Gravitational $p \to \Delta^+$ transition form factors in ChPT

Bao-Dong Sun (孙保东)

Ruhr-Universität Bochum (RUB)

In collaboration with: H. Alharazin, E. Epelbaum, J. Gegelia, and U.-G. Meißner

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Chiral Dynamics (CD2024), 29 Aug. 2024

Outline：

1. Introduction
2. Building ChPT in curved space–time
3. Numerical results
4. Summary

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CD 24 BOCHUM

Definition & Usage: Gravitational Form Factors (GFFs)

$P = (p' + p)/2$
$\Delta = p' - p$
$t = \Delta^2$

Electromagnetic Form Factors of proton:

$$\langle p', s'|\hat{j}^\mu(x)|p, s\rangle = \bar{u}'\left[\gamma^\mu F_1(t) + \frac{1}{2m} i\sigma^{\mu\nu}\Delta_\nu F_2(t)\right] u\, e^{i(p'-p)x}$$

Dirac FF — Pauli FF

Sachs FFs

$$G_E(t) = F_1(t) + \frac{t}{4M^2} F_2(t) \xrightarrow{t\to 0} \text{charge}$$

$$G_M(t) = F_1(t) + F_2(t) \longrightarrow \text{magnetic moment}$$

Charge density in Breit frame:

$$\rho(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3} G_E\left(-\vec{\Delta}^2\right) e^{-i\vec{\Delta}\cdot\vec{r}}$$

Charge radius:

$$\langle r^2\rangle = \frac{\int d^3\vec{r} r^2 \rho(\vec{r})}{\int d^3\vec{r}\rho(\vec{r})}$$

see H.-W. Hammer's talk on Wednesday
L. Dai's talk on Thu., S. Collins's on Fri.

Energy-Momentum Tensor Form Factors (or, gravitational form factors GFFs) of proton: Ji 1995 &1997; Polyakov, 1999
(pion: Kobzarev & Okun 1962; Pagels 1966;)

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\left[A^a(t)\frac{P_\mu P_\nu}{m} + J^a(t)\frac{i P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t) g_{\mu\nu}\right] u\, e^{i(p'-p)x}$$

$(a = q, g)$

- $A^a(t) \xrightarrow{t\to 0}$ mass — external properties
- $J^a(t) \longrightarrow$ spin — external properties
- $D^a(t) \longrightarrow$ D-term — "internal" property
- $\bar{c}^a(t)$: **"Druck"= pressure**

→Trace anomaly: mass budget
see C. Weiss's talk on Tuesday

→spin and orbital angular momentum
e.g. Leader, Lorcé, Phys.Rept. 541(2014)3

→spatial densities of hadrons
see J. Panteleeva's talk on Thursday

- Ji's sum: $A^q(t) + B^q(t) = 2J^q(t)$
- Free fermion: $D_{\text{fermion}} = 0 \rightarrow \neq 0$: interaction! Hudson & Schweitzer, 2018

“Probe” GFFs

Operator product expansion（OPE）/ Mellin moments:

（Diehl, 2003; Belitsky, Radyushkin, 2005）

$$(P^+)^{n+1}\int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\right]_{z^+=0,\, \boldsymbol{z}=0}$$

$$= \left(i\frac{\mathrm{d}}{\mathrm{d}z^-}\right)^n \left[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\right]\Big|_{z=0} = \bar{q}(0)\,\gamma^+(i\overleftrightarrow{\partial}^+)^n\, q(0)$$

$\downarrow n \to 0$ $\quad\downarrow n \to 1$

probe $|N\rangle$ by $\hat{J}^\mu_{\rm em}$ $\quad \hat{T}^{\mu\nu}_{\rm grav}$

$\partial^\mu \to D^\mu$: by Wilson line $L[-\frac{z}{2}, \frac{z}{2}]$.

- DVCS @ HERMES & HERA (DESY), CLAS(JLab), EIC, etc
- TCS @ KEKB, P̄ANDA etc.

GPDs∋{EM form factor, PDFs}

$$\int \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t)$$

$$\lim_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$$

(Three Dim.) **(One Dim.)**

GPDs ↔ GFFs（polynomiality）（Ji, 1996）

$$\int \mathrm{d}x\; x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)$$

$$\int \mathrm{d}x\; x\, E^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t)$$

Sum-rules for other cases:

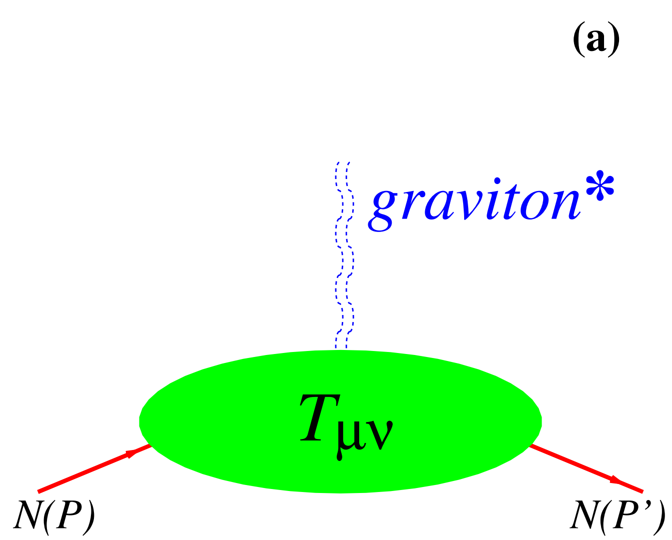
spin-1: Polyakov, BDS, Phys.Rev.D 100 (2019) 3

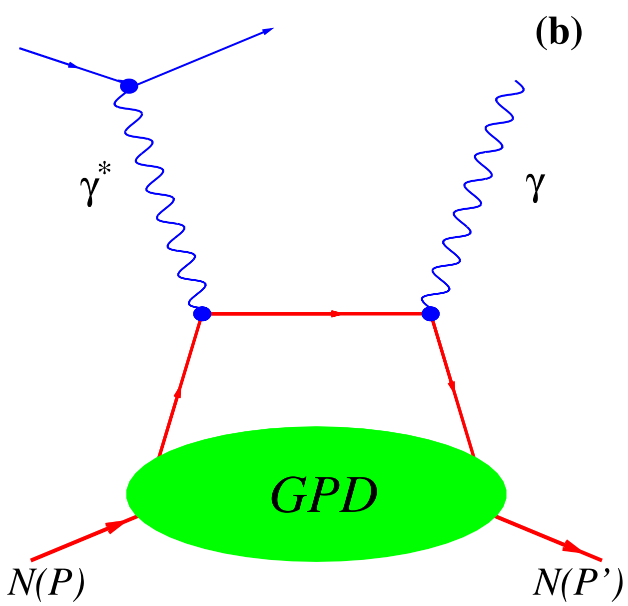
spin-3/2: Fu, BDS, Dong, Phys.Rev.D 106 (2022) 11

