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# Gravitational local spatial densities for hadrons

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in collaboration with

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Based on:
M.V. Polyakov, P. Schweizer [Int. J. Mod. Phys. A 33] (2018)
V.Burkert, L. Elouadrhiri, F.X.Girod, C. Lorce, P. Schweitzer [Rev. Mod. Phys. 95, (2023)]
Epelbaum, Gegelia, Lange, Meißner, Polyakov [Phys.Rev.Lett.129, 012001] (2022)
Panteleeva, Epelbaum, Gegelia, Meißner [PhysRevD.106, 056019] (2022)
Panteleeva, Epelbaum, Gegelia, Meißner [Eur.Phys.J.C 83, 617] (2023)
Alharazin, Sun, Epelbaum, Gegelia, Meißner [*JHEP* 02 163] (2023)
Panteleeva, Epelbaum, Gegelia, Meißner [*JHEP* 07 237] (2023)

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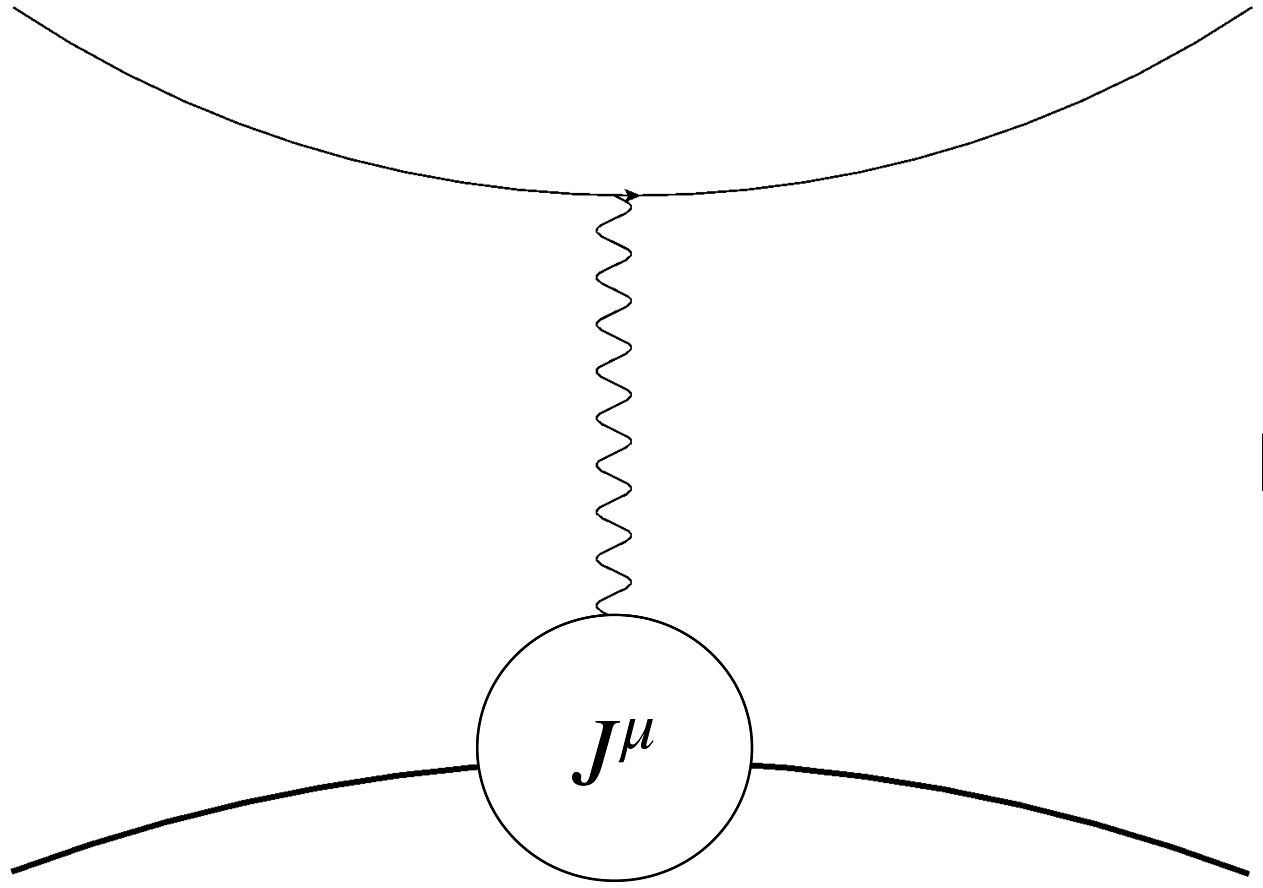
# EM structure of a particle




virtual photon

$$d\sigma/d\Omega = \left(d\sigma/d\Omega\right)_{pointlike} \times \left( F_1^2(q^2) + \frac{q^2}{4m^2}(F_2^2(q^2) + \ldots) \right)$$

For spin-1/2

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \bar{u}(p', s') \left[ \gamma^\mu F_1(q^2) + \frac{1}{2m} i\sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p, s)$$

