Perturbative chiral nuclear forces

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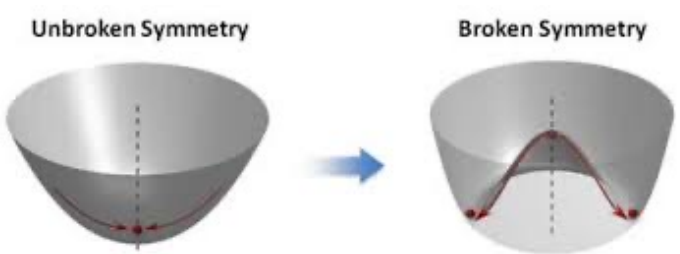
QCD → Chiral Lagrangian

External sources: v, a, s, p_s

$$\mathcal{L}_{QCD} = \bar{q}_R i\gamma_\mu D^\mu q_R + \bar{q}_L i\gamma_\mu D^\mu q_L - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}(s - i\gamma_5 p_s)q + \ldots$$

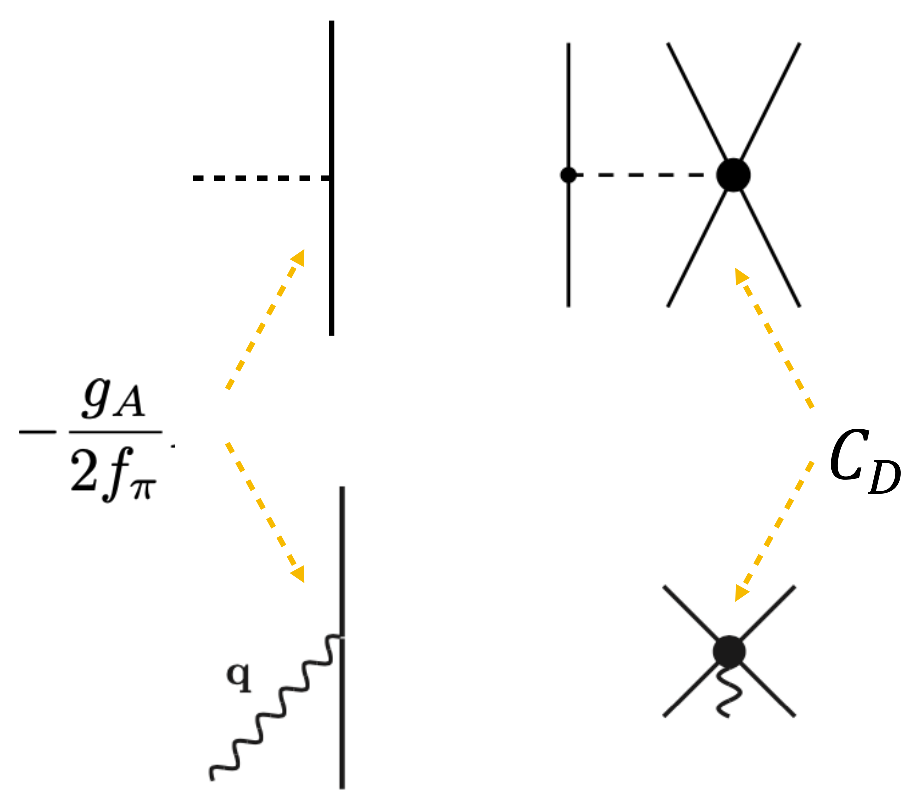
$$r_\mu = v_\mu + a_\mu = -eQA_\mu\,,$$
$$l_\mu = v_\mu - a_\mu = -eQA_\mu - \frac{g}{\sqrt{2}}(W^\dagger_\mu T_+ + h.c)\,,$$
$$s = \mathcal{M} \equiv \mathrm{diag}(m_q, m_q)\,,$$
$$p_s = 0\,。$$



nonlinear realization

CCWZ; Weinberg; ...

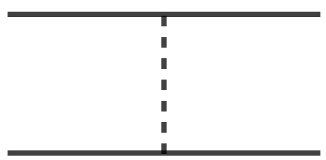
Constraints by chiral symmetry



Wavy lines: axial currents

One-pion exchange

- One-pion exchange



Tensor F

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[T(r)S_{12} + Y(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

- Tensor force dominant

$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right]$$

$$r = m_\pi^{-1}: \quad \frac{\langle {}^3S_1|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\hat{S}_{12}T(1)|{}^3D_1\rangle}{\langle {}^1S_0|\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\vec{\sigma}_1\cdot\vec{\sigma}_2Y(1)|{}^1S_0\rangle} = 14\sqrt{2}\,,$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

- Breaks spin-isospin SU(4) symmetry (Wigner)
- Has several scales, unitarity limit not manifest

