The Transition Density Formalism in the First Compton Computation on ^{4}He, and Beyond

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Daniel R. Phillips (Ohio U.)

How do constituents of the nucleon react to external fields?

How to reliably extract proton, neutron, spin polarisabilities?

How to plan effective experiments & test theory?

Exp-Th Compton Roadmap in "Next-Gen γ Source": IJMPG49 (2022) 010502

Review: +G. Feldman: Prog. Part. Nucl. Phys. 67 (2012) 841

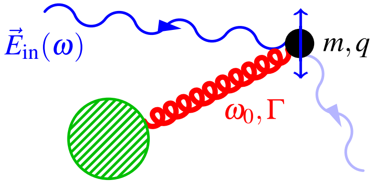
Transition Density Formalism and ^{3}He: hg/JMcG/AN/DRP: Few-Body Syst. 61 (2020) 61 [2005.12207] [nucl-th]

^{4}He $\mathcal{O}(e^2\delta^3)$: Liao/hg/JMcG/AN/DRP: EPJA 60 (2024) 132 [2401.16995] [nucl-th]

1. Two-Photon Response Explores System Dynamics

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905



$$\vec{d}_{\text{ind}}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{\omega_0^2 - \omega^2 - \text{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)}\,\vec{E}_{\text{in}}(\omega)$$

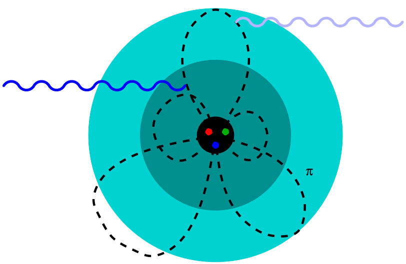
$=: 4\pi\,\alpha_{E1}(\omega)$ "displaced volume" $[10^{-4}\ \text{fm}^3]$

Clean, perturbative probe: χiral symmetry of pion-cloud & its breaking, $\Delta(1232)$, spin-constituents.

$$2\pi\Big[\underbrace{\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2}_{\text{electric, magnetic scalar dipole}}\Big]$$

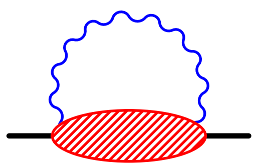
electric, magnetic scalar dipole

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2\rangle\dots$ PDG



Community Goal: Unified framework, reliable errors for p, d, ^{3}He,...

A2@MAMI, Compton@HIγS, MAXlab Roadmap: IJMPG49 (2022) 010502



Cottingham ΣR explains $M_\gamma^{\text{p}} - M_\gamma^{\text{n}}$ with

$$\alpha_{E1}^{p-n} = [-1.7 \pm 0.4_{\text{tot}}]$$

Hoferichter/Gasser/Leutwyler/Rusetzky 2015

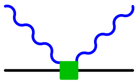
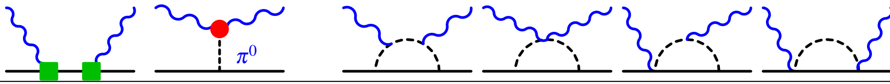
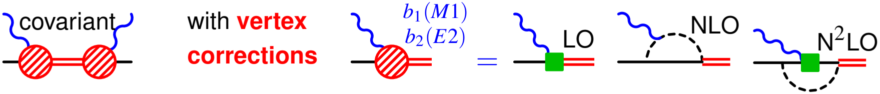
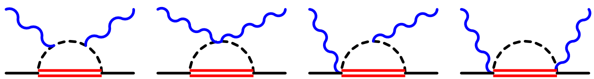
data of p-n difference: $[-0.6 \pm 1.2_{\text{tot}}]$

(b) All 1N Contributions to N^{4}LO

Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/... 1998
McGovern 2001, hg/Hemmert/Hildebrandt/Pasquini 2003
McGovern/Phillips/hg 2013

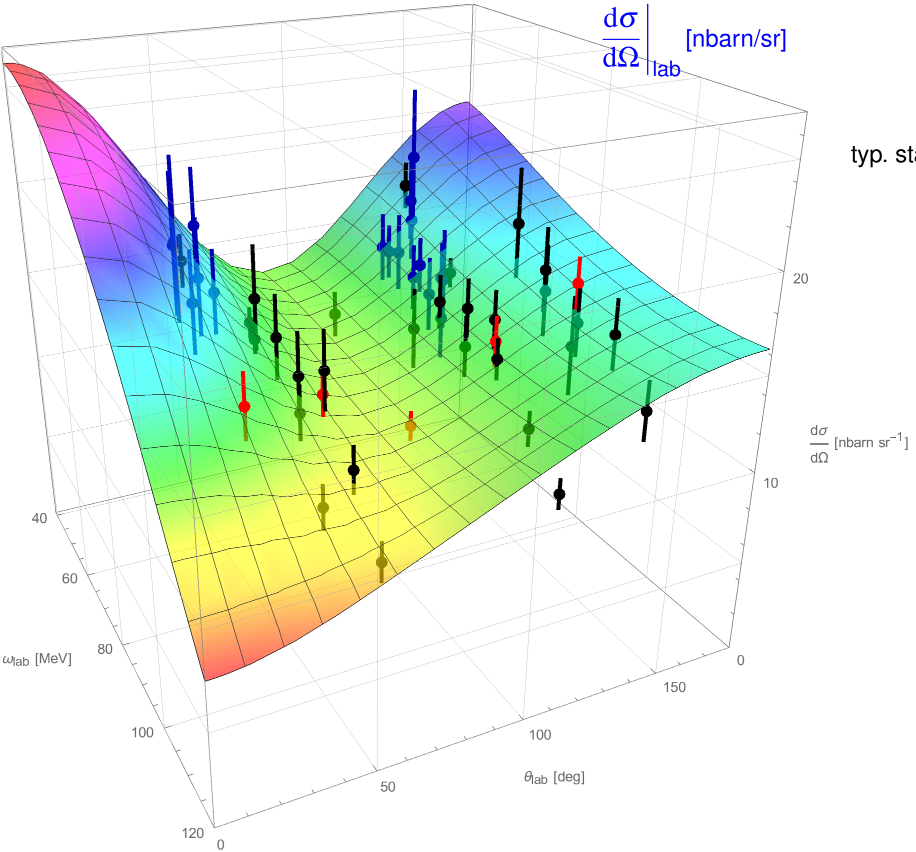
Unified Amplitude: **accuracy decreases with ω:**

