Deciphering the mechanism of $J/\psi N$ scattering

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Based on arXiv: 2409.xxxxx

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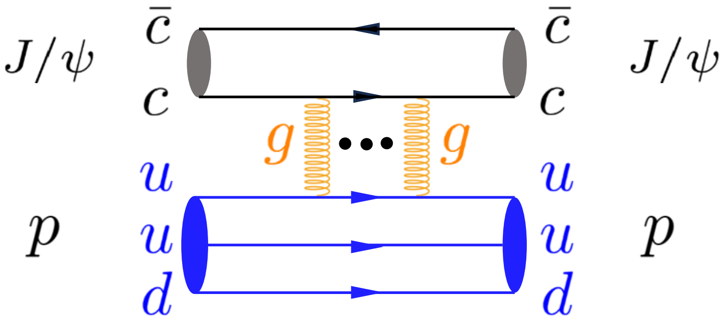
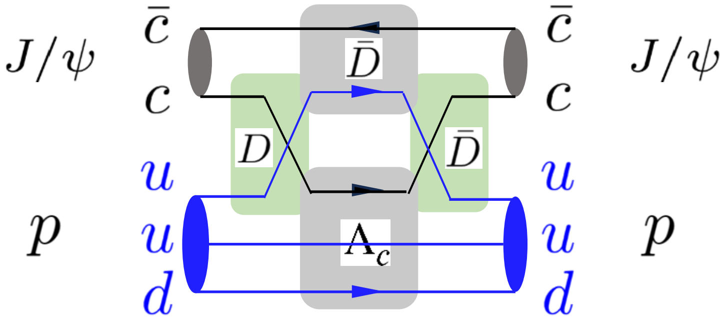
1 Background

- **The interaction between J/ψ and nucleons**
  - Typical OZI suppression process
  - Interaction through meson exchange is suppressed $(1/N_c)$ G. 't Hooft, Nucl. Phys. B 72 (1974) 461
  - Interaction mechanism



Gluon exchanges



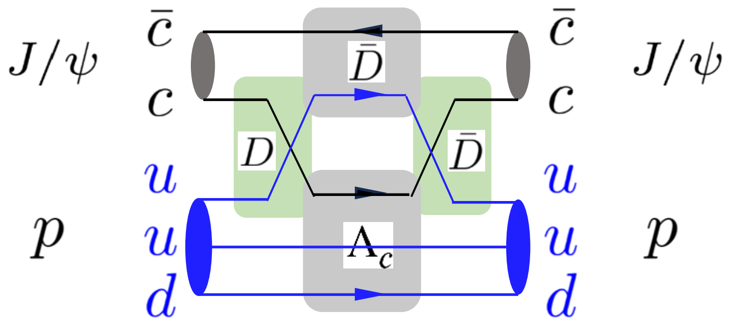
Coupled-channel

$\langle J/\psi|GG|J/\psi\rangle$ Chromopolarizability
A. Sibirtsev et al., PRD 71 (2005) 076005
Y.-H. Chen et al., PRD 93 (2016) 034030

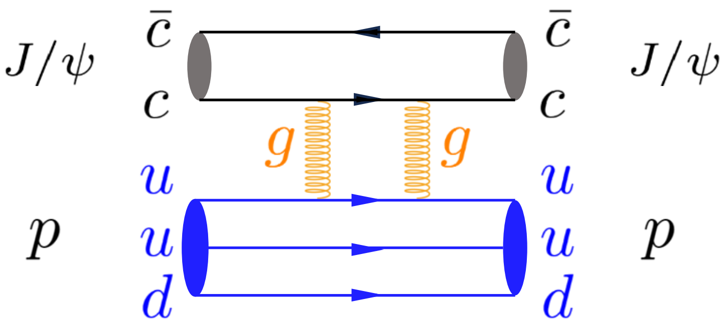
$\langle N|GG|N\rangle$ gluon trace anomaly contribution to the nucleon mass
Y. Hatta et al., JHEP 12 (2018) 008
Y.-B. Yang et al., PRD 104 (2021) 074507

$J/\psi N$-$\Lambda_c \bar{D}^{(*)}/\Sigma_c^{(*)}\bar{D}^{(*)}$-$J/\psi N$
coupled-channel mechanism (hadronic loops) evade the OZI suppression
H. Lipkin, B.-S. Zou, PRD 53 (1996) 6693-6696

- $J/\psi N$ **scattering length**



Coupled-channel



Gluon exchanges

- ☐ Coupled-channel mechanism
  - $\Lambda_c\bar{D}^{(*)}$ contribution: 0.2 ... 3 am
    M.-L. Du et al., EPJC 80 (2020) 11, 1053
  - $\Sigma_c^{(*)}\bar{D}^{(*)}$ contribution: 0.1 ... 10 am
    M.-L. Du et al., JHEP 08 (2021) 157

- ☐ How about gluon exchange?

Motivation: Calculate the scattering length of $J/\psi N$ under gluon exchange, and then comparing the contributions of gluon exchange and coupled-channel processes.