NN potentials in NDA

- Contact forces doing heavy lifting

$$V_{1S0} = c_0^{1S0} + c_2^{1S0}(p^2 + p'^2) + \cdots$$

$$V_{3P0} = c_0^{3P0} pp' + \cdots$$

$\ldots$

24 contact terms in NDA up to Q^4

NDA = naive dim. analysis

van Kolck et al. '92

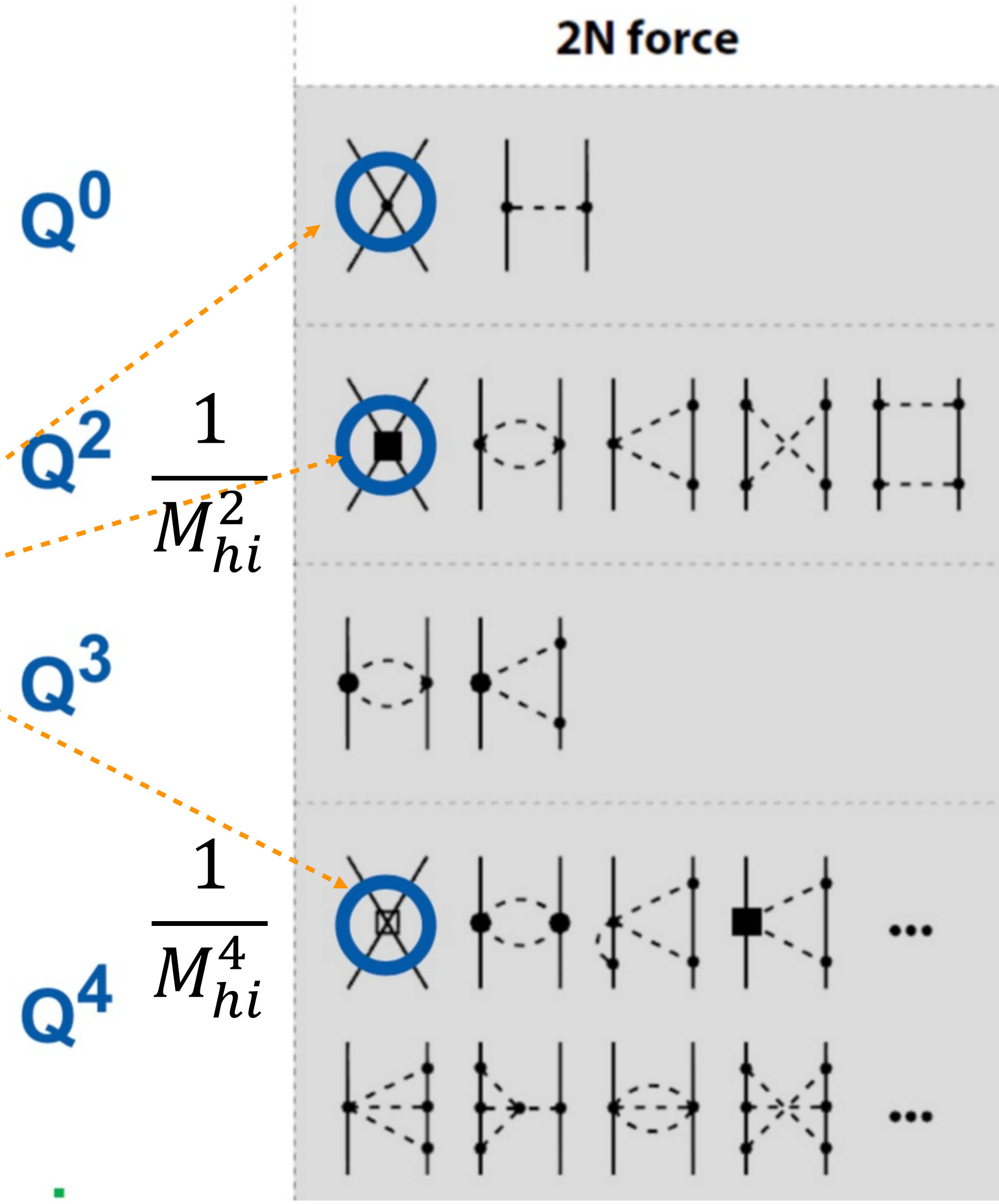
Entem & Machleidt '03

Epelbaum et al. '99

Pastore, Piarulli, et al. '09

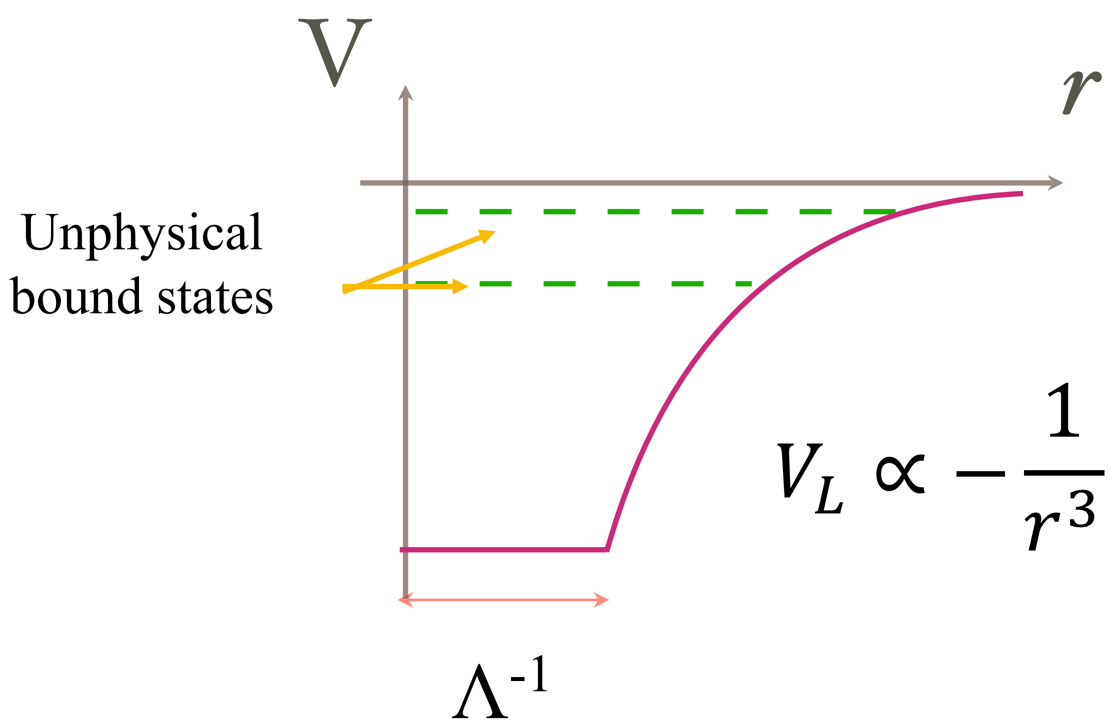
Ekstrom, et al. '13

…



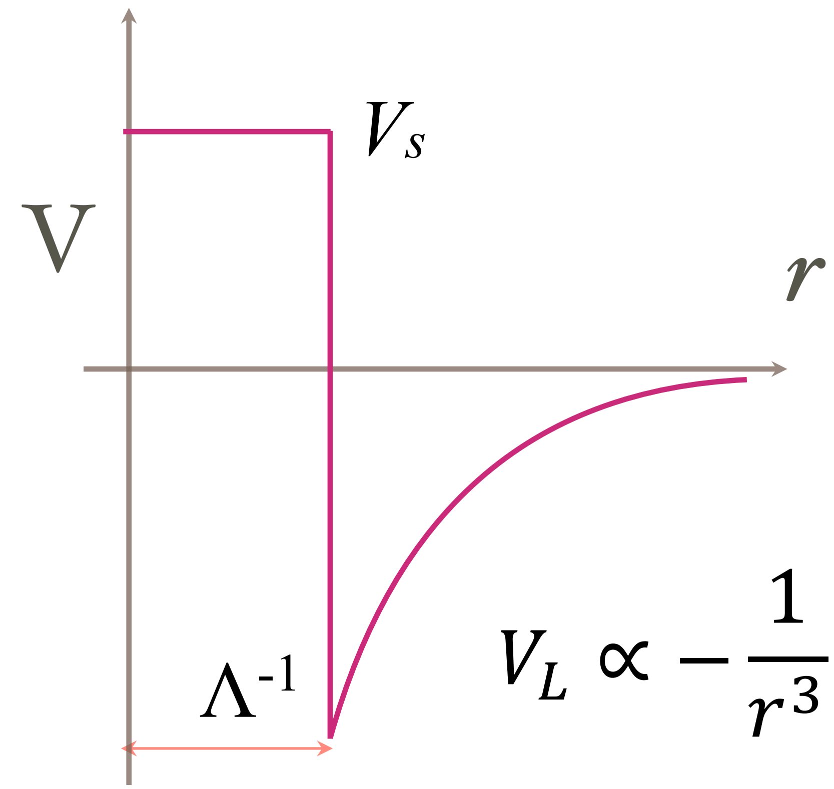