	in low régime $\omega \lesssim m_\pi$	at least N^{4}LO $(e^2\delta^4)$: accuracy $\delta^5 \lesssim 2\%$;
or	in high régime $\omega \sim M_\Delta - M_N$	at least NLO $(e^2\delta^0)$: accuracy $\delta^2 \lesssim 20\%$.

	$\omega \lesssim m_\pi$	$\omega \sim M_\Delta - M_N \approx 300\ \text{MeV}$
 Thomson term: $-\frac{Z^2\alpha_{EM}}{M}$	$e^2\delta^0$ ***LO***	$e^2\delta^0 \searrow$NLO
	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1} \nearrow$***LO***
	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ static α_{E1}, β_{M1} (offset) $\Longrightarrow$ ω-dependence *predicted*.

hg/. . . 2005-2010
hg/McGovern/Phillips/Feldman PPNP 2012
Myers/. . . 2014



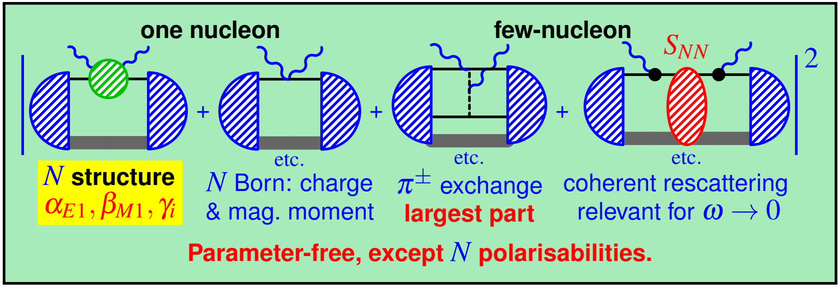
52 points Illinois, Saskatoon, Lund

typ. stat.+uncorrel. sys.: $\pm[7\dots10]\%$

typ. correl. sys.: $\pm[3\dots5]\%$

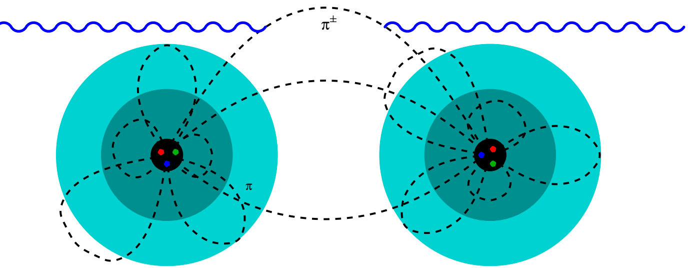
limited energy & angle coverage

deuteron: hg/.../+Phillips/+McGovern 2004-; **MECs:** Beane/... 1999-2005
^{3}**He:** Shukla/... 2009 +Strandberg/Margaryan/hg/... [1804.01206]
^{4}**He:** hg/.../+Nogga EPJA 60 (2024) 132 [2401.16995]



Deuteron, ^{4}**He**: sensitive to $\alpha^p_{E1} + \alpha^n_{E1}$, $\beta^p_{M1} + \beta^n_{M1}$ $\Longrightarrow$ **neutron pols**

^{3}**He**: sensitive to $2\alpha^p_{E1} + \alpha^n_{E1}$, $2\beta^p_{M1} + \beta^n_{M1}$ $\Longrightarrow$ **neutron pols**



Model-independently subtract binding effects.

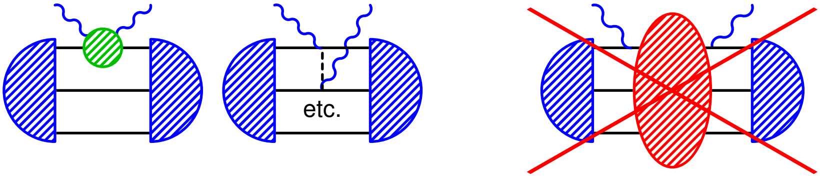
Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

Test charged-pion component of NN force.

