JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Nuclear Structure in Two-Photon Exchange in (Muonic) Deuterium

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Two-Photon Exchange in (Muonic) Atoms

Proton radius puzzle

2 languages

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The **proton radius puzzle** is an unanswered problem in physics relating to the size of the proton.[1] Historically the proton charge radius was measured by two independent methods, which converged to a value of about 0.877 femtometres (1 fm = 10^{-15} m). This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this, at 0.842 femtometres.[2] New experimental results reported in the autumn of 2019 agree with the smaller measurement, as does a re-analysis of older data published in 2022. While some believe that this difference has been resolved,[3][4] this opinion is not yet universally held.[5][6]

Two-Photon Exchange in (Muonic) Atoms

- The „third method“: spectroscopy of muonic atoms

Antognini, Pohl, many others (CREMA), 2010-...

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Two-Photon Exchange in (Muonic) Atoms

See F. Hagelstein's talk on Fri for more on it

- The „third method": spectroscopy of muonic atoms

Antognini, Pohl, many others (CREMA), 2010-...

- More sensitive to nuclear charge radii
- But also greater sensitivity to subleading nuclear response

Bohr radius

$$a = (Z\alpha m_r)^{-1}$$

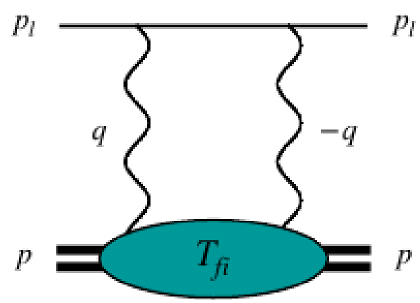
R_E: charge radius

R_F: Friar radius

Lamb Shift: $$\Delta E_{nS} = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \ldots$$

- Dominant nuclear structure effect: Two-Photon Exchange (TPE)

$\propto \alpha^5$



- Also its contribution in the uncertainty is dominant

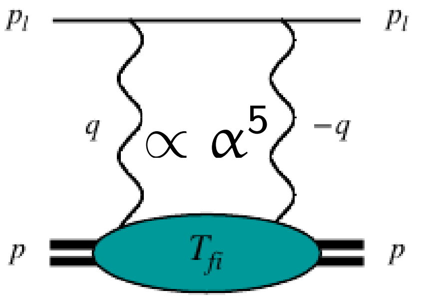
Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl – theory review (2022)
[a] experiment: CREMA (2013-2023)

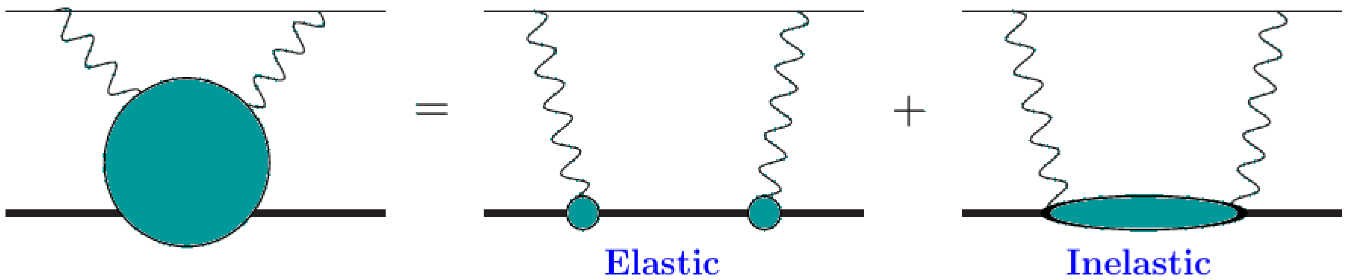
	Correction		μH	μD	μ^3He$^+$	μ^4He$^+$
$E_{\rm QED}$	point nucleus		206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size		$-5.225\,9\, r_p^2$	$-6.107\,4\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
$E_{\rm NS}$	nuclear structure		0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
E_L(exp)	experiment[a]		202.370 6(23)	202.878 5(34)	1258.612(86)	1378.521(48)

TPE gives > 90% to the shown theory uncertainties

TPE and VVCS

- TPE is naturally described in terms of (doubly virtual fwd) Compton scattering (VVCS)
- Elastic ($\nu = \pm Q^2/2M_{\text{target}}$, elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)





- Forward spin-1/2 VVCS amplitude

$$\alpha_{\text{em}}\, M^{\mu\nu}(\nu, Q^2) = -\left\{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\nu, Q^2)\right.$$
$$\left. + \frac{i}{M}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu, Q^2) + \frac{i}{M^3}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\ s_\beta - s\cdot q\ p_\beta)\, S_2(\nu, Q^2)\right\}$$

~HFS

Lamb Shift: $$E_{nS}^{2\gamma} = -8i\pi\alpha m\, [\phi_n(0)]^2 \int_s \frac{d^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

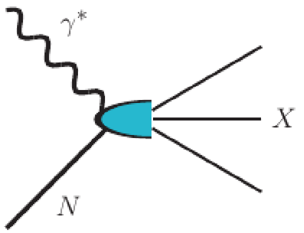
For the HFS (mostly in μH), see talks of F. Hagelstein, D. Ruth (Fri)

VVCS and Structure Functions

Lamb Shift: $$E_{nS}^{2\gamma} = -8i\pi\alpha m\,[\phi_n(0)]^2 \int_s \frac{d^4q}{(2\pi)^4} \frac{(Q^2-2\nu^2)\,T_1(\nu,Q^2) - (Q^2+\nu^2)\,T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

- Unitarity and analyticity, data-driven: dispersive relations

Structure functions $F_1(x,Q^2)$, $F_2(x,Q^2)$



$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi M\nu^2}{Q^4}\int_0^1 dx\,\frac{x\,F_1(x,Q^2)}{1-x^2(\nu/\nu_{el})^2 - i0^+},$$

$$T_2(\nu,Q^2) = \frac{16\pi M}{Q^2}\int_0^1 dx\,\frac{F_2(x,Q^2)}{1-x^2(\nu/\nu_{el})^2 - i0^+}$$

