Status of two-baryon scattering in lattice QCD

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The 11th International Workshop on Chiral Dynamics
Ruhr University Bochum, Germany

Questions in nuclear physics

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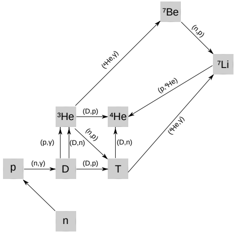

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Big Bang nucleosynthesis has *deuterium bottleneck*:
low deuteron binding energy 2.2 MeV delays onset of nucleosynthesis.
→ controls abundances of light elements.

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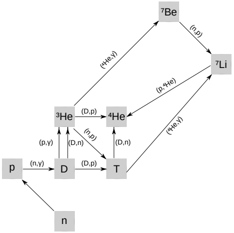

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How strongly does deuteron binding depend on quark masses?
Could pp or nn bind?

Nuclei as tools in experiments

In practice, nuclei instead of free nucleons are often used.

- Argon in neutrino experiments (MicroBooNE, DUNE).
- Xenon for dark matter direct detection (XENONnT, LUX-ZEPLIN).

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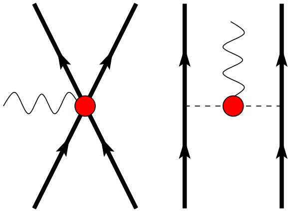
distribution of quarks is different inside nucleus compared with proton and neutron



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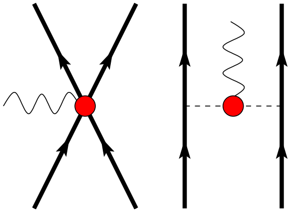

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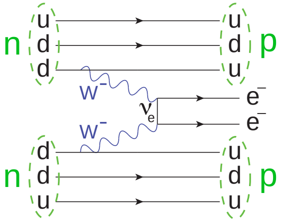

Long-term challenge: neutrinoless double beta decay.

Are neutrinos Majorana?

Hyperon interactions

NN interaction thoroughly studied in experiments. What about strange baryons (*hyperons*)?
Hyperon interactions with $S = -1$ or -2 less well known.

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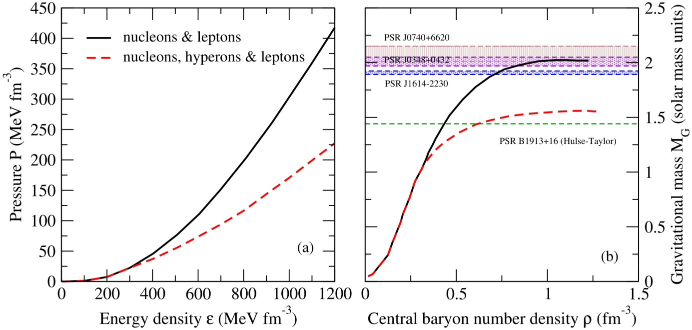

I. Vidaña, EPJ Web Conf. **271**, 09001 (2022)

Λ baryons can reduce Fermi pressure in neutron stars.

Contradicted by detection of neutron stars with $M \approx 2M_\odot$.

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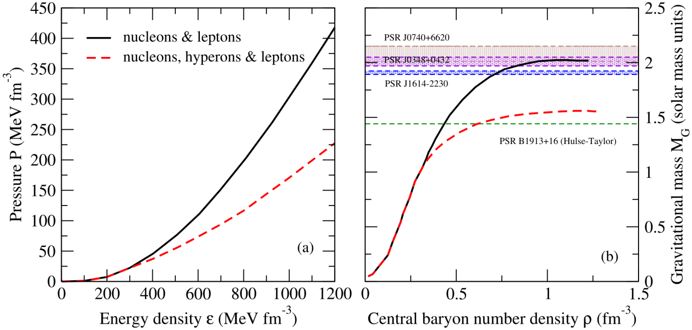

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Do hyperon-hyperon (YY) or NNY interactions play a role?

Outline

1. Methodology and challenges
2. NN: old versus new calculations
3. H dibaryon
4. Outlook

Methods for baryon-baryon scattering

Standard approach:

1. Compute the finite-volume spectra for various quantum numbers: flavour, total momentum $\boldsymbol{P}$, little-group irrep Λ.
2. Use finite-volume quantization to constrain model for scattering amplitude.
3. Find bound-state poles, resonances, etc. in model.

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   - Discretization effects: lattice spacing $a \to 0$.
   - Residual finite-volume effects: box size $L \to \infty$.

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Compute the two-point function

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$$\begin{aligned} C(t) &= \sum_n e^{-E_n t} \left|\langle n|\mathcal{O}^\dagger|\Omega\rangle\right|^2 \\ &\xrightarrow{t\to\infty} e^{-E_0 t} \left|\langle 0|\mathcal{O}^\dagger|\Omega\rangle\right|^2. \end{aligned}$$

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Then take the effective mass,

$$\begin{aligned} m_{\text{eff}}(t) &= \frac{1}{\Delta}\log\frac{C(t)}{C(t+\Delta)} \\ &\longrightarrow E_0 + O(e^{-(E_1-E_0)t}). \end{aligned}$$

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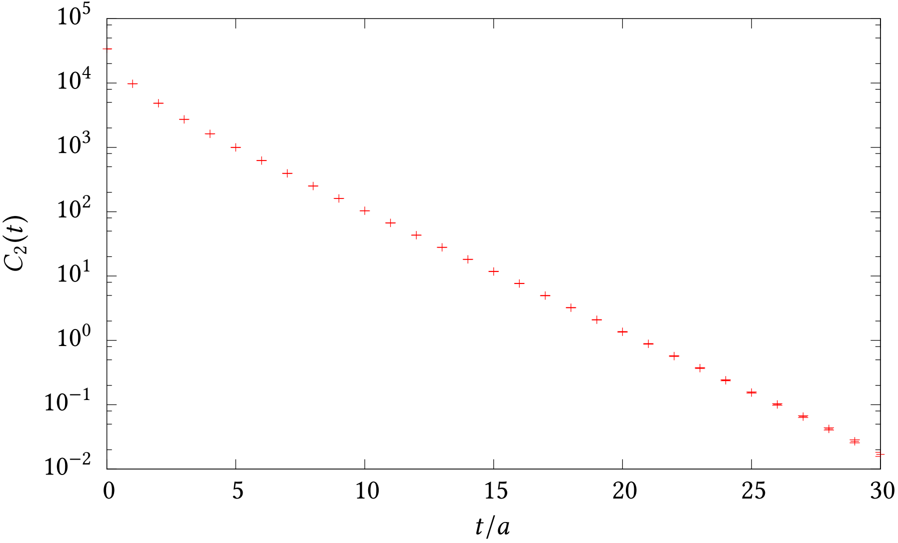

proton
two-point function

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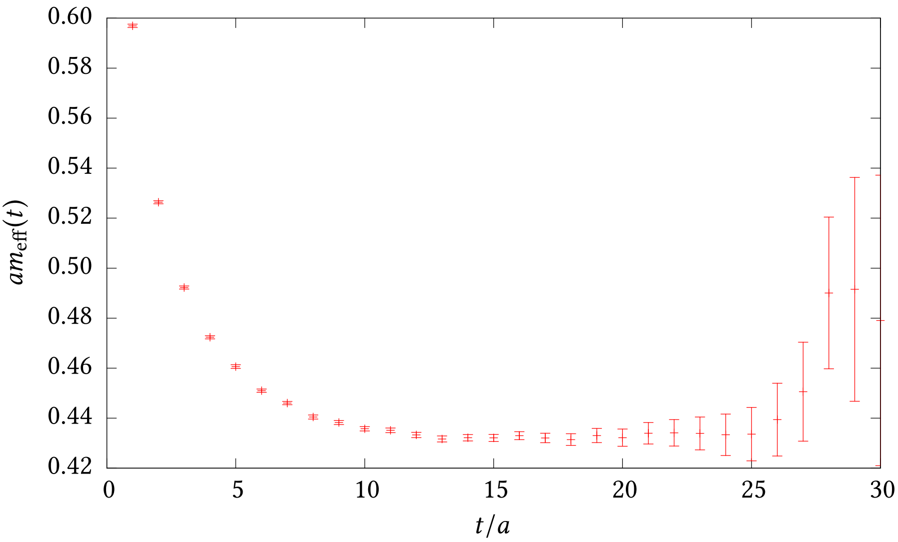

proton
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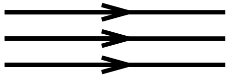
Signal-to-noise problem

Nucleon correlator:

$$C_{2\text{pt}}(t) = \left\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \right\rangle \sim \langle\!\langle \mathfrak{R}[S(t,0)^3] \rangle\!\rangle$$
$$\to e^{-m_N t}$$



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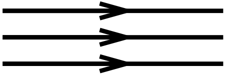

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In general $\sigma^2(X) = \langle\!\langle X^2\rangle\!\rangle - \langle\!\langle X\rangle\!\rangle^2$.

