

SHORT RANGE CORRELATIONS IN LIGHT NUCLEI

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In collaboration with:

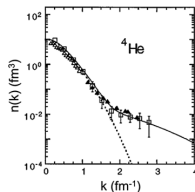
Laura Elisa Marcucci (UniPi & INFN-Pisa)

Michele Viviani (UniPi)

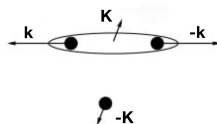
Short-Range Correlations (SRCs) in Nuclei

Mean field approximation: shell model

Two-nucleon short-range correlations



[O. Hen et al., Rev. Mod. Phys. **89**, 045002 (2017)]



large k and small K
compared to k_F
 $\Rightarrow$ SRC

Two-body Momentum Distributions (2bmd) - Density Functions (2bdf)

$$n^{N_1 N_2}(k) = \int d\hat{\mathbf{k}} \int d\mathbf{K} \psi^\dagger(\mathbf{k}, \mathbf{K}) P^{N_1 N_2} \psi(\mathbf{k}, \mathbf{K})$$

$$n^{ST}(k) = \int d\hat{\mathbf{k}} \int d\mathbf{K} \psi^\dagger(\mathbf{k}, \mathbf{K}) P^{ST} \psi(\mathbf{k}, \mathbf{K})$$

$$\rho^{N_1 N_2}(r) = \int d\hat{\mathbf{r}} \int d\mathbf{R} \psi^\dagger(\mathbf{r}, \mathbf{R}) P^{N_1 N_2} \psi(\mathbf{r}, \mathbf{R})$$

$$\rho^{ST}(r) = \int d\hat{\mathbf{r}} \int d\mathbf{R} \psi^\dagger(\mathbf{r}, \mathbf{R}) P^{ST} \psi(\mathbf{r}, \mathbf{R})$$

$P^{N_1 N_2}$: projector on the nucleon pair $N_1 N_2$

P^{ST} : projector on the spin-isospin states ST

Generalized Contact Formalism (GCF)

$$n_A^{N_1 N_2, \alpha}(k) \longrightarrow \tilde{C}_{A, N_1 N_2}^{\alpha} \times |\tilde{\varphi}_{N_1 N_2}^{\alpha}(k)|^2$$

$$\rho_A^{N_1 N_2, \alpha}(r) \longrightarrow C_{A, N_1 N_2}^{\alpha} \times |\varphi_{N_1 N_2}^{\alpha}(r)|^2$$

$\varphi_{N_1 N_2}^{\alpha}(r)$ with $\alpha = \ell, s, j \rightarrow$ universal function

- ▶ does NOT depend on the nucleus
- ▶ does depend on the interaction
- ▶ is the solution of the zero-energy two-body problem
- ▶ $\tilde{\varphi}_{N_1 N_2}^{\alpha}(k) \rightarrow$ FT of $[\varphi_{N_1 N_2}^{\alpha}(r)e^{-(r/c)^2}]$ ($c \sim 50\text{fm}$)

Nucleus-dependent Contact Coefficients

In k-space, for $k \rightarrow \infty$

$$\tilde{C}_{pp/nn}^{s=0} = n^{pp/nn}(k) / |\tilde{\varphi}_{pp/nn}^{s=0}(k)|^2$$

$$\tilde{C}_{pn}^{s=1} = n^{ST=10}(k) / |\tilde{\varphi}_{pn}^{s=1}(k)|^2$$

$$\tilde{C}_{pn}^{s=0} = [n^{pn}(k) - \tilde{C}_{pn}^{s=1} |\tilde{\varphi}_{pn}^{s=1}(k)|^2] / |\tilde{\varphi}_{pn}^{s=0}(k)|^2$$

In r-space, for $r \rightarrow 0$

$$C_{pp/nn}^{s=0} = \rho^{pp/nn}(r) / |\varphi_{pp/nn}^{s=0}(r)|^2$$

$$C_{pn}^{s=1} = \rho^{ST=10}(r) / |\varphi_{pn}^{s=1}(r)|^2$$

$$C_{pn}^{s=0} = [\rho^{pn}(r) - C_{pn}^{s=1} |\varphi_{pn}^{s=1}(r)|^2] / |\varphi_{pn}^{s=0}(r)|^2$$

[R. Weiss et al., Phys. Lett. B **780**, 211 (2018)]

(nn and pp cases are the same: without Coulomb)

Contact Coefficients for $A=2,3$ with Hyperspherical Harmonics (HH) method

HH method: ab-initio accurate method, both in r - and in k -space $\rightarrow$ any potential model for $A=2,3,4,6$ bound states

Local interactions:

- ▶ AV18/UIX (phenomenological) $\rightarrow$ to compare with [R. Cruz-Torres et al., Nat. Phys. **17**, 306 (2021)] [NP17]
- ▶ Norfolk chiral interactions: NV2+3/Ia*, NV2+3/Ib* and NV2+3/IIa*, NV2+3/IIb* [M. Piarulli et al., Phys. Rev. C **91**, 024003 (2015)] [A. Baroni et al., Phys. Rev. C **98**, 044003 (2018)]
I $\rightarrow$ fit for $E_{lab} \leq 125$ MeV;
II $\rightarrow$ fit for $E_{lab} \leq 200$ MeV;
[(R_s, R_L)=(0.8,1.2)fm or (0.7,1.0)fm for a/b]

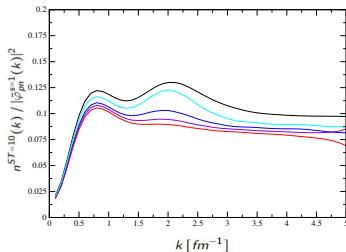
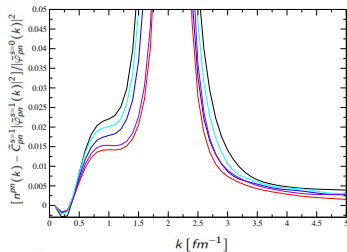
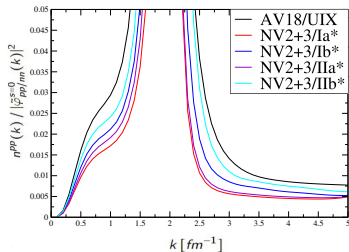
Non-local interactions:

- ▶ Chiral N2LO-N3LO-N4LO ($\Lambda = 450, 500, 550$ MeV) with N2LO TNI [D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003)] [A. Gnech et al., Phys. Rev. C **106**, 044001 (2022)]

In all cases $\rightarrow c_D - c_E$ fitted to $B(A=3)$ and Gamow-Teller matrix element in tritium β -decay

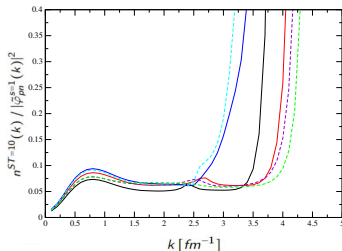
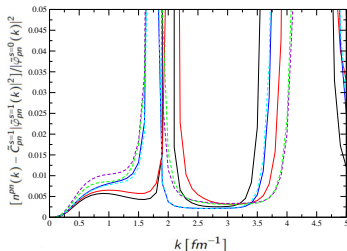
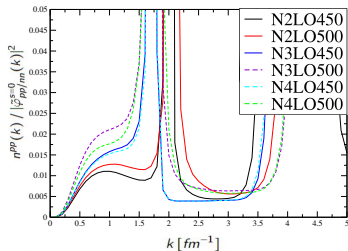
Contact Coefficients in GCF: ^3He with local interactions

Two-body momentum distributions



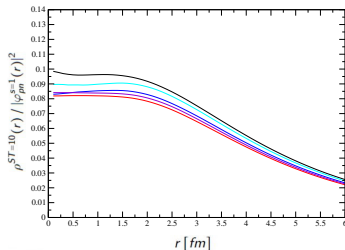
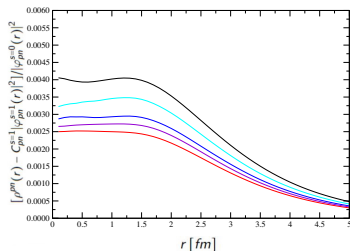
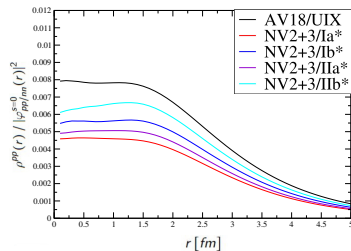
Contact Coefficients in GCF: ^3He with non-local interactions

Two-body momentum distributions



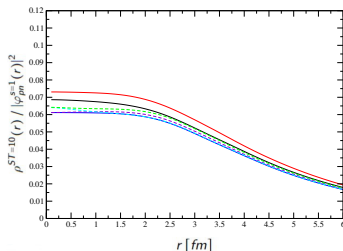
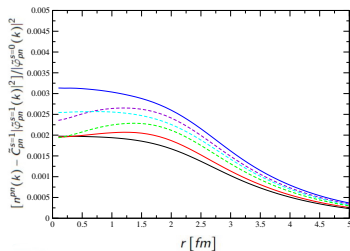
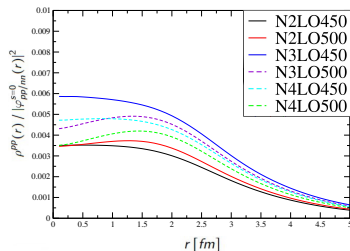
Contact Coefficients in GCF: ^3He with local interactions

Two-body density functions

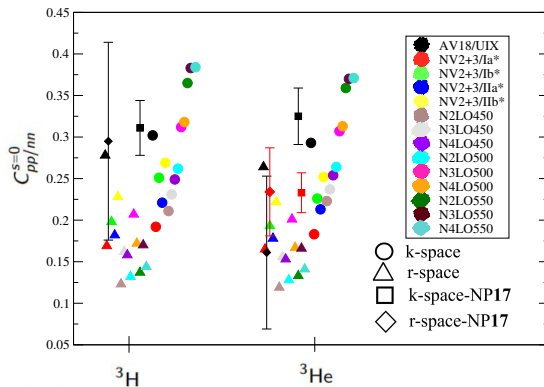


Contact Coefficients in GCF: ^3He with non-local interactions

Two-body density functions

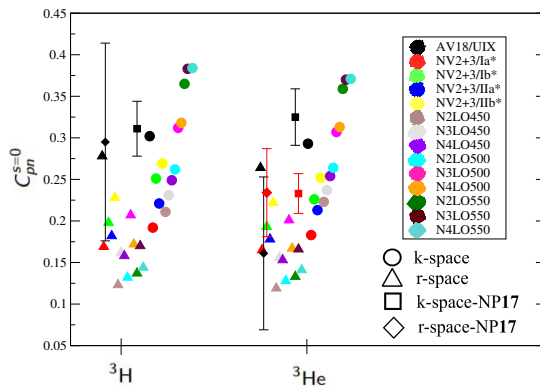


Contact Coefficients



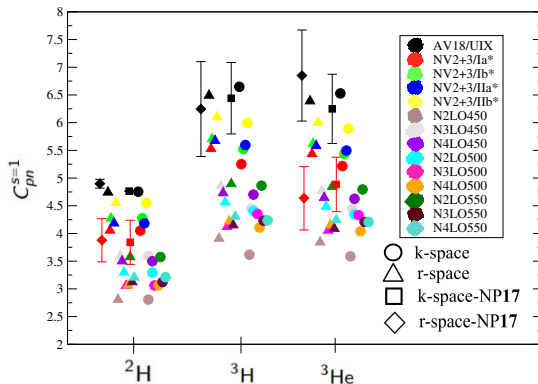
- ▶ Reasonable agreement between r - and k -space coefficients for local potentials
- ▶ Some differences between r - and k -space coefficients for non-local potentials
- ▶ → also in local chiral interaction (N2LO) in [NP17]

Contact Coefficients



- Same comments as for $C_{pp/nn}^{s=0}$

Contact Coefficients



- ▶ Reasonable agreement between r - and k -space coefficients for all the potentials
- ▶ SRCs are mainly in np $s = 1$ pairs
- ▶ the single Contact Coefficients are quite model-dependent

Further test:

- ▶ Until now, all calculations assume $\ell = 0 \Rightarrow \begin{matrix} S = 0 \rightarrow T = 1 \\ S = 1 \rightarrow T = 0 \end{matrix}$
- ▶ → Beyond this assumption:

In k-space, for $k \rightarrow \infty$

$$\tilde{C}_{pp/nn}^{s=0} = n_{pp/nn}^{s=0}(k) / |\tilde{\varphi}_{pp/nn}^{s=0}(k)|^2$$

$$\tilde{C}_{pn}^{s=1} = n_{pn}^{s=1}(k) / |\tilde{\varphi}_{pn}^{s=1}(k)|^2$$

$$\tilde{C}_{pn}^{s=0} = n_{pn}^{s=0}(k) / |\tilde{\varphi}_{pn}^{s=0}(k)|^2$$

$$\tilde{C}_{pp/nn}^{s=0} = n^{pp/nn}(k) / |\tilde{\varphi}_{pp/nn}^{s=0}(k)|^2$$

$$\tilde{C}_{pn}^{s=1} = n^{ST=10}(k) / |\tilde{\varphi}_{pn}^{s=1}(k)|^2$$

$$\tilde{C}_{pn}^{s=0} = [n^{pn}(k) - \tilde{C}_{pn}^{s=1} |\tilde{\varphi}_{pn}^{s=1}(k)|^2] / |\tilde{\varphi}_{pn}^{s=0}(k)|^2$$

In r-space, for $r \rightarrow 0$

$$C_{pp/nn}^{s=0} = \rho_{pp/nn}^{s=0}(r) / |\varphi_{pp/nn}^{s=0}(r)|^2$$

$$C_{pn}^{s=1} = \rho_{pn}^{s=1}(r) / |\varphi_{pn}^{s=1}(r)|^2$$

$$C_{pn}^{s=0} = \rho_{pn}^{s=0}(r) / |\varphi_{pn}^{s=0}(r)|^2$$

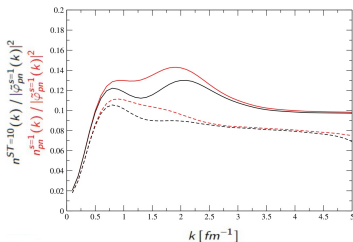
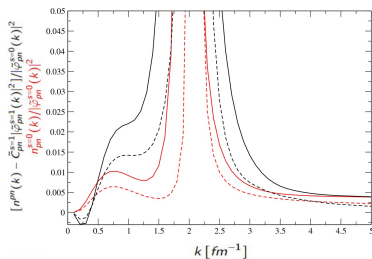
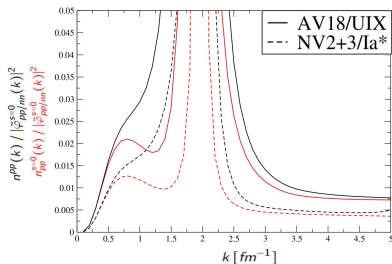
$$C_{pp/nn}^{s=0} = \rho^{pp/nn}(r) / |\varphi_{pp/nn}^{s=0}(r)|^2$$

$$C_{pn}^{s=1} = \rho^{ST=10}(r) / |\varphi_{pn}^{s=1}(r)|^2$$

$$C_{pn}^{s=0} = [\rho^{pn}(r) - C_{pn}^{s=1} |\varphi_{pn}^{s=1}(r)|^2] / |\varphi_{pn}^{s=0}(r)|^2$$

Contact Coefficients in GCF: ^3He with local interactions

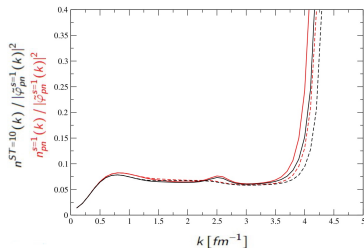
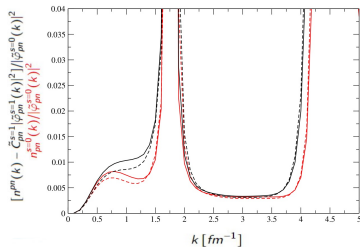
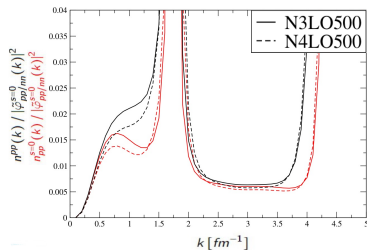
Two-body momentum distributions



More stable plateau without the assumption $\ell = 0$

Contact Coefficients in GCF: ^3He with non-local interactions

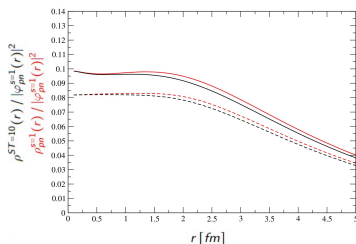
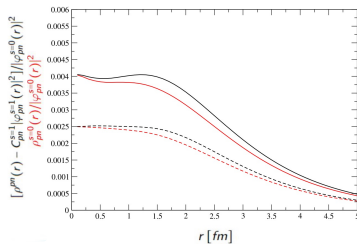
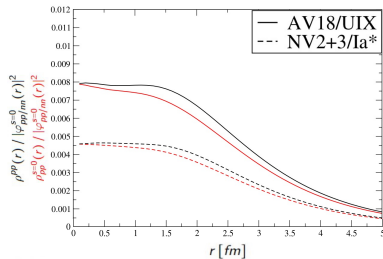
Two-body momentum distributions



More stable plateau without the assumption $\ell = 0$

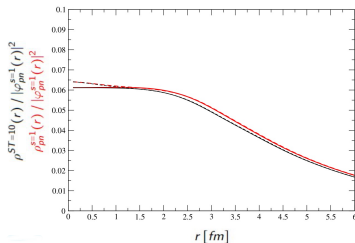
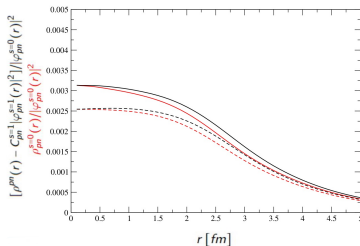
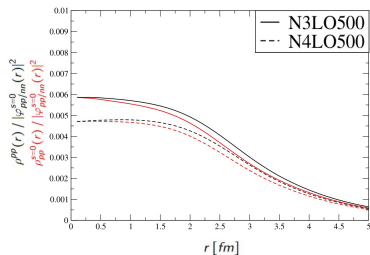
Contact Coefficients in GCF: ^3He with local interactions