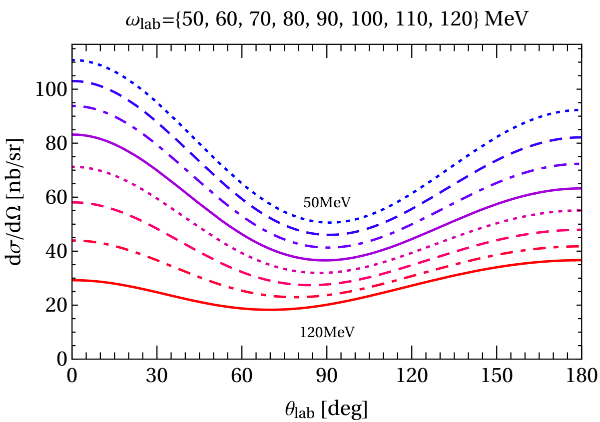
(e) The Hard Way: ^{3}He Theory

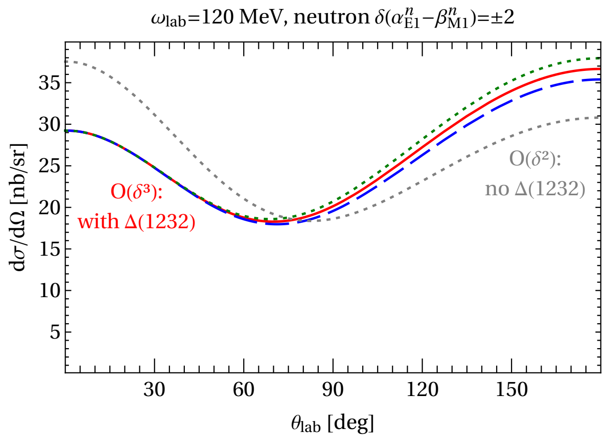
Shukla/Phillips/Nogga 2009, err. PRL **120** (2018) 249901
+Strandberg/Margaryan/hg/McG/Ph [1804.01206]

$\mathcal{O}(e^2\delta^3)$ **by adding** $\Delta(1232)$ $\Longrightarrow$ χ**EFT appears convergent; angle-dependence changes substantially.**



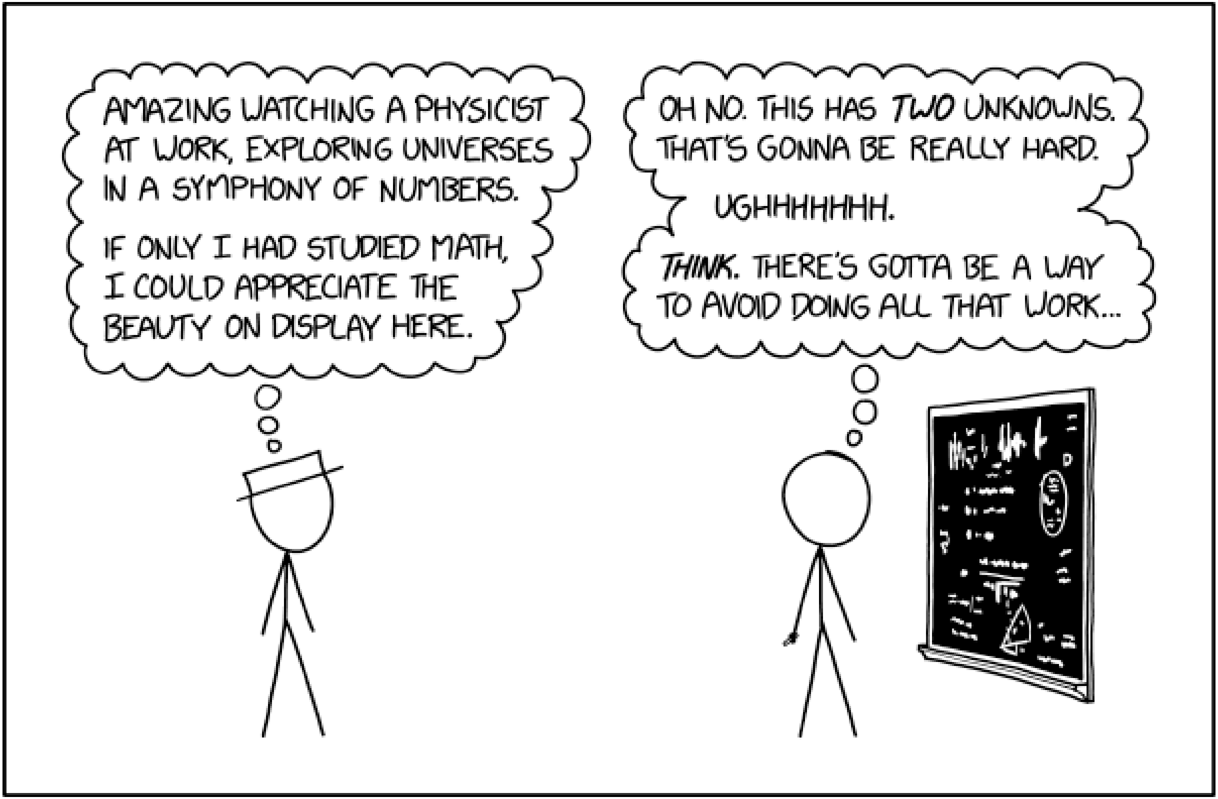
Ongoing: – implement rescattering for Thomson Walet/Singh/Kirscher/Birse/hg/…FewB Sys 64 (2023) 56 [2303.09361];
– implement $\mathcal{O}(e^2\delta^4)$: sub-leading π cloud.





Observables available as *Mathematica* notebook.

certification of hgrie by an eminent chiral physicist, 2006



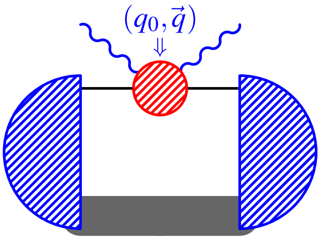
xkcd 2019

2. A New Hope

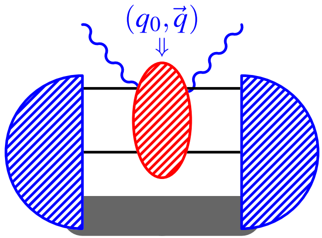
hg/McGovern/Phillips/Nogga
FewB Sys 61 (2020) 61 [2005.12207]
Alex Long: PhD Project 2022-?

