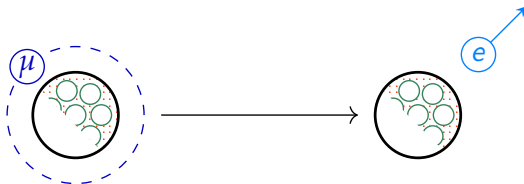


$\mu \rightarrow e$ conversion in nuclei and nuclear charge distributions

Frederic Noël

Universität Bern
Institute for Theoretical Physics



29.08.2024

The 11th International Workshop on Chiral Dynamics

[Hoferichter, Menéndez, Noël; Phys. Rev. Lett. 130 (2023)]

[Noël, Hoferichter; JHEP 08 (2024)]

[Heinz, Hoferichter, Miyagi, Noël, Schwenk; in preparation]

Lepton Flavour Violation

- Search for **new physics** (NP):
one prominent probe $\rightarrow$ **LFV**

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Lepton Flavours

e	μ	τ
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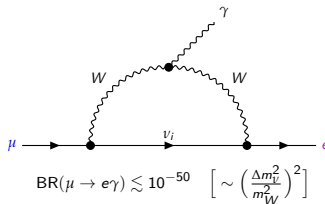
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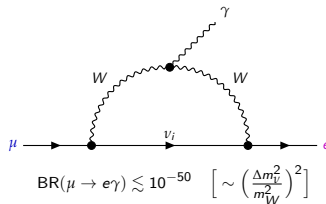


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- Observation of CLFV would be NP

Lepton Flavours

e	μ	τ
ν_e	ν_μ	ν_τ



Very **clean BSM signal** (no competing SM)

LFV Experiments and current limits

LFV process	current limit	(planned) experiments
$\mu \rightarrow e\gamma$	$< 4.2 \cdot 10^{-13}$ [MEG]	MEG II
$\mu \rightarrow 3e$	$< 1.0 \cdot 10^{-12}$ [SINDRUM]	Mu3e
$\tau \rightarrow \ell\gamma, 3\ell, \ell P, \dots$	$\lesssim 10^{-8}$ [Belle, LHCb, ...]	Belle 2, ...
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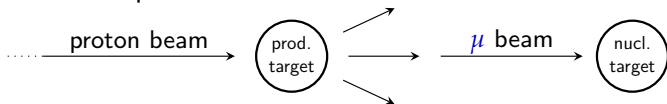
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$\text{Au } \mu^- \rightarrow \text{Au } e^-$	$< 7 \cdot 10^{-13}$ [SINDRUM II]	Mu2e, COMET
$\text{Ti } \mu^- \rightarrow \text{Ti } e^-$	$< 6.1 \cdot 10^{-13}$ [SINDRUM II]	
$\text{Al } \mu^- \rightarrow \text{Al } e^-$	$\lesssim 10^{-17}$ (projected)	

Major **experimental improvements** expected
 → **stringent bounds** on LFV

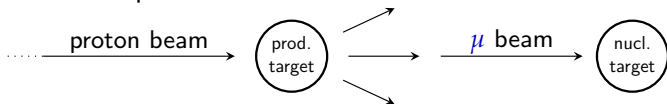
What is $\mu \rightarrow e$ conversion? (theorist's perspective)

- Experimental Setup:



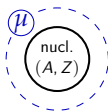
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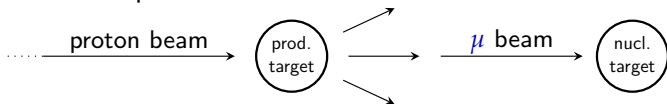
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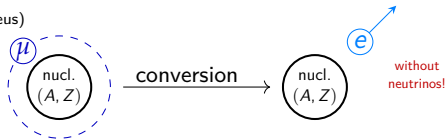
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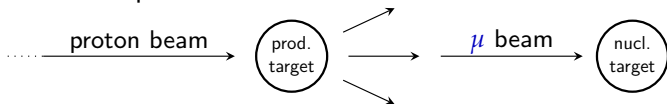
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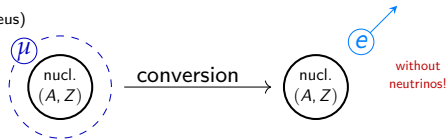
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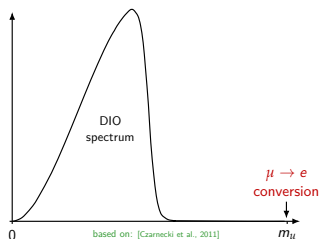
e^- with $q \approx m_\mu$

Only background: decay in orbit

$$\mu^- \rightarrow \nu_\mu \bar{\nu}_e e^-$$

Normalisation: muon capture

$$\mu (A, Z) \rightarrow \nu_\mu (A, Z - 1)$$



How to describe LFV from the theory side?

Standard Model EFT:

- **Model-independent** effective field theory description of BSM physics with **higher dimensional operators** obeying all SM symmetries:

$$\mathcal{L}^{\text{SM EFT}} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}^{(5)} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$

- Can be seen as the **low-energy effective theory** of any theory introducing new physics at high energies

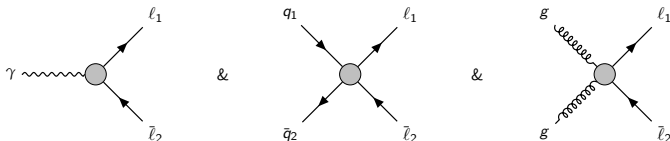
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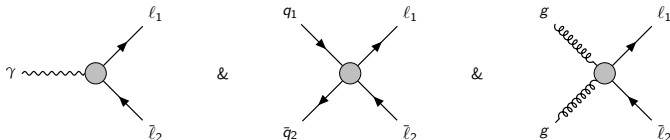
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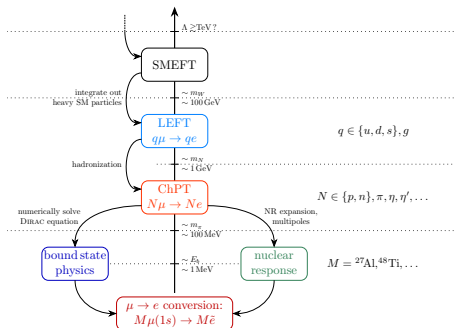
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Can be used to describe all LFV processes in a model-independent way

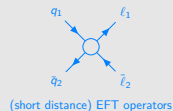
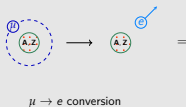
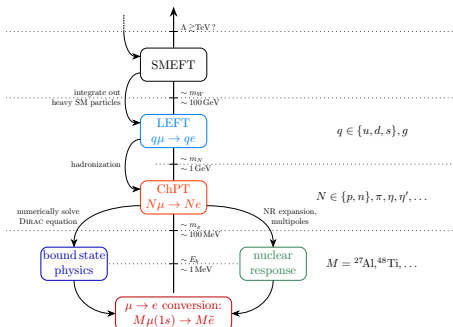
How to use this for $\mu \rightarrow e$ conversion?

