral order

      A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002.

      B. Long and C.-J. Yang, PRC84(2011)057001 …

      U. van Kolck, Front. in Phys. 8 (2020) 79

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    B. Long and C.-J. Yang, PRC84(2011)057001 …

    U. van Kolck, Front. in Phys. 8 (2020) 79

- **Lorentz invariant framework to reformulate chiral force**
  - ✓ Fundamental symmetry of our nature

Chiral forces in Lorentz invariant framework

- Initial idea: modified Weinberg approach — E. Epelbaum and J. Gegelia, PLB716(2012)338-344
  - Use Weinberg power counting to expand the NN potential
    - ✓ Relativistic corrections are perturbatively included

$$V(p', p) = \bar{u}_1 \bar{u}_2 \, \mathcal{A} \, u_1 u_2, \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

  - Use Kadyshevsky equation to calculate the scattering T-matrix

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{2(\boldsymbol{k}^2 + m_N^2)} \frac{1}{\sqrt{\boldsymbol{p}^2 + m_N^2} - \sqrt{\boldsymbol{k}^2 + m_N^2} + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

  - **LO study: a renormalizable framework** (except $^3P_0$ channel)

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  - **LO study: a renormalizable framework** (except $^3P_0$ channel)

- Based on this idea, we proposed **a systematic framework** within the time-ordered perturbation theory (TOPT) using covariant chiral Lagrangians

  - Formulate the NN interaction up to next-to-next-to-leading order

    V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798, 134987 (2019)
    XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101, 034001 (2020)
    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)
    XLR et al., in preparation (2024)

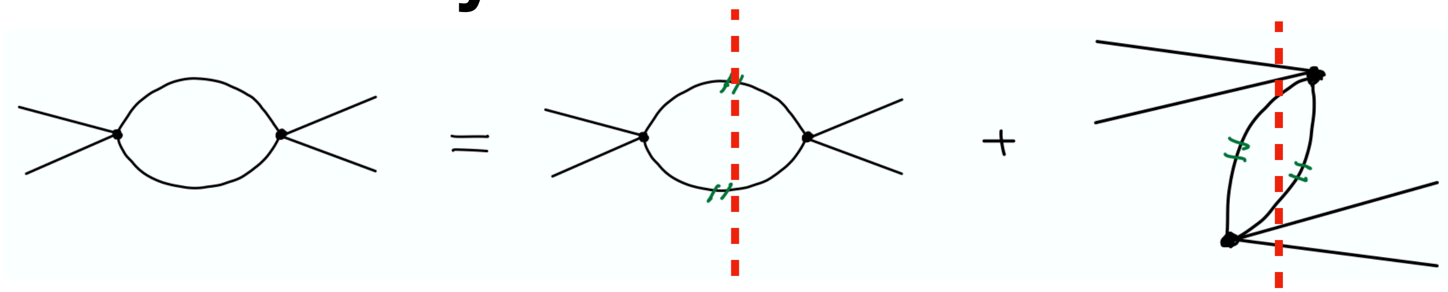
Theoretical framework

Time-ordered perturbation theory

Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
  - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
- **TOPT or old-fashioned perturbation theory**



Advantages

- Explicitly show the unitarity
- Easily to tell the contributions of a particular diagram

Obtain the rules for time-ordered diagrams

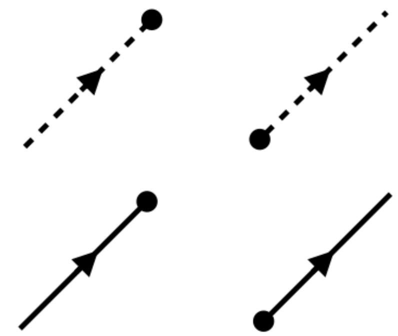
- Perform Feynman integrations over the zeroth components of the loop momenta
- Decompose Feynman diagram into sums of time-ordered diagrams
- Match to the rules of time-ordered diagrams

Diagrammatic rules in TOPT

XLR, PoS(CD2021)007

- **External lines**

Spin 0 boson (in, out)



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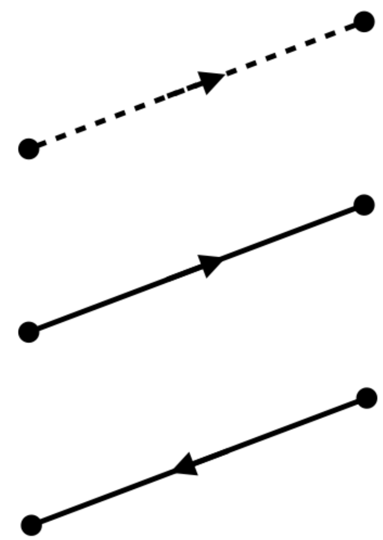
Spin 1/2 fermion (in, out)

$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

- **Internal lines**



Spin 0 (anti-)boson

$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion

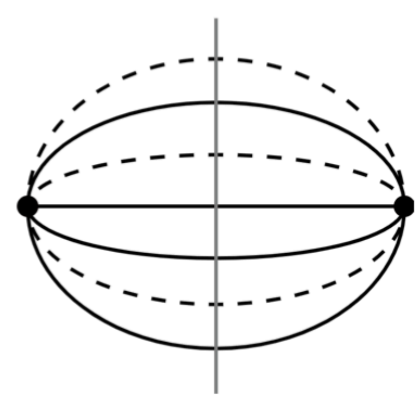
$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

anti-fermion

$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

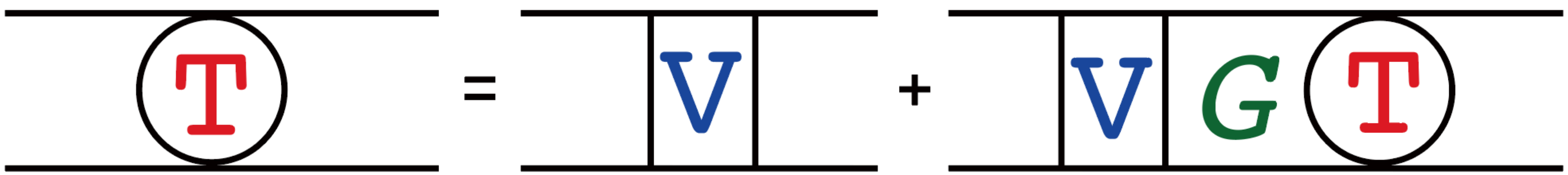
- **Interaction vertices:** the standard Feynman rules
  - Zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Nucleon-nucleon scattering in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the **two-nucleon irreducible** time-ordered diagrams
  - **Weinberg power counting:** systematic ordering of all graphs
- Scattering equation



- **Two-nucleon Green function** $G(E,k) = \frac{m_N^2}{k^2+m_N^2}\frac{1}{E-2\sqrt{k^2+m_N^2}+i\epsilon}$
- **Uniquely determined** the scattering equation

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{m_N^2}{k^2+m_N^2} \frac{1}{E-2\sqrt{k^2+m_N^2}+i\epsilon} T(\boldsymbol{k},\boldsymbol{p})$$