- The subtraction function is not directly accessible in experiment
  - less of a problem for composite nuclei, more for the proton
- Data on structure functions is deficient for anything heavier than proton

For the proton, see talks by V. Biloshytskyi (Thu), F. Hagelstein, D. Ruth (Fri)

- Nuclear Effective Field Theories (EFTs) are doing a better job here

EFTs for TPE (and *vice versa*)

Lamb Shift: $$E^{2\gamma}_{nS} = -8i\pi\alpha m\,[\phi_n(0)]^2 \int_s \frac{d^4q}{(2\pi)^4} \frac{(Q^2-2\nu^2)\,T_1(\nu,Q^2)-(Q^2+\nu^2)\,T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

- Typical energies in (muonic) atoms are small: natural to use EFTs
- Chiral EFT (covariant, HB, …) or (even) pionless EFT for nuclear effects
- Expansion in powers of a small parameter, order-by-order uncertainty
- TPE effect is needed to high precision to extract radii

	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
$E_{\rm QED}$	point nucleus	206.0344(3)	228.7740(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\,r_C^2$	finite size	$-5.2259\,r_p^2$	$-6.1074\,r_d^2$	$-103.383\,r_h^2$	$-106.209\,r_\alpha^2$
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  - a rather high order calculation of these effects is typically needed

- If TPE can be extracted (e.g. isotope shifts and/or known radii), this provides a benchmark for the theory
- We will concentrate on the deuteron/μD

F. Hagelstein's talk on Fri for other light muonic atoms

Deuteron VVCS in Pionless EFT

Kaplan, Savage, Wise (1998)
Chen, Rupak, Savage (1999)
Phillips, Rupak, Savage (1999)

- Nucleons are non-relativistic → $E \simeq p^2/M = O(p^2)$
- Loop integrals $dE\, d^3p = O(p^5)$
- Nucleon propagators $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45$ MeV
- Photon momenta $|\vec{q}| \sim p, \quad \nu \sim p^2$
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
- *NN* system has a low-lying bound/virtual state → enhance S-wave coupling constants, resum the LO *NN* S-wave scattering amplitude
- z-parametrization (reproducing deuteron S-wave asymptotics at NLO)

- Easy to solve (analytic results for *NN*)
- Explicit gauge invariance and renormalisability

Counting for VVCS and TPE: Predictive Powers

- Longitudinal and Transverse amplitudes

$$f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2), \qquad f_T(\nu, Q^2) = T_1(\nu, Q^2)$$

Lamb Shift:

$$\Delta E_{nl} = -8i\pi m\, [\phi_{nl}(0)]^2 \int_s \frac{d^4 q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

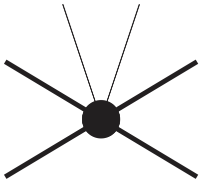
$f_L = O(p^{-2}), \qquad f_T = O(p^0)$ in the VVCS amplitude

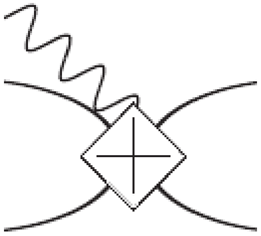
$\alpha_{E1} = 0.64 \text{ fm}^3$

$\beta_{M1} = 0.07 \text{ fm}^3$

$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$ in TPE

- Transverse contribution to TPE starts only at N4LO
- N4LO: ΔE_{nl} needs to be regularised, an unknown lepton-NN LEC
- We go up to N3LO in f_L, and up to (relative) NLO in f_T [cross check]
- One unknown LEC at N3LO in f_L
  - important for the charge form factor
  - extracted from the H-D isotope shift and proton R_E

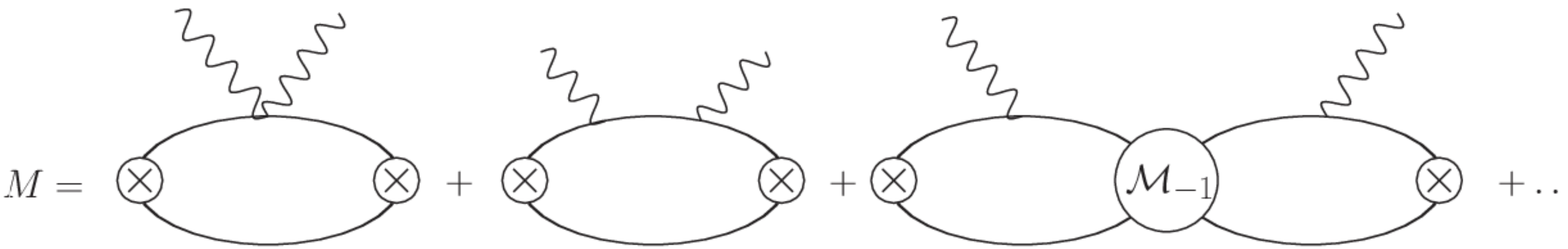




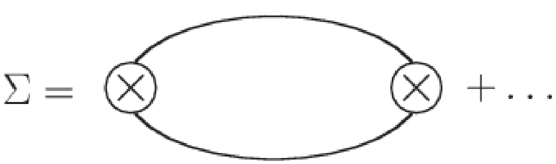
Amplitude with Deuterons

- The reaction amplitude is given by the LSZ reduction

$$T = M \left[\left. \frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E} \right|_{E=E_d} \right]^{-1}$$



– irreducible VVCS graphs (here full LO for f_L ; crossed not shown)



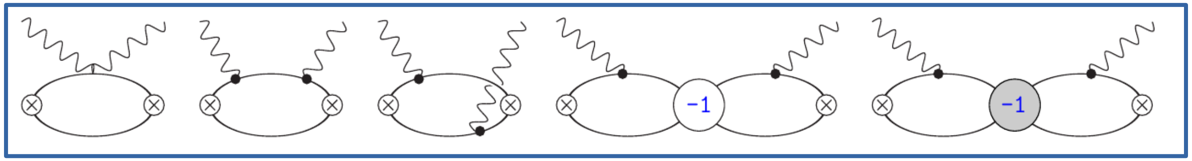
– deuteron self-energy (here at LO)