Many different scales matter:



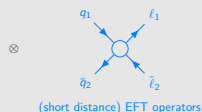
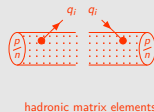
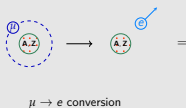
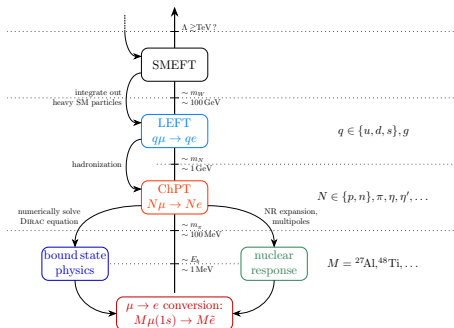
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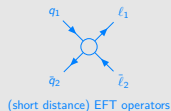
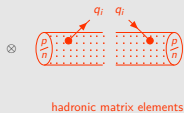
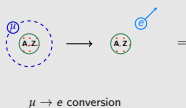
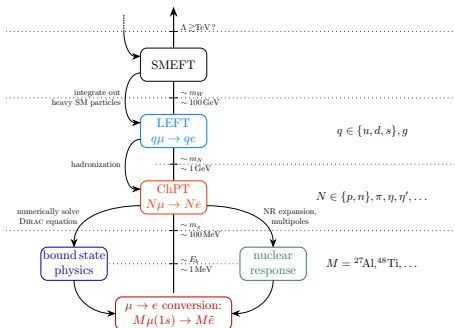
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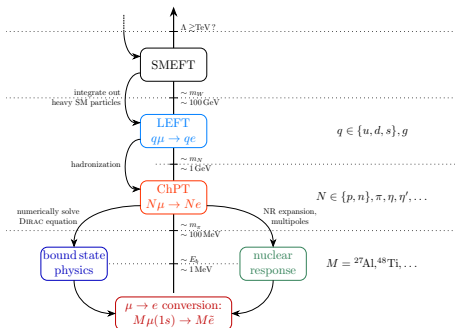
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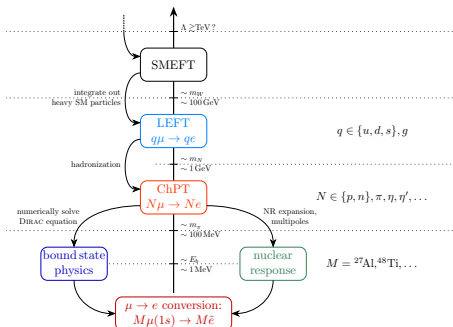
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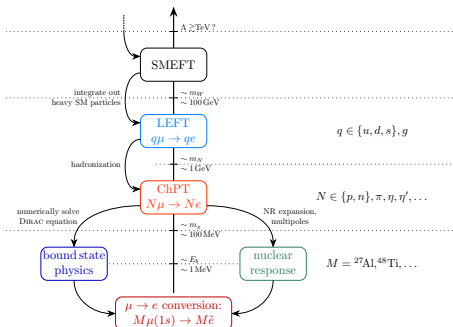
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- Compare different probes:
e.g.: $\mu \rightarrow e$ vs. $P \rightarrow \bar{\mu} e$
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Objectives:

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- Discriminate BSM operators
- Control theory uncertainties:
 - Hadronic matrix elements
 - Nuclear response
 - Coulomb corrections
- RG corrections



