# Weak nonleptonic hyperon decays in relativistic $\chi$PT

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# Nonleptonic decays

Transition amplitude

$$\mathcal{M}(B_i \to B_f \pi) = G_F m_{\pi^+}^2 \bar{u}_f (A^{(S)} + A^{(P)} \gamma_5) u_i$$

Dimensionless $l$-wave amplitudes

$$\text{parity-violating: } A^{(S)} \equiv S$$

$$\text{parity-conserving: } A^{(P)} \equiv \frac{|\vec{\mathbf{p}}_f|}{E_f + m_f} P$$

Decay observables

$$\alpha = \frac{2\Re(S^* P)}{|S|^2 + |P|^2} \qquad \beta = \frac{2\Im(S^* P)}{|S|^2 + |P|^2} = \sqrt{1 - \alpha^2} \sin\phi$$

$$\Gamma = \mathrm{kin}(|\vec{\mathbf{p}}_f|, E_f, m_f)(|S|^2 + |P|^2)$$

# Motivation: new data landscape

nature physics | LETTERS
https://doi.org/10.1038/s41567-019-0494-8

**Polarization and entanglement in baryon–antibaryon pair production in electron–positron annihilation**

**The BESIII Collaboration***

[Nature Phys. 15, 631 (2019)]

Article | Open Access | Published: 01 June 2022

**Probing CP symmetry and weak phases with entang double-strange baryons**

The BESIII Collaboration

*Nature* **606**, 64–69 (2022) | Cite this article

**11k** Accesses | **7** Citations | **96** Altmetric | Metrics

[Nature 606, 6469 (2022)]

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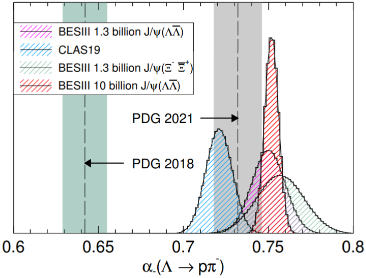
Precise Measurements of Decay Parameters and $CP$ Asymmetry with Entangled $\Lambda$–$\bar{\Lambda}$ Pairs

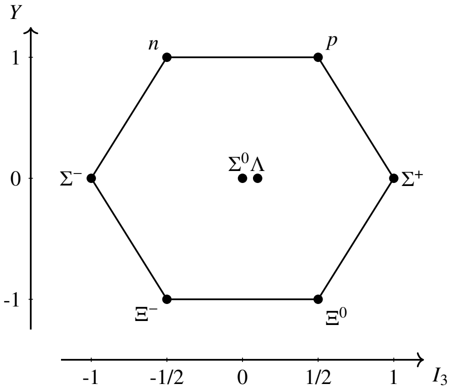
M. Ablikim *et al.* (BESIII Collaboration)
Phys. Rev. Lett. **129**, 131801 – Published 22 September 2022

[PRL 129, 131801 (2022)]



# Subject of study

- $\Delta S = 1$ transitions:
  - $\Sigma^+ \to n\pi^+$
  - $\Sigma^- \to n\pi^-$
  - $\Lambda \to p\pi^-$
  - $\Xi^- \to \Lambda\pi^-$




$$\sqrt{2}A(\Sigma^+ \to p\pi^0) - A(\Sigma^+ \to n\pi^+) + A(\Sigma^- \to n\pi^-) = 0$$

$$A(\Lambda \to p\pi^-) + \sqrt{2}A(\Lambda \to n\pi^0) = 0$$

$$A(\Xi^- \to \Lambda\pi^-) + \sqrt{2}A(\Xi^0 \to \Lambda\pi^0) = 0$$

# Extrapolate data points

## Next step

Use $\alpha$, $\phi$, $\Gamma$ data to extrapolate updated experimental $L$-wave amplitude values.