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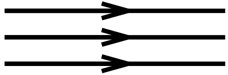

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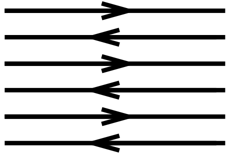
$$\sigma^2(C_{2pt}(t)) \sim \langle\!\langle S(t,0)^3 S^*(t,0)^3\rangle\!\rangle + \cdots$$
$$\to e^{-3m_\pi t}$$



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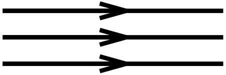

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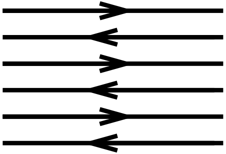
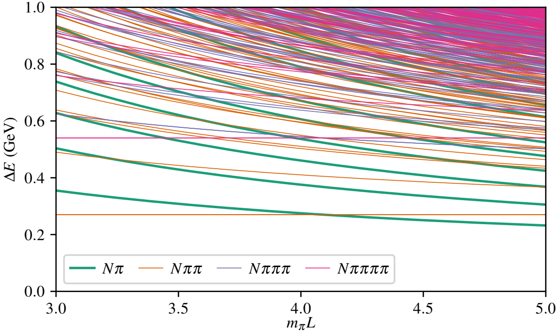


Signal-to-noise ratio:

$$S/N \equiv \frac{C_{2pt}(t)}{\sigma(C_{2pt}(t))} \to e^{-(m_N - \frac{3}{2}m_\pi)t} \quad \text{single nucleon}$$
$$\to e^{-2(m_N - \frac{3}{2}m_\pi)t} \quad \text{two nucleons}$$

Excited-state spectrum (noninteracting)

N



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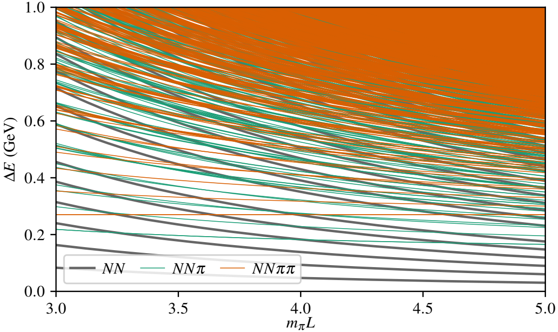

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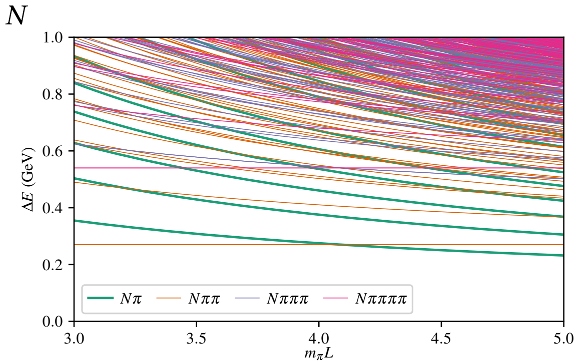
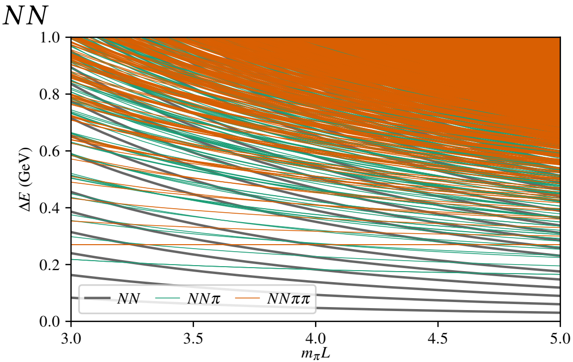


$$S/N \sim e^{-(m_N - \frac{3}{2} m_\pi)t}$$



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Given a set of N interpolating operators $\{\mathcal{O}_i\}$,
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$$C(t+\Delta) v_n = \lambda_n C(t) v_n.$$

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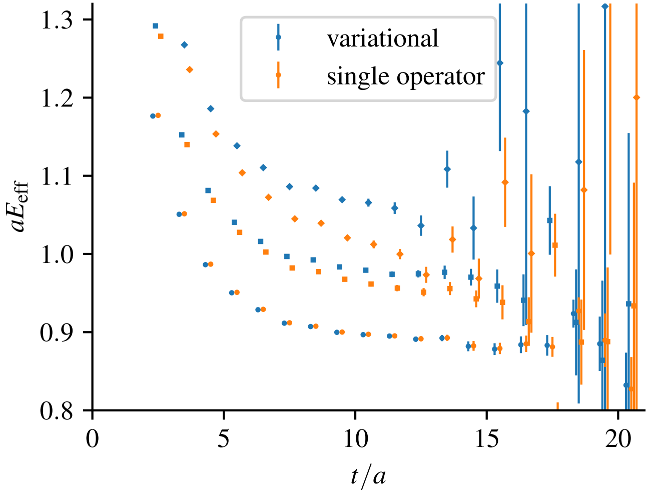
For each of the lowest N states, this gives an effective mass
and an optimized interpolating operator:

$$m_{\text{eff},n} = \frac{-1}{\Delta}\log\lambda_n, \qquad \tilde{\mathcal{O}}_n = v_{ni}^\dagger \mathcal{O}_i,$$

with faster approach to plateau $\sim e^{-(E_N - E_n)t}$.

Importance of variational method



Variational approach essential for excited states.

Single operators can also fail to obtain ground state.

Interpolating operators for dibaryon

Typically use "smeared" quark fields with Gaussian-like profile. Simplest choices:

Hexaquark

$$\mathcal{O}_H(t, \boldsymbol{P}) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{P}\cdot\boldsymbol{x}} (qqqqqq)(t, \boldsymbol{x})$$

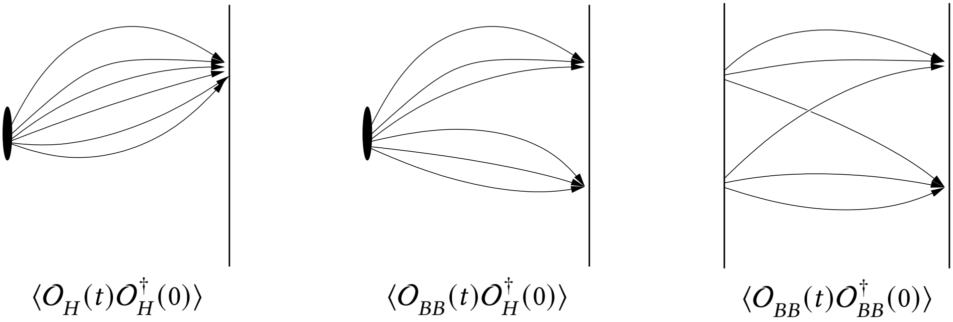
- Looks like quark-model state.

Two-baryon

$$\mathcal{O}_{BB}(t, \boldsymbol{P}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} e^{-i(\boldsymbol{P}-\boldsymbol{p}_1)\cdot\boldsymbol{y}} (qqq)(t, \boldsymbol{x})(qqq)(t, \boldsymbol{y})$$

- Looks like noninteracting baryon-baryon state.
- Varying $\boldsymbol{p}_1$ yields many different operators with same total $\boldsymbol{P}$.

Correlation functions



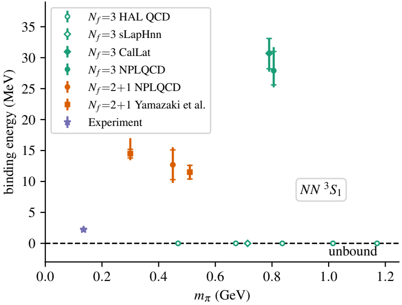
How to compute?