Renormalizing singular attraction

Beane et al ’01
Pavon Valderrama & Ruiz Arriola ’05, ’07
Nogga et al ’05
BwL & van Kolck ’07



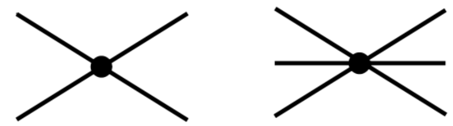
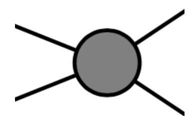
Modify PC

$\Longrightarrow$

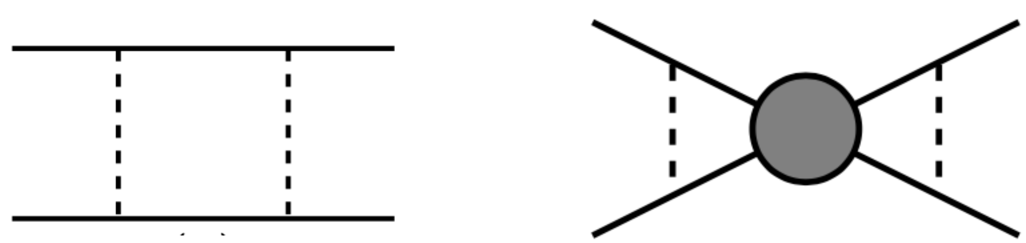


Perturbative pions?

Kaplan, Savage & Wise ’98
Fleming, Mehen & Stewart ’99
Beane, Kaplan & Vuorinen ’08

- Same LO as pionless: 
- LO of NN = bubble sums = 
- Subleading orders = distorted-wave expansion in OPE

