Possible scenario of dynamical chiral symmetry breaking in the instanton liquid

Based on YS, Jido, Phys. Rev. D 110, 014037 (2024)

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Introduction

- **Vacuum of strong interaction**
  - non-trivial structure
    - confinement
    - dynamical chiral symmetry breaking (χSB)

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  - non-Abelian gauge theory (QCD) has topological configurations
    - instantons
    - sphalerons

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    - confinement
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- **non-trivial structure due to non-Abelian gauge theory**
  - non-Abelian gauge theory (QCD) has topological configurations
    - instantons
    - sphalerons

- **Despite their significance, it's difficult to detect them experimentally**
  - chiral magnetic effect in QGP by RHIC
  - mass spectrum of light scalar and pseudoscalar mesons

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A. **There might be.** We focus on:

- **topological configuration (instanton)**
- **chiral ($\mathrm{U}(1)_A$) anomaly**
- **chiral symmetry breaking (χSB)**

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A. **There might be.** We focus on:

- **topological configuration (instanton)**
- **chiral ($\mathrm{U}(1)_A$) anomaly**
- **chiral symmetry breaking (χSB)**

- **topological configuration relates to chiral ($\mathrm{U}(1)_A$) anomaly**
  - cf. the Atiyah-Singer index theorem

- **chiral ($\mathrm{U}(1)_A$) anomaly** has long been also discussed in the context of χSB by chiral effective theories

- **discussion for chiral symmetry breaking** with chiral ($\mathrm{U}(1)_A$) anomaly advances our understanding of the QCD vacuum structure

Outline

- **Chiral symmetry breaking (χSB) in NJL model**
- **Chiral symmetry breaking (χSB) in the instanton liquid [our work]**
- **Summary**

Chiral symmetry breaking in NJL model: critical coupling

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- **NJL model** explains spontaneous chiral symmetry breaking with large coupling $g_S > g_S^{\mathrm{crit}}$

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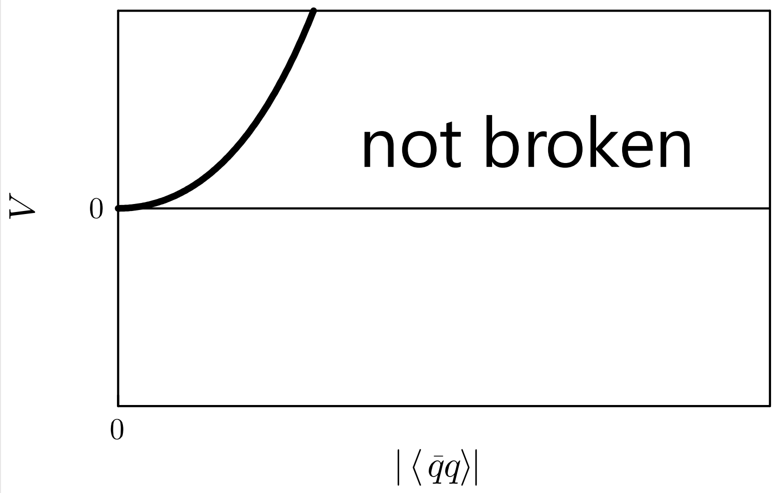

Example

NJL model with chiral limit $\mathcal{L}_{\text{int}} = \sum_{a=0}^{8} \frac{g_S}{2} \left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2 \right]$

Below g_S^{crit}

- chiral symmetry not broken
- order parameter $\langle \bar{q}q \rangle = 0$ at the vacuum

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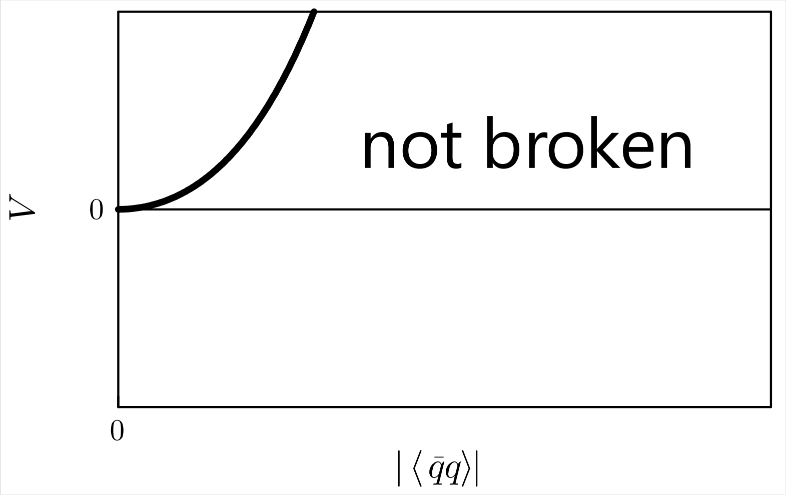
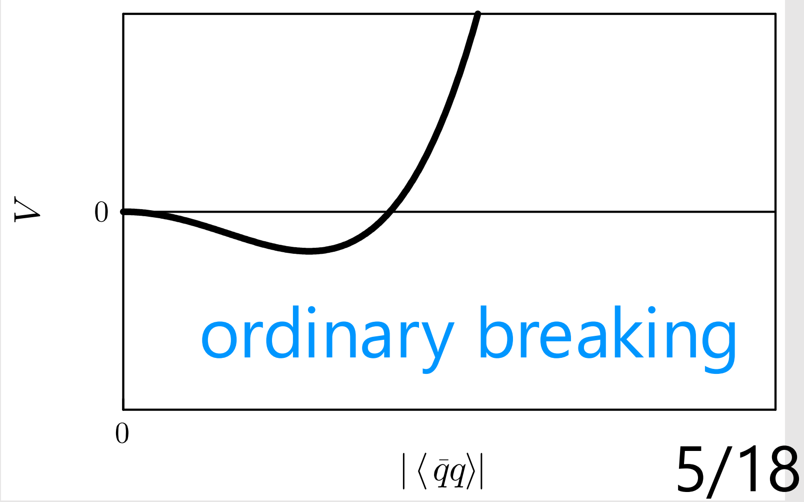
Above $g_S^{\rm crit}$

- chiral symmetry broken
- order parameter $\langle \bar{q}q \rangle \neq 0$ at the vacuum

→ ***Ordinary chiral symmetry breaking***





Realization of $\mathrm{U}(1)_A$ anomaly in chiral effective theories

- **chiral ($\mathbf{U}(\mathbf{1})_A$) anomaly** has been introduced
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Kobayashi-Maskawa-'t Hooft (KMT) term

- respects $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ but violates $\mathrm{U}(1)_A$

$$\propto \{\det [\bar{q}_i (q - \gamma_5) q_j] + \mathrm{H.c.}\}$$

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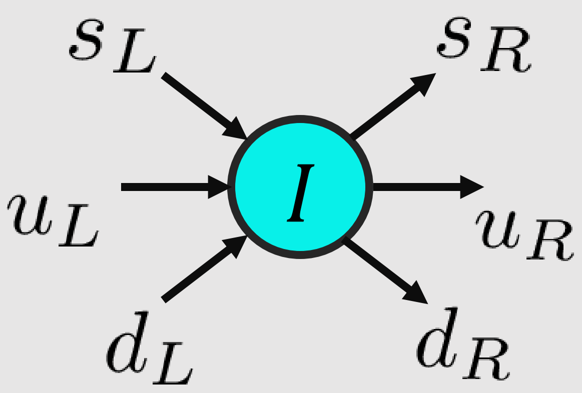
't Hooft vertex

- effective Lagrangian induces KMT term as a its part
- originates from instanton
- flips quark's chirality → chiral ($\mathrm{U}(1)_A$) anomaly



[4] E. Shuryak, "Nonperturbative Topological Phenomena in QCD and Related theory" (2021)

Chiral symmetry breaking in NJL model: induced by anomaly

[5] S. Kono, et al., PTEP **2021**, 093D02 (2021)

- strength of the chiral ($\mathrm{U}(1)_A$) anomaly **changes χSB pattern**

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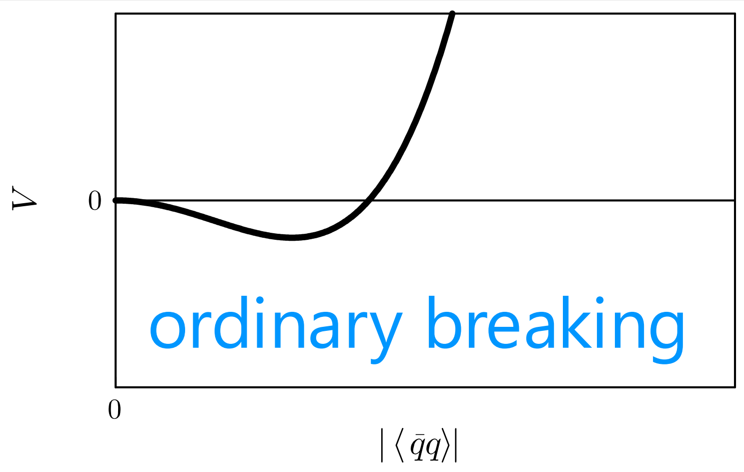
- chiral symmetry broken in "ordinary" way



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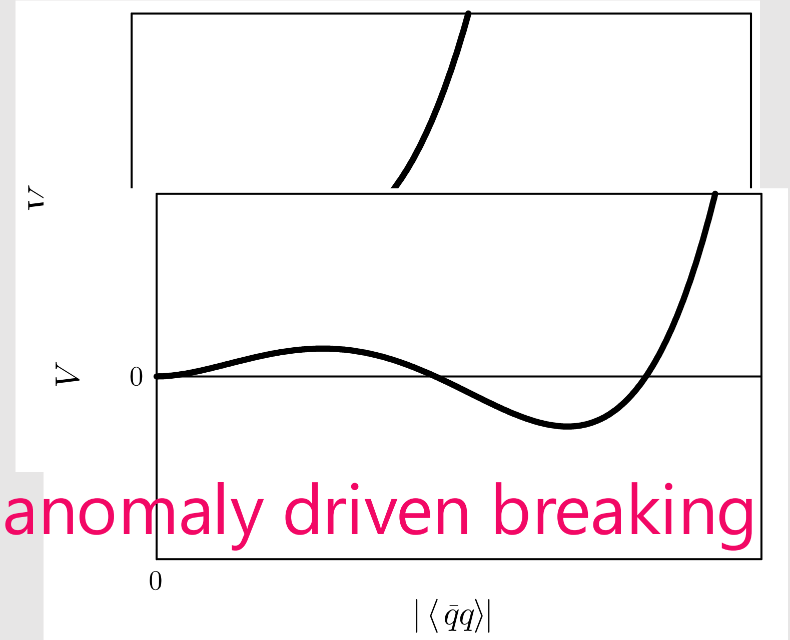
Below $g_S^{\rm crit}$

- even if $g_S < g_S^{\rm crit}$, chiral symmetry *can be* broken due to sufficiently large chiral ($\mathrm{U}(1)_A$) anomaly

