Modified Power Counting in Chiral Effective Field Theory up to N^3LO

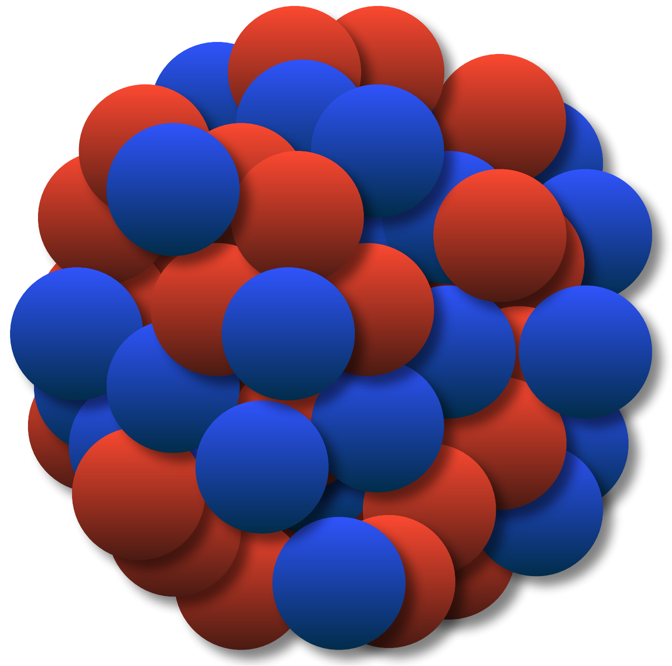
The 11th International Workshop on Chiral Dynamics, Ruhr University Bochum, Germany 2024

Swedish Research Council
European Research Council
Established by the European Commission

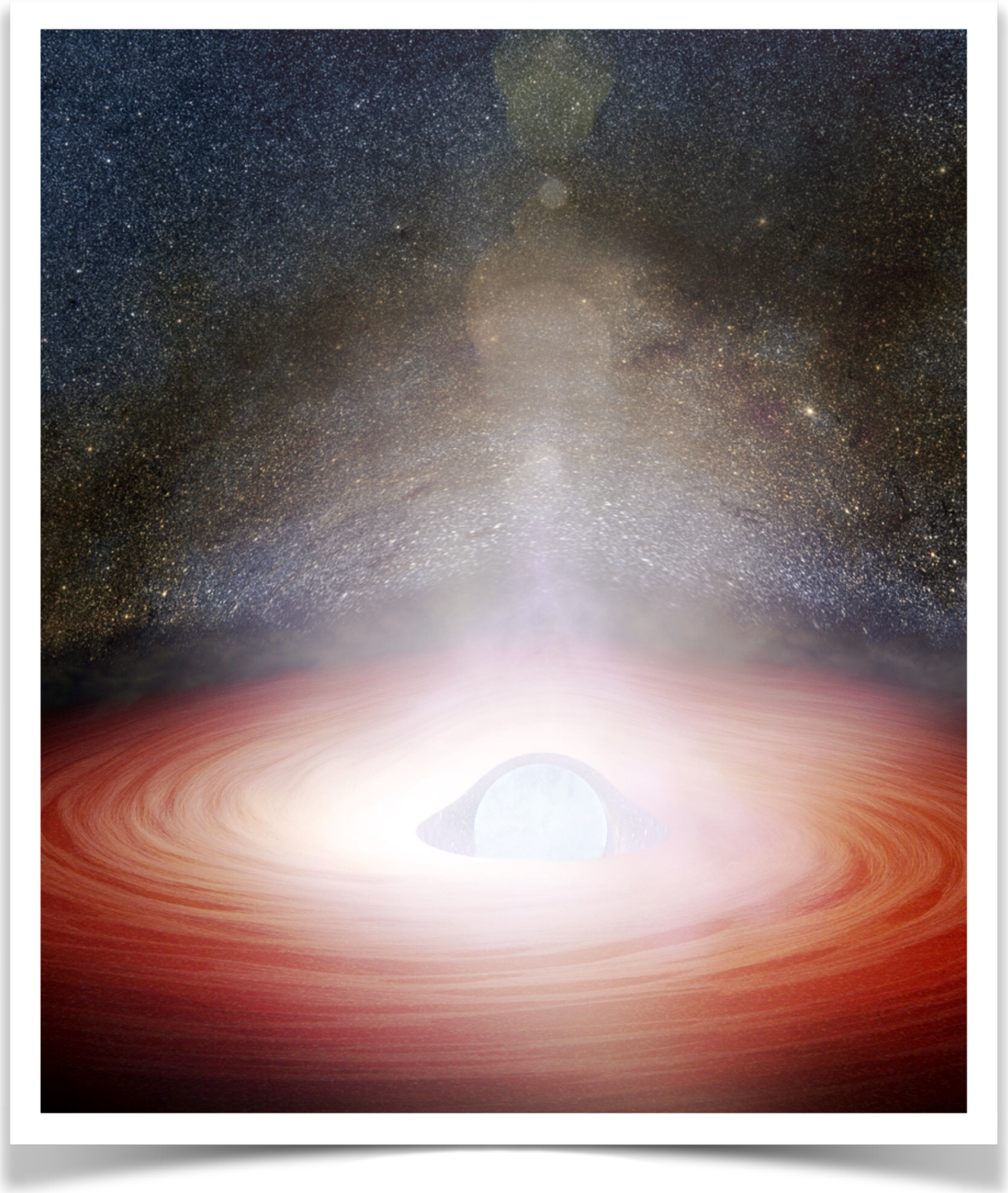
Oliver Thim | Theoretical Subatomic Physics | Chalmers University of Technology

