Neutrino-induced pion-production off the nucleon in covariant ChPT

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11th International Workshop on Chiral Dynamics

Bochum August 27, 2024

Outline

Introduction

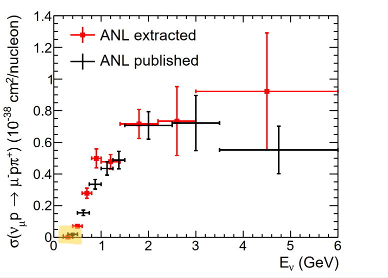
Motivation:

- neutrino oscillation experiments use ν_ℓ-nucleus interactions to reconstruct ν_ℓ energy
  - ν_e appearance $\Rightarrow$ $\nu_e n \to e^- p$ background: (NC) $\nu_\mu N \to \nu_\mu N' \pi^0$
  - ν_μ disappearance $\Rightarrow$ $\nu_\mu n \to \mu^- p$ background: (CC) $\nu_\mu N \to \mu^- N' \pi$

[Formaggio and Zeller, Reviews of Modern Physics, 2012]

Experimental data:

Argonne National Laboratory



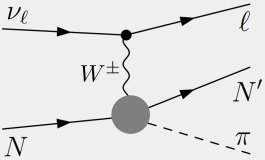
[Wilkinson et al., Phys.Rev.D, 2014]

Other theoretical work:

- covariant ChPT

  [Yao et al., Phys.Rev.D, 2018]
  - one-loop
  - inclusion of Δ on tree-level
- HNV-model (Hernández, Nieves, Valverde)

  [Hernandez et al., Phys.Rev.D, 2007]
  - tree diagrams with Δ from $SU(2)$ non-linear σ model
  - form factors of $WN\Delta$ and $N\Delta\pi$ vertices fitted to data



Amplitude:

$$\mathcal{T}_{fi} = \frac{G_F|V_{ud}|}{\sqrt{2}} \bar{u}_\ell(k_2)\gamma^\mu(1-\gamma_5)u_\nu(k_1)H^{ba}_\mu$$

$$\text{with} \quad H^{ba}_\mu = \langle N'\pi^b \,|\, V^a_\mu(0) - A^a_\mu(0) \,|\, N\rangle$$

Isospin decomposition:

$$H^{ba}_\mu(s_2, t_1, t) = \chi^\dagger_{N'}(\delta^{ab}H^+_\mu + i\epsilon^{bac}\tau^c H^-_\mu)\chi_N \quad \begin{cases} H^+_\mu - H^-_\mu, & \nu_\ell p \to \ell^- p\pi^+ / \bar{\nu}_\ell n \to \ell^+ n\pi^- \\ H^+_\mu + H^-_\mu, & \nu_\ell n \to \ell^- n\pi^+ / \bar{\nu}_\ell p \to \ell^+ p\pi^- \\ \sqrt{2}H^-_\mu, & \nu_\ell n \to \ell^- p\pi^0 / \bar{\nu}_\ell p \to \ell^+ n\pi^0 \end{cases}$$

Lorentz vector and axial-vector operators:

$$H^\pm_\mu = \bar{u}_{N'}\Big[\sum_{i=1}^{8} A^\pm_i \mathcal{O}^A_{\mu,i} + V^\pm_i \mathcal{O}^V_{\mu,i}\Big]u_N \quad \text{with} \quad \mathcal{O}^V_{\mu,i} = \mathcal{O}^A_{\mu,i}\gamma_5$$

CVC: $\#\mathcal{O}^V_\mu$ reduces to 6

$$\mathcal{O}^A_{\mu,1} = q_\mu, \quad \mathcal{O}^A_{\mu,2} = p_{i,\mu}, \quad \mathcal{O}^A_{\mu,3} = p_{f,\mu}, \quad \mathcal{O}^A_{\mu,4} = \not{q} q_\mu,$$

$$\mathcal{O}^A_{\mu,5} = \not{q} p_{i,\mu}, \quad \mathcal{O}^A_{\mu,6} = \not{q} p_{f,\mu}, \quad \mathcal{O}^A_{\mu,7} = \gamma_\mu \not{q}, \quad \mathcal{O}^A_{\mu,8} = \gamma_\mu$$