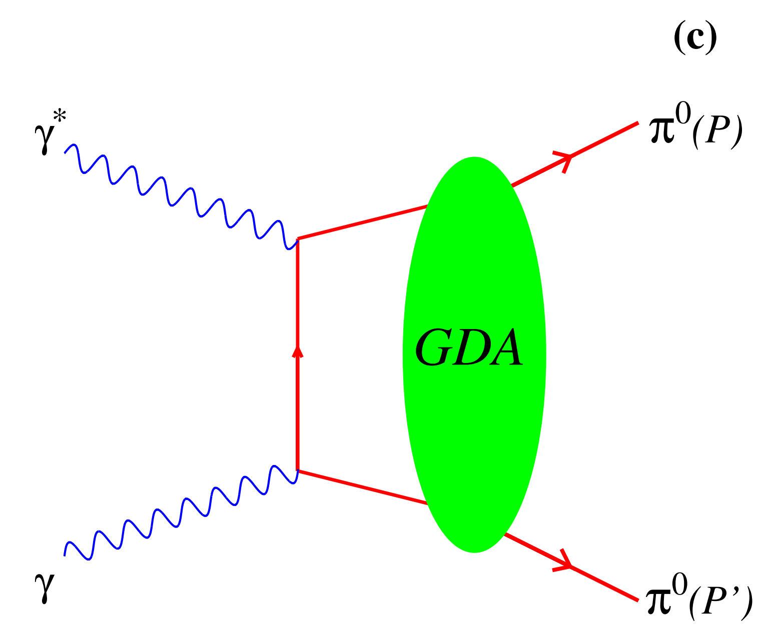
$N \to N\pi$: Polyakov, Stratmann, arXiv:hep-ph/0609045

Not easy to extract GPDs! Zhang, Ji, arXiv:2408.04133

Polyakov, Schweitzer, Int.J.Mod.Phys.A 33（2018）26







$N \to \Delta$ transition and One pion production ($\gamma^* N \to \gamma(M) N\pi$)

Why is it interesting:

1. $N \to \Delta$ electromagnetic transition form factors are required to explore geometry (shape) of the nucleon: spatial transverse charge densities of quarks which are active in the excitation

   see M. Paolone's talk on Monday

2. $N \to \Delta$ (or πN) GPDs access the longitudinal momentum distributions of the active quarks in this transition

   Semenov-Tian-Shansky, Vanderhaeghen, Phys.Rev.D 108 (2023) 3

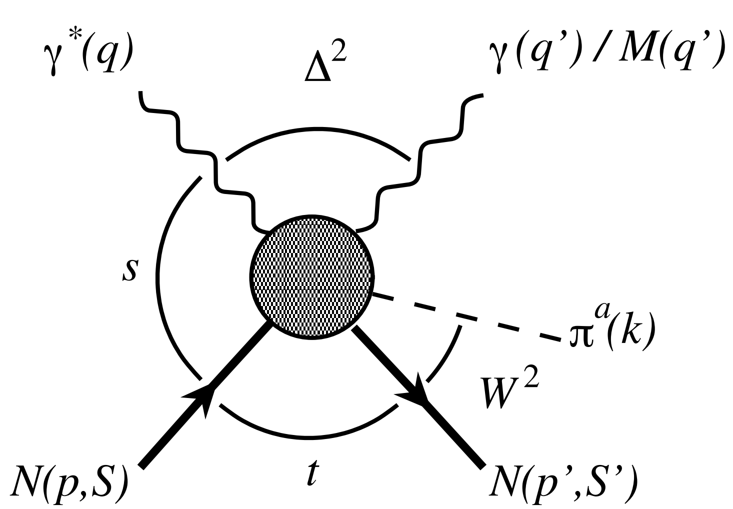
3. $D_N = c_8 m_N$ (in chiral limit) with c_8 as low energy constant from pure grav. interactions (higher chiral order)

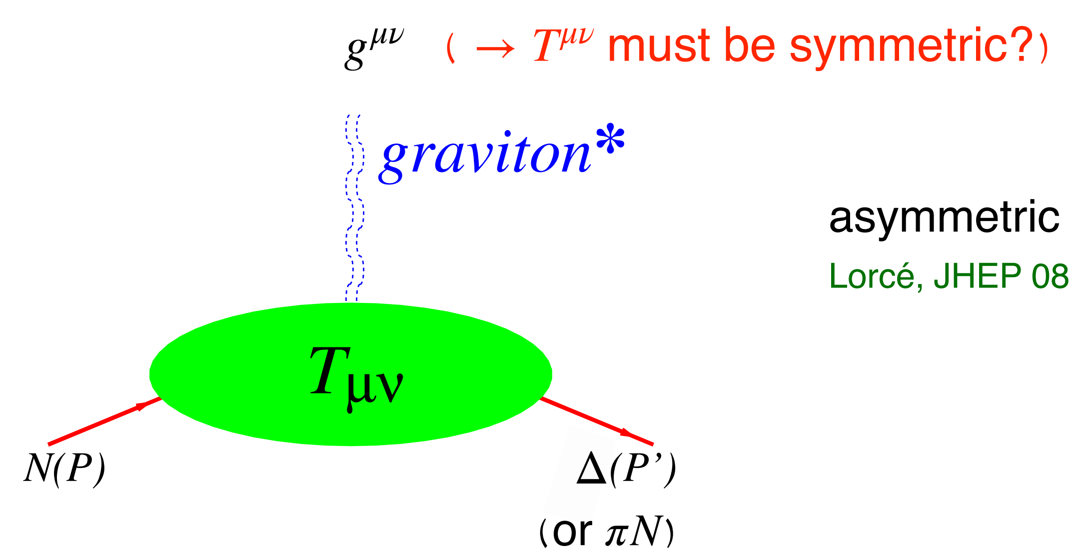
   Gegelia, Polyakov, Phys.Lett.B 820 (2021) 136572

   see H. Alharazin's talk on Monday

CLAS@JLab recent target: N* N transition GPDs etc.
Brodsky et al, 2020; Joo and Diehl 2022.

Polyakov, Stratmann, arXiv:hep-ph/0609045





asymmetric $T^{\mu\nu}$?
Lorcé, JHEP 08 (2015) 045

For sure one needs a systematic way to get EMT: e.g. ChPT

$p \to \Delta^+$ transition processes

GPDs for $N \to \Delta$ transition are given by Semenov-Tian-Shansky, Vanderhaeghen 2023:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R, s_R)| \bar{q}\left(-\frac{\lambda n}{2}\right) \gamma \cdot n\, q\left(\frac{\lambda n}{2}\right) |N(p, s_N)\rangle$$
$$= \left(\frac{C_{iso}}{6}\right) \bar{R}_\beta (p_R, s_R) \left\{ h_M(x,\xi,\Delta^2)\Gamma_M^{\beta\nu} + h_E(x,\xi,\Delta^2)\Gamma_E^{\beta\nu} + h_C(x,\xi,\Delta^2)\Gamma_C^{\beta\nu} + h_4(x,\xi,\Delta^2)\Gamma_4^{\beta\nu} \right\} n_\nu N(p, s_N)$$

- $\bar{R}_\beta$ is Δ field. Sum-rules: $\int dx h_4(x,\xi,\Delta^2) = 0$ and $\int dx h_{M,E,C} = g_{M,E,C}(\Delta^2)$ which are electromagnetic transition FFs.
- Transition EMT form factors are second Mellin moment of GPDs, i.e. Gravitational Transition Form Factors（GTFFs）