- Point-source propagator $\longrightarrow \langle \mathcal{O}_H(t)\mathcal{O}_H^\dagger(0)\rangle$ or $\langle \mathcal{O}_{BB}(t)\mathcal{O}_H^\dagger(0)\rangle$.
- Nonlocal methods like *distillation* $\longrightarrow \langle \mathcal{O}_{BB}(t)\mathcal{O}_{BB}^\dagger(0)\rangle$.

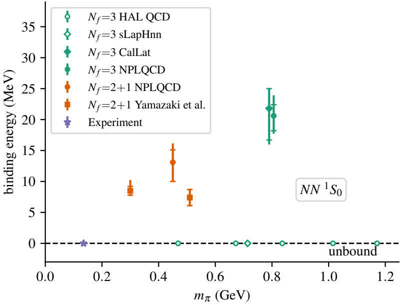
Many early calculations used only $\langle \mathcal{O}_{BB}(t)\mathcal{O}_H^\dagger(0)\rangle$ asymmetric correlators.

Nucleon-nucleon scattering from LQCD: past calculations

Decade-long controversy over presence of bound states at heavy quark masses.



deuteron



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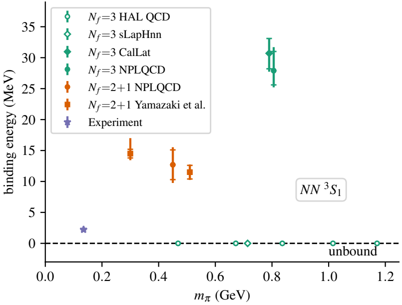

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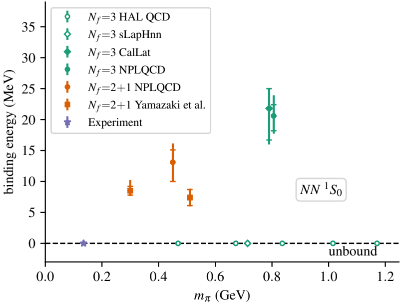

dineutron

Disagreement about simplest warm-up problem for nuclear physics on the lattice.

Experiment: $B_d \approx 2.2$ MeV known for 90 years. J. Chadwick and M. Goldhaber, Nature **134**, 237–238 (1934)

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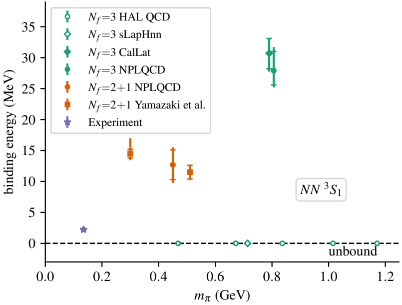

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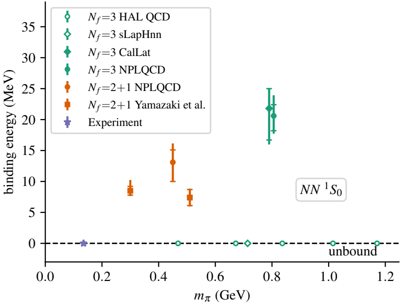

dineutron

No calculation performed using more than one lattice spacing.

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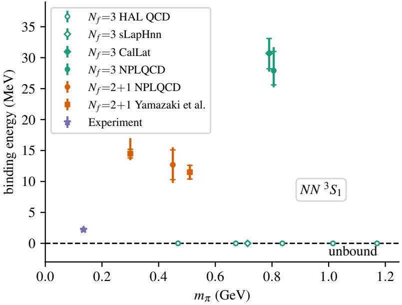

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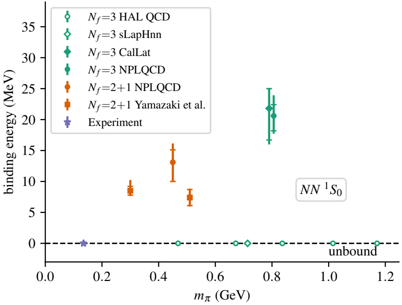

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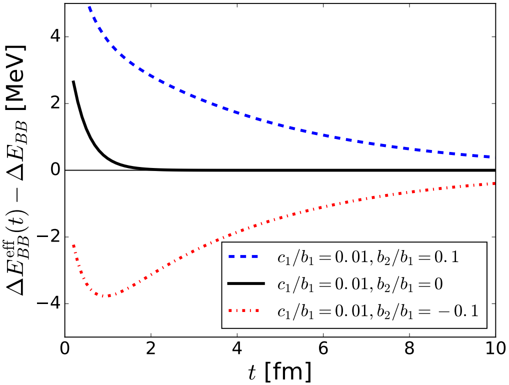
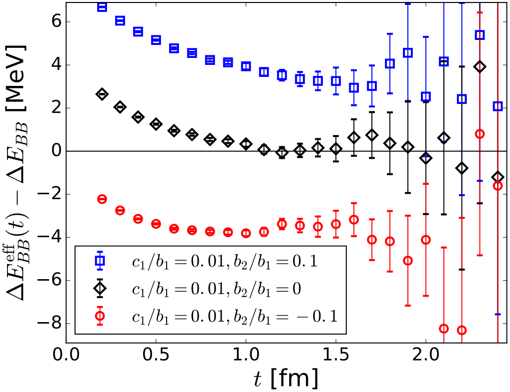
All calculations that obtain bound states use $\langle O_{BB}(t) O_H^\dagger(0)\rangle$ asymmetric correlation functions.

What can go wrong?

T. Iritani *et al.* (HAL QCD), Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD, JHEP **2016**, 101 (2016) [1607.06371] (CC BY 4.0)

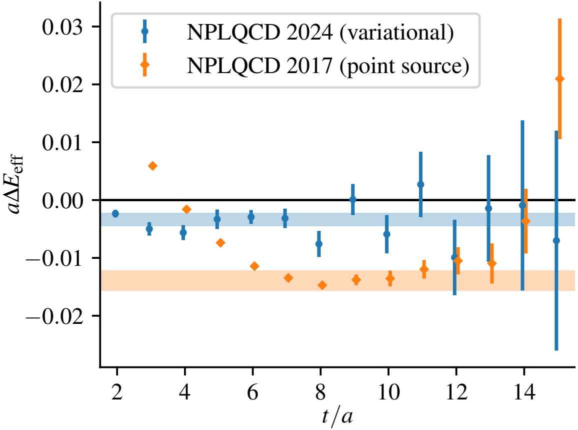




Mock data: effective mass for correlator $C(t) = b_1 + b_2 e^{-\delta E_{\rm el} t} + c_1 e^{-\delta E_{\rm inel} t}$.
"elastic" excitation $\delta E_{\rm el} = 50$ MeV
"inelastic" excitation $\delta E_{\rm inel} = 500$ MeV

Point sources versus variational method with bilocal interpolators

Old and new methods used on same ensemble.



$m_\pi \approx 800$ MeV.

Old calculation:

$^1S_0$ bound state with $B_{nn} \approx 21$ MeV.

New calculation consistent with unbound NN.