Two-body density functions



Contact Coefficients in GCF: ^3He with non-local interactions

Two-body density functions

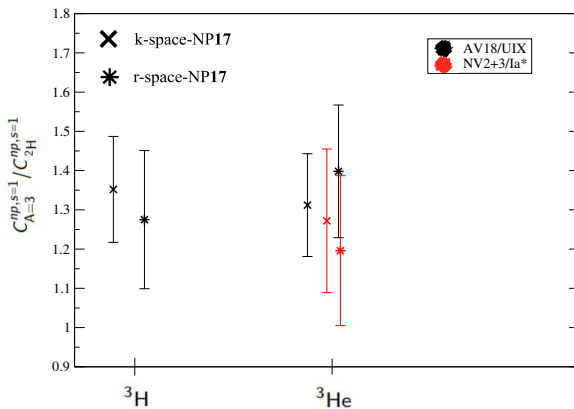


→ s-wave gives the dominant contribution as expected

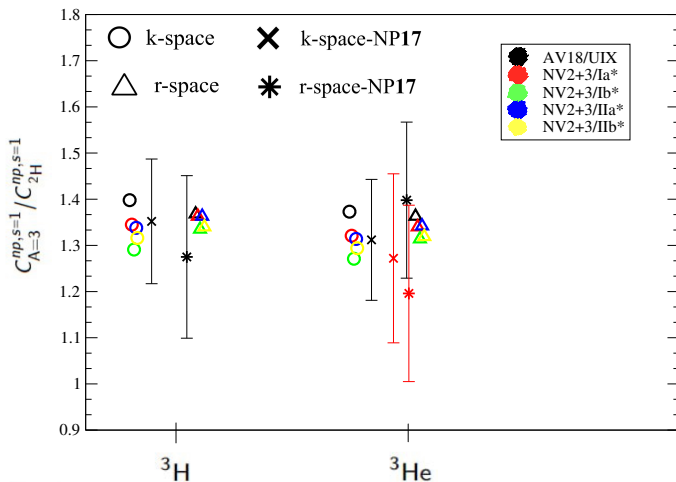
Ratios:

$$\begin{aligned} \rho_A^{N_1 N_2, \alpha}(r) / \rho_{A_0}^{N_1 N_2, \alpha}(r) &\xrightarrow{r \rightarrow 0} C_A^{N_1 N_2, \alpha} / C_{A_0}^{N_1 N_2, \alpha} \\ n_A^{N_1 N_2, \alpha}(k) / n_{A_0}^{N_1 N_2, \alpha}(k) &\xrightarrow{k \rightarrow \infty} \tilde{C}_A^{N_1 N_2, \alpha} / \tilde{C}_{A_0}^{N_1 N_2, \alpha} \quad [\text{NP17}] \end{aligned}$$

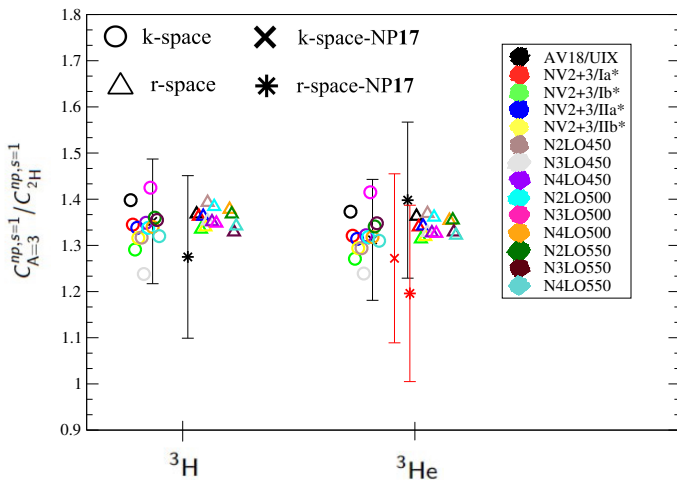
$A_0 = \text{reference nucleus} \Rightarrow A_0 = {}^2\text{H} \text{ for } s = 1$
 $A_0 = {}^4\text{He} \text{ for } s = 0 \rightarrow \text{work in progress}$



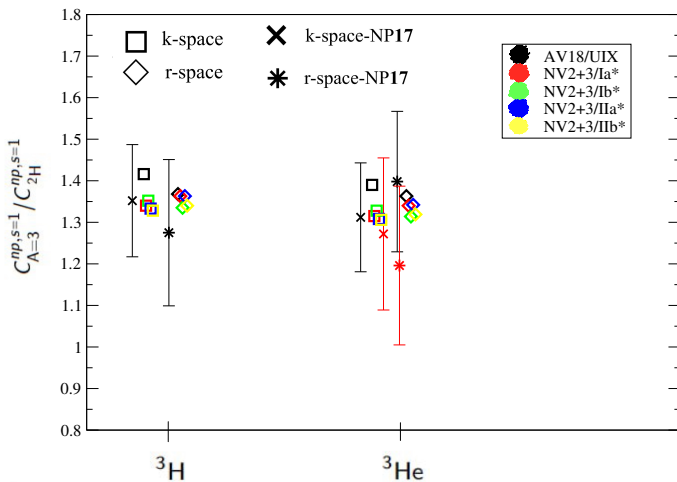
$C_{A=3}^{np,s=1} / C_{2H}^{np,s=1}$ with the assumption $\ell=0$ for local potential



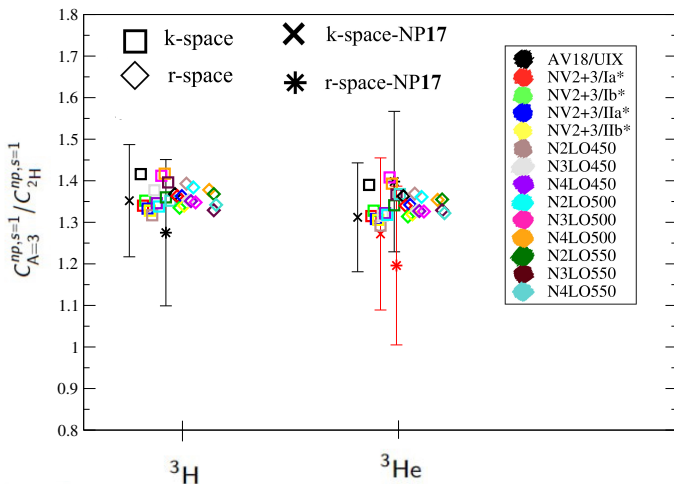
$C_{A=3}^{np,s=1} / C_{2H}^{np,s=1}$ with the assumption $\ell=0$