→ ***Anomaly driven chiral symmetry breaking***



Anomaly driven χSB may link to physical observables

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- show that **size of the sigma mass** depends on the breaking pattern

ordinary breaking $\Rightarrow m_\sigma > 800\ \mathrm{MeV}/c^2$

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anomaly driven breaking $\Rightarrow m_\sigma < 800\ \mathrm{MeV}/c^2$

- here, the sigma is introduced as chiral partner of pion (chiral sigma)
- to compare the f0(500) resonance, we need to know its composition

Determination procedure for χSB: from coupling to curvature

[6] YS and D. Jido, PRD **110**, 014037 (2024)

- so far use **model specific coupling g_S**
  to discuss χSB and chiral $(\mathrm{U}(1)_A)$ anomaly effect
  → invalid except for NJL model

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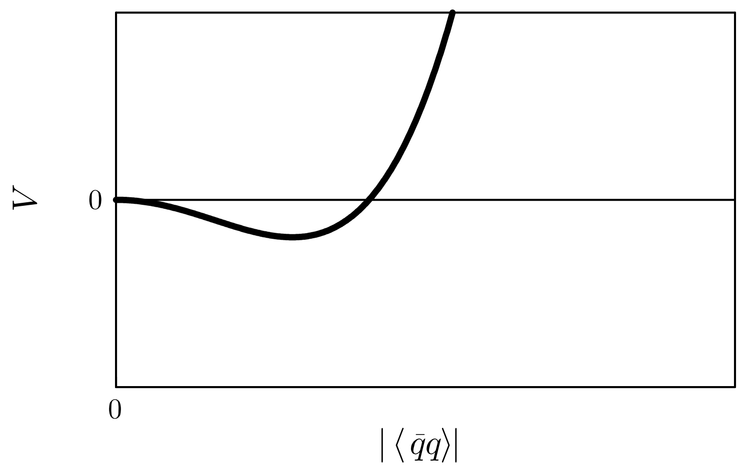
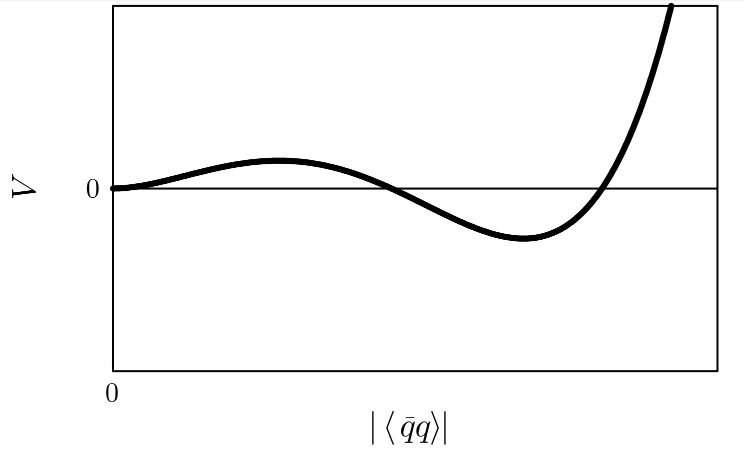


ordinary breaking

Below g_S^{crit} but large anomaly



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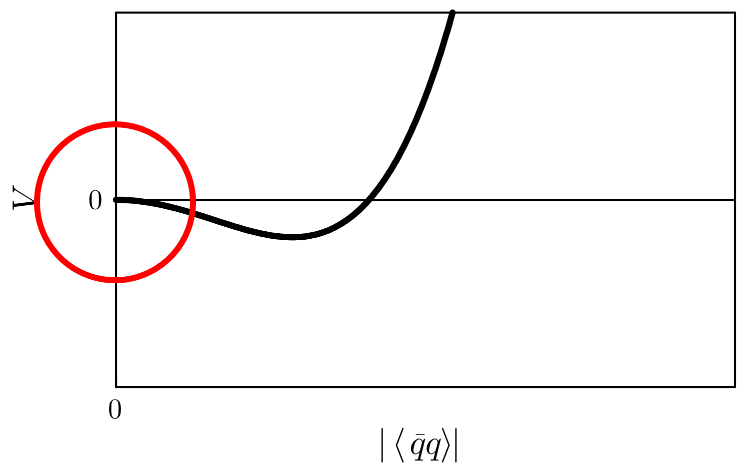
  → invalid except for NJL model
- as generalization use **curvature of the effective potential at the origin**
  → valid for other systems

Above g_S^{crit}



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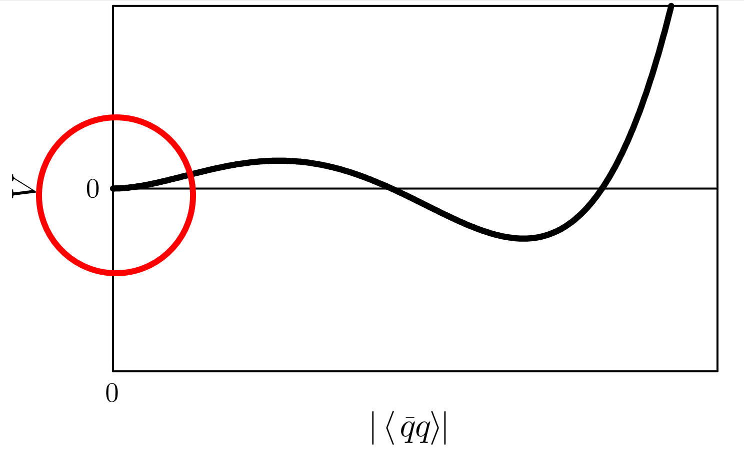

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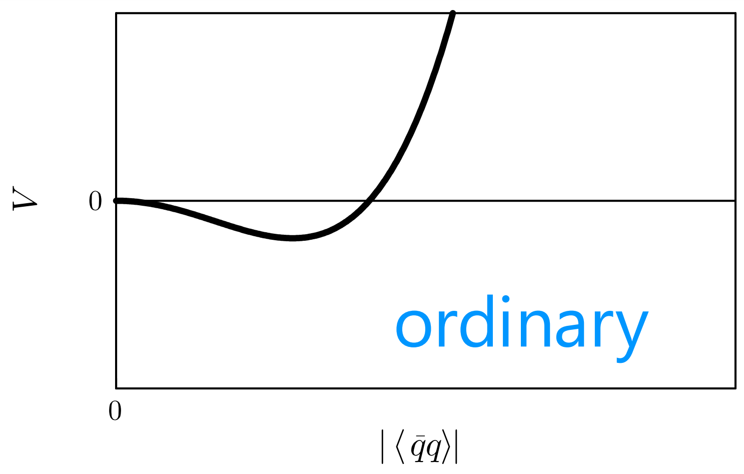
coupling

Above g_S^{crit}



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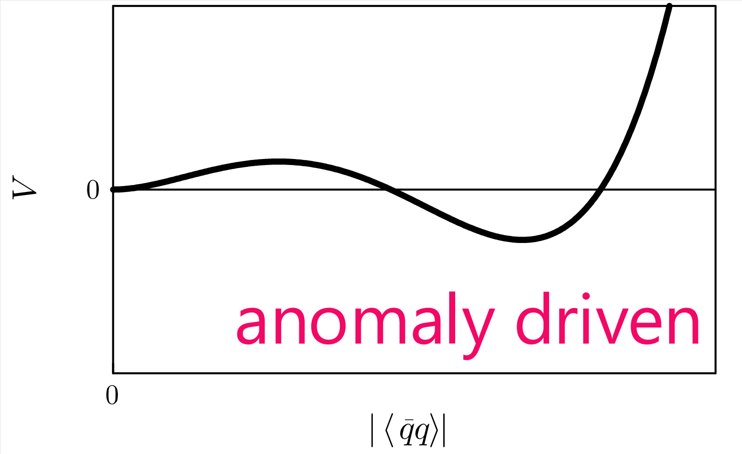

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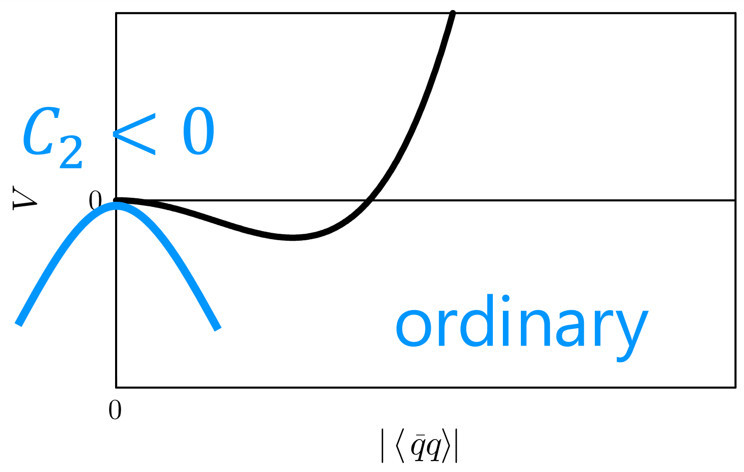
coupling

▼

curvature:

$$C_2 \equiv \left. \frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2} \right|_{\langle \bar{q}q \rangle = 0}$$

Above g_S^{crit}



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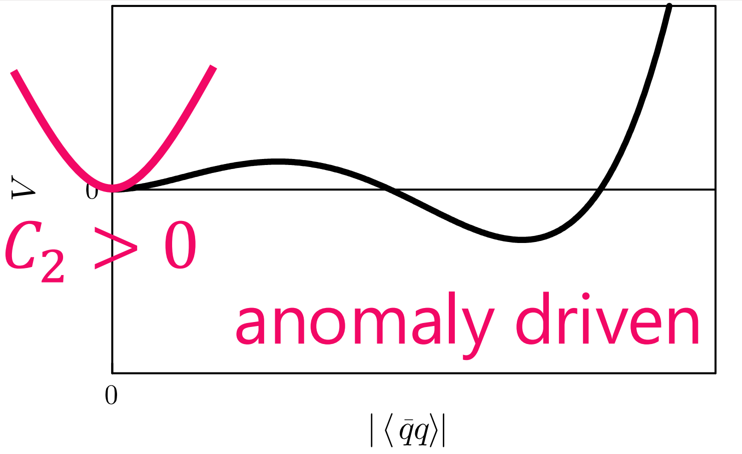

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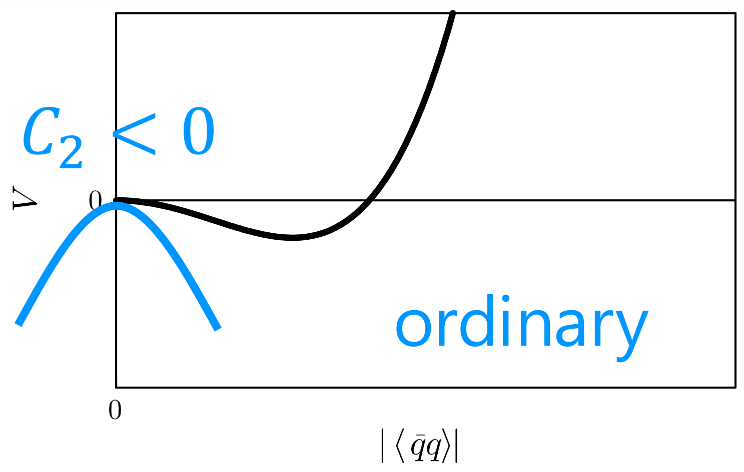