Data extracted from

W. Detmold *et al.* (NPLQCD), 2404.12039

M. L. Wagman *et al.* (NPLQCD), PRD 96, 114510 (2017) [1706.06550]

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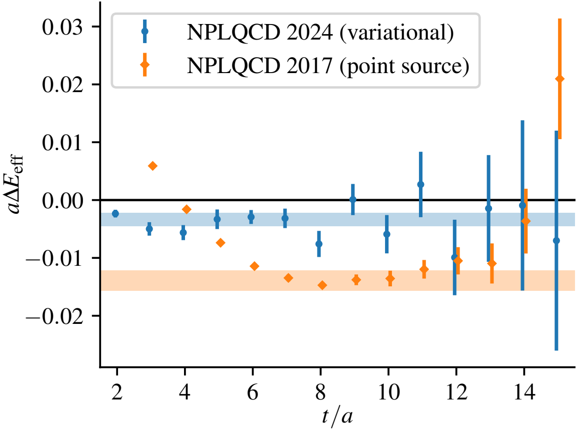

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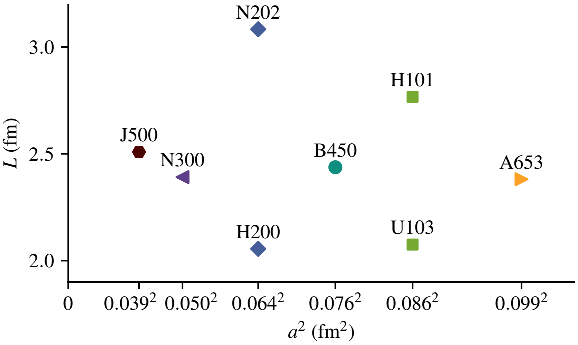
Several variational baryon-baryon calculations done:
A. Francis, JRG *et al.*, PRD 99, 074505 (2019) [1805.03966]
B. Hörz *et al.* (sLapHnn), PRC 103, 014003 (2021) [2009.11825]
JRG *et al.*, PRL 127, 242003 (2021) [2103.01054]
S. Amarasinghe *et al.* (NPLQCD), PRD 107, 094508 (2023) [2108.10835]
W. Detmold *et al.* (NPLQCD), 2404.12039
Z.-Y. Wang @ Lattice 2024 Y. Geng (CLQCD) @ Lattice 2024

Largely consistent picture:
no NN bound state at heavy m_π.

Calculations at light SU(3)-symmetric point

JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig: Phys. Rev. Lett. **127**, 242003 (2021); PoS LATTICE **2021**, 294; PoS LATTICE **2022**, 200;
M. Padmanath, J. Bulava, JRG, A. D. Hanlon, B. Hörz, P. Junnarkar, C. Morningstar, S. Paul, H. Wittig, PoS LATTICE **2021**, 459
+ ongoing work (BaSc collaboration)



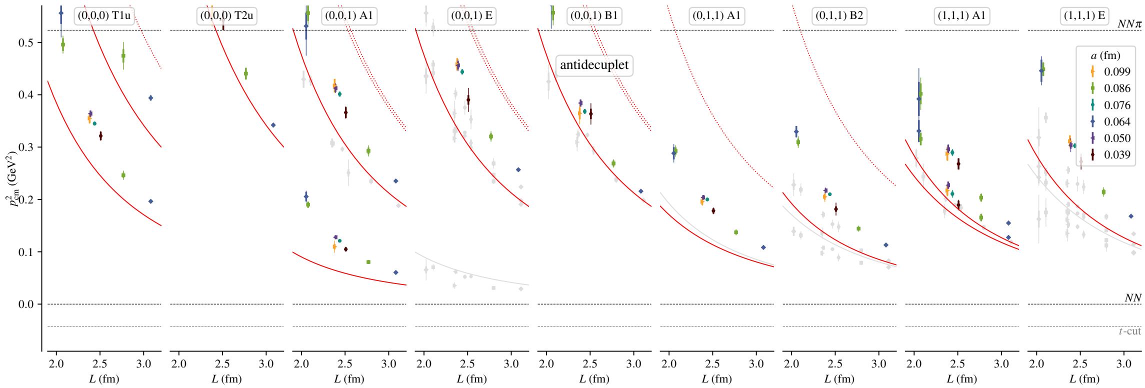
Ensembles with $O(a)$ improved Wilson-clover fermions from CLS.

SU(3)-symmetric point with physical $m_u + m_d + m_s$.

$m_\pi = m_K = m_\eta \approx 420$ MeV.

Two octet baryons: $(\mathbf{8} \otimes \mathbf{8})_S = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$, $\quad (\mathbf{8} \otimes \mathbf{8})_A = \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$.
H dibaryon: $\mathbf{1}$; $\quad NN$: $\mathbf{27}$, $\overline{\mathbf{10}}$.

Antidecuplet ($NN\ I = 0$): spin 0 spectrum

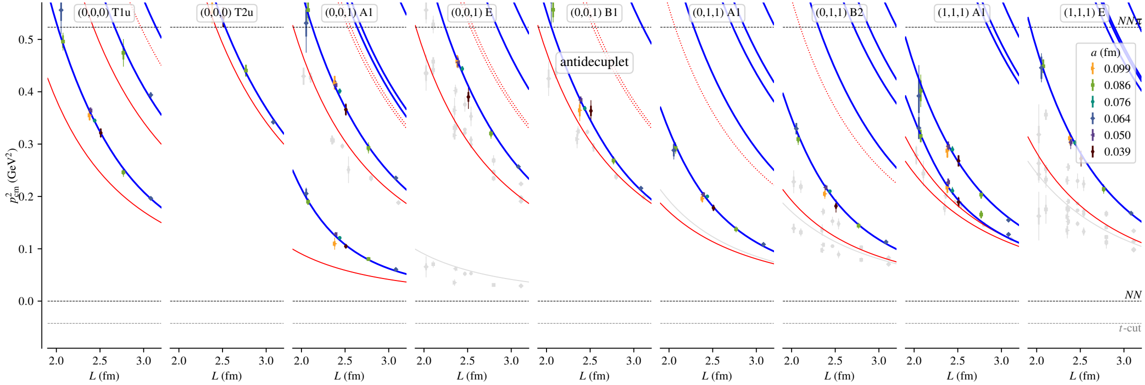


Operators constructed with definite spin. Spin-1 states (gray) identified via overlaps.

Quantization condition factorizes in spin. Here $^1P_1$ and $^1F_3$ are relevant.

Red curves: noninteracting levels.

Antidecuplet ($NN\ I = 0$): spin 0 spectrum, example fit 1

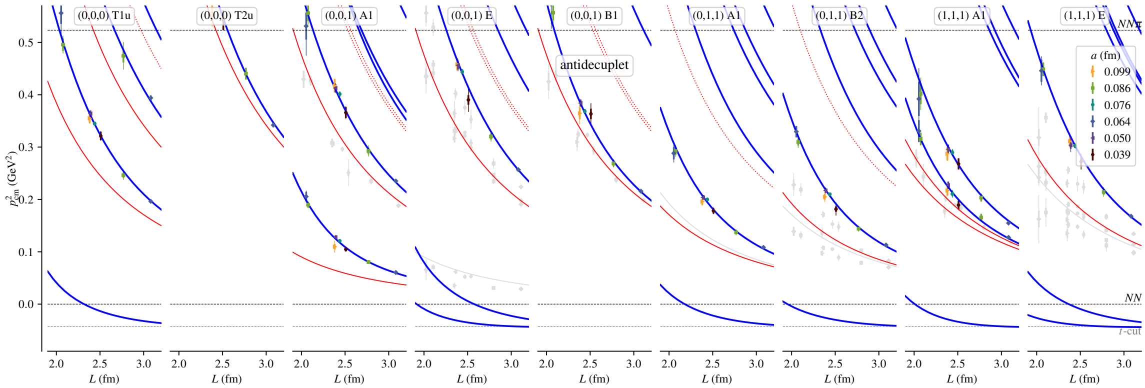


Fit ansatz:

$$p^3 \cot \delta_{^1P_1} = c_1 + c_2 p^2, \qquad p^7 \cot \delta_{^1F_3} = c_3 + c_4 p^8,$$

assuming no discretization effects.

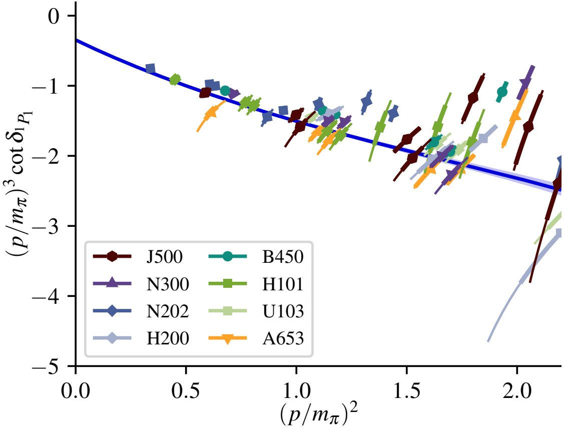
Antidecuplet (NN $I = 0$): spin 0 spectrum, example fit 2

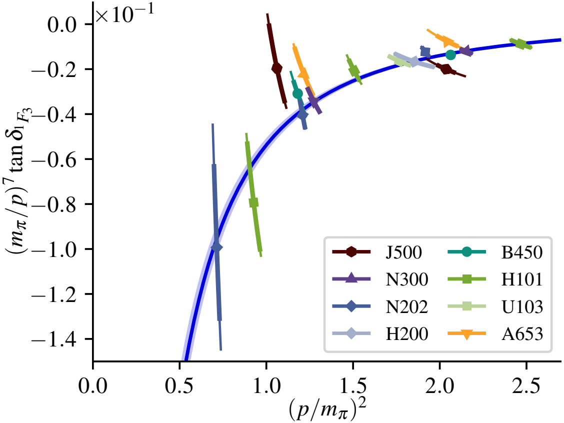


Fit ansatz: solutions to Lippmann-Schwinger equation for ${}^1P_1$ and ${}^1F_3$ with one-pion-exchange potential and contact terms, $\Lambda = 1.5m_\pi$, assuming no discretization effects.