- The expression for the residue is very simple up to N3LO:

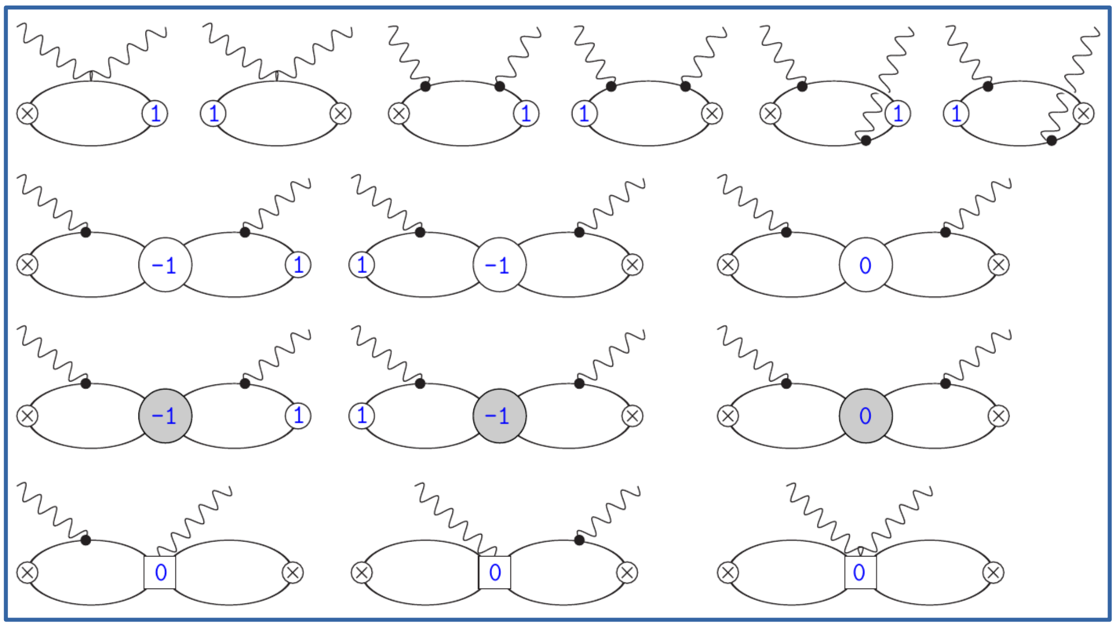
$$\left[\left. \frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E} \right|_{E=E_d} \right]^{-1} = \frac{8\pi\gamma}{M^2} \left[1 + (Z - 1) + 0 + 0 + \ldots\right]$$