$L$-wave amplitude extraction: assuming $CP$ conservation, $\Delta I = 1/2$

$$L = \sum_j L_j \exp(i\delta_j^L), \; j \in \{2\Delta I, 2I\}$$

and final-interaction phase shifts [PRD105, 116022 (2022)]

| | $\lvert\mathbf{q}\rvert$ [MeV/c] | $\delta_1^S$ [°] | $\delta_3^S$ [°] | $\delta_1^P$ [°] | $\delta_3^P$ [°] |
|---|---|---|---|---|---|
| $\Lambda \to N\pi$ | 103 | 6.52(9) | −4.60(7) | −0.79(8) | −0.75(4) |
| $\Sigma \to N\pi$ | 190 | 9.98(23) | −10.70(13) | −0.04(33) | −3.27(15) |

Relative sign between amplitudes fixed by Lee-Sugawara relation

$$\frac{3}{\sqrt{6}} A^{(S)}(\Sigma^- \to n\pi^-) + A^{(S)}(\Lambda \to p\pi^-) + 2A^{(S)}(\Xi^- \to \Lambda\pi^-) = 0$$

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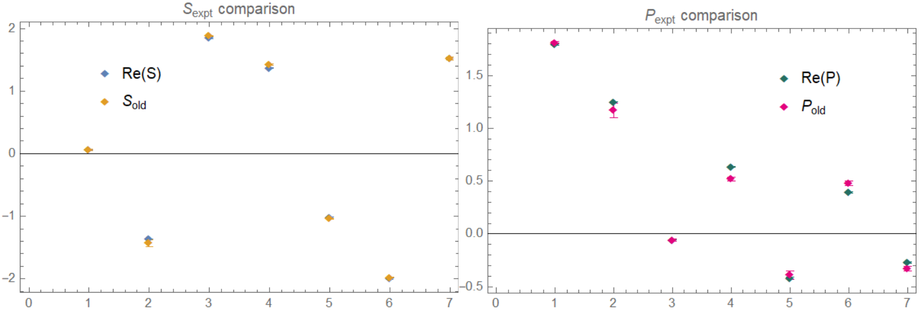
Use $\alpha$, $\phi$, $\Gamma$ data to extrapolate updated experimental $L$-wave amplitude values.

New reference values extracted from current data compared to [NPB 375 (1992) 561-581]:




Previous values extracted on the assumption of real-valued amplitudes:
updated to complex-valued $l$-waves on most recent data.

# Framework

- At low-energy regime, $\alpha_s$ too large for a perturbative description of hadron interactions from QCD.
- $\chi$PT: EFT with hadrons as DF parametrizes meson-baryon interactions [Physica A 96 (1979) 1-2, 327-340], [Annals Phys. 158 (1984) 142].

Weak nonleptonic hyperon decays previously studied in

Nuclear Physics B261 (1985) 185–198
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**ON THE VALIDITY OF CHIRAL PERTURBATION THEORY FOR WEAK HYPERON DECAYS***

J BIJNENS[1], H SONODA and Mark B WISE[2]

*California Institute of Technology, Pasadena, CA 91125, USA*

Received 23 January 1985
(Revised 14 June 1985)

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DOI 10.1007/s100529800896

THE EUROPEAN PHYSICAL JOURNAL C


**Non-leptonic hyperon decays in chiral perturbation theory**

B. Borasoy, Barry R. Holstein

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Received: 16 December 1997 / Revised version: 18 February 1998 / Published online: 15 June 1998

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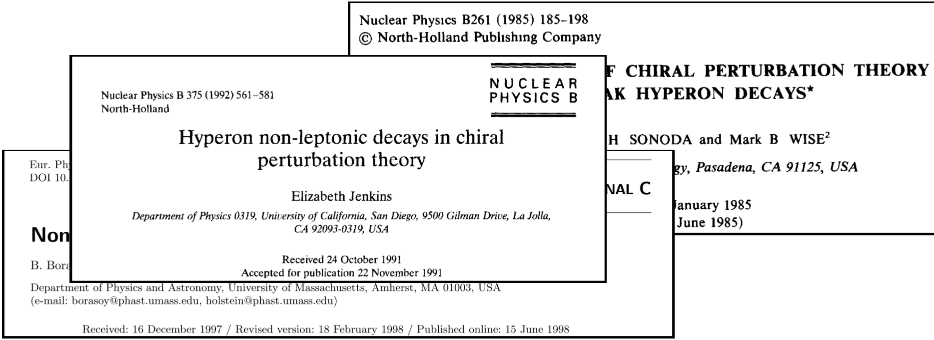
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June 1985)