(a) Idea Of The Transition-Density Formalism

more in next talk: Alex Long
application to Dark Matter in d, ^{3}He, ^{4}He:
de Vries/Köber/Nogga/Shain [2310.11343]

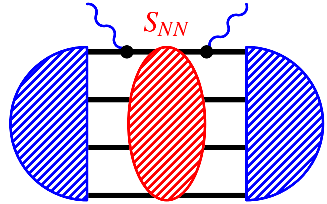


$A-1$ **spectator nucleons**
1 **Active Nucleon:**
one-body density



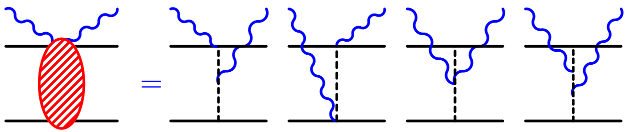
$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density

only depends on quantum numbers of actives and mom. transfer



no spectators
All Nucleons active:
rescattering
important as $\omega \lesssim \frac{1}{R}$

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
Compton



Beane/…1999-2005

hg/McGovern/Phillips/Nogga
FewB Sys 61 (2020) 61 [2005.12207]
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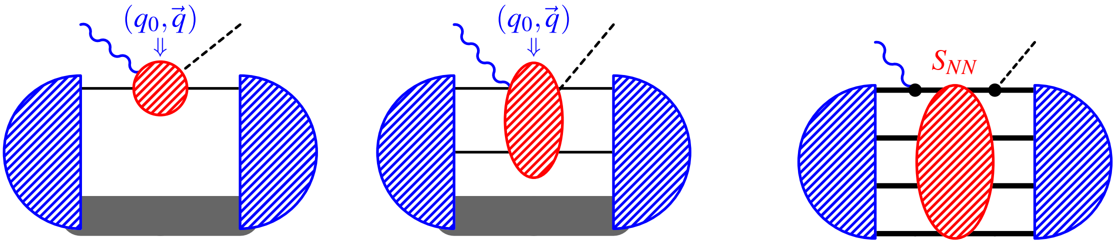

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$A-2$ spectator nucleons
2 Active Nucleons:
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All Nucleons active:
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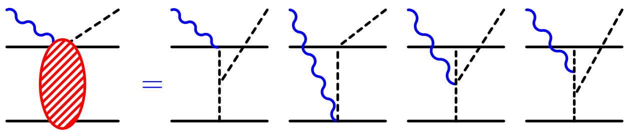
$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π production



Beane/...1995-97

hg/McGovern/Phillips/Nogga
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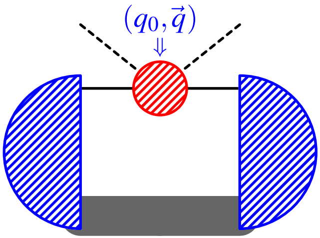

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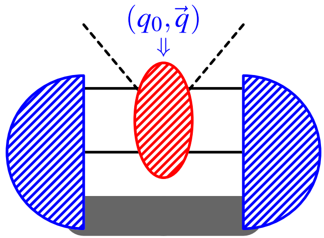

$A-2$ **spectator nucleons**
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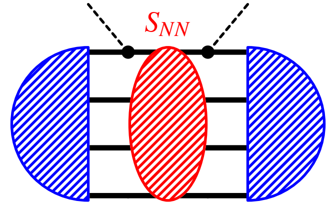
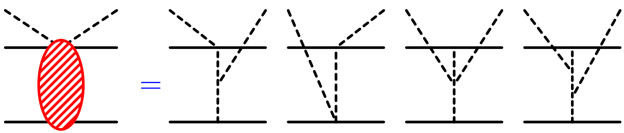
two-body density



no spectators
All Nucleons active:
rescattering
important as $\omega \lesssim \frac{1}{R}$

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π scattering



Beane/. . . 1998

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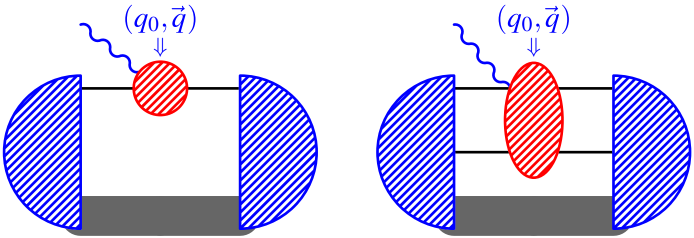
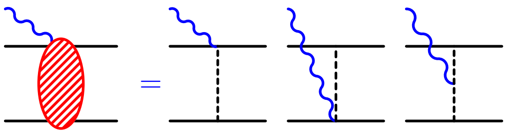


$A-1$ **spectator nucleons**	$A-2$ **spectator nucleons**
1 **Active Nucleon:**	2 **Active Nucleons:**
one-body density	**two-body density**

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
form factor



checked for d, ^{3}He, ^{4}He in de Vries/Köber/Nogga/Shain [2310.11343]

2. A New Hope

hg/McGovern/Phillips/Nogga
FewB Sys 61 (2020) 61 [2005.12207]
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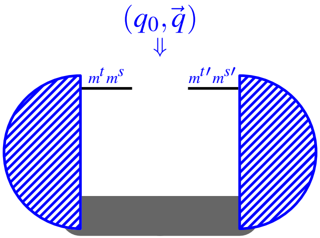

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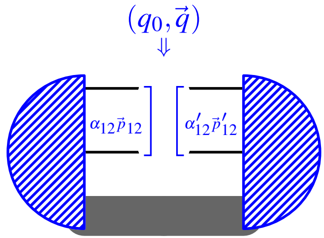