V. Kadyshevsky, NPB (1968)

✓ SELF-CONSISTENTLY obtained in our TOPT framework

✓ Milder UV behaviour than the Lippmann-Schwinger equation

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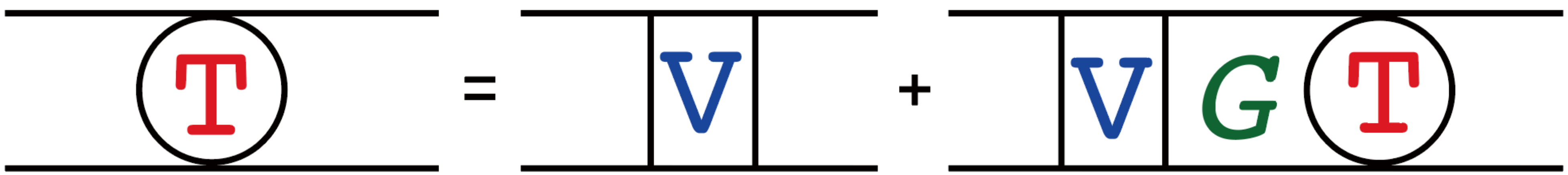

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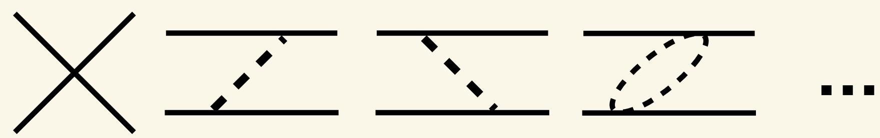
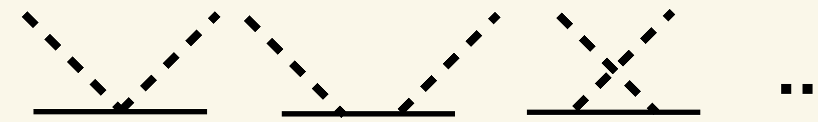
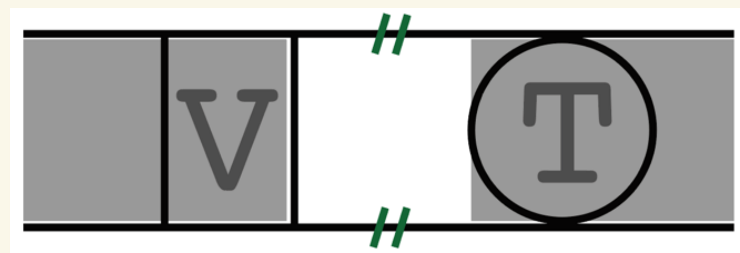
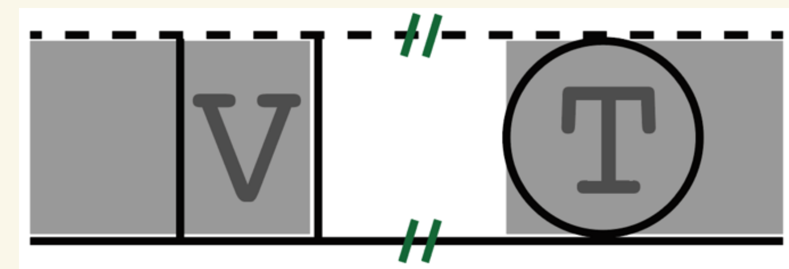
Potential and scattering equation are obtained on an equal footing!

Extend to BB and MB scatterings

	Baryon-baryon scattering	Meson-baryon scattering
Potential TOPT diagrams		
Green function	 $G_{ij}^{BB}(E) = \frac{m_i m_j}{\omega_{m_i}\omega_{m_j}} \frac{1}{E - \omega_{m_i} - \omega_{m_j} + i\epsilon}$	 $G^{MB}(E) = \frac{m}{2\omega_M \omega_m} \frac{1}{E - \omega_M - \omega_m + i\epsilon}$

- **Unify the description of SU(3) baryon-baryon and meson-baryon scatterings within our TOPT framework**
  - $S = -1$ baryon-baryon interaction at LO

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 101, 034001 (2020)
  - $S = -1$ meson-baryon interaction at LO and NLO / $\Lambda(1405)$

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, EPJC 80 (2020) 406; 81 (2021) 582;
    XLR, Phys. Lett. B 855,138802 (2024)
    XLR et al., work in progress

Results and discussion

Chiral Lagrangian up to NNLO

Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

- Purely pionic sector *J.Gasser, H. Leutwyler, Ann.Phys.(1984)*

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

- One-nucleon sector *J. Gasser, M. E. Sainio, and A. Svarc, NPB(1988)*

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

✓ $f_\pi = 92.4$ MeV, $g_A = 1.267$, $c_{1,2,3,4}$ determined by πN scattering data

- Two-nucleon sector (with unknown LECs) *N.Fettes, U.-G. Meißner, S. Steininger, NPA(1998)*

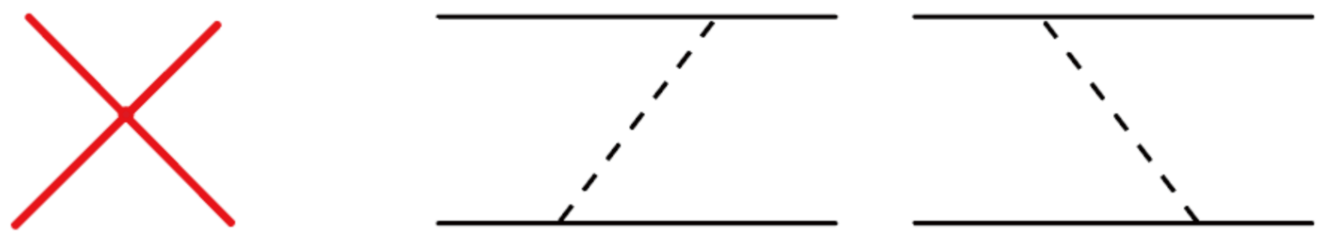
$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2} \left[C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) + C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \Psi_N \right) \right.$$
$$\left. + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N \right) + C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N \right) \left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N \right) \right]$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N \mathcal{O}_i \Psi_N \Psi_N$$

L.Girlanda, S. Pastore, R. Schiavilla, M. Viviani, PRC(2010)
Yang Xiao, Li-Sheng Geng, XLR, PRC(2019)
E. Filandri, L. Girlanda, PLB (2023)

Leading order potentials

- Follow TOPT rules



- Perform the expansion for the nucleon energies (Weinberg P.C.)

$$V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

S. Weinberg, PLB251(1990)288-292

- Consistent with the non-relativistic contact terms

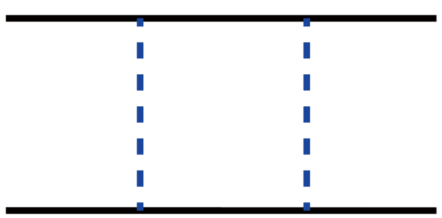
$$V_{\rm OPE} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{4m_N^2}{\omega(q, M_\pi)\,(m_N + \omega(p, m_N))\,(m_N + \omega(p', m_N))} \times \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\omega(p, m_N) + \omega(p', m_N) + \omega(q, M_\pi) - E - i\epsilon}$$

- Milder UV behaviour than that of the non-relativisitc OPEP

$V_{\rm OPE}(p', k) \xrightarrow{k \longrightarrow \infty}$ Our $\frac{1}{k}$ vs. Non-Rel. $\frac{1}{1}$