At all steps **uncertainties** need to be **controlled!**

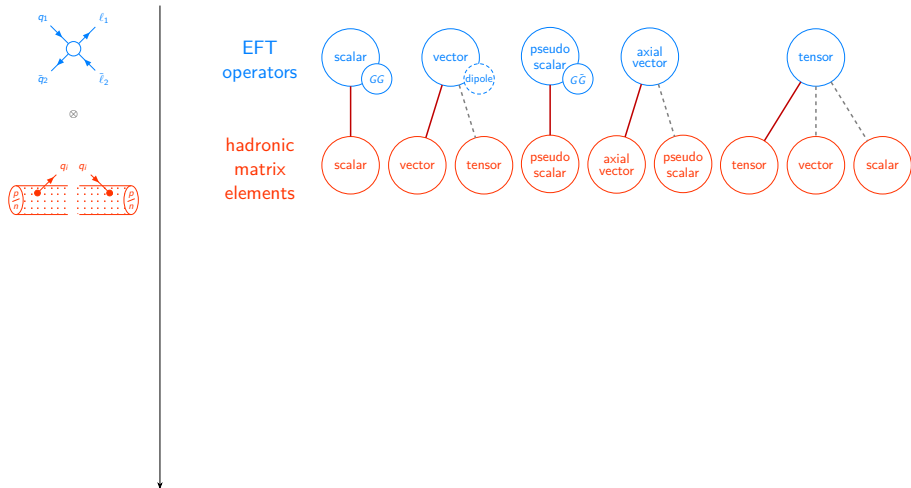
Decomposition of the hadronic side



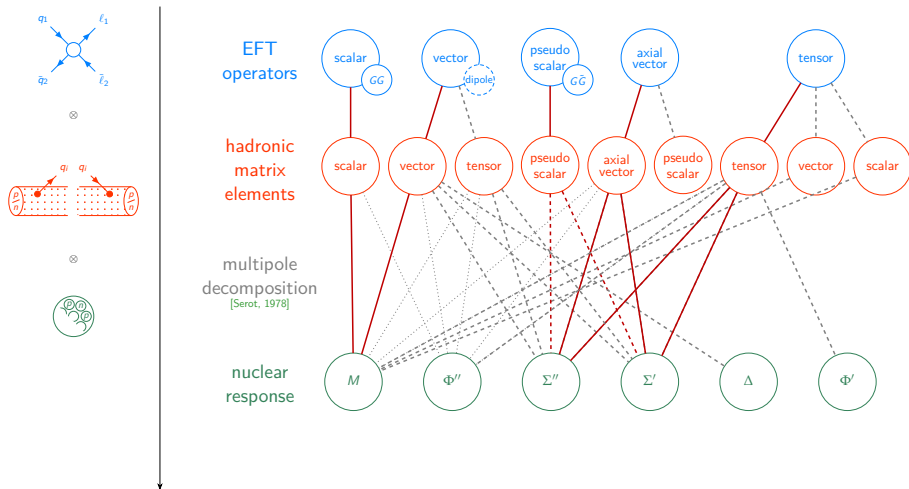
EFT
operators



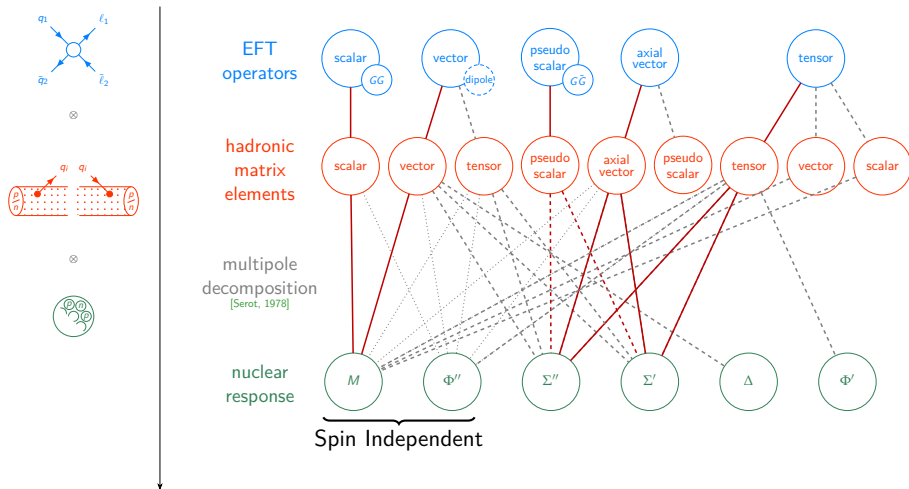
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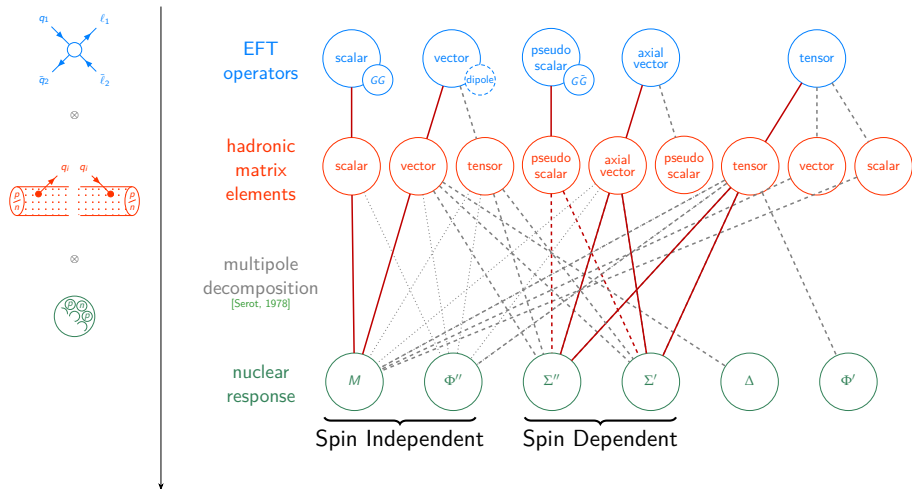


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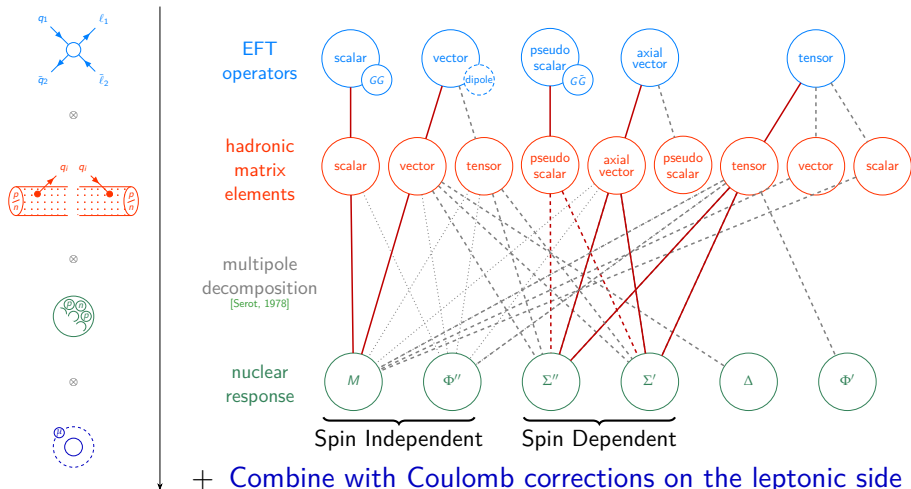
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- Same operators probe SD $\mu \rightarrow e$ conversion and $P \rightarrow \bar{\mu}e$: [Gan et al., 2022]

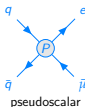
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Decay $P \rightarrow \bar{\mu}e$:

Decay
Rate =



hadronic matrix elements

$\otimes$

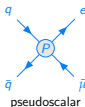


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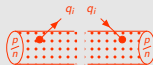
bound state physics

⊗



nuclear response

⊗



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⊗



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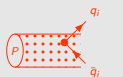
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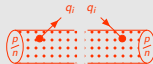
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nuclear response

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hadronic matrix elements

⊗



(short distance) EFT operator

Can use $\mu \rightarrow e$ conversion limits to derive limits on $P \rightarrow \bar{\mu}e$

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$\mu \rightarrow e$ (exp.)	$P \rightarrow \bar{\mu}e$ (derived)	current limit
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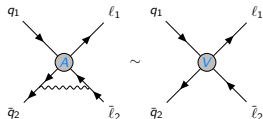
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- For $\eta^{(\prime)} \rightarrow \bar{\mu}e$: in principle, no strict limits
- Cancellation easily lifted by **RG corrections**



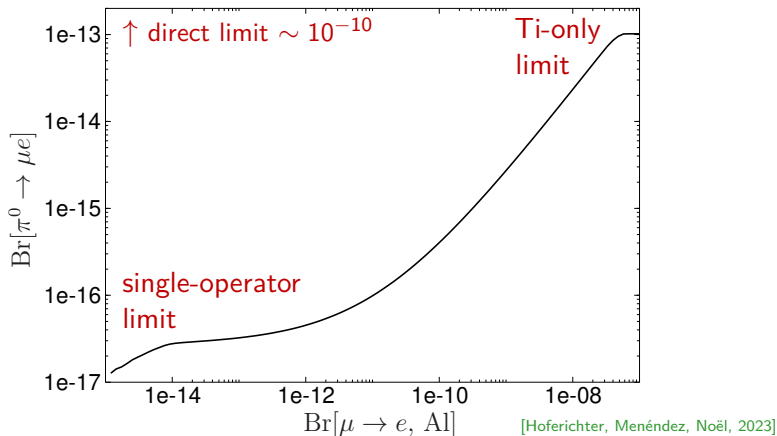
Future projection for $\pi^0 \rightarrow \bar{\mu}e$