Nuclear Physics B 375 (1992) 561–581
North-Holland

NUCLEAR PHYSICS B

Hyperon non-leptonic decays in chiral perturbation theory

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DOI 10.

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# Starting point

Computing 1-loop corrections using *Heavy-Baryon* $\chi$PT (nonrelativistic approach):

1. Bijnens, Sonoda & Wise [NPB 261 (1985) 185-198]:
   - baryon decuplet fields not included; terms up to $O(M_K^2 \log M_K)$.
2. Jenkins [NPB 375 (1992) 561-581]:
   - inclusion of decuplet, 3-meson vertex $h_\pi$; terms up to $O(M_K^2 \log M_K)$.
3. Borasoy & Holstein [EPJC 6 (1999) 85-107]
   - decuplet not included, 3-meson vertex $h_\pi$; terms up to $M_K^2(a + b \log M_K)$.
4. Abd El-Hady & Tandean [PRD 61, 114014 (2000)]:
   - same conclusions as 2, contradicting results in $h_\pi$ terms.

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   - same conclusions as 2, contradicting results in $h_\pi$ terms.

## General conclusions

- LO chiral corrections to *S*-waves are in good agreement with experiment.
- Conversely, *P*-waves are not well-described.
- Results from simultaneous fitting are presumed unsatisfactory: yet unexplored.

# Relativistic $\chi$PT and EOMS

- Dimensional regularization + modified minimal subtraction do not work for baryons;
- inclusion of baryon masses breaks power-counting for the loop diagrams.

**Previous approach: HB$\chi$PT** Jenkins & Manohar [PLB 255 (1991) 558-562]

- Expansion of the Lagrangian in powers of $1/m_B$ in the chiral limit.
- Power-counting is manifest, but Lorentz invariance is not.
- Issues with analyticity (reproducing the correct positions of poles).

**Extended On-Mass-Shell ren. scheme** Gegelia & Japaridze [PRD 60 (1999) 114038]

- Subtract the power-counting-violating terms, i.e. choosing appropriate renormalization conditions.
- Manifest Lorentz invariance: the standard approach for B$\chi$PT nowadays.

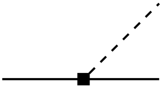
# Procedure

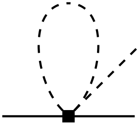
Compute 1-loop corrections from relativistic LO Lagrangian

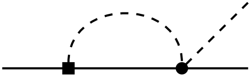
$$\mathcal{L}^{\mathrm{s}}_{\phi B} + \mathcal{L}^{\mathrm{w}}_{\phi B}$$

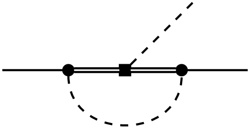
$$\mathcal{L}^{\mathrm{w}}_{\phi B} = h_D \,\mathrm{tr}\bar{B}\{\xi^\dagger h \xi, B\} + h_F \,\mathrm{tr}\bar{B}[\xi^\dagger h \xi, B] + h_C \,\mathrm{tr}\,\bar{T}^\mu (\xi^\dagger h \xi) T_\mu$$

E.g. $S$-wave contributions:





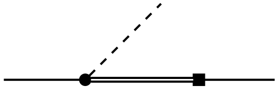




# Procedure

Inclusion of lower-lying $\frac{1}{2}^{\pm}$ resonances [PRD 59, 094025 (1999)]

$$\mathcal{L}^{\mathrm{w}}_{\mathrm{res}} \propto d^{*} \left[\mathrm{tr}(\bar{R}^{+}\{\xi^{\dagger}h\xi, B\}) + \mathrm{tr}(\bar{B}\{\xi^{\dagger}h\xi, R^{+}\})\right] + f^{*} \left[\mathrm{tr}(\bar{R}^{+}[\xi^{\dagger}h\xi, B]) + \mathrm{tr}(\bar{B}[\xi^{\dagger}h\xi, R^{+}])\right]$$
$$+ i w_{d} \left[\mathrm{tr}(\bar{R}^{-}\{\xi^{\dagger}h\xi, B\}) - \mathrm{tr}(\bar{B}\{\xi^{\dagger}h\xi, R^{-}\})\right] + i w_{f} \left[\mathrm{tr}(\bar{R}^{-}[\xi^{\dagger}h\xi, B]) - \mathrm{tr}(\bar{B}[\xi^{\dagger}h\xi, R^{-}])\right]$$



The resulting amplitudes:

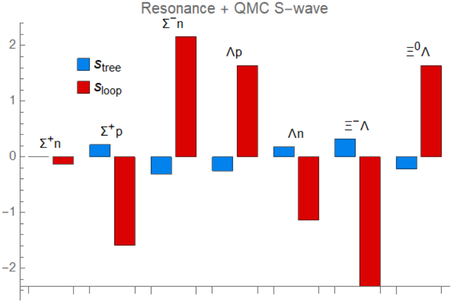
$$S_{\mathrm{theory}},\ P_{\mathrm{theory}} = l.c.(h_{D,F,C},\ w_{d,f},\ d^{*},\ f^{*})$$

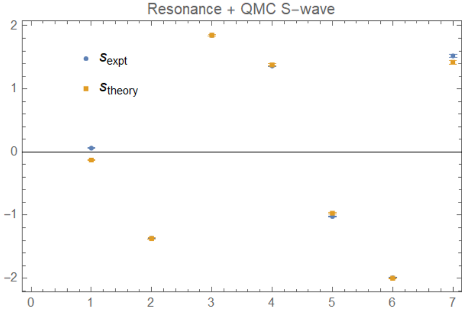
## Goal

To fit $L_{\mathrm{theory}}$ to $L_{\mathrm{expt}}$ using least squares method to obtain LEC's values.

# Preliminary results - $S$-waves

LECs extracted from fit to ***S*-waves only**:




| LEC | $[G_F m_\pi^2 \sqrt{2} f_\pi]$ |
|---|---|
| $h_D$ | $-1/3 h_F$ |
| $h_F$ | $-0.233 \pm 0.007$ |
| $h_C$ | $1.99 \pm 0.04$ |
| $w_f$ | $4.63 \pm 0.07$ |
| $w_d$ | $-14.72 \pm 0.11$ |

## EOMS + resonances

- Good agreement with experiment.

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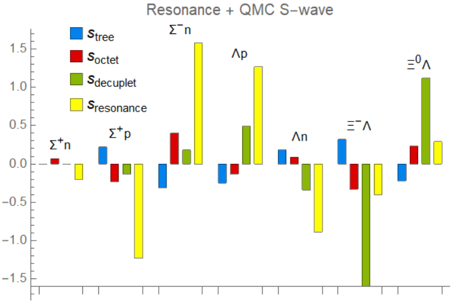


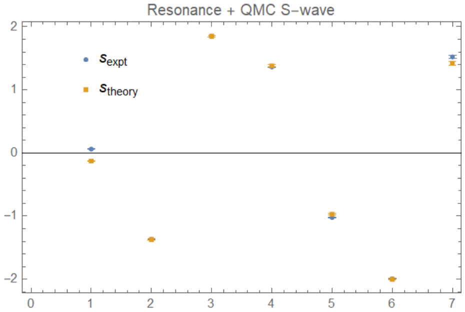


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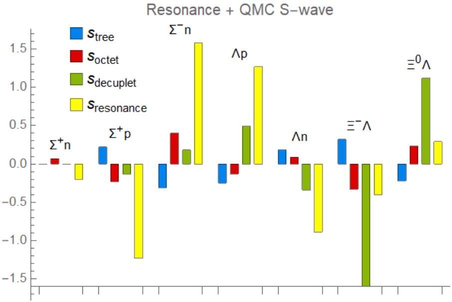