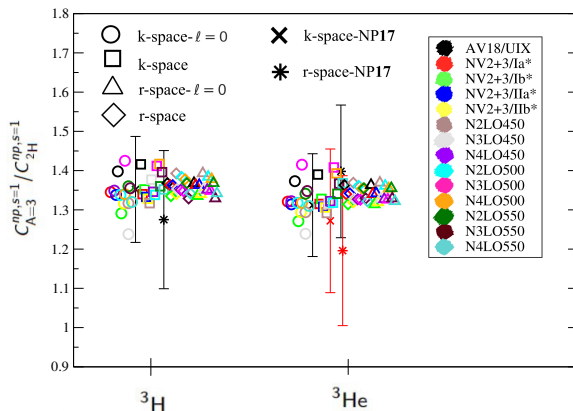
$C_{A=3}^{np,s=1}/C_{2H}^{np,s=1}$ without the assumption $\ell=0$ for local potential



$C_{A=3}^{np,s=1} / C_{2H}^{np,s=1}$ without the assumption $\ell=0$



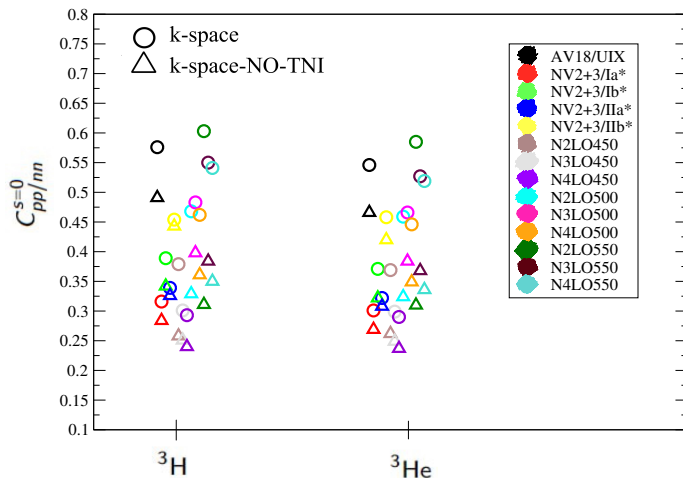
$$C_{A=3}^{np,s=1} / C_{2H}^{np,s=1}$$



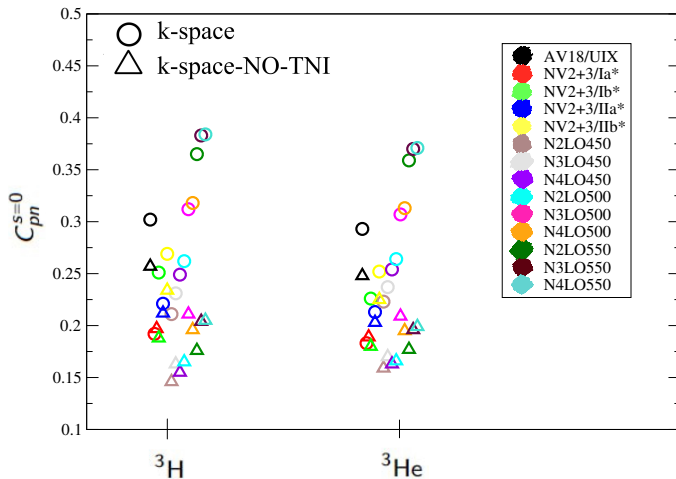
- ▶ Independent of k/r scheme $\rightarrow$ for small r /large k
- ▶ Model independent
- ▶ Nucleus independent
- ▶ Comparable with and without the assumption $\ell = 0$

Another check: $C_{pp/nn}^{s=0}$

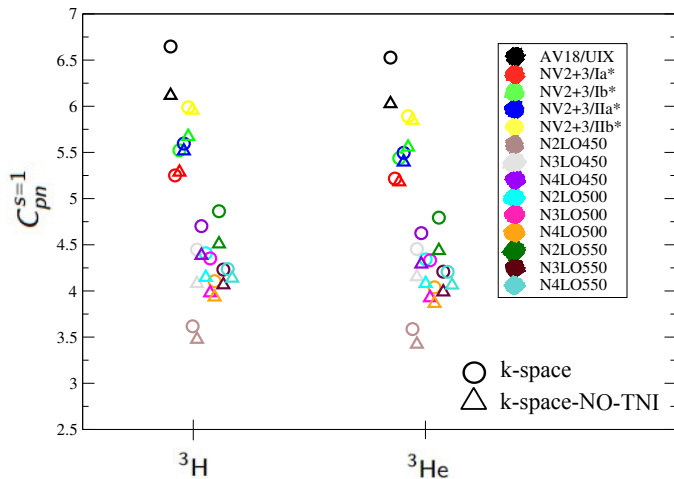
Calculation without three-nucleon interactions (TNI) in k -space $\rightarrow$ differences of about 10%



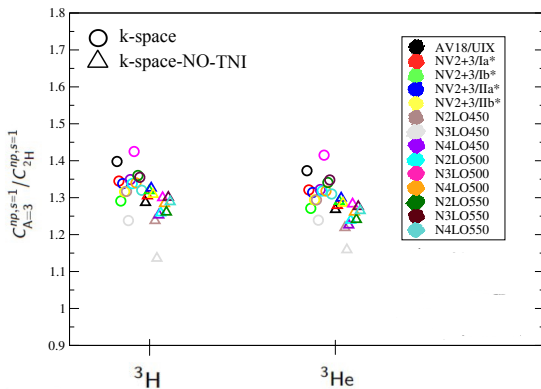
Another check: $C_{pn}^{s=0}$



Another check: $C_{pn}^{s=1}$



$$C_{A=3}^{np,s=1} / C_{2H}^{np,s=1}$$



- ▶ Without TNI → results shifted toward lower values
- ▶ TNI are needed

Conclusions and Outlooks

Conclusions:

- ▶ HH Method $\rightarrow A=2,3$ 2bdf and 2bmd with local and non-local potentials
- ▶ Contact Coefficients in k -space and in r -space
 $\rightarrow$ TNI needs to be included
- ▶ Ratios of Contact Coefficients $C_{A=3}^{np,s=1}/C_{2H}^{np,s=1}$
 $\rightarrow$ Model-independent
- ▶ Beyond the assumption $\ell = 0 \rightarrow \ell > 0$ contributions are included

Short-term outlook:

- ▶ Estimate of the errors associated with the extraction of the Contact Coefficients

- ▶ Repeat the study for $^4\text{He} \Rightarrow$ calculate $C_{3H}^{nn,s=0}/C_{4He}^{nn,s=0}$
 $C_{3He}^{pp,s=0}/C_{4He}^{pp,s=0}$
 $C_{A=3}^{np,s=0}/C_{4He}^{np,s=0}$

Long-term outlook:

- ▶ Extend the study to $A = 6$ nuclei (the HH method $\rightarrow A = 6$)
[A. Gnech et al., Phys. Rev. C **102**, 014001 (2020)]

Thank you for your attention!