- electric charge

  $F_1(0) = e$

- anomalous magnetic moment

  $1/2(F_1(0) + F_2(0)) = \mu$

- conserved

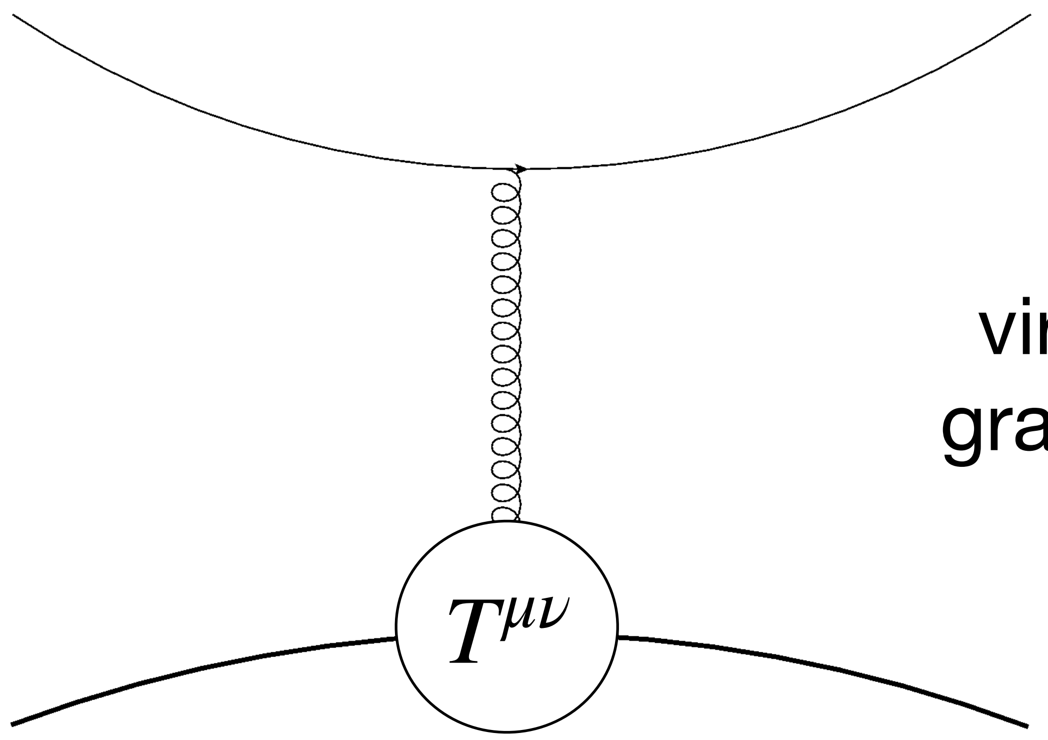
  $\partial^\mu j_\mu = 0$

- gauge invariant

[Rosenbluth, 1950
Hofstadter et al. 1953]

# Gravitational structure of hadrons

[Kobzarev, Okun (1962)
Pagels (1966)]




virtual graviton

*For spin-1/2*

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = \bar{u} \left[ A(q^2) \frac{P_\mu P_\nu}{m} + iJ(q^2) \frac{(P_\mu \sigma_{\nu\alpha} + P_\nu \sigma_{\mu\alpha}) q^\alpha}{4m} + D(q^2) \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \right] u$$

- **mass** $m = \int d^3 r T_{00}(r)$

$$A(0) = 1$$

- **spin** $J^i = \epsilon^{ijk} \int d^3 r r^j T_{0k}(r)$

$$J(0) = 1/2$$

- **anomalous magnetic moment**

$$2J(t) = A(t) + B(t) \quad B(0) = 0$$

- symmetric
- conserved

$$\partial^\mu T_{\mu\nu} = 0$$

- gauge invariant

No direct experiment for detection of the matter-graviton interaction

$$T_{\mu\nu}(x) \sim \left. \frac{\delta S_M}{\delta g^{\mu\nu}(x)} \right|_{g=\eta}$$

Gravity couples to matter due to EMT

- **D-term:**

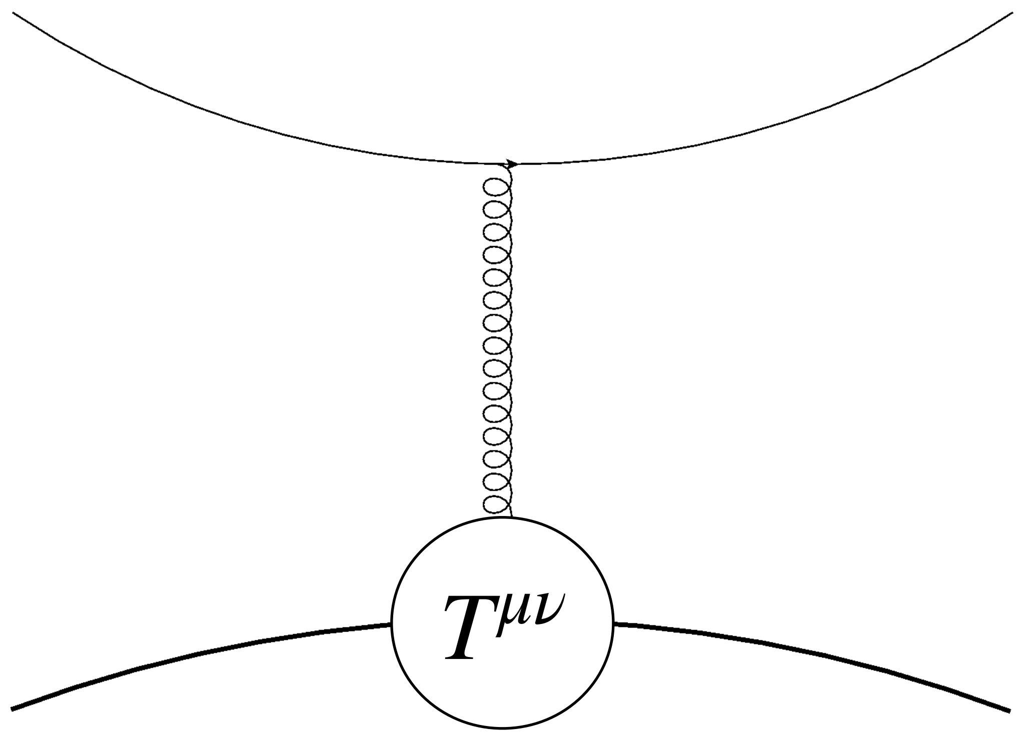
$$D = D(0) = -\frac{m}{2} \int d^3 r \left( r^i r^j - \frac{1}{3} \delta_{ij} \right) T_{ij}(r)$$

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

*…D-term as fundamental as mass and spin!*
*It is necessary connected with the true gravity.*

[Xiang-Dong Ji, Phys.Rev.D 58 (1998)
Xiang-Dong Ji, Phys.Rev.Lett. 78 (1997)]