NLO:  $+ \ldots$

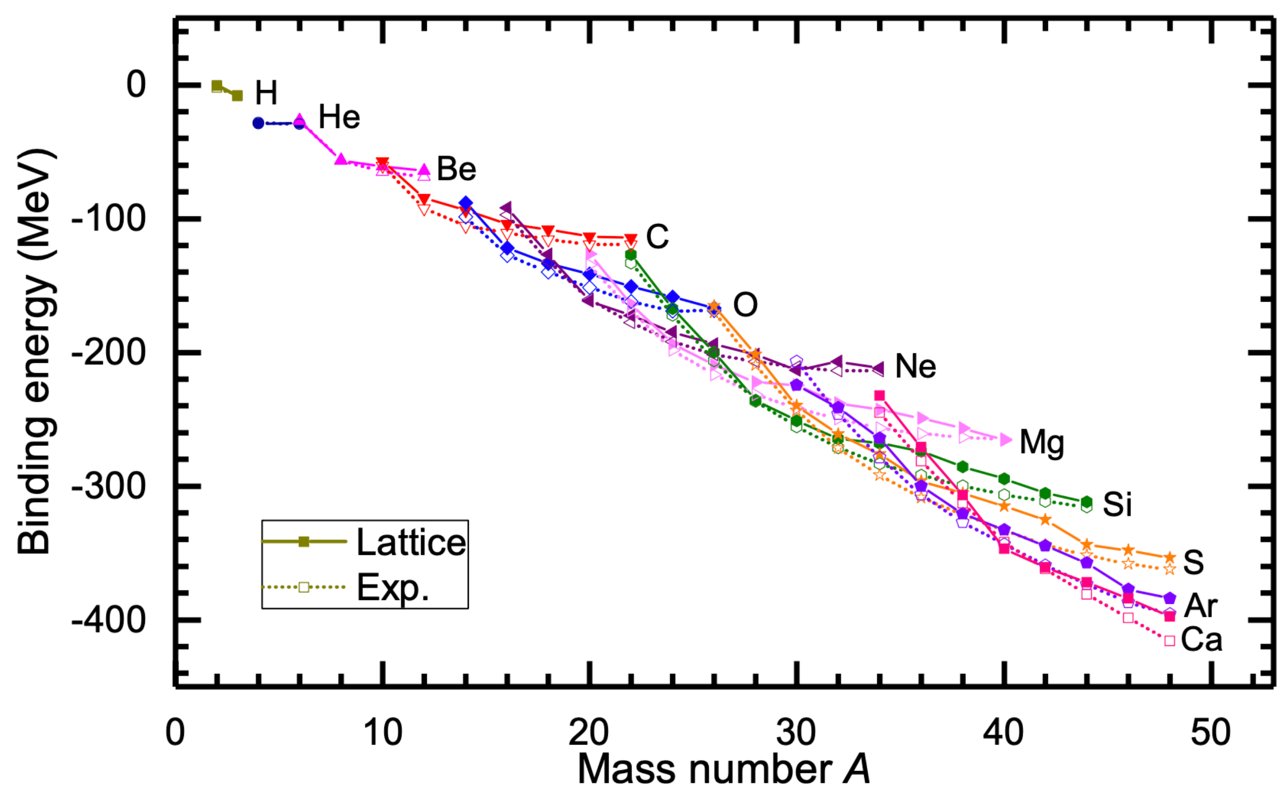
N2LO:  $+ \ldots$

Would-be benefits of a PPI

- Microscopic explanation for recent pionless-like (short-ranged) structure calculations:

  Koenig et al. ’16
  Lyu BN et al. ’18
  Gattobigio et al. ‘19
  …

Lyu et al. ’18, Lattice EFT calaboration

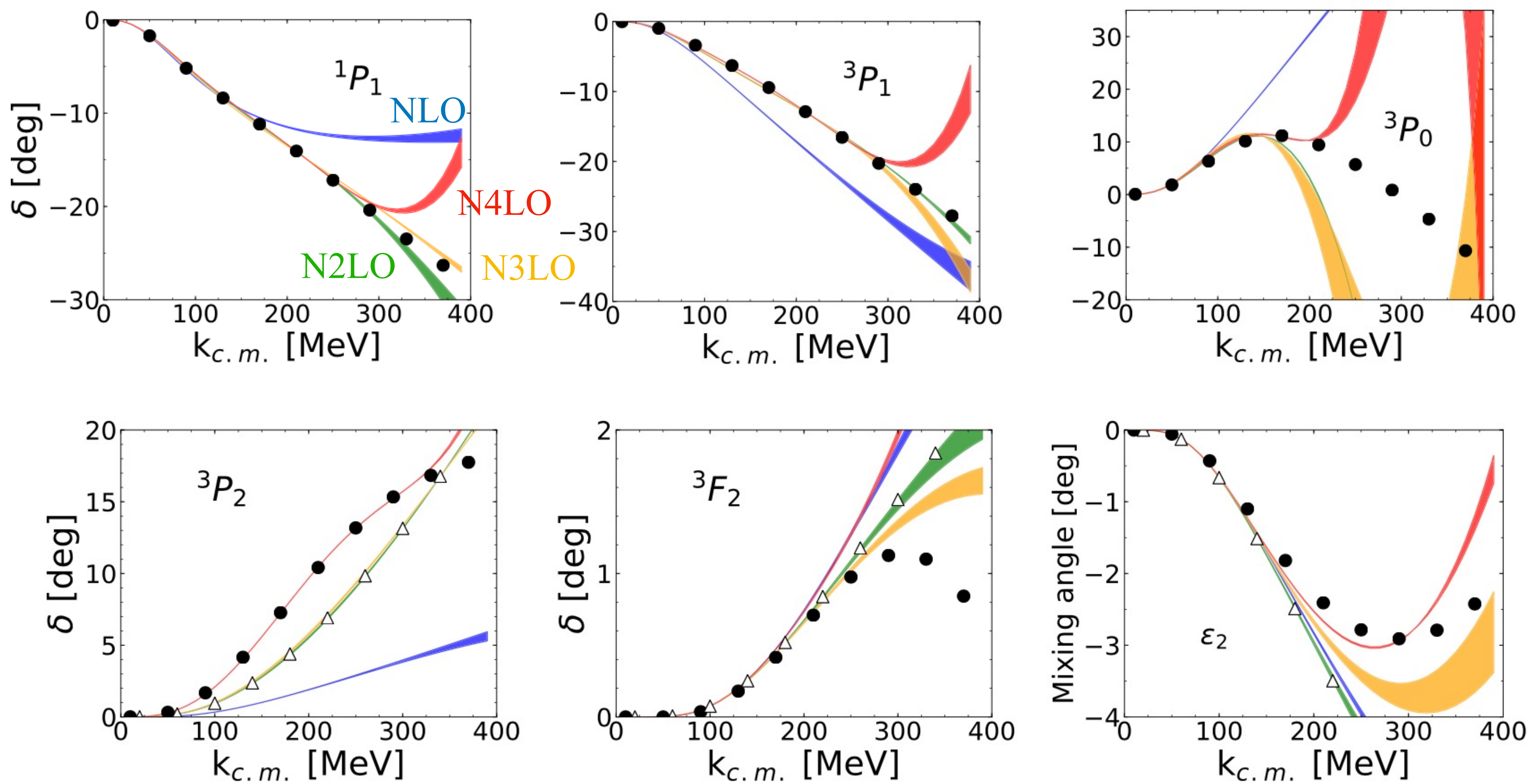


- LO forces can be made SU(4) inv, unitarity limit, … accidental symmetries are great
- Implying 3NF is relevant in a larger kinematic domain