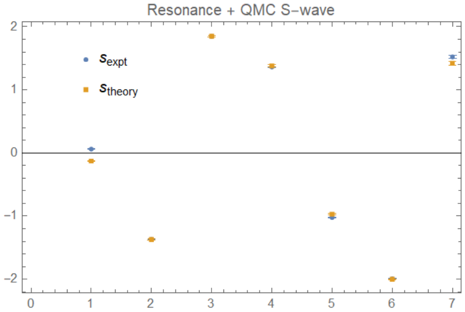


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## EOMS + resonances

- Good agreement with experiment.
- Corrections to LO $\chi$PT are large.
- The importance of resonances is confirmed.

# Preliminary results - *P*-waves

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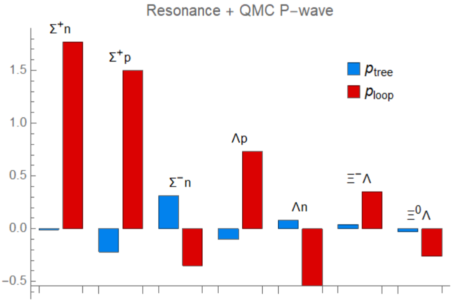


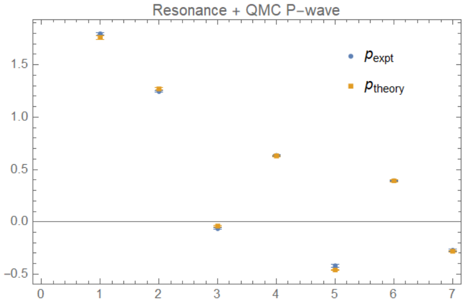


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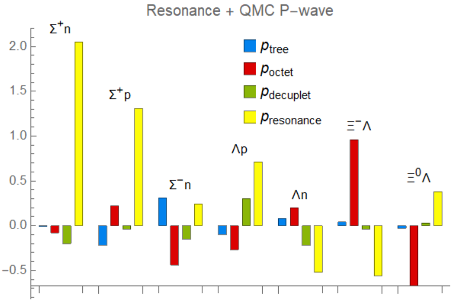


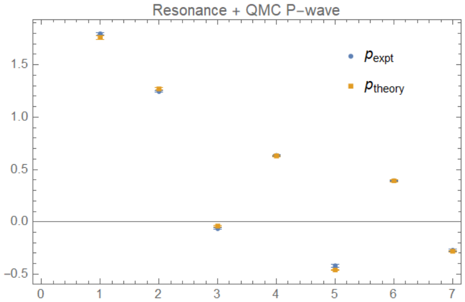


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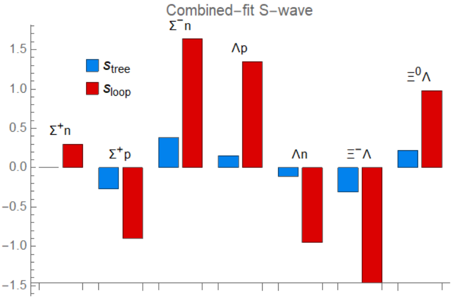
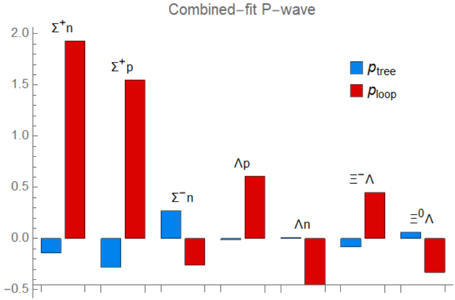
## EOMS + resonances

- Agreement with experiment better than in *S*-waves.
- Similar relative size of resonance terms to tree-level.