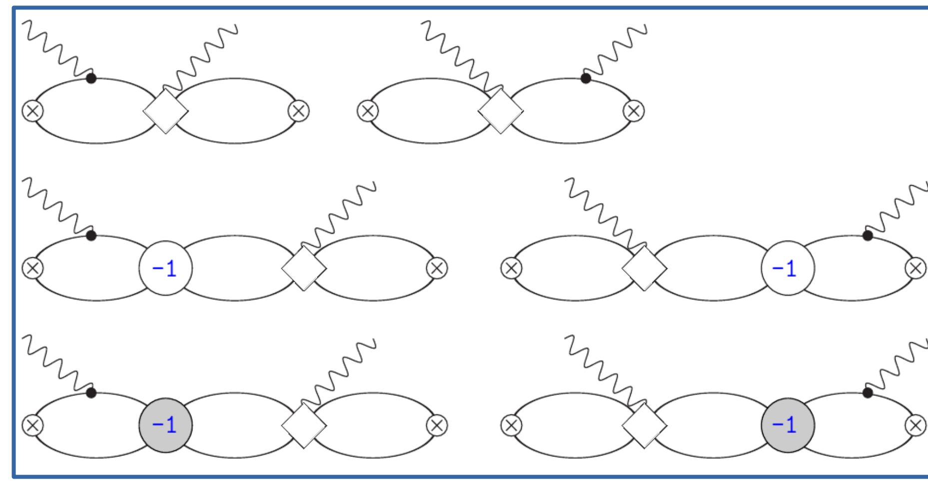
Deuteron VVCS: Feynman Graphs

LO



NLO

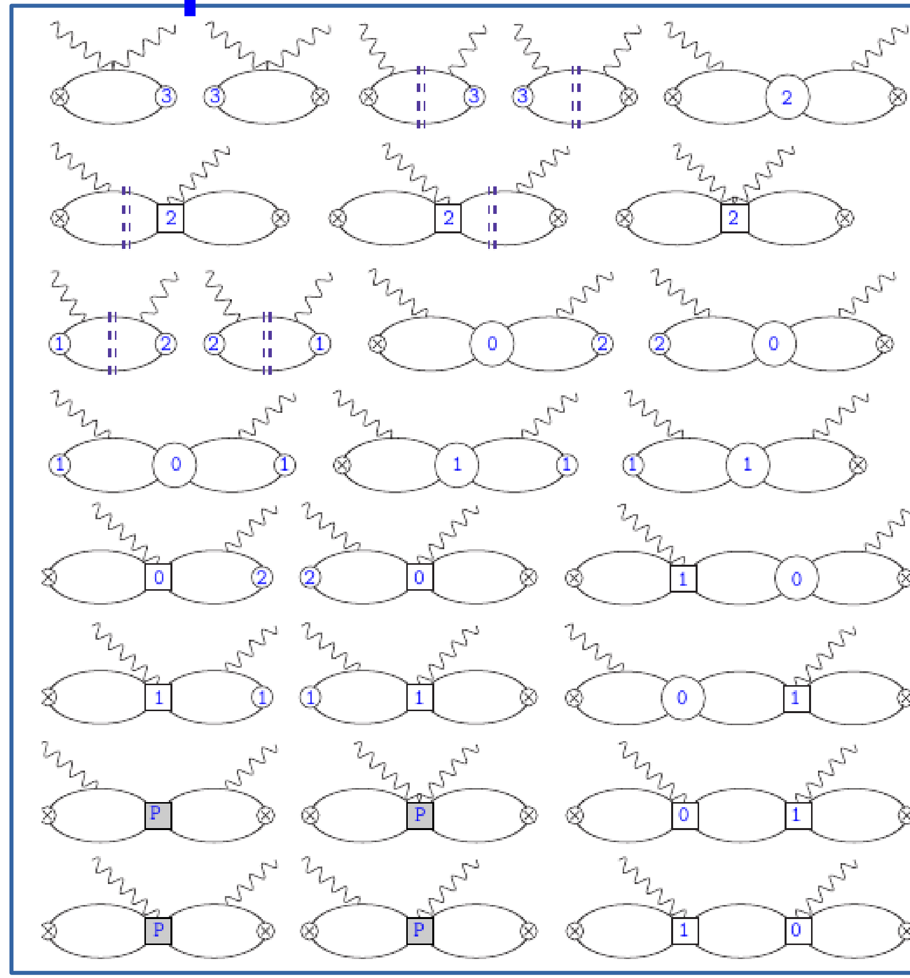




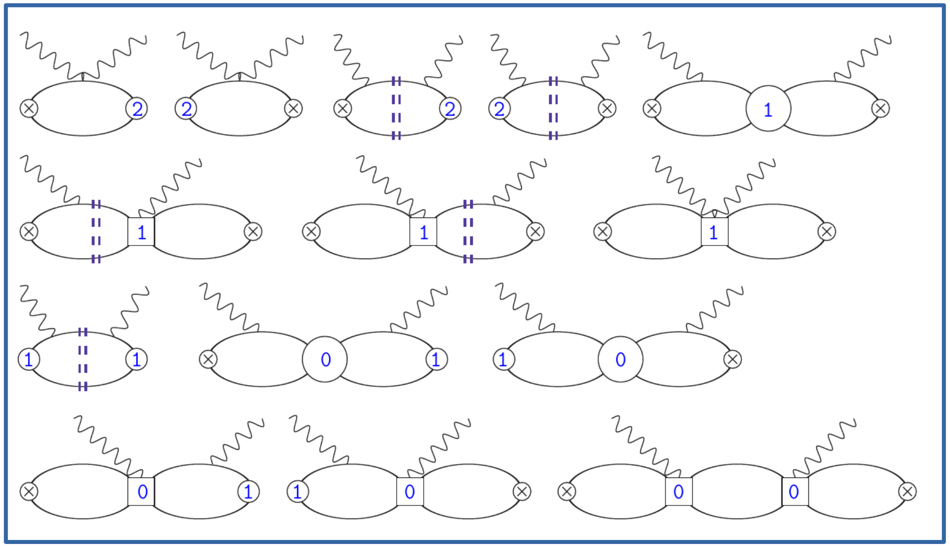
- Amplitudes are calculated analytically (dimreg+PDS) Kaplan, Savage, Wise (1998)
- Checks:
  - the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - regularisation scale dependence has to vanish

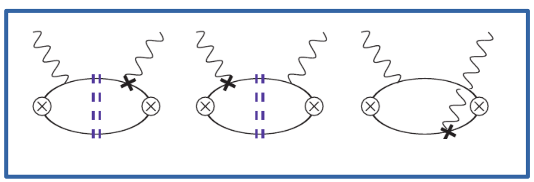
Deuteron VVCS: Feynman Graphs

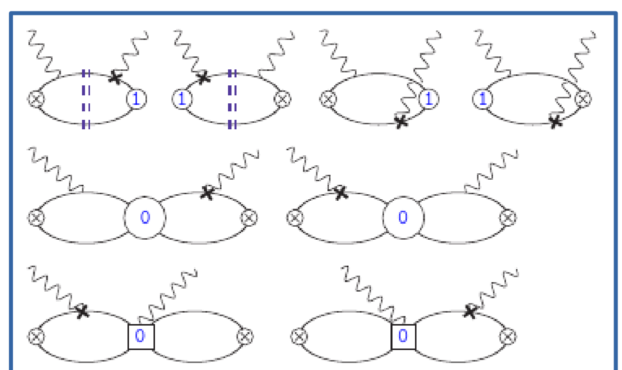
N3LO

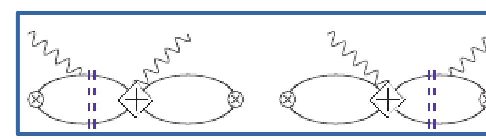


NNLO







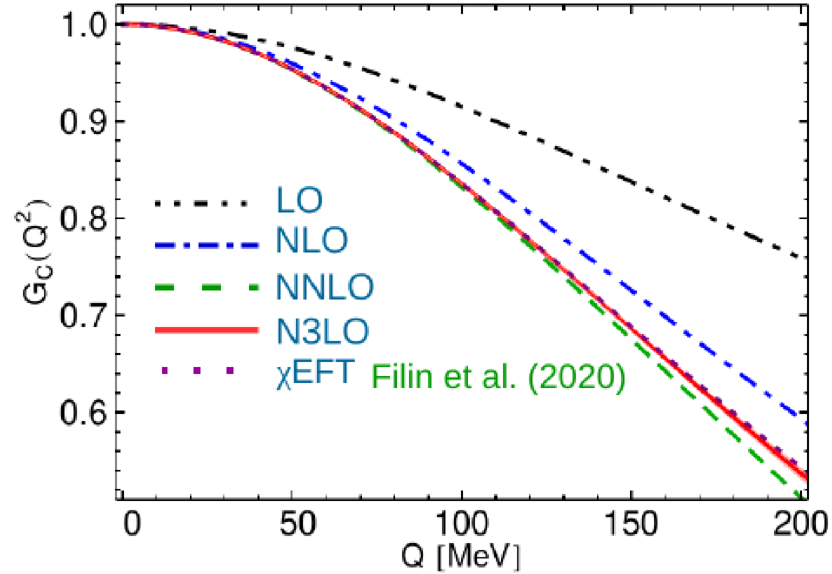


Many interesting results obtained from the VVCS amplitude, e.g., the deuteron (generalised) polarisabilities