The atomic nucleus

$\sim 10^{-15}$ m

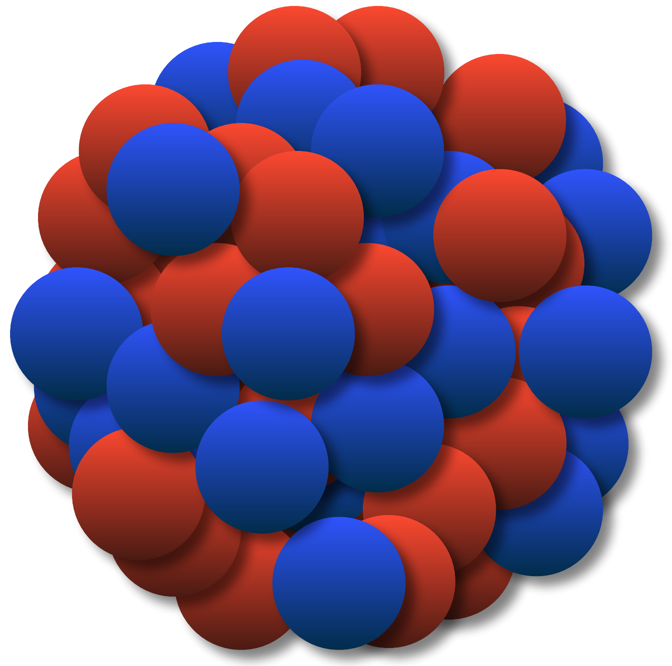


The atomic nucleus

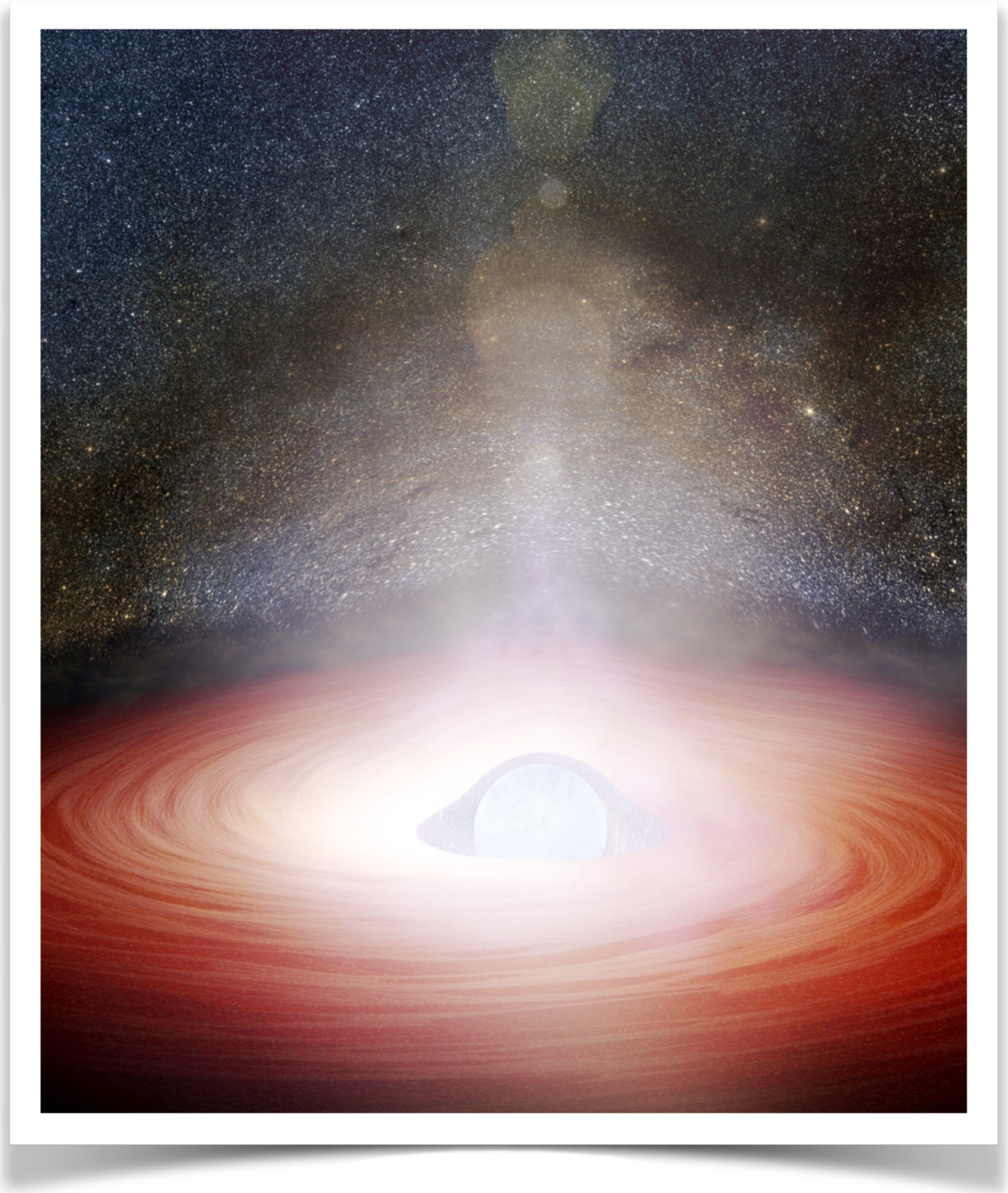


$\sim 10^4$ m

$\sim 10^{-15}$ m

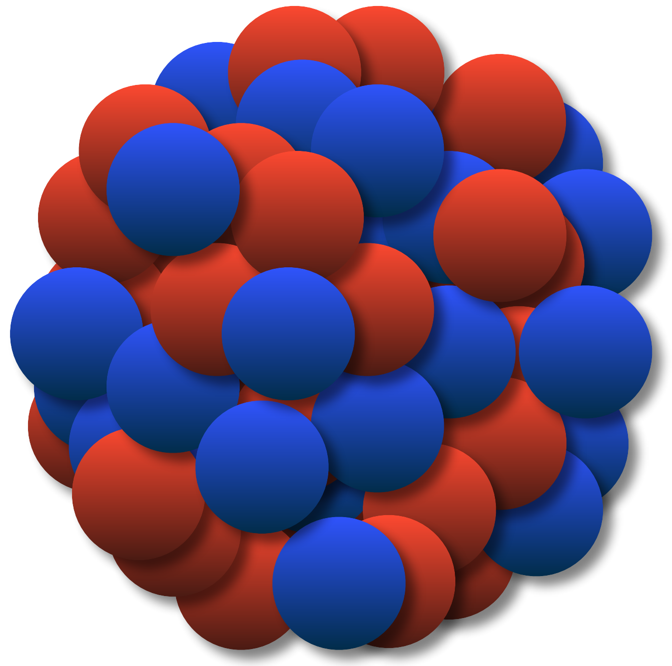


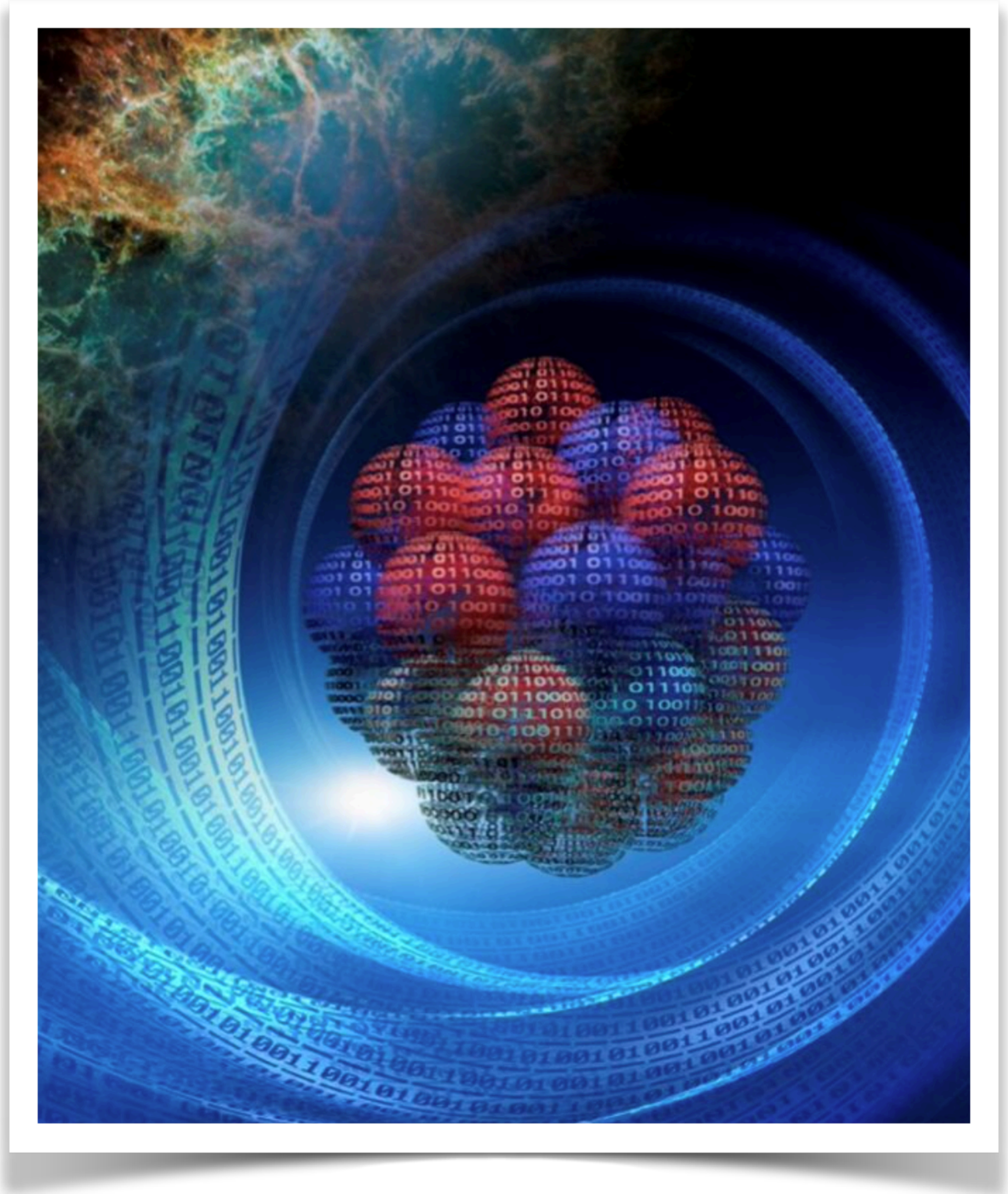
The atomic nucleus



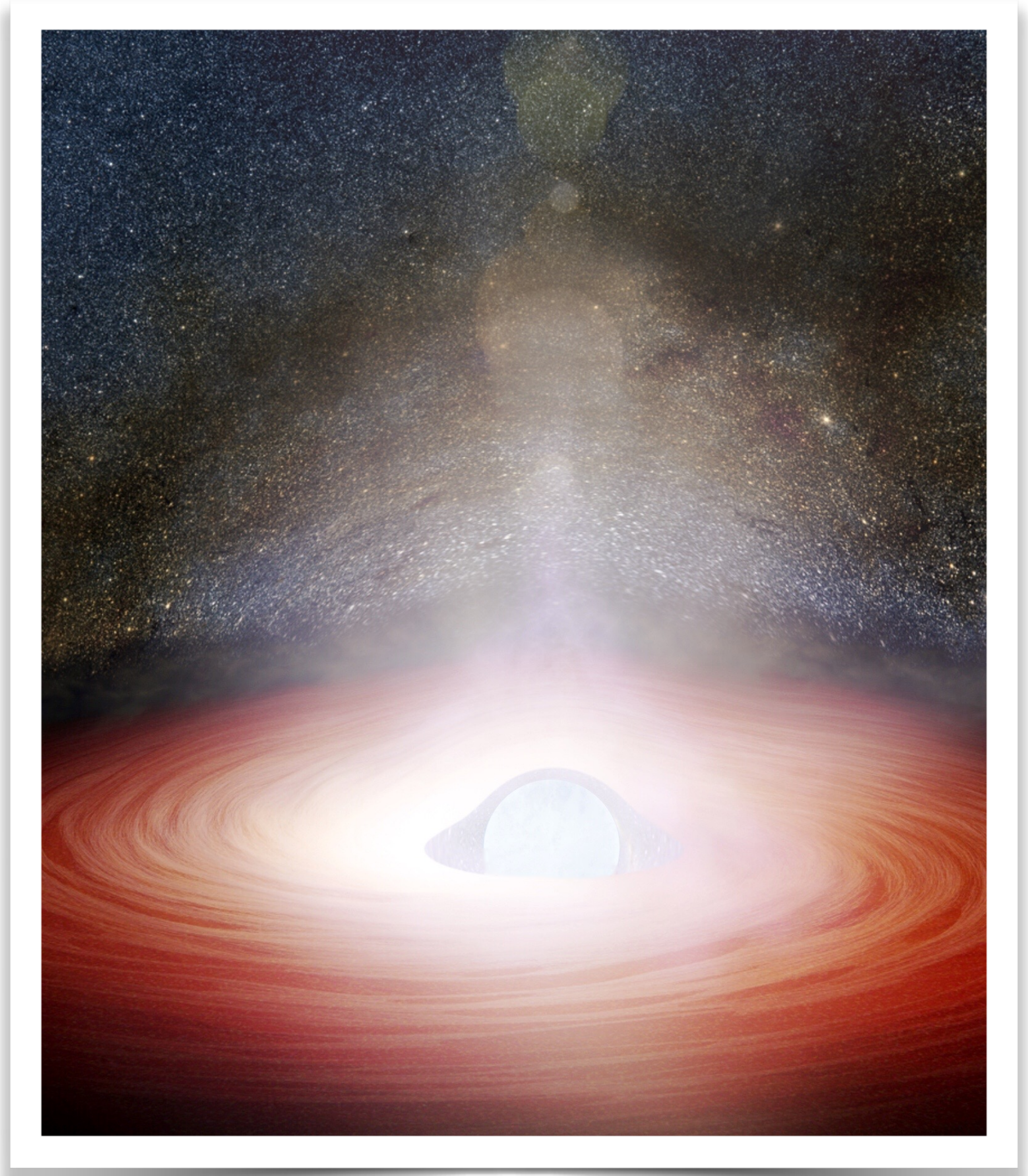
$\sim 10^4$ m

$\sim 10^{-15}$ m



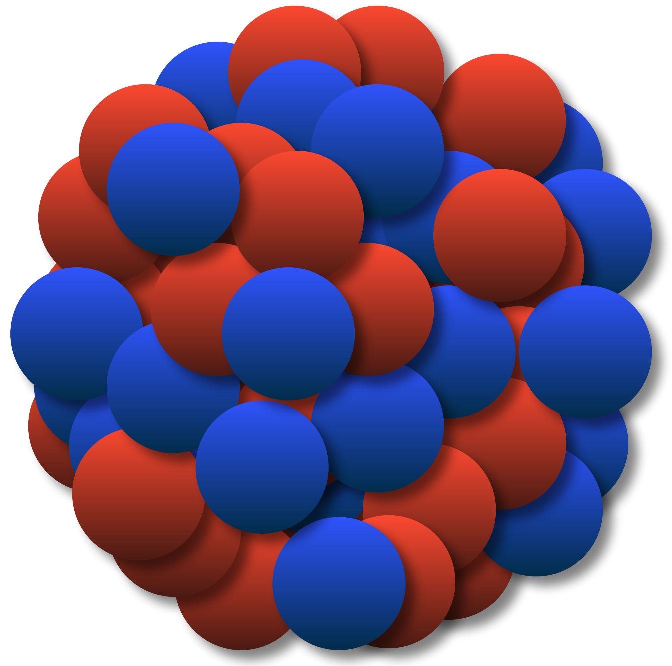


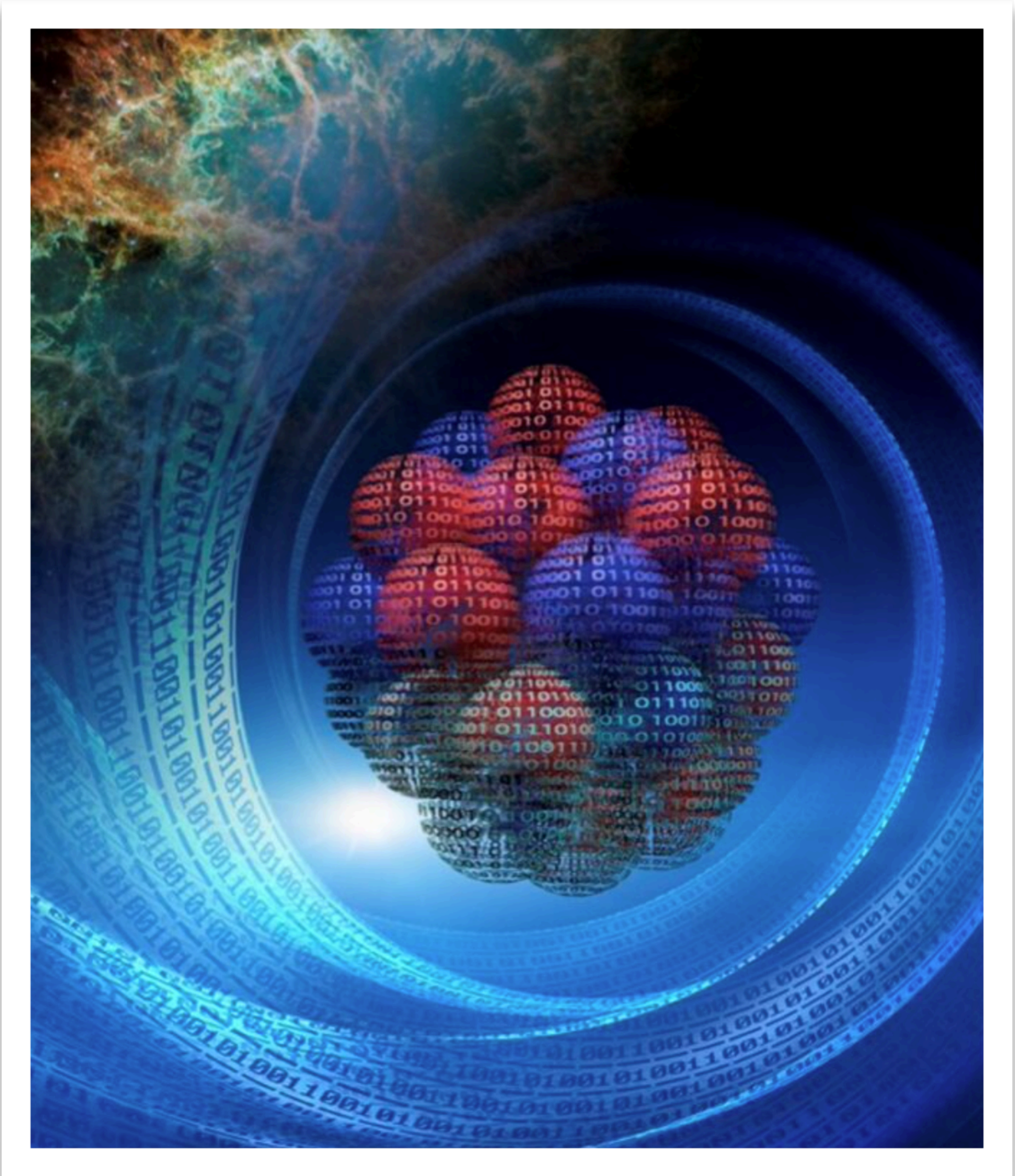
The atomic nucleus

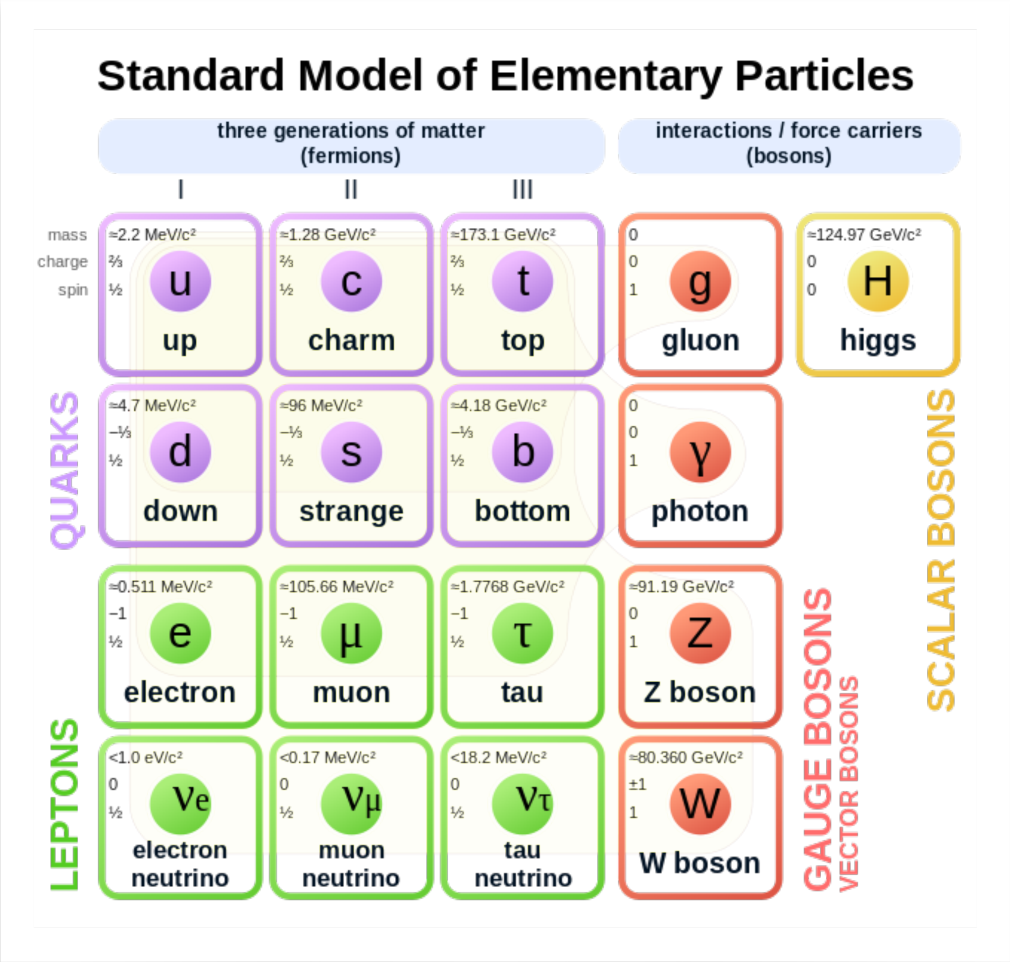


$\sim 10^{4}$ m

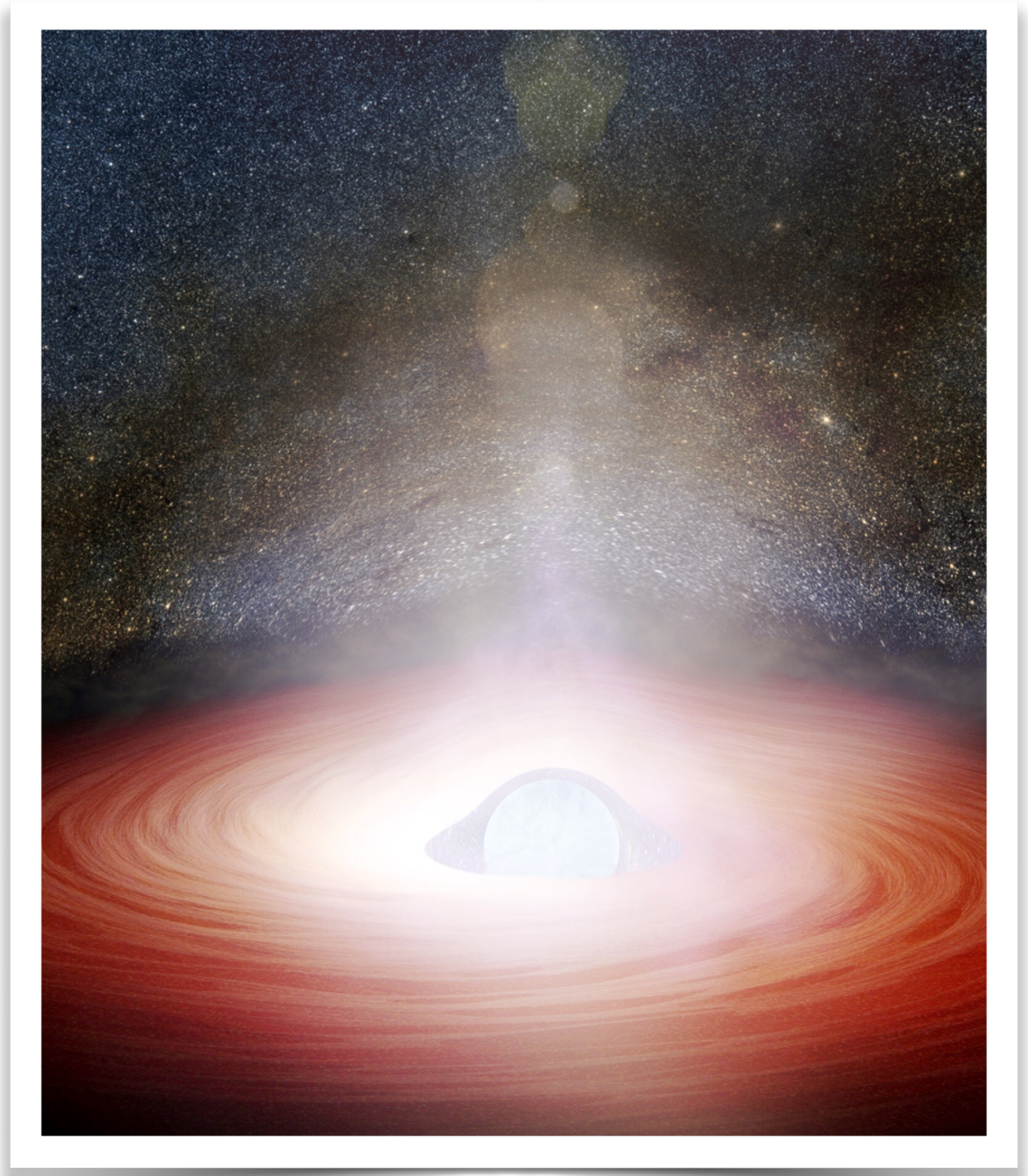
$\sim 10^{-15}$ m





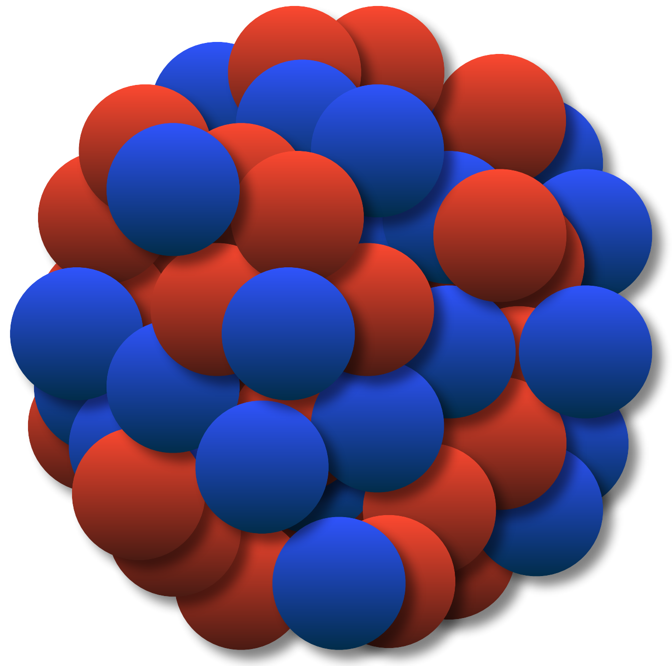


The atomic nucleus

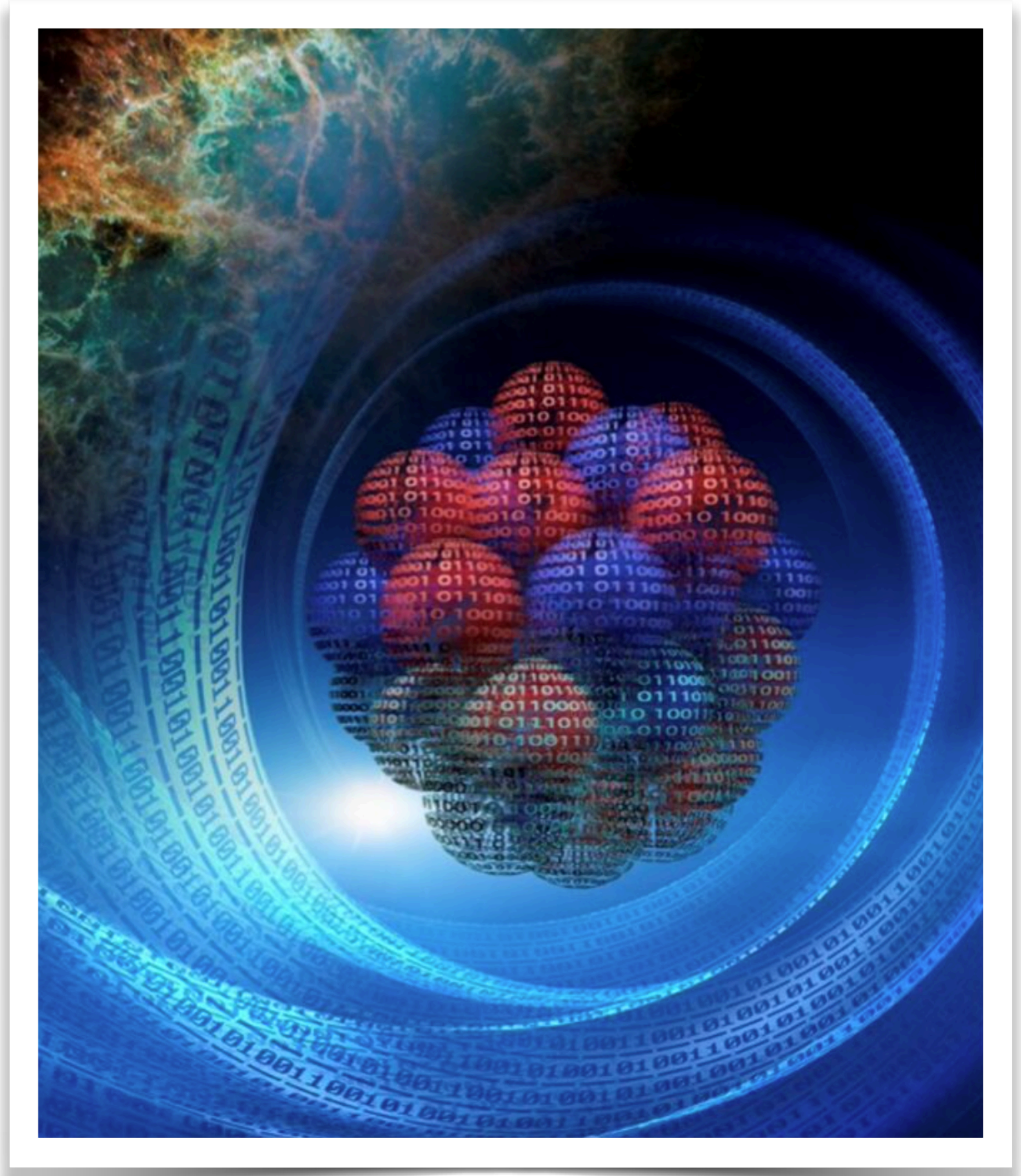


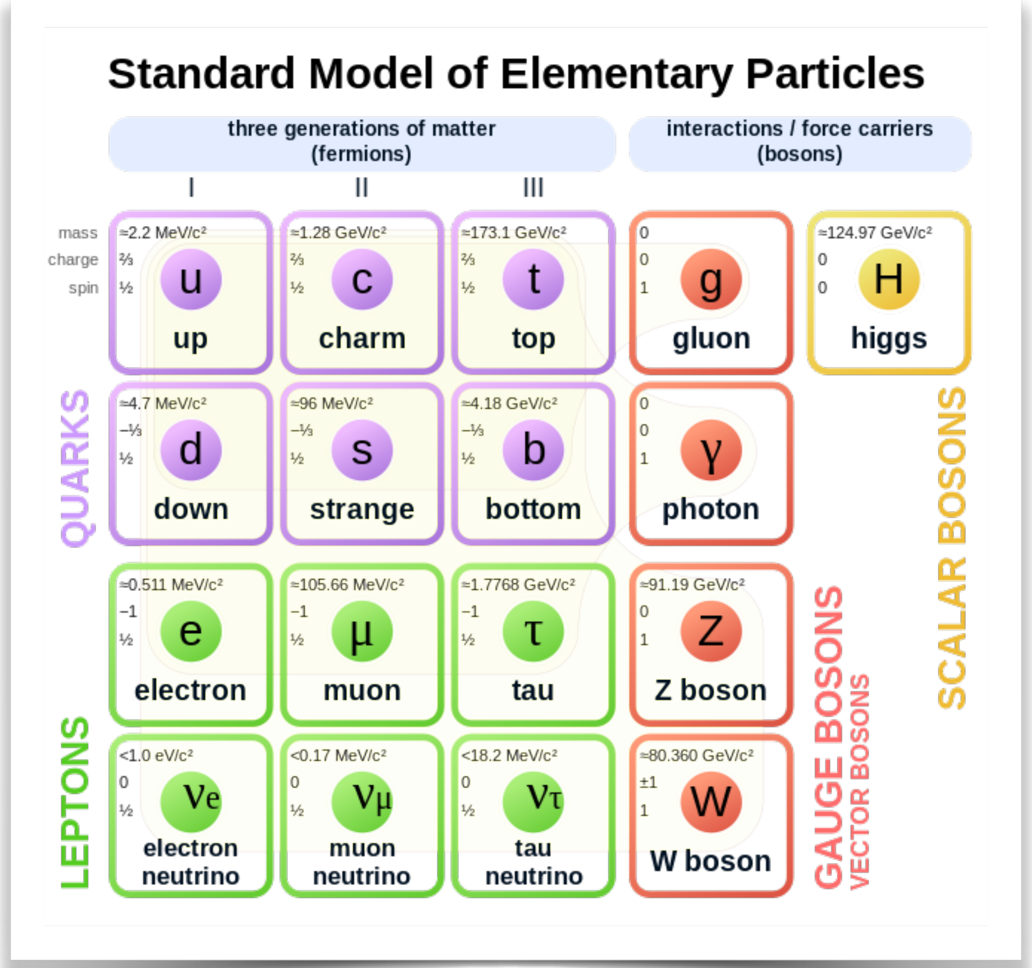
$\sim 10^{4}\ \mathrm{m}$

$\sim 10^{-15}\ \mathrm{m}$



$$H\left|\psi\right\rangle = E\left|\psi\right\rangle$$



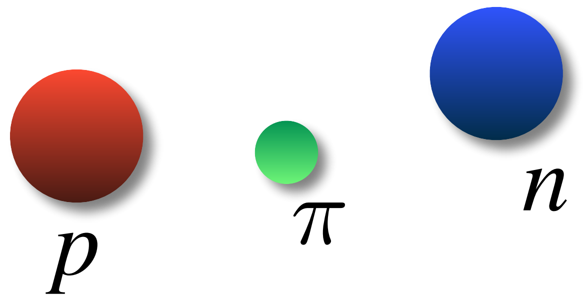


Questions

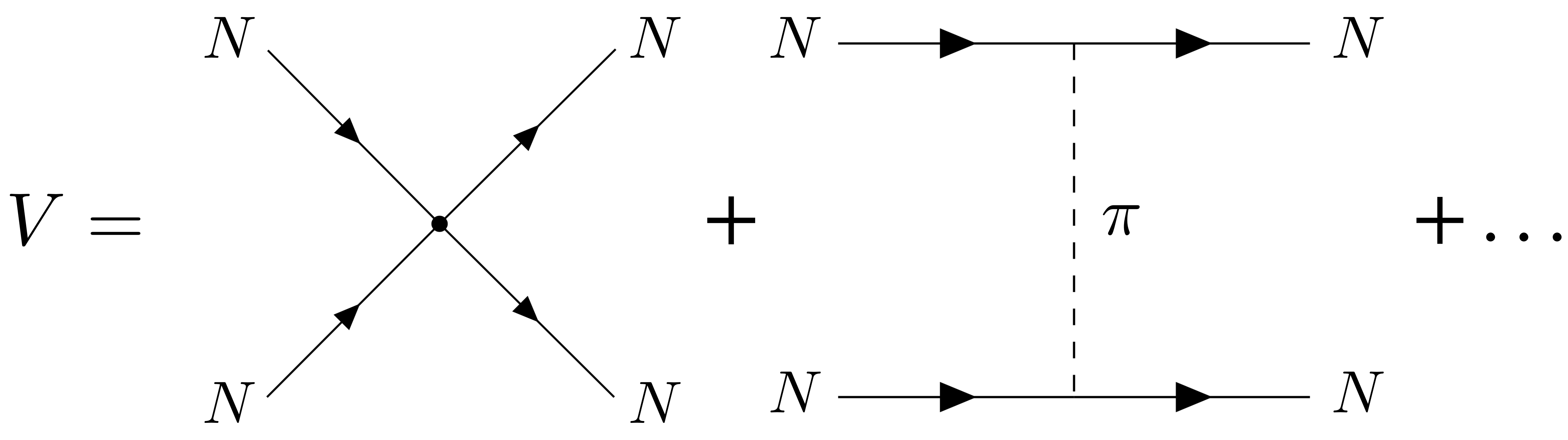
- How to construct H and keep the connection to QCD?
- How to obtain precise predictions for nuclear observables with quantified theoretical error?

The nuclear force from EFT

- Weinberg, 90's: S. Weinberg, (1979), (1990), (1991)
  - Use protons, neutrons and pions as degrees of freedom.