With values from Mu2e or COMET the limits become even stronger

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- Combining the limits from Ti and Al we find:



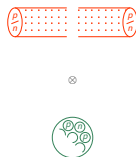
Controlling uncertainties

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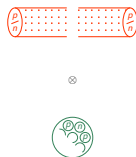
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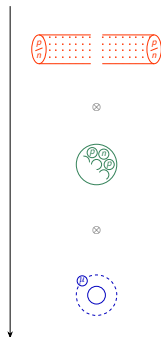
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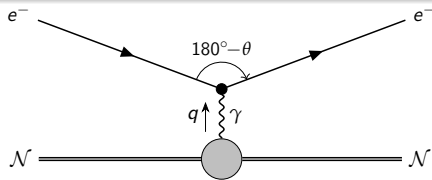


Charge densities with **quantified uncertainties** required

So far: As Fourier-Bessel series **without uncertainties** [Vries et al., 1987]
 → Redo extraction from **elastic electron nucleus scattering**

How to describe elastic electron scattering?

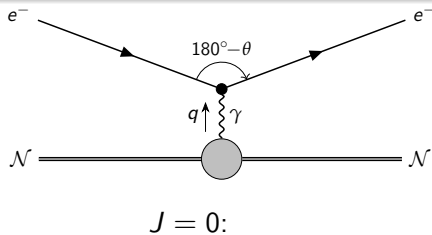
Typical description via Plane Wave Born Approximation



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \times \frac{E'_e}{E_e} \times |F(q, \theta)|^2$$

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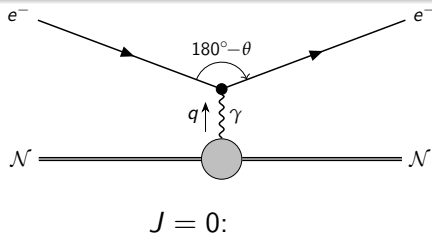
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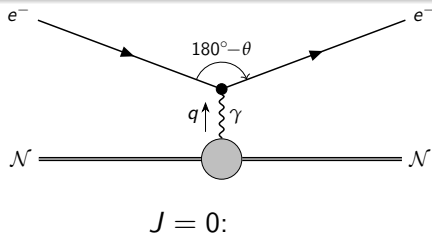
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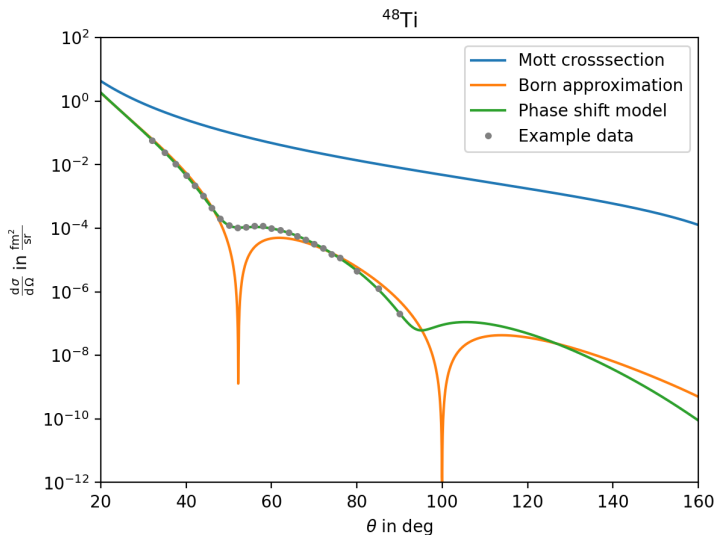
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Even for $J = 0$ insufficient → Coulomb corrections

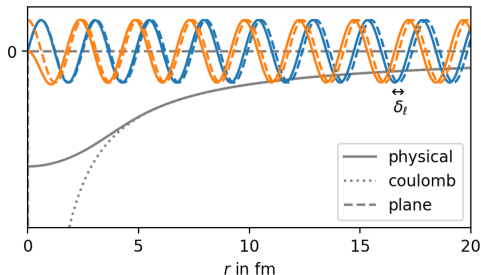
Coulomb corrections



- Coulomb corrections **fill out minima** and shift the crosssection

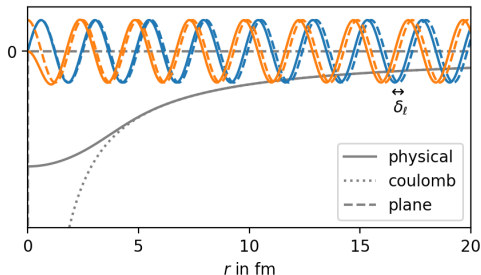
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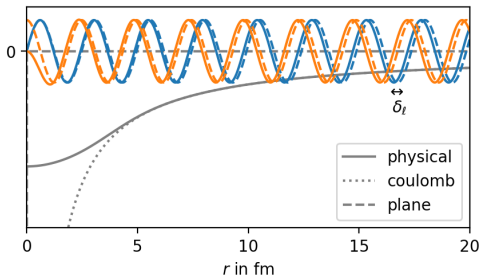


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phase-shift model: Solve Dirac-eq. $\forall \ell : \left[\psi_\ell \sim \begin{pmatrix} g_\ell(r) \\ if_\ell(r) \end{pmatrix} \right] \rightarrow \delta_\ell = \delta_\ell^{\text{C}} + \bar{\delta}_\ell$

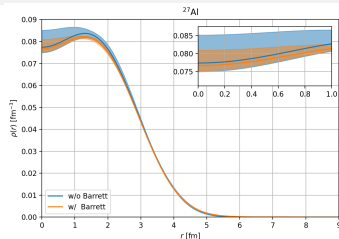
$$\Rightarrow \frac{d\sigma}{d\Omega} \sim (1 + \tan^2(\frac{\theta}{2})) |f(\theta)|^2 \quad \text{with} \quad f(\theta) \sim \sum_{\ell} P_{\ell}(\cos(\theta)) e^{2i\delta_\ell}$$

Extracting charge densities from electron scattering

- **Fourier-Bessel** parameterization:

($q_n = \frac{n\pi}{R}$ s.t. $j_0(q_n R) = 0$) [Dreher et al., 1974]

$$\rho_0(r) = \begin{cases} \sum_{n=1}^N a_n j_0(q_n r) & , r \leq R \\ 0 & , r > R \end{cases}$$



[FN, Hoferichter, 2024]

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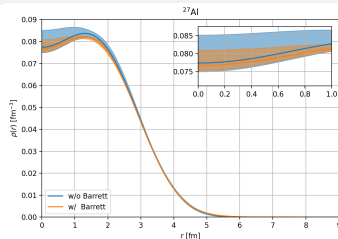
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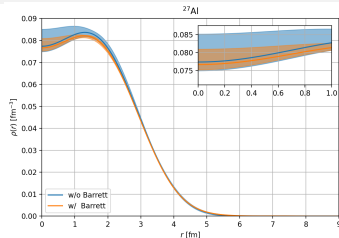
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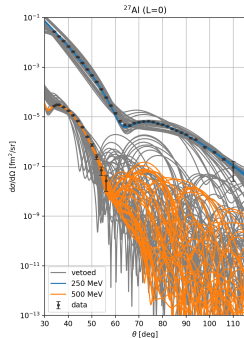
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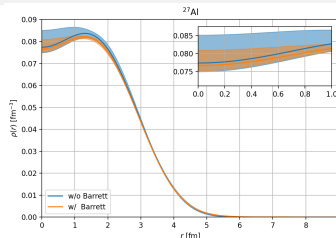
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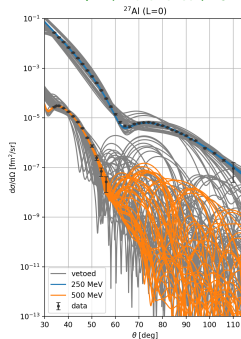
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Carried out for ^{27}Al , $^{40,48}\text{Ca}$, $^{48,50}\text{Ti}$

Results available in **python notebook** [2406.06677]



[FN, Hoferichter, 2024]



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Conversion
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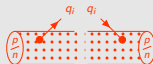
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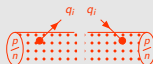
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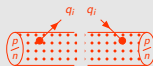
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[Kitano et al., 2002]

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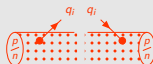
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Employ extracted **charge densities** with uncertainties

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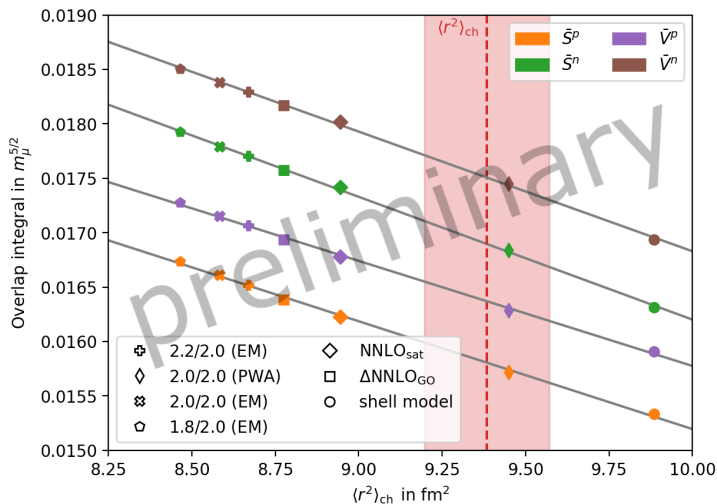
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Alternative: Correlate using ab-initio methods

Correlations



○ Calculated using IMSRG for ^{27}Al

[see also: talk by Takayuki Miyagi, Tu, 16:30, WG3]

Conclusion

Summary:

- LFV is a promising BSM probe with lots of experimental developments
- EFT for $\mu \rightarrow e$ conversion in nuclei
 - Discriminate LFV mechanisms
 - Controlled uncertainty estimates

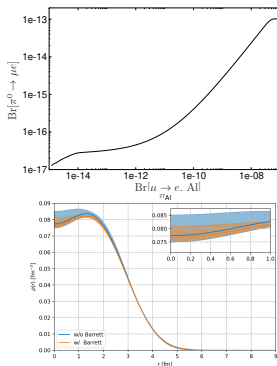
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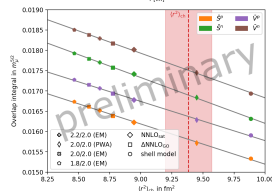
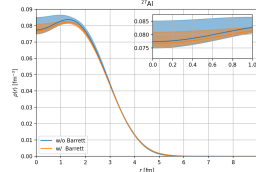
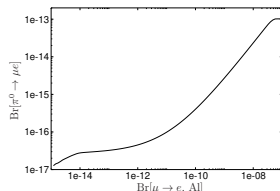
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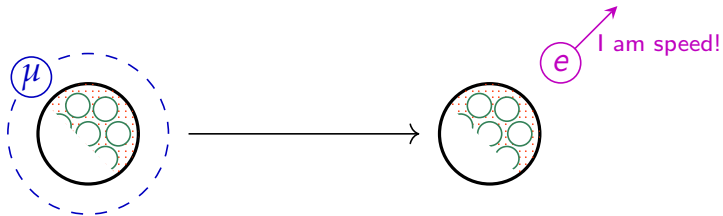
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Outlook:

- **Overlap integrals** from ab-initio calculations
- **Phase-shift model** python package
- Subleading nuclear responses; two-body currents



Thank you for your attention!



References I

- Hoferichter, M., J. Menéndez, and F. Noël (Apr. 2023).
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- Noël, F. and M. Hoferichter (2024). “Uncertainty quantification for $\mu \rightarrow e$ conversion in nuclei: charge distributions”.
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Backup-Slides

Description of $\mu \rightarrow e$ conversion

Effective description by separation of the appearing scales



- EFT operators** from Lagrangian: $L^\Gamma \in \{e\bar{\gamma}\mu, e\bar{\gamma}\gamma_\mu\mu, e\bar{\gamma}\sigma_{\mu\nu}\mu\}, \quad (\Gamma = S, P, V, A, T, D, GG, G\tilde{G})$
 $\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_\Gamma \mathcal{C}_q^\Gamma (L^\Gamma \cdot Q^{\Gamma,q}) \quad Q^{\Gamma,q} \in \{\bar{q}q, \bar{q}\gamma^5 q, \bar{q}\gamma^\mu q, \bar{q}\gamma^\mu\gamma^5 q, \bar{q}\sigma^{\mu\nu}q, F^{\mu\nu}, G_{\mu\nu}^a G_a^{\mu\nu}, G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}\}$