VL, Hiller Blin, Pascalutsa (2021)

Deuteron Charge Form Factor and TPE in μD

- The deuteron charge form factor obtained from the residue of the VVCS amplitude
- The result is consistent with χEFT
- Correlation between R_F and R_E
  - generated by the N3LO LEC



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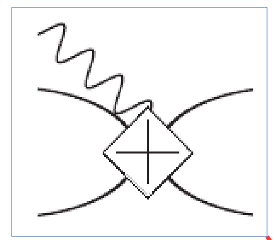

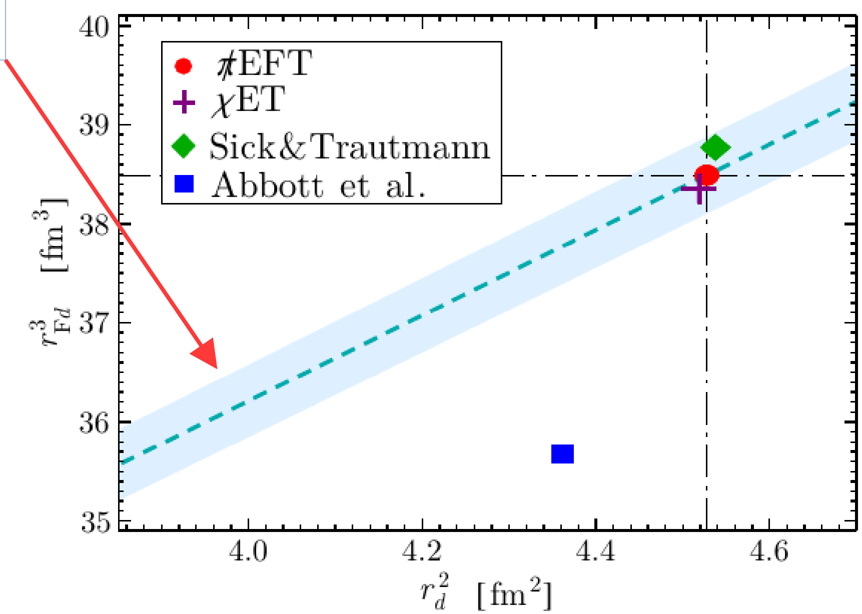

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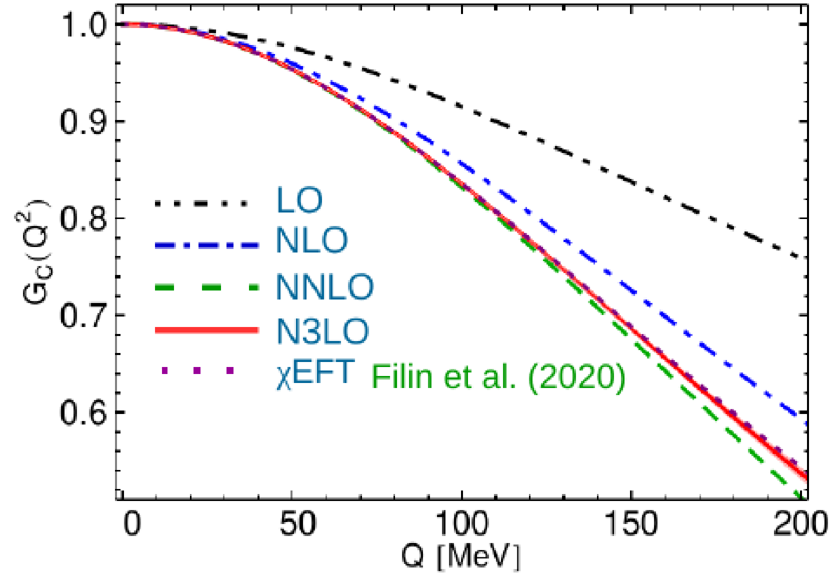


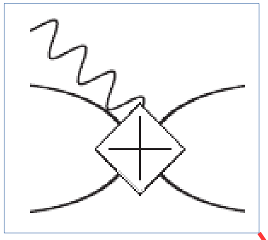
VL, Hiller Blin, Pascalutsa (2021)

$$R_{\mathrm{F}}^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

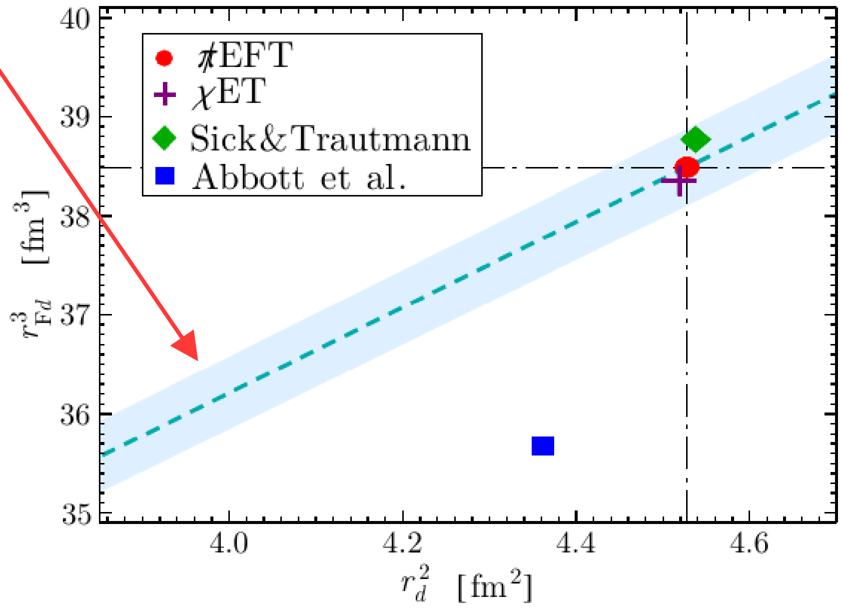
$$= \frac{3}{80\gamma^3}\Big\{Z\left[5 - 2Z(1 - 2\ln 2)\right]$$