  - Formulate the most general dynamics consistent with low-energy symmetries of QCD.
  - Perturbative expansion in (Q/Λ_b) .



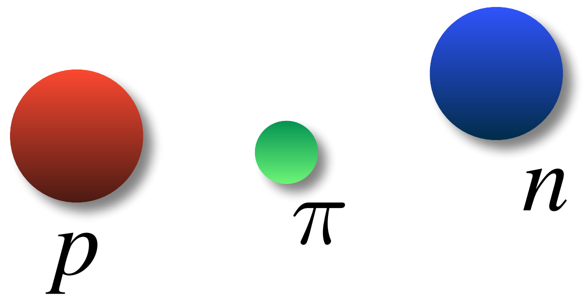
✓ EFT description rooted in QCD.

✓ Systematic expansion with **quantifiable theoretical error**:

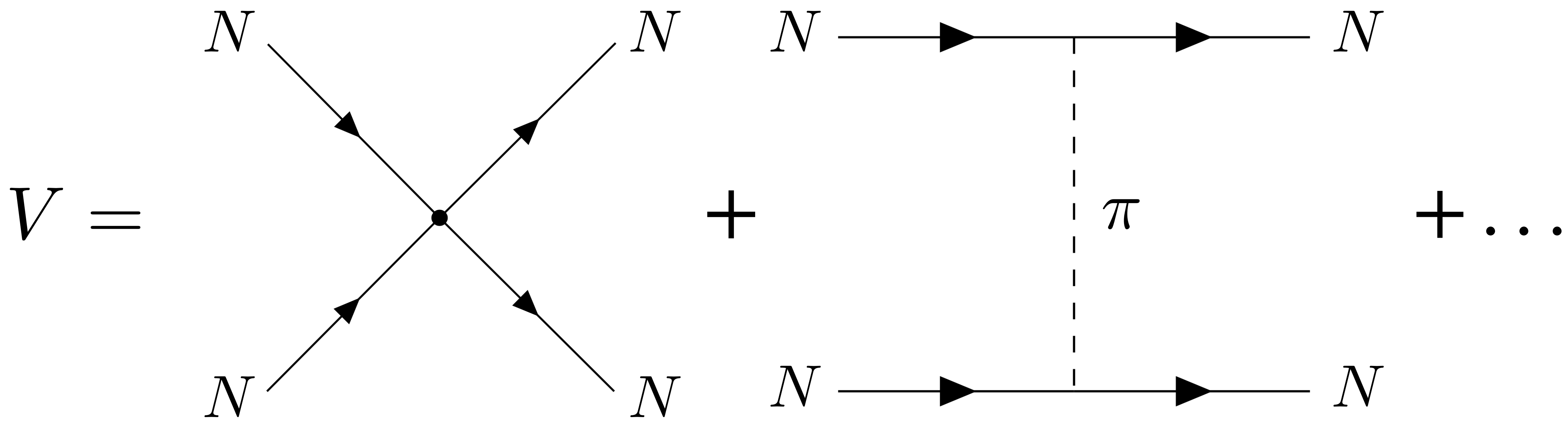
$$y_{\mathrm{th}}^{(\nu)} = \sum_{n=0}^{\nu} y_n \left(\frac{Q}{\Lambda_b}\right)^n + \mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^{\nu+1}$$

The nuclear force from EFT

- Weinberg, 90's: S. Weinberg, (1979), (1990), (1991)
  - Use protons, neutrons and pions as degrees of freedom.



  - Formulate the most general dynamics consistent with low-energy symmetries of QCD.
  - Perturbative expansion in (Q/Λ_b) .



✓ EFT description rooted in QCD.

✓ Systematic expansion with **quantifiable theoretical error**:

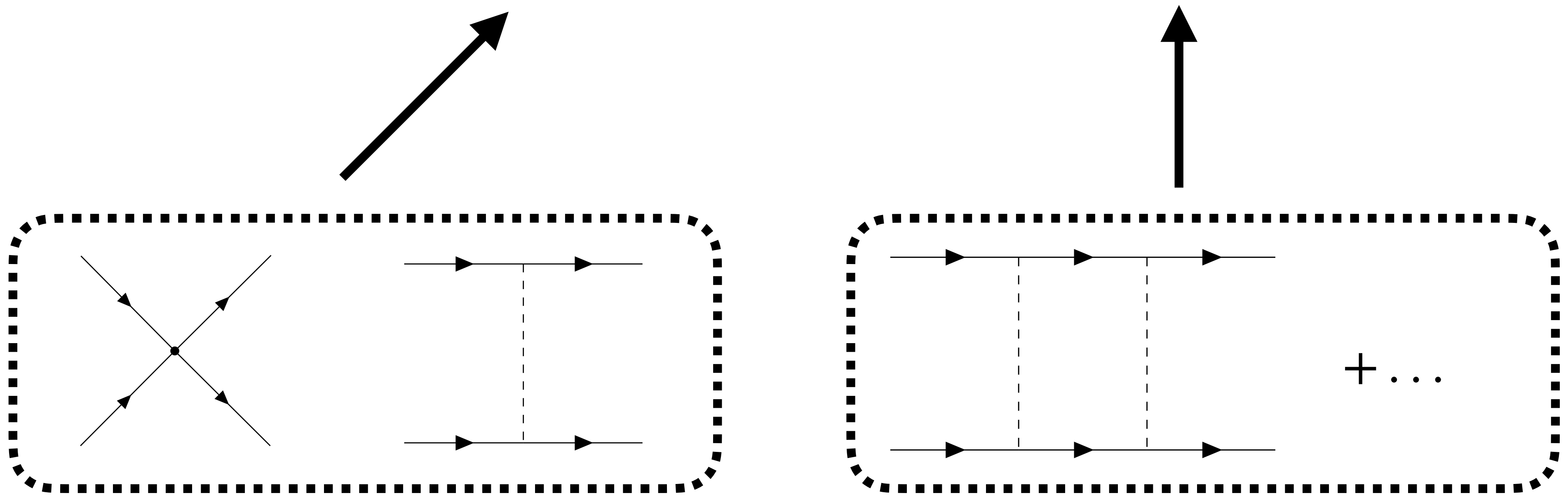
$$y_{\rm th}^{(\nu)} = \sum_{n=0}^{\nu} y_n \left(\frac{Q}{\Lambda_b}\right)^n + \mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^{\nu+1}$$

χEFT

Weinberg PC

- Construct nucleon-nucleon **potentials**:

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) \; + \; \dots$$



- Use dimensional analysis to organize diagrams.
- Resum potential nonperturbatively in LS-equation.