# How to measure GFFs?




No direct experiment to measure GFFs

$$H, E \sim d\sigma/d\Omega$$

**Details in**

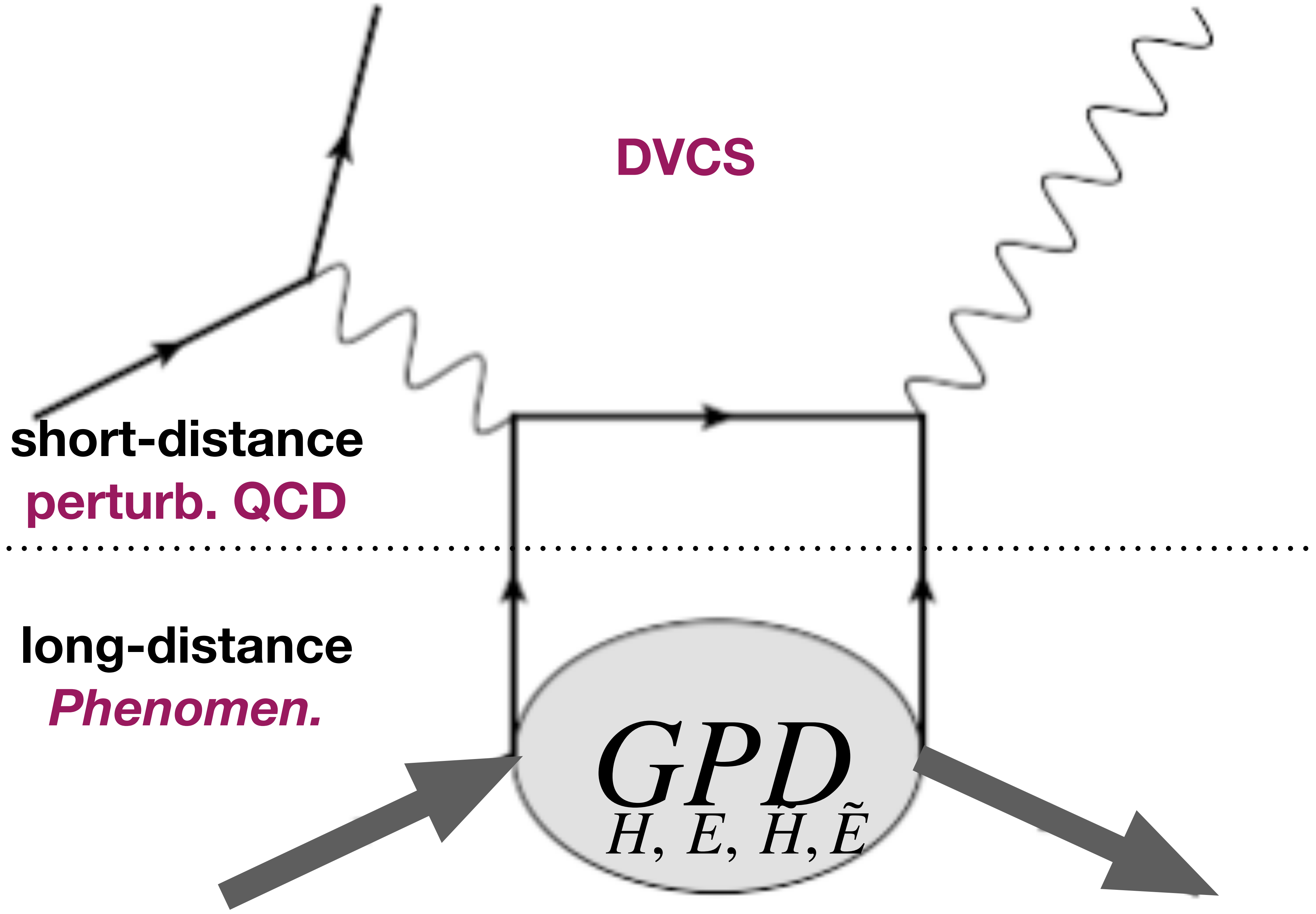
M.V.Polyakov, PLB 555 (2003)

Anikin, Teryaev, PRD76 (2007)

Diehl and Ivanov, EPJC52 (2007)

Radyushkin, PRD83, 076006 (2011)

Bertone et al., PRD 103 (2021)




**However, it is possible with 2 photons**

$$\int_{-1}^{1} dx\ xH(x,\xi,t) = A(t) + \xi^2 D(t)$$

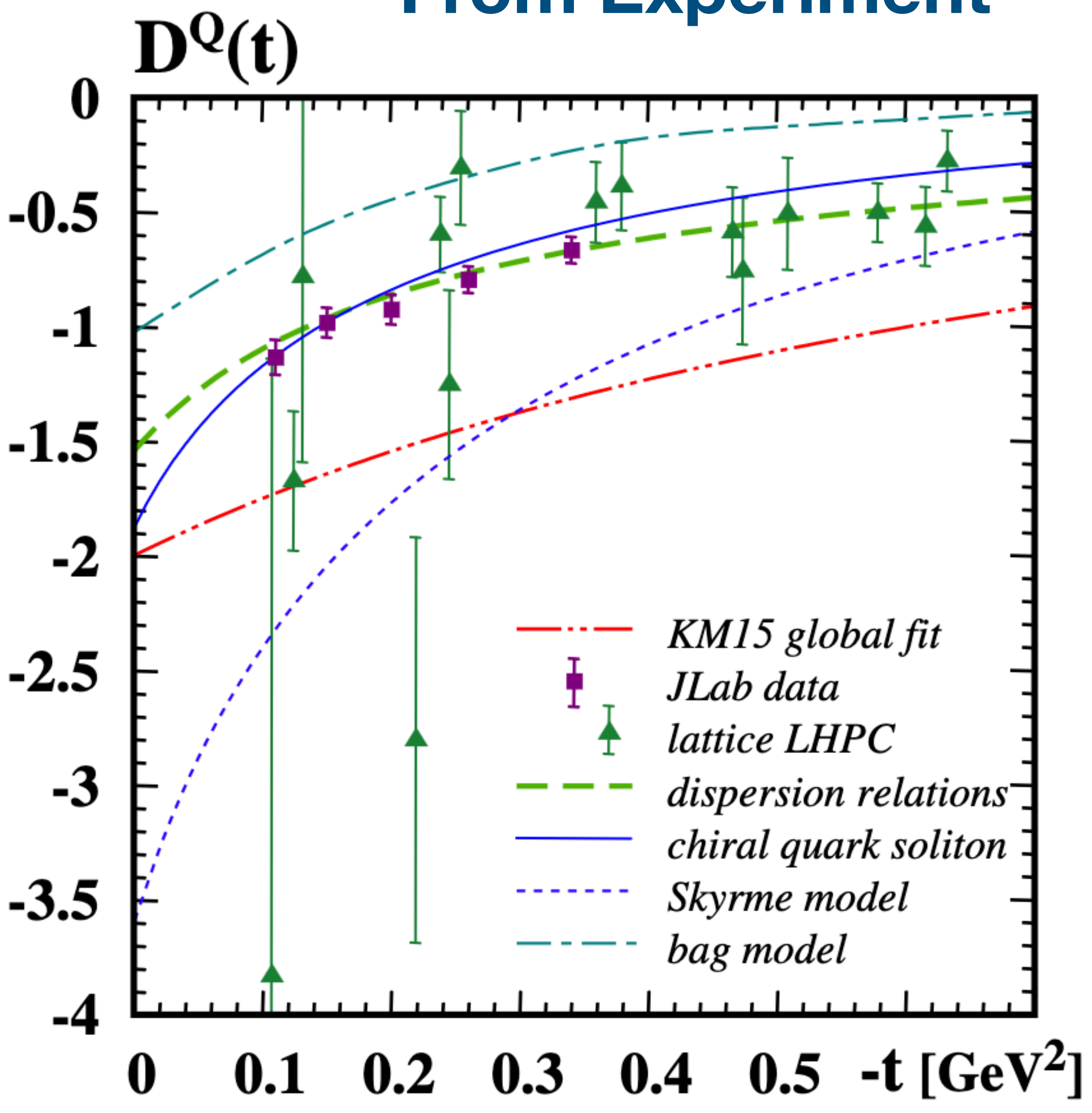
$$\int_{-1}^{1} dx\ xE(x,\xi,t) = B(t) - \xi^2\ D(t)$$

