Study of dark matter scattering off $^{2}\mathrm{H}$ and $^{4}\mathrm{He}$ nuclei within Chiral effective field theory

Elena Filandri[1], Michele Viviani [1]

[1]Università di Pisa, INFN Sezione Pisa

August 26-30, 2024

The 11th International Workshop on Chiral Dynamics (CD2024)

Istituto Nazionale di Fisica Nucleare

Università di Pisa

Motivations

- On galactic scales and over, a great number of gravitational anomalies has been detected
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]
- In order to analyze the results of the various direct detection experiments an accurate description of the nuclear response is required
- Light nuclei are great testing laboratories
  - Helium isotopies and $^6\mathrm{Li}$ are potential experimental target [W. Guo *et al.*, Phys. Rev. D 87 (2013)],[CRESST Collab., Eur.J. Phys.C 82 (2022)]

Our purpose is the study of the $^2\mathrm{H}$-$^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and transferred energy are small
- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Motivations

- On galactic scales and over, a great number of gravitational anomalies has been detected
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]
- In order to analyze the results of the various direct detection experiments an accurate description of the nuclear response is required
- Light nuclei are great testing laboratories
  - Helium isotopies and $^6\mathrm{Li}$ are potential experimental target [W. Guo *et al.*, Phys. Rev. D 87 (2013)],[CRESST Collab., Eur.J. Phys.C 82 (2022)]

Our purpose is the study of the $^2\mathrm{H}$-$^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and transferred energy are small
- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Motivations

- On galactic scales and over, a great number of gravitational anomalies has been detected
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]
- In order to analyze the results of the various direct detection experiments an accurate description of the nuclear response is required
- Light nuclei are great testing laboratories
  - Helium isotopies and $^6\mathrm{Li}$ are potential experimental target [W. Guo *et al.*, Phys. Rev. D 87 (2013)],[CRESST Collab., Eur.J. Phys.C 82 (2022)]

Our purpose is the study of the $^2\mathrm{H}$-$^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and transferred energy are small
- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Motivations

- On galactic scales and over, a great number of gravitational anomalies has been detected
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]

- In order to analyze the results of the various direct detection experiments an accurate description of the nuclear response is required

- Light nuclei are great testing laboratories
  - Helium isotopies and $^6\mathrm{Li}$ are potential experimental target [W. Guo *et al.*, Phys. Rev. D 87 (2013)],[CRESST Collab., Eur.J. Phys.C 82 (2022)]

Our purpose is the study of the $^2\mathrm{H}$-$^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic $|\frac{v_\chi}{c}| \sim 10^{-3}$

- The typical momentum and transferred energy are small

- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Motivations

- On galactic scales and over, a great number of gravitational anomalies has been detected
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]
- In order to analyze the results of the various direct detection experiments an accurate description of the nuclear response is required
- Light nuclei are great testing laboratories
  - Helium isotopies and $^6\mathrm{Li}$ are potential experimental target [W. Guo *et al.*, Phys. Rev. D 87 (2013)],[CRESST Collab., Eur.J. Phys.C 82 (2022)]

Our purpose is the study of the $^2\mathrm{H}$-$^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and transferred energy are small
- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Effective quark-WIMP interactions

We start from the general dimension 6 effective Lagrangian for the interaction between quark and WIMP, the latter assumed to be a Dirac fermion [M. Hoferichter et al., Phys. Rev. Lett. (2016)]

$$\begin{aligned}\mathcal{L}_q &= \mathcal{L}_{\mathrm{QCD}}^{\mathcal{M}=0} + \bar{q}(x)\gamma^\mu \left(v_\mu(x) + \frac{1}{3} v_\mu^{(s)}(x) + \gamma^5 a_\mu(x) \right) q(x) \\ & \quad - \bar{q}(x)\left(s(x) - i\gamma^5 p(x)\right) q(x) + \bar{q}(x)\sigma^{\mu\nu} t_{\mu\nu}(x) q(x)\end{aligned}$$

$$s(x) = -\frac{1}{\Lambda_S^2}\left(C_{S+} + C_{S-}\tau_z\right)\bar{\chi}\chi \qquad p(x) = \frac{1}{\Lambda_S^2}\left(C_{P+} + C_{P-}\tau_z\right)\bar{\chi} i\gamma_5\chi$$

$$\frac{1}{3} v^{\mu(s)}(x) = \frac{1}{\Lambda_S^2} C_{V+}\bar{\chi}\gamma^\mu\chi \qquad v^\mu(x) = \frac{1}{\Lambda_S^2} C_{V-}\tau_z\bar{\chi}\gamma^\mu\chi$$

$$t^{\mu\nu}(x) = \frac{1}{\Lambda_S^2}\left(C_{T+} + C_{T-}\tau_z\right)\bar{\chi}\sigma^{\mu\nu}\chi \qquad a^\mu(x) = \frac{1}{\Lambda_S^2} C_{A-}\tau_z\bar{\chi}\gamma^\mu\gamma_5\chi$$