Note: spurious solutions to quantization condition near left-hand cut.

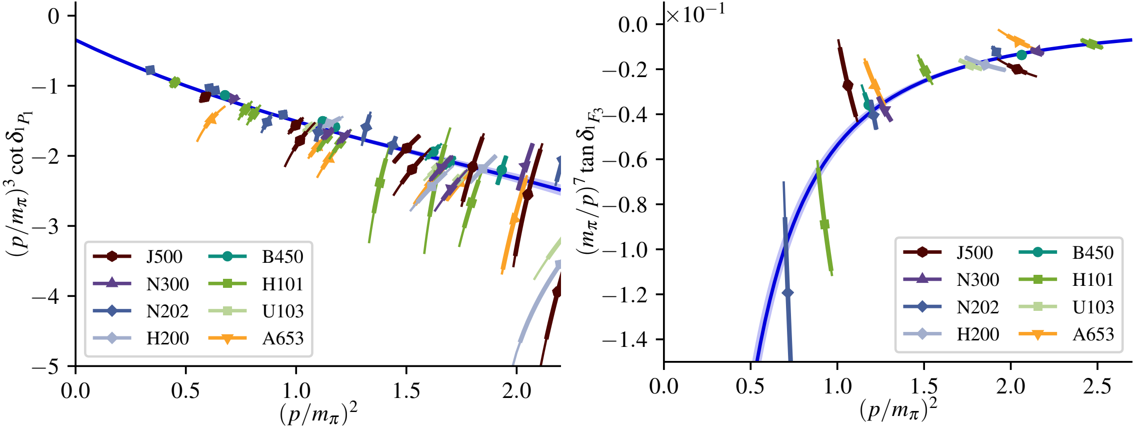
Spin 0 phase shifts: P and F waves (fit 2)





Points: energy levels under single-partial-wave approximation.

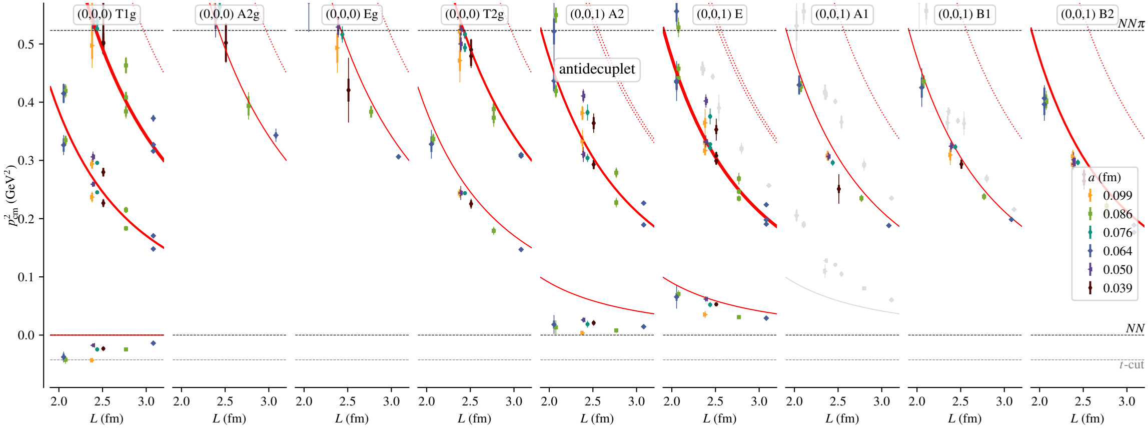
Spin 0 phase shifts: P and F waves (fit 2)



Points: energy levels taking other partial wave into account.

Data lie on single curve. Nontrivial consistency check of spectrum!

Antidecuplet ($NN\ I = 0$): spin 1 spectrum (1)



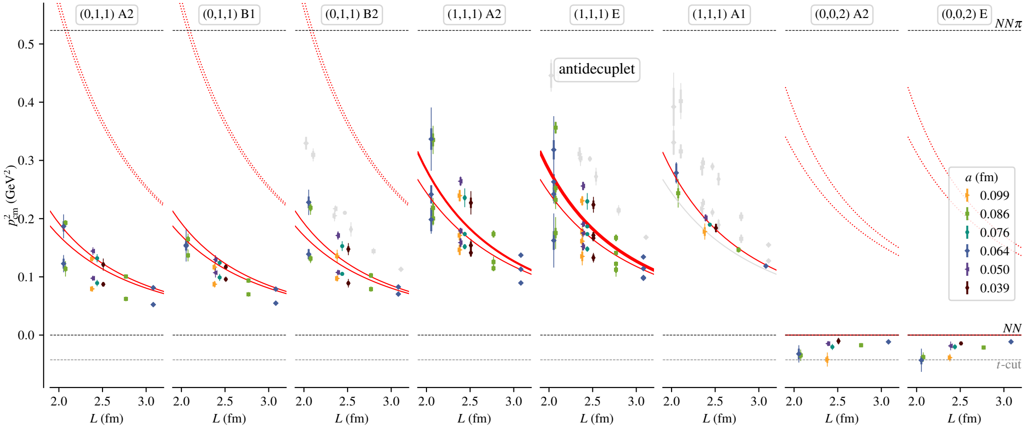
Spin-zero states shown in gray.

Thickness of red curves proportional to degeneracy of noninteracting level.

(39 levels) × (8 ensembles) = 312, although some lie above $NN\pi$ threshold.

${}^3S_1$, ${}^3D_1$, ${}^3D_2$, ${}^3D_3$ are relevant. Possibly also G waves.

Antidecuplet (NN $I = 0$): spin 1 spectrum (2)



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Analyzing coupled ${}^3S_1$ and ${}^3D_1$

Quantization condition: $\det(\tilde{K}^{-1} - B) = 0$. Briceño, Davoudi, Luu 2013; Morningstar *et al.* 2017

Blatt-Biedenharn parametrization including $i^{\ell-\ell'}$ due to convention mismatch:

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} p\cot\delta_{1\alpha} & 0 \\ 0 & p\cot\delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}.$$

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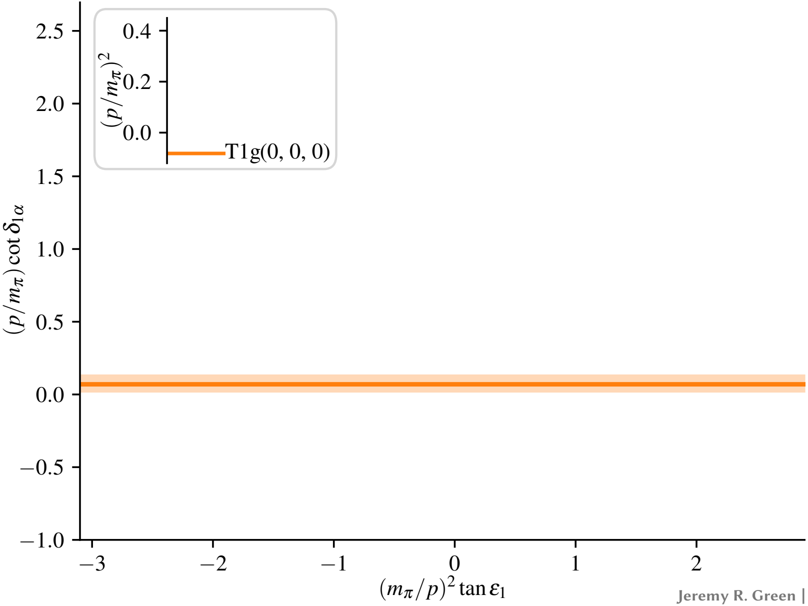
Start with assumption $\delta_{1\beta} = 0$. Then ϵ_1 causes splitting of helicity states.