**Details in**

[D. Müller et al., F.Phys. 42,1994,
X. Ji, PRL 78, 610, 1997
A. Radyushkin, PLB 380, 1996 ]

# Results for GFFs

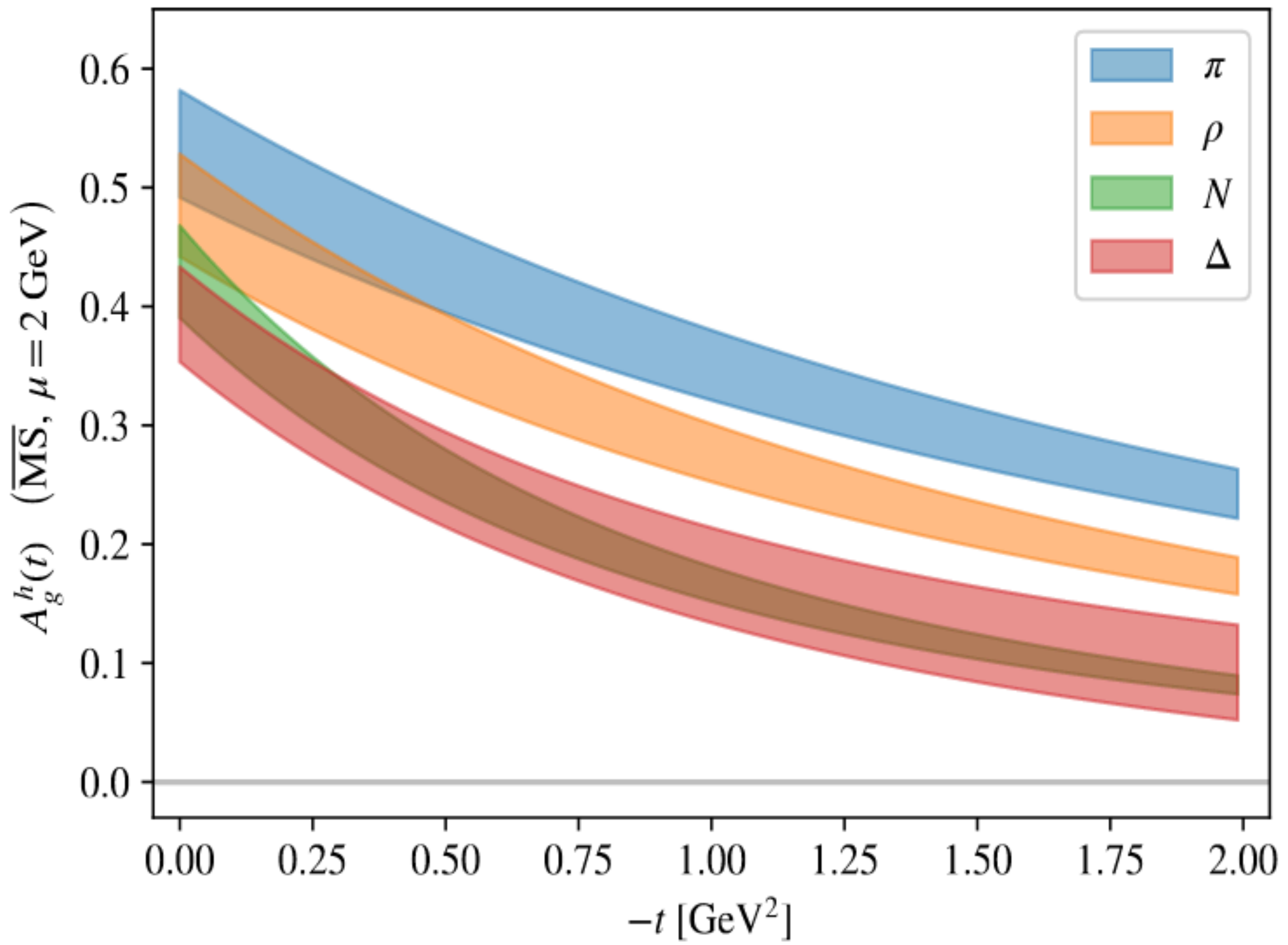
## From Experiment




Comparison of experimental data with lattice data and model calculations
[M.V. Polyakov, P. Schweitzer, Int.J.Mod.Phys.A 33 (2018)]

**Details in**
[Burkert et al., Nature 557 (2018)
Kumeticki, Nature 570 (2019)
Dutrieux et al., Eur.Phys.J C 81

## From lattice




Gluon contribution to GFF A(t) for various hadrons from lattice QCD study
with pion mass mπ = 450(5) MeV
[Pefkou et all. *Phys.Rev.D* 105 (2022)]