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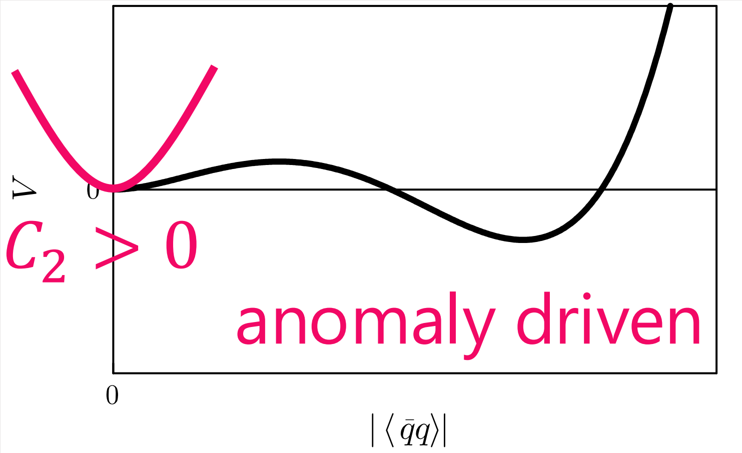


our definition

$C_2 < 0 \ \Rightarrow$ ordinary

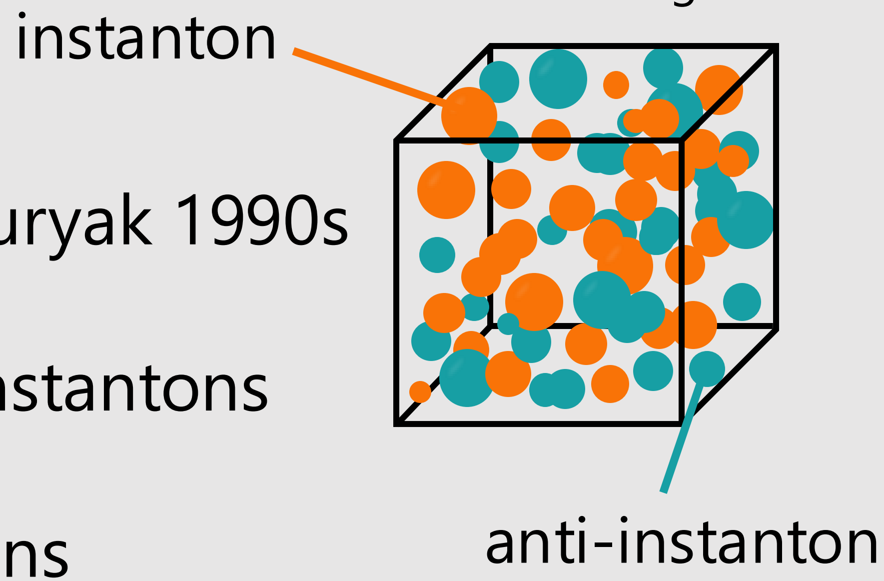
$C_2 > 0 \ \Rightarrow$ anomaly driven

Model

[7] T. Schafer and E. Shuryak, Rev. Mod. Phys. **70** 323 (1998).

- **I**nteracting **I**nstanton **L**iquid **M**odel (IILM), E. Shuryak 1990s
- allows us to treat the QCD vacuum as statistical mechanics of instantons and anti-instantons
- described by Euclidean QCD partition function that is saturated by instantons and anti-instantons

*schematic image of IILM



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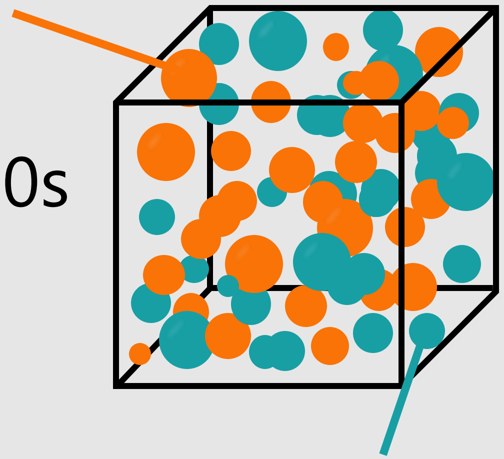


IILM partition function:

Instanton-instanton interaction

Instanton-quark interaction

$$Z_{\mathrm{IILM}} = \frac{1}{N_+! N_-!} \int \left(\prod_{i=1}^{N_+ + N_-} d\Omega_i \, f(\rho_i) \right) \exp(-S_{\mathrm{int}}) \prod_{f=1}^{N_f} \mathrm{Det}\,(\gamma_\mu D_\mu + m_f)$$

Semiclassical Instanton amplitude

Collective coordinates of instantons

Simulation detail

[6] YS and D. Jido, PRD **110**, 014037 (2024)

Model action:

full

quench

$$S_{\text{eff}} = -\sum_{i=1}^{N} \log\left[f(\rho_i)\right] + S_{\text{int}} - \sum_{f=1}^{N_f} \log\left[\text{Det}\left(\gamma_\mu D_\mu + m_f\right)\right]$$

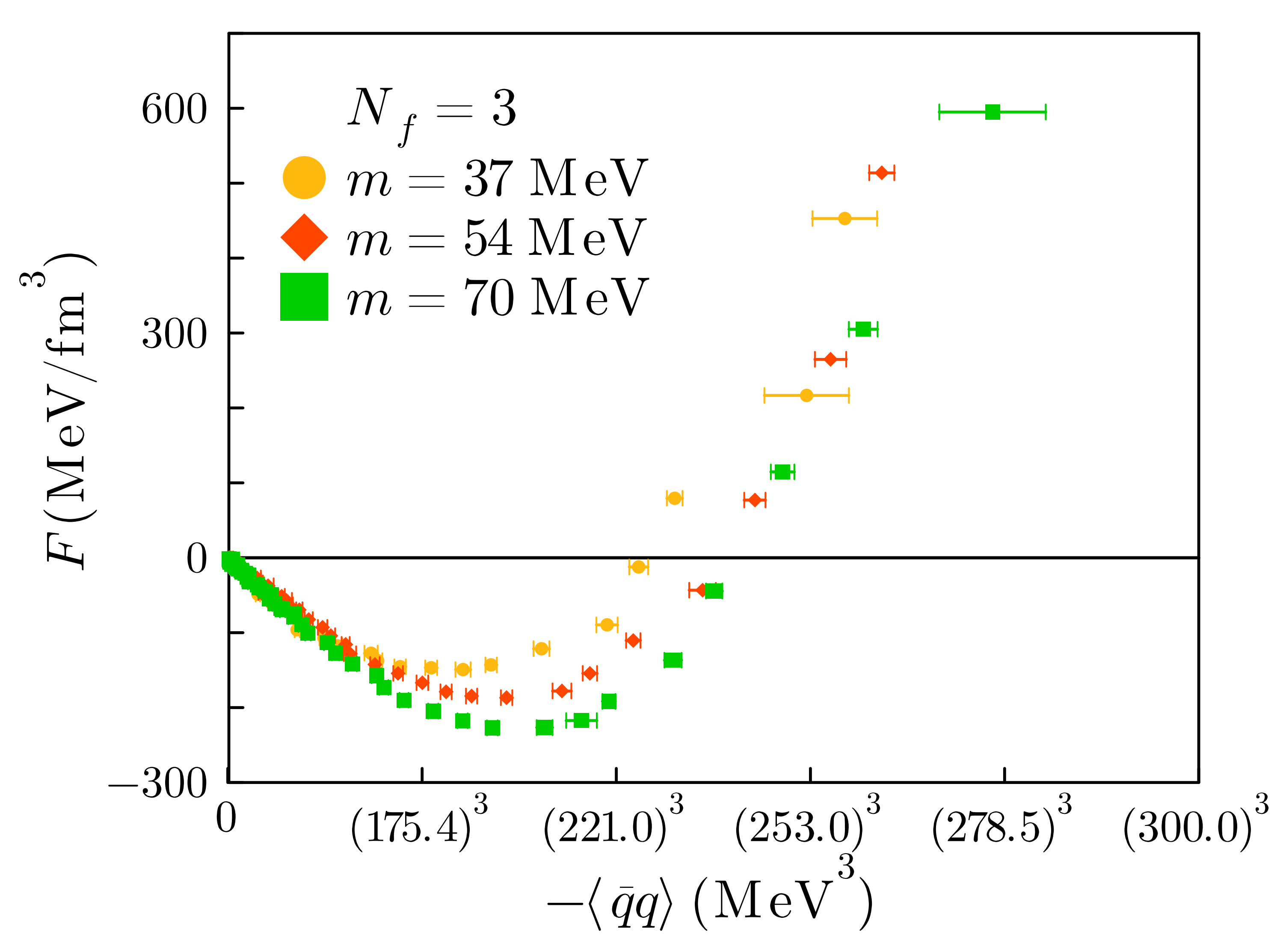
Setup:

	flavor	m_f	# of $I\&\bar{I}$ (fixed)	# of conf.
full	$N_f = 3$	37 MeV~70 MeV	$32 = 16 + 16$	5000
quench	$N_f = 0$	2.8 MeV~28 MeV		

Calculated quantities:

	description
$F = -\ln Z_{\text{IILM}} / V_4$	vacuum energy at zero temp. corresponding to eff. potential
$\langle \bar{q} q \rangle$	quark condensate w/o free contrib. per flavor

Result: F vs. $\langle \bar{q}q \rangle$ [full]