$C_{X\pm}$ adimensional coupling constant to be determined by experimental data and $\Lambda_S = 1$ GeV

Differences with respect to other approaches

- Presence of an isoscalar part $a_\mu^{(s)}(x) \to SU(3)$
- Presence of tensor current $t^{\mu\nu} \to$ new terms in the χEFT Lagrangian

Nucleon-WIMP interactions

[Bishara et al. J. High Energ. Phys. (2016)]

- The nucleon-WIMP interaction vertices are derived substituting the expression of the currents s,p,v, . . . in the hadron χEFT Lagrangians
  - NR expansion of H_{int} up to $O(1/M^2)$

Nucleon-WIMP interactions

[Bishara et al. J. High Energ. Phys. (2016)]

- The nucleon-WIMP interaction vertices are derived substituting the expression of the currents s,p,v, ... in the hadron χEFT Lagrangians
  - NR expansion of H_{int} up to $O(1/M^2)$
- The T-matrix for the elastic scattering of a WIMP by a two-nucleon system is obtained using the time-ordered perturbation theory (TOPT) method
  - Each term contributing to the T-matrix can be visualised as a TOPT diagram: A chiral order ν can be assigned to each diagram $\Rightarrow$ $(Q/\Lambda_\chi)^\nu$ ($Q \sim m_\pi$ = nucleon momentum, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV)

$\Rightarrow$ In this study we will consider contributions up to the next-to-next-lo-leading-order (N2LO)

Nucleon-WIMP interactions

[Bishara et al. J. High Energ. Phys. (2016)]

- The nucleon-WIMP interaction vertices are derived substituting the expression of the currents s,p,v, ... in the hadron χEFT Lagrangians
  - NR expansion of H_{int} up to $O(1/M^2)$
- The T-matrix for the elastic scattering of a WIMP by a two-nucleon system is obtained using the time-ordered perturbation theory (TOPT) method
  - Each term contributing to the T-matrix can be visualised as a TOPT diagram: A chiral order ν can be assigned to each diagram $\Rightarrow$ $(Q/\Lambda_\chi)^\nu$ ($Q \sim m_\pi$ = nucleon momentum, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV)

$\Rightarrow$ In this study we will consider contributions up to the next-to-next-lo-leading-order (N2LO)

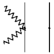
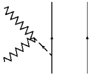
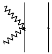
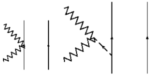
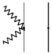
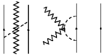
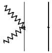
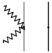
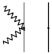
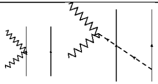
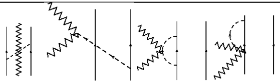
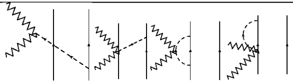
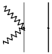
The T-matrix has the following general form

$$T_{fi} = \left\{ \frac{1}{\Omega} \left(J^{(1)}_{\alpha_1,\alpha_1'} \delta_{\mathbf{p}_1'+\mathbf{k}',\mathbf{p}_1+\mathbf{k}} \delta_{\alpha_2',\alpha_2} + J^{(1)}_{\alpha_2,\alpha_2'} \delta_{\mathbf{p}_2'+\mathbf{k}',\mathbf{p}_2+\mathbf{k}} \delta_{\alpha_1',\alpha_1} \right) + \frac{1}{\Omega^2} J^{(2)}_{\alpha_1,\alpha_1',\alpha_2,\alpha_2'} \delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}-\mathbf{k}'} \right\} \cdot L_{\mathbf{k}r,\mathbf{k}'r'}$$

α_i (α_i')$\equiv$ $\{\mathbf{p}_i, s_i, t_i\}$ = nucleon state i, $\mathbf{k}$ ($\mathbf{k}'$)=initial (final) WIMP momentum and r (r') its spin projection, $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$, $\mathbf{K}_i = (\mathbf{p}_i + \mathbf{p}_i')/2$, $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ and $\mathbf{Q} = (\mathbf{k} + \mathbf{k}')/2$

- $J^{(1)}$ ($J^{(2)}$) is the so-called one-body (two-body) current, while L is the so-called WIMP current
- We neglect eventual three-body transition currents