**Details in**
[Detmold et al. Phys.Rev.Lett. 126 (2021)
Alexandrou et al., Phys.Rev.D 105 (2022)
Hacket et al., arXiv:2310.08484v1 (2023)]

## From ChPT

**Details in**
[Alharazin, Djukanovic, Gegelia, Polyakov, *Phys.Rev.D* 102 (2020)
Epelbaum, Gegelia, Meißner, Polyakov*, Phys.Rev.D* 105 (2022)
Alharazin, Epelbaum, Gegelia, Meißner, Sun, *Eur.Phys.J.C* 82 (2022)]

# How to use FFs?

**for non-relativistic (heavy) systems**

[Hofstadter et. all,
Rev. Mod. Phys. 30, 482 (1958)]

$$F(Q^2) = \int d^3 r \rho(\mathbf{r}) e^{i\vec{Q}\cdot\vec{r}}$$

charge density of proton

[Sachs,
Phys. Rev.126, 2256-2260 (1962)]

Breit frame
$Q^2 = -\vec{q}^{\,2}$

$$\rho(r) \equiv \int \frac{d^3Q}{(2\pi)^3} G_E(Q^2) e^{-i\vec{Q}\cdot\vec{r}}$$

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

$$T_{\mu\nu}(\mathbf{r}, s) = \frac{1}{2E}\int \frac{d^3Q}{(2\pi)^3} e^{i\vec{Q}\cdot\vec{r}} \langle p', s' | \hat{T}_{\mu\nu}(0) | p, s\rangle$$

| | | | | |
|---|---|---|---|---|
| **em:** $\partial_\mu J^\mu_{\text{em}} = 0$ | $\langle N' \vert J^\mu_{\text{em}} \vert N\rangle$ | $\longrightarrow$ | $Q$ | $= 1.602176487(40) \times 10^{-19}\text{C}$ |
| | | | $\mu$ | $= 2.792847356(23)\mu_N$ |
| **weak:** PCAC | $\langle N' \vert J^\mu_{\text{weak}} \vert N\rangle$ | $\longrightarrow$ | $g_A$ | $= 1.2694(28)$ |
| | | | $g_p$ | $= 8.06(55)$ |
| **gravity:** $\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$ | $\langle N' \vert T^{\mu\nu}_{\text{grav}} \vert N\rangle$ | $\longrightarrow$ | $m$ | $= 938.272013(23)\,\text{MeV}/c^2$ |
| | | | $J$ | $= \frac{1}{2}$ |
| | | | $\boldsymbol{D}$ | $=$ **?** |

**Last global unknown property**

*…Sachs's derivation assumes delocalised wave packet, resulting in moments of the charge density governed by the size of the wave packet*

[M. Burkardt
Phys. Rev. D 66 (2002), 119903(E)]
[G. Miller
Phys. Rev. Lett. 99, 112001 (2007)
Phys. Rev. C 79, 055204 (2009)
Ann. Rev. Nucl. Part. Sci. 60 (2010), 1-25
Phys. Rev. C99, no.3, 035202 (2019)]
[A. Freese and G. Miller
Phys. Rev. D103, 094023 (2021)
*Phys.Rev.D* 108 (2023)]
[R.L.Jaffe, Phys. Rev. D103 no.1, 016017 (2021)]
…..

*…the meaningful way to obtain the fully relativistic spatial densities is through 2D Fourier transform at fixed light-front time*

*…this interpretation is not valid for light system*

## How to define spatial densities?

- **3D Breit frame approach is not exact, valid only for heavy system with $\Delta > 1/m$**
- **2D light-front approach is exact, valid for all systems**
- **the 3D phase-space approach is exact, valid for all systems, but has no probabilistic interpretation**
- **3D novel approach of sharp localisation**

C. Lorce,
Phys. Rev. Lett. 125, no.23, 232002 (2020),
C. Lorce, P. Schweitzer and K. Tezgin,
Phys.Rev. D 106, 014012 (2022)
Y. Guo, X. Ji and K. Shiells,
Nucl. Phys. B 969, 115440 (2021),
C. Lorce, H. Moutarde and A. P. Trawinski,
Eur. Phys. J. C 79, no.1, 89 (2019).1, 016017 (2021)
….

# Construction of electromagnetic densities for a spin-1/2 particle

Matrix element of electromagnetic current operator at t=0:

$$\langle p', s' | \hat{j}^\mu(\mathbf{x},0) | p, s\rangle = e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}}\,\bar{u}(p',s')\left[\gamma^\mu F_1(q^2) + \frac{1}{2} i\sigma^{\mu\nu} q_\nu F_2(q^2)\right] u(p,s)$$

Normalised Heisenberg-picture state: $|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\,\phi(s,\mathbf{p})\,e^{-i\mathbf{p}\cdot\mathbf{X}}\,|p,s\rangle$

$$j^\mu_\phi(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' | \hat{j}^\mu(\mathbf{x},0) | \Phi, \mathbf{X}, s\rangle$$

ZAMF - zero average momentum frame, where $\langle \Phi, \mathbf{X}, s | \mathbf{p} | \Phi, \mathbf{X}, s\rangle = 0$

$\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2,\ \mathbf{q} = \mathbf{p}' - \mathbf{p}$

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P\,d^3q}{(2\pi)^3\sqrt{4EE'}}\bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2 - \mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$E = \sqrt{m^2 + \mathbf{P}^2 - \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4} \qquad E' = \sqrt{m^2 + \mathbf{P}^2 + \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4}$$

### Calculation

$F_1(0) = 1,\ F_2(0) = \kappa/m$

$q = p' - p$

Profile function: **spherically symmetric**

$\phi(s,\mathbf{p}) = \phi(\mathbf{p}) = R^{3/2}\tilde{\phi}(R|\mathbf{p}|)$

sharp localization: $R \to 0$

$\int d^3p\,|\phi(s,\mathbf{p})|^2 = 1$

$\mathbf{X}$ — position of the charge and magnetisation center

# Current densities in static approximation

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2-\mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2-\mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

[R.L. Jaffe, 2021]

taking $m \to \infty$ and after that $R \to 0$ using method of dimensional counting (= strategy of regions):