$A-2$ spectator nucleons
2 Active Nucleons:
two-body density

n-body transition density amplitude:
n nucleons with intrinsic momenta
and specific quantum numbers α
– absorb momentum transfer $(q_0, \vec{q})$;
– re-arrange quantum numbers to α';
– get absorbed back into nucleus.

only depends on quantum numbers of actives and mom. transfer

Idea: Split calculation into

kernel: interaction with n active nucleons	|	**structure: $A-n$ spectators**
recycle same reaction for different nuclei	|	**recycle same nucleus for different reactions**
Compton on ^{3}He, ^{3}H, ^{4}He, ^{6}Li, ...	|	**^{4}He Compton, π prod., FFs, dark matter,...**

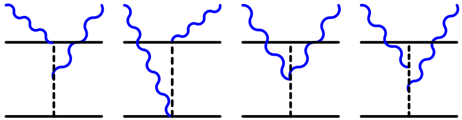
χEFT hierarchy of few-body interactions: onebody, twobody $\gg$ threebody $\gg$ fourbody...

Computationally highly efficient: well-developed, sophisticated numerical few-body techniques.
^{3}He, ^{4}He: CPU time reduced from days to hours; extensive checks; same result as traditional.

$\Longrightarrow$ Compute to higher numerical accuracy (integration mesh, j_{12},...): $\approx 1\%$ change in ^{3}He.

$\Longrightarrow$ **Density repository for ^{3}He, ^{4}He at `pypi.org/project/nucdens/` contains family of EFT-inspired potentials χSMS[400-550]N4LO+N2LO3NI** **Reinert/...2018 Maris 2021**, plus IdahoN3LO+3NIa/b, AV18+UIX,...

Compton matrix elements



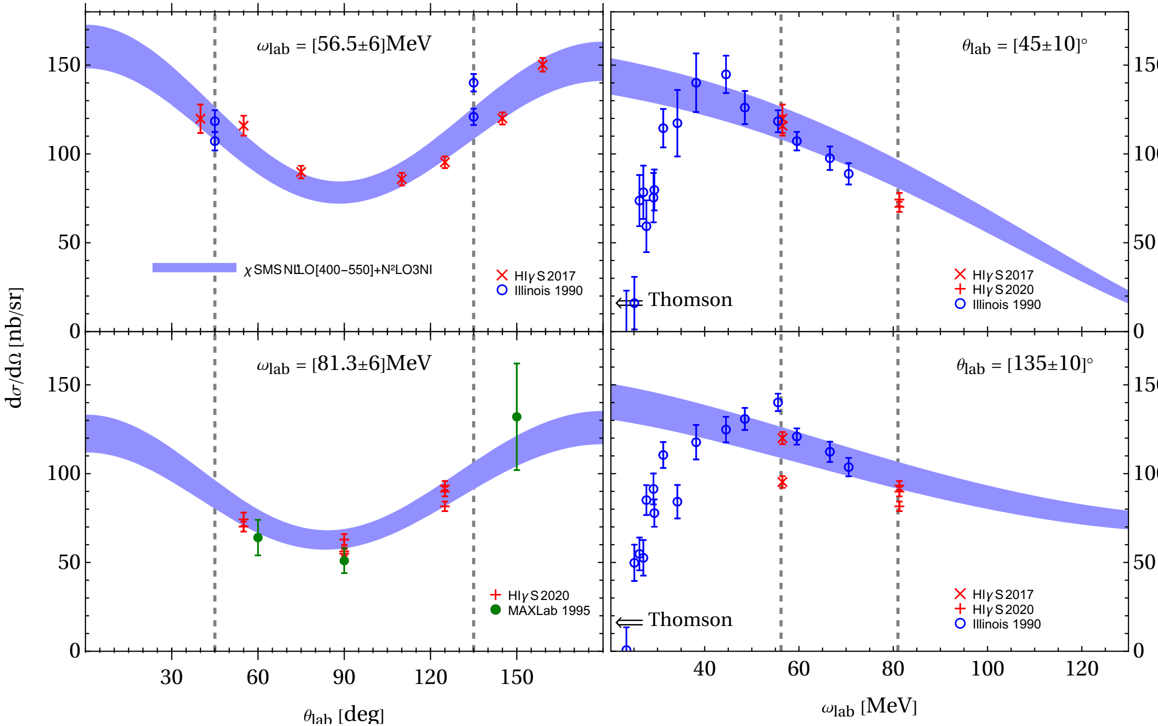
	$\omega = 50$ MeV, $\theta = 30°$				$\omega = 120$ MeV, $\theta = 165°$			
	Idaho N^{3}LO+3NFb		AV18+UIX		Idaho N^{3}LO+3NFb		AV18+UIX	
$\{M', M; \lambda', \lambda\}$	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.
$\{\frac{1}{2}, \frac{1}{2}; 1, 1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%
$\{\frac{1}{2}, \frac{1}{2}; -1, 1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; 1, -1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; -1, -1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%

Table 7: Comparison of two-body matrix elements in the "density" approach and the "traditional" approach for potentials Idaho N^{3}LO+3NFb and AV18+UIX with $j_{12} \leq 2$ at $\omega = 50$ MeV, $\theta = 30°$ (where mostly diagonal matrix elements are probed) and $\omega = 120$ MeV, $\theta = 165°$ (where off-diagonal matrix elements are probed more strongly). See also text and captions to tables 5 and 3 for further details.