3. Calculation of hadronic current

Effective Lagrangians:

$\mathcal{L}_{\pi\pi}^{(2)},\ \mathcal{L}_{\pi\pi}^{(4)} \to\ F,\ \ell_3,\ \ell_4,\ \ell_6$ [Gasser, Annals of Physics, 1984]

$\mathcal{L}_{\pi N}^{(1)} \to \mathring{g}_A \qquad \mathcal{L}_{\pi N}^{(2)} \to\ c_1,\ c_2,\ c_3,\ c_4,\ c_6$ [Fettes et al., Annals of Physics, 2000]

$\mathcal{L}_{\pi N}^{(3)} \to d_1,\ d_2,\ d_3,\ d_4,\ d_5,\ d_6,\ d_8,\ d_{14},\ d_{15},\ d_{18},\ d_{20},\ d_{21},\ d_{22},\ d_{23}$

$\mathcal{L}_{\pi\Delta}^{(1)},\ \mathcal{L}_{\pi N\Delta}^{(1)},\ \to \mathring{g}_1,\ \mathring{h}_A \qquad \mathcal{L}_{\pi N\Delta}^{(2)} \to b_1,\ b_2,\ \not{b}_3,\ b_7,\ \not{b}_8$ [Rijneveen, 2020]

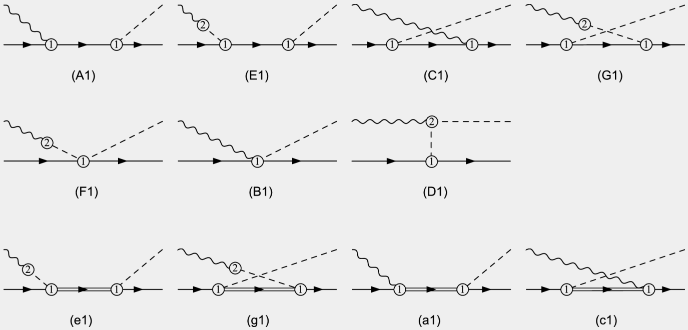
$\mathcal{L}_{\pi N\Delta}^{(3)} \to \underbrace{f_1,\ \not{f}_2}_{\text{[Yao et al., JHEP, 2016]}},\ \underbrace{h_1,\ h_4,\ h_7,\ \not{h}_{15},\ \not{h}_{16},\ h_{31},\ \not{h}_{32},}_{\text{[Zöller, 2014]}}\ \underbrace{\not{f}_{19}}_{\text{[Jiang et al., Phys.Rev.D, 2018]}}$

num. values for LEC's of $\mathcal{O}(p^2)$ & $\mathcal{O}(p^3)$: [Yao et al., Phys.Rev.D, 2018] [Rijneveen, 2020]

$W^{\pm}$ enters through external fields $\ell^{\mu} = -\frac{g_W V_{ud}}{2} \ell_{\mu}^{a} \tau_a$ with $W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(\ell_{\mu}^{1} \mp i\ell_{\mu}^{2})$

Feynman diagrams:

$\mathcal{O}(p)$:



$\mathcal{O}(p^2)$: $4 + 8\Delta$ & $\mathcal{O}(p^3)$: $19 + 16\Delta + 89$ loop $+ 170$ Δ-loop

- chiral order $= 4L + \sum_i iV^i - 2I_\pi - I_N - I_\Delta$
- calculation of diagrams: *FeynArts* + *FeynCalc* + custom routines