Each energy level imposes constraint on $(p^{-2}\tan\epsilon_1, p\cot\delta_{1\alpha})$ plane:

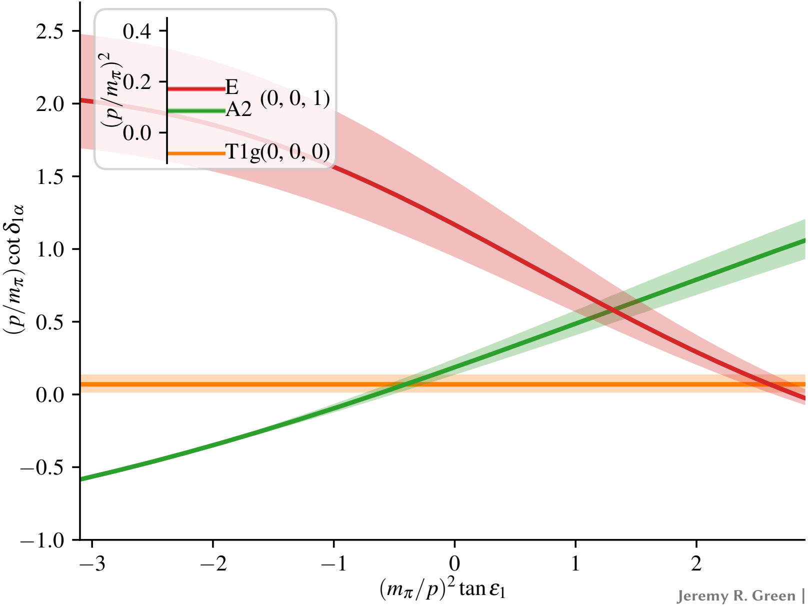
$$p\cot\delta_{1\alpha} = \frac{B_{00} - (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4x^2}, \qquad x = p^{-2}\tan\epsilon_1.$$

$\delta_{1\alpha}$ and ϵ_1 on N202



Assume $\delta_{1\beta} = 0$.
Also neglect ${}^3D_2$, ${}^3D_3$.

$\delta_{1\alpha}$ and ϵ_1 on N202



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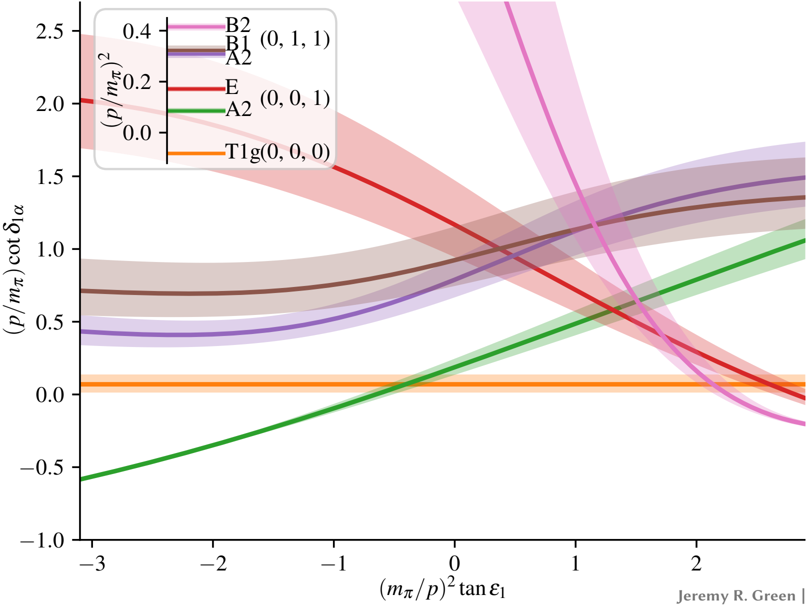

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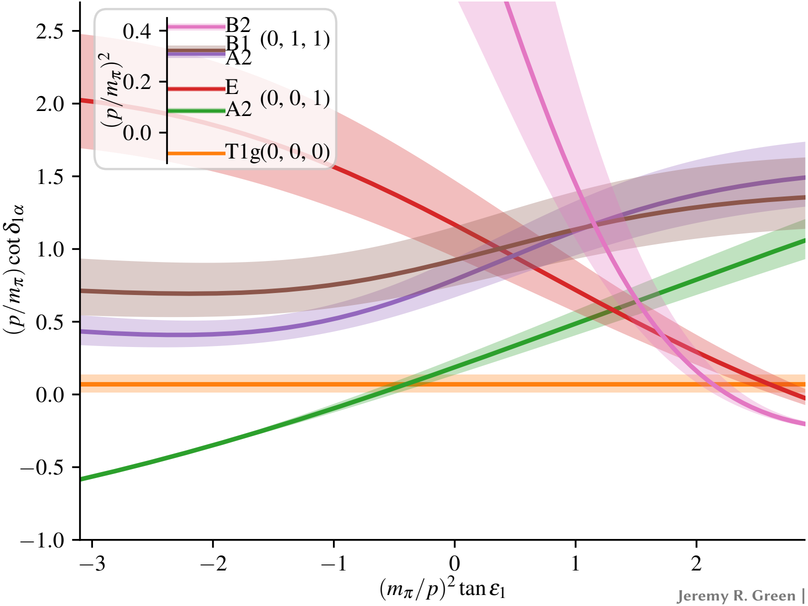
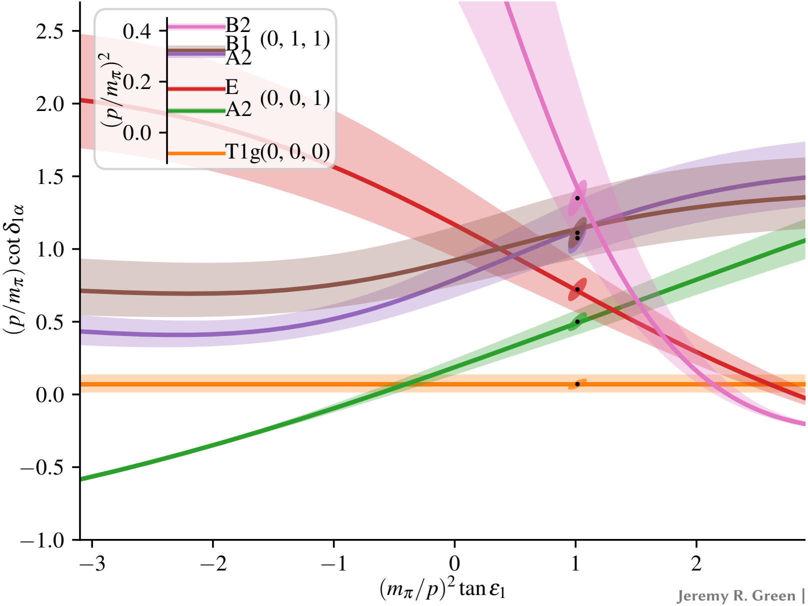


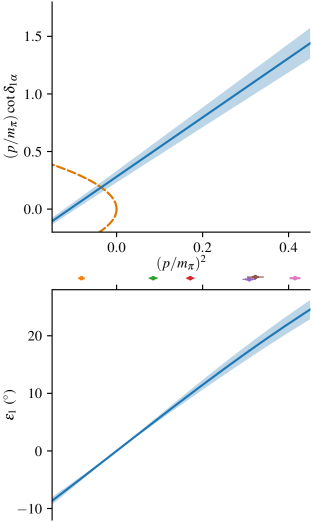
Assume $\delta_{1\beta} = 0$.
Also neglect ${}^3D_2$, ${}^3D_3$.

Fit spectrum using

$$p\cot\delta_{1\alpha} = c_1 + c_2 p^2,$$
$$p^{-2}\tan\epsilon_1 = c_3.$$

$\delta_{1\alpha}$ and ϵ_1 on N202





Deuteron is virtual state.

H dibaryon

VOLUME 38, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1977

Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P=0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P=1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

TABLE I. Quantum numbers and masses of S-wave dibaryons.