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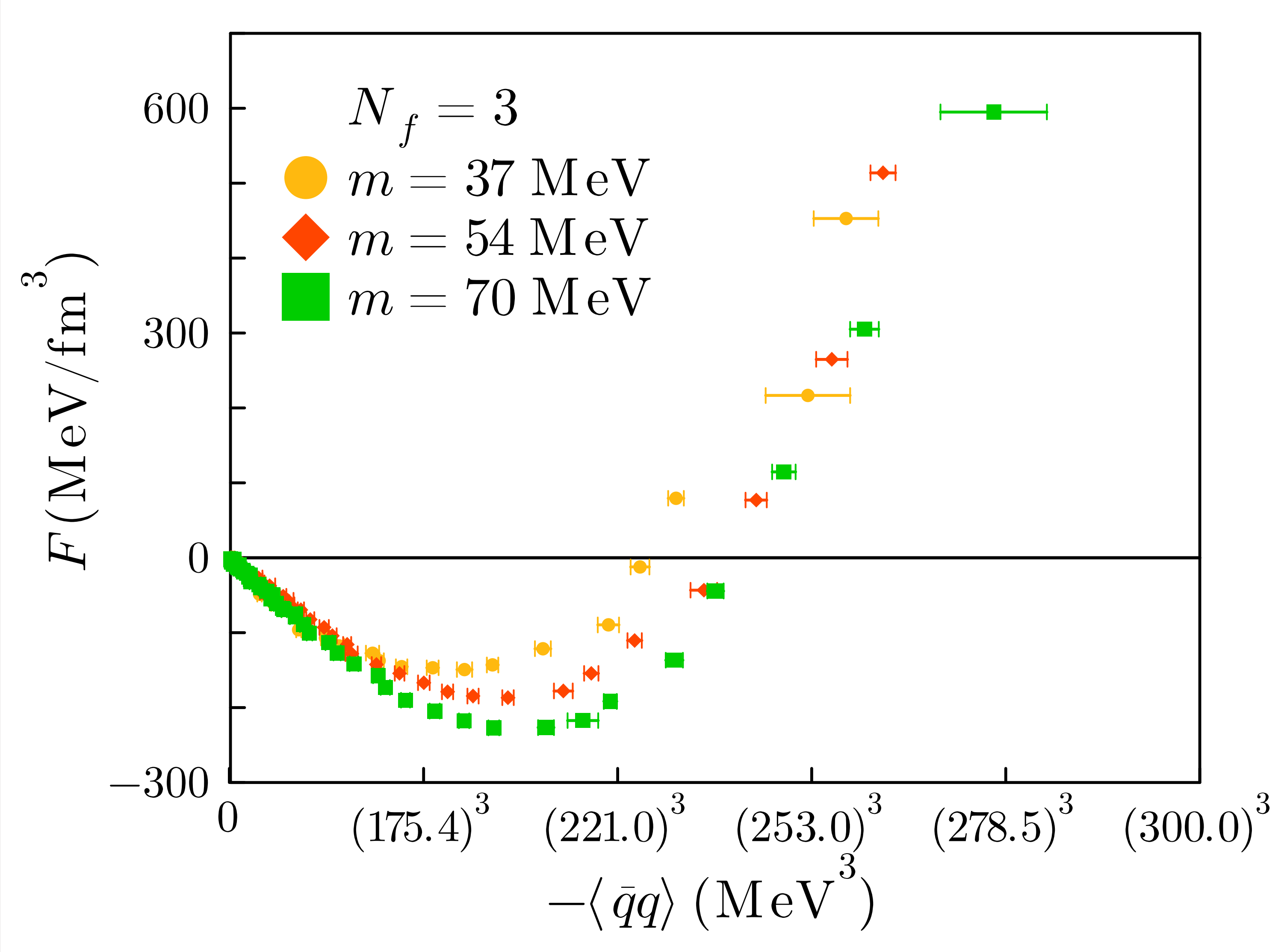

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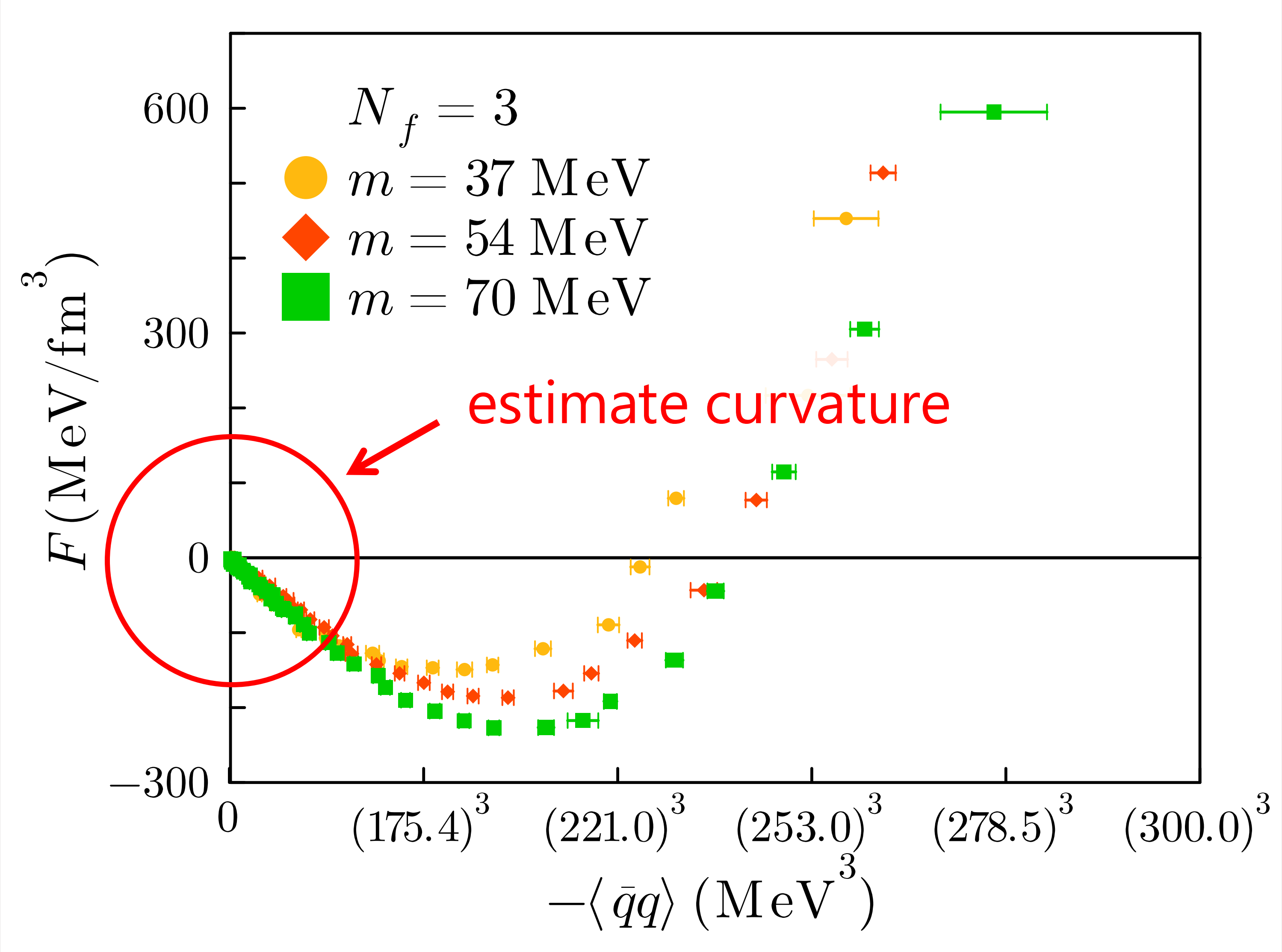

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estimate curvature at the origin

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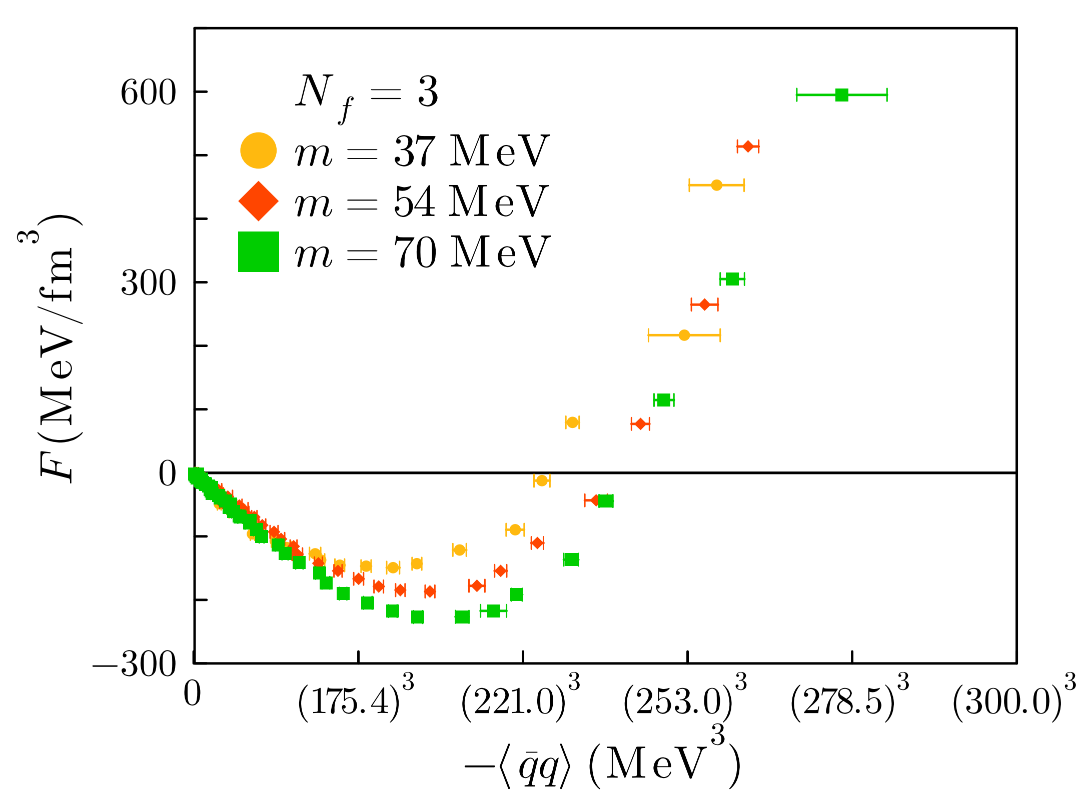

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Result: Curvature [full]

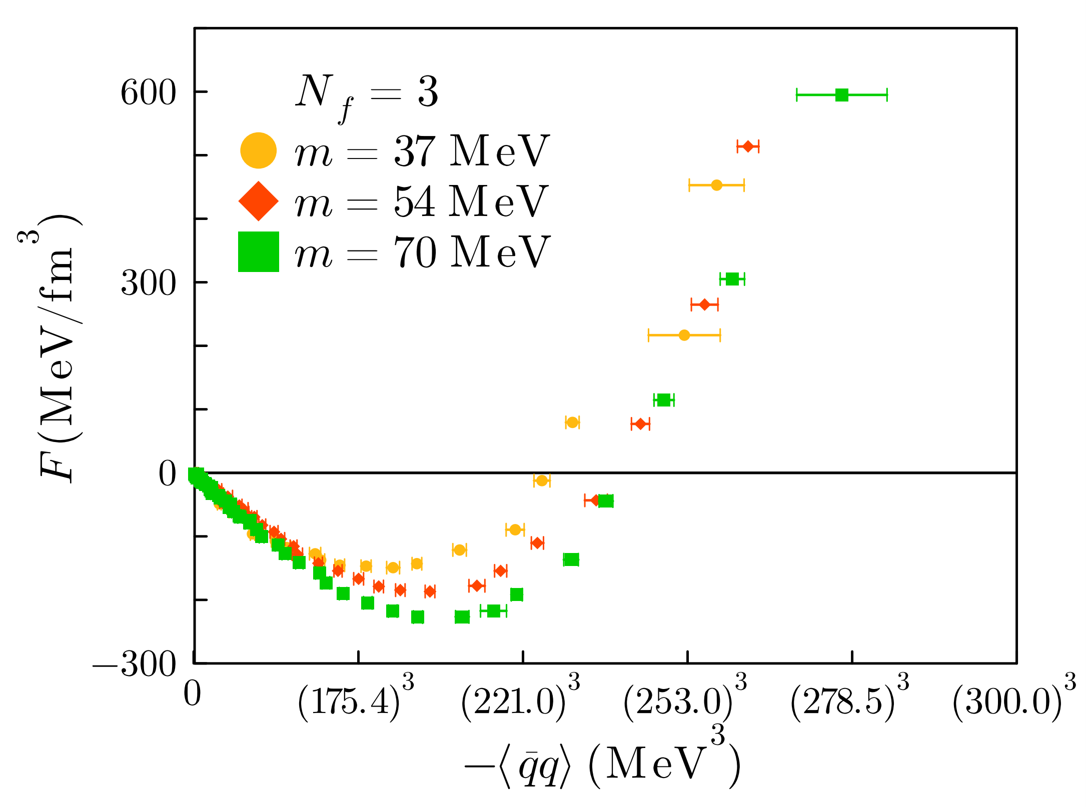
1. obtain F vs. $\langle \bar{q}q \rangle$ (as previous slide)