GTFFs are given by: Kim, 2022

$$\langle \Delta, p_f, s_f | T_a^{\mu\nu}(x) | N, p_i, s_i \rangle$$
$$= \bar{u}_\alpha(p_f, s_f) \Bigg\{ F_1^a(t) \left(g^{\alpha\{\mu} P^{\nu\}} + \frac{m_{\Delta^+}^2 - m_p^2}{\Delta^2} g^{\mu\nu}\Delta^\alpha - \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2} g^{\alpha\{\mu}\Delta^{\nu\}} - \frac{1}{\Delta^2} P^{\{\mu}\Delta^{\nu\}}\Delta^\alpha \right)$$
$$+ F_2^a(t) \left(P^\mu P^\nu \Delta^\alpha + \frac{(m_{\Delta^+}^2 - m_p^2)^2}{4\Delta^2} g^{\mu\nu}\Delta^\alpha - \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2} P^{\{\mu}\Delta^{\nu\}}\Delta^\alpha \right)$$
$$+ F_3^a(t) \left(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu} \right) \Delta^\alpha$$
$$+ F_4^a(t) \left(g^{\alpha\{\mu}\gamma^{\nu\}} + \frac{2(m_p + m_{\Delta^+})}{\Delta^2} g^{\mu\nu}\Delta^\alpha - \frac{m_p + m_{\Delta^+}}{\Delta^2} g^{\alpha\{\mu}\Delta^{\nu\}} - \frac{1}{\Delta^2}\gamma^{\{\mu}\Delta^{\nu\}}\Delta^\alpha \right)$$
$$+ F_5^a(t) \left(P^{\{\mu}\gamma^{\nu\}}\Delta^\alpha + \frac{(m_{\Delta^+}^2 - m_p^2)(m_p + m_{\Delta^+})}{\Delta^2} g^{\mu\nu}\Delta^\alpha - \frac{m_p + m_{\Delta^+}}{\Delta^2} P^{\{\mu}\Delta^{\nu\}}\Delta^\alpha - \frac{m_{\Delta^+}^2 - m_p^2}{2\Delta^2}\gamma^{\{\mu}\Delta^{\nu\}}\Delta^\alpha \right)$$
$$+ \left(\bar{C}_1^a(t) g^{\mu\nu}\Delta_\alpha + \bar{C}_2^a(t)\Delta^{\{\mu}P^{\nu\}}\Delta_\alpha + \bar{C}_3^a(t)\gamma^{\{\mu}\Delta^{\nu\}}\Delta_\alpha + \bar{C}_4^a(t) g_\alpha^{\{\mu}\Delta^{\nu\}} \right) \Bigg\} \gamma^5 u(p_i, s_i)\, e^{-i\Delta\cdot x}$$

- $\partial_\mu T^{\mu\nu} = 0 \to$ 5 conserving FFs
- 4 non-conserving FFs cancels in total FFs

ChPT in curved space-time

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

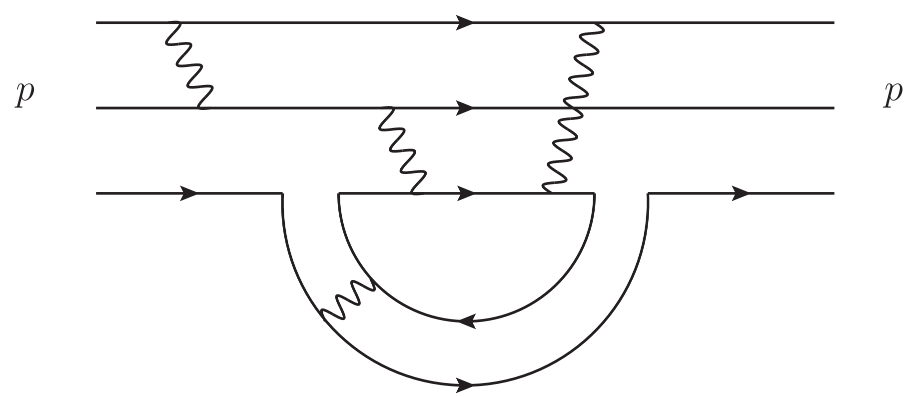
A systematic way to study of hadronic processes induced by gravity in the low-energy domain

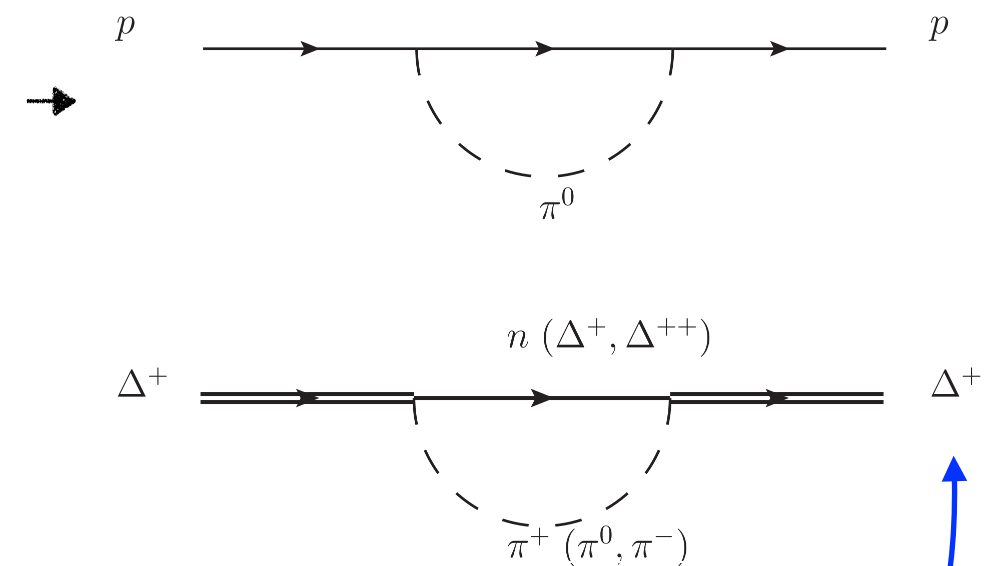
QCD

↓

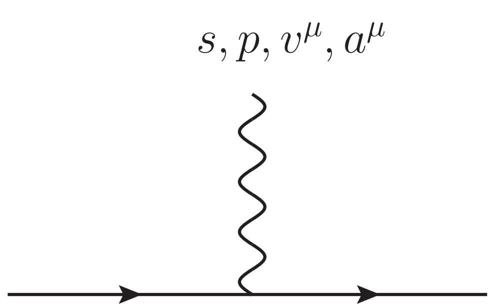
SU(2) ChPT:

quark & gluon → pions

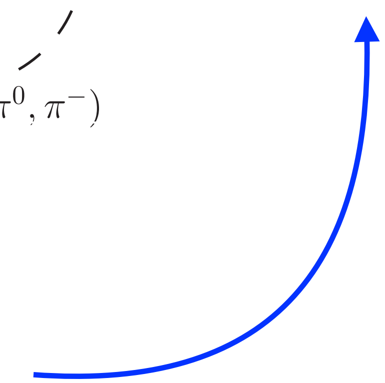




in presence of External Fields:



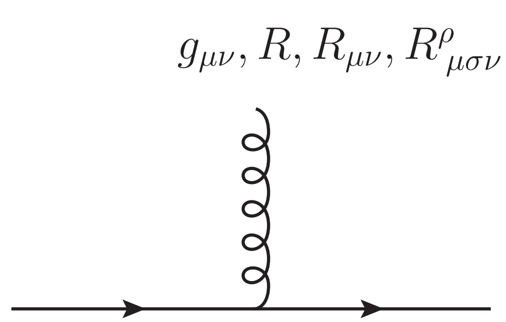
- Two sources for isospin symmetry breaking:
  1. $s = \text{diag}[m_u, m_d]$, when $m_u \neq m_d$
  2. electromagnetic interaction



$p \to \Delta^+$

$I = 1/2 \to I = 3/2$

in curved space-time:



- Flat limit: $g_{\mu\nu}(x) \to \eta_{\mu\nu}$
- To connect Lorentz and Dirac indices: $\gamma_\mu = e^a_\mu \gamma_a$
- Vielbein fields satisfy: $e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$

Building blocks for ChPT actions

$$\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$U(x) = \exp\left[i\frac{\phi}{F_0}\right]$$

$$u^2(x) = U(x)$$

$$u^\mu = i\left\{u^\dagger\left(\partial^\mu - ir^\mu\right)u - u\left(\partial^\mu - il^\mu\right)u^\dagger\right\}$$