2 Formalism

2.1 The definition of scattering length

$J/\psi\ N$ scattering length: $$a_0^{J/\psi N} = -\frac{T_{J/\psi N \to J/\psi N,0}(s_{th})}{8\pi\sqrt{s_{th}}}$$ $$s_{th} = (m_{J/\psi} + m_N)^2$$

where: $$\mathrm{Im}[T_{J/\psi N \to J/\psi N,0}(s)] = |T_{J/\psi N \to J/\psi N,0}(s)|^2\ \rho_{J/\psi N}(s)\theta(\sqrt{s} - m_{J/\psi} - m_N)$$

$$\rho_{J/\psi N}(s) = \frac{1}{16\pi}\sqrt{\frac{(s-(m_{J/\psi}+m_N)^2)(s-(m_{J/\psi}-m_N)^2)}{s^2}}$$

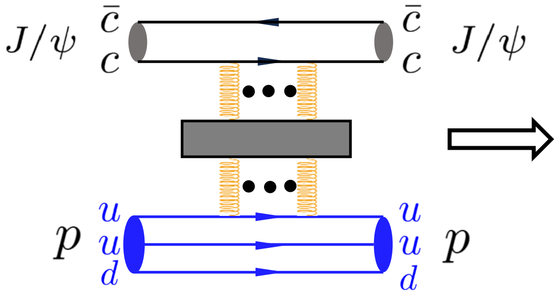
BS equation: $$T_l(s) = V_l(s) + V_l(s)G(s)T_l(s) = [1 - V_l(s)G(s)]^{-1}V_l(s)$$

Kernel 1: The S-wave $J/\psi N$ interaction potential V_0 through gluon exchange

2.2 The S-wave $J/\psi N$ interaction potential

- At long distances, the exchanged soft gluons hadronize into exchanging $\boldsymbol{\pi}$ and heavier mesons

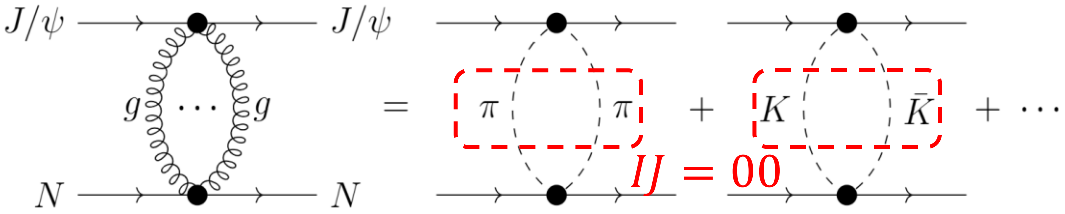
N. Brambilla et al., Phys. Rev. D 93 (2016) 054002
X.-K. Dong et al., Sci. Bull. 66 (2021) 2462-2470

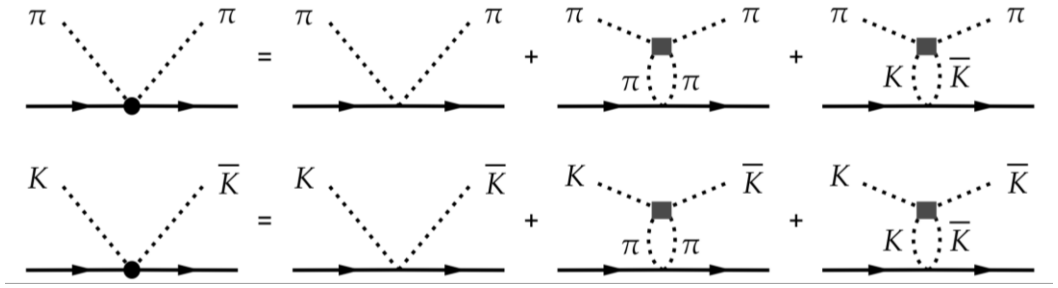


$\Longrightarrow$ All possible color-singlet states that can couple to gluons: $\pi\pi$, $K\bar{K}$, ...