$SU(6)_{cs}$ representation	C_6	J	$SU(3)_f$ representation	Mass in the limit $m_s=0$ (MeV)
490	144	0	$\underline{1}$	1760
896	120	1,2	$\underline{8}$	1986
280	96	1	$\underline{10}$	2165
175	96	1	$\underline{10}$*	2165
189	80	0,2	$\underline{27}$	2242
35	48	1	$\underline{35}$	2507
1	0	0	$\underline{28}$	2799

Proposed *uuddss* flavour-singlet dibaryon with $J^P=0^+$.

Bound state of two Λ hyperons with $B_H \approx 80$ MeV.

H dibaryon: Experimental searches

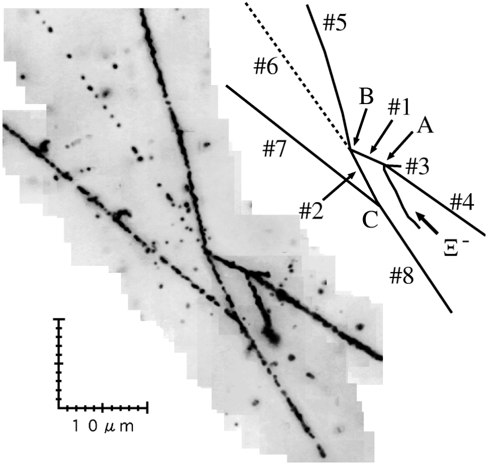


FIG. 2. Photograph and schematic drawing of NAGARA event. See text for detailed explanation.

H. Takahashi *et al.*, PRL **87**, 212502 (2001)

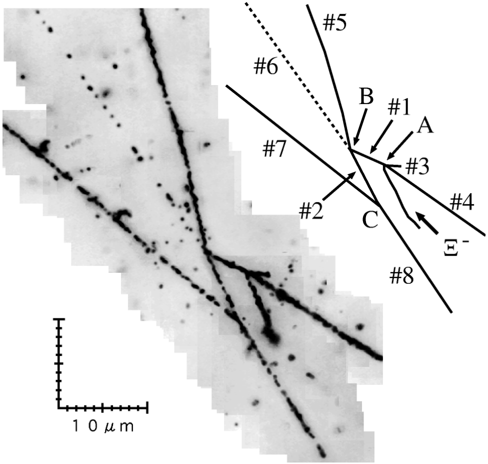
Strongest constraint comes from “Nagara” event from E373 at KEK, which found a ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ double-hypernucleus with ΛΛ separation energy

$$B_{\Lambda\Lambda}^{\mathrm{Nagara}} = 6.91 \pm 0.16\ \mathrm{MeV}.$$

Absence of strong decay ${}^{6}_{\Lambda\Lambda}\mathrm{He} \to {}^{4}\mathrm{He} + H$ implies

$$B_H < B_{\Lambda\Lambda}^{\mathrm{Nagara}}.$$

H dibaryon: Experimental searches



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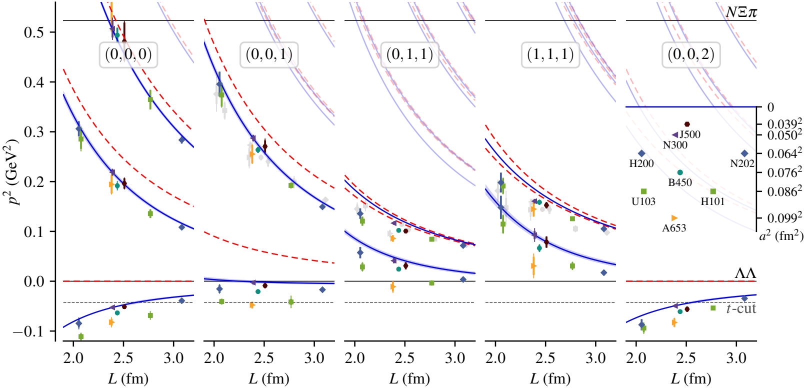
Also studied using "femtoscopy" method at LHC.

ALICE, PLB **797**, 134822 (2019)

H dibaryon: spectrum summary

Weakly bound H dibaryon from SU(3)-flavor-symmetric QCD
JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig, Phys. Rev. Lett. **127**, 242003 (2021)



SU(3) singlet.

Trivial (A1g or A1) irreps.

p^2 is back-to-back scattering momentum: $E_{\text{cm}} = 2\sqrt{p^2 + m^2}$

Points: lattice energy levels.

Red dashed curves: noninteracting levels.

Blue curves: interacting levels in continuum.

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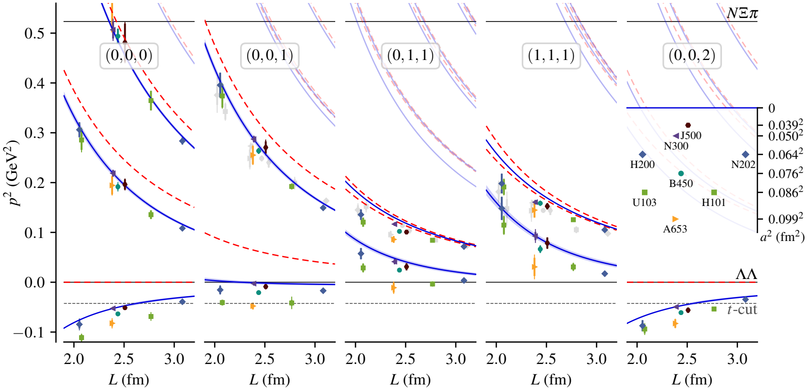

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Strong dependence on a^2!

Levels lie on left-hand cut!

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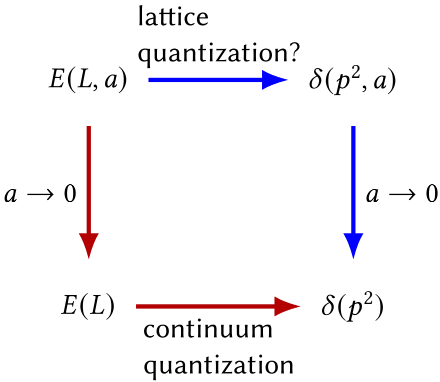
Quantization condition and continuum limit



Continuum extrapolation:
follow blue path, applying continuum quantization condition at nonzero lattice spacing.

Combined fits to multiple lattice spacings: let

$$p \cot \delta(p^2, a) = \sum_{i=0}^{N-1} c_i(a) p^{2i}, \quad c_i(a) = c_{i0} + c_{i1} a^2.$$

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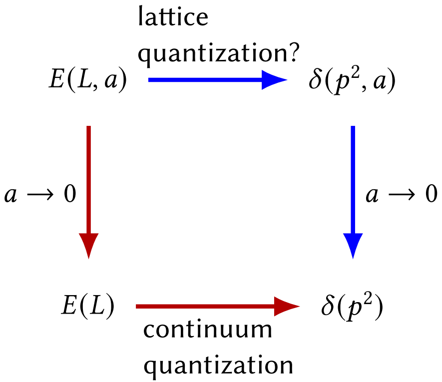

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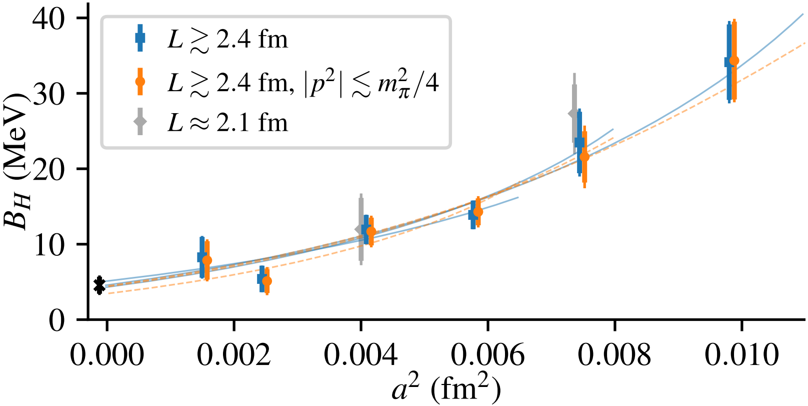
Recent work on including discretization effects in quantization condition:

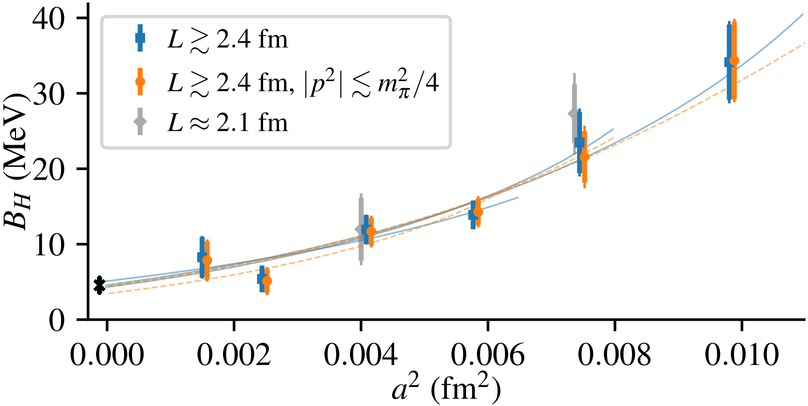
M. T. Hansen and T. Peterken, 2408.07062

H dibaryon binding energy versus lattice spacing



Fits to spectrum with different cuts on a and p^2.
Strong dependence on lattice spacing.