$\dots$

X	Dim	P	C	H.c.
u^μ	1	$-u^\mu$	$(u^\mu)^T$	u^μ
u_i^μ	2	$-u_i^\mu$	$c_{ij}u_j^\mu$	u_i^μ
$h^{\mu\nu}$	2	$h^{\mu\nu}$	$(h^{\mu\nu})^T$	$h^{\mu\nu}$
$h_i^{\mu\nu}$	2	$-h_i^{\mu\nu}$	$c_{ij}h_j^{\mu\nu}$	$h_i^{\mu\nu}$
$\chi_\pm$	2	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$\chi_{\pm,i}$	2	$\pm\chi_{\pm,i}$	$c_{ij}\chi_{\pm,j}$	$\pm\chi_{\pm,i}$
$\chi_{\pm,s}$	2	$\pm\chi_{\pm,s}$	$\chi_{\pm,s}$	$\pm\chi_{\pm,s}$
$f_\pm^{\mu\nu}$	2	$\pm f_\pm^{\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
$f_{\pm,i}^{\mu\nu}$	2	$\pm f_{\pm,i}^{\mu\nu}$	$\mp c_{ij}f_{\pm,j}^{\mu\nu}$	$f_{\pm,i}^{\mu\nu}$
$f_{+,s}^{\mu\nu}$	2	$f_{+,s}^{\mu\nu}$	$-f_{+,s}^{\mu\nu}$	$f_{+,s}^{\mu\nu}$

Table 1: Chiral dimensions, transformation behavior P, C and H.c. of the building blocks.

Derivatives on fields:

π $$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$$

N $$\vec{\nabla}_\mu\Psi = \partial_\mu\Psi + \frac{i}{2}\omega_\mu^{ab}\sigma_{ab}\Psi + \left(\Gamma_\mu - iv_\mu^{(s)}\right)\Psi$$

Δ $$\nabla_\mu^{ij}\Psi_\nu^j = \left[\delta^{ij}\partial_\mu + \delta^{ij}\Gamma_\mu - i\delta^{ij}v_\mu^{(s)} - i\epsilon^{ijk}\mathrm{Tr}\left(\tau^k\Gamma_\mu\right) + \frac{i}{2}\delta^{ij}\omega_\mu^{ab}\sigma_{ab}\right]\Psi_\nu^j - \Gamma_{\mu\nu}^\alpha\Psi_\alpha^i$$

- Spin connection: $\omega_\mu^{ab} = -1/2\, g^{\nu\lambda}e_\lambda^a\left(\partial_\mu e_\nu^b - e_\sigma^b\Gamma_{\mu\nu}^\sigma\right)$
- Christoffel symbol: $\Gamma_{\alpha\beta}^\lambda = 1/2 g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right)$

Rarita-Schwinger fields: $$\Psi_\mu(x) = \sum_{s_\Delta}\int\frac{d^3p}{(2\pi)^3}\frac{M_\Delta}{E}\left[b\left(\vec{p},s_\Delta\right)u_\mu\left(\vec{p},s_\Delta\right)e^{-ip\cdot x} + d^\dagger\left(\vec{p},s_\Delta\right)v_\mu\left(\vec{p},s_\Delta\right)e^{ip\cdot x}\right]$$

Power counting, Renormalization Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

ϵ-counting scheme:
(i.e. small scale expansion)

electric charge e:	1
Pion mass M:	1
Derivatives on N or Δ:	0
Masses m_Δ, m_N:	0
$m_\Delta - m_N$:	1
Momentum transfer:	1

Loop momenta:	1
Pion lines:	-2
Nucleon lines:	-1
Delta lines:	-1
$L^{(N)}$ vertices :	N

- To calc delta matrix elements of order 3 (up-to one loop)
- Other counting schemes, e.g, δ-scheme where $M \sim \delta^2$, just shift some digs to higher orders

Pascalutsa, Phillips, Phys. Rev. C 67, 055202 (2003)

- Use EOMS(extended on-mass-shell) scheme
  (remove divergent + power counting violating pieces (PCBs) only)
- Strategy of regions to check possible PCBs
  [Beneke and Smirnov, Nucl.Phys.B 522 321-344, 1998; Gegelia, Japaridze and Turashvili Theor.Math.Phys. 101 (1994) 1313]:

see D.-L. Yao's talk on Monday
N. D. Conrad's talk on Tuesday

Actions including Δ Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$S_\gamma^{(2)} = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_\gamma^2}{2}A_\mu A^\mu\right\}$$

$$S_\pi^{(2)} = \int d^4x\sqrt{-g}\left\{\frac{F^2}{4}g^{\mu\nu}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\}$$

$$S_{\mathrm{N}\pi}^{(1)} = \int d^4x\sqrt{-g}\left\{\bar\Psi\, i\gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu \Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\mu\gamma_5 u_\mu\Psi\right\},$$

see N. Döpper's talk on Tuesday

Actions including Δ Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

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$$\begin{aligned} S_{\pi\Delta}^{(1)} = -\int d^4x\sqrt{-g}\Big[& g^{\mu\nu}\,\bar\Psi_\mu^i\, i\gamma^\alpha\overleftrightarrow{\nabla}_\alpha\Psi_\nu^i - m_\Delta\, g^{\mu\nu}\bar\Psi_\mu^i\Psi_\nu^i - g^{\lambda\sigma}\left(\bar\Psi_\mu^i i\gamma^\mu\overleftrightarrow{\nabla}_\lambda\Psi_\sigma^i + \bar\Psi_\lambda^i i\gamma^\mu\overleftrightarrow{\nabla}_\sigma\Psi_\mu^i\right) \\ & + i\bar\Psi_\mu^i\gamma^\mu\gamma^\alpha\gamma^\nu\overleftrightarrow{\nabla}_\alpha\Psi_\nu^i + m_\Delta\bar\Psi_\mu^i\gamma^\mu\gamma^\nu\Psi_\nu^i + \frac{g_1}{2}g^{\mu\nu}\bar\Psi_\mu^i u_\alpha\gamma^\alpha\gamma_5\Psi_\nu^i + \frac{g_2}{2}\bar\Psi_\mu^i\left(u^\mu\gamma^\nu + u^\nu\gamma^\mu\right)\gamma_5\Psi_\nu^i \\ & + \frac{g_3}{2}\bar\Psi_\mu^i u_\alpha\gamma^\mu\gamma^\alpha\gamma_5\gamma^\nu\Psi_\nu^i\Big] \end{aligned}$$

$$S_{\Delta\mathrm{N}\pi}^{(1,2)} = \int d^4x\sqrt{-g}\left\{-g_{\pi N\Delta}\bar\Psi\left(g^{\mu\nu} - \gamma^\mu\gamma^\nu\right)u_{\mu,i}\Psi_{\nu,i} + d_3^{(2)} i\bar\Psi f_+^{i\mu\nu}\gamma_5\gamma_\mu\left(g_{\nu\lambda} - \left[z_n + \frac{1}{2}\right]\gamma_\nu\gamma_\lambda\right)\Psi^{i\lambda} + \mathrm{H.c.}\right\}.$$

- $m_\gamma \to 0$, no IR divergences
- Choose off-shell parameters as $A = -1$, $z_n = 0$
- Constrain on LECs $g_2 = g_3 = -g_1$
- $g_{\pi N\Delta}$ and $d_3^{(2)}$ are fixed by $\Delta \to p$ decay and other LECs given by PDG or refs:
  →No free parameters!

see N. Döpper's talk on Tuesday

EMT vertices

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

Actions in curved space-time: $S = \int d^4x \sqrt{-g}\, \mathcal{L}$

Calc EMT: $T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\rm m}}{\delta g^{\mu\nu}} \quad \Rightarrow \quad T_{\mu\nu} = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\, e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\, e^a_\mu\right]$