3. Calculation of hadronic current

Renormalization

- UV divergencies substracted in modified $\overline{MS}$ scheme
- power counting breaking terms (PCBT) subtracted in EOMS scheme
  - extraction of PCBT´s: expansion of integrand in small quantities before integration of Schwinger parameters $\xi_1 \& \xi_2$
  - integrals including Δ often require Mellin-Barnes representation:

$$(\delta\xi_2 + m_N^2(1-\xi_1))^{-X} = \int_{-i\infty}^{i\infty} \frac{\Gamma(-t)\Gamma(X+t)}{2\pi\mathrm{i}\,\Gamma(X)} (m_N^2(1-\xi_1))^t (\delta\xi_2))^{X-t} dt$$

$$\text{with} \quad \delta = m_\Delta^2 - m_N^2$$

  - expansion of hypergeometrical functions around $D=4$ using *HypExp*
    [Huber and Maître, CPC, 2008]
  - PCBT´s are given in terms of basic Passarino-Veltman integrals
    $A_0(m_N^2),\ A_0(m_\Delta^2),\ B_0(m_N^2, 0, m_\Delta^2),\ B_0(m_\Delta^2, 0, m_N^2),$
    $C_0(m_N^2, 0, m_\Delta^2, 0, m_\Delta^2, m_N^2),\ C_0(m_N^2, 0, m_\Delta^2, 0, m_\Delta^2, m_\Delta^2),\ C_0(m_N^2, 0, m_\Delta^2, 0, m_N^2, m_N^2)$

3. Calculation of hadronic current

Renormalization

- masses: $m_N = \mathring{m}_N - \Sigma_N(m_N^2),\ M_\pi = M - \Sigma_\pi(M_\pi^2)$
  and $\mathring{m}_\Delta = z_\Delta + \Sigma_\Delta(z_\Delta^2)$ with $z_\Delta = m_\Delta - \mathrm{i}\frac{\Gamma_\Delta}{2}$ (complex mass scheme)
- LO couplings
  - axial vector coupling g_A

$$\langle N \left| A_i^\mu(0) \right| N' \rangle = \bar{u}(p_f) \left[\gamma^\mu G_A(q^2) + \frac{q^\mu}{2m_N} G_P(q^2) \right] \gamma_5 \frac{\tau_i}{2} u(p_i)$$

$$G_A(0) = g_A = \mathring{g}_A + 4d_{16} M_\pi^2 + g_A \delta_{\mathcal{Z}_N} + \delta_{g_A}^{loop}$$

  - nucleon-to-delta axial vector coupling h_A

$$\langle \Delta \left| A_i^\mu(0) \right| N \rangle = \xi^{ia} \bar{u}_\lambda(p_f) \sum_{j=3,4,5,6} \mathcal{O}_j^{\lambda\mu} C_j^A(q^2) u(p_i)$$

$$C_5^A(0) = h_A = \mathring{h}_A + 2h_4 M_\pi^2 + \frac{1}{2} h_A \delta_{\mathcal{Z}_\Delta} + \frac{1}{2} h_A \delta_{\mathcal{Z}_N} + \delta_{h_A}^{loop}$$

  - pion decay constant F_π

$$\langle 0 \left| L_a^\mu(0) \right| \pi_b(q) \rangle = -iq^\mu \frac{F_\pi}{2} \delta_{ab} \qquad F_\pi = F + \frac{\ell_4 M_\pi^2}{F_\pi} + \frac{A_0(M_\pi^2)}{16\pi^2 F_\pi}$$

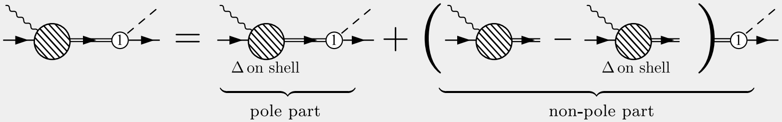
3. Calculation of hadronic current

Renormalization

- $H_\mu(s_2, t, t_1) = \sqrt{\mathcal{Z}_\pi}\,\mathcal{Z}_N \hat{H}_\mu(s_2, t, t_1)$
- change to CES-basis $\mathcal{O}^{A/V}_{\mu,i} \to \tilde{\mathcal{O}}^{A/V}_{\mu,i}$ [Yao et al., 2018, Adler, 1968]
  - example:

$$\bar{u}(\underbrace{\mathcal{O}^A_{\mu,1}}_{=p_f^\mu})u \sim \mathcal{O}(p^0), \quad \bar{u}(\underbrace{\mathcal{O}^A_{\mu,7}}_{=\gamma_\mu \not{q}})u \sim \mathcal{O}(p), \quad \bar{u}(\mathcal{O}^A_{\mu,7} - \mathcal{O}^A_{\mu,1})u = \bar{u}(\underbrace{\tilde{\mathcal{O}}^A_{\mu,7}}_{=\frac{1}{2}[\gamma_\mu, \not{q}]})u \sim \mathcal{O}(p)$$