Considered diagrams

Chiral order	S	P	V	A	T
Q^{-3}					
Q^{-2}		—		—	
Q^{-1}					

Nucleus-WIMP cross-section

$T_{f,i}$ between the initial nucleus+WIMP state $|\mathbf{P}_i, M_i, \mathbf{k}, r\rangle$ and final nucleus+WIMP state $|\mathbf{P}_f, M_f, \mathbf{k}', r'\rangle$ is

$$\langle \mathbf{P}_f \mathbf{k}' | T | \mathbf{P}_i \mathbf{k} \rangle = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i, \mathbf{P}_f} \int e^{i\mathbf{q}\cdot\mathbf{x}} \langle \Psi_A^{J_A, s_A'} | J^{Xa}{}_c(\mathbf{x}) | \Psi_A^{J_A, s_A} \rangle (L^{Xa})^c d\mathbf{x}\,, \qquad J_c^{Xa}(\mathbf{x}) = \sum_{i=1,A} J^{Xa,(1)}{}_c(i)\delta(\mathbf{x}-\mathbf{r}_i) + \sum_{i<j} J^{Xa,(2)}{}_c(i,j)\delta(\mathbf{x}-\mathbf{R}_{ij})\,.$$

Performing the multipole expansion of the current

$$T_{f,i} = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i, \mathbf{P}_f} (-1)^{J-J_z} \Bigg(\sum_{l\geq 0}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,0}(\varphi, \theta, -\varphi) \sqrt{4\pi} (J' J_z' J - J_z | lm) \{ L_0 X_l^C(q,r) + L_z X_l^L(q,r) \}$$

$$+ \sum_{l=1}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,\lambda}(\varphi, \theta, -\varphi) \sqrt{2\pi} (J' J_z' J - J_z | lm) \times L_{-\lambda} \left\{ -\lambda X_l^M(q,r) - X_l^E(q,r) \right\} \Bigg)$$

$$\mathcal{D}^l_{m,m'}(\alpha, \beta, \gamma) = \text{rotation matrices}, \quad \theta, \varphi = \text{angles of } \mathbf{q}, \quad \mathbf{q} = \mathbf{k} - \mathbf{k}', \quad X_l^Y = \text{RMEs}$$

Nucleus-WIMP cross-section

$T_{f,i}$ between the initial nucleus+WIMP state $|\mathbf{P}_i, M_i, \mathbf{k}, r\rangle$ and final nucleus+WIMP state $|\mathbf{P}_f, M_f, \mathbf{k}', r'\rangle$ is

$$\langle \mathbf{P}_f \mathbf{k}' | T | \mathbf{P}_i \mathbf{k} \rangle = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} \int e^{i\mathbf{q}\cdot\mathbf{x}} \langle \Psi_A^{J_A,s'_A} | J^{Xa}{}_c(\mathbf{x}) | \Psi_A^{J_A,s_A} \rangle (L^{Xa})^c \, d\mathbf{x} \,, \qquad J_c^{Xa}(\mathbf{x}) = \sum_{i=1,A} J^{Xa,(1)}{}_c(i)\delta(\mathbf{x}-\mathbf{r}_i) + \sum_{i<j} J^{Xa,(2)}{}_c(i,j)\delta(\mathbf{x}-\mathbf{R}_{ij}) \,.$$

Performing the multipole expansion of the current

$$T_{f,i} = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} (-1)^{J-J_z} \Bigg(\sum_{l\geq 0}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,0}(\varphi,\theta,-\varphi)\sqrt{4\pi}(J' J'_z J - J_z | lm) \{ L_0 X_l^C(q,r) + L_z X_l^L(q,r) \}$$

$$+ \sum_{l=1}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,\lambda}(\varphi,\theta,-\varphi)\sqrt{2\pi}(J' J'_z J - J_z | lm) \times L_{-\lambda} \left\{ -\lambda X_l^M(q,r) - X_l^E(q,r) \right\} \Bigg)$$

$$\mathcal{D}^l_{m,m'}(\alpha,\beta,\gamma) = \text{rotation matrices}, \quad \theta,\varphi = \text{angles of } \mathbf{q}, \quad \mathbf{q} = \mathbf{k} - \mathbf{k}', \quad X_l^Y = \text{RMEs}$$

- Nuclear wave function evaluated using the Hyperspheric Harmonics method with both phenomenological (Av18) and chiral N4LO potentials [M. Viviani et al., Phys. Rev. Lett (2023)]

Nucleus-WIMP cross-section

$T_{f,i}$ between the initial nucleus+WIMP state $|\mathbf{P}_i, M_i, \mathbf{k}, r\rangle$ and final nucleus+WIMP state $|\mathbf{P}_f, M_f, \mathbf{k}', r'\rangle$ is

$$\langle \mathbf{P}_f \mathbf{k}' | T | \mathbf{P}_i \mathbf{k} \rangle = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} \int e^{i\mathbf{q}\cdot\mathbf{x}} \langle \Psi_A^{J_A, s'_A} | J^{Xa}{}_c(\mathbf{x}) | \Psi_A^{J_A, s_A} \rangle (L^{Xa})^c d\mathbf{x}\,, \qquad J_c^{Xa}(\mathbf{x}) = \sum_{i=1,A} J^{Xa,(1)}{}_c(i)\delta(\mathbf{x}-\mathbf{r}_i) + \sum_{i<j} J^{Xa,(2)}{}_c(i,j)\delta(\mathbf{x}-\mathbf{R}_{ij})\,.$$