- hadronic matrix elements:**

$$\langle N | Q^{\Gamma,q} | N \rangle \rightarrow \sim \mathcal{F}_{q,N}^{\Gamma,i} \bar{u}_N \mathcal{O}_i u_N \xrightarrow{\text{non.rel.}} \sim \bar{u}_N^{\text{NR}} \mathcal{O}_i^{\text{NR}} u_N^{\text{NR}}$$

- nuclear multipoles** (shell-model):

$$\langle M | \mathcal{O}_i^{\text{NR}} | M \rangle \rightarrow \sim \mathcal{F}^S \mathcal{N}$$

$$\mathcal{O}_i^{\text{NR}} \in \{\mathbb{1}, \vec{\sigma}, \vec{\nabla}, \dots \text{and all combinations}\}$$

$$S \in \{M, \Sigma^{(n)}, \Phi^{(n)}, \Delta^{(n)}, \Omega^{(n)}, \Gamma^{(n)}, \Pi^{(n)}, \Theta^{(n)}\}$$

- bound state physics** (numerical):

$$\langle \tilde{e} | L^\Gamma | \mu(1s) \rangle \rightarrow \sim \overline{\Psi}_e \mathcal{O}_\Gamma \Psi_\mu \text{ with } \Psi_e, \Psi_\mu \xleftarrow{\text{Dirac-eq.}} V(r) \leftarrow \rho_{\text{ch}}(r)$$

Master Formula: $P \rightarrow \bar{\mu} e$

Decay
Rate =



hadronic matrix elements



(short distance) EFT operator

$$\text{Br}_{P \rightarrow \mu^\mp e^\pm} = \frac{(M_P^2 - m_\mu^2)^2}{16\pi\Gamma_P M_P^3} \sum_{Y=L,R} |C_Y^P|^2$$

$$C_Y^P = \sum_q \frac{b_q}{\Lambda^2} \left(\pm C_Y^{A,q} f_P^q m_\mu - C_Y^{P,q} \frac{h_P^q}{2m_q} \right) + \frac{4\pi}{\Lambda^3} C_Y^{G\tilde{G}} a_P$$

- only contributions from:

$P, A, G\tilde{G}$

- hadronic matrix elements from lattice-QCD and phenomenology
- Ward identity:

$$b_q f_P^q M_P^2 = b_q h_P^q - a_P$$

	π	η		η'	
		Pheno	Lattice	Pheno	Lattice
$\frac{b_u f_P^u}{F_\pi}$	1	0.80	0.77	0.66	0.56
$\frac{b_d f_P^d}{F_\pi}$	-1	0.80	0.77	0.66	0.56
$\frac{b_s f_P^s}{F_\pi}$	0	-1.26	-1.17	1.45	1.50
$a_P [\text{GeV}^3]$	0	-	-0.017	-	-0.038
$a_P^{\text{FKS}} [\text{GeV}^3]$	0	-0.022	-0.021	-0.056	-0.048
h_P^q	Ward identity				

Phenomenology: [Escribano et al., 2016]

Lattice-QCD: [Bali et al., 2021]

Master Formula: SD $\mu \rightarrow e$ conversion



$$\text{Br}_{\mu \rightarrow e}^{\text{SD}} = \frac{4m_\mu^5 \alpha^3 Z^3}{\pi \Gamma_{\text{cap}} (2J+1)} \left(\frac{Z_{\text{eff}}}{Z} \right)^4 \times \sum_{\substack{Y=L,R \\ \tau=L,T}} \left[C_Y^{\tau,00} S_{00}^\tau + C_Y^{\tau,11} S_{11}^\tau + C_Y^{\tau,01} S_{01}^\tau \right]$$

$$C_Y^{T,ij} = \left[\bar{C}_Y^{A,i} (1 + \delta')^i \pm 2 \bar{C}_Y^{T,i} \right] \times (i \leftrightarrow j); \quad C_Y^{L,ij} = \left[\bar{C}_Y^{A,i} (1 + \delta'')^i - \frac{m_\mu}{2m_N} \bar{C}_Y^{P,i} \pm 2 \bar{C}_Y^{T,i} \right] \times (i \leftrightarrow j)$$

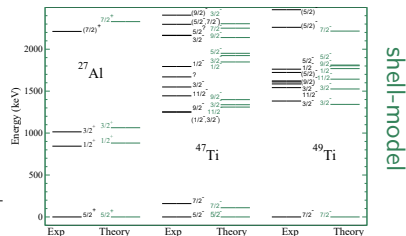
$$\bar{C}_Y^{P,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{P,q} \frac{m_N}{m_q} g_5^{q,N} - \frac{4\pi}{\Lambda^3} C_Y^{G\bar{G}} \bar{a}_N; \quad \bar{C}_Y^{A,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{A,q} g_A^{q,N}; \quad \bar{C}_Y^{T,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{T,q} f_{1,T}^{q,N}$$

- numerical solution of Dirac equation:

$$Z_{\text{eff}}^{\text{Al}} = 11.64, \quad Z_{\text{eff}}^{\text{Ti}} = 17.65 \quad [\text{Kitano et al., 2002}]$$

- corr. from NLO chiral EFT and 2-body currents: $\delta' = -0.28(5)$, $\delta'' = -0.44(4)$

[Hoferichter et al., 2020]



$g_A^{u,p}$	$g_A^{d,p}$	$g_A^{s,N}$	$\bar{a}_N [\text{GeV}]$	$g_5^{q,N}$
0.842(12)	-0.427(13)	-0.085(18)	-0.39(12) [$N_C \rightarrow \infty$]	Ward identity
[HERMES, 2007]				

Outlook: Full Masterformula for $\mu \rightarrow e$ conversion



- effective Lagrangian with all possible **quark and gluon operators**:

$$\Gamma \in S, P, V, A, T, D, GG, G\tilde{G}$$

- hadronic matrix elements** (including higher order terms): $F_{q,N}^{\Gamma,i}$
- nuclear multipoles** (beyond SD and SI):

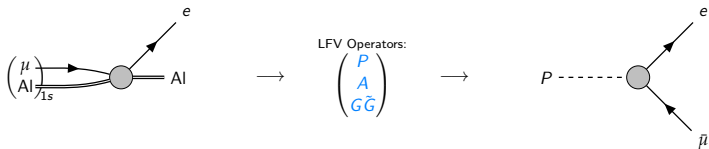
$$\mathcal{S} \in M, \Sigma^{(n)}, \Phi^{(n)}, \Delta^{(n)}, \Omega^{(n)}, \dots$$