The longest-distance (lightest exchange particles) contribution

the strong $\pi\pi$-$K\bar{K}$ coupling

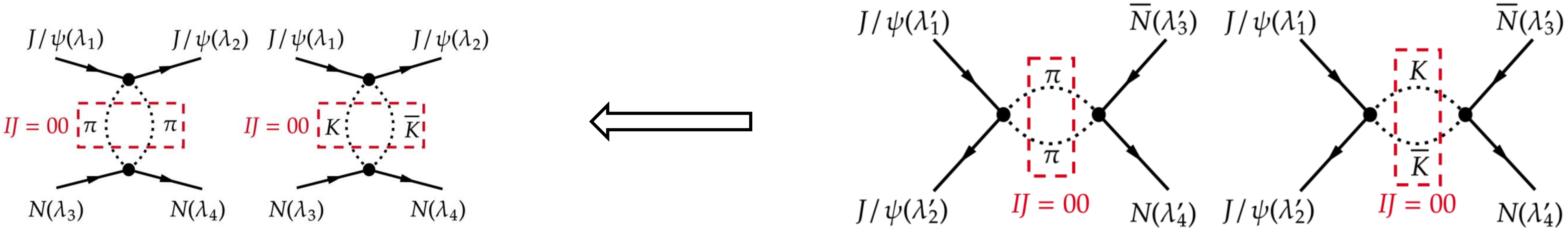




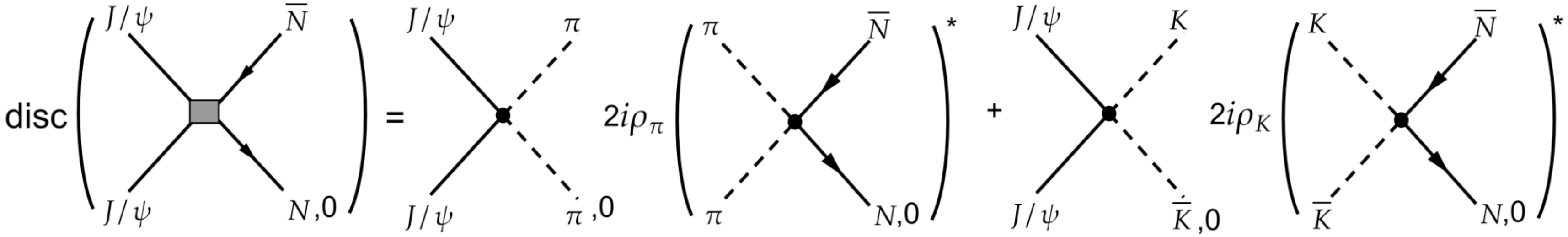
Kernel 2: The S-wave $J/\psi N$ interaction potential through correlated $\pi\pi$-$K\bar{K}$ exchange

- **Crossing relation** A. D. Martin, T. D. Spearman, Elementary particle theory

$$T^{(s)}_{cd;ab}(s,t) = \sum d^{s_a}_{a'a}(\chi_a) d^{s_b}_{b'b}(\chi_b) d^{s_c}_{c'c}(\chi_c) d^{s_d}_{d'd}(\chi_d) T^{(t)}_{c'a';d'b'}(s,t)$$



- **Unitary and dispersion relation**



$$\rho_\pi(s) = \frac{1}{16\pi}\sqrt{\frac{s-4M_\pi^2}{s}}\theta(\sqrt{s}-2M_\pi)$$

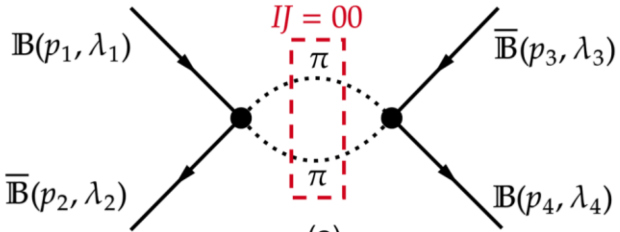
$$\rho_K(s) = \frac{1}{16\pi}\sqrt{\frac{s-4M_K^2}{s}}\theta(\sqrt{s}-2M_K)$$

factor arising from kinematic singularities

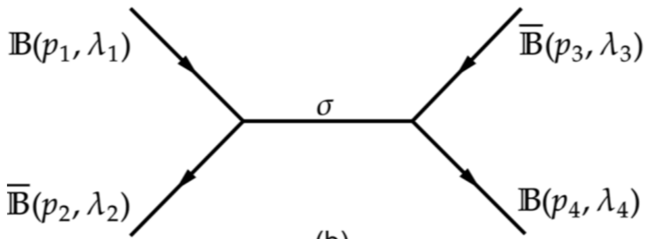
$$T_{J/\psi J/\psi\to\bar{N}(\lambda_3)N(\lambda_4),0}(s,\Lambda) = \frac{(\lambda_3+\lambda_4)\sqrt{s-4m_N^2}}{2\pi i}\int_{4M_\pi^2}^{+\infty} \mathrm{d}z \frac{\mathrm{disc}\left[T_{\pi\pi,0}(z)+T_{K\bar{K},0}(z)\right]}{z-s}$$

Kernel 3: The S-wave amplitude of $J/\psi\, J/\psi \to \pi\pi/K\bar{K}$ and $\bar{N}N \to \pi\pi/\bar{K}K$, incorporating $\pi\pi$-$K\bar{K}$ FSI

- **Muskhelishvili-Omnes representation (with LHC)**
  - ☐ Single-channel: B. Wu et al., PRD 109 (2024) 034026



(a)



(b)

$$T_{\mathbb{B}\bar{\mathbb{B}}\to\pi\pi,0}(s) = L_{\mathbb{B},0}(s) + \Omega_0(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4m_\pi^2}^{+\infty} dz \frac{L_{\mathbb{B},0}(z)\sin\delta_0(z)}{(z-s)z^n|\Omega_0(z)|}\right)$$

$$\Omega_L(s) \equiv \exp\left[\frac{s}{\pi}\int_{s_{\rm th}}^{\infty} dz \frac{\delta_L(z)}{z(z-s)}\right]$$