$$-320/9\; r_0^2\gamma^2\left[Z(1 - 4\ln 2) - 2 + 2\ln 2\right]$$

$$+80(Z-1)^3\; l_1^{C0_S}\Big\}$$



$$R_E^2 = \frac{1}{8\gamma^2} + \frac{Z-1}{8\gamma^2} + 2r_0^2 + \frac{3(Z-1)^3}{\gamma^2}\; l_1^{C0_S}$$



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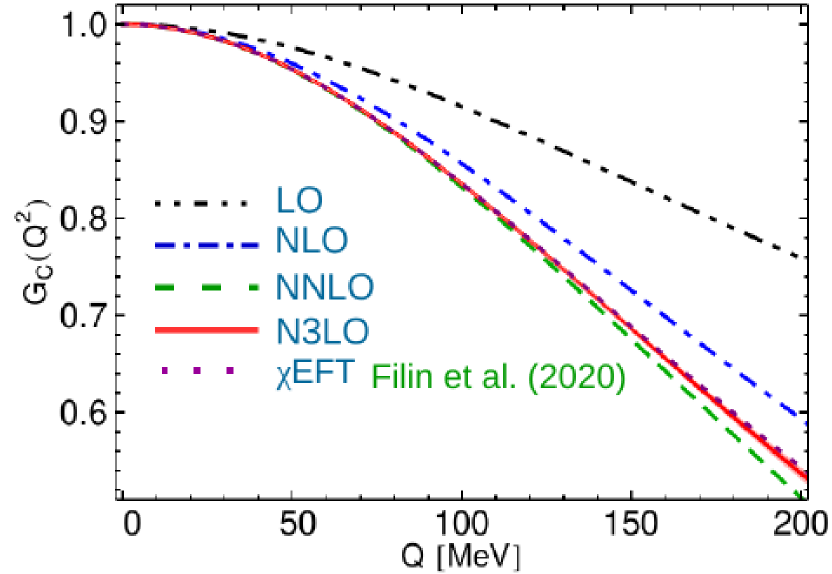

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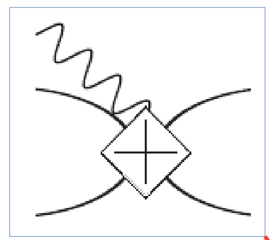
$$R_F^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

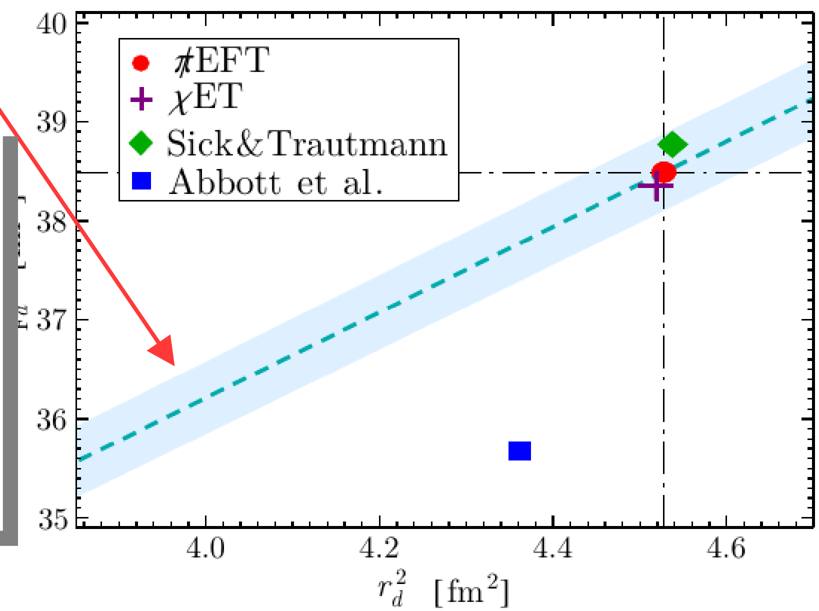
$= 3$



- Benchmark: EFTs work better at low Q than at least some empirical parametrizations
- Not only R_E but also higher derivatives need to be reproduced correctly!

$R_E^2 =$

- Agreement with χEFT vindicates both EFTs



See talk of A. Filin in this session for χEFT of nuclear form factors

VL, Hagelstein, Pascalutsa (2022)

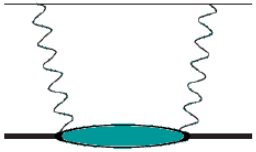
TPE in μD: Higher-Order Corrections

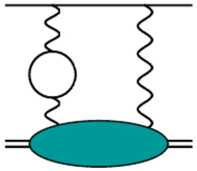
$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$

- Higher-order in α terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$

    taken from elsewhere $\Delta E_{2S}^{\text{Coulomb}} = 0.2625(15)\ \text{meV}$

non-forward





  - eVP $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\text{eVP}} = -0.027\ \text{meV}$

See talk by I. Reis in this session for different radiative corrections, potentially important as well

- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from χEFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\text{hadr}} = -0.032(6)\ \text{meV}$

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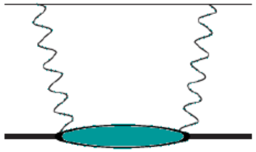