H dibaryon binding energy versus lattice spacing

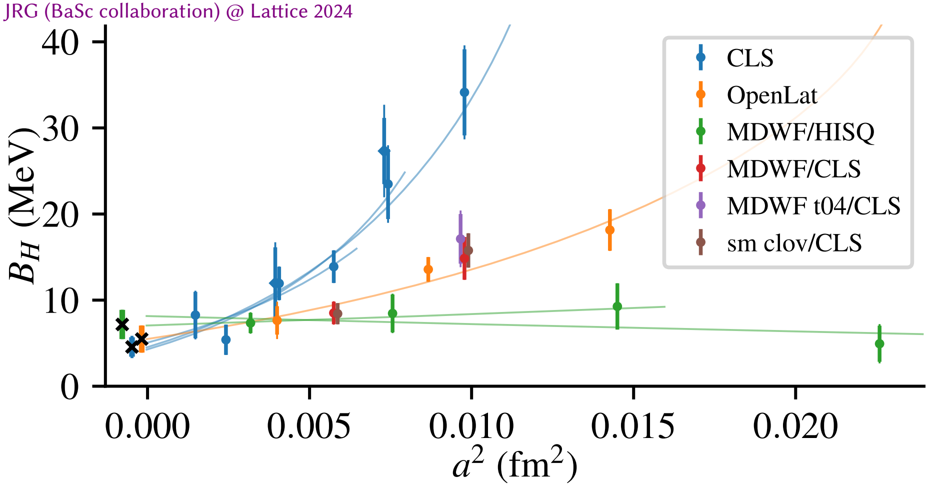


Strong dependence on a^2 also found by HAL QCD and NPLQCD at heavier pion mass.

T. Inoue, Few Body Syst. **65**, 34 (2024)
R. Perry @ Lattice 2024

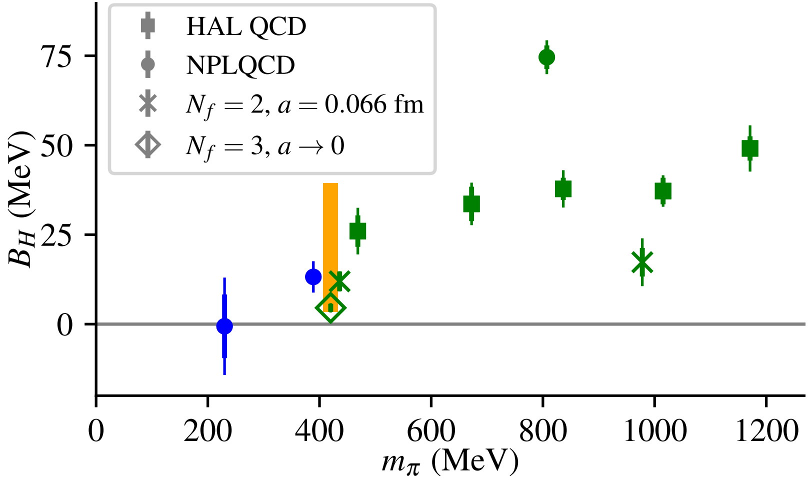
Fits to spectrum with different cuts on a and p^2.
Strong dependence on lattice spacing.

Binding energy of H dibaryon: different lattice actions



Three independent $a \to 0$ extrapolations agree. Size of lattice artifacts varies significantly.

H dibaryon binding energy: comparison with literature



green: $m_s = m_{ud}$

blue: $m_s = m_s^{\text{phys}} > m_{ud}$

Summary and outlook

Findings:

- Variational methods are essential for obtaining correct finite-volume spectrum.
- Contrary to earlier calculations, probably no NN bound state at heavy m_π.
- H dibaryon is bound by ~ 5 MeV at SU(3)-symmetric point.
- Discretization effects can be surprisingly important, particularly in S waves.

Summary and outlook

Findings:

- Variational methods are essential for obtaining correct finite-volume spectrum.
- Contrary to earlier calculations, probably no NN bound state at heavy m_π.
- H dibaryon is bound by ~ 5 MeV at SU(3)-symmetric point.
- Discretization effects can be surprisingly important, particularly in S waves.

Important next steps:

- Better understanding of lattice artifacts.
- Inclusion of left-hand cut in finite-volume quantization.
- More detailed cross-checks between collaborations and with HAL QCD.
- Lighter quark masses.

Combined phase shift fits

S-wave quantization condition:

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi L \gamma}} Z_{00}^{\boldsymbol{P}L/(2\pi)}\left(1, \left(\frac{pL}{2\pi}\right)^2\right)$$

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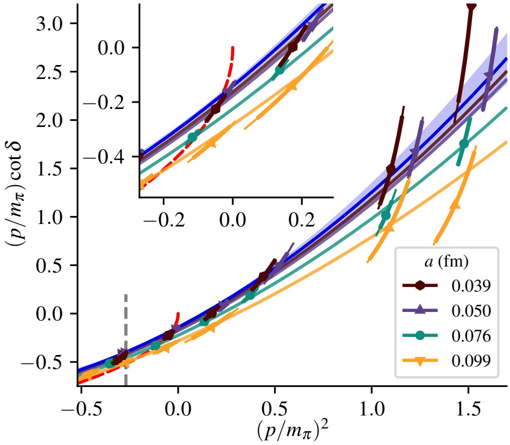

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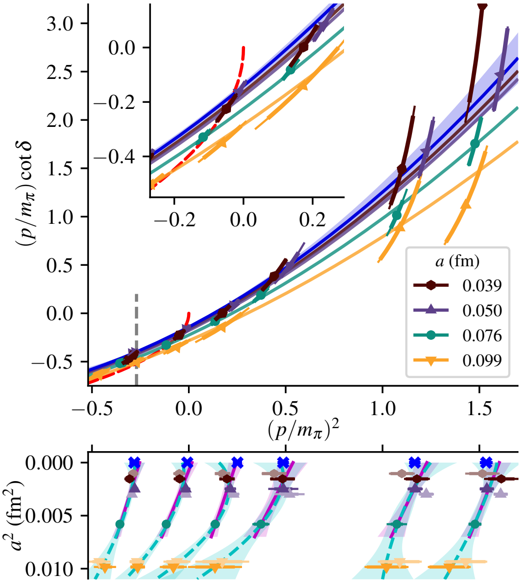
Cross check: extrapolate energies at fixed volume.



Symanzik theory: EFT describing lattice QCD at $a > 0$

With $O(a)$ improved action, corrections start at a^2:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + a^2 \sum_i \mathcal{O}_i + O(a^3).$$

Dimension-six operators $\mathcal{O}_i$ are gluonic, $\bar{q}q$, or $(\bar{q}q)^2$ satisfying symmetries of lattice action:

- Some break $O(4)$ rotational symmetry $\longrightarrow$ modified dispersion relations.
- Some break chiral symmetry.

Logarithmic corrections also understood. N. Husung *et al.*, 2022

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Logarithmic corrections also understood. N. Husung *et al.*, 2022

We see percent-level effects on baryon-baryon energies
but O(100%) effects on scattering observables such as the scattering length.

Can we understand what is causing these large effects? Study using different actions.