Variations need to be considered: $\delta g_{\mu\nu}\,,\, \delta\sqrt{-g}\,,\, \delta\omega_\mu^{ab}\,,\, \delta e^a_\mu\,,\, \delta\Gamma^\rho_{\mu\nu}\,,\, \delta R\,,\, \delta R_{\mu\nu}\,,\, \delta {R^\rho}_{\mu\lambda\nu}\,,\, \dots$

see H. Alharazin's talk on Monday

Results:

$$T^{(2)}_{\pi,\mu\nu} = \frac{F^2}{4}\,{\rm Tr}(D_\mu U (D_\nu U)^\dagger) - \frac{\eta_{\mu\nu}}{2}\left\{\frac{F^2}{4}\,{\rm Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\,{\rm Tr}(\chi U^\dagger + U\chi^\dagger)\right\} + (\mu \leftrightarrow \nu)$$

$$T^{(1)}_{\pi N,\mu\nu} = \frac{i}{2}\bar\Psi\,\gamma_\mu \overleftrightarrow{D}_\nu \Psi + \frac{g_A}{4}\bar\Psi\,\gamma_\mu\gamma_5 u_\nu \Psi - \frac{\eta_{\mu\nu}}{2}\left(\bar\Psi\, i\gamma^\alpha \overleftrightarrow{D}_\alpha \Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\alpha\gamma_5 u_\alpha \Psi\right) + (\mu \leftrightarrow \nu)$$

$$T^{(1)}_{\pi N\Delta,\mu\nu} = \frac{1}{2} g_{\pi N\Delta}\ \eta_{\mu\nu}\left[\bar\Psi^i_\alpha u^\alpha_i \Psi + \bar\Psi u^\alpha_i \Psi^i_\alpha - \bar\Psi^i_\alpha \gamma^\alpha\gamma^\beta u^i_\beta \Psi - \bar\Psi\gamma^\beta\gamma^\alpha u^i_\beta \Psi^i_\alpha\right] - g_{\pi N\Delta}\ \left(\bar\Psi^i_\mu u^i_\nu \Psi + \bar\Psi u^i_\nu \Psi^i_\mu\right) + \dots$$

$p \to \Delta^+$ gravitational transition FFs

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

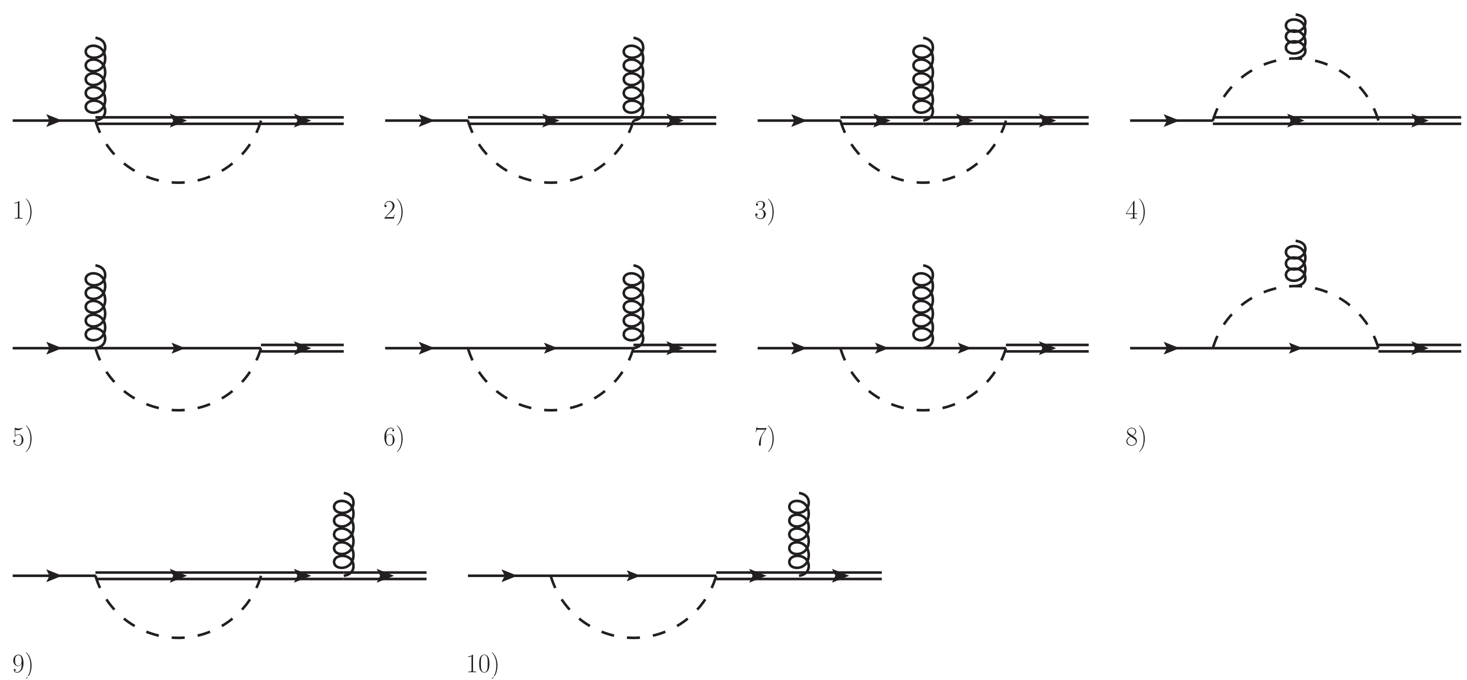
Leading tree order contributions to GTFFs starts from chiral order Q^4: Not tree contributions in this work(Q^3)!

$$S_{\Delta N}^{(4)} = \int d^4 \sqrt{-g}\, d_4\, e^2\, R^{\alpha\beta}\, \bar{\Psi}\delta^{i3}\left(\tau^3 + 1\right)\gamma_\beta\gamma^5\Psi_{\alpha,i} \; + \; \langle\chi_+\rangle \text{ terms}$$

(gravitational interaction itself preserves isospin symmetry!)

One-loop contributions to GTFFs

1. Strong contributions:



- Gravitational-source-baryon-baryon vertex $T_N^{\mu\nu}$, $T_\Delta^{\mu\nu}$ has both chiral order Q^0, and Q^1.
- No power-counting-violating terms and UV divergences up to Q^3: It has to be this way!
- All LECs are fixed, i.e., no free parameters involved: Predictions of ChPT!

2. Electromagnetic contributions: pion lines replaced by photon lines in above diags
   - Every diagram always company with $e^2(\sim Q^2)$
   - Starts at chiral order Q^3