Performing the multipole expansion of the current

$$T_{f,i} = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} (-1)^{J-J_z} \Bigg(\sum_{l\geq 0}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,0}(\varphi,\theta,-\varphi) \sqrt{4\pi} (J' J'_z J - J_z | l m) \{ L_0 X_l^C(q,r) + L_z X_l^L(q,r) \}$$

$$+ \sum_{l=1}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,\lambda}(\varphi,\theta,-\varphi) \sqrt{2\pi} (J' J'_z J - J_z | l m) \times L_{-\lambda} \left\{ -\lambda X_l^M(q,r) - X_l^E(q,r) \right\} \Bigg)$$

$$\mathcal{D}^l_{m,m'}(\alpha,\beta,\gamma) = \text{rotation matrices}, \quad \theta,\varphi = \text{angles of } \mathbf{q}, \quad \mathbf{q} = \mathbf{k} - \mathbf{k}', \quad X_l^Y = \text{RMEs}$$

- Nuclear wave function evaluated using the Hyperspheric Harmonics method with both phenomenological (Av18) and chiral N4LO potentials [M. Viviani et al., Phys. Rev. Lett (2023)]

The non-polarized cross section for this process is calculated from Fermi golden rule, $d^2\sigma_{fi}$ can be written as

$$\frac{d^2\sigma}{dE'_A d\hat{\mathbf{P}}'_A} = \frac{4\pi^2}{(2J_A+1)(2\pi)^3} \frac{C_{Xa}^2}{\Lambda_S^4} \sum_{r'r} M_A\, \delta\left(\mathbf{v}\cdot\hat{\mathbf{P}}'_A - \frac{\mathbf{P}'_A}{2\mu} \right) \frac{1}{v} \Bigg\{ \sum_{l\geq 0} \left[L_0 L_0{}^* |X_l^C|^2 + L_z L_z{}^* |X_l^L|^2 - 2 L_0 L_z{}^* Re(X_l^C X_l^{L*}) \right] + \sum_{l\geq 1} L_1 L_1^* (|X_l^M|^2 + |X_l^E|^2) \Bigg\}$$

J_A = the spin of the target nuclei, $E'_A = \sqrt{M_A^2 + \mathbf{P}'^2_A} - M_A$ and $\mathbf{v} = \mathbf{k}/M_\chi$.

Interaction rate

The double-differential interation rate is given by,

$$\frac{d^2R}{dE'_A \hat{\mathbf{P}'_A}} = N_\chi N_A \int d^3\mathbf{v}\, v\, f(\mathbf{v}) \frac{d^2\sigma}{dE'_A d\hat{\mathbf{P}'_A}}$$

$$N_A = \frac{100\ \mathrm{ton}}{\mathrm{Nucleus\ Mass}}\ \text{(arbitarily chosen)}$$

$$N_\chi = \frac{0.6\ \mathrm{GeV/cm^3}}{\mathrm{WIMP\ Mass}}$$ [*Cadeddu et al.* (2017)]

We assume the Standard Halo Model (SHM) [Cadeddu *et al.*, Astro-ph (2017)]

$$f(\mathbf{v}) = \frac{1}{\sqrt{(2\pi\sigma_v^2)^3}} e^{-\frac{1}{2}\left(\frac{\mathbf{v}+\mathbf{V}}{\sigma_v}\right)^2} \qquad \sigma_V = \frac{V}{\sqrt{2}} \qquad V \approx 220\,\mathrm{km/s}\ \text{(Sun velocity)}$$

Sum over the WIMP spin

$$\sum_{r'r} L_i L_j^* = a + \mathbf{b}\cdot\mathbf{u} + cu^2 + (\mathbf{d}\cdot\mathbf{u})^2 + O(\mathbf{u})^3 \qquad a, \mathbf{b}, c, \mathbf{d} = \text{constant terms} \qquad \mathbf{u} = \mathbf{v} + \mathbf{V}$$

After integrating over $\mathbf{u}$

$$\frac{d^2R}{dE'_A d\hat{\mathbf{P}'_A}} = \frac{N_\chi N_A M_A}{(2J_A+1)4\pi^2} \frac{C_{Xa}^2}{\Lambda_S^4} \frac{e^{-\frac{A^2}{2\sigma_v^2}}}{\sqrt{2\pi\sigma_v^2}} \left(a + \mathbf{b}\cdot\hat{\mathbf{q}}A + 2c\sigma_v^2 + cA^2 + d^2\sigma_v^2 - (\mathbf{d}\cdot\hat{\mathbf{q}})^2(\sigma_v^2 - A^2)\right) \sum_{\alpha=1,4} F_\alpha^X(q),$$