- $\underbrace{(m_\Delta - m_N)}_{=\delta} \sim \mathcal{O}(p^0), \quad M_\pi \sim (s_2 - m_N^2) \sim (u - m_N^2) \sim \mathcal{O}(p), \quad t \sim t_1 \sim \mathcal{O}(p^2)$
- seperation into pole & non-pole part consistent with $\delta \sim \mathcal{O}(p^0)$



4. Total cross section

$$\sigma_{tot}(E_\nu) = \frac{1}{(4\pi)^4 E_\nu m_N} \int_{\omega_\ell^-}^{\omega_\ell^+} d\omega_\ell \int_{-1}^{1} \mathrm{d}\cos\theta_\ell \int_{\omega_\pi^-}^{\omega_\pi^+} d\omega_\pi \int_0^\pi d\phi'_\pi |\mathcal{T}_{fi}|^2$$

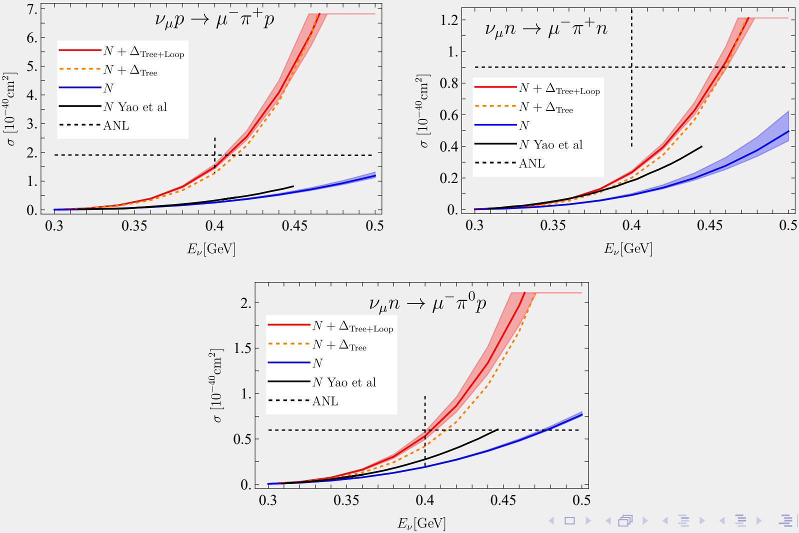
$$\text{with} \quad s = m_N^2 + 2m_N E_\nu, \quad \omega_\ell^- = m_\ell, \quad \omega_\ell^+ = \frac{s + m_\ell^2 - (M_\pi + m_N)^2}{2\sqrt{s}}$$

$$\text{and} \quad \omega_\pi^\pm = \frac{1}{2(m_\ell^2 - 2\sqrt{s}\omega_\ell + s)} \Big[(\sqrt{s} - \omega_\ell)(m_\ell^2 - m_N^2 + M_\pi^2 - 2\sqrt{s}\omega_\ell + s) \pm \sqrt{\omega_\ell^2 - m_\ell^2} \sqrt{(m_\ell^2 - m_N^2 - M_\pi^2 - 2\sqrt{s}\omega_\ell + s)^2 - 4m_N^2 M_\pi^2} \Big]$$