- full numerical solution of **muon and electron wave functions**

$$\mathcal{M} \sim \int \frac{d^3 q}{(2\pi)^3} \sum_{\Gamma, q, i, N, S} K_{q,N}^{\Gamma,i,S_N}(\vec{q}) \cdot \mathcal{C}_q^\Gamma \cdot F_{q,N}^{\Gamma,i}(\vec{q}) \cdot \mathcal{F}^{S_N}(\vec{q}) \cdot \overline{\Psi_e} \mathcal{O}_\Gamma \Psi_\mu(\vec{q})$$

Deduced Limits (individual)

- Use limits on $\mu \rightarrow e$ conversion to derive limits on $P \rightarrow \bar{\mu}e$



- In general the operators do **not** appear in the same linear combinations
- If we consider **one operator at a time**, the transition is immediate:

$\mu \rightarrow e$ (exp.)	$P \rightarrow \bar{\mu}e$ (derived)	current limit
$\text{BR}_{\text{Ti}} < 6.1 \times 10^{-13}$	$\text{BR}_{\pi^0} \lesssim 4 \times 10^{-17}$	$< 3.6 \times 10^{-10}$
	$\text{BR}_{\eta} \lesssim 5 \times 10^{-13}$	$< 6.0 \times 10^{-6}$
	$\text{BR}_{\eta'} \lesssim 7 \times 10^{-14}$	$< 4.7 \times 10^{-4}$

(scan over all "one operator at a time"-scenarios and choices for constants)

Derived limits are several **orders of magnitude** better!

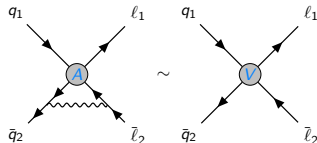
Deduced Limits (rigorous)

For rigorous limits we need to **scan over all Wilson coefficients**:

- Maximise:
$$\frac{\Gamma_{P \rightarrow \bar{\mu} e}(C_P, C_A, C_{G\tilde{G}})}{\Sigma_{\mu \rightarrow e}(C_P, C_A, C_{G\tilde{G}})}$$
 - $\rightarrow \exists$ **fine-tuned** solution: $\Sigma_{\mu \rightarrow e} \stackrel{!}{=} 0$
- In this scenario $\Gamma_{\pi^0 \rightarrow \bar{\mu} e}$ **vanishes** as well:

rigorous limit: $\text{Br}_{\pi^0 \rightarrow \bar{\mu} e} < 1.0 \times 10^{-13}$ (exp: $< 3.6 \cdot 10^{-10}$)

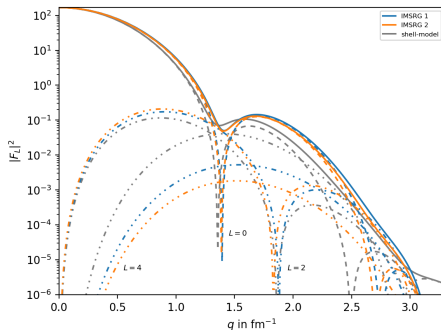
- However, $\Gamma_{\eta^{(\prime)} \rightarrow \bar{\mu} e}$ **can still be non-zero**:
 - $\rightarrow \text{Br}_{\eta^{(\prime)} \rightarrow \bar{\mu} e}$ with sufficient fine-tuning **in principle unbound**
- easily spoilt by **RG corrections**
- contributing to SI $\mu \rightarrow e$ conversion



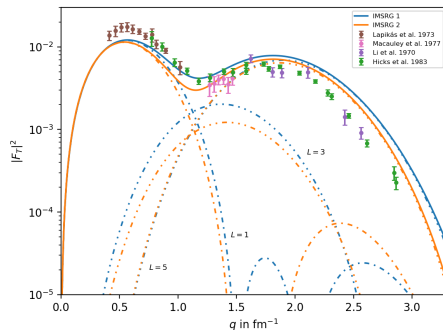
Ab initio inputs

$^{27}\text{Al} (J = \frac{5}{2})$ requires $L > 0$ contributions

ZF_L^{ch}

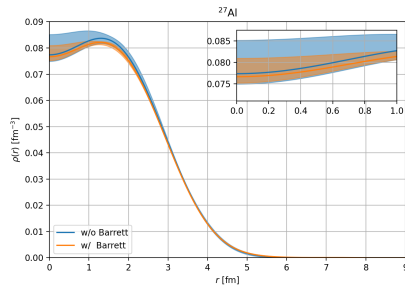
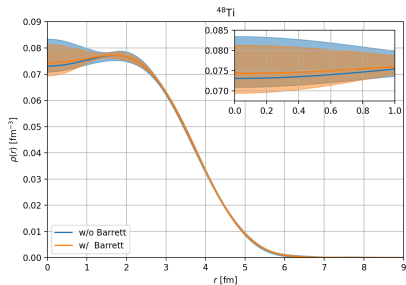
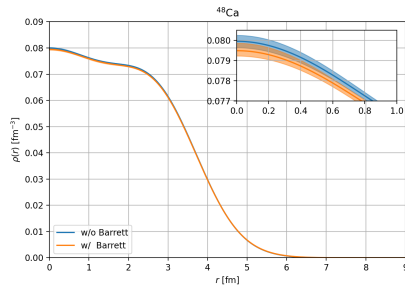
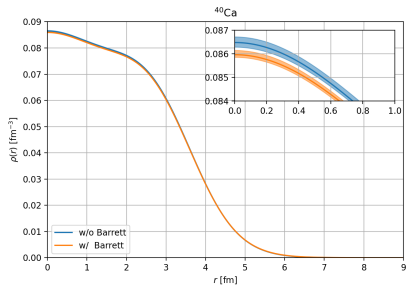


F_L^{mag}



- Subtract and remove data points dominated by $L > 0$
- So far: No Coulomb corrections for $L > 0$ (requires DWBA)

Charge density results



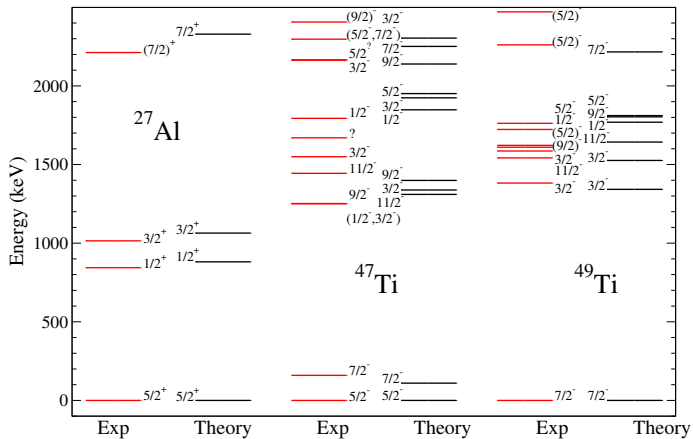
Radii

- Qualitative radii for the considered nuclei
- Statistical uncertainties
 - based on fit statistics and data uncertainties
- Systematical uncertainties
 - based on different R , N with two strategies