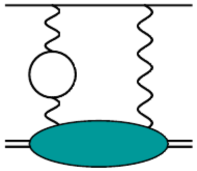

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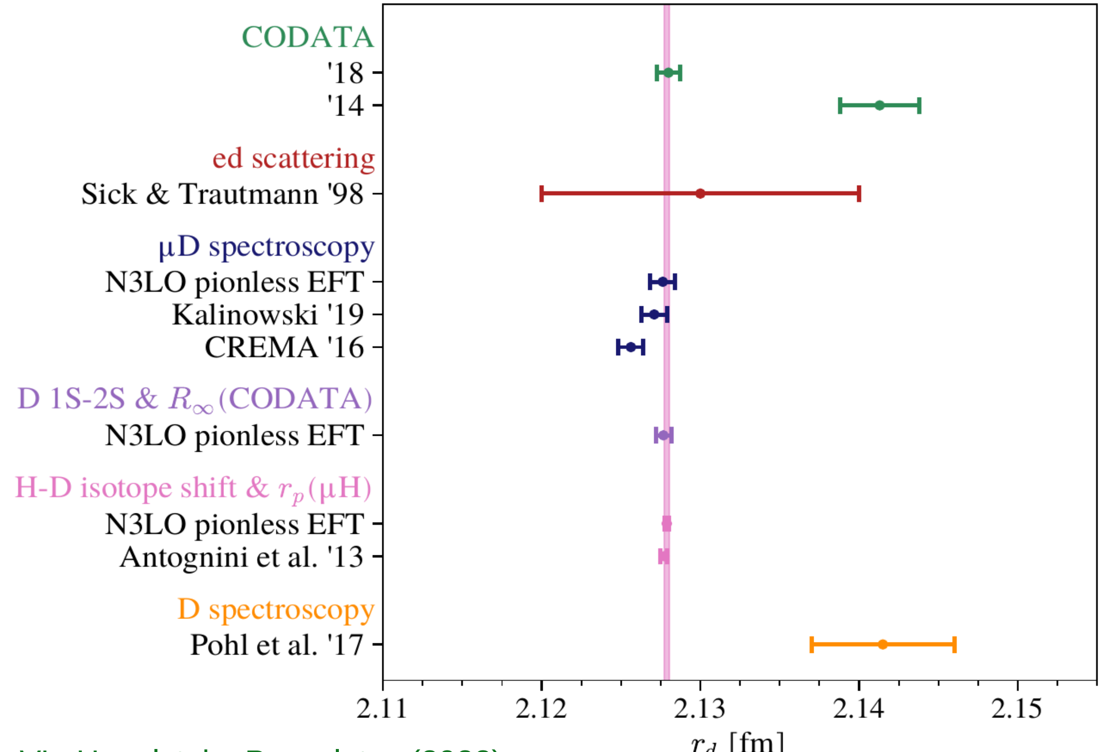
$$\boxed{\Delta E_{2S}^{2\gamma} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} + \Delta E_{2S}^{\text{hadr}} + \Delta E_{2S}^{\text{eVP}} + \Delta E_{2S}^{\text{Coulomb}} = -1.752(20)\ \text{meV}}$$

Deuteron Charge Radius and TPE in μD

- μD, D, and H-D isotope shift all consistent with one another
- Agreement with the very precise empirical value of 2γ exchange

	$E^{2\gamma}_{2S}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	$-1.7096(200)$
Kalinowski '19 [6, Eq. (6) + (19)]	$-1.740(21)$
$\not{\pi}$EFT (this work)	$-1.752(20)$
Empirical (μH + iso)	
Pohl et al. '16 [3]	$-1.7638(68)$
This work	$-1.7585(56)$



VL, Hagelstein, Pascalutsa (2022)

- Agreement with other calculations [most of those evaluate structure functions (using χEFT/model NN interactions) and use dispersion relations to get the TPE]

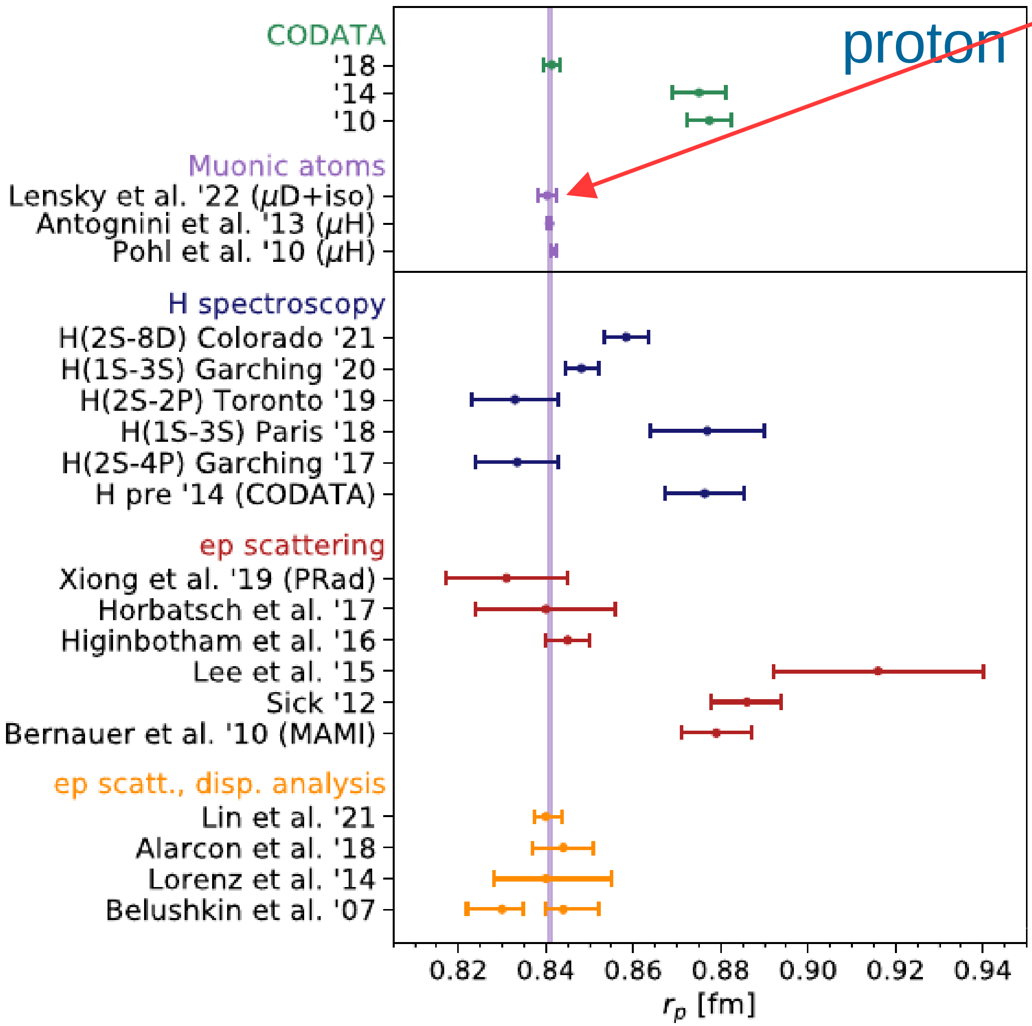
	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
$E_{\rm QED}$	point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size	$-5.225\,9\, r_p^2$	$-6.107\,4\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
$E_{\rm NS}$	nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
E_L(exp)	experiment[a]	202.370 6(23)	202.878 5(34)	1258.612(86)	1378.521(48)