Where KSW works

- OK for $k < \Delta \simeq 290$ MeV for $l > 0$ except 3P0 Wu & Long '18 Kaplan '19
- TPEs included in N3LO & N4LO

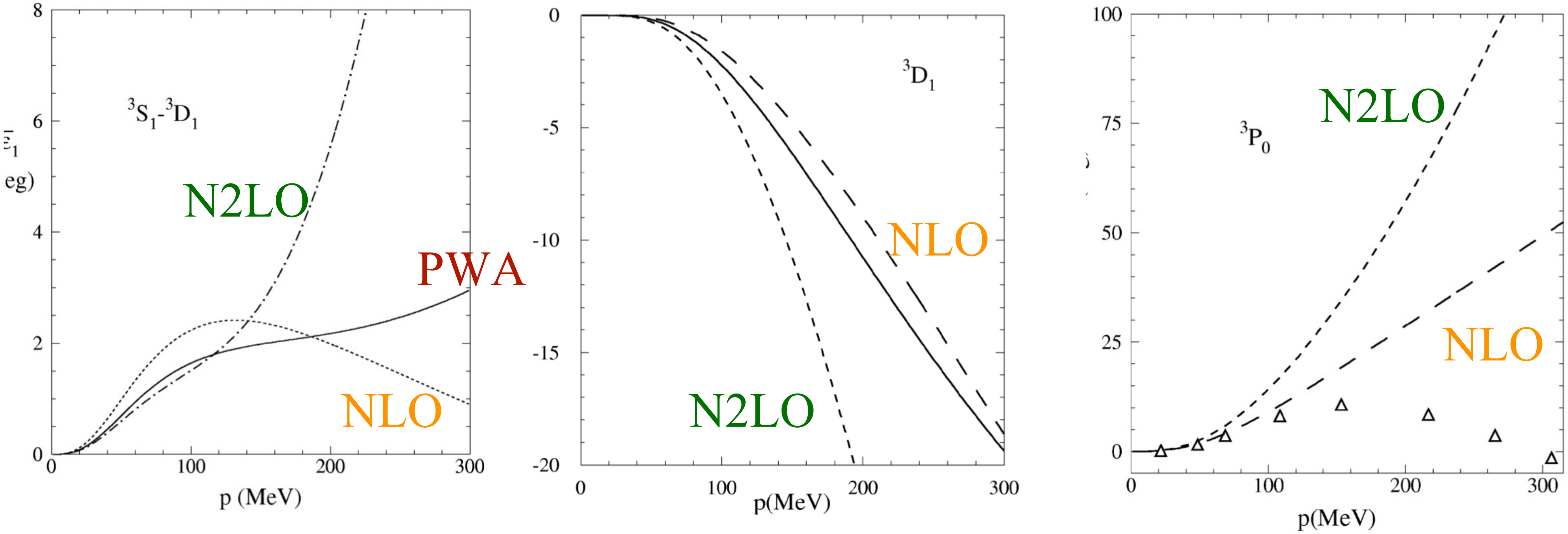
Wu & Long '18



KSW

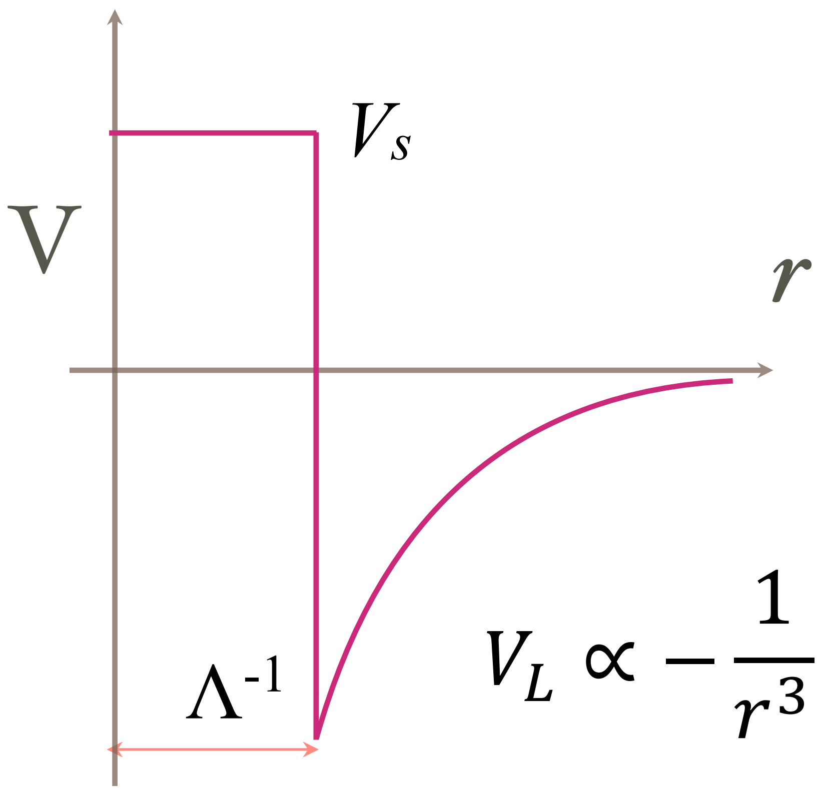
Fleming, Mehen & Stewart ’99

- Convergence not better than pionless, esp. for higher waves



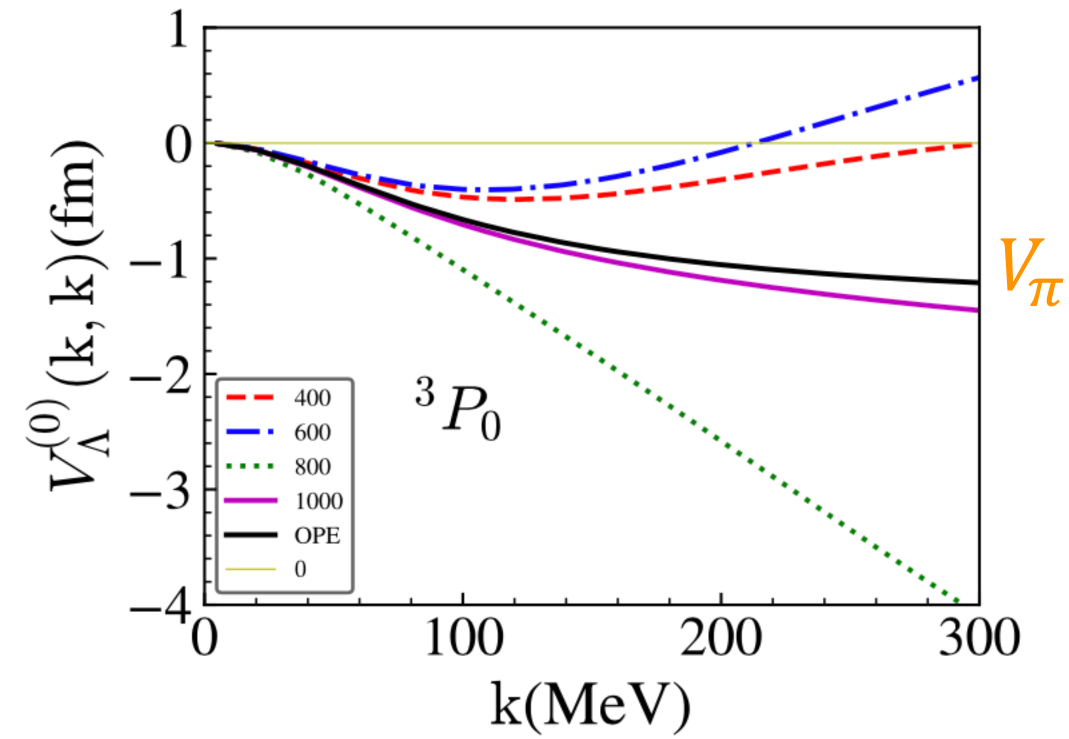
Pushing pert-pion interactions

- Re-organizing higher contact terms



Nonpert ren of OPE

$V_\pi + C_{3P0}\, p' p$



- From nonpert renormalization: V_s always repulsive (towards lower Λ)

- Using V_s to moderate the OPE tensor force

PPI in 3P0

Peng, Lyu & BwL ’20

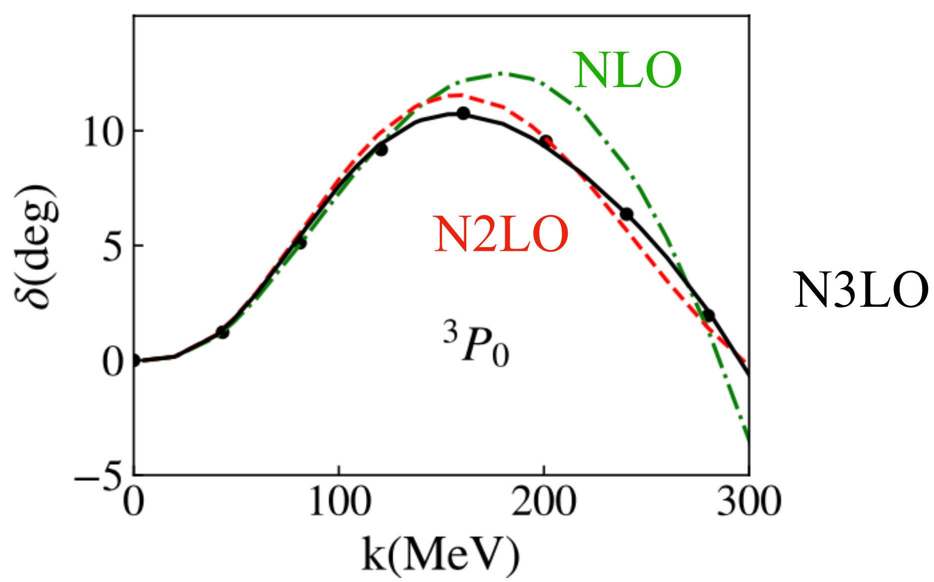
- Expansion in $V_\pi + C_{3P0}p'p$ (Born approximation)

$$V^{(1)}(^3P_0) = V_\pi + C_2^{^3P_0}p'p$$