2. fit data to polynomial

$$F(\langle \bar{q}q \rangle) = C_0 + \boxed{C_1} \langle \bar{q}q \rangle + \boxed{C_2} \langle \bar{q}q \rangle^2 + \cdots + C_K \langle \bar{q}q \rangle^K$$

slope $\propto m_q$
$\because$ finite quark mass

curvature
= what we want



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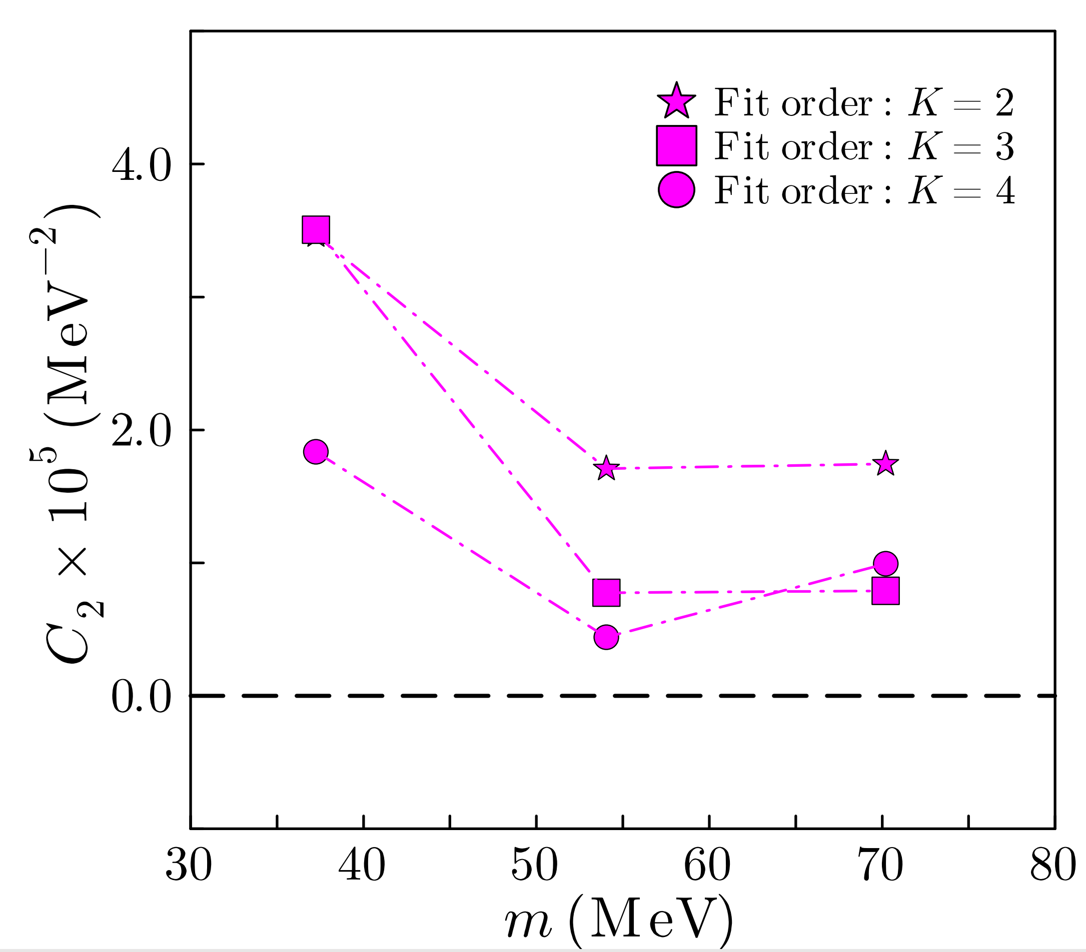
C_2 values for several current quark masses and polynomial orders to fit



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3. extract the curvature C_2 (right figure)
   - $C_2 > 0$ for wide quark mass range
     → ***Anomaly driven breaking!!***
   - different orders K give systematic errors
   - small statistic errors are omitted

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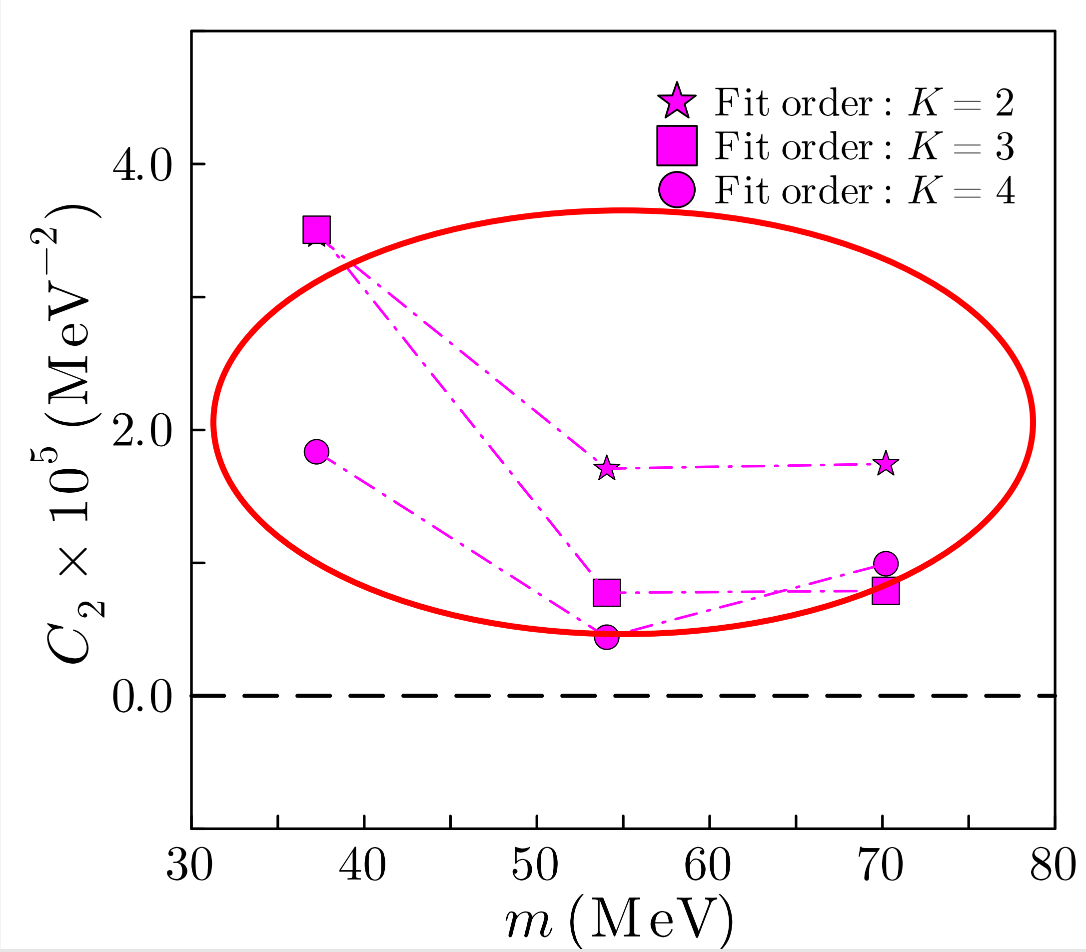

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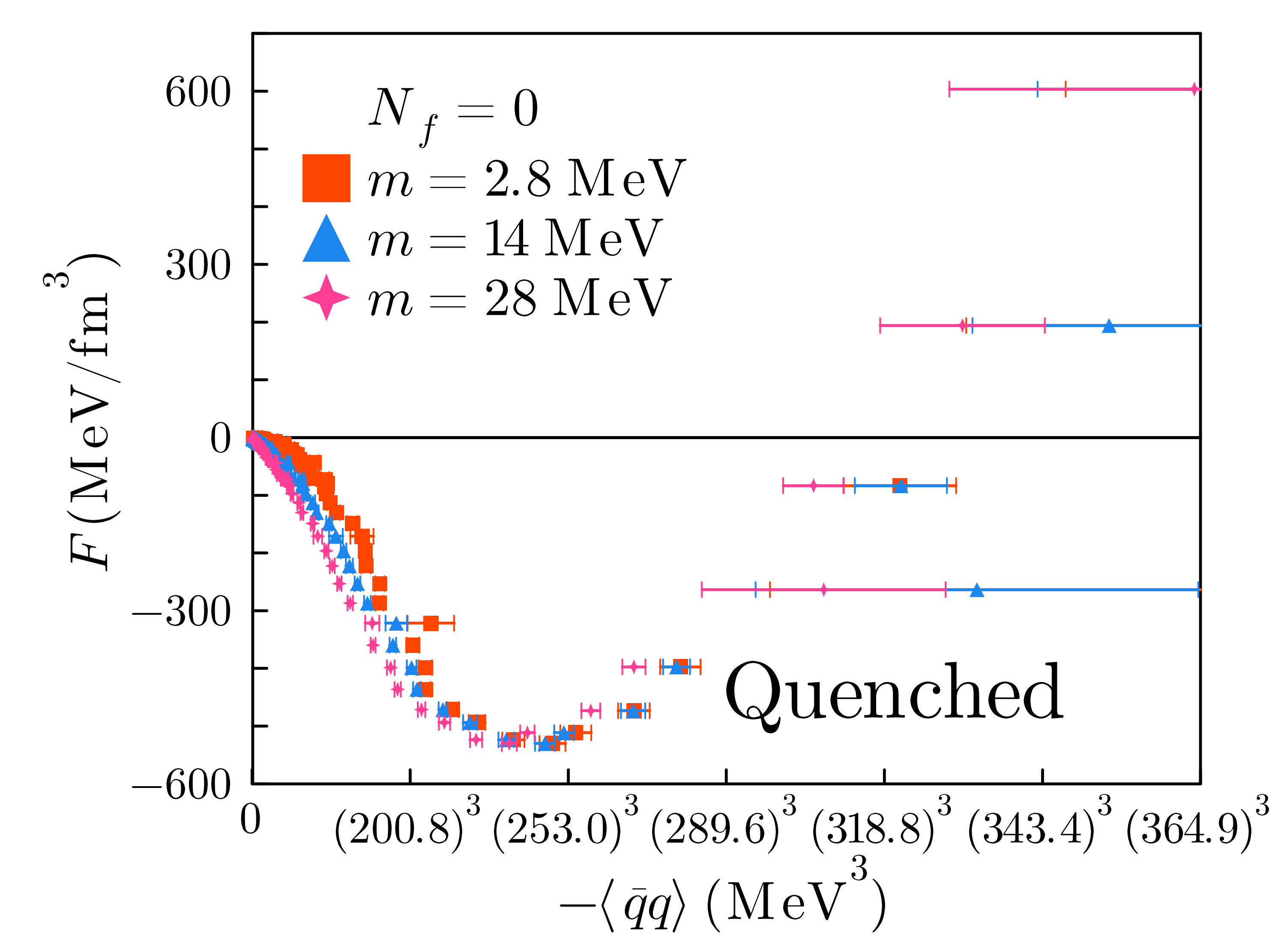
Result: F vs. $\langle \bar{q}q \rangle$ [quench]