Backup

The Hyperspherical Harmonics (HH) method

- ▶ $|\psi(1, \dots, A)\rangle = \sum_{\mu} c_{\mu} |\Phi_{\mu}\rangle$
- ▶ Appropriate choice of $|\Phi_{\mu}\rangle$
- ▶ Appropriate variational principle $\rightarrow c_{\mu} \quad [\sum_{\mu} \langle \Phi_{\mu} | H - E | \Phi_{\mu} \rangle c_{\mu} = 0]$

Jacobi coordinates $\mathbf{x}_1, \mathbf{x}_2$
 $\rightarrow$ Hyperspherical coordinates

Jacobi conjugate momenta $\mathbf{k}_1, \mathbf{k}_2$
 $\rightarrow$ Hyperspherical coordinates

$$[\rho, \Omega_r], \quad \Omega_r = [\hat{x}_1, \hat{x}_2, \phi]$$

$$\rho = \sqrt{x_1^2 + x_2^2}, \quad \tan\phi = \frac{x_1}{x_2}$$

$$[Q, \Omega_k], \quad \Omega_k = [\hat{k}_1, \hat{k}_2, \phi_k]$$

$$Q = \sqrt{k_1^2 + k_2^2}, \quad \tan\phi_k = \frac{k_1}{k_2}$$

$$|\Phi_{\mu}\rangle = f_l(\rho) Y_{\{G\}}(\Omega_r)$$

$Y_{\{G\}}(\Omega_r)$ = spin-isospin HH functions

$$|\Phi_{\mu}\rangle = g_{G,l}(Q) Y_G(\Omega_k)$$

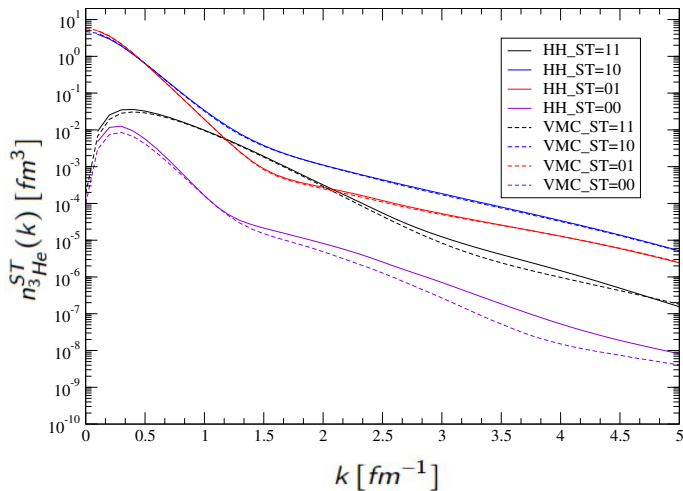
FT of $f_l(\rho) Y_{\{G\}}(\Omega_r)$

$f_l(\rho)$ = hyperradial functions [Laguerre polynomials]

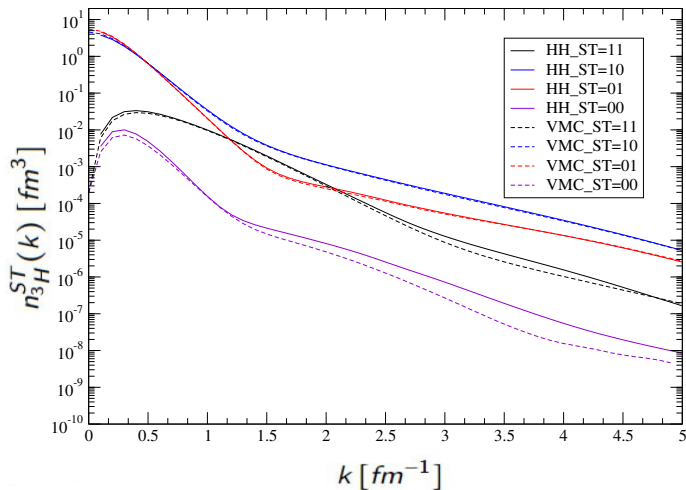
- ▶ We can use ANY potential (local and non-local)
- ▶ We can calculate $n(k)$ and $\rho(r)$ directly in k - and r -space (use $|\Phi_{\mu}\rangle$ in k - or r -space, the c_{μ} are the same)

M. Viviani et al., Few-Body Syst. **39**, 159 (2006)

2bmd with the HH method for ^3He : comparison with VMC for AV18/UIX

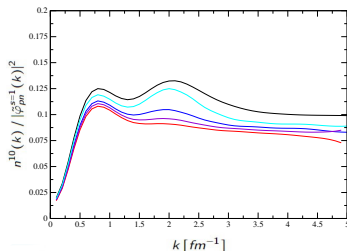
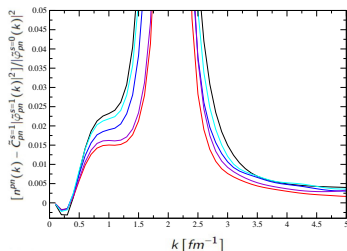
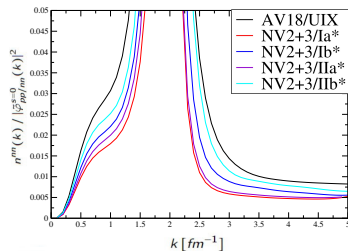