# Preliminary results - Combining $S$ and $P$

LECs extracted from fit to **combined *S*- and *P*-waves**:

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## EOMS + resonances

- LEC's size somewhat consistent with other results.

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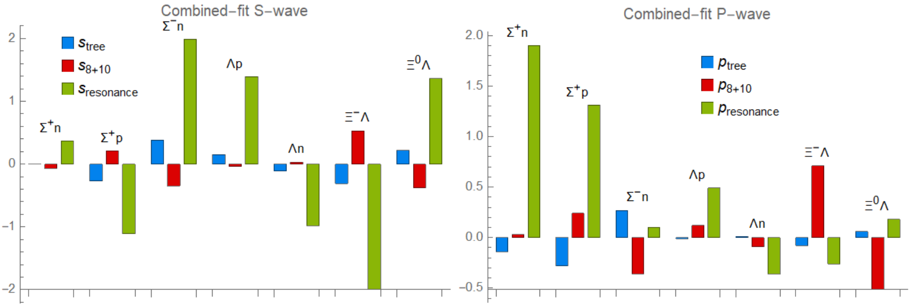


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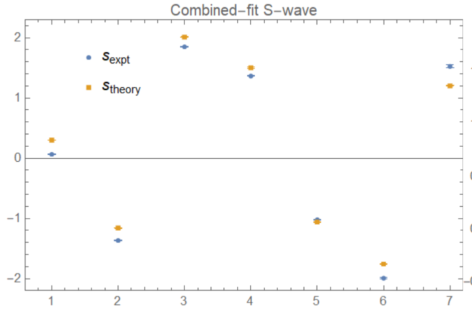


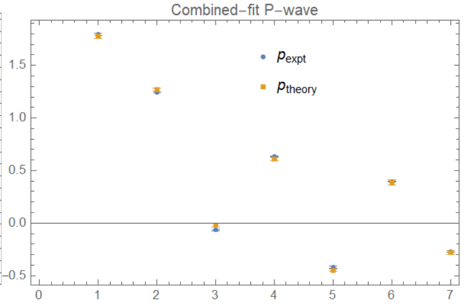


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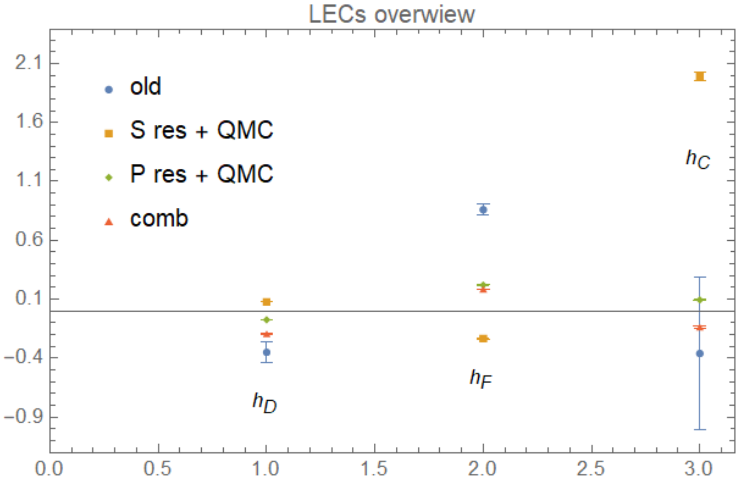
- Resonances dominate almost always over "true" loops.
- $S$ agreement with experiment slightly worse, $P$ remains good.

# LECs results



# Summary

- Hyperon nonleptonic decays were never studied in relativistic $\chi$PT: following previous attempts in HB$\chi$PT and updated measurement of $\Lambda$ decay asymmetry from BESIII.
- Established a limit to HB$\chi$PT from the full relativistic amplitude and resolved conflict between previous works (three-meson vertex terms, eventually neglected).
- Included resonance saturation terms at tree-level; they are the most relevant.
- Simultaneous fit to $S$- and $P$-waves is performed: good agreement with experiment, loss of convergent behavior.
- Outlook: two-loops, $\Lambda(1405)$ as hadron molecule state...