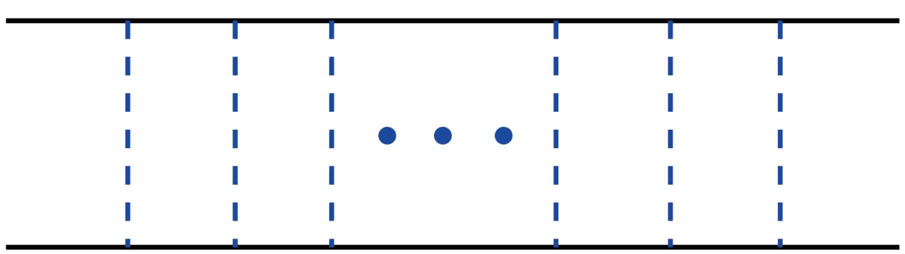
UV behavior of the OPE potential

- Once-iterated OPEP: $V\,G\,V$ *XLR, PoS(CD2021)007*



$$\begin{cases} I_{VGV}^{\mathrm{Our}} \to \int dk^3 \frac{1}{k}\frac{1}{k^3}\frac{1}{k} = \int dk^3 \frac{1}{k^5} & \textbf{UV convergent} \\ I_{VGV}^{\mathrm{NR}} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3 \frac{1}{k^2} & \textbf{UV divergent} \end{cases}$$

- Iteration of our OPEP



$k \to \infty$ → Finite

- Scattering amplitude from OPEP is **cutoff independent**

$$T_{\mathrm{OPE}} = V_{\mathrm{OPE}} + V_{\mathrm{OPE}}\,G\,T_{\mathrm{OPE}}$$

UV behavior of the OPE potential

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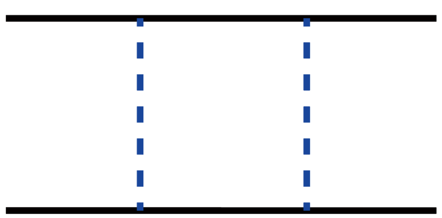

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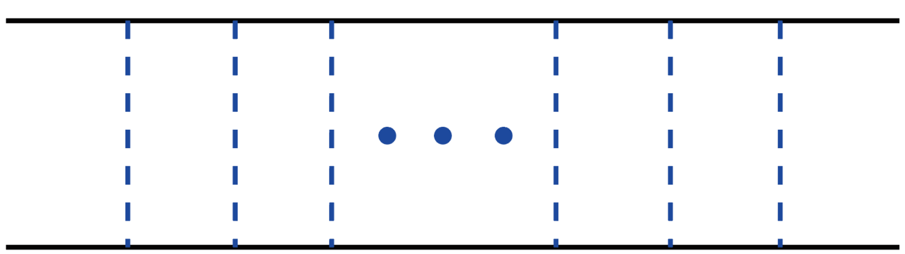


$\xrightarrow{k \to \infty}$ Finite

- Scattering amplitude from OPEP is **cutoff independent**

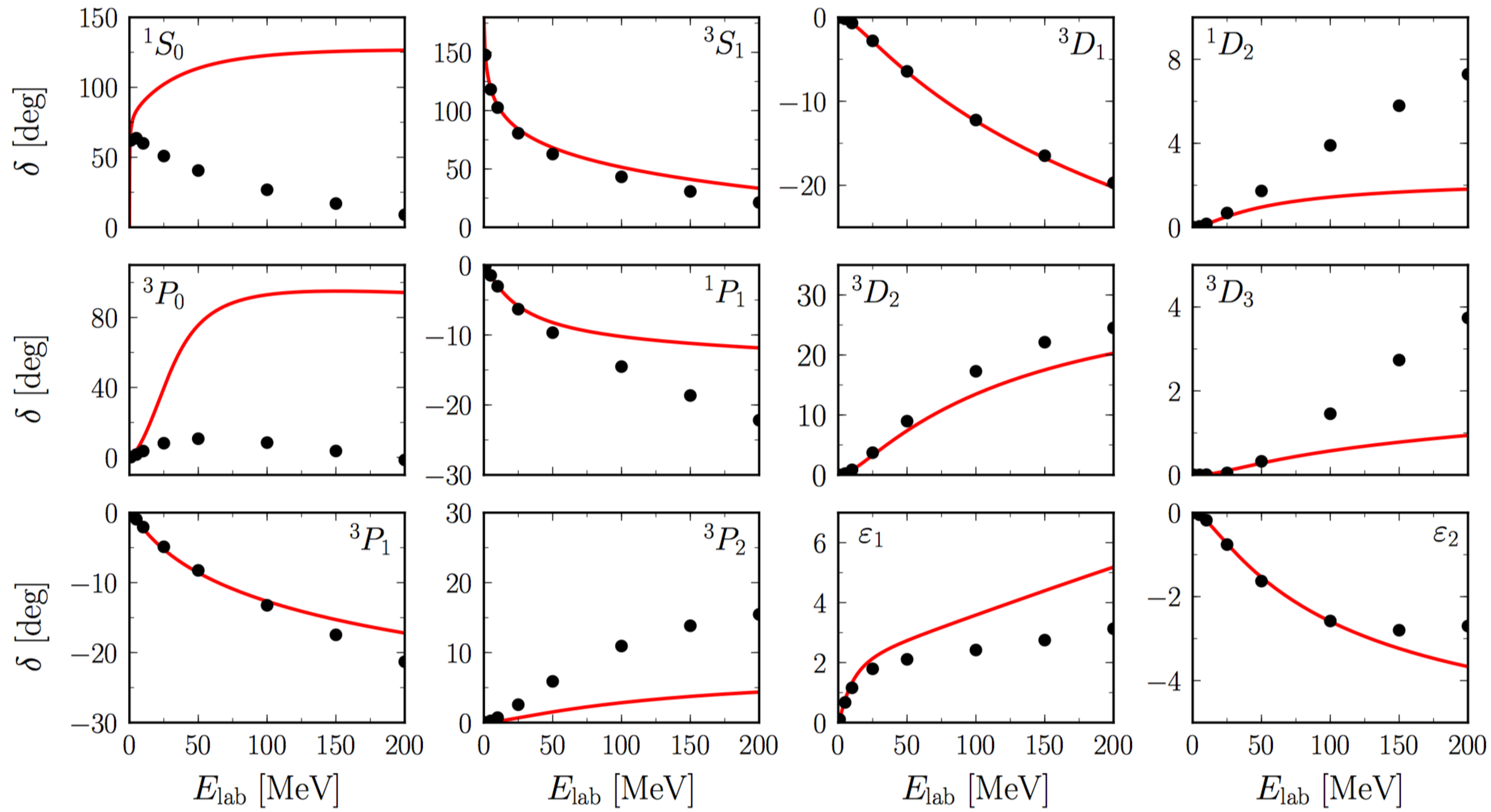
$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}\,G\,T_{\rm OPE}$$

☑ **Our LO potential is renormalizable!**

- Unique solutions for all partial waves, no limit-cycle behavior
- Avoid finite-cutoff artefacts inherent to the conventional NR framework

Phase shifts at LO

- Two LECs: fixed by scattering lengths of $^1S_0$ and $^3S_1$ ($\Lambda = 20$ GeV)