R. Machleidt and D. R. Entem, Phys. Rep. **503** (2011)

E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81,** (2009)

H.-W Hammer, S. König, and U. van Kolck, Rev. Mod. Phys. **92,** (2020)

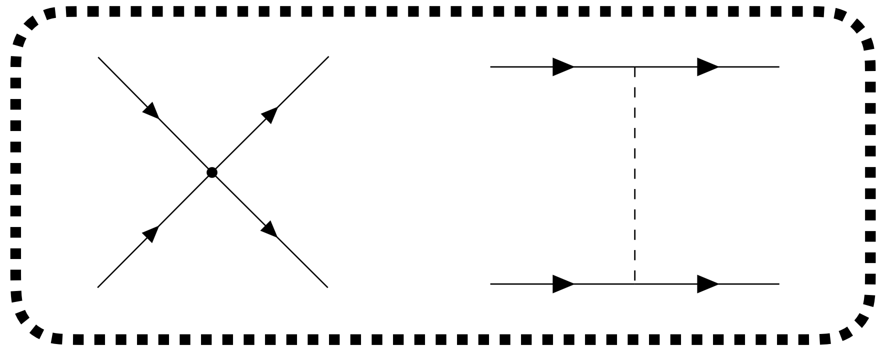
- Calibrate unknown LECs using **data.**

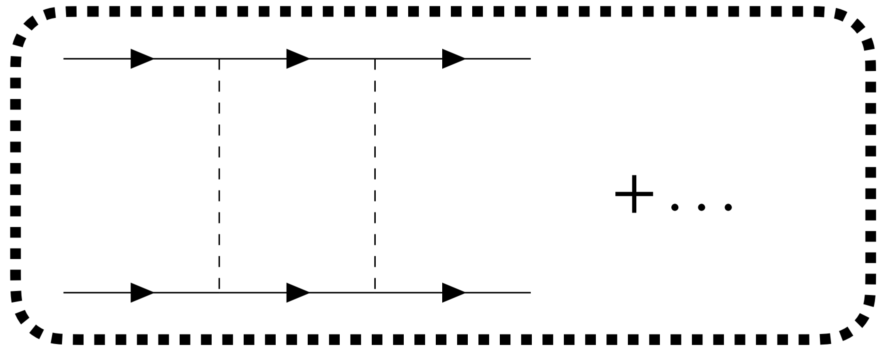
- Compute **predictions.**

Weinberg PC

- Construct nucleon-nucleon **potentials**:

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) \ + \ \dots$$





- Use dimensional analysis to organize diagrams.
- Resum potential nonperturbatively in LS-equation.

- Calibrate unknown LECs using **data.**

- Compute **predictions.**

- χEFT with WPC: Successful descriptions of two- and three-nucleon forces and interaction currents.

R. Machleidt and D. R. Entem, Phys. Rep. **503** (2011)

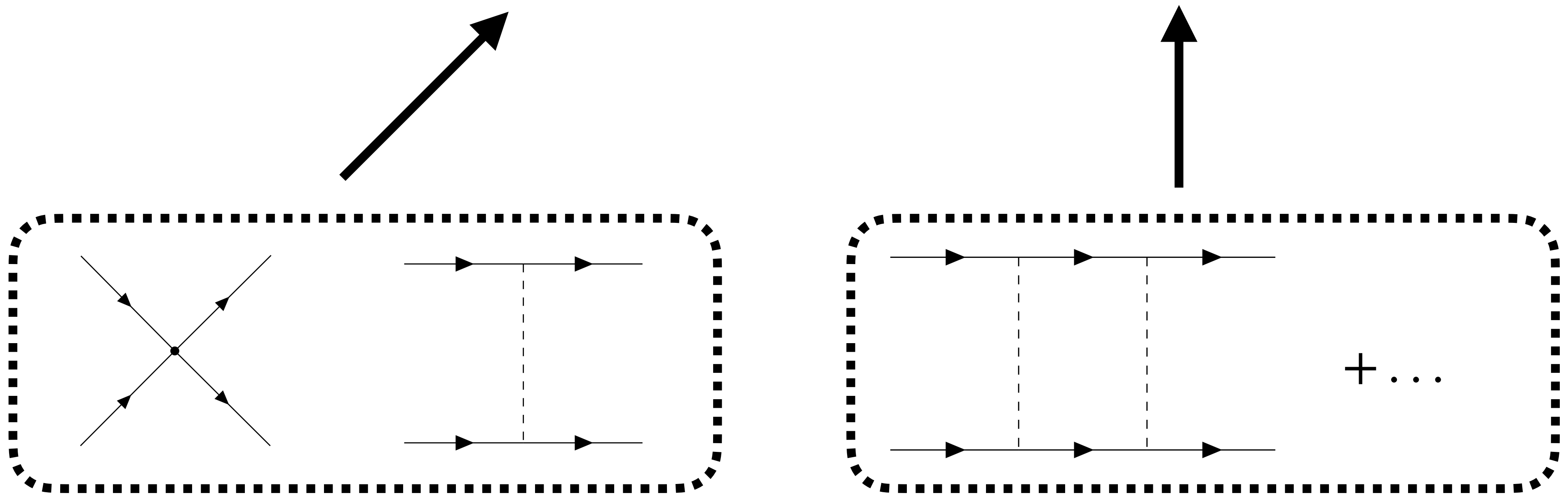
E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81,** (2009)

H.-W Hammer, S. König, and U. van Kolck, Rev. Mod. Phys. **92,** (2020)

Weinberg PC

- Construct nucleon-nucleon **potentials**:

$$V = V^{(0)}_{\rm NN}(\boldsymbol{\alpha}^{(0)}) + V^{(2)}_{\rm NN}(\boldsymbol{\alpha}^{(2)}) \; + \; \dots$$



- Use dimensional analysis to organize diagrams.
- Resum potential nonperturbatively in LS-equation.