[J. Gegelia, G.Sh. Dzaparidze and K.Sh. Turashvili, Theor. Math. Phys.101, 1313-1319 (1994)]

$$J^0_{\rm static}(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \left(F_1(-\mathbf{q}^2) + \frac{\mathbf{q}^2}{4m} F_2(-\mathbf{q}^2)\right) \equiv \rho^{\rm ch}_{\rm static}(r)$$

$$\mathbf{J}_{\rm static}(\mathbf{r}) = \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \left(F_1(-\mathbf{q}^2) + mF_2(-\mathbf{q}^2)\right) \equiv \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m}\rho^{mag}_{static}(r)$$

[Sachs,
Phys. Rev.126, 2256-2260 (1962)]

- coincide with Breit Frame expressions
- no dependence on wave packet
- valid for heavy systems with $\Delta \gg R \gg 1/m$
- this approximation is doubtful for light hadrons, $\Delta \lesssim 1/m$

[R.L. Jaffe, Phys. Rev. D103 no.1, 016017, (2021)]

# Novel definition of the current densities

$$j^{\mu}_{\phi}(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p',s')\left[\gamma^{\mu}F_1((E-E')^2-\mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2}F_2((E-E')^2-\mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^{\star}\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

taking $R \to 0$ for arbitrary $m$, using method of dimensional counting:

*[Epelbaum et al. [Phys.Rev.Lett.129, 012001](2022) ]*

$$\mathbf{J}(\mathbf{r}) = \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{4}\left(1+\alpha^2\right) m F_2\left[(\alpha^2-1)\mathbf{q}^2\right] \equiv \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m}\rho_2(r)$$

★

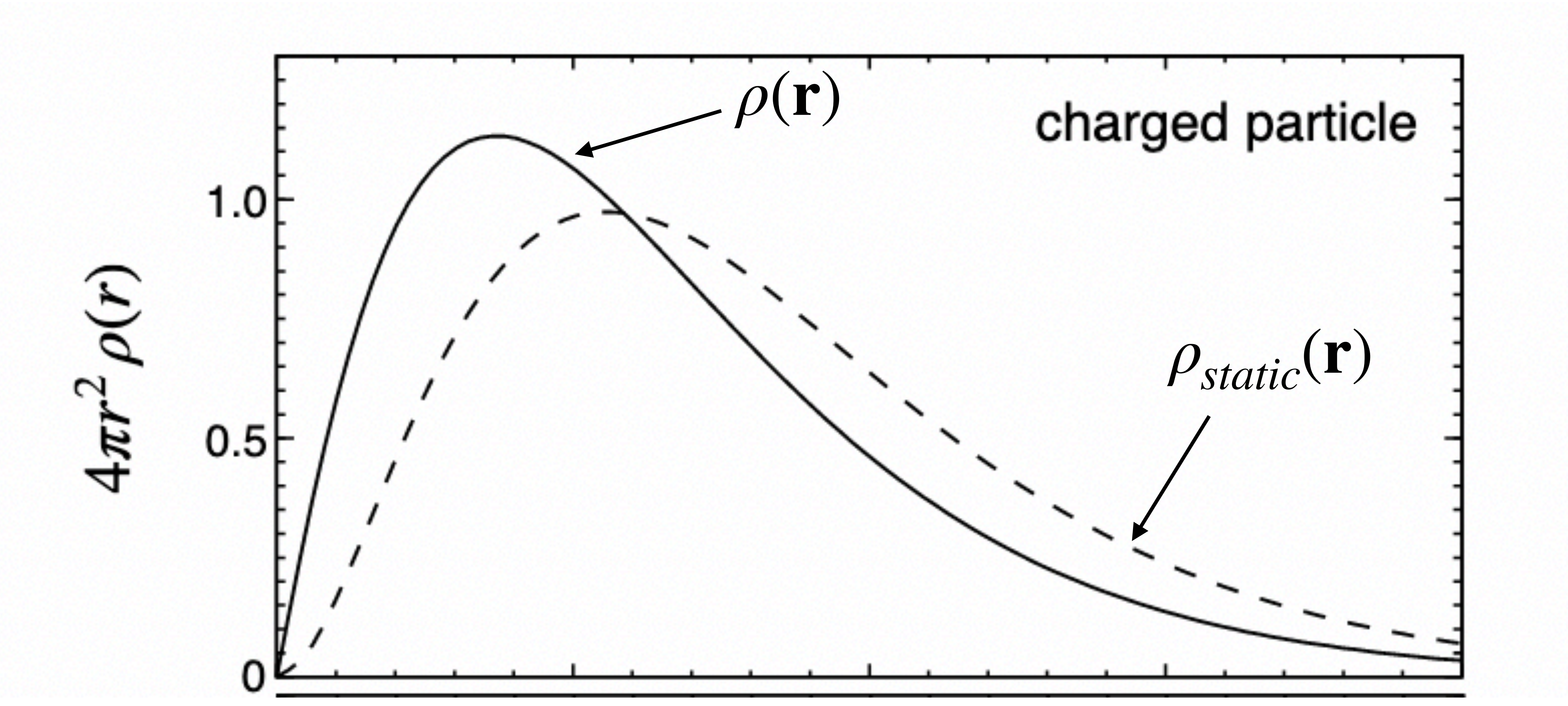
$$J^0(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{2} F_1\left[(\alpha^2-1)\mathbf{q}^2\right] \equiv \rho_1(r)$$

[G.N.Fleming, Physical Reality Math. Descrip., 357 (1974)]

$$\sqrt{\langle r^2\rangle_{\text{static}}} = \sqrt{6\left(F_1'(0)\right)} \simeq 0.8409(4)\ , \quad \sqrt{\langle r^2\rangle} = \sqrt{4F_1'(0)} \simeq 0.62649,$$

$\Delta \gg R \gg 1/m$ $\qquad$ $R \to 0$

[G. A. Miller, Phys. Rev. C 99, no.3, 035202 (2019).]