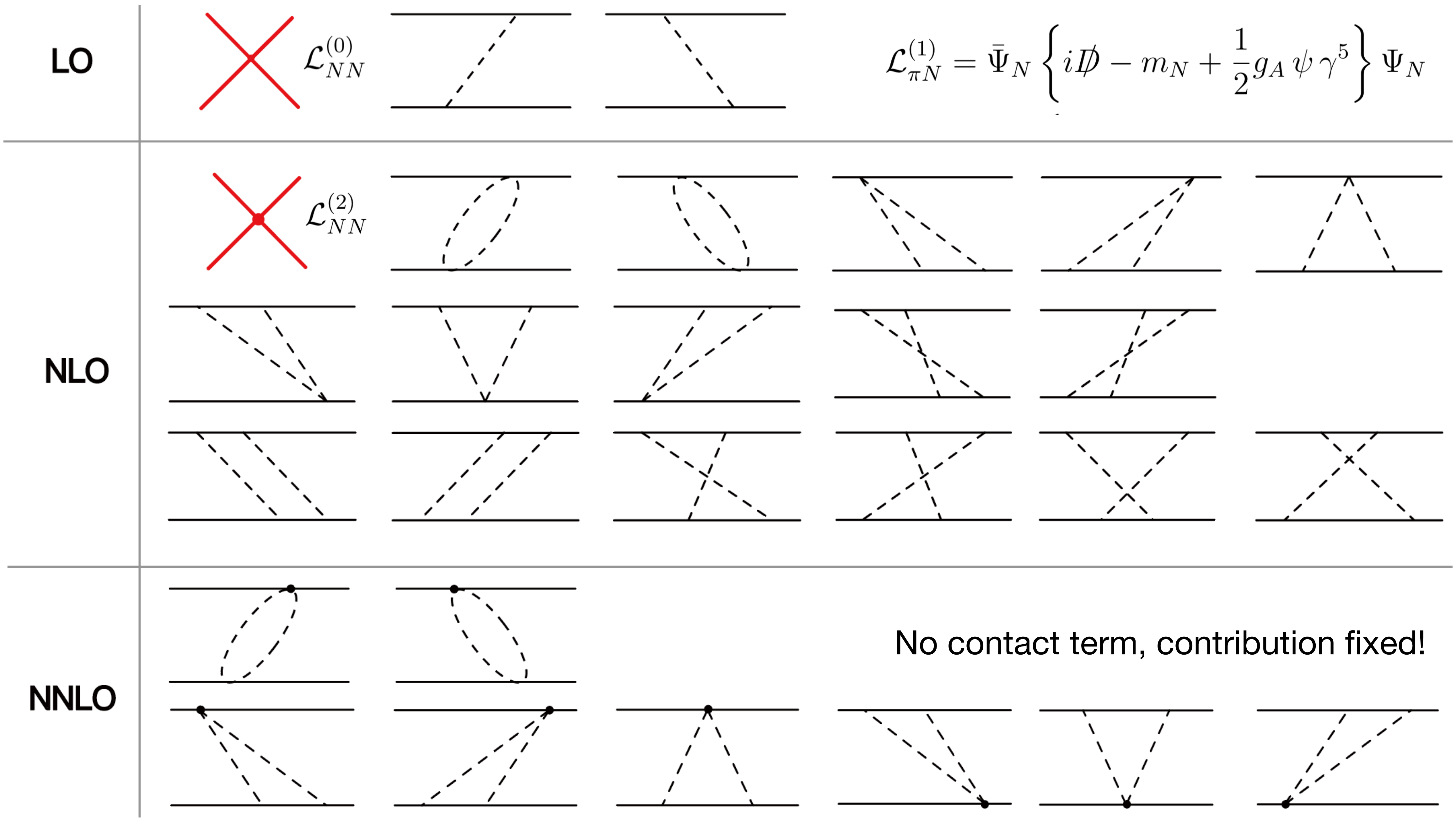
- Provides a reasonable description of the empirical phase shifts
  - ✓ $^1S_0$ and $^3P_0$: Large deviation
  - ✓ Part of the subleading corrections must be treated non-perturbatively

Beyond LO

V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798, 134987 (2019)

NNLO potential in TOPT

- Time ordered diagrams up to NNLO



$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i \not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

No contact term, contribution fixed!

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$

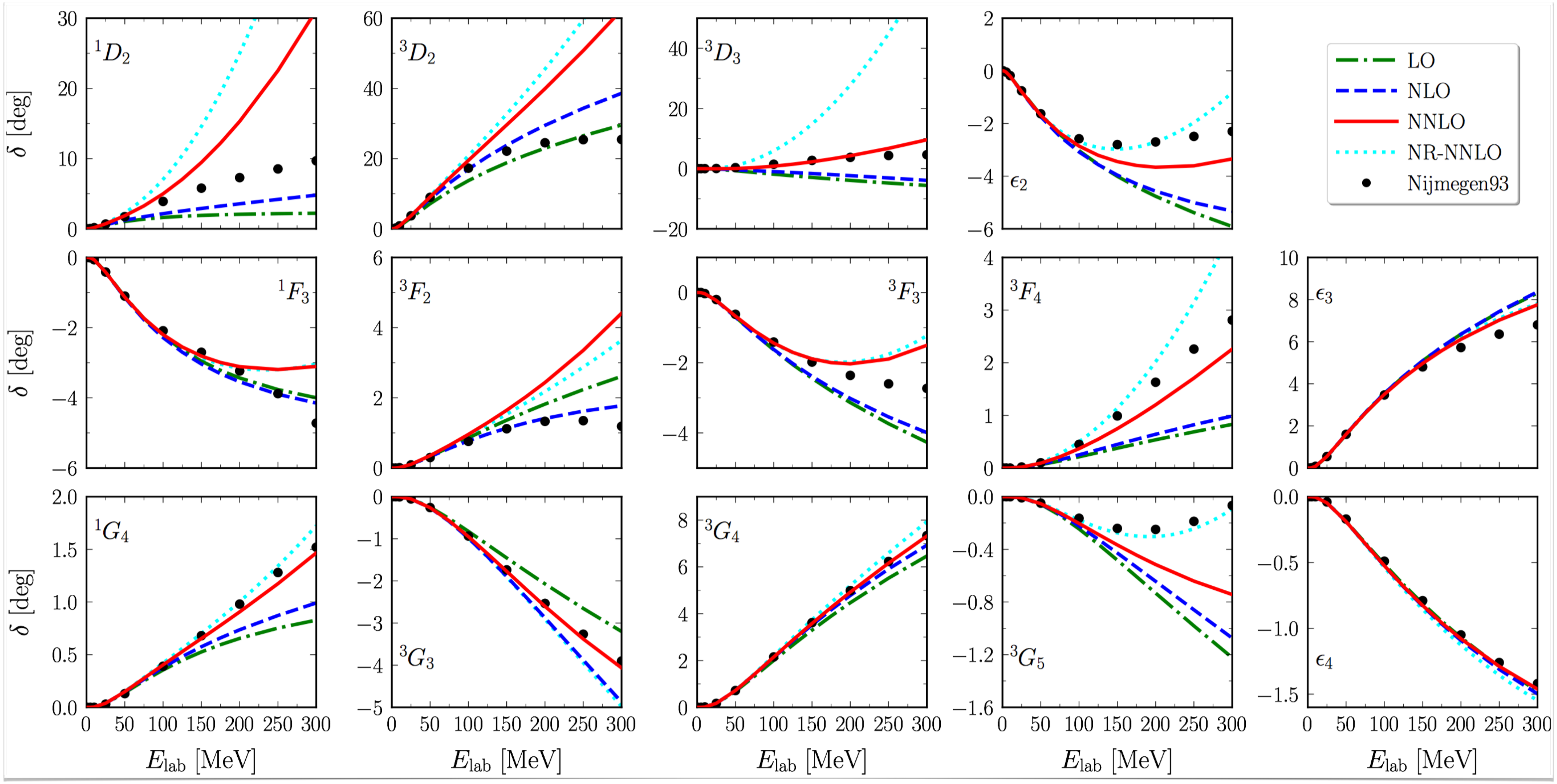
D. Siemens, et al.,PLB 770 (2017) 27-34

Pion-exchange contribution

- On-shell T-matrix under the Born approximation

$$T(p',p) = V_{\rm OPE}(p',p) + V^{(2)}_{2\pi,irr}(p',p) + V^{(3)}_{2\pi,irr}(p',p) + V_{\rm OPE}\,G\,V_{\rm OPE}$$