R. Machleidt and D. R. Entem, Phys. Rep. **503** (2011)

E. Epelbaum, H.-W. Hammer, and U.-G. Meissner, Rev. Mod. Phys. **81,** (2009)

H.-W Hammer, S. König, and U. van Kolck, Rev. Mod. Phys. **92,** (2020)

- Calibrate unknown LECs using **data.**

- Compute **predictions.**

- χEFT with WPC: Successful descriptions of two- and three-nucleon forces and interaction currents.

- Predictions of observables **depend on Λ** (= not RG invariant) A. Nogga *et al.*, Phys. Rev. C **72**, (2005)

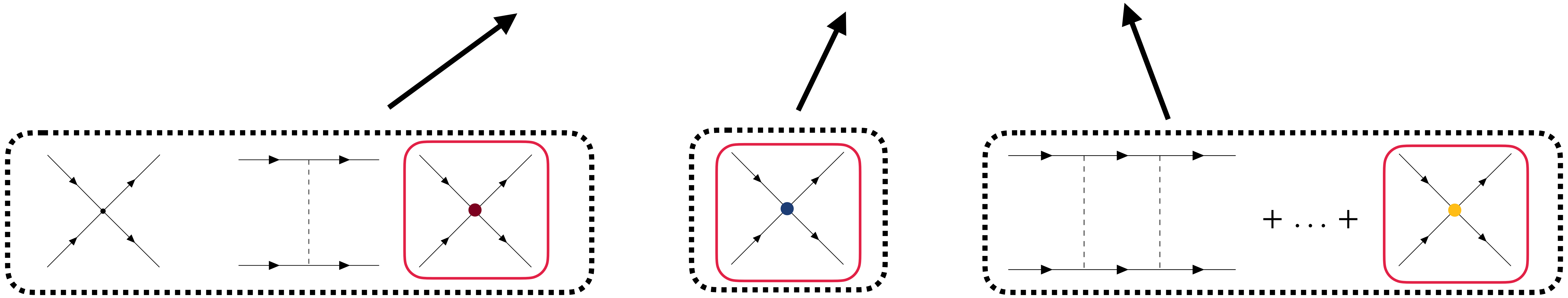
Modified Weinberg PC

- What happens if we impose Λ-independence and promote counterterms to lower orders to achieve this?

Modified Weinberg PC

- What happens if we impose Λ-independence and promote counterterms to lower orders to achieve this?

$$V = V_{\rm NN}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\rm NN}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\rm NN}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\rm NN}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \dots$$



B. Long and U. van Kolck, Ann. Phys. **323**, (2008)

B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)

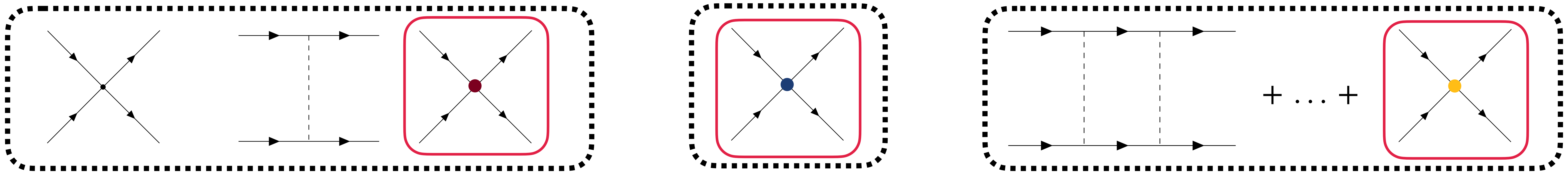
A. Nogga *et al.,* Phys. Rev. C **72**, (2005)

Modified Weinberg PC

- What happens if we impose Λ-independence and promote counterterms to lower orders to achieve this?

Treated perturbatively

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\mathrm{NN}}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \dots$$



B. Long and U. van Kolck, Ann. Phys. **323**, (2008)

B. Long, C. J. Yang, Phys. Rev. C **84**, (2011), Phys. Rev. C **85**, (2012), Phys. Rev. C **86**, (2012)

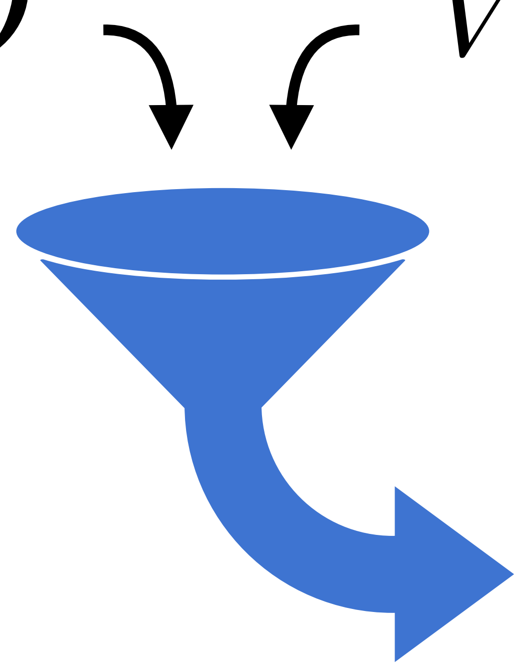
A. Nogga *et al.,* Phys. Rev. C **72**, (2005)

Calibrating the LO potential

- Use *NN* scattering observables to calibrate LO LECs.
- Bayesian inference: Treat LECs as random variables.

R. J. Furnstahl, N. Klco, D. R. Phillips, and S. Wesolowski, Phys. Rev. C **92**, (2015)

D $V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)})$

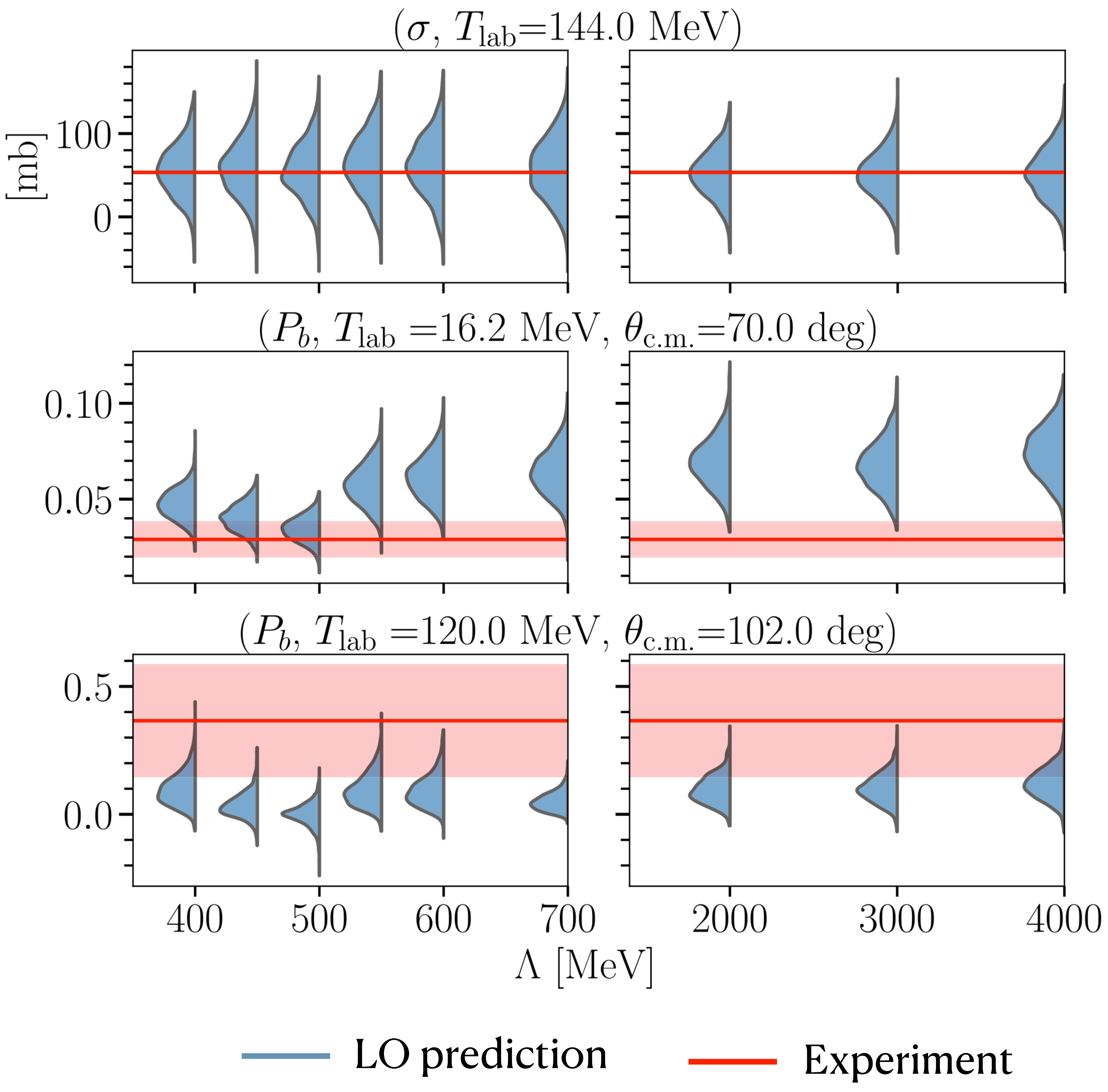


Likelihood, Prior, Posterior, Evidence

Bayes' rule:
$$\frac{\mathrm{pr}(D|\boldsymbol{\alpha}^{(0)}, I) \cdot \mathrm{pr}(\boldsymbol{\alpha}^{(0)}|I)}{\mathrm{pr}(D|I)} = \mathrm{pr}(\boldsymbol{\alpha}^{(0)}|D, I)$$

$$\mathrm{pr}\left(\boldsymbol{\alpha}^{(0)}|D, I\right)$$

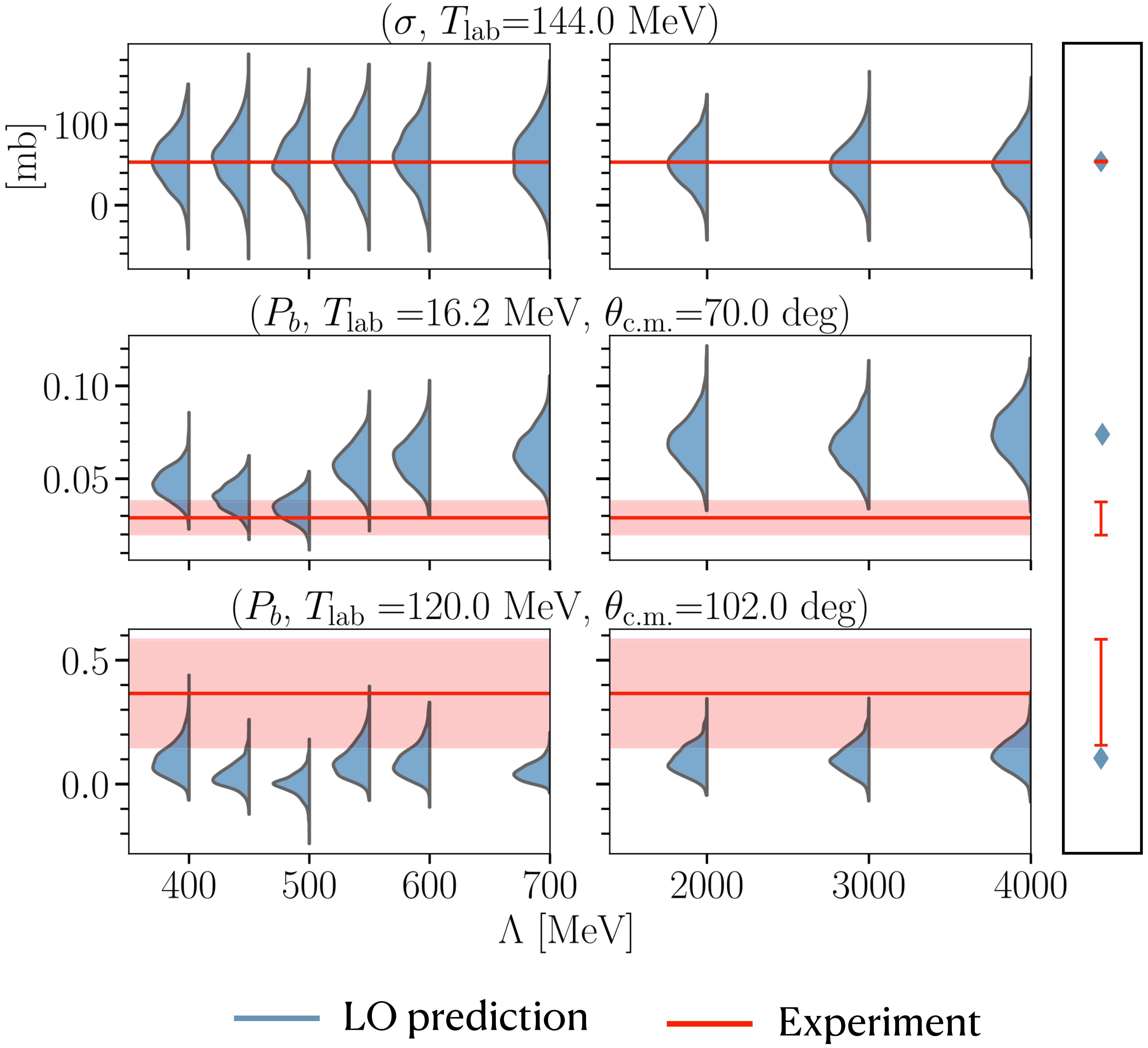
Predicted scattering observables



- Accurate, but not very precise (high energy, LO).
- Not very accurate, but within LO uncertainty.
- Quite accurate, but the experimental error is large.

OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, (2023)

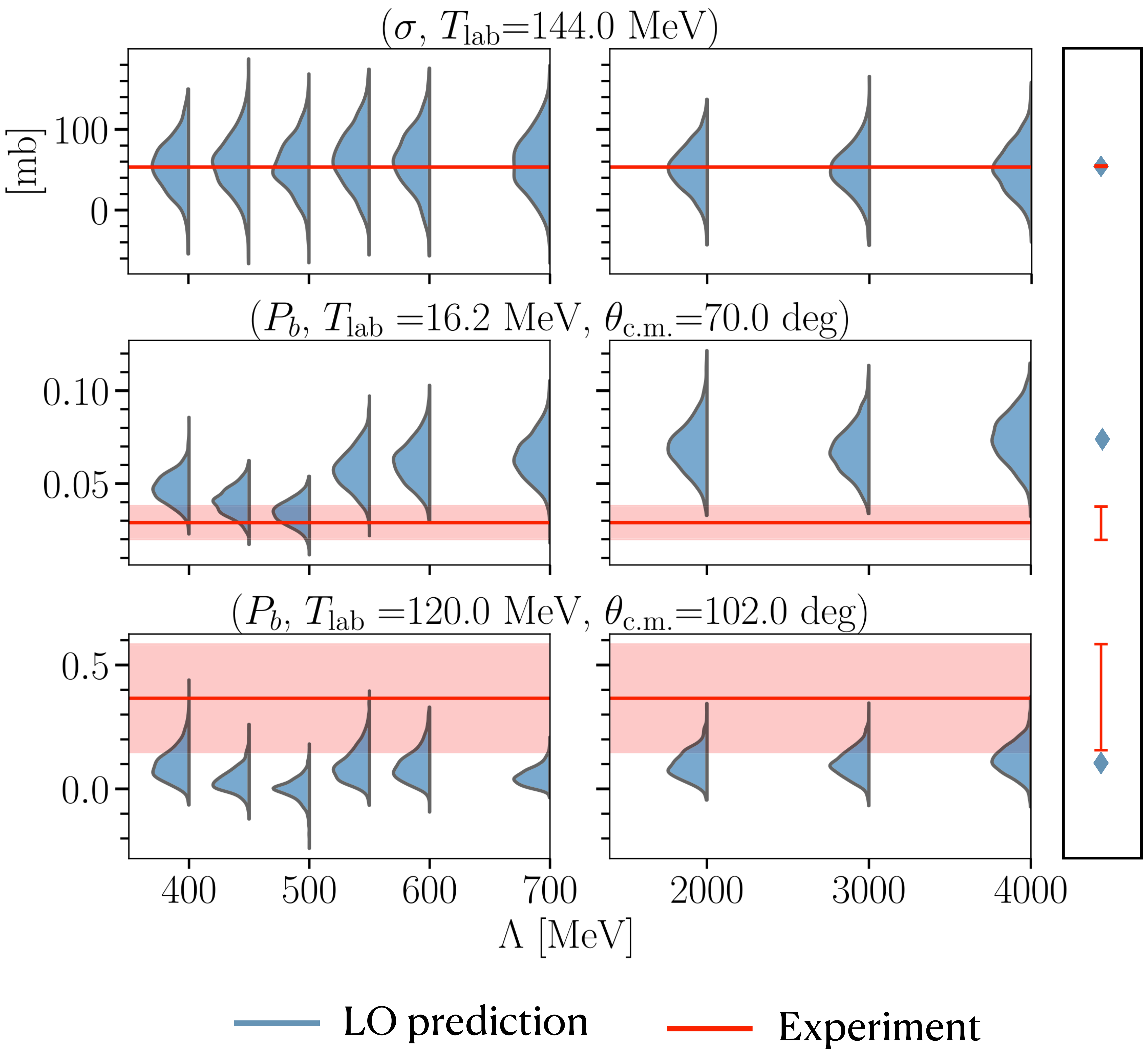
Predicted scattering observables



- Accurate, but not very precise (high energy, LO).
- Not very accurate, but within LO uncertainty.
- Quite accurate, but the experimental error is large.

OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, (2023)

Predicted scattering observables



- Accurate, but not very precise (high energy, LO).
- Not very accurate, but within LO uncertainty.
- Quite accurate, but the experimental error is large.

- Predictions are RG-invariant.
- Uncertainties are crucial for conclusions!
- The error model used is insufficient, higher orders are needed.

OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, (2023)

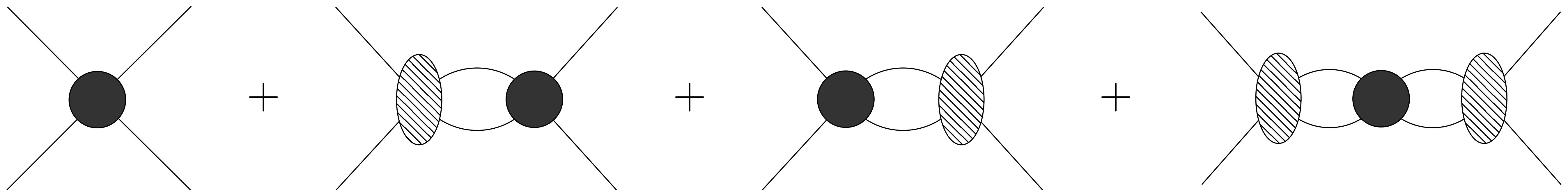
Adding perturbative corrections

$$V = V_{\mathrm{NN}}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\mathrm{NN}}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\mathrm{NN}}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\mathrm{NN}}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \dots$$

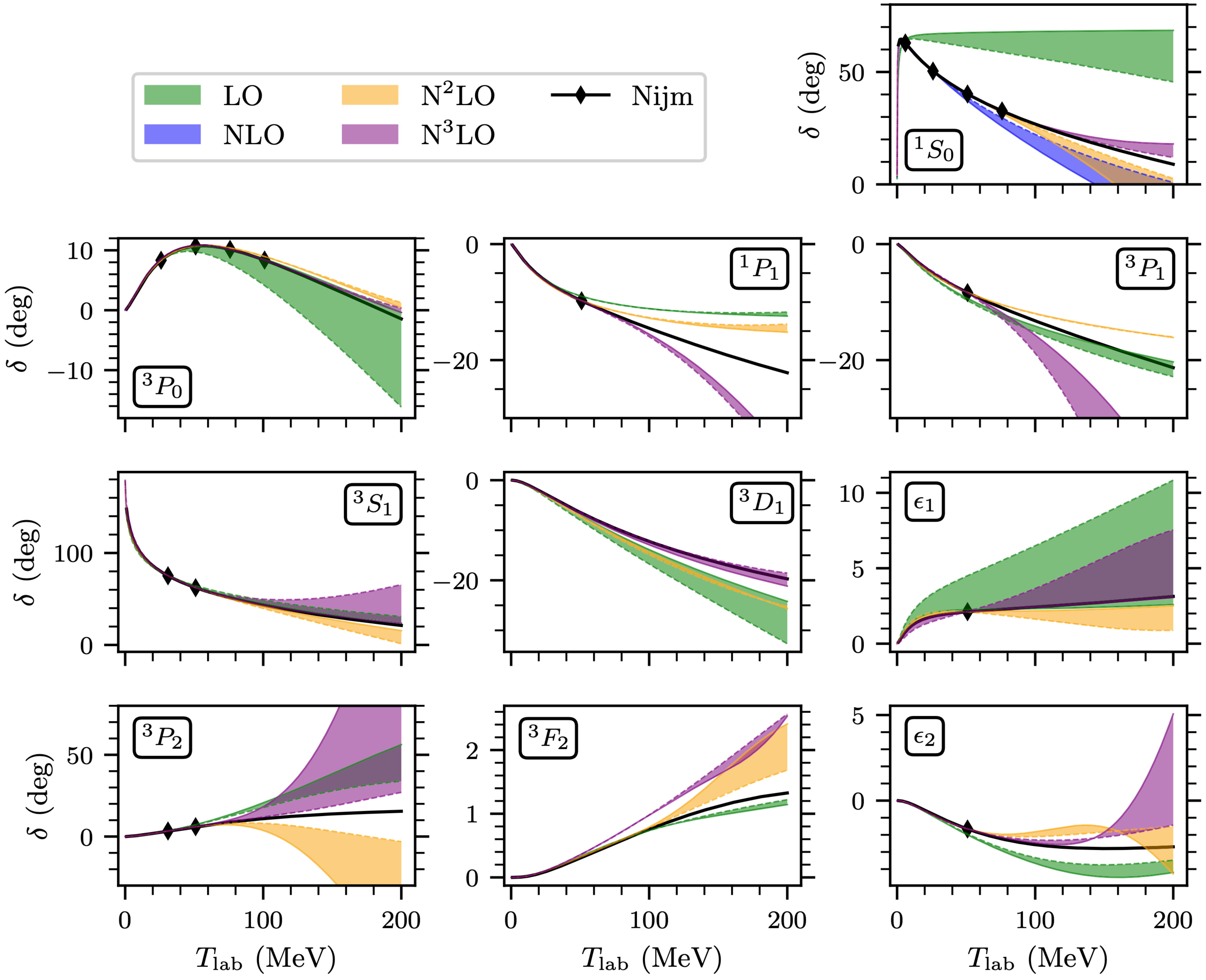
LO — Perturbative corrections

- More LECs: $\boldsymbol{\alpha}^{(0)}, \boldsymbol{\alpha}^{(1)}, \boldsymbol{\alpha}^{(2)}, \boldsymbol{\alpha}^{(3)}$.
- A first step: Calibrate LECs using **phase shifts** and compute predictions for **scattering observables**.

NLO: $T^{(1)} =$



Calibrate LECs using *np* phase shifts



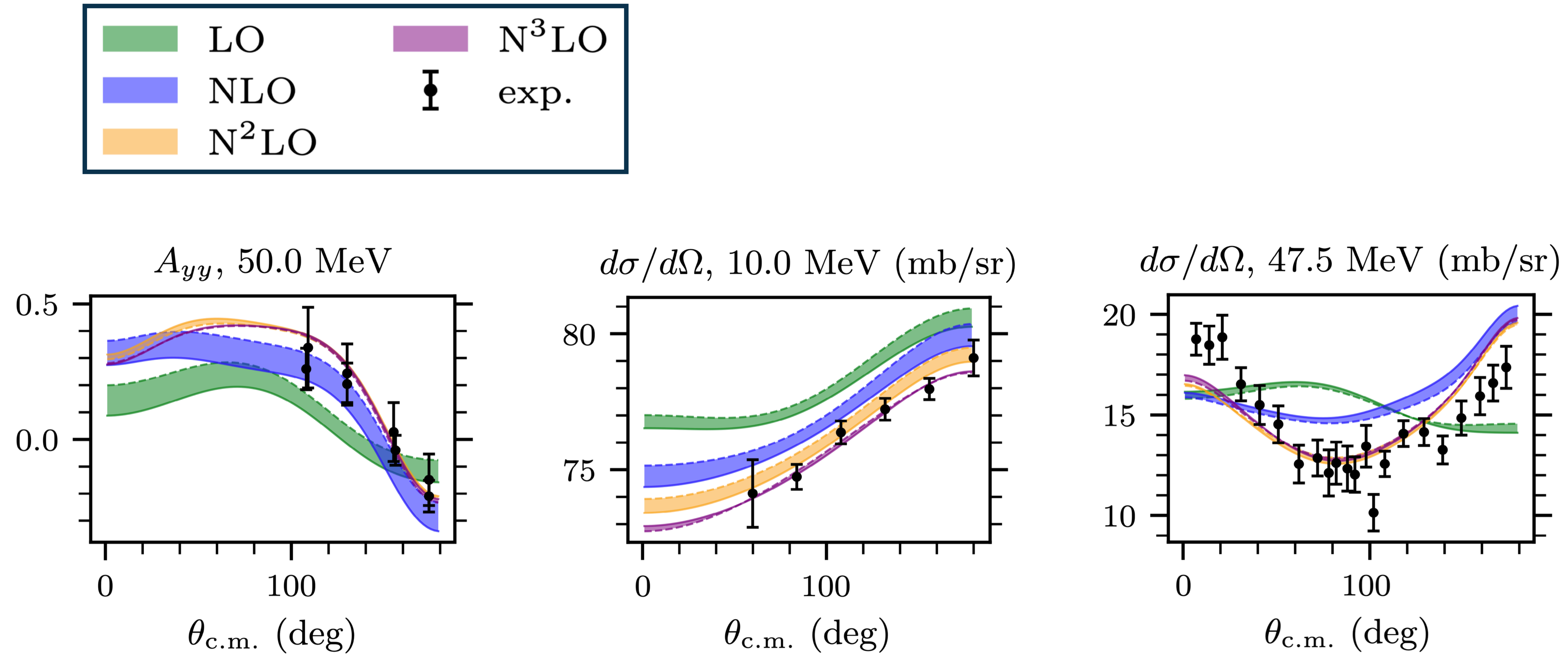
OT, A. Ekström, and C. Forssén, Phys. Rev. C **109**, (2024)

- Phase shifts are computed perturbatively.
- LECs are inferred by reproducing phase shifts at specific energies (♦).
- Two cutoffs:

$$\Lambda = 500 \text{ MeV}, \quad \Lambda = 2500 \text{ MeV}$$

- Note: NLO = LO except in $^1S_0$.