be determined by matching to the tree-level chiral amplitudes

	LHC	RHC	Total	[33]	[18]	[34]	[37]	[36]	[24]	[23]	m_σ
$g_{\Sigma\Sigma\sigma}$	$1.8^{+0.5}_{-0.5}$	$3.5^{+2.0+0.8}_{-1.8-0.9}$	$3.5^{+1.8+0.4}_{-1.3-0.4}$	…	…	10.85(8.92)	4.65	…	…	…	519^{+50}_{-48}
$g_{\Xi\Xi\sigma}$	$0.2^{+0.1}_{-0.1}$	$2.6^{+1.5+0.5}_{-1.4-0.6}$	$2.5^{+1.5+0.5}_{-1.3-0.6}$	…	…	…	…	…	3.4	…	614^{+56}_{-81}
$g_{\Lambda\Lambda\sigma}$	$1.2^{+0.4}_{-0.3}$	$6.7^{+1.0+1.4}_{-1.1-1.7}$	$6.8^{+1.0+1.1}_{-1.0-1.4}$	…	…	8.18(6.54)	4.37	…	…	6.59	596^{+41}_{-51}
$g_{NN\sigma}$	$2.9^{+0.9}_{-0.8}$	$8.8^{+1.4+1.9}_{-1.4-2.3}$	$8.7^{+1.3+1.1}_{-1.3-1.4}$	12.78	8.46	8.46	8.58	13.85	10.2	9.86	558^{+33}_{-42}
$g_{NN\sigma}^{\rm SU(2)}$	$2.7^{+0.8}_{-0.8}$	$12.5^{+0.2+2.6}_{-0.2-3.2}$	$12.2^{+0.2+1.9}_{-0.2-2.3}$								586^{+38}_{-48}

For $g_{NN\sigma}$, see also M. Hoferichter et al., PLB 853 (2024) 138698 and Jacobo Ruiz de Elvira's report yesterday

◻ Coupled-channel:

For $N\bar{N} \to \pi\pi/K\bar{K}$:

$$\vec{T}_0(s) = \vec{L}_0(s) + \Omega_0(s)\left[\vec{P}_{n-1}(s) - \frac{s^n}{\pi}\int_{4M_\pi^2}^{+\infty} \mathrm{d}z \frac{\mathrm{Im}\left[\Omega_0^{-1}(z)\right]\vec{L}_0(z)}{(z-s)z^n}\right]$$

where

$$\vec{T}_0(s) = \begin{pmatrix} T_{N\bar{N}\to\pi\pi,0}(s) \\ T_{N\bar{N}\to K\bar{K},0}(s) \end{pmatrix} \quad \vec{L}_0(s) = \begin{pmatrix} L_{N\bar{N}\to\pi\pi,0}(s) \\ L_{N\bar{N}\to K\bar{K},0}(s) \end{pmatrix} \quad \vec{P}_{n-1}(s) = \begin{pmatrix} P_{n-1}^{N\bar{N}\to\pi\pi}(s) \\ P_{n-1}^{N\bar{N}\to K\bar{K}}(s) \end{pmatrix} \quad \Omega_0(s) = \begin{pmatrix} \Omega_{0,11}(s) & \Omega_{0,12}(s) \\ \Omega_{0,21}(s) & \Omega_{0,22}(s) \end{pmatrix}$$