$A = \mathbf{V}\cdot\hat{\mathbf{q}} + \frac{q}{2\mu}$, μ =reduced mass and

$$F_1^X(q) = 4\pi \sum_l |X_l^C|^2,\ F_2^X(q) = -4\pi \sum_l 2Re\left(X_l^C X_l^{L*}\right),\ F_3^X(q) = 4\pi \sum_l |X_l^L|^2,\ F_4^X(q) = 4\pi \sum_l \left(|X_l^M|^2 + |X_l^E|^2\right).$$

Deuteron-DM scattering

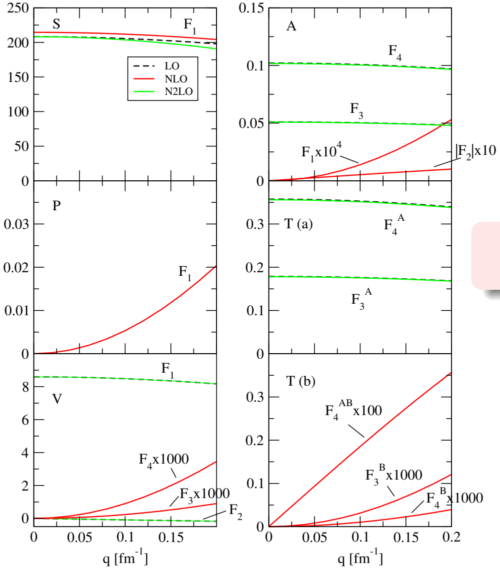
Contribution of the RMEs

$q = 0.05\ \text{fm}^{-1}$

Int.	RME	order	AV18			N4LO500		
S			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	LO	$-0.144+02$		$-0.229-02$	$-0.144+02$		$-0.231-02$
		NLO	$-0.218+00$		$+0.328-03$	$-0.806-01$		$+0.336-03$
		N2LO	$+0.153+00$		$-0.467-05$	$+0.103+00$		$-0.829-05$
P			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	NLO		$-0.367-01$			$-0.377-01$	
V			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	LO	$-0.293+01$		$-0.465-03$	$-0.293+01$		$-0.470-03$
		N2LO	$+0.289-04$		$+0.320-05$	$+0.294-04$		$+0.320-05$
	X_ℓ^L	NLO	$-0.769-02$		$-0.120-05$	$-0.769-02$		$-0.120-05$
	X_ℓ^M	NLO		$-0.150-01$			$-0.152-01$	
A			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	NLO		$+0.593-03$			$+0.609-03$	
	X_ℓ^L	LO		$+0.226+00$			$+0.232+00$	
	X_ℓ^L	N2LO		$-0.469-03$			$-0.781-03$	
	X_ℓ^E	LO		$-0.319+00$			$-0.328+00$	
	X_ℓ^E	N2LO		$+0.660-03$			$+0.110-02$	
T			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	$X_\ell^L(A)$	LO		$-0.422+00$			$-0.433+00$	
	$X_\ell^L(B)$	NLO	$-0.280E-02$		$-0.290E-03$	$-0.285E-02$		$-0.314E-03$
	$X_\ell^L(A)$	N2LO		$+0.875-03$			$+0.687-03$	
	$X_\ell^E(A)$	LO		$+0.597+00$			$+0.613+00$	
	$X_\ell^M(B)$	NLO		$+0.157-02$			$+0.161-02$	
	$X_\ell^E(A)$	N2LO		$-0.124-03$			$-0.978-03$	

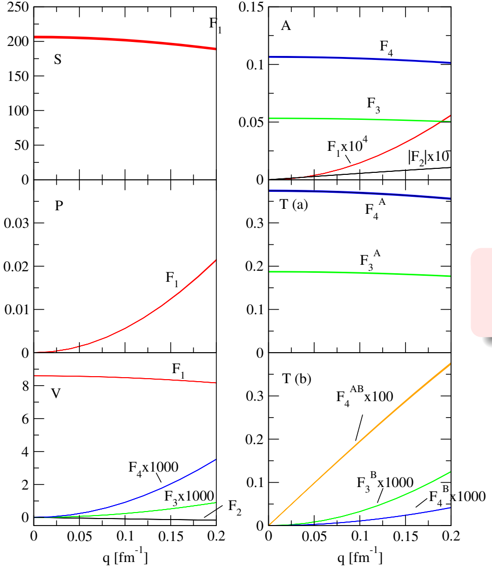
- LO transition operators give largest RMEs
- RMEs dependence from nuclear interaction is rather weak

Structure Functions calculated with AV18 potential



- NLO and N2LO components in the transition operators are tiny

Structure Functions calculated with chiral potential



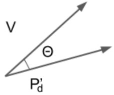
- Width band reflects the spread of theoretical results using $\Lambda = 450$, 500, or 550 MeV cutoff values