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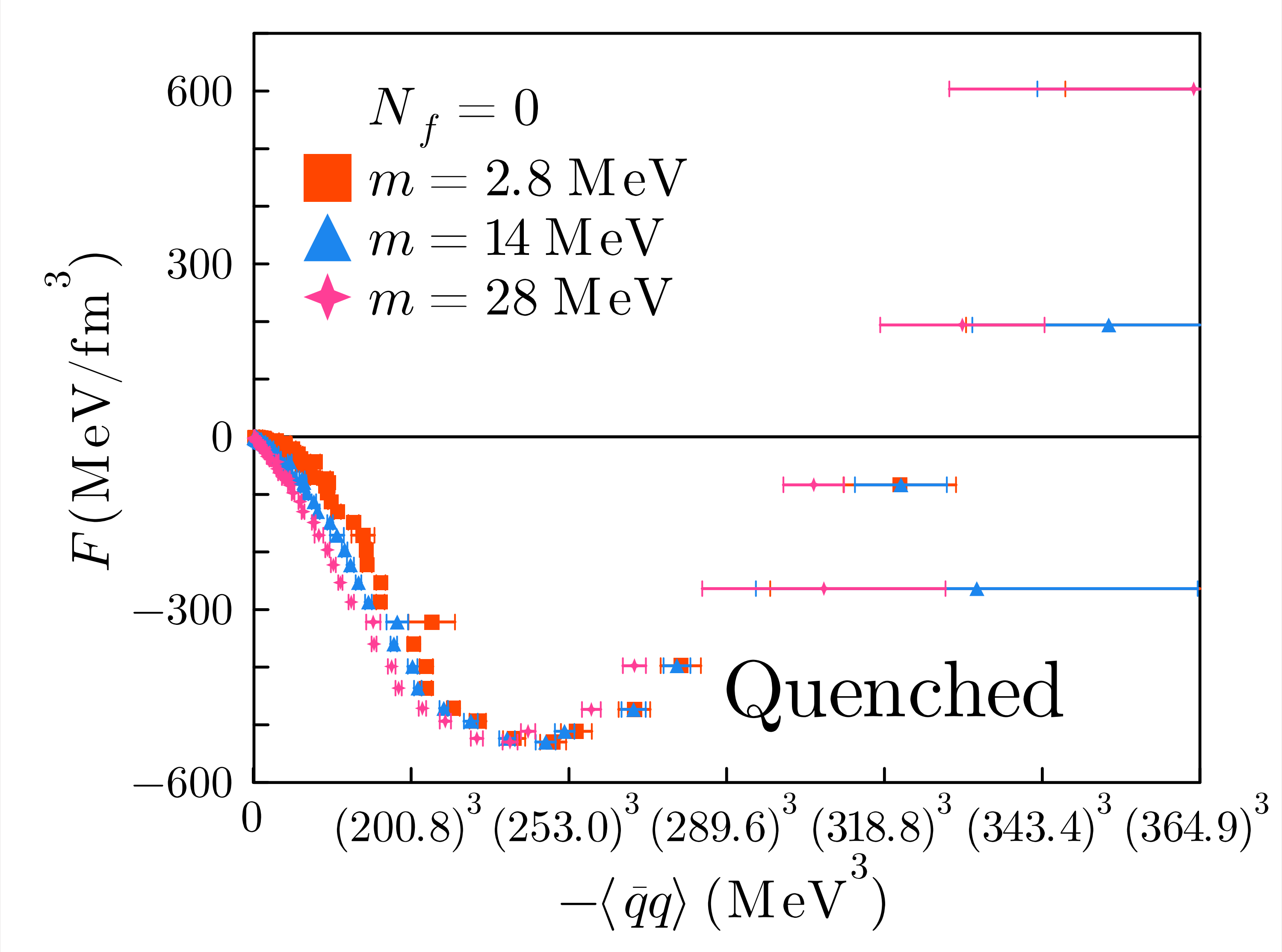

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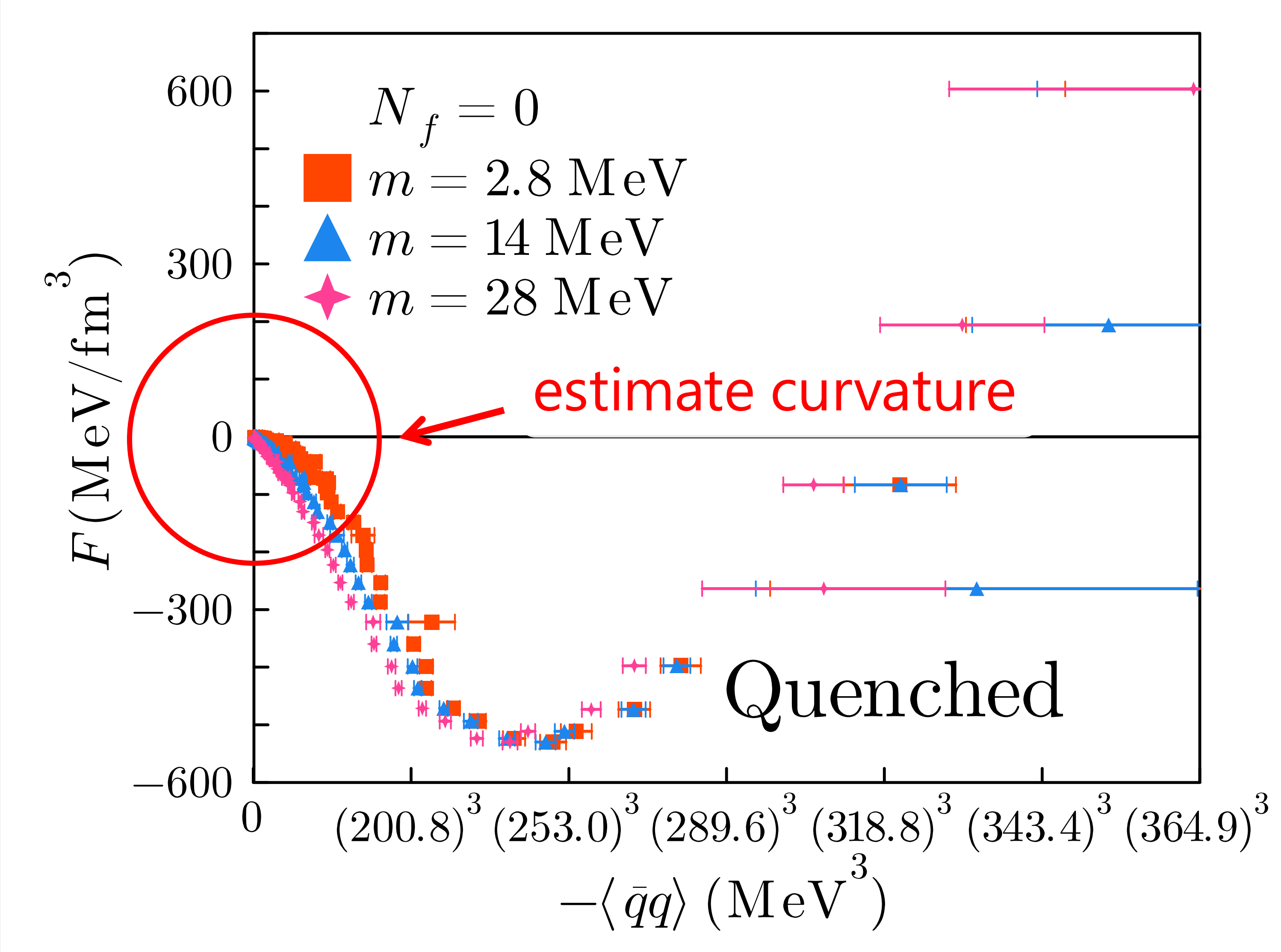
Result: F vs. $\langle \bar{q}q \rangle$ [quench]



- F has min. at $\langle \bar{q}q \rangle \neq 0$
- chiral symmetry broken

estimate curvature at the origin

[7] YS and D. Jido, PRD **110**, 014037 (2024)

Result: Curvature [quench]

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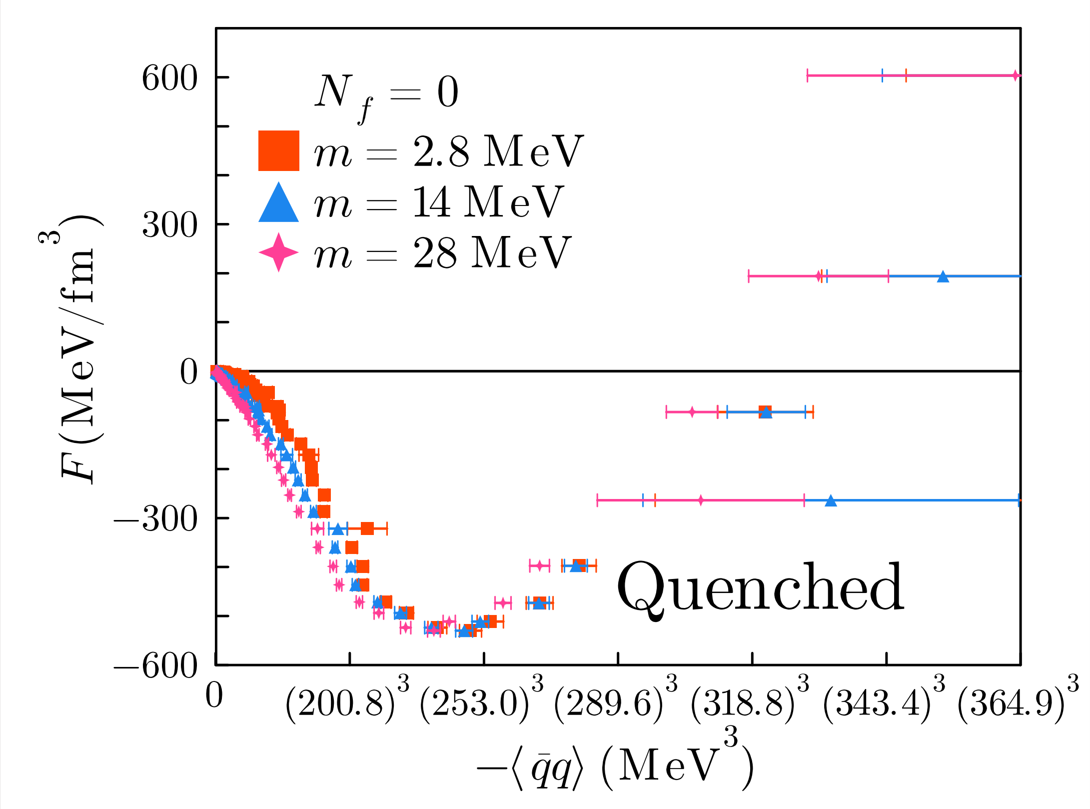