- $\frac{1}{2} \sum_{\text{spins}} |\mathcal{T}_{fi}|^2 = \frac{G_F^2 |V_{ud}|^2}{2} L_{\mu\nu} H^{\mu\nu}$
  - $H^{\mu\nu} = H_s^{\mu\nu} + \mathrm{i} H_a^{\mu\nu}$ (10 symmetric and 9 antisymmetric pieces)
  - $L_{\mu\nu} = 8 \left(k_{1\,\mu} k_{2\,\nu} + k_{2\,\mu} k_{1\,\nu} - k_1 \cdot k_2 g_{\mu\nu} \pm \mathrm{i}\epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right)$
  - write Mandelstam variables & $\epsilon_{\mu\nu\alpha\beta} k_1^\mu p_f^\nu p_i^\alpha q^\beta$ in terms of ω_ℓ, cosθ_ℓ, ω_π, ϕ'_π & E_ν
- rough uncertainty estimation: $\delta\mathcal{O}(p^4) \approx (\mathcal{O}(p^3)$ loop contribution$)\frac{\sqrt{-t_1}}{4\pi F_\pi}$
- numerical values for loop-functions: *LoopTools* [Hahn, 1999] & *XPackage* [Patel, 2017]
- numerical 3-particle phase space integration: *NIntegrate*

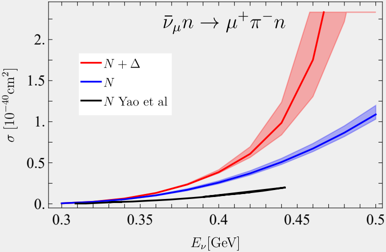
4. Total cross section

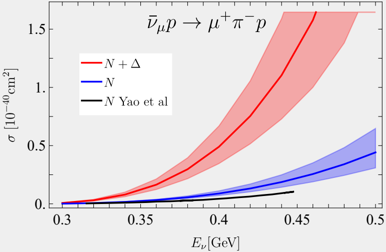
preliminary results: neutrino induced π production

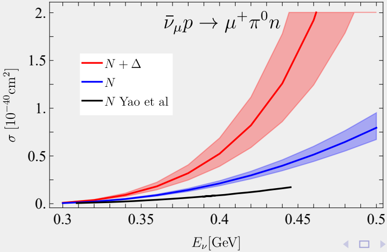


4. Total cross section

preliminary results: anti-neutrino induced π production







5. Summary/Outlook

- $\nu/\bar{\nu}$ induced π production calculated in ChPT with inclusion of $\Delta(1232)$ to one-loop order
- calculation of total cross section for

$$\begin{aligned} \nu_\ell p \to \ell^- p\pi^+, &\quad \bar{\nu}_\ell n \to \ell^+ n\pi^- \\ \nu_\ell n \to \ell^- n\pi^+, &\quad \bar{\nu}_\ell p \to \ell^+ p\pi^- \\ \nu_\ell n \to \ell^- p\pi^0, &\quad \bar{\nu}_\ell p \to \ell^+ n\pi^0 \end{aligned}$$

  - only minor contribution from Δ-loop diagrams
- comparison to results of [Yao et al., Phys.Rev.D, 2018]
  - reason for deviations still needs to be resolved
- further steps: calculation of differential cross section & comparison to other theoretical predictions

Thank you!

References I

Stephen L Adler. Photo-, electro-, and weak single-pion production in the (3,3) resonance region. *Annals of Physics*, 50(2):189–311, 1968. ISSN 0003-4916. doi: https://doi.org/10.1016/0003-4916(68)90278-9. URL https://www.sciencedirect.com/science/article/pii/0003491668902789.

Nadia Fettes, Ulf-G. Meißner, Martin Mojžiš, and Sven Steininger. The chiral effective pion-nucleon lagrangian of order p4. *Annals of Physics*, 283(2):273–307, August 2000. ISSN 0003-4916. doi: 10.1006/aphy.2000.6059. URL http://dx.doi.org/10.1006/aphy.2000.6059.

J. A. Formaggio and G. P. Zeller. From ev to eev: Neutrino cross sections across energy scales. *Reviews of Modern Physics*, 84(3): 1307–1341, September 2012. ISSN 1539-0756. doi: 10.1103/revmodphys.84.1307. URL http://dx.doi.org/10.1103/RevModPhys.84.1307.

References II

Leutwyler Gasser. Chiral perturbation theory to one loop. *Annals of Physics*, 158(1):142–210, 1984. ISSN 0003-4916. doi: https://doi.org/10.1016/0003-4916(84)90242-2. URL https://www.sciencedirect.com/science/article/pii/0003491684902422.