Proton and Deuteron Radii and Isotope Shift

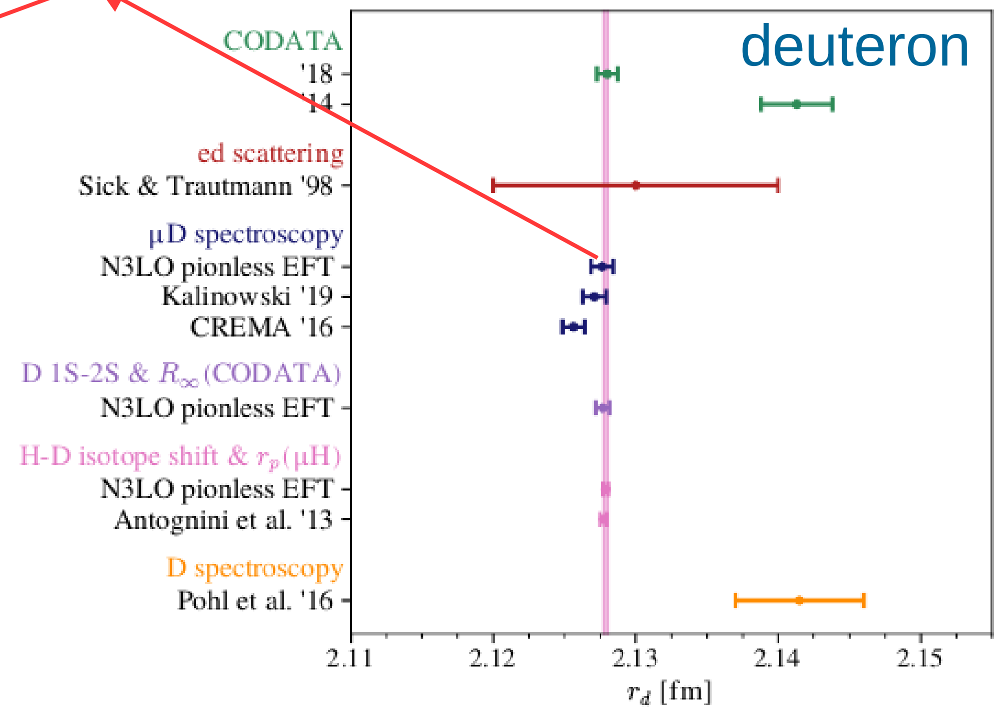
- H-D isotope shift: $E(H, 1S-2S) - E(D, 1S-2S)$

$$r_d^2 - r_p^2 = 3.820\,61(31)\,\text{fm}^2$$

Jentschura et al. (2011)



Antognini, Hagelstein, Pascalutsa (2022)



VL, Hagelstein, Pascalutsa (2022)

- Muonic Deuterium and H-D isotope shift are consistent with the small proton radius

HFS in μD: It's More Tricky

- Nuclear contribution to TPE in HFS is about 20 times smaller

$$\Delta E^{2\gamma}_{2S,\mathrm{HFS}} = 0.0966(73)\ \mathrm{meV}$$

Pohl et al. (2016), Pachucki, Kalinowski, Yerokhin (2018)

- Existing recent theoretical evaluations disagree

$$\Delta E^{2\gamma,\mathrm{theor}}_{2S,\mathrm{HFS}} = 0.0383(86)\ \mathrm{meV}$$

Pachucki, Kalinowski, Yerokhin (2018)

$$\Delta E^{2\gamma,\mathrm{theor}}_{2S,\mathrm{HFS}} = 0.1180(90)\ \mathrm{meV}$$

Ji, Zhang, Platter (2023)

- The smallness of the nuclear HFS contribution is the result of cancellations between different contributions
- Cancellations at the VVCS amplitude level make its spin-dependent part suppressed
- Cancellations between nuclear and single-nucleon terms
- No χEFT-based calculation exists (?)
- An alternative high-order EFT calculation (possibly accounting for relativistic corrections) is needed

Summary and Outlook

- μD and H-D isotope shift in pionless EFT consistent with each other
  - small proton radius
- Agreement with the very precise empirical value of 2γ exchange
  - experimental precision: both a challenge and a benchmark for theory
- Correlation between charge and Friar radius
  - another benchmark to check form factor parametrizations
- Single-nucleon effects are starting to be sizeable
  - more importaint in heavier nuclei
- Higher-order radiative corrections are also becoming important

- HFS in μD: more difficult (cancellations!), different results disagree at the moment, alternative calculations desirable
- A better accuracy can hopefully be achieved in high-order (N4LO+) χEFT calculations (also relies on progress in single-nucleon)
- A lot of room for improvement in heavier nuclei

Thank You for Your Attention!