# Connection with IMF densities

**In moving frame:** $j^{\mu}_{\phi,v}(\mathbf{r}) = {}_{\mathbf{v}}\langle \Phi, \mathbf{X}, s' \,|\, \hat{j}^{\mu}(\mathbf{r},0) \,|\, \Phi, \mathbf{X}, s \rangle_{\mathbf{v}}$

$$\mathbf{r}_\perp = \mathbf{r} - (\mathbf{r}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}$$
$$\mathbf{r}_\parallel = (\mathbf{r}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}$$
$$r_\parallel = |\mathbf{r}_\parallel|$$

Computed for spin- 0, 1/2 and 1 systems

**Charge density**

$$J^0_{ZAMF}(\mathbf{r}) = \frac{1}{4\pi}\int d\hat{\mathbf{v}}\, J^0_{IMF}(\mathbf{r}_\perp)\delta(r_\parallel)\,,$$

**Magnetic density**

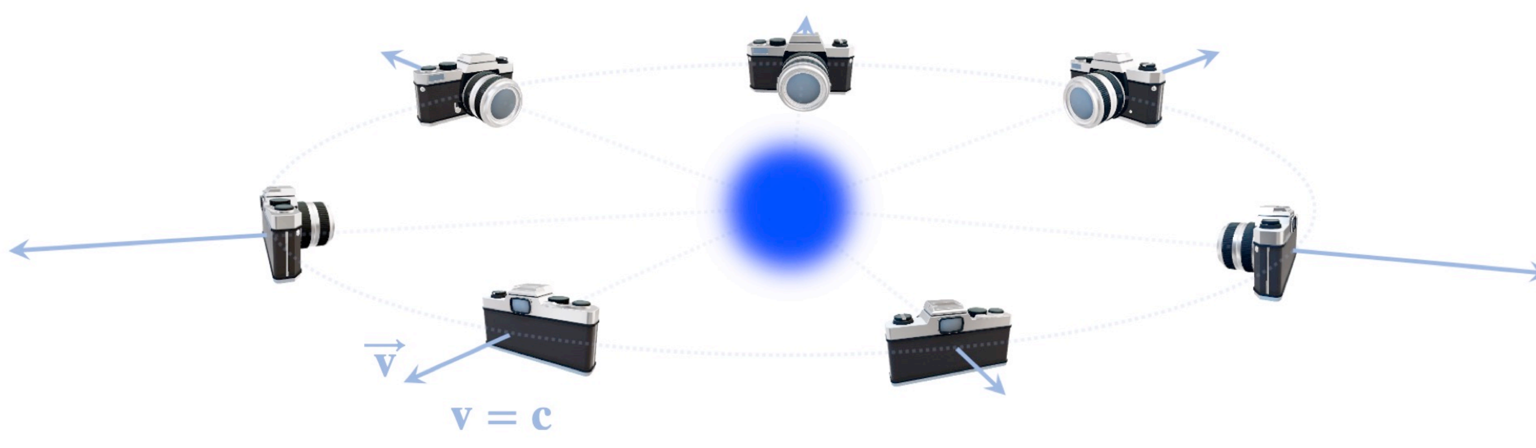
$$\mathbf{J}_{ZAMF}(\mathbf{r}) = 2\times\frac{1}{4\pi}\int d\hat{\mathbf{v}}\,\mathbf{J}_{IMF}(\mathbf{r}_\perp)\delta(r_\parallel)\,.$$

**There is no connection for the quadrupole density**

Panteleeva, Epelbaum, Gegelia, Meißner [*JHEP* 07 237] (2023)

- no dependence on the radial form of the wave packet
- no dependence on the Compton wavelength $1/m$
  - -> valid for light hadrons
  - -> static densities do not emerge from ZAMF densities
- holographic-like relation between ZAMF and IMF

[Epelbaum et al. [Phys.Rev.Lett.129, 012001](2022) ]



# Gravitational spatial densities for spin-1/2

$$\langle p', s' | \hat{T}_{\mu\nu}(\mathbf{x},0) | p, s\rangle = e^{-i\mathbf{q}\cdot\mathbf{x}}\, \bar{u}(p', s') \left[ A(q^2)\frac{P_\mu P_\nu}{m} + iJ(q^2)\frac{P_\mu \sigma_{\nu\alpha} q^\alpha + P_\nu \sigma_{\mu\alpha} q^\alpha}{2m} + D(q^2)\frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \right] u(p, s)$$

$$t_\phi^{\mu\nu}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{x},0) | \Phi, \mathbf{X}, s\rangle$$

Panteleeva, Epelbaum, Gegelia, Meißner, [Eur.Phys.J.C 83, 617] (2023)

$$t_\phi^{00}(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$N_{\phi,\infty} = \frac{1}{R}\int d^3\tilde{P}\tilde{P}\, |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

$$N_{\phi,0} = R\int d^3\tilde{P}\frac{1}{\tilde{P}}\, |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2$$

$$t_\phi^{0i}(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left[ \frac{iJ\left(-\mathbf{q}_\perp^2\right)}{2m} \left( (\boldsymbol{\sigma}_\perp \times \mathbf{q})^i + \hat{\mathbf{n}}\cdot(\boldsymbol{\sigma}_\perp \times \mathbf{q})\hat{n}^i \right) \right] e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$t_\phi^{ij}(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \hat{n}^i\hat{n}^j A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}} + \frac{1}{4}\frac{N_{\phi,0}}{4\pi} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left(q^i q^i - \delta^{ij}\mathbf{q}_\perp^2\right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

***flow tensor*** ***stress tensor***

★

# Mass and energy distribution

***Interpretation***

$$t_\phi^{00}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' \,|\, \hat{T}^{00}(\mathbf{r},0) \,|\, \Phi, \mathbf{X}, s \rangle$$

For sharply localised packet $R \to 0$ and arbitrary $m$

$$t_\phi^{00}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2\hat{n}\, d^2 q_\perp}{(2\pi)^2 (4\pi)} A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} \delta(r_\parallel)$$