Predicted scattering observables



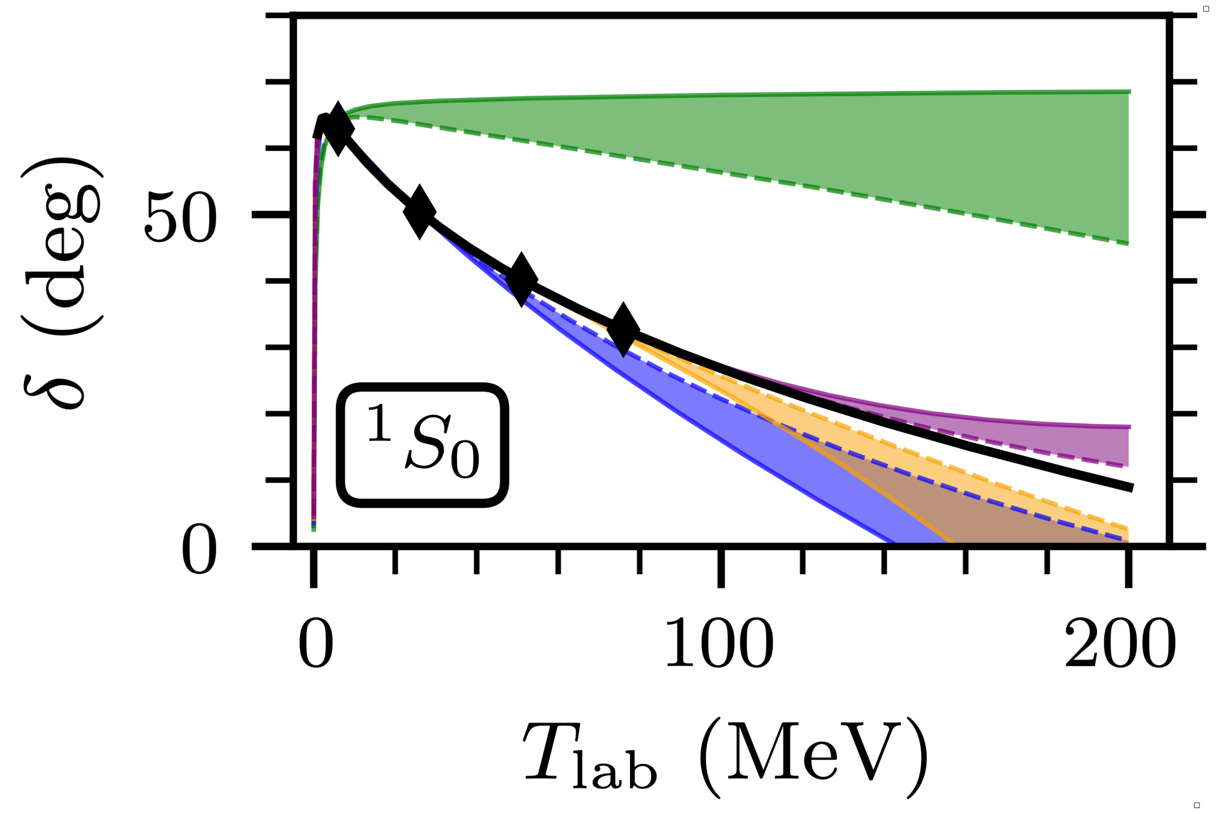
- Clear improvement order-by-order.
- **Accurate** cross sections up to at least $T_{\text{lab}} = 100$ MeV.

OT, A. Ekström, and C. Forssén, Phys. Rev. C **109**, (2024)

Low-energy theorems (LETs)

- Is pion dynamics being treated properly?
- LET: Predicted higher-order coefficients in the effective-range expansion.

$$F(k) \equiv k \cot \delta(k) = \underbrace{-\frac{1}{a} + \frac{1}{2} r k^2}_{\text{Fit}} + \underbrace{v_2 k^4 + v_3 k^6 + v_4 k^8}_{\text{Predict}} + \mathcal{O}\left(k^{10}\right)$$

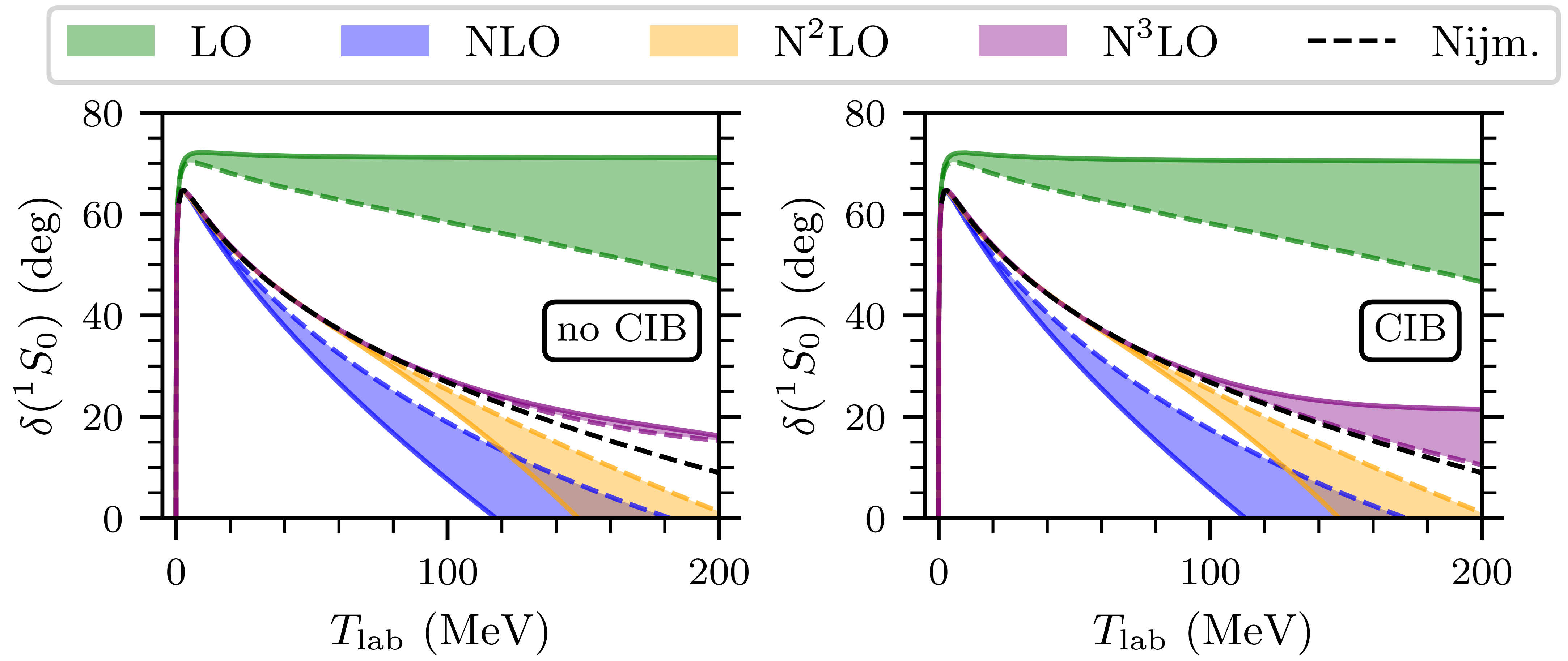


- Predictions are clear indicators of correctly captured pion dynamics.

T.D. Cohen, J.M. Hansen, Phys. Rev. C **59**, (1999)

Low-energy theorems: $^1S_0$

Phase shifts in $^1S_0$



$$F(k) \equiv k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \mathcal{O}\left(k^{10}\right)$$

$$F(k) - ik = -\frac{2}{\pi m_N T^{(0)}} \left[1 - \frac{T^{(1)}}{T^{(0)}} + \left(\left[\frac{T^{(1)}}{T^{(0)}}\right]^2 - \frac{T^{(2)}}{T^{(0)}} \right) + \right.$$
$$\left. + \left(2\frac{T^{(1)}T^{(2)}}{\left(T^{(0)}\right)^2} - \frac{T^{(3)}}{T^{(0)}} - \left[\frac{T^{(1)}}{T^{(0)}}\right]^3 \right) + \mathcal{O}\left(\frac{Q^4}{\Lambda_b^4}\right) \right].$$

Predicted effective range parameters (LETs)

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	−23.735(16)	2.68(3)	−0.48(2)	3.9(1)	−19.6(5)
$\Lambda = 500$ MeV, (no CIB)					
LO	*	1.71(0)	−1.77(0)	8.54(0)	−47.0(3)
NLO	*	*	−0.64(0)	4.79(0)	−29.9(2)
N^{2}LO	*	2.72(0)	−0.71(0)	5.05(0)	−29.3(2)
N^{3}LO	*	2.69(0)	−0.66(0)	5.42(0)	−31.0(2)
$\Lambda = 500$ MeV, (CIB)					
LO	*	1.68(0)	−1.55(0)	6.63(0)	−31.64(8)
NLO	*	*	−0.45(0)	3.42(0)	−18.95(8)
N^{2}LO	*	2.70(0)	−0.55(0)	3.77(0)	−18.8(2)
N^{3}LO	*	2.68(0)	−0.50(0)	4.02(0)	−19.8(2)

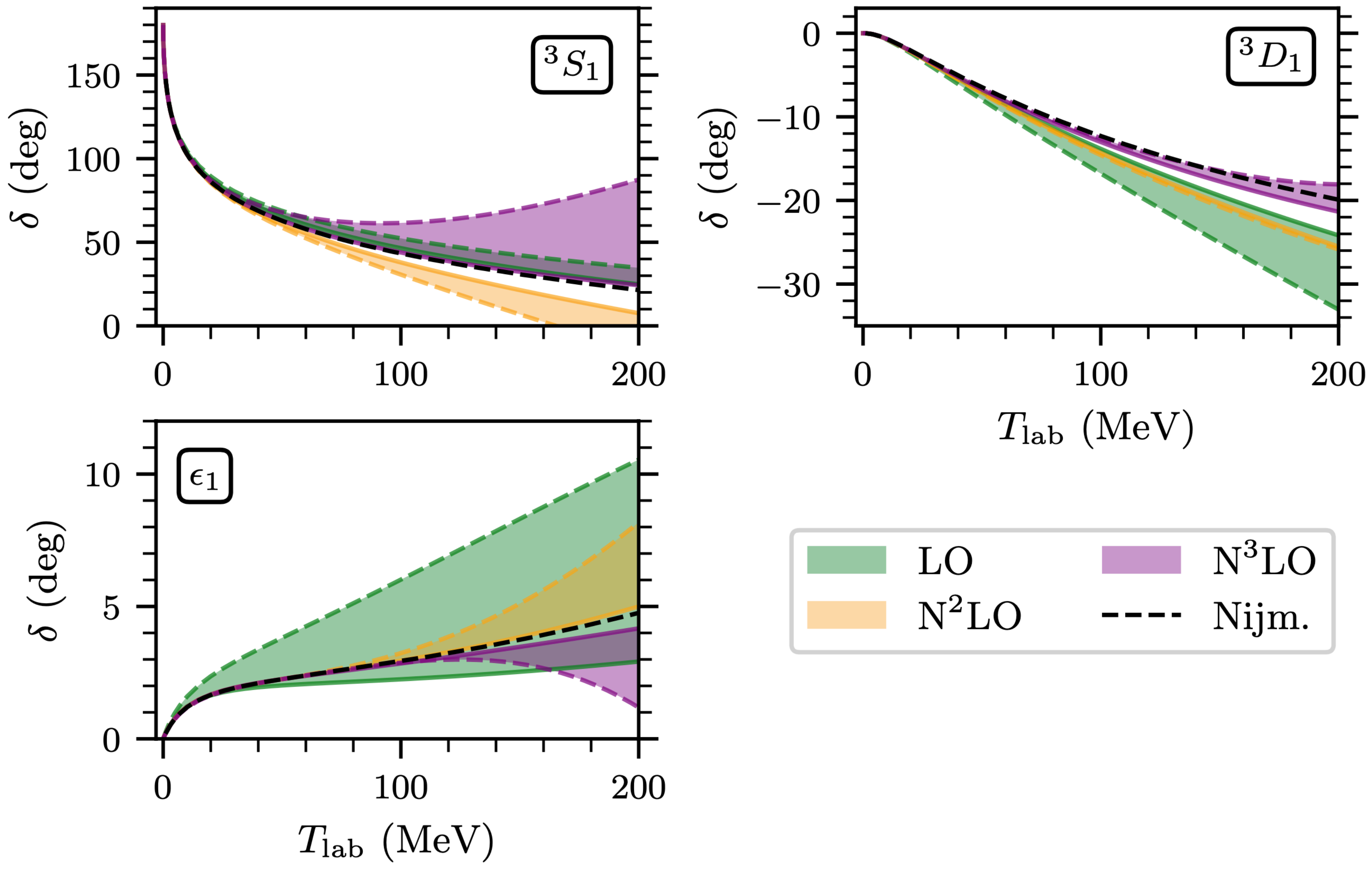
- CIB in one-pion exchange is **significant** in $^1S_0$.

✓ Both phase shift and LETs are accurate.