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**Thank you!**

# Relativistic chiral LO Lagrangian

Meson-baryon LO Lagrangian

$$\mathcal{L}^{\rm s}_{\phi B} = i \,\mathrm{tr}\, \bar{B} \not{D} B - m_B \,\mathrm{tr}\, \bar{B} B + D \,\mathrm{tr}\, \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} + F \,\mathrm{tr}\, \bar{B} \gamma^\mu \gamma_5 [A_\mu, B] - i \,\bar{T}^\mu \not{D} T_\mu$$
$$+ m_T \,\bar{T}^\mu T_\mu + \mathcal{C} \left( \bar{T}^\mu A_\mu B + \bar{B} A_\mu T^\mu \right) + \mathcal{H} \,\bar{T}^\mu \gamma_\nu \gamma_5 A^\nu T_\mu + \frac{f^2}{4} \,\mathrm{tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger$$

Inclusion of $\frac{1}{2}^{\mp}$ resonances [PRD59, 094025 (1999)]

$$\begin{aligned} \mathcal{L}^{\rm s}_{RB} &= 2 s_d \left[ \mathrm{tr}(\bar{R} \gamma_\mu \{A_\mu, B\}) - \mathrm{tr}(\bar{B} \gamma_\mu \{A_\mu, R\}) \right] \\ &\quad + 2 s_f \left[ \mathrm{tr}(\bar{R} \gamma_\mu [A_\mu, B]) - \mathrm{tr}(\bar{B} \gamma_\mu [A_\mu, R]) \right] \end{aligned}$$

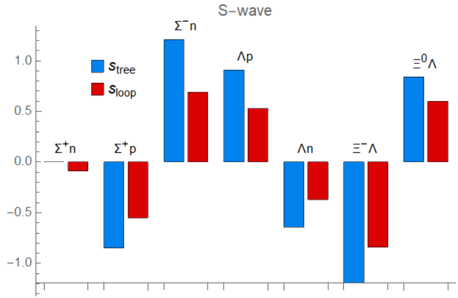
$$\begin{aligned} \mathcal{L}^{\rm s}_{B^* B} &= \frac{D^*}{2} \left[ \mathrm{tr}(\bar{B}^* \gamma_\mu \gamma_5 \{A_\mu, B\}) + \mathrm{tr}(\bar{B} \gamma_\mu \gamma_5 \{A_\mu, B^*\}) \right] \\ &\quad + \frac{F^*}{2} \left[ \mathrm{tr}(\bar{B}^* \gamma_\mu \gamma_5 [A_\mu, B]) + \mathrm{tr}(\bar{B} \gamma_\mu \gamma_5 [A_\mu, B^*]) \right] \end{aligned}$$

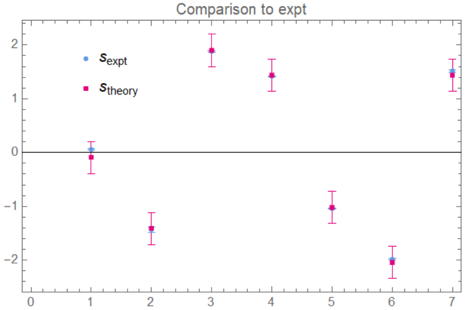
$$V^\mu = \frac{1}{2} \left( \xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi \right), \quad A^\mu = \frac{i}{2} \left( \xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi \right)$$

$$\xi = \exp \frac{i\pi}{f}, \quad \Sigma = \xi^2 = \exp \frac{2i\pi}{f}$$

# Previous work results - *S*-waves

LECs extracted from fit to ***S*-waves only**:




| **LEC** | $[G_F m_\pi^2 \sqrt{2} f_\pi]$ |
|---|---|
| $h_D$ | $-0.35 \pm 0.09$ |
| $h_F$ | $0.86 \pm 0.05$ |
| $h_C$ | $-0.36 \pm 0.65$ |

[NPB 375 (1992) 561-581]

- Good agreement with experiment.