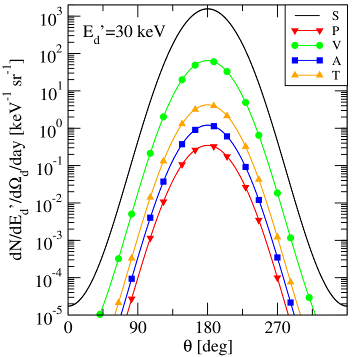
Events per day

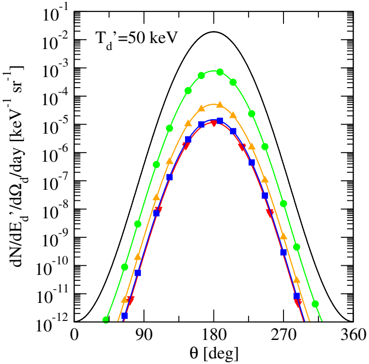
$$\frac{d^2N}{d\hat{\mathbf{P}}'_d dE'_d} \equiv \frac{d^2R}{d\hat{\mathbf{P}}'_d dE'_d} \times 1 \ \text{ day} = \text{events per day per unit of deuteron recoil energy and per steradiant}$$

(indipendent of φ)

$C_{\chi+} = 10^{-4}$ and $M_\chi = 10$ GeV

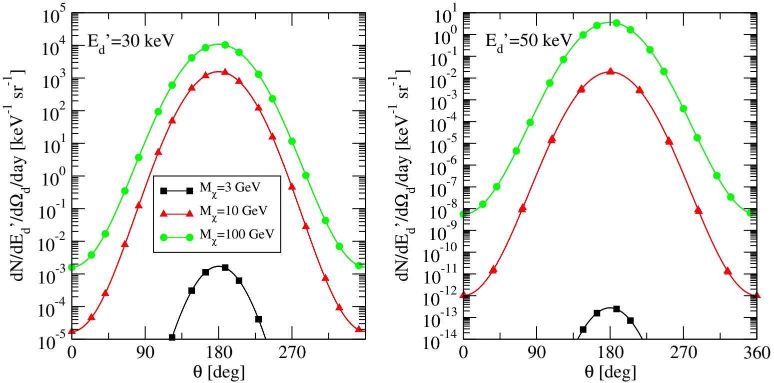






- The rates are peaked at $\theta = 180$ deg
- The number of events also depends critically on the kinetic energy
- It also depends on the assumed interaction type
- The results shown in the figure are bands: Rates calculated with the N4LO450, N4LO500, and N4LO550 interactions and obtained with transition currents from LO to N2LO

Number of events for three different values of the WIMP mass M_χ for scalar interaction



- For lighter WIMP the number of recoiling deuterons at a given energy decreases noticeably
- Dependence on the WIMP mass is particularly critical for light WIMP, with mass around 1 to 10 GeV

^{4}He-DM scattering

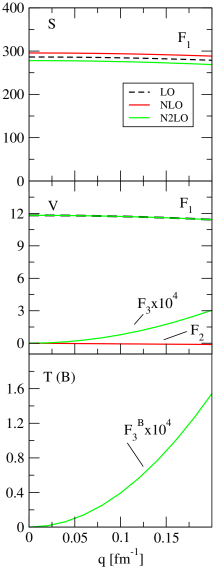
RMEs

$q = 0.05\ \mathrm{fm}^{-1}$

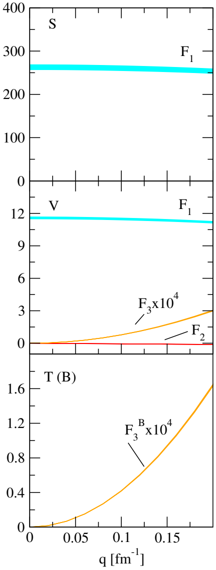
Int.	RME	order	AV18/UIX	N4LO500/3N
S			$\ell = 0$	$\ell = 0$
	X_ℓ^C	LO	$-0.169+02$	$-0.168+02$
		NLO	$-0.273+00$	$+0.144+00$
		N2LO	$+0.521+00$	$+0.313+00$
		N2LO	$+0.451-02$	$+0.448-02$
V			$\ell = 0$	$\ell = 0$
	X_ℓ^C	LO	$-0.343+01$	$-0.341+01$
		N2LO	$+0.325-04$	$+0.336-04$
	X_ℓ^L	NLO	$-0.444-02$	$-0.438-02$
T			$\ell = 0$	$\ell = 0$
	$X_\ell^L(B)$	NLO	$-0.315-02$	$-0.325-02$