- Higher-order contacts are identified when needed for renormalization at second, third-order Born approximation

	LO	NLO	N^{2}LO	N^{3}LO
π	OPE			TPE0
$^3P_0$		C_2	C_4	C_6

Updated fit $\Lambda = 800$ MeV



PPI: 3S1-3D1

Lyu, Zuo, Peng, Koenig & BwL
(in preparation)

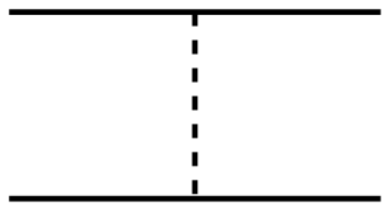
- Mixing angle vanishing at unitarity & chiral limits despite strong OPE tensor force

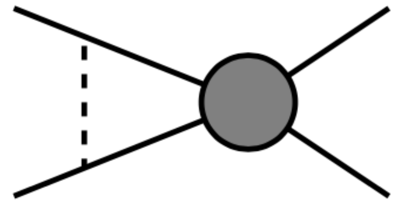
$$\epsilon_1 = \frac{m_N k}{4\pi} \frac{k}{(a_t^{-2} + k^2)^{\frac{1}{2}}}$$

$$\times \left[\frac{1}{a_t k} \langle k, {}^3S_1 | V_\pi | k, {}^3D_1 \rangle - \frac{g_A^2}{\sqrt{2} f_\pi^2} \Pi(0, \frac{m_\pi}{k}) \right]$$

$$\Pi(kr, \frac{m_\pi}{k}) \equiv m_\pi^3 \int_r^\infty dr' r'^2 n_0(kr') T(m_\pi r') j_2(kr')$$