matching to the tree-level chiral amplitudes

LO Chiral Lagrangian

A. Krause, Helv. Phys. Acta 63 (1990) 3-70

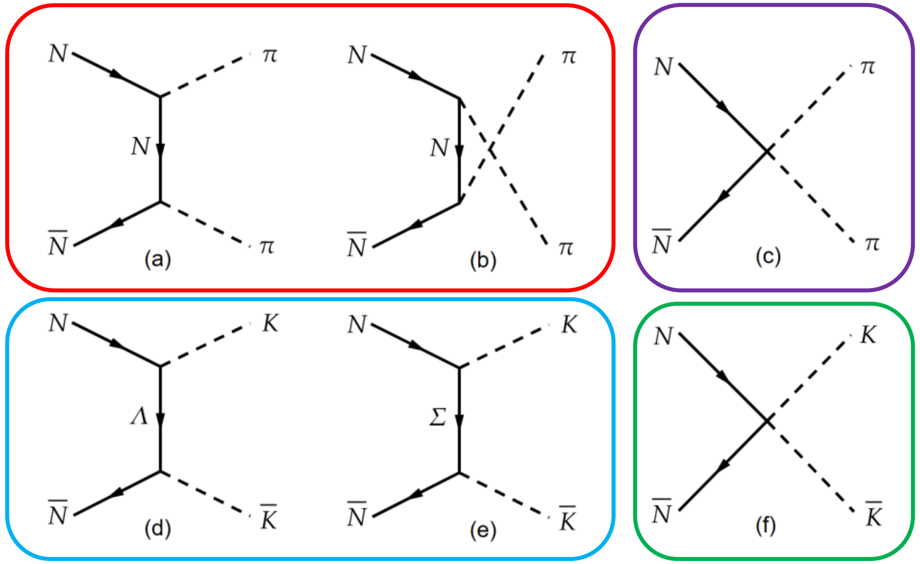
NLO Chiral Lagrangian

M. Frink and U.-G. Meißner, JHEP 07, 028   J. A. Oller et al., JHEP 09, 079

LECs taken from X.-L. Ren et al., JHEP 12 (2012) 073

$$L_{N\bar{N}\to\pi\pi,0}(s) \sim \hat{A}_{N\bar{N}\to\pi\pi,0}(s)$$

$$L_{N\bar{N}\to K\bar{K},0}(s) \sim \hat{A}_{N\bar{N}\to K\bar{K},0}(s)$$



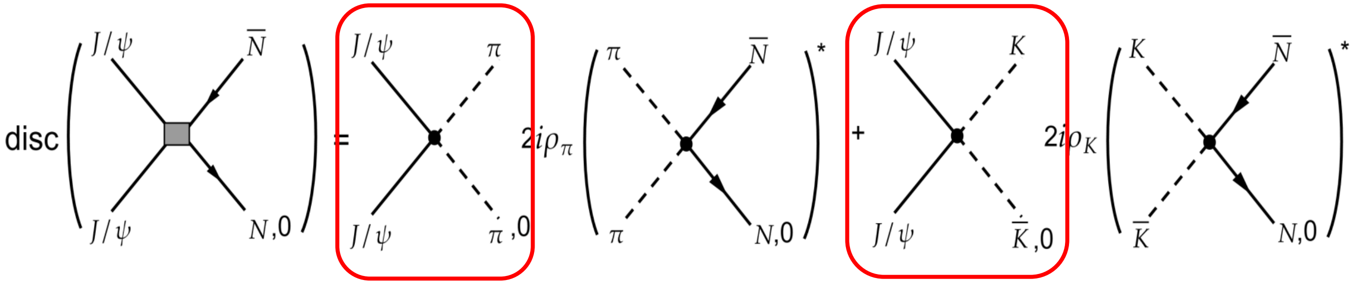
$$P_{n-1}^{N\bar{N}\to\pi\pi}(s) \sim A_{N\bar{N}\to\pi\pi,0}(s)$$

$$P_{n-1}^{N\bar{N}\to K\bar{K}}(s) \sim A_{N\bar{N}\to K\bar{K},0}(s)$$

FIG. 2. The tree-level Feynman diagrams for the process of $N\bar{N}\to\pi\pi$ and $N\bar{N}\to K\bar{K}$.

For $J/\psi J/\psi \to \pi\pi/K\bar{K}$:

X.-K. Dong et al., Sci. Bull. 66 (2021) 2462-2470



$$T_{J/\psi J/\psi\to|i\rangle,0}(s) = \Omega_{0,i1}(s)\mathcal{M}(s,M_\pi) + \Omega_{0,i2}(s)\frac{2}{\sqrt{3}}\mathcal{M}(s,M_K)$$

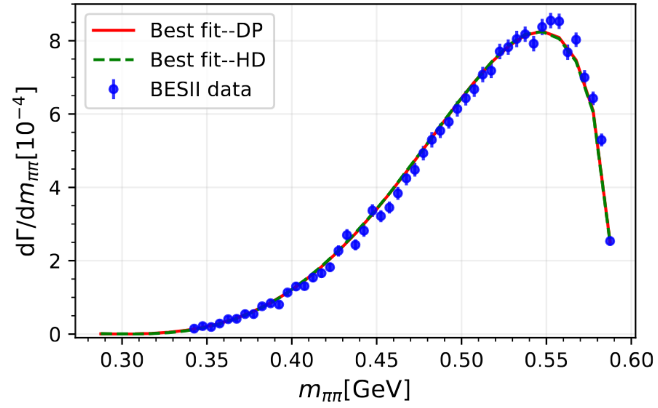
$$\mathcal{M}(s,M_p) = -\frac{2}{F_\pi^2}\sqrt{\frac{3}{2}}\left\{c_1^{(21)}(s-2m_p^2) + \frac{c_2^{(21)}}{2}\left[s + \frac{s(s-4M_{J/\psi}^2)}{4M_{J/\psi}^2}\left(1-\frac{s-4M_p^2}{3s}\right)\right]\right\}$$

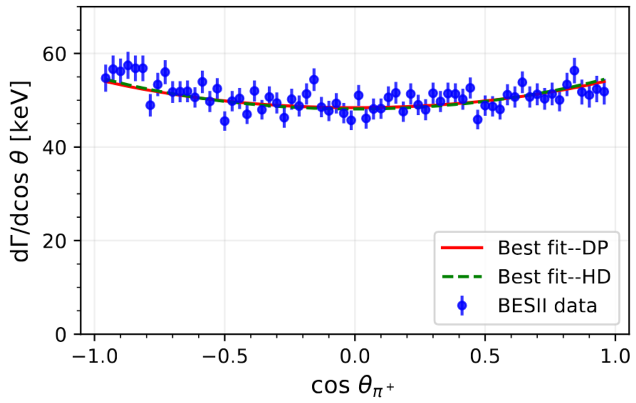
$$\to -\frac{2I_{\mathcal{P}}}{F_\pi^2}\left\{c_1^{(11)}(s-2M_{\mathcal{P}}^2) + 2c_m^{(11)}M_{\mathcal{P}}^2 + \frac{c_2^{(11)}}{12M_{J/\psi}^2}\left[s^2+2s(M_{\mathcal{P}}^2+M_{J/\psi}^2) - 8M_{\mathcal{P}}^2M_{J/\psi}^2\right]\right\}$$

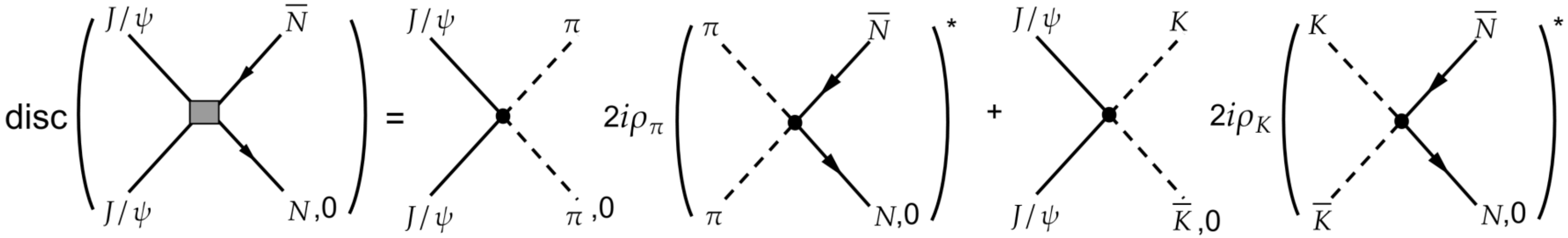
determine the low-energy constants(LECs) by fitting the BESII data on the $\psi(2S)\to J/\psi\pi\pi$

the updated values:

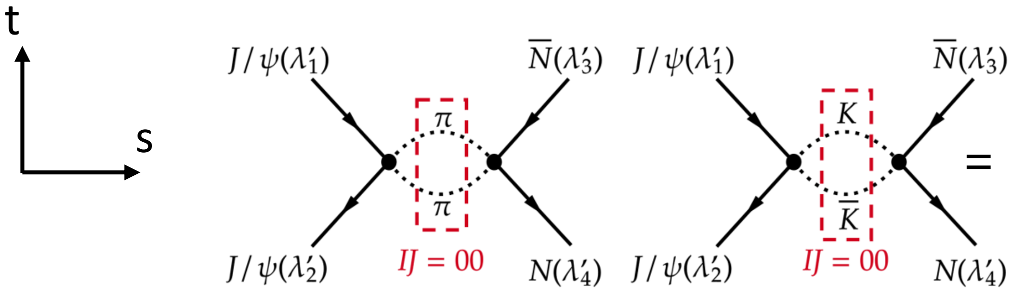
$$c_1^{(21)} = 0.178 \pm 0.002,\ c_2^{(21)} = -0.122 \pm 0.002,\ c_m^{(21)} = 0.222 \pm 0.002$$







dispersion relation

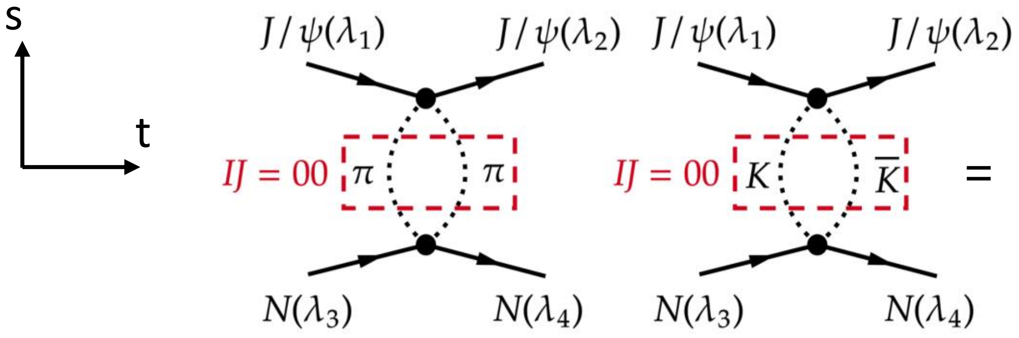


$$= \quad T_{J/\psi J/\psi\to\bar N(\lambda_3)N(\lambda_4),0}(s,\Lambda)=\frac{(\lambda_3+\lambda_4)\sqrt{s-4m_N^2}}{2\pi i}\int_{4M_\pi^2}^{+\infty}\mathrm{d}z\,\frac{\mathrm{disc}\,[T_{\pi\pi,0}(z)+T_{K\bar K,0}(z)]}{z-s}\,\boxed{e^{-\frac{z-s}{\Lambda^2}}}$$

Gaussian form factor to regularize the integral

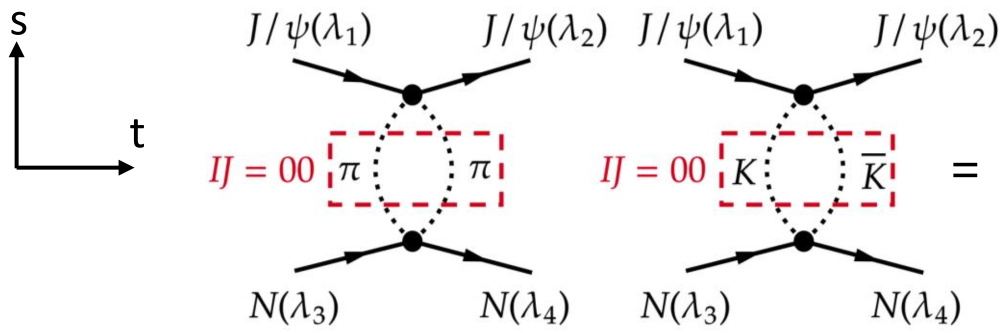
crossing relation

X.-K. Dong et al., Sci. Bull. 66 (2021) 2462-2470
P. Reinert et al., EPJA 54, 86 (2018)



$$= \quad T_{J/\psi(\lambda_1)N(\lambda_3)\to J/\psi(\lambda_2)N(\lambda_4),0}(s,t,\Lambda)$$
$$=\epsilon(\lambda_1,0,0)\cdot\epsilon^*(\lambda_2,\theta,0)e^{-i\pi\lambda_4}(-1)^{\lambda_3+\lambda_4}\sum_{\lambda_3'\lambda_4'}d^{\frac{1}{2}}_{\lambda_3\lambda_3'}(\omega)d^{\frac{1}{2}}_{\lambda_4\lambda_4'}(\omega+\pi)T_{J/\psi J/\psi\to\bar N(\lambda_3')N(\lambda_4'),0}(s,\Lambda)$$