2bmd with the HH method for ^3H : comparison with VMC for AV18/UIX



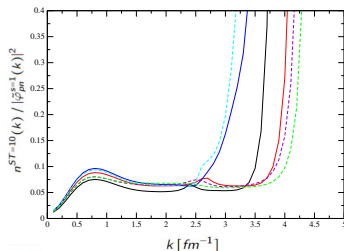
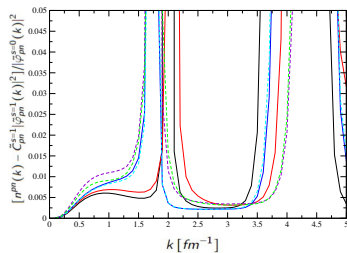
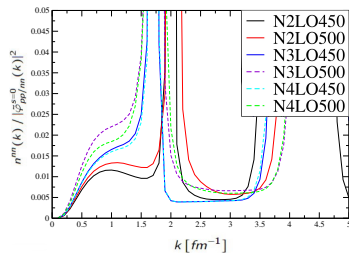
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Two-body momentum distributions



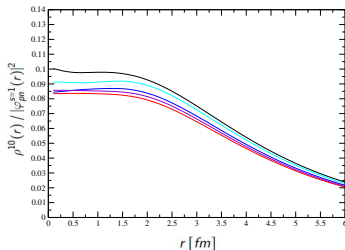
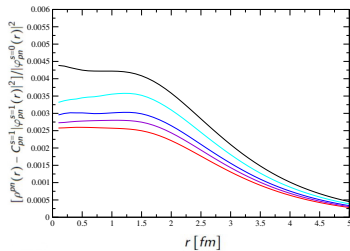
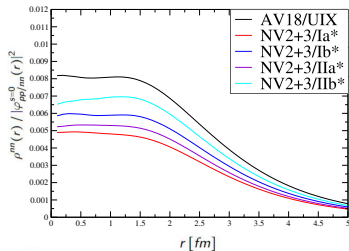
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Two-body momentum distributions



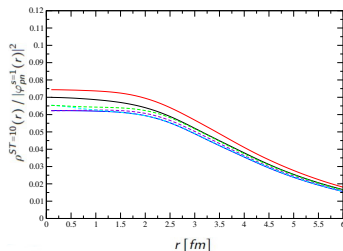
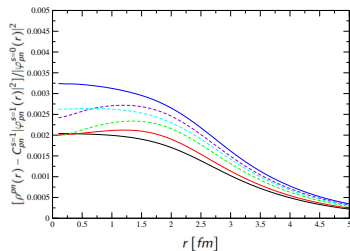
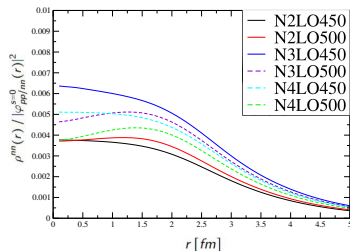
Contact Coefficients in GCF: ^3H with local interactions

Two-body density functions



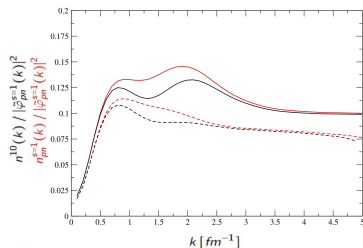
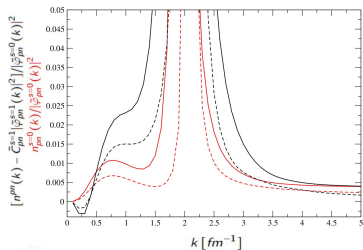
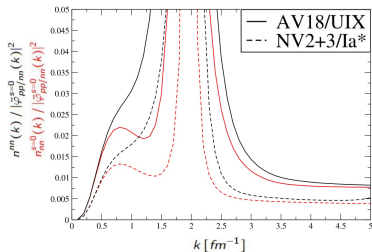
Contact Coefficients in GCF: ^3H with non-local interactions

Two-body density functions



Contact Coefficients in GCF: ^3H with local interactions

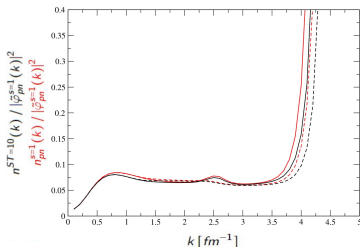
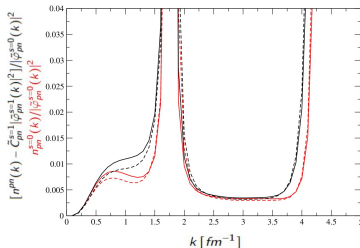
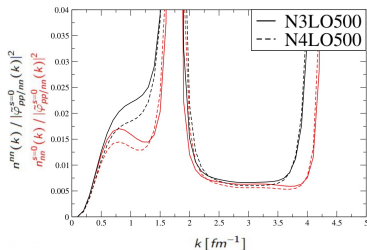
Two-body momentum distributions



More stable plateau without the assumption $\ell = 0$

Contact Coefficients in GCF: ^3H with non-local interactions

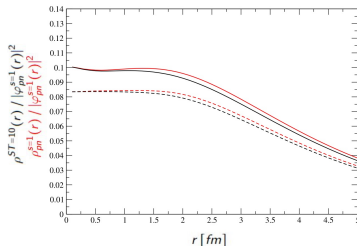
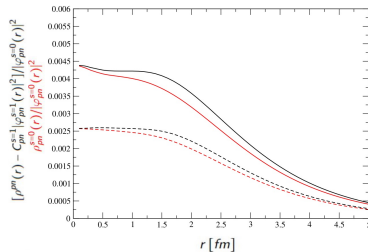
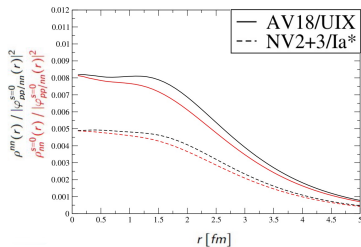
Two-body momentum distributions