- **Prediction: phase shifts of D, F, G waves**



✓ Improve the description of D waves; globally similar results for F, G waves

- $^3G_5$: non-rel. result is accidental, c_i/m_N effect (N^{4}LO) is large *D. Entem, et al., PRC 91, 014002 (2015)*

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

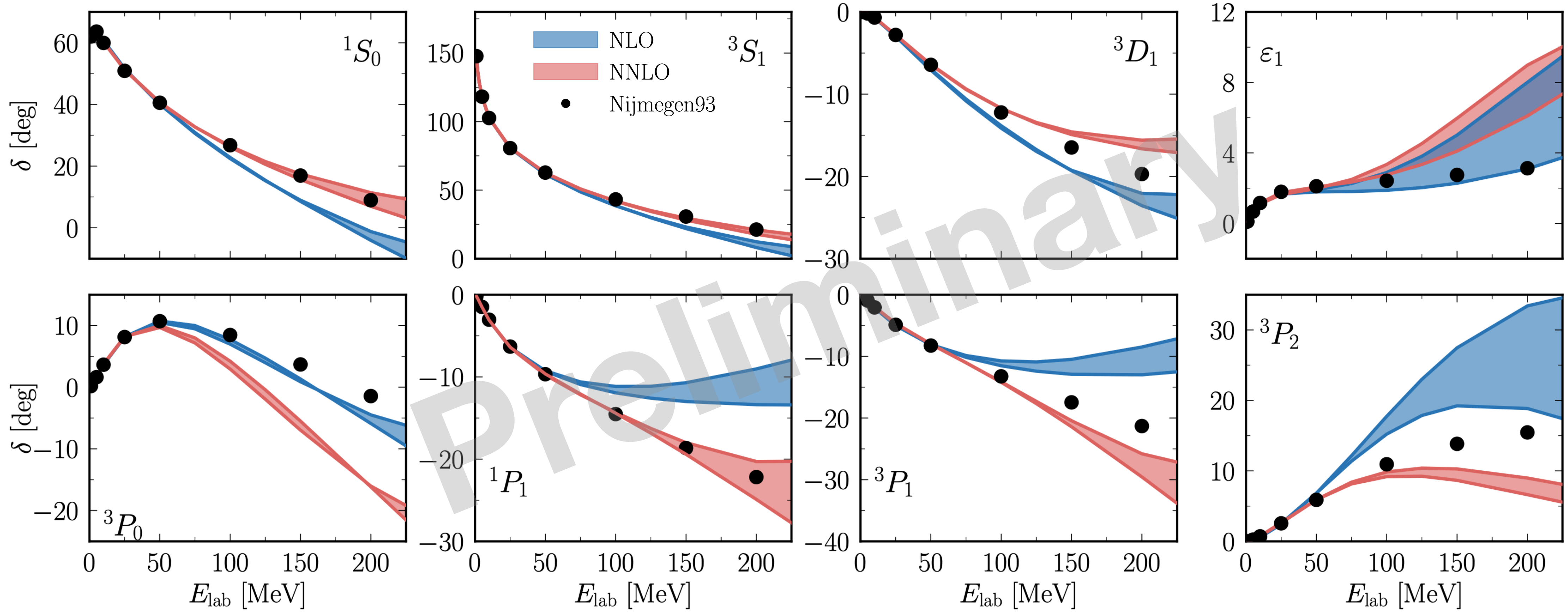
NNLO: contact + pion exchanges

- Partial wave T-matrix
  - V_{NNLO} non-perturbatively iterated in the Kadyshevsky equation

$$T^{sj}_{ll'}(p',p) = V^{sj}_{ll'}(p',p) + \sum_{l''} \int \frac{d^3k}{(2\pi)^3} V^{sj}_{ll''}(p',k) \frac{m_N^2}{2(k^2+m_N^2)} \frac{1}{\sqrt{p^2+m_N^2} - \sqrt{k^2+m_N^2} + i\epsilon} T^{sj}_{l''l'}(k,p)$$

  - Pion-loop potential: cutoff regularization with $k_{\text{max.}} = 500$ MeV
  - Exponential regulator: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$, with $n = 2$, $\Lambda = 400 \sim 550$ MeV

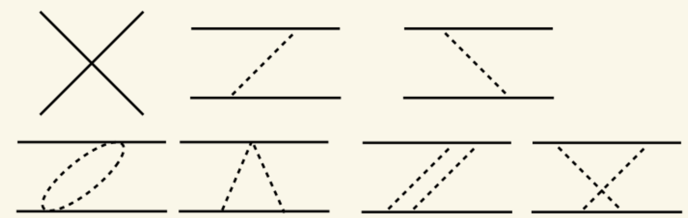
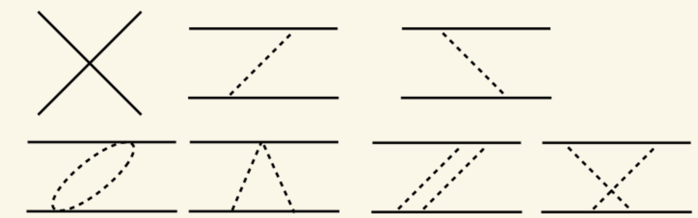
- Phase shifts: Fit NPWA ($E_{\text{lab}} \leq 100$ MeV)



- Deuteron binding energy NLO -2.16 MeV; NNLO -2.18 GeV; no deeply bound states

Summary

- Proposed a systematic framework to formulate chiral forces

Time-ordered perturbation theory	Non-relativistic (Heavy-baryon)	Manifestly Lorentz invariant
Chiral Lagrangians	$N^\dagger \left[i(v\cdot D)+g_A(S\cdot u)\right]N$ $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$	$\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$
Potential TOPT diagrams		
Scattering equations (T = V + VGT)	Lippmann-Schwinger eq.	Kadyshevsky eq.
Power counting	Weinberg p.c.	Weinberg p.c.