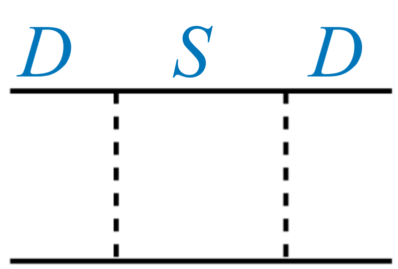
$$m_\pi \to 0 : \quad \int_0^\infty dr r^2 n_0(kr) \frac{1}{r^3} j_2(kr) = 0$$

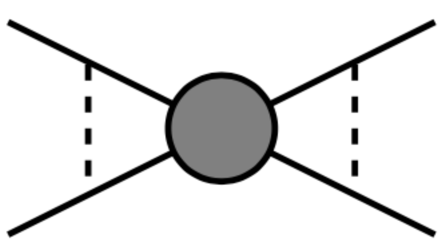


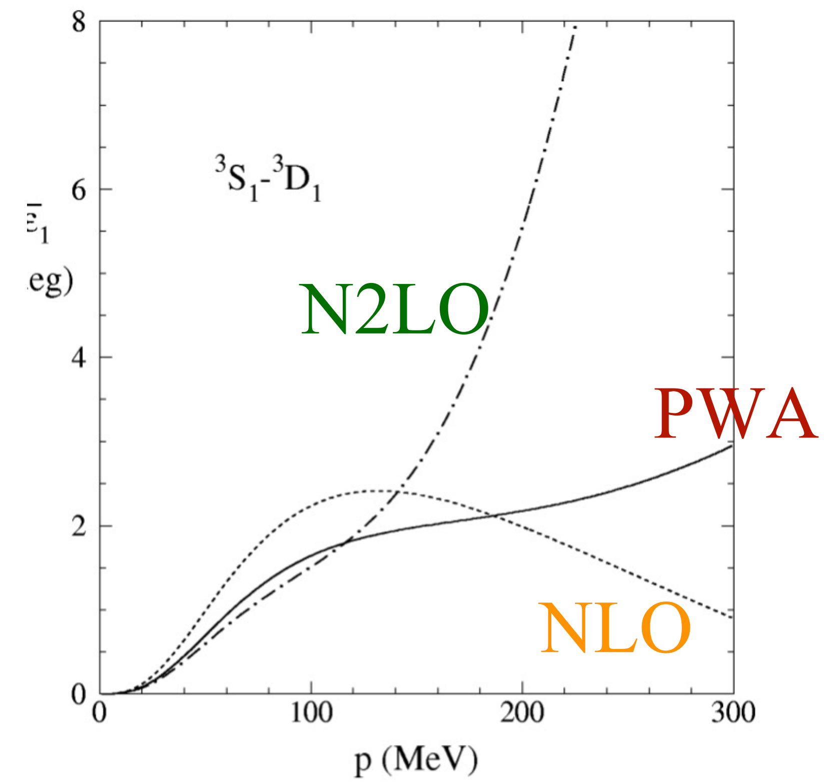


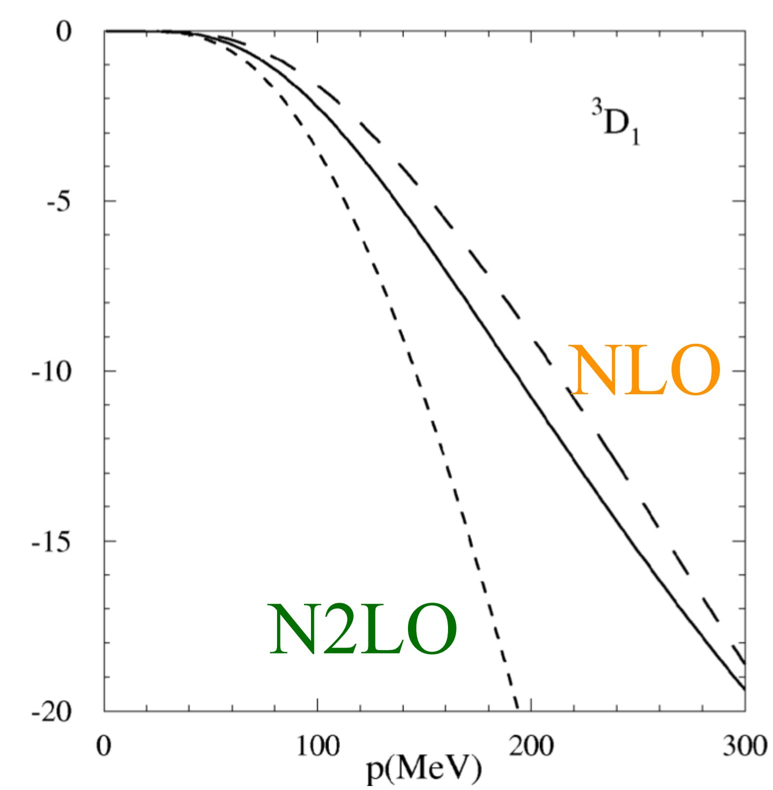
PPI: 3S1-3D1

- Not enough though
- Only works for on-shell kinematics
- N2LO 3D1, mixing angle are large