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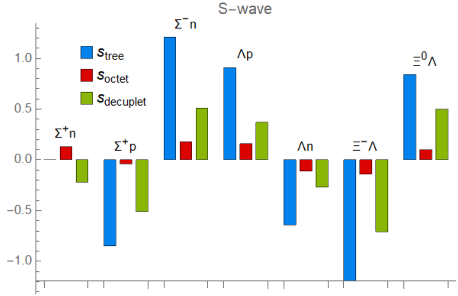


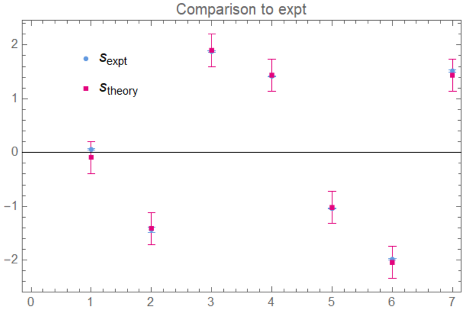


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[NPB 375 (1992) 561-581]

- Good agreement with experiment.
- Decuplet contribution dominates over octet.

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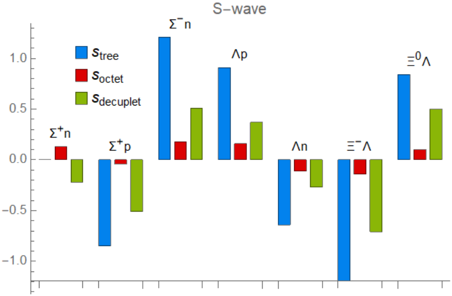


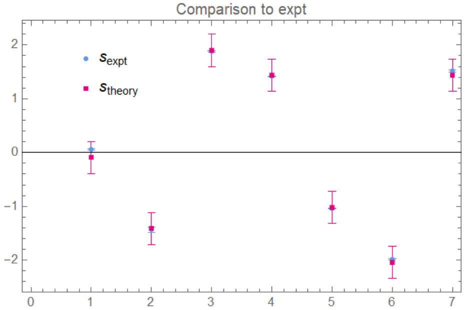


| LEC | $[G_F m_\pi^2 \sqrt{2} f_\pi]$ |
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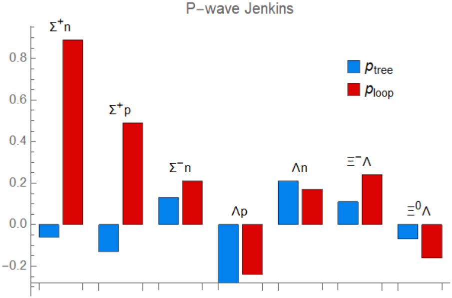
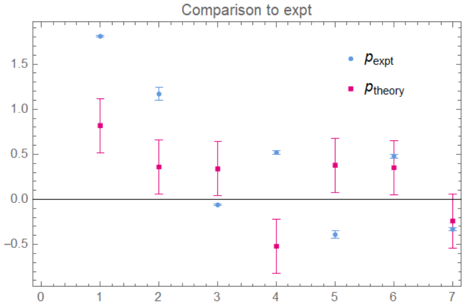
[NPB 375 (1992) 561-581]

- Good agreement with experiment.
- Decuplet contribution dominates over octet.
- $h_C$ not well determined by 1-loop fit.

# Previous work results - $P$-waves

Using the LECs from $S$-wave fit:




| LEC | $[G_F m_\pi^2 \sqrt{2} f_\pi]$ |
|---|---|
| $h_D$ | $-0.35 \pm 0.09$ |
| $h_F$ | $0.86 \pm 0.05$ |
| $h_C$ | $-0.36 \pm 0.65$ |

[NPB 375 (1992) 561-581]

- $P$-waves are poorly described.
- Simultaneous fitting seems unfeasible.