(a) ^{4}He: Confronting Data

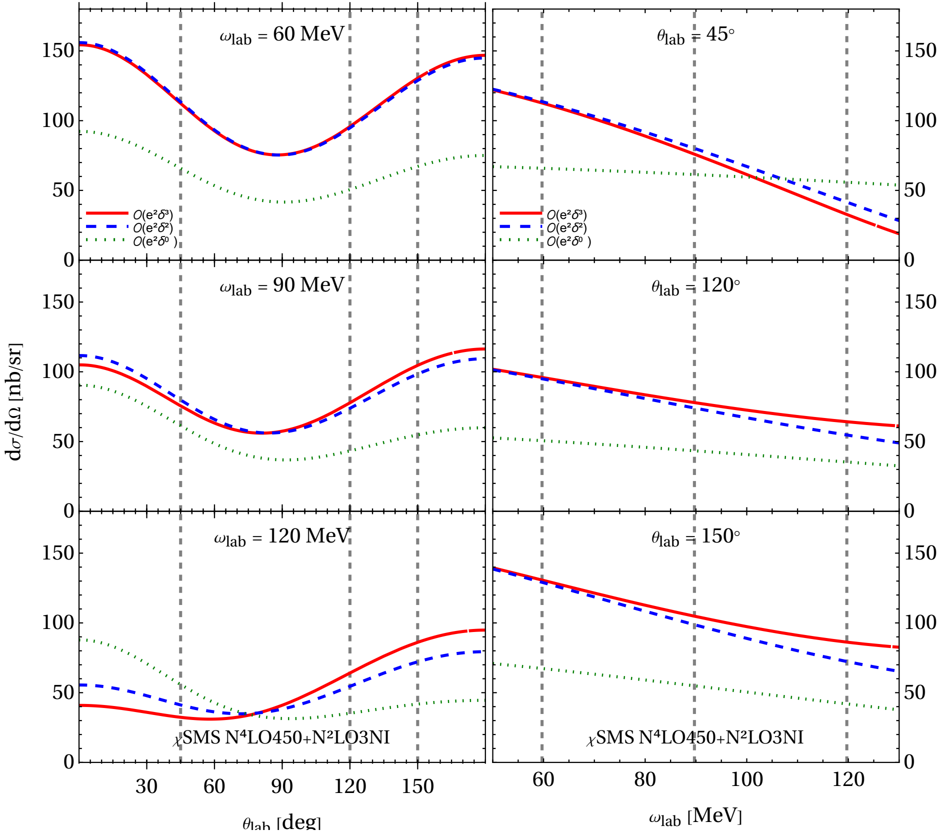
$\Longrightarrow$ In principle very clean if binding effects under control: $\mathcal{O}(e^2\delta^3)$ with established α_{E1}, β_{M1}.

Promising but wave-function/potential dependence; some data issues. $\Longrightarrow$ No polarisability extraction (yet).



(b) Order-By-Order Convergence

hg/J. Liao/JAMcG/A. Nogga/DRP:
EPJA 60 (2024) 132 [2401.16995], math.nb



$\mathcal{O}(e^2\delta^0)$: 1N Thomson

$\mathcal{O}(e^2\delta^2)$: $+\pi$N cloud

$\mathcal{O}(e^2\delta^3)$: $+\Delta+\pi\Delta$

$\Delta(1232)$ increases angular tilt, but not as much as in ^{3}He.

Reasonable convergence.

Error estimate:

$\pm 8\%$ order-by-order;

$\pm 8\%(12\%)$ cutoff band;

$\Longrightarrow \pm 10\%$

($\pm 12\%$ for $\omega \nearrow 120$ MeV $\wedge\, \theta \to 180°$)

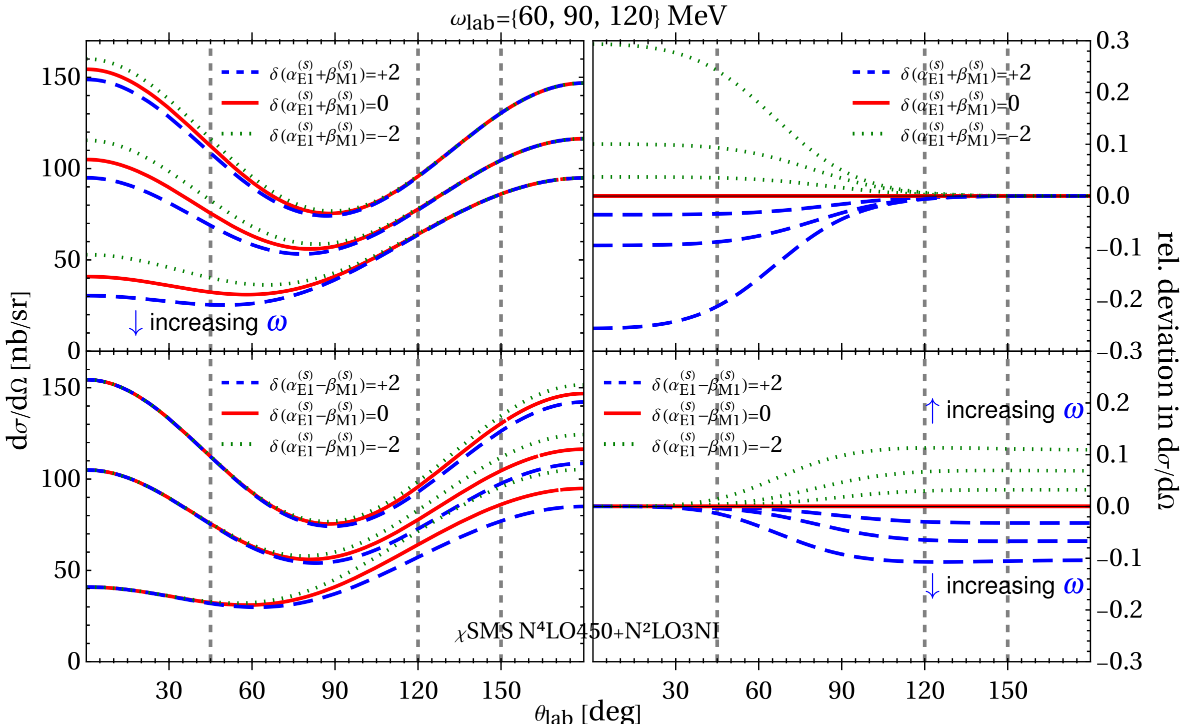
suffices for rates, not for extractions.