More stable plateau without the assumption $\ell = 0$

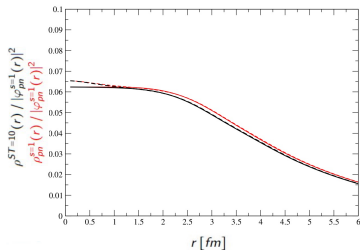
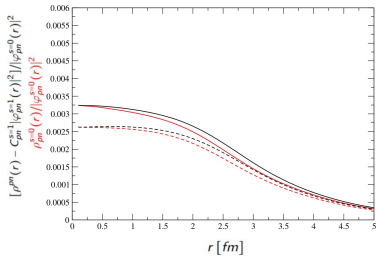
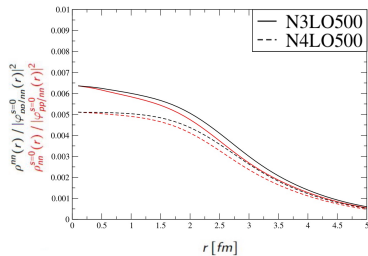
Contact Coefficients in GCF: ^3H with local interactions

Two-body density functions



Contact Coefficients in GCF: ^3H with non-local interactions

Two-body density functions



		Present study						[R. Cruz-Torres et al., Nat. Phys. 17, 306 (2021)]					
		$C_{np}^{s=1}$		$C_{nn}^{s=0}$		$C_{np}^{s=0}$		$C_{np}^{s=1}$		$C_{nn}^{s=0}$		$C_{np}^{s=0}$	
		r	k	r	k	r	k	r	k	r	k	r	k
^2H	AV18/UIX	4.743	4.754					4.898	4.764				
	NV2+3/1a*	4.056	4.051					3.878	3.840				
	NV2+3/1b*	4.274	4.277										
	NV2+3/11a*	4.185	4.183										
	NV2+3/11b*	4.554	4.552										
	N2LO450:	2.806	2.807										
	N3LO450	3.594	3.593										
	N4LO450	3.501	3.499										
	N2LO500	3.298	3.294										
	N3LO500	3.058	3.061										
	N4LO500	3.064	3.064										
	N2LO550	3.576	3.575										
	N3LO550	3.123	3.123										
	N4LO550	3.209	3.211										
^3H	AV18/UIX	6.490	6.647	0.532	0.576	0.278	0.302	6.246	6.441	0.549	0.590	0.295	0.311
	NV2+3/1a*	5.528	5.251	0.317	0.316	0.169	0.192						
	NV2+3/1b*	5.706	5.523	0.399	0.389	0.198	0.251						
	NV2+3/11a*	5.674	5.596	0.346	0.339	0.182	0.221						
	NV2+3/11b*	6.102	5.989	0.448	0.454	0.228	0.269						
	N2LO450	3.910	3.617	0.224	0.379	0.123	0.211						
	N3LO450	4.861	4.448	0.309	0.301	0.162	0.231						
	N4LO450	4.731	4.702	0.301	0.293	0.158	0.249						
	N2LO500	4.564	4.408	0.241	0.468	0.132	0.262						
	N3LO500	4.122	4.352	0.401	0.483	0.207	0.312						
	N4LO500	4.222	4.106	0.332	0.462	0.172	0.318						
	N2LO550	4.894	4.863	0.251	0.603	0.137	0.365						
	N3LO550	4.150	4.231	0.324	0.550	0.170	0.383						
	N4LO550	4.306	4.237	0.270	0.541	0.144	0.384						
^3He	AV18/UIX	6.388	6.527	0.516	0.546	0.264	0.293	6.851	6.249	0.536	0.570	0.161	0.325
	NV2+3/1a*	5.436	5.217	0.309	0.301	0.165	0.183	4.637	4.885	0.310	0.336	0.234	0.233
	NV2+3/1b*	5.618	5.437	0.369	0.371	0.193	0.226						
	NV2+3/11a*	5.582	5.496	0.329	0.322	0.178	0.213						
	NV2+3/11b*	6.006	5.892	0.423	0.458	0.222	0.252						
	N2LO450	3.841	3.587	0.212	0.369	0.119	0.223						
	N3LO450	4.774	4.454	0.290	0.299	0.156	0.237						
	N4LO450	4.647	4.626	0.284	0.290	0.153	0.254						
	N2LO500	4.488	4.342	0.228	0.459	0.128	0.264						
	N3LO500	4.054	4.332	0.377	0.466	0.201	0.307						
	N4LO500	4.150	4.039	0.313	0.446	0.167	0.313						
	N2LO550	4.845	4.794	0.238	0.585	0.133	0.359						
	N3LO550	4.089	4.209	0.307	0.527	0.166	0.370						
	N4LO550	4.243	4.207	0.257	0.519	0.141	0.371						

The extracted Contact Coefficients have been divided by $A/2$ and multiplied by 100

		$C_{A=3}^{np,s=1}/C_{2H}^{np,s=1}$		$C_{A=3}^{np,s=1}/C_{2H}^{np,s=1}$	
		$\ell = 0$		$\ell \geq 0$	
		r	k	r	k
^3H	AV18/UIX	1.368	1.398	1.385	1.416
	NV2+3/Ia*	1.363	1.345	1.379	1.340
	NV2+3/Ib*	1.335	1.291	1.349	1.352
	NV2+3/IIa*	1.363	1.338	1.371	1.333
	NV2+3/IIb*	1.340	1.316	1.356	1.328
	N2LO450	1.393	1.317	1.375	1.317
	N3LO450	1.353	1.238	1.377	1.377
	N4LO450	1.351	1.349	1.360	1.346
	N2LO500	1.384	1.338	1.389	1.338
	N3LO500	1.348	1.425	1.412	1.412
	N4LO500	1.378	1.340	1.385	1.417
	N2LO550	1.368	1.360	1.360	1.360
	N3LO550	1.329	1.355	1.337	1.396
	N4LO550	1.342	1.320	1.348	1.352
^3He	AV18/UIX	1.347	1.373	1.363	1.390
	NV2+3/Ia*	1.340	1.321	1.356	1.315
	NV2+3/Ib*	1.314	1.271	1.328	1.328
	NV2+3/IIa*	1.342	1.314	1.348	1.308
	NV2+3/IIb*	1.319	1.328	1.348	1.306
	N2LO450	1.369	1.292	1.334	1.292
	N3LO450	1.328	1.239	1.316	1.316
	N4LO450	1.327	1.322	1.336	1.322
	N2LO500	1.361	1.318	1.365	1.318
	N3LO500	1.326	1.415	1.408	1.408
	N4LO500	1.354	1.318	1.361	1.393
	N2LO550	1.355	1.341	1.341	1.341
	N3LO550	1.309	1.348	1.316	1.367
	N4LO550	1.322	1.310	1.329	1.368