$p \to \Delta^+$ gravitational transition FFs

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

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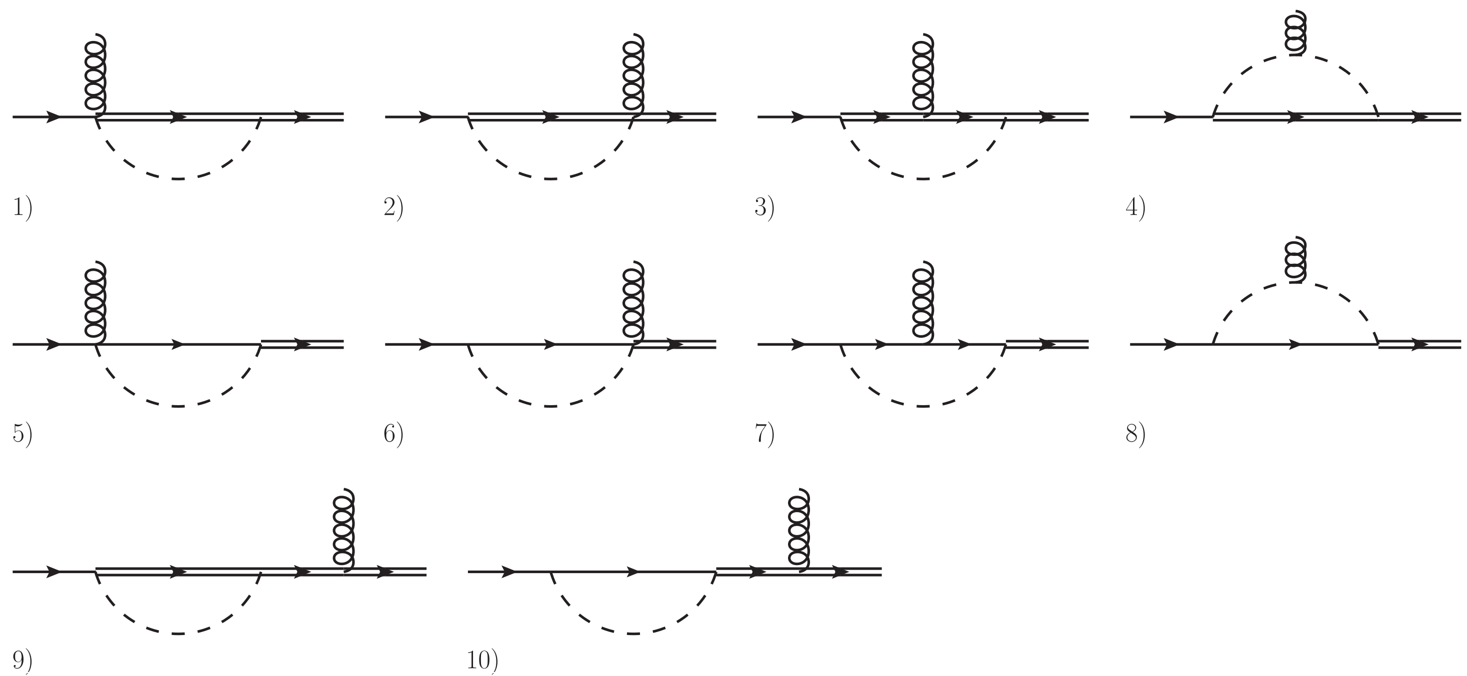
(gravitational interaction itself preserves isospin symmetry!)

One-loop contributions to GTFFs

1. Strong contributions:



- Gravitational-source-baryon-baryon vertex $T_N^{\mu\nu}$, $T_\Delta^{\mu\nu}$ has both chiral order Q^0, and Q^1.
- No power-counting-violating terms and UV divergences up to Q^3: It has to be this way!
- All LECs are fixed, i.e., no free parameters involved: Predictions of ChPT!

2. Electromagnetic contributions: pion lines replaced by photon lines in above diags

- Every diagram always company with $e^2(\sim Q^2)$
- Starts at chiral order Q^3

Real part of GTFFs

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

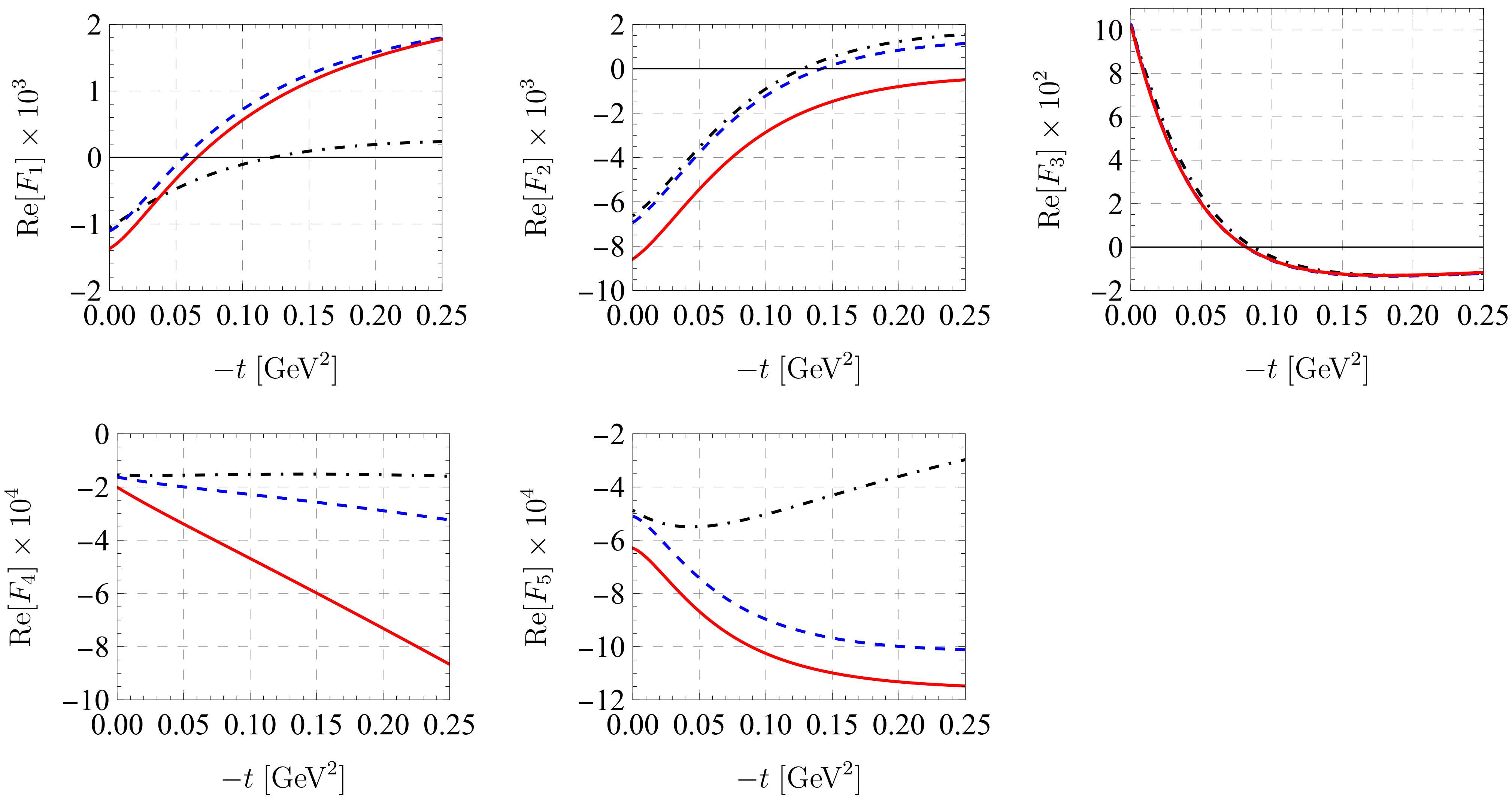
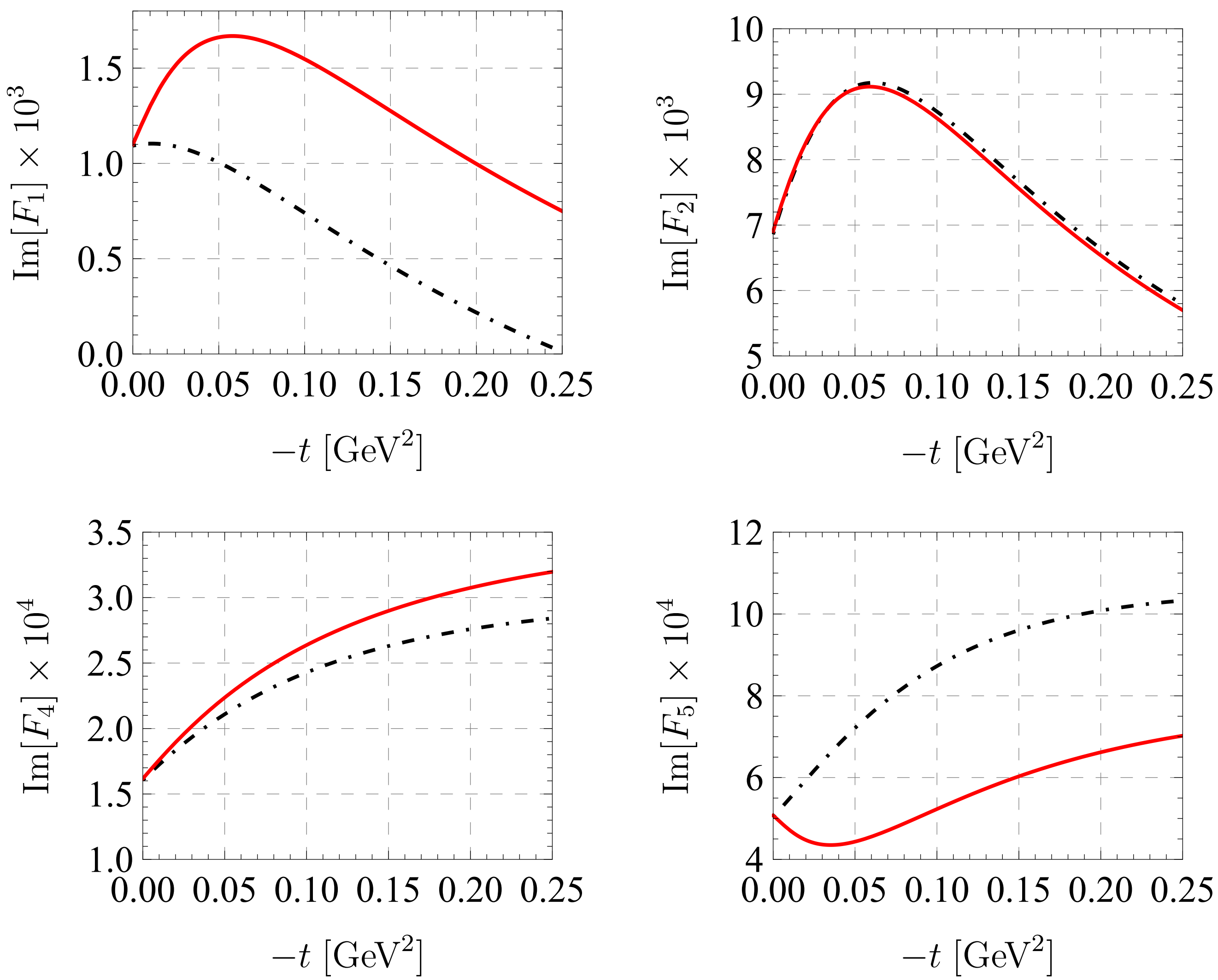