(c) ^{4}He: Dependence on Scalar-Isoscalar Polarisabilities

^{4}He: perfect scalar-isoscalar $\Longrightarrow$ sensitive *only* to α^{p+n}, β^{p+n}, not to spin-polarisabilities γ_i.

$\mathcal{O}(e^2\delta^3)$ with χSMSN^{4}LO 450+N^{2}LO 3NI (other potentials: $\lesssim \pm 8\%$). **Suffices for *planning*, not for *analysis*!**

Increasing ω gives bigger relative signals, but smaller rates.

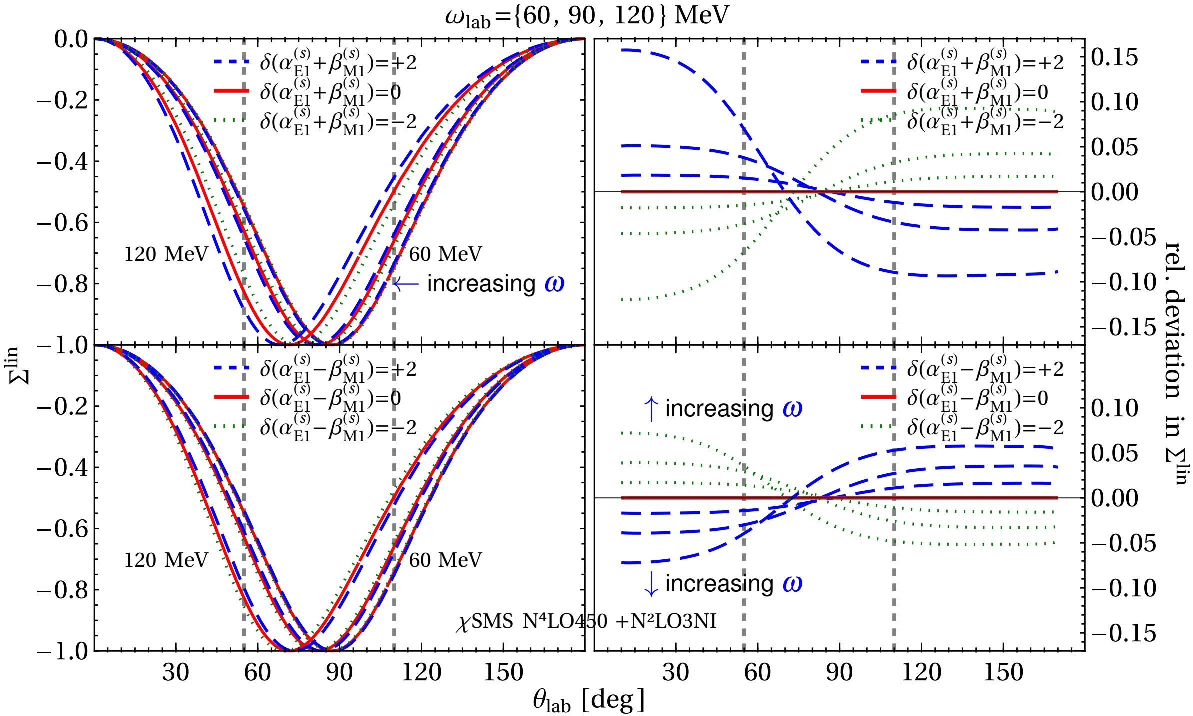


(d) The "Only" Other Observable: Beam Asymmetry Σ^{lin}

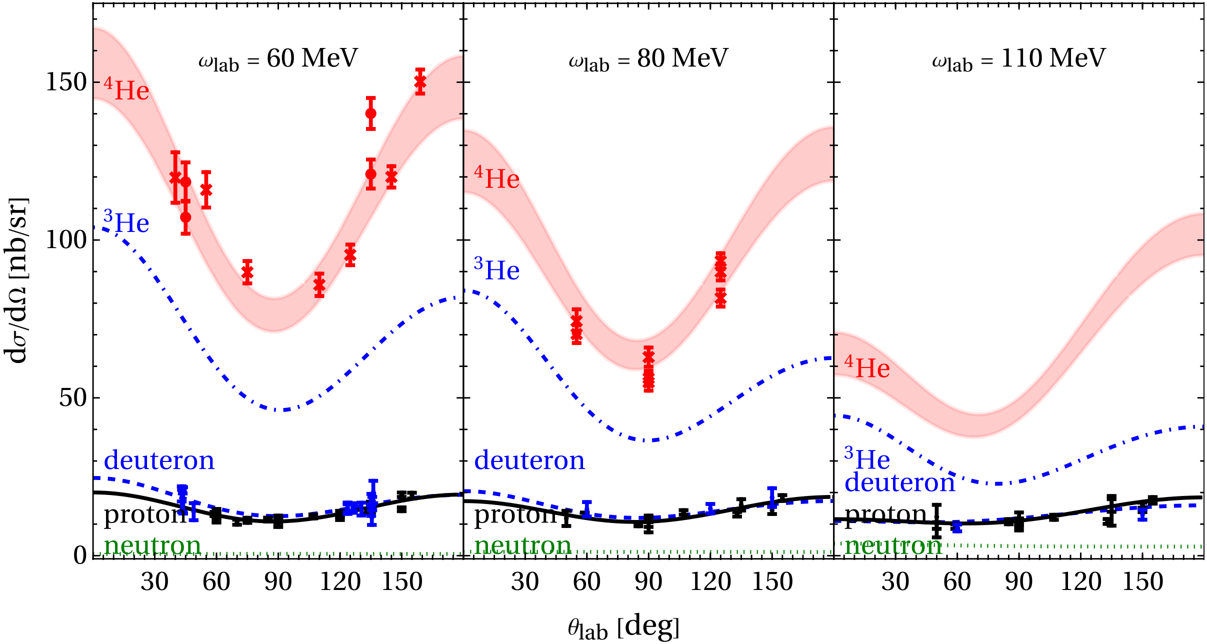
^{4}He: perfect scalar-isoscalar $\Longrightarrow$ sensitive *only* to α^{p+n}, β^{p+n}, not to spin-polarisabilities γ_i.