- Obtained **the non-singular LO potential**, achieve the cutoff independence
- Formulated the chiral potential up to **NNLO**
  - Calculated the complicated two-pion-exchange potential at one-loop level
  - Achieved a rather reasonable description of phase shifts

Summary

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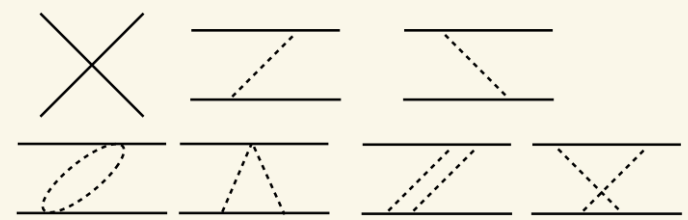
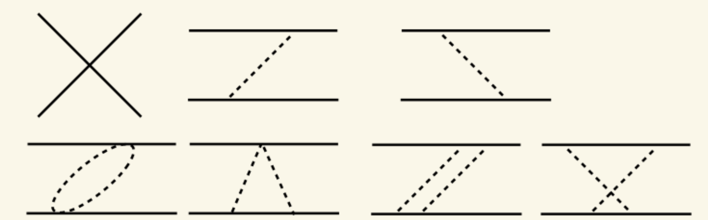
Potential TOPT diagrams		
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Thank you for your attention!

Additional slides

Additional slides

NNLO: contact + pion exchanges

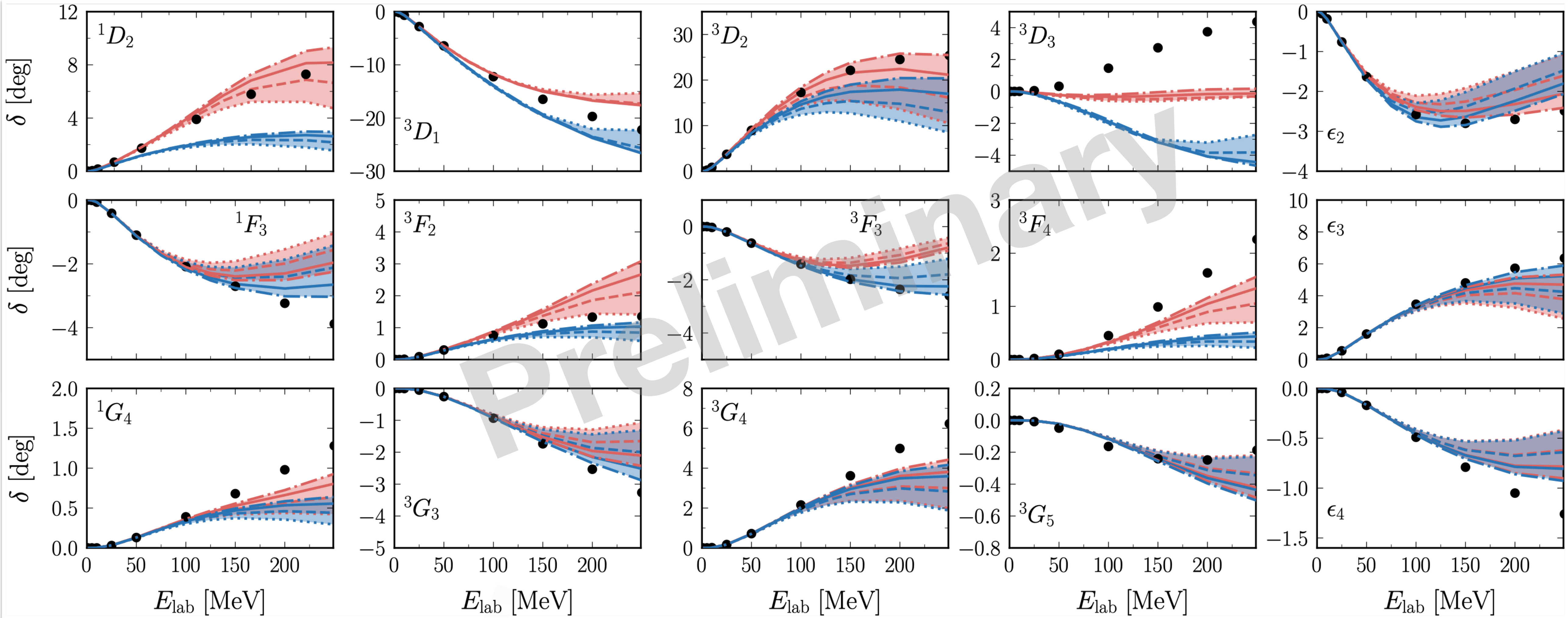
Partial wave T-matrix

- $V_{\rm NNLO}$ non-perturbatively iterated in the Kadyshevsky equation

$$T_{ll'}^{sj}(p',p) = V_{ll'}^{sj}(p',p) + \sum_{l''}\int \frac{d^3k}{(2\pi)^3} V_{ll''}^{sj}(p',k)\, \frac{m_N^2}{2(k^2+m_N^2)}\, \frac{1}{\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon}\, T_{l''l'}^{sj}(k,p)$$

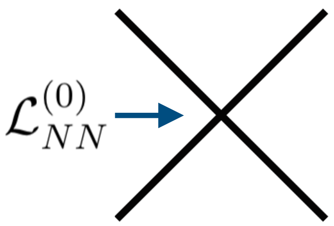
- Pion-loop potential: cutoff regularization: $k_{\rm max.} = 500$ MeV
- Exponential regulator: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$, with $n = 2$, $\Lambda = 400 \sim 600$ MeV

Prediction of D,F,G partial wave phases



Contact terms up to NNLO

LO contact term (5 LECs)



$$V_{\rm LO} = C_S(\bar u_3\, u_1)(\bar u_4\, u_2) + C_A(\bar u_3\, \gamma_5\, u_1)(\bar u_4 \gamma_5 u_2) + C_V(\bar u_3\, \gamma_\mu\, u_1)(\bar u_4\, \gamma^\mu\, u_2)$$
$$+ C_{AV}(\bar u_3\, \gamma_\mu \gamma_5\, u_1)(\bar u_4\, \gamma^\mu \gamma_5\, u_2) + C_T(\bar u_3\, \sigma_{\mu\nu}\, u_1)(\bar u_4\, \sigma^{\mu\nu}\, u_3)$$

- Expand the nucleon energy up to $\mathcal{O}(p^2)$ / NLO

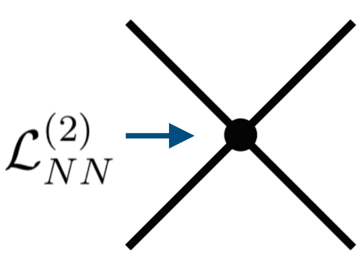
$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2 m_N} + \frac{p^2}{4\sqrt{2}\, m_N^{3/2}} + \mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for LO contact terms

➡ Keep the full form of Dirac spinors

NLO contact term

- Expand the nucleon energy $\sqrt{\omega(p, m_N) + m_N} = \sqrt{2 m_N} + \mathcal{O}(p^2)$
- Same form as the non-relativistic case with 7 LECs



$$V_{\rm NLO} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{P}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{P}^2\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{i}{2} C_5 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}$$
$$+ C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7 (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{P} \cdot \boldsymbol{\sigma}_2)$$