Figure 3. The real parts of the $p \to \Delta^+$ transition form factors. Dash-dotted (black), dashed (blue) and solid (red) lines correspond to the form factors containing contributions of loop diagrams with inner pion and nucleon lines only, diagrams with inner pion and nucleon lines plus radiative corrections, and all loop contributions, respectively.

Imaginary part of GTFFs

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022



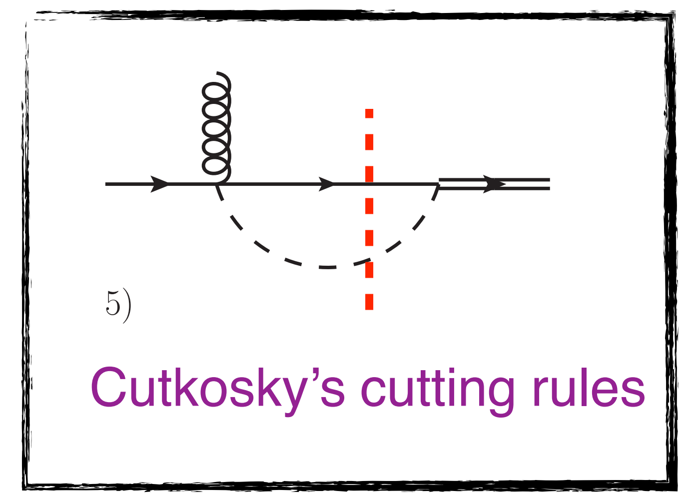


Figure 4. Imaginary parts of the $p \to \Delta^+$ transition form factors. Dash-dotted (black), and solid (red) lines correspond to the form factors containing contributions of loop diagrams with inner pion and nucleon lines only, and diagrams with inner pion and nucleon lines plus radiative corrections, respectively.

Leading contributions: by pion mass differences

Alharazin, et al, arXiv:2312.05193

1. Terms proportional to pion mass differences

$$\sim \frac{1}{p^2 - M_{\pi^+}^2} - \frac{1}{p^2 - M_{\pi^0}^2} \simeq \frac{M_{\pi^+}^2 - M_{\pi^0}^2}{\left(p^2 - M_{\pi^+}^2\right)\left(p^2 - M_{\pi^0}^2\right)}$$