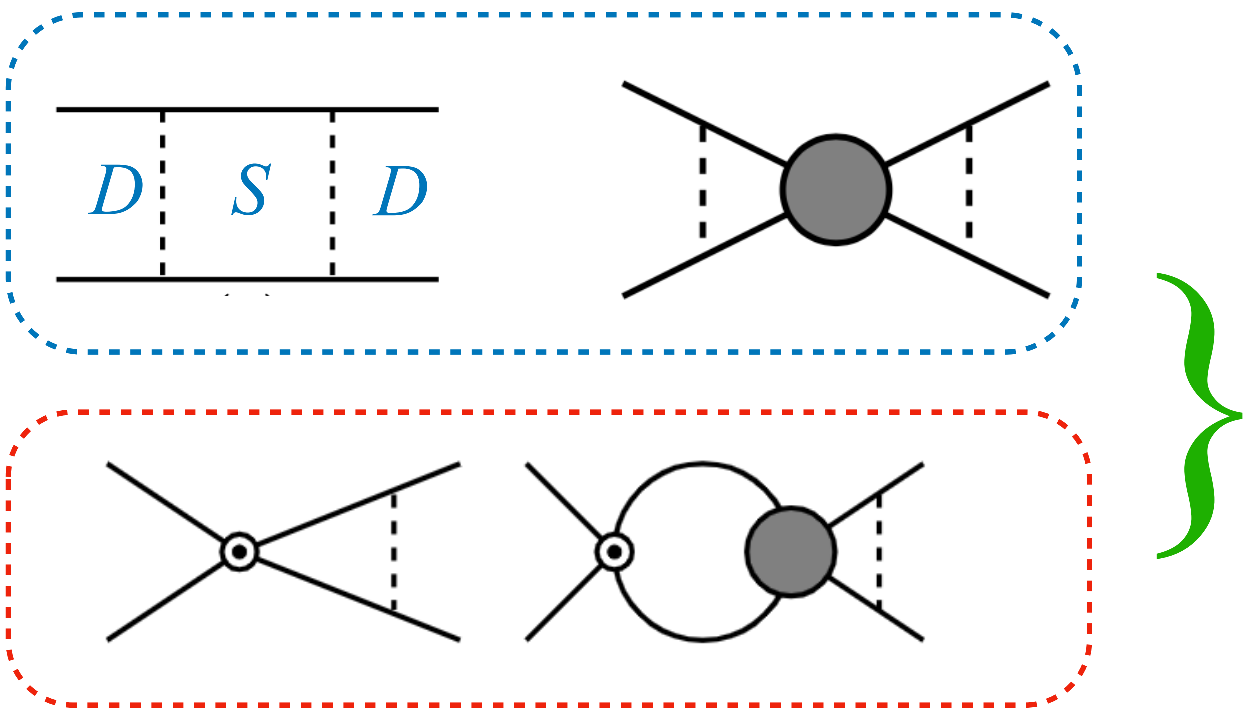




PPI: 3S1-3D1

- Expansion in (V_π + SD mixing contact)

$$V^{(1)}({}^3S_1 - {}^3D_1) = V_\pi$$
$$+ \begin{pmatrix} C_0^{(1)} + C_2^{{}^3S_1}(p'^2 + p^2) & \boxed{-C_2^{SD}}\,p^2 \\ \boxed{-C_2^{SD}}\,p'^2 & 0 \end{pmatrix}$$



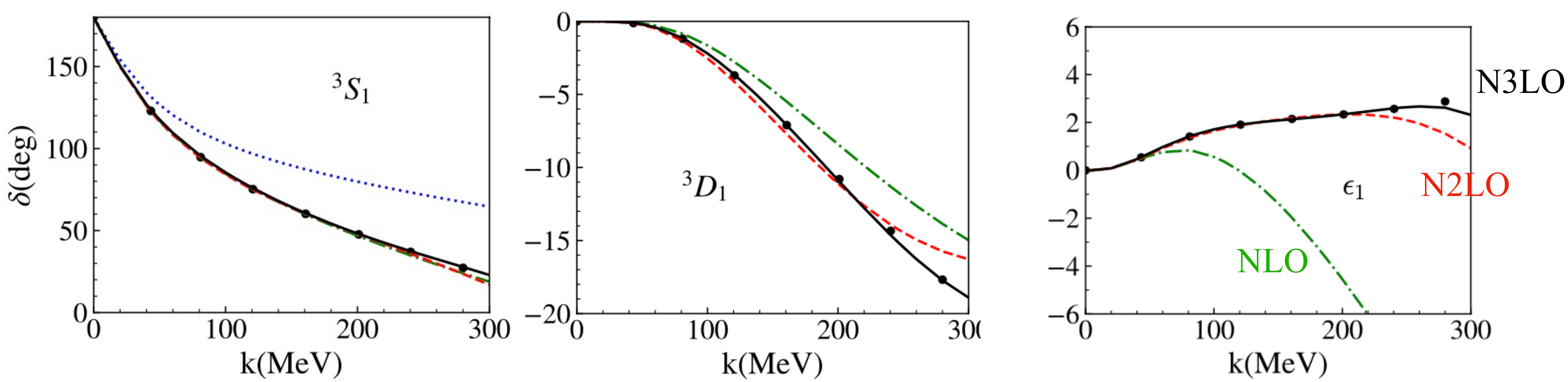
Cancellation

PPI: 3S1-3D1

- PC of high-order contacts by counting divergence

	LO	NLO	$\mathrm{N^2LO}$	$\mathrm{N^3LO}$
π		OPE		TPE0
$^3S_1$ $^1S_0$	C_0	C_2	C_4	C_6
SD		C_2	C_4	C_6
$^3D_1$				C_4

$\Lambda = 800$ MeV



Preliminary

Lyu, Zuo, Peng, Koenig & BwL

Summary

- A new perturbative-pion scheme with re-organized contacts
- Simplify LO nuclear interactions to emphasize accidental symmetries like SU(4)-Wigner, unitarity limits, etc.
- W/ larger validity range and 3NF at LO, implies 3NF important even for Q > m_π