helicity amplitude



$$= T_{J/\psi(\lambda_1)N(\lambda_3)\to J/\psi(\lambda_2)N(\lambda_4),0}(s,t,\Lambda)$$
$$= \epsilon(\lambda_1,0,0)\cdot\epsilon^*(\lambda_2,\theta,0)e^{-i\pi\lambda_4}(-1)^{\lambda_3+\lambda_4}\sum_{\lambda_3'\lambda_4'} d^{\frac{1}{2}}_{\lambda_3\lambda_3'}(\omega)d^{\frac{1}{2}}_{\lambda_4\lambda_4'}(\omega+\pi)T_{J/\psi J/\psi\to\bar{N}(\lambda_3')N(\lambda_4'),0}(s,\Lambda)$$

the relation between the helicity state and LS-coupling state

$$|JM;LS\rangle = \sum_{\mu_1\mu_2}\sqrt{\frac{2L+1}{2J+1}}C^{S(\mu_1-\mu_2)}_{s_1\mu_1;s_2(-\mu_2)}C^{J(\mu_1-\mu_2)}_{L0;S(\mu_1-\mu_2)}|JM;\mu_1\mu_2\rangle$$

$$V^{|^{2S+1}0_S\rangle}_{J/\psi N}(t,\Lambda) = \sum_{\mu_1\mu_2}\sum_{\mu_1'\mu_2'}\frac{1}{2S+1}C^{S(\mu_1'-\mu_2')}_{1\mu';\frac{1}{2}(-\mu_2')}C^{S(\mu_1-\mu_2)}_{1\mu_1;\frac{1}{2}(-\mu_2)}\int\frac{d\Omega}{4\pi}d^S_{\mu,\mu'}(\theta)T_{J/\psi(\mu_1)N(\mu_2)\to J/\psi(\mu_1')N(\mu_2'),0}(s,t,\Lambda)$$

$$a_0 = -\frac{T_{ii,0}(s_{th})}{8\pi\sqrt{s_{th}}}$$

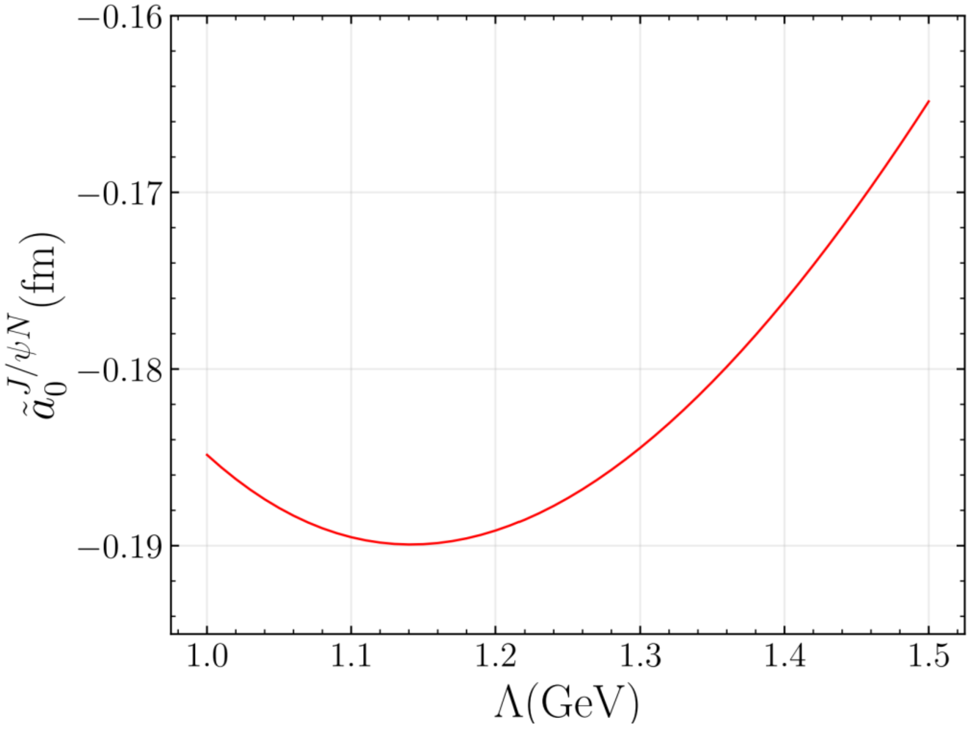
$$T_{J/\psi N\to J/\psi N,0}(s) = \frac{V_{J/\psi N\to J/\psi N,0}(s)}{1-V_{J/\psi N\to J/\psi N,0}(s)G(s)}$$

$$V^{|^{2S+1}0_S\rangle}_{J/\psi N}(t_{\rm th},\Lambda) = -iT_{J/\psi J/\psi\to\bar{N}(\frac{1}{2})N(\frac{1}{2}),0}(s=0,\Lambda)$$

$$a_0^{J/\psi N} \approx -\frac{V^{|^{2S+1}0_S\rangle}_{J/\psi N}(t=t_{\rm th},\Lambda)}{8\pi\,(m_{J/\psi}+m_N)}$$

3 Numerical Result

- Numerical result with LECs arising from the BESII data on the $\psi(2S) \to J/\psi\pi\pi$