[7] YS and D. Jido, PRD **110**, 014037 (2024)

Result: Curvature [quench]

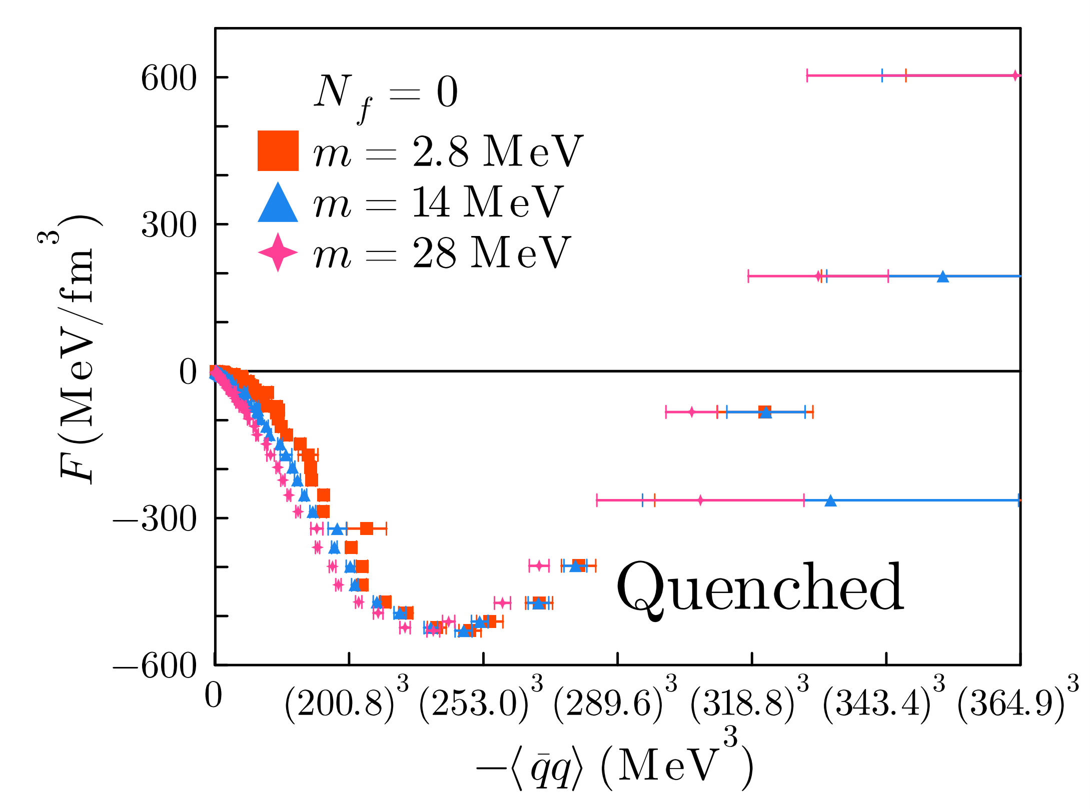
1. obtain F vs. $\langle \bar{q}q \rangle$ (as previous slide)

2. fit data to polynomial

$$F(\langle \bar{q}q \rangle) = C_0 + \boxed{C_1} \langle \bar{q}q \rangle + \boxed{C_2} \langle \bar{q}q \rangle^2 + \cdots + C_K \langle \bar{q}q \rangle^K$$

slope $\propto m_q$
$\because$ finite quark mass

curvature
= what we want



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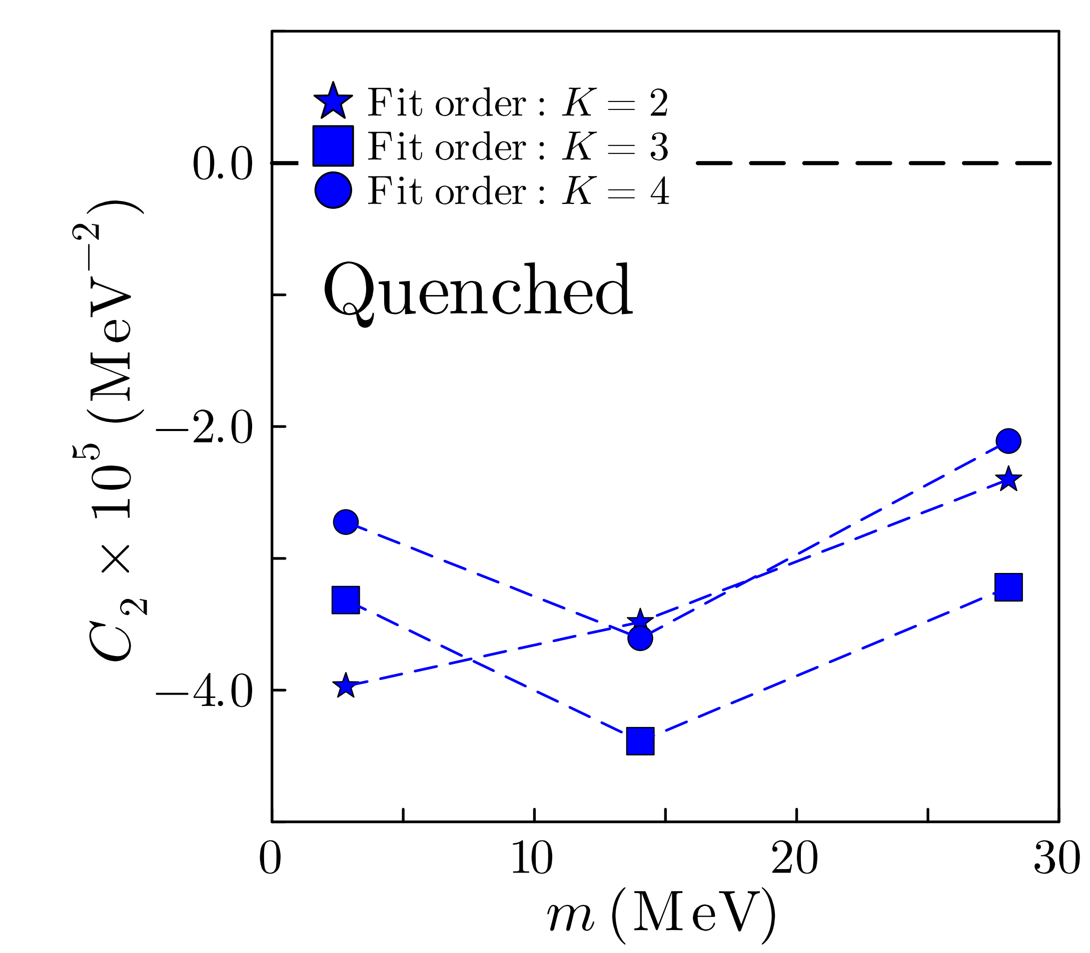
3. extract the curvature C_2 (right figure)

C_2 values for several current quark masses and polynomial orders to fit



[7] YS and D. Jido, PRD **110**, 014037 (2024)

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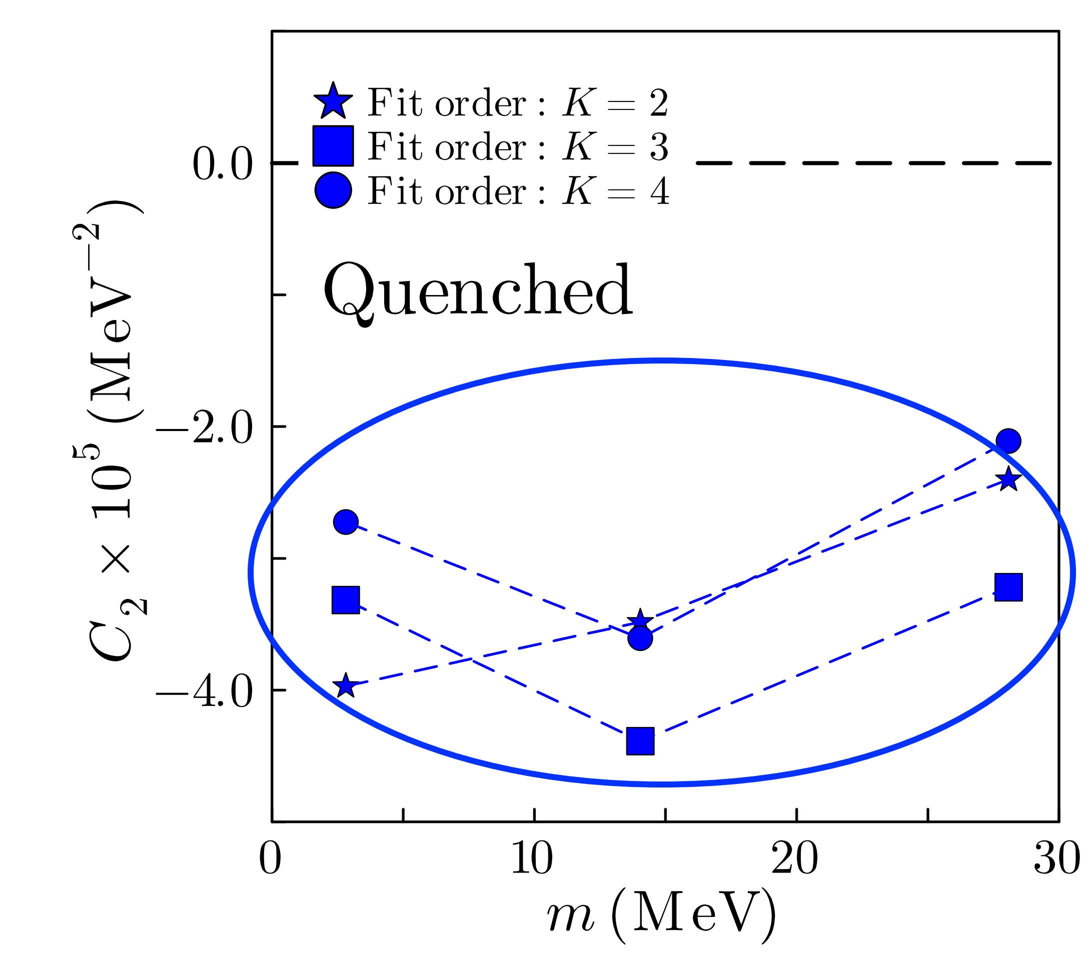
= what we want

3. extract the curvature C_2 (right figure)
   - Interestingly, for quench calculations, $C_2 < 0$ for wide quark mass range
     → ***Ordinary breaking!!***
   - errors are estimated as in the full calculations

C_2 values for several current quark masses and polynomial orders to fit



[7] YS and D. Jido, PRD **110**, 014037 (2024)