Partial wave contact terms

- J=0: 1S0 and 3P0 partial waves

$$V(^1S_0) = \xi_N C^{LO}_{^1S_0}\left(1+R_p^2R_{p'}^2\right) + \xi_N\left[\hat{C}^{LO}_{^1S_0} + 4m_N^2 C^{NLO}_{^1S_0}\right]\left(R_p^2+R_{p'}^2\right)$$
$$= \xi_N C^{LO}_{^1S_0}\left(1+R_p^2R_{p'}^2\right) + \xi_N\tilde{C}_{^1S_0}\left(R_p^2+R_{p'}^2\right)$$

$$V(^3P_0) = -2\xi_N\left[C^{LO}_{^3P_0} - 2m_N^2 C^{NLO}_{^3P_0}\right]R_pR_{p'}$$
$$= -2\xi_N\tilde{C}_{^3P_0}R_pR_{p'} = -2\tilde{C}_{^3P_0}\frac{p\,p'}{4m_N^2}$$

$$\xi_N = \frac{(\omega_p+m_N)(\omega_{p'}+m_N)}{4m_N^2},\quad R_p = \frac{p}{\omega_p+m_N},\ \text{and}\ R_{p'} = \frac{p'}{\omega_{p'}+m_N}$$

- J=1: 1P1, 3P1, 3S1-3D1 partial waves

$$V(^1P_1) = \frac{2}{3}\xi_N\left[-C^{LO}_{^1P_1} - 6m_N^2\,C^{NLO}_{^1P_1}\right]R_pR_{p'}$$
$$= \frac{2}{3}\xi_N\tilde{C}_{^1P_1}R_pR_{p'} = \frac{2}{3}\tilde{C}_{^1P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3P_1) = -\frac{4}{3}\xi_N\left[C^{LO}_{^3P_1} - 3m_N^2 C^{NLO}_{^3P_1}\right]R_pR_{p'}$$
$$= -\frac{4}{3}\xi_N\tilde{C}_{^3P_1}R_pR_{p'} = -\frac{4}{3}\tilde{C}_{^3P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3S_1) = \frac{\xi_N}{9}C^{LO}_{^3S_1}\left(9+R_p^2R_{p'}^2\right) + \frac{\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 36m_N^2 C^{NLO}_{^3S_1}\right]\left(R_p^2+R_{p'}^2\right)$$
$$= \frac{\xi_N}{9}C^{LO}_{^3S_1}\left(9+R_p^2R_{p'}^2\right) + \xi_N\tilde{C}_{^3S_1}\left(R_p^2+R_{p'}^2\right)$$

$$V(^3D_1) = \frac{8\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

$$V(^3S_1-{}^3D_1) = \frac{2\sqrt{2}\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 9\sqrt{2}m_N^2 C^{NLO}_{^3D_1-{}^3S_1}\right]R_p^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$
$$= \xi_N\tilde{C}_{^3D_1-{}^3S_1}R_p^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

$$V(^3D_1-{}^3S_1) = \xi_N\tilde{C}_{^3D_1-{}^3S_1}R_{p'}^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

- J=2: 3P2 partial wave $V(^3P_2) = C^{NLO}_{^3P_2}\,p\,p' = \tilde{C}_{^3P_2}\frac{p\,p'}{4m_N^2}$

- Finally, we have **9 LECs to be fixed:** $C^{LO}_{^1S_0}, C^{LO}_{^3S_1},\ \tilde{C}_{^1S_0}, \tilde{C}_{^3P_0}, \tilde{C}_{^1P_1}, \tilde{C}_{^3P_1}, \tilde{C}_{^3S_1}, \tilde{C}_{^3D_1-{}^3S_1}, \tilde{C}_{^3P_2}$

Same number of contact terms as the non-relativistic NLO case

OPE correction up to NNLO

OPE potential

$$V_{\rm OPE} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{\omega_q}\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p+\omega_{p'}+\omega_q-E-i\epsilon}$$

- Expand the nucleon energy expansion for OPEP at NLO

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}\,m_N^{3/2}} + \mathcal{O}(p^4)$$

  ✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for OPE potential

  ➡ Keep the full form of Dirac spinors

- Eliminate the energy dependence of OPEP (avoid the pole contribution)

  ✓ Expand E at $\omega_p+\omega_p'$, then, we obtain contribution of OPEP at NLO

$$V_{\rm OPE}^{\not E} = -\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\frac{1}{\omega_q^2}(\bar{u}_3\gamma_\mu\gamma_5 q^\mu u_1)(\bar{u}_4\gamma_\nu\gamma_5 q^\nu u_2) \longrightarrow \textbf{LO correction } V^{(0)}_{{\rm OPE},\not E}$$

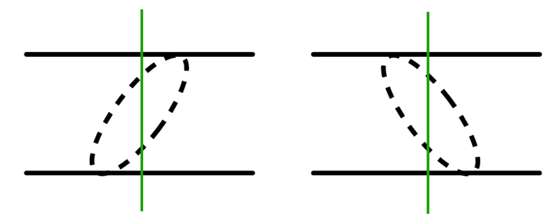
NLO correction $V^{(2)}_{2\pi,\not E}$

$$+\frac{1}{2}\left(\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\right)^2\int\frac{d^3k}{(2\pi)^3}\frac{m_N^2}{k^2+m_N^2}\frac{\omega_{p'-k}+\omega_{p-k}}{\omega^3_{p'-k}\omega^3_{p-k}}$$
$$\times\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}-\boldsymbol{p})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}-\boldsymbol{p})\right].$$

Two-pion exchange potential at NLO

- Follow our TOPT rules:
  - Football diagram

$$V_F = \frac{1}{16 f_\pi^4}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \int \frac{d^3k}{(2\pi)^3}\,\frac{(\omega_k+\omega_{k+q})(\omega_p+\omega_{p'}) + 4\omega_k\omega_{k+q} - E(\omega_k+\omega_{k+q})}{2\omega_k\omega_{k+q}\,(\omega_k+\omega_{k+q}+\omega_p+\omega_{p'}-E)}\,.$$

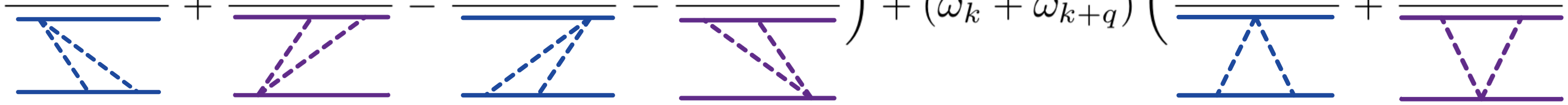


Energy denominator of football diagram

  - Triangle diagrams

$$V_{T+\tilde{T}}^{NN} = \frac{4 m_N g_A^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{128 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[\left(\boldsymbol{k}^2 + (\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k}\right) + \frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}(a+b)\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

$$\times\left[(\omega_{k+q}-\omega_k)\left(\frac{1}{\cdot}+\frac{1}{\cdot}-\frac{1}{\cdot}-\frac{1}{\cdot}\right)+(\omega_k+\omega_{k+q})\left(\frac{1}{\cdot}+\frac{1}{\cdot}\right)\right]$$

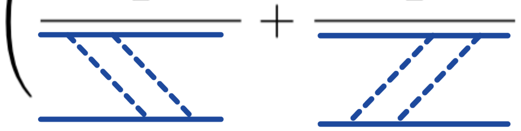


Energy denominator

  - Planar and crossed box diagrams

$$V_B = \frac{m_N^2 g_A^4 (3 - 2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{64 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_1 + X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + X_3\frac{i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2} + X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n}) + X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}^2}\left(\frac{1}{\cdot}+\frac{1}{\cdot}\right)$$