Q^{-2} Q^{-2} Q^{-2}

Left and right sides have same chiral order

2. Terms proportional to proton-neutron (Δ's) mass differences

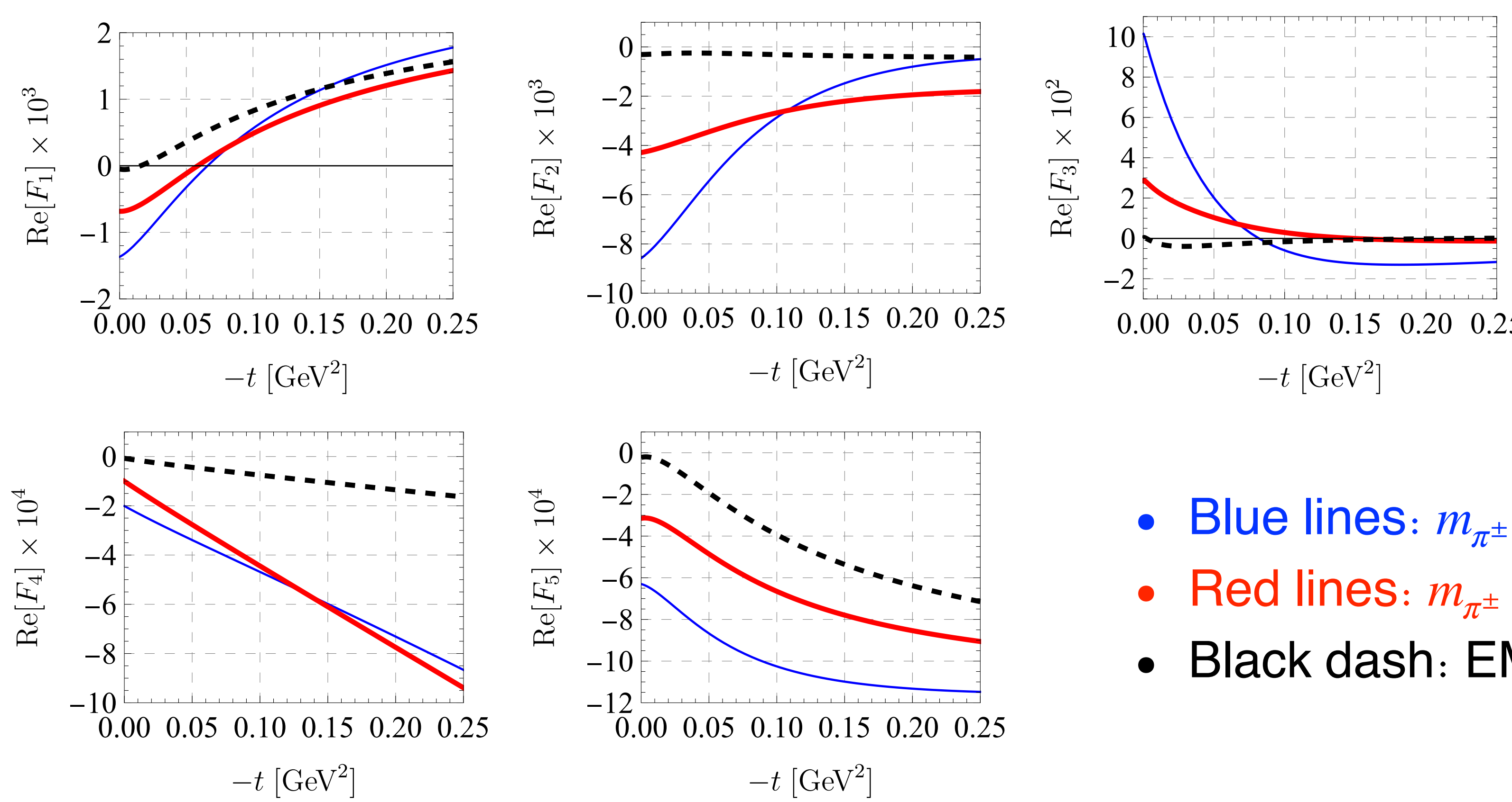
$\sim m_d - m_u \sim m_\pi^2 \sim Q^2$

$$\sim \frac{1}{\not{p} - m_p} - \frac{1}{\not{p} - m_n} \simeq \frac{m_p - m_n}{\left(\not{p} - m_p\right)\left(\not{p} - m_n\right)}$$

Q^{-1} Q^{-1} Q^{0}

Right side is one order higher than left!

3. Electromagnetic contributions are negligible in chiral limit



- Blue lines: $m_{\pi^\pm} \neq m_{\pi^0}$
- Red lines: $m_{\pi^\pm} = m_{\pi^0}$
- Black dash: EM

Note: Separating strong and electromagnetic isospin breaking is anyway afflicted with some uncertainties: running couplings

Meißner and Rusetsky, Effective Field Theories (2022)

Summary

1. $N \to \Delta$ (or πN) GPDs access the longitudinal momentum distributions of the active quarks in this transition, but it's not easy to extract GPD from experiments.
2. Sum-rules connect GPDs and GTFFs which can provide some constraints/inputs on how to extract GPDs.
3. ChPT actions with Δ degree of freedom in curved space-time is constructed and $p \to \Delta^+$ EMT are obtained systematically, then GTFFs are calculated up to the leading one-loop contributions.
4. Electromagnetic and strong isospin violating effects give contributions of comparable sizes.
5. All LECs for gravitational $N - \Delta$ transition are fixed (no free parameters involved), so our results can be regarded as predictions of ChPT.
6. Outlook: ChPT calculation for transition GFFs of the one-pion-graviproduction off the nucleon is undergoing (also not easy...).

Thanks for your attention!

Backup

$$g_A = 1.289, \quad g = 1.35, \quad m_{\pi^0} = 0.135, \quad m_p = 0.938, \quad m_n = 0.940,$$
$$m_\Delta = 1.232, \quad F = 0.092, \quad m_{\pi^+} = 0.140, \quad m_{\Delta^{++}} = 1.231, \quad m_{\Delta^+} = m_\Delta,$$
$$m_{\Delta^0} = 1.233, \quad g_1 = 9g_A/5, \quad e = 0.303\,, \quad d_3^{(2)} = 2.72\,\mathrm{GeV}^{-1}\,,$$

D-term for spin-0

Kobzarev & Okun 1962; Pagels 1966;

Definition: $\langle p' | \hat{T}^a_{\mu\nu}(x) | p \rangle = \left[2P_\mu P_\nu \, A^a(t) + \frac{1}{2}(\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2)\, D^a(t) + 2\, m^2\, \bar{c}^a(t)\, g_{\mu\nu} \right] e^{i(p'-p)x}$ 2+1

Free Klein-Gordon field (no interaction):

Callan, Coleman, Jackiw 1970
Collins, 1976,
Hudson & Schweitzer, 2017

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - V_0(\Phi)\,, \quad V_0(\Phi) = \frac{1}{2} m^2 \Phi^2$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \;\Rightarrow\; D \equiv \lim_{t \to 0} D(t) = -1$$

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Action in cured spacetime with conformal symmetry requires a non-minimal coupling term:

$$S_{\rm grav} = \int \mathrm{d}^n x\, \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} (\partial_\mu \Phi)(\partial_\nu \Phi) - V(\Phi) - \frac{1}{2} h\, R\, \Phi^2 \right)\,, \qquad h = \frac{1}{4}\left(\frac{n-2}{n-1}\right)$$

Generate one "improvement term" in EMT (not vanish in flat limit)

$$\theta^{\mu\nu}_{\rm improve} = -h(\partial^\mu \partial^\nu - g^{\mu\nu}\Box)\, \Phi(x)^2$$

$$(\text{with } \Box = g^{\mu\nu}\nabla_\mu \nabla_\nu)$$

$$T^{\mu\nu} \;\Rightarrow\; T^{\mu\nu} + \theta^{\mu\nu}_{\rm improve} \;\Rightarrow\; D = -\frac{1}{3}$$

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- Even infinitesimally small interactions can drastically impact D-term
- Cannot arbitrarily add "total derivatives" to the EMT
- h removes UV divergences up to three loops in dimensional regularization

Actions from "pure" gravitational interaction

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

Riemann tensor: $R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$

Ricci tensor: $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$

Ricci scalar: $R = g^{\mu\nu} R^{\lambda}{}_{\mu\lambda\nu}$

Chiral order $= Q^2$

$$
\begin{aligned}
S^{(2)}_{\pi\Delta,b} = \int d^4x \sqrt{-g} \Big[& h_1 \; R \; g^{\alpha\beta} \bar{\Psi}^i_\alpha \Psi^i_\beta + \dots \\
& + h_4 \; R^{\mu\nu} \; \bar{\Psi}^i_\mu \Psi^i_\nu + \dots \\
& + i h_{10} \; R^{\mu\nu\alpha\beta} \bar{\Psi}^i_\alpha \sigma_{\mu\nu} \Psi^i_\beta + \dots \Big]
\end{aligned}
$$

- There are 15 terms in total
- 8 out of total 15 terms (h_i's) actually contribute at tree order
- h_i's provide counter terms, absorb power-counting violating terms
- EMT surface terms DO matters, same as the spin-0 case

  Hudson,Schweitzer, 2017

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\end{aligned}
$$

One-loop finite parts of counter terms :

$$
\begin{aligned}
\delta h_1 &= \frac{\delta h_{12} m_N}{2} - \frac{\left(1575\, g^2_{\pi N\Delta} + 172\, g_1^2\right) m_N}{207360\pi^2 F^2}\,, \\
\delta h_4 &= -2\,\delta h_{10} - \delta h_{12} m_N - \frac{m_N \left(45\, g^2_{\pi N\Delta} + 2336\, g_1^2\right)}{51840\pi^2 F^2}\,, \\
\delta h_5 &= -\frac{\delta h_{12}}{2} - \frac{11\left(135\, g^2_{\pi N\Delta} + 124\, g_1^2\right)}{207360\pi^2 F^2}\,, \\
\delta h_{13} &= 2\,\delta h_{10} - \delta h_{12} m_N + \frac{\left(9\, g^2_{\pi N\Delta} + 490\, g_1^2\right) m_N}{10368\pi^2 F^2}
\end{aligned}
$$

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