Discussion

Q. How to interpret these results?

A. IILM would, by definition, include the effect of chiral ($\mathrm{U}(1)_A$) anomaly

$\mathrm{SU}(3)_f$ NJL model including chiral ($\mathrm{U}(1)_A$) anomaly

IILM for flavor $\mathrm{SU}(3)$

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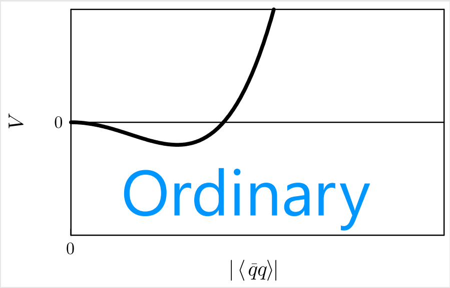
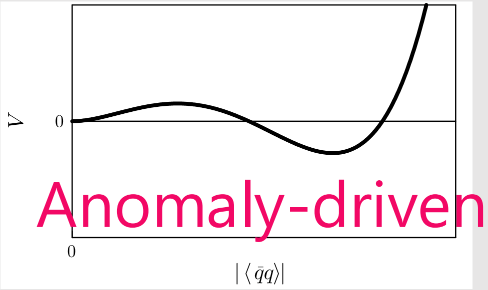
includes the KMT term:

$$\propto \{\det [\bar{q}_i(q - \gamma_5)q_j] + \mathrm{H.c.}\}$$

its strength changes the χSB pattern





IILM for flavor $\mathrm{SU}(3)$

Discussion

Figure: E. Shuryak, "Nonperturbative Topological Phenomena in QCD and Related theory" (2021) DOI:10.1007/978-3-030-62990-8

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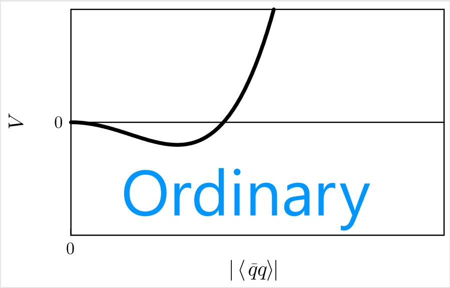
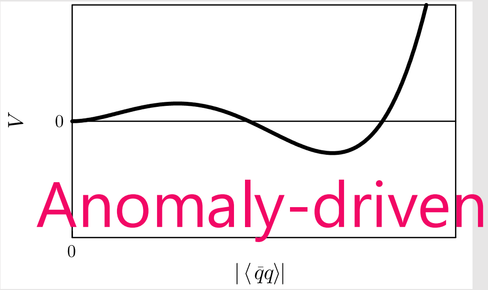


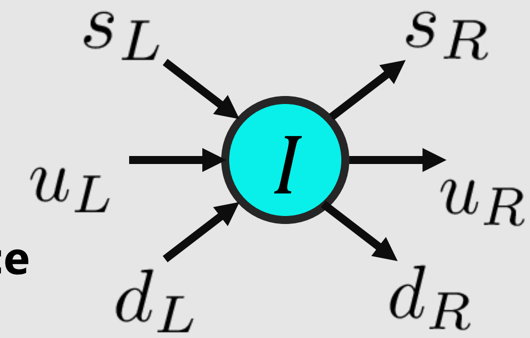


IILM for flavor $\mathrm{SU}(3)$

sums all orders of 't Hooft vertex in the quark det. part:

$$S_{\mathrm{eff}} = -\sum_{i=1}^{N} \log[f(\rho_i)] + S_{\mathrm{int}} - \sum_{f=1}^{N_f} \boxed{\log[\mathrm{Det}(\gamma_\mu D_\mu + m_f)]}$$

implicitly includes
6-quark interaction
without any parameters →



thus, it is **natural to reproduce anomaly driven χSB in IILM** as in the NJL model with the KMT term

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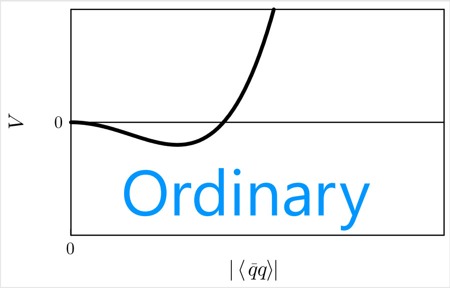

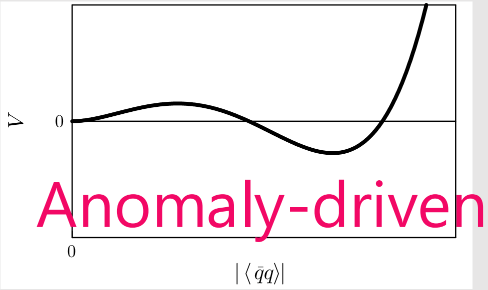

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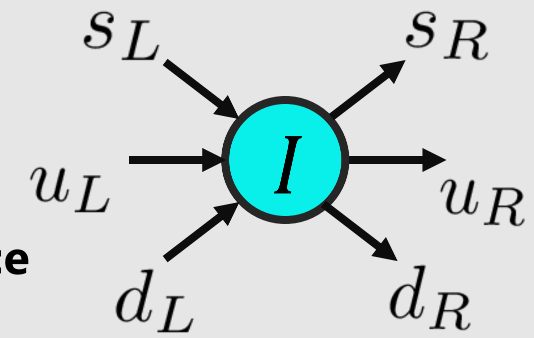


thus, it is **natural to reproduce anomaly driven χSB in IILM** as in the NJL model with the KMT term

suggestion: to explore the phenomena linked to the anomaly driven χSB topological configuration would be important

Summary

- **We studied the χSB pattern driven by chiral ($\mathrm{U}(1)_A$) anomaly in IILM**
  - ✓ PRD110(2024)014037

- To determine the χSB pattern we used the curvature of the energy density w.r.t. the quark condensate at the origin:

  curvature: $C_2 \equiv \left.\frac{\partial^2 V}{\partial\langle\bar{q}q\rangle^2}\right|_{\langle\bar{q}q\rangle=0}$
  - ordinary breaking: $C_2 < 0$
  - anomaly driven breaking: $C_2 > 0$
- IILM calculations show that the curvature is positive $C_2 > 0$
- **implies the anomaly driven breaking may be taken place in IILM**

- Further calculations are running with the aim of the systematic study
  - $N_f = 2 + 1$ situation: SU(3) breaking effect
  - $N_f = 2$ situation: interpolation between SU(3) and quench calculations

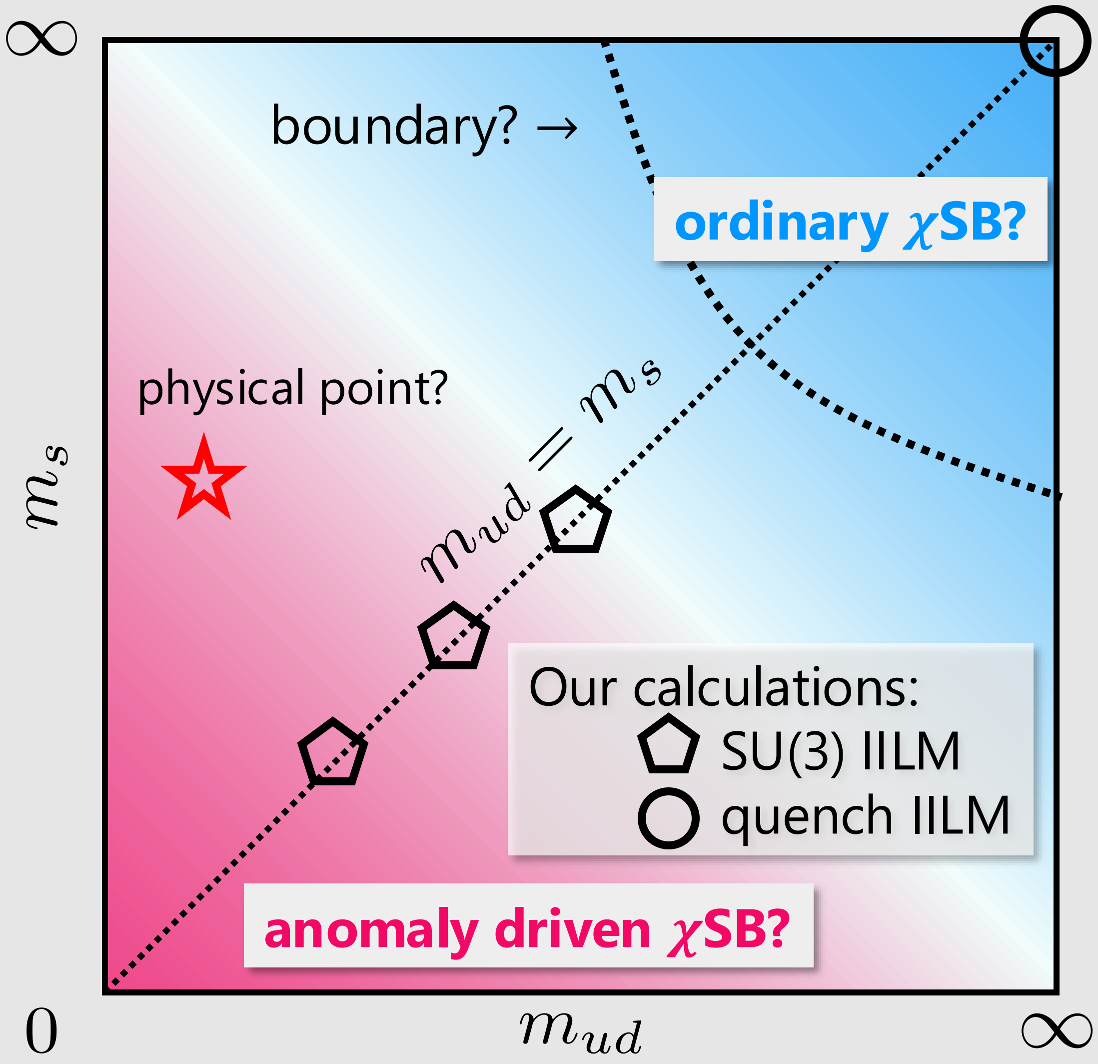
Backups

Backup: Nf-dependence

[8] K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74** (2011) 014011

NOTE: This is NOT the Columbia phase diagram
schematic figure of the phase diagram
for the ordinary/anomaly driven χSB pattern in IILM

- if dominant chiral ($\mathrm{U}(1)_A$) anomaly exists, ordinary/anomaly driven χSB pattern would depend on quark mass
- impact should manifest itself in some form for the cases $N_f = 3, N_f = 2 + 1$ and $N_f = 2$
- expect that these impacts also realizes in QCD



Backup: Why the instanton liquid model?

[7] YS and D. Jido, PRD **110**, 014037 (2024)

- if the anomaly-driven χSB is a universal feature of strong interaction
  $\Rightarrow$ could provide some way to detect
  the topological structure of the QCD vacuum

- verify whether the anomaly driven breaking takes place or not
  in other systems rather than chiral effective theories

- **instanton liquid model model** is a suitable model to check it:
  - can describe the QCD vacuum in terms of topological configuration
  - can reproduce the dynamical χSB
  - can compute effective potential (vacuum energy)
  - can compute quark condensate