All parameterizations with uncertainties and correlations are made available in a complementary python notebook

Nucleus	$\sqrt{\langle r^2 \rangle}$ [fm]	Refs.
^{27}Al	$2.996(11) \begin{smallmatrix} (43) & [44] \\ (+26) & [-33] \end{smallmatrix} [35]$	3.035(2)
	$3.063(3) \begin{smallmatrix} (30) & [31] \\ (+0) & [-1] \end{smallmatrix} [3]$	3.0610(31)
^{40}Ca	$3.452(3) \begin{smallmatrix} (8) & [9] \\ (+1) & [-9] \end{smallmatrix} [10]$	3.450(10)
	$3.4771(17) \begin{smallmatrix} (17) & [24] \\ (+0) & [-5] \end{smallmatrix} [17]$	3.4776(19)
^{48}Ca	$3.4499(29) \begin{smallmatrix} (31) & [42] \\ (+42) & [-52] \end{smallmatrix} [60]$	3.451(9)
	$3.475(2) \begin{smallmatrix} (10) & [10] \\ (+0) & [-3] \end{smallmatrix} [4]$	3.4771(20)
^{48}Ti	$3.62(3) \begin{smallmatrix} (8) & [8] \\ (+2) & [-3] \end{smallmatrix} [4]$	3.597(1)
	$3.596(3) \begin{smallmatrix} (57) & [57] \\ (+1) & [-1] \end{smallmatrix} [3]$	3.5921(17)

Shell model spectrum



Formulas I

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(k) \rangle = i b_q f_P^q k^\mu, \quad (1)$$

$$\langle 0 | m_q \bar{q} i \gamma_5 q | P(k) \rangle = \frac{b_q h_P^q}{2}, \quad (2)$$

$$\langle 0 | \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | P(k) \rangle = a_P, \quad (3)$$

$$\langle N | \bar{q} \gamma^\mu \gamma_5 q | N \rangle = g_A^{q,N} \langle N | \bar{N} \gamma^\mu \gamma_5 N | N \rangle, \quad (4)$$

$$m_q \langle N | \bar{q} i \gamma_5 q | N \rangle = m_N g_5^{q,N} \langle N | \bar{N} i \gamma_5 N | N \rangle, \quad (5)$$

$$\langle N | \bar{q} \sigma^{\mu\nu} q | N \rangle = f_{1,T}^{q,N} \langle N | \bar{N} \sigma^{\mu\nu} N | N \rangle, \quad (6)$$

$$\langle N | \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} | N \rangle = \tilde{a}_N \langle N | \bar{N} i \gamma_5 N | N \rangle, \quad (7)$$

Formulas II

$$\text{Br}_{\text{SI}}[\mu \rightarrow e] = \frac{4m_\mu^5}{\Gamma_{\text{cap}}} \sum_{Y=L,R} \left| \sum_{\substack{N=p,n \\ \mathcal{O}=S,V}} \bar{c}_Y^{\mathcal{O},N} \mathcal{O}^{(N)} \right|^2, \quad (8)$$

$$\bar{c}_Y^{S,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{S,q} \frac{m_N}{m_q} f_q^N + \frac{4\pi}{\Lambda^3} C_Y^{GG} a_N, \quad (9)$$

$$\bar{c}_Y^{V,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{V,q} f_{V_q}^N, \quad (10)$$

$$S^{(N)} = V^{(N)} = \frac{(\alpha Z)^{3/2}}{4\pi} \left(\frac{Z_{\text{eff}}}{Z} \right)^2 \mathcal{F}_N^M(m_\mu^2), \quad (11)$$

Formulas III

$$\bar{C}^0 = \frac{\bar{C}^p + \bar{C}^n}{2}, \quad \bar{C}^1 = \frac{\bar{C}^p - \bar{C}^n}{2}, \quad (12)$$

$$g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}, \quad (13)$$

$$\tilde{a}_N = -2m_N g_A^{u,0} = -0.39(12) \text{ GeV}, \quad (14)$$

Formulas IV

$$C_Y^{A,u} = C_Y^{A,d}, \quad C_Y^{A,s} = -\frac{2C_Y^{A,u}g_A^{u,0}}{g_A^{s,N}}, \quad (15)$$

$$\frac{C_Y^{P,u}}{m_u} = \frac{C_Y^{P,d}}{m_d}, \quad \frac{C_Y^{P,s}}{m_s} = \frac{4\pi}{\Lambda} C_Y^{G\tilde{G}} \frac{2g_A^{u,0}}{g_A^{u,0} - g_A^{s,N}}. \quad (16)$$

Formulas V

$$S_{00}^{\mathcal{T}} = \sum_L \left[\mathcal{F}_+^{\Sigma'_L}(q^2) \right]^2, \quad S_{00}^{\mathcal{L}} = \sum_L \left[\mathcal{F}_+^{\Sigma''_L}(q^2) \right]^2, \quad (17)$$

$$S_{11}^{\mathcal{T}} = \sum_L \left[\mathcal{F}_-^{\Sigma'_L}(q^2) \right]^2, \quad S_{11}^{\mathcal{L}} = \sum_L \left[\mathcal{F}_-^{\Sigma''_L}(q^2) \right]^2, \quad (18)$$

$$S_{01}^{\mathcal{T}} = \sum_L 2 \mathcal{F}_+^{\Sigma'_L}(q^2) \mathcal{F}_-^{\Sigma'_L}(q^2), \quad (19)$$

$$S_{01}^{\mathcal{L}} = \sum_L 2 \mathcal{F}_+^{\Sigma''_L}(q^2) \mathcal{F}_-^{\Sigma''_L}(q^2), \quad (20)$$

Table

	π^0	η	η'
$C_Y^{A,3}$	1.3×10^{-17}	—	—
$C_Y^{A,8}$	—	1.5×10^{-17}	4.0×10^{-20}
$C_Y^{A,0}$	—	2.9×10^{-19}	2.1×10^{-19}
$C_Y^{P,3}$	4.1×10^{-17}	—	—
$C_Y^{P,8}$	—	1.6×10^{-12}	2.1×10^{-14}
$C_Y^{P,0}$	—	4.1×10^{-12}	5.4×10^{-13}
$C_Y^{G\tilde{G}}$	—	5.8×10^{-15}	4.7×10^{-16}