OT, Few-Body Syst. 65, 69 (2024)

Low-energy theorems: $^3S_1$

Phase shifts in $^3S_1$— $^3D_1$



Predicted effective range parameters (LETs)

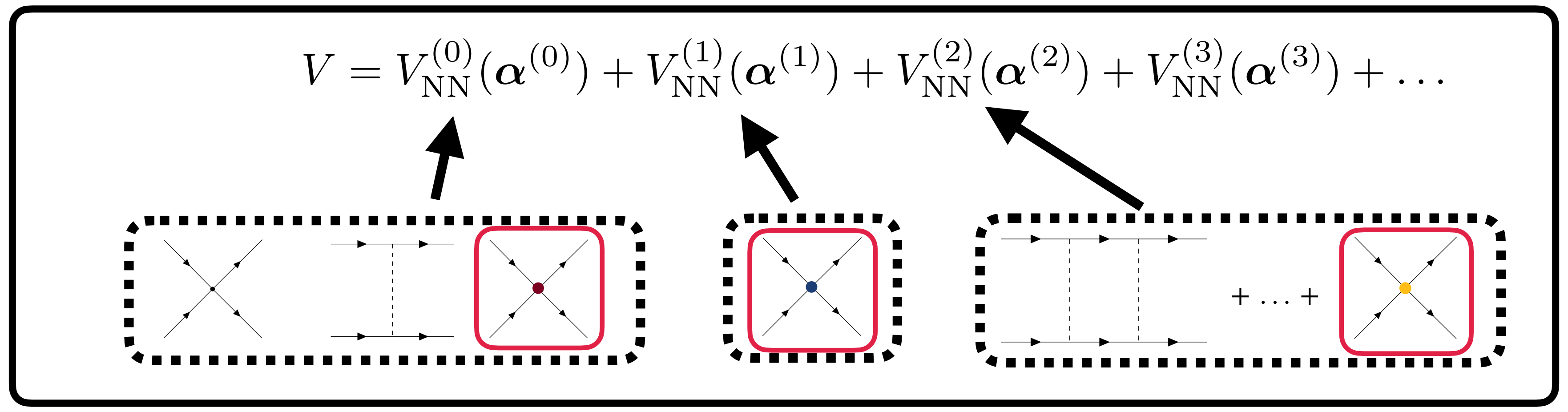
$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
Empirical (Ref. [27])	5.42	1.75	0.045	0.67	−3.94
$\Lambda = 500$ MeV					
LO	*	1.58(0)	−0.10(0)	0.89(0)	−5.5(2)
N^{2}LO	*	*	0.14(0)	0.80(0)	−4.2(2)
N^{3}LO	*	*	−0.06(0)	0.46(0)	−3.7(2)
$\Lambda = 2500$ MeV					
LO	*	1.66(0)	−0.01(0)	0.79(0)	−4.7(2)
N^{2}LO	*	*	0.09(0)	0.74(0)	−4.2(7)
N^{3}LO	*	*	0.04(0)	0.67(2)	−4.0(9)

- CIB in one-pion exchange is **not** significant in $^3S_1$.
- Cutoff independence for $\Lambda \gtrsim 750$ MeV.

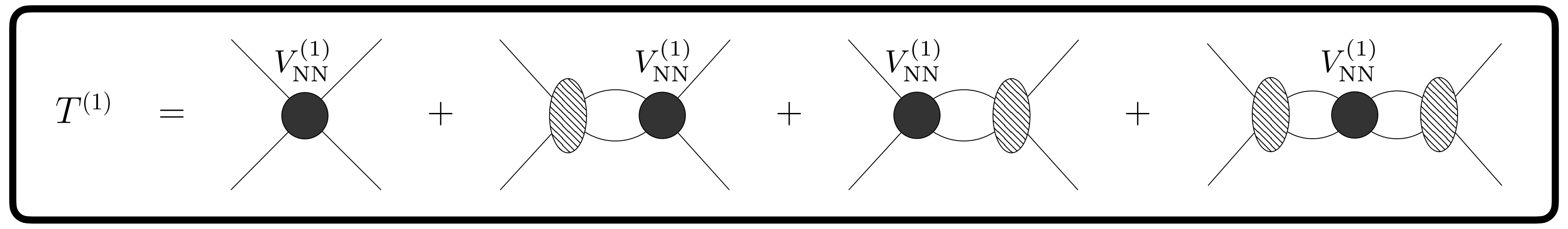
✓ Both phase shift and LETs are accurate, and improved for high cutoffs.

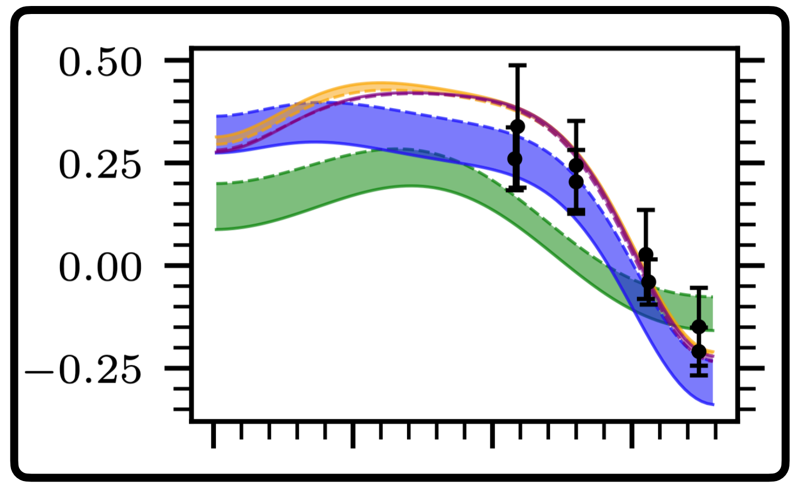
OT, Few-Body Syst. 65, 69 (2024)

Summary



- Modified PC:
  - Extra counterterms to absorb Λ - dependence.
  - Potential corrections added perturbatively beyond LO.



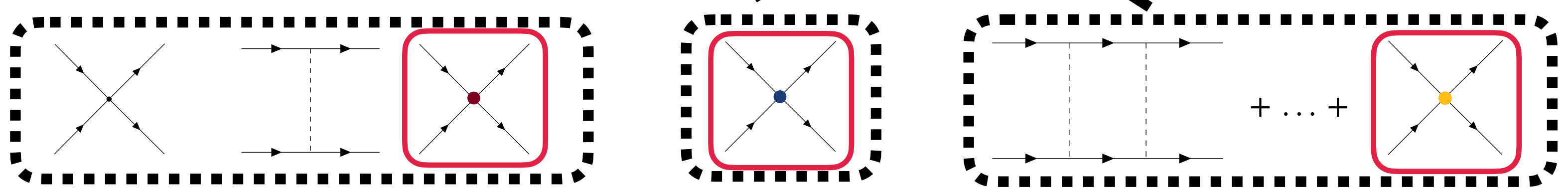


- We have found:
  - A Bayesian approach is advantageous to infer LECs at LO.
  - Accurate description of *np* scattering up to 100 MeV at N^3LO.
  - Satisfactory low-energy behavior of amplitudes.

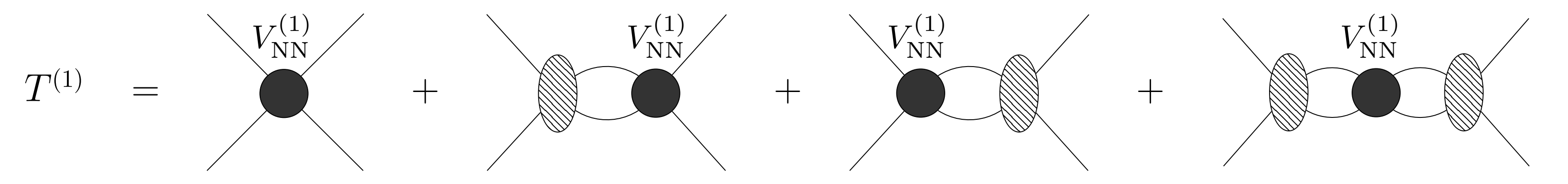
$$F(k) \equiv k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \mathcal{O}\left(k^{10}\right)$$

Summary

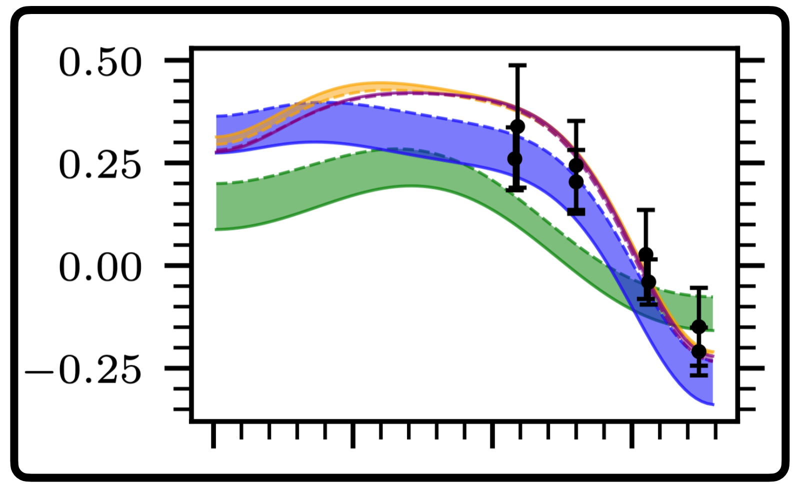
$$V = V_{\rm NN}^{(0)}(\boldsymbol{\alpha}^{(0)}) + V_{\rm NN}^{(1)}(\boldsymbol{\alpha}^{(1)}) + V_{\rm NN}^{(2)}(\boldsymbol{\alpha}^{(2)}) + V_{\rm NN}^{(3)}(\boldsymbol{\alpha}^{(3)}) + \ldots$$



- Modified PC:
  - Extra counterterms to absorb Λ - dependence.
  - Potential corrections added perturbatively beyond LO.



- We have found:
  - A Bayesian approach is advantageous to infer LECs at LO.
  - Accurate description of *np* scattering up to 100 MeV at N^3LO.
  - Satisfactory low-energy behavior of amplitudes.



$$F(k) \equiv k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \mathcal{O}\left(k^{10}\right)$$

Thank you!

Extra slides

Perturbation theory for amplitudes

$$
\begin{aligned}
T^{(1)} &= \Omega_-^\dagger V^{(1)} \Omega_+, \\
T^{(2)} &= \Omega_-^\dagger \left(V^{(2)} + V^{(1)} G_1^+ V^{(1)} \right) \Omega_+, \\
T^{(3)} &= \Omega_-^\dagger \Big(V^{(3)} + V^{(2)} G_1^+ V^{(1)} + V^{(1)} G_1^+ V^{(2)} + \\
&\quad + V^{(1)} G_1^+ V^{(1)} G_1^+ V^{(1)} \Big) \Omega_+
\end{aligned}
$$

$$
\begin{aligned}
\Omega_+ &= \mathbb{1} + G_0^+ T^{(0)} \\
\Omega_-^\dagger &= \mathbb{1} + T^{(0)} G_0^+
\end{aligned}
$$

MWPC by Long and Yang

Order	Pion contribution	Contact terms
LO	$V_{1\pi}^{(0)}$	$V_{\rm ct}^{(0)}$: $C_{^1S_0}^{(0)}$, $\begin{pmatrix} C_{^3S_1}^{(0)} & 0 \\ 0 & 0 \end{pmatrix}$, $D_{^3P_0}^{(0)} p'p$, $\begin{pmatrix} D_{^3P_2}^{(0)} p'p & 0 \\ 0 & 0 \end{pmatrix}$
NLO	-	$V_{\rm ct}^{(1)}$: $D_{^1S_0}^{(0)}(p'^2+p^2)$, $C_{^1S_0}^{(1)}$
$\mathrm{N}^2\mathrm{LO}$	$V_{2\pi}^{(2)}$	$V_{\rm ct}^{(2)}$: $E_{^1S_0}^{(0)} p'^2p^2$, $D_{^1S_0}^{(1)}(p'^2+p^2)$, $C_{^1S_0}^{(2)}$, $\begin{pmatrix} D_{^3S_1}^{(0)}(p'^2+p^2) & D_{SD}^{(0)}p^2 \\ D_{SD}^{(0)}p'^2 & 0 \end{pmatrix}$, $\begin{pmatrix} C_{^3S_1}^{(1)} & 0 \\ 0 & 0 \end{pmatrix}$, $E_{^3P_0}^{(0)} p'p(p'^2+p^2)$, $D_{^3P_0}^{(1)} p'p$, $p'p \begin{pmatrix} E_{^3P_2}^{(0)}(p'^2+p^2) & E_{PF}^{(0)}p^2 \\ E_{PF}^{(0)}p'^2 & 0 \end{pmatrix}$, $\begin{pmatrix} D_{^3P_2}^{(1)} p'p & 0 \\ 0 & 0 \end{pmatrix}$, $D_{^1P_1}^{(0)} p'p$, $D_{^3P_1}^{(0)} p'p$
$\mathrm{N}^3\mathrm{LO}$	$V_{2\pi}^{(3)}$, (include πN LECs: c_1, c_3, c_4)	$V_{\rm ct}^{(3)}$: $F_{^1S_0}^{(0)} p'^2p^2(p'^2+p^2)$, $E_{^1S_0}^{(1)} p'^2p^2$, $D_{^1S_0}^{(2)}(p'^2+p^2)$, $C_{^1S_0}^{(3)}$, $\begin{pmatrix} D_{^3S_1}^{(1)}(p'^2+p^2) & D_{SD}^{(1)}p^2 \\ D_{SD}^{(1)}p'^2 & 0 \end{pmatrix}$, $\begin{pmatrix} C_{^3S_1}^{(2)} & 0 \\ 0 & 0 \end{pmatrix}$, $E_{^3P_0}^{(1)} p'p(p'^2+p^2)$, $D_{^3P_0}^{(2)} p'p$, $p'p \begin{pmatrix} E_{^3P_2}^{(1)}(p'^2+p^2) & E_{PF}^{(1)}p^2 \\ E_{PF}^{(1)}p'^2 & 0 \end{pmatrix}$, $\begin{pmatrix} D_{^3P_2}^{(2)} p'p & 0 \\ 0 & 0 \end{pmatrix}$, $D_{^1P_1}^{(1)} p'p$, $D_{^3P_1}^{(1)} p'p$