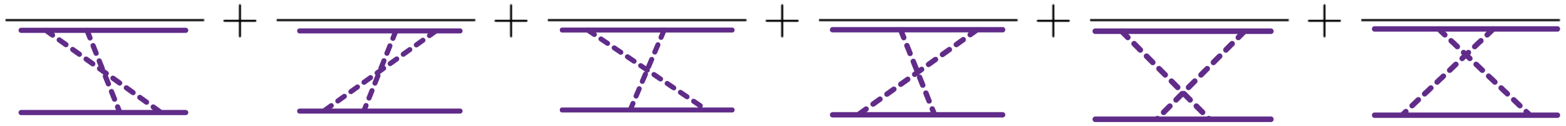


$$\boldsymbol{k} = a\,\boldsymbol{p} + b\,\boldsymbol{p}' + c\,(\boldsymbol{p}'\times\boldsymbol{p})$$

$X_1 = [\boldsymbol{k}^2 + \boldsymbol{q}\cdot\boldsymbol{k}]^2$, $X_2 = -c^2\boldsymbol{q}^2\left[\boldsymbol{P}^2\boldsymbol{q}^2 - (\boldsymbol{q}\cdot\boldsymbol{P})^2\right]$, $X_3 = -2(a+b)\left(\boldsymbol{k}^2 + (\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k}\right)$,
$X_4 = -(a+b)^2 + c^2\boldsymbol{q}^2$, $X_5 c^2\left[\boldsymbol{P}^2\boldsymbol{q}^2 - (\boldsymbol{q}\cdot\boldsymbol{P})^2\right]$

$$V_{\tilde{B}} = \frac{m_N^2 g_A^4 (3 + 2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{64 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_1 + X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n}) + X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

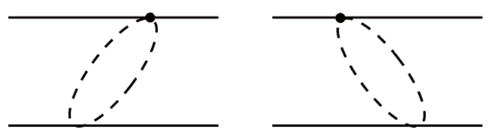
$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}\omega_{p'+k}}\left(\frac{1}{\cdot}+\frac{1}{\cdot}+\frac{1}{\cdot}+\frac{1}{\cdot}+\frac{1}{\cdot}+\frac{1}{\cdot}\right)$$



UV Divergent terms and power counting breaking terms are removed by using **the subtractive renormalization**

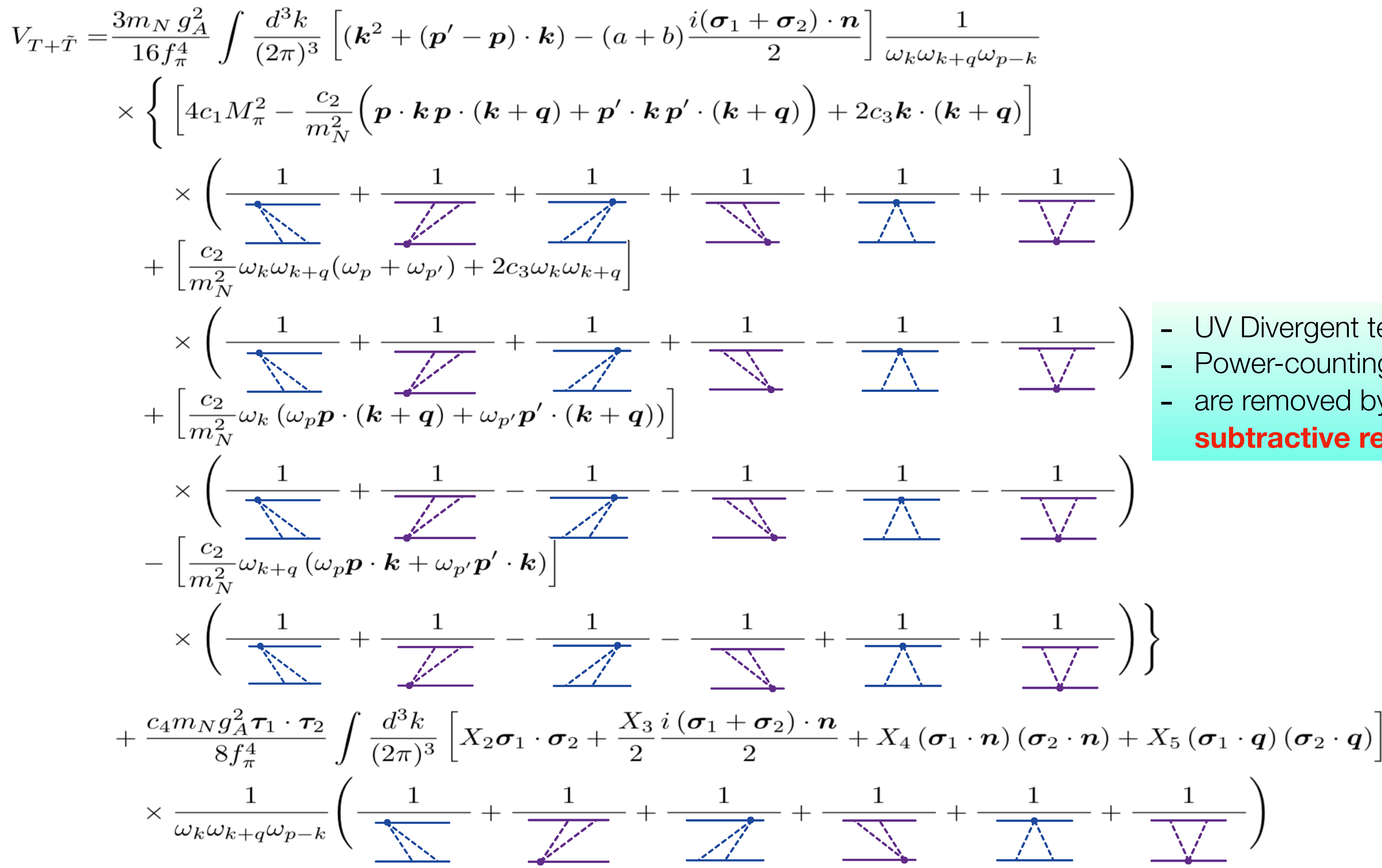
Two-pion exchange potential at NNLO

- Follow our TOPT rules:
  - Football diagrams



No contribution!

  - Triangle diagrams



$$V_{T+\tilde{T}} = \frac{3m_N g_A^2}{16 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k})-(a+b)\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

$$\times\left\{\left[4c_1M_\pi^2-\frac{c_2}{m_N^2}\Big(\boldsymbol{p}\cdot\boldsymbol{k}\,\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\boldsymbol{p}'\cdot\boldsymbol{k}\,\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\Big)+2c_3\boldsymbol{k}\cdot(\boldsymbol{k}+\boldsymbol{q})\right]\right.$$

$$+\left[\frac{c_2}{m_N^2}\omega_k\omega_{k+q}(\omega_p+\omega_{p'})+2c_3\omega_k\omega_{k+q}\right]$$

$$+\left[\frac{c_2}{m_N^2}\omega_k\left(\omega_p\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\omega_{p'}\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\right)\right]$$

$$-\left[\frac{c_2}{m_N^2}\omega_{k+q}\left(\omega_p\boldsymbol{p}\cdot\boldsymbol{k}+\omega_{p'}\boldsymbol{p}'\cdot\boldsymbol{k}\right)\right]$$

$$+\frac{c_4 m_N g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{8f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{X_3}{2}\frac{i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}+X_4\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

- UV Divergent terms
- Power-counting breaking terms
- are removed by using **the subtractive renormalization**