T. Hahn. Automated one-loop calculations in four and d dimensions. *Computer Physics Communications*, 118(2):153–165, 1999. ISSN 0010-4655. doi: https://doi.org/10.1016/S0010-4655(98)00173-8. URL https://www.sciencedirect.com/science/article/pii/S0010465598001738.

E. Hernandez, J. Nieves, and M. Valverde. Weak Pion Production off the Nucleon. *Phys. Rev. D*, 76:033005, 2007. doi: 10.1103/PhysRevD.76.033005.

References III

Tobias Huber and Daniel Maître. Hypexp 2, expanding hypergeometric functions about half-integer parameters. *Computer Physics Communications*, 178(10):755–776, May 2008. ISSN 0010-4655. doi: 10.1016/j.cpc.2007.12.008. URL http://dx.doi.org/10.1016/j.cpc.2007.12.008.

Shao-Zhou Jiang, Yan-Rui Liu, and Hong-Qian Wang. Chiral lagrangians with $\delta(1232)$ to one loop. *Physical Review D*, 97(1), jan 2018. doi: 10.1103/physrevd.97.014002. URL https://doi.org/10.1103%2Fphysrevd.97.014002.

Hiren H. Patel. Package -x 2.0: A mathematica package for the analytic calculation of one-loop integrals. *Computer Physics Communications*, 218:66–70, September 2017. ISSN 0010-4655. doi: 10.1016/j.cpc.2017.04.015. URL http://dx.doi.org/10.1016/j.cpc.2017.04.015.

References IV

Jan Rijneveen. *Radiative pion photoproduction on the nucleon in covariant chiral perturbation theory*. doctoral thesis, Ruhr-Universität Bochum, Universitätsbibliothek, 2020.

Callum Wilkinson, Philip Rodrigues, Susan Cartwright, Lee Thompson, and Kevin McFarland. Reanalysis of bubble chamber measurements of muon-neutrino induced single pion production. *Physical Review D*, 90(11), December 2014. ISSN 1550-2368. doi: 10.1103/physrevd.90.112017. URL `http://dx.doi.org/10.1103/PhysRevD.90.112017`.

De-Liang Yao, D. Siemens, V. Bernard, E. Epelbaum, A. M. Gasparyan, J. Gegelia, H. Krebs, and Ulf-G. Meißner. Pion-nucleon scattering in covariant baryon chiral perturbation theory with explicit delta resonances. *Journal of High Energy Physics*, 2016(5), may 2016. doi: 10.1007/jhep05(2016)038. URL `https://doi.org/10.1007%2Fjhep05%282016%29038`.

References V

De-Liang Yao, Luis Alvarez-Ruso, Astrid N. Hiller Blin, and M.J. Vicente Vacas. Weak pion production off the nucleon in covariant chiral perturbation theory. *Physical Review D*, 98(7), October 2018. ISSN 2470-0029. doi: 10.1103/physrevd.98.076004. URL http://dx.doi.org/10.1103/PhysRevD.98.076004.

Christian Zöller. *Effective Chiral Nucleon-Delta Lagrangian at Order Q^3*. master thesis, Ruhr-Universität Bochum, Universitätsbibliothek, 2014.

$$\mathcal{L}^{(2)}_{\pi N\Delta} = \bar{\Psi}^{i,\mu}\xi^{\frac{3}{2}}_{ij}\left[\frac{\mathrm{i}b_1}{2}F^{+,j}_{\mu\alpha}\gamma^\alpha\gamma^5 + \mathrm{i}b_2 F^{-,j}_{\mu\alpha}\gamma^\alpha + \mathrm{i}b_3\omega^j_{\mu\alpha}\gamma^\alpha - \frac{b_7}{m}F^{-,j}_{\mu\alpha}D^\alpha - \frac{b_8}{m}\omega^j_{\mu\alpha}D^\alpha\right]\Psi_N + \text{ h.c.}$$