- Contributions of isovector operators neglected since they are very small

Structure Functions



Av18

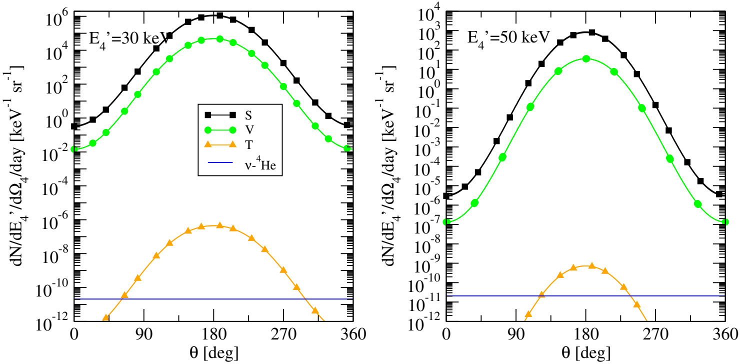


Chiral N4LO/N2LO interactions

NLO and N2LO contributions are sizeable for s case very tiny for the other cases

Events per day with Chiral potential

$C_{X+} = 10^{-4}$ and $M_\chi = 10$ GeV



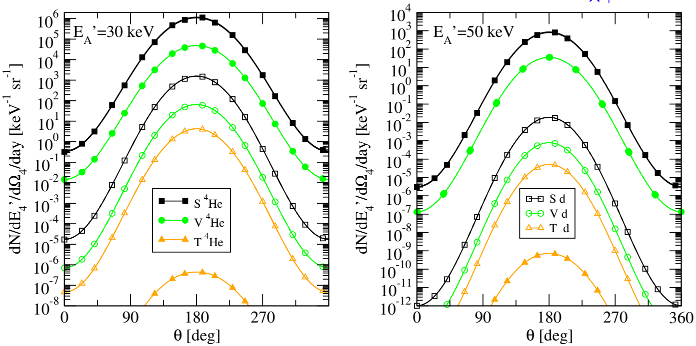
Rate of ${}^4\mathrm{He}$ recoils due to the flux of atmospheric neutrinos

Minimal C_{X+} so that the rate of recoiling ${}^4\mathrm{He}$ nuclei with kinetic energy T due to DM be greater than that due to the neutrino scattering

T (keV)	C_{S+}	C_{V+}	C_{T+}
30	$4.5\,10^{-13}$	$2.1\,10^{-12}$	$7.0\,10^{-7}$
50	$1.6\,10^{-7}$	$7.7\,10^{-7}$	$1.7\,10^{-1}$

Comparison between the rates for $^2\mathrm{H}$ and $^4\mathrm{He}$

$C_{X+} = 10^{-4}$ and $M_\chi = 10$ GeV



$^4\mathrm{He}$ rate higher than the $^2\mathrm{H}$ case $\Rightarrow$ J_A and $e^{-\frac{(\mathbf{V}\cdot\hat{\mathbf{q}}+\frac{q}{2\mu})^2}{2\sigma_v^2}}$

Conclusion & Future perspectives

With respect to other calculations in the literature

- Inclusion of two body currents, and treated $a_\mu^{(s)}$ and $t^{\mu\nu}$ currents
- Systematically study of the interation rate for each interactione type

In conclusion

- Scalar interaction dominant over the others
- NLO and N2LO contributions are suppressed
- Little dependence of the results on the nuclear interaction

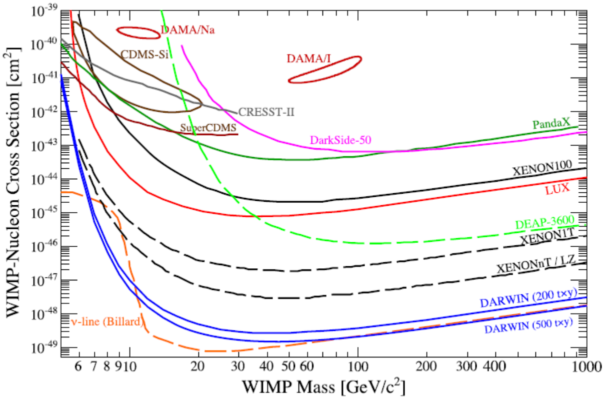
Future goals

- Inclusion of parity/charge conjugation violating terms
- Treatement of other DM cases
- Apply this study to other nuclei ($^6\mathrm{Li}$)

Backup slides

Experimental situation

WIMP-nucleon cross section limits



[Baudis, 2018]

Invariant terms under local G_χ [Fettes, 2000]:

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\left\langle D_\mu U(x)(D^\mu U(x))^\dagger\right\rangle + \frac{f_\pi^2}{4}\left\langle \xi(x)U^\dagger(x) + U(x)\xi^\dagger(x)\right\rangle$$

$$r_\mu = v_\mu + a_\mu\,, \quad l_\mu = v_\mu - a_\mu$$

$$D_\mu U(x) = \partial_\mu U(x) - ir_\mu(x)U(x) + il_\mu(x)U(x)$$

$$\xi(x) = 2B_c(s(x) + ip(x))$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\not{D} - M + \frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu\right)N$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N} = \; & c_1\bar{N}\langle\xi_+\rangle N \\ & -\frac{c_2}{8M^2}\left[\bar{N}\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}N + \text{h.c.}\right] \\ & +\frac{c_3}{2}\bar{N}\langle u_\mu u^\mu\rangle N \\ & +\frac{i c_4}{4}\bar{N}[u_\mu, u_\nu]\sigma^{\mu\nu}N \\ & +c_5\bar{N}\hat{\xi}_+N \\ & +\frac{c_6}{8M}\bar{N}\sigma^{\mu\nu}F^+_{\mu\nu}N \\ & +\frac{c_7}{4M}\bar{N}\sigma^{\mu\nu}F^{(s)}_{\mu\nu}N\end{aligned}$$

$$u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu u^\dagger)\}$$

$$\xi_\pm = u^\dagger\xi\, u^\dagger \pm u\,\xi^\dagger u$$

$$F^{(s)}_{\mu\nu} = \partial_\mu v^{(s)}_\nu - \partial_\nu v^{(s)}_\mu$$

$$F^\pm_{\mu\nu} = u^\dagger F^R_{\mu\nu}u \pm u\,F^L_{\mu\nu}u^\dagger$$

where $\langle\ldots\rangle$ indicates the trace of the matrices and $\hat{\xi}_+ = \xi_+ - \frac{1}{2}\langle\xi_+\rangle$

- The unknown quark dynamics is parametrized via the so called low-energy constants (LECs) B_c, c_1, c_2,..which can be fixed in general from experiments

Incorporation of the isoscalar axial current

Due to the $U_A(1)$ anomaly, it is not possible to introduce isoscalar axial current in $SU(2)$ space$\rightarrow$ $SU(3)$ [Scherer & Schindler, 2004]

$$\langle N|\bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d|N\rangle \rightarrow \langle N|A_\mu^{(8)}|N\rangle$$

where

$$A_\mu^{(8)} = \bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d - 2\bar{s}\gamma_\mu\gamma^5 s = \sqrt{3}\bar{q}\gamma_\mu\gamma^5\lambda_8 q \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix} \quad q(x) = \begin{pmatrix}u(x)\\ d(x)\\ s(x)\end{pmatrix}$$

Valid in the hypothesis that the content of the strange quark in the nucleon vanishes [Ellis, Nagata & Olive, 2018]

The axial current part of the quark Lagrangian in the $SU(3)$ space is

$$\mathcal{L}_q^{axial} = \sum_i \alpha_i \bar{q}\gamma_\mu\gamma^5\lambda_i q\bar{\chi}\gamma^\mu\gamma^5\chi \qquad a_\mu = a_{\mu i}\lambda_i = \alpha_i\bar{\chi}\gamma_\mu\gamma^5\chi\lambda_i$$

where the costants α_i are zero except

$$\alpha_3 = \frac{C_{A-}}{\Lambda_S^2} \qquad \alpha_8 = \frac{C_{A+}}{\Lambda_S^2} \qquad \lambda_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

- Since now $\langle\lambda_i\rangle = 0 \rightarrow$ no anomaly

Tensor current

Not treated in literature

$$t^{\mu\nu} = t_0^{\mu\nu}\mathbf{1} + t_a^{\mu\nu}\tau_a$$

Assuming

$$t^{\mu\nu} \to t^{\mu\nu\prime} = L t^{\mu\nu} R^\dagger$$

$$\mathcal{L}_q^{tens} = \bar{q}\sigma_{\mu\nu}t^{\mu\nu}q = \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L + \bar{q}_L\sigma_{\mu\nu}(t^{\mu\nu})^\dagger q_R$$

In the hadron Lagrangian we construct the terms invariant under G_χ:

- Lowest order $O(Q^2)$

$$\bar{N}\sigma_{\mu\nu}T_\pm^{\mu\nu}N \qquad T_\pm^{\mu\nu} = ut^{\mu\nu\dagger}u \pm u^\dagger t^{\mu\nu}u^\dagger$$

Only term C, P invariant $\to T_+^{\mu\nu}$

$$\mathcal{L}_{\pi N}^{(2)} = \tilde{c}_1\bar{N}\sigma_{\mu\nu}\langle T_+^{\mu\nu}\rangle N + \tilde{c}_2\bar{N}\sigma_{\mu\nu}\hat{T}_+^{\mu\nu}N$$

where $\tilde{c}_1$ and $\tilde{c}_2$ are new LECs

- Other terms $O(Q^3)$

$$\bar{N}\gamma^\mu\gamma_5[u^\nu, T_{+\mu\nu}]N + \bar{N}\gamma^\mu\{u^\nu, T_{-\mu\nu}\}N$$

  not considered for simplicity