$\mathcal{O}(e^2\delta^3)$ with χSMSN^{4}LO 450+N^{2}LO 3NI (other potentials: $\lesssim \pm 4\%$). **Suffices for *planning*, not for *analysis*!**

Increasing ω gives bigger relative signals, but only around centres of rise/fall.



(e) Comparing ^{4}He, ^{3}He, Deuteron, Proton, Neutron



Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

***Sweet-spot*: Complementing Targets of Opportunity.**

Comprehensive *mathematica* notebooks for observables and sensitivities on p, d, ^{3}He, fourHe available.

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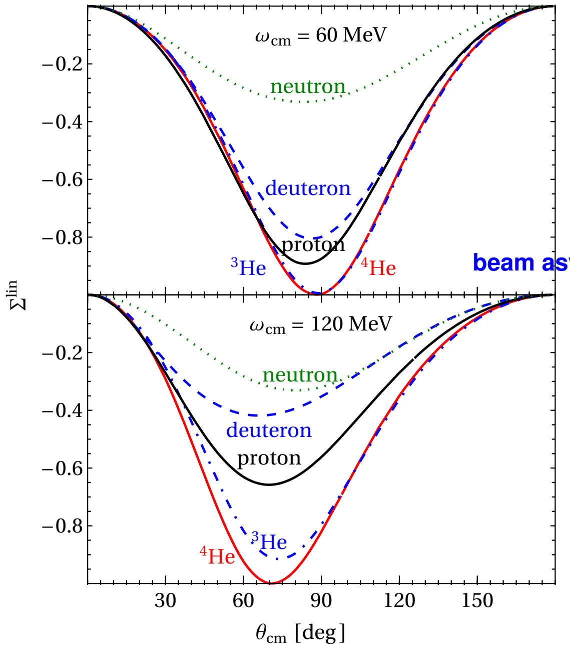
hg/J. Liao/JAMcG/A. Nogga/DRP:
EPJA 60 (2024) 132 [2401.16995], math.nb



beam asymmetry Σ_{lin}

Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

***Sweet-spot*: Complementing Targets of Opportunity.**

Comprehensive *mathematica* notebooks for observables and sensitivities on p, d, ^{3}He, fourHe available.

4. Concluding Questions

Transition Density Method: separate kernels with n actives from spectators in external probes.

Efficient recycling: χSMS[400-550]N4LO+N2LO3NI densities from `pypi.org/project/nucdens/`.

Polarisabilities: **χiral symmetry of pion-cloud, $\Delta(1232)$ properties. Impact on $M_p - M_n$,...**

χEFT: systematic, parameter-free predictions with uncertainties; lattice QCD catching up.

Target	**Opportunities**	**Theory Status for All Observables**
proton & neutron	**nucleon spin polarisabilities**	**"done": N^4LO $\omega \lesssim 230$ MeV for pols.** math.nb jupyter.py
deuteron	**sensitive to $p+n$ average polarised, d-wave interference: mixed spin pols $\gamma_{E1M2}, \gamma_{M1E2}$**	**$\omega < m_\pi$ N^3LO done, N^4LO this year** math.nb **$\omega \gtrsim m_\pi$ needs resources**
^{3}He: increased rates	**unpolarised: sensitive to $2p+n$ polarised: "n-spin" $\Longrightarrow \gamma_i^n$ only**	**densities method [2005.12207]** ^{3}He, ^{4}He: math.nb **$\omega \in [50\text{ MeV}; m_\pi]$ N^3LO✓, N^4LO like d**
^{4}He: increased rates	**sensitive to $p+n$ average, not γ_i's**	**$\omega \to 0$ under way — $\omega \gtrsim m_\pi$ needs resources**
$\gamma X \to NY\gamma$ quasifree	**tag n or p directly – both at once?**	**$\gamma d \to np\gamma$ N^4LO done; more needs resources**

We Need Data: **elastic & inelastic cross-sections & asymmetries –** ***reliable systematics!***

Low-ω for scalar, high-ω for spin-polarisabilities, **but always** $\omega \lesssim 230$ MeV.

Only ***combination of dedicated experiments*** **meaningful!** (Not "one datum for one answer".)

$\Longrightarrow$ ***Synergy of Experiment, Low-Energy Theory & Lattice QCD, competitive uncertainties!***

$\Longrightarrow$ **Compton Community programme outlined in White Paper for a** ***Next Generation Laser Compton Gamma-ray Beam Facility*** **[2012.10843] and DOE.**

The efficient person gets the job done right. The effective person gets the right job done.