$$\mathcal{L}^{(3)}_{\pi N\Delta} = \bar{\Psi}^{i,\mu}\xi^{\frac{3}{2}}_{ij}\left[\frac{\mathrm{i}f_1}{m}[D_\mu, \omega^j_{\alpha\beta}]\gamma^\alpha D^\beta - \frac{f_2}{2m^2}[D_\mu, \omega^j_{\alpha\beta}]\{D^\alpha, D^\beta\} + \frac{h_1}{2m}\langle F^+_{\mu\alpha}\tau^j\rangle\gamma_5 D^\alpha + \frac{h_4}{2}\langle\chi_+\rangle\omega^j_\mu + ih_7\langle[D_\mu, \chi_-]\tau^j\rangle - \frac{ih_{15}}{2}\langle[D_\alpha, F^+_{\mu\beta}]\tau^j\rangle\sigma^{\alpha\beta}\gamma_5 + \frac{\mathrm{i}h_{16}}{2m}\langle[D_\alpha, F^+_{\mu\beta}]\tau^j\rangle\gamma^\beta\gamma_5 D^\alpha + \frac{\mathrm{i}h_{31}}{2}\langle[D_\mu, F^-_{\alpha\beta}]\tau^j\rangle\sigma^{\alpha\beta} + \frac{\mathrm{i}h_{32}}{m}\langle[D_\mu, F^-_{\alpha\beta}]\tau^j\rangle\sigma^{\alpha\beta} + \frac{f_{19}}{2}D^\alpha\langle F^-_{\alpha\mu}\tau^j\rangle\right]\Psi_N + \text{ h.c.}$$

Backup

$$H^{\mu\nu} = H_s^{\mu\nu} + iH_a^{\mu\nu} \quad \text{with}$$

$$\begin{aligned}H_a^{\mu\nu} =& (p_f^\nu p_i^\mu - p_f^\mu p_i^\nu) H_a^1 + (p_f^\nu q^\mu - p_f^\mu q^\nu) H_a^2 + (p_i^\nu q^\mu - p_i^\mu q^\nu) H_a^3 \\ &+ p_f^\alpha p_i^\beta q^\gamma (p_f^\nu \epsilon^{\alpha\beta\gamma\mu} - p_f^\mu \epsilon^{\alpha\beta\gamma\nu}) H_a^4 + p_f^\alpha p_i^\beta q^\gamma (p_i^\nu \epsilon^{\alpha\beta\gamma\mu} - p_i^\mu \epsilon^{\alpha\beta\gamma\nu}) H_a^5 \\ &+ p_f^\alpha p_i^\beta q^\gamma (q^\nu \epsilon^{\alpha\beta\gamma\mu} - q^\mu \epsilon^{\alpha\beta\gamma\nu}) H_a^6 + p_f^\alpha p_i^\beta \epsilon^{\alpha\beta\mu\nu} H_a^7 \\ &+ p_f^\alpha q^\beta \epsilon^{\alpha\beta\mu\nu} H_a^8 + p_i^\alpha q^\beta \epsilon^{\alpha\beta\mu\nu} H_a^9\end{aligned}$$

and

$$\begin{aligned}H_s^{\mu\nu} =& g^{\mu\nu} H_s^1 + p_i^\mu p_i^\nu H_s^2 + p_f^\mu p_f^\nu H_s^3 + q^\mu q^\nu H_s^4 \\ &+ (p_f^\nu p_i^\mu + p_f^\mu p_i^\nu) H_s^5 + (p_f^\nu q^\mu + p_f^\mu q^\nu) H_s^6 + (p_i^\nu q^\mu + p_i^\mu q^\nu) H_s^7 \\ &+ p_f^\alpha p_i^\beta q^\gamma (p_f^\nu \epsilon^{\alpha\beta\gamma\mu} + p_f^\mu \epsilon^{\alpha\beta\gamma\nu}) H_s^8 + p_f^\alpha p_i^\beta q^\gamma (p_i^\nu \epsilon^{\alpha\beta\gamma\mu} + p_i^\mu \epsilon^{\alpha\beta\gamma\nu}) H_s^9 \\ &+ p_f^\alpha p_i^\beta q^\gamma (q^\nu \epsilon^{\alpha\beta\gamma\mu} + q^\mu \epsilon^{\alpha\beta\gamma\nu}) H_s^{10}.\end{aligned}$$