$\boldsymbol{S}$-wave $J/\psi N$ scattering length through soft gluon exchange:

$$\tilde{a}_0^{J/\psi N} = -0.16 \ldots - 0.19 \text{ fm}$$

- ☐ The interaction between J/ψ and N is attractive
- ☐ The strength is comparable to the S-wave isospin-0 $\pi\pi$ interaction

$$a_{00}^{\pi\pi} = -0.2210 \pm 0.0047 \pm 0.0015 \text{ fm}$$

J.R. Batley et al., PLB 633 (2006) 173-182
B. Bloch-Devaux, PoS KAON09 (2009) 033

- Inequality of Scattering Lengths

  A. Sibirtsev et al., PRD 71 (2005) 076005

  - The chromopolarisability

$$\mathcal{M}(A \to B\pi\pi) = \alpha_{AB} \langle \pi\pi | \vec{E}^a \cdot \vec{E}^a | 0 \rangle$$

α_{AB}: chromopolarisability; $\vec{E}^a$: chromoelectric field

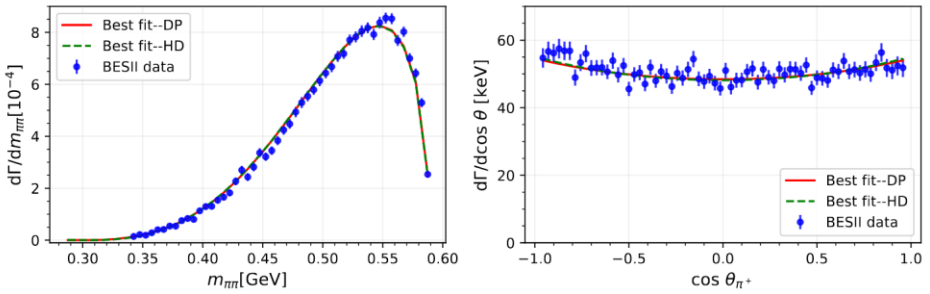


FIG. 4. Fit to the BESII data on $\psi(2S) \to J/\psi\pi^+\pi^-$ [71]. "Best fit–DP" represents the fit results using the Omnès matrix from Ref. [62], while "Best fit–HD" represents those from Ref. [50].

  - The previous numerical result is based on $\alpha_{\psi(2S)J/\psi}$

$$\tilde{a}_0^{J/\psi N} = -0.16\ldots-0.19 \text{ fm}$$

$\tilde{a}_0^{J/\psi N} \to \alpha_{\psi(2S)J/\psi}$

  - $\alpha_{\psi(2S)\psi(2S)}\alpha_{J/\psi J/\psi} \geq |\alpha_{\psi(2S)J/\psi}|^2$

  - Keep the previous calculation unchanged and replace $m_{J/\psi}$ with $m_{\psi(2S)}$

$$\tilde{a}_0^{\psi(2S)N} = -0.14\ldots-0.17 \text{ fm}$$

$\tilde{a}_0^{\psi(2S)N} \to \alpha_{\psi(2S)J/\psi}$

  - Scattering length inequality

$$a_0^{J/\psi N} a_0^{\psi(2S)N} \geq \tilde{a}_0^{J/\psi N}\tilde{a}_0^{\psi(2S)N} \approx (-0.15 \text{ fm})^2$$

- Compared with Coupled-channel mechanism

  - $\Lambda_c\bar{D}^{(*)}$ contribution: 0.2 ... 3 am

    M.-L. Du et al., EPJC 80 (2020) 11, 1053

  - $\Sigma_c^{(*)}\bar{D}^{(*)}$ contribution: 0.1 ... 10 am

At least one order of magnitude smaller than the result from the soft gluon exchange!

4 Summary

- We estimate the S-wave scattering length of $J/\psi N$ through soft gluon exchange (correlated $\pi\pi$-$K\bar{K}$)

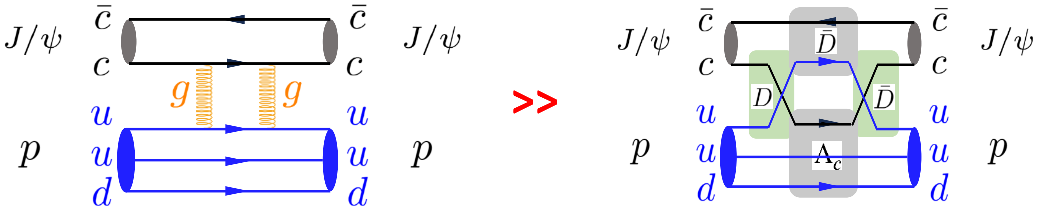
$$\tilde{a}_0^{J/\psi N} = -0.16\ldots -0.19 \text{ fm}$$

The same process to estimate the scattering length for $\psi(2S)N$ through soft gluon exchange:

$$\tilde{a}_0^{\psi(2S)N} = -0.14\ldots -0.17 \text{ fm}$$

After chromopolarisability correction：

$$a_0^{J/\psi N} a_0^{\psi(2S)N} \geq \tilde{a}_0^{J/\psi N} \tilde{a}_0^{\psi(2S)N} \approx (-0.15 \text{ fm})^2$$

- 

In the low-energy interaction of $J/\psi N$, the contribution from soft gluon exchange is dominant

Thank you very much for your attention!