Energy distribution

For static approximation $(m \to \infty, R \to 0)$: $R \gg 1/m$

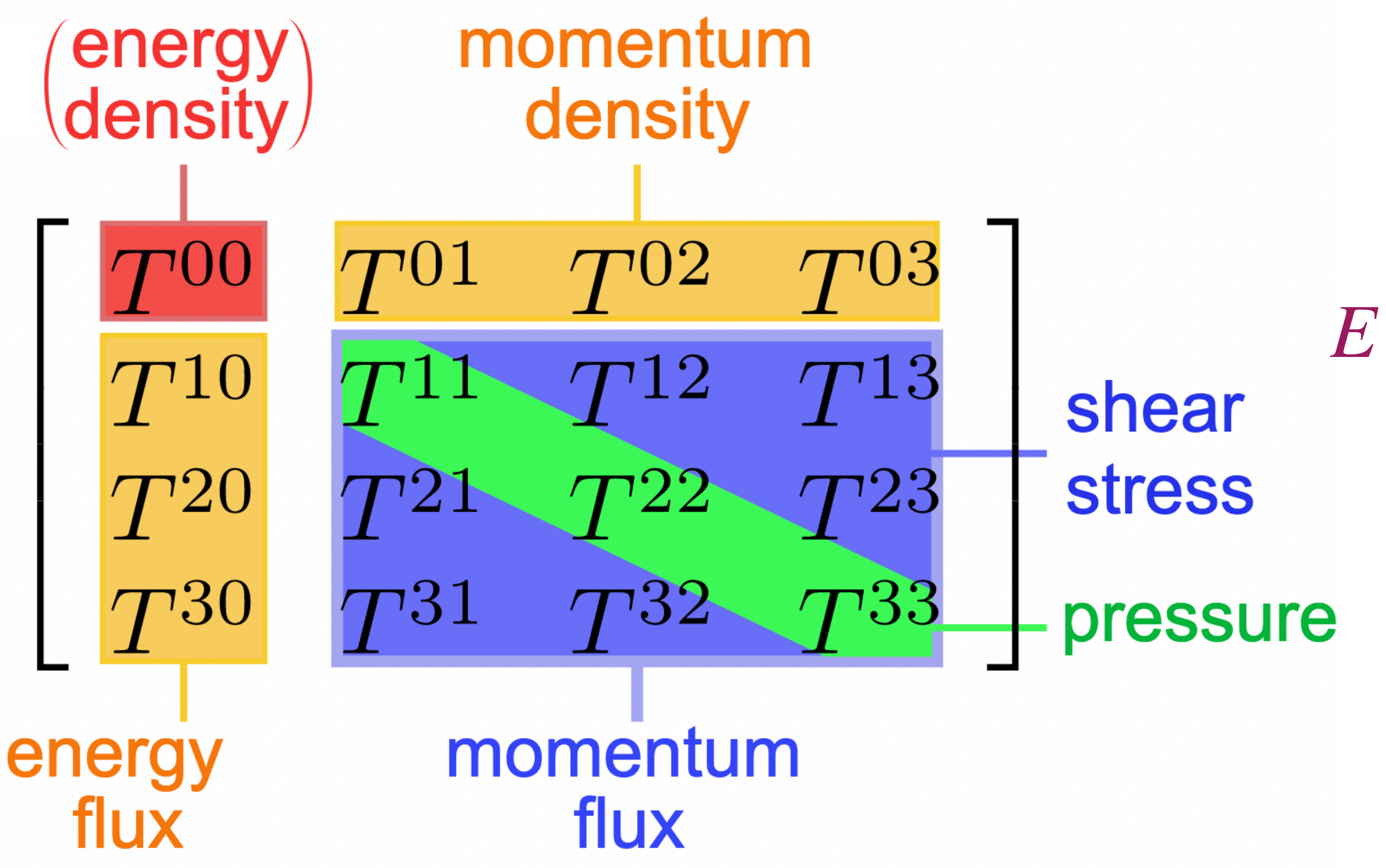
$$t_{static}^{00}(\mathbf{r}) = m \int \frac{d^3 q}{(2\pi)^3} A(-\mathbf{q}^2) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Mass distribution

$$N_{\phi,\infty} = \frac{1}{R} \int d^3\tilde{P} \tilde{P} \,|\, \tilde{\phi}(\,|\,\tilde{\mathbf{P}}\,|\,)\,|^2 = \langle E \rangle$$

for $R \to 0$ and $\mathbf{P} \sim 1/R$

the energy $E = \sqrt{m^2 + \mathbf{P}^2} \sim \dfrac{1}{R}$




for $m \to \infty$ $R \gg 1/m$,
$\mathbf{P} \sim 1/R \ll m$

$$E = \sqrt{m^2 + \mathbf{P}^2} \simeq m + O(\mathbf{P}^2/(2m))$$

# Pressure and shear force distributions

$$t^{ij}_{\phi}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{ij}(\mathbf{x},0) | \Phi, \mathbf{X}, s \rangle = t^{ij}_{\phi,0}(\mathbf{r}) + t^{ij}_{\phi,2}(\mathbf{r})$$

For sharply localised packet ($R \to 0$ and arbitrary $m$)

$$t^{ij}_{\phi,2}(\mathbf{r}) = \frac{1}{4} N_{\phi,0} \int \frac{d^2\hat{n}}{4\pi} \frac{d^3q}{(2\pi)^3} \left(q^i q^j - \delta^{ij}\mathbf{q}^2_{\perp}\right) D\left(-\mathbf{q}^2_{\perp}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

For static approximation ($m \to \infty, R \to 0$): $R \gg 1/m$

$$t^{ij}_{static,2}(\mathbf{r}) = \frac{1}{4m} \int \frac{d^3q}{(2\pi)^3} \left(-\mathbf{q}^2\delta^{ij} + q^i q^j\right) D\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

**pressure** / **shear force**

$$t^{ij}_2(r) = \delta^{ij} p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r)$$

$$p(\mathbf{r}) = \frac{N_{\phi,0}}{4} \int \frac{d^2\hat{n}}{4\pi} \left(\frac{1}{r^2_{\perp}} \frac{d}{dr_{\perp}} r^2_{\perp} \frac{d}{dr_{\perp}} - \frac{1}{3}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\right) \left(\delta(r_{\parallel}) \tilde{D}\left[\mathbf{r}_{\perp}\right]\right)$$

$$s(\mathbf{r}) = -\frac{N_{\phi,0}}{4} \int \frac{d^2\hat{n}}{4\pi} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \left(\delta(r_{\parallel}) \tilde{D}\left[\mathbf{r}_{\perp}\right]\right)$$

2D Fourier transformation

$$p_{static}(\mathbf{r}) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}\left[\mathbf{r}\right]$$

$$s_{static}(\mathbf{r}) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}\left[\mathbf{r}\right]$$

3D Fourier transformation

$$\langle r_E^2 \rangle = \frac{\int d^3r r^2 t_{00}^{static}(r)}{\int d^3r t_{00}^{static}(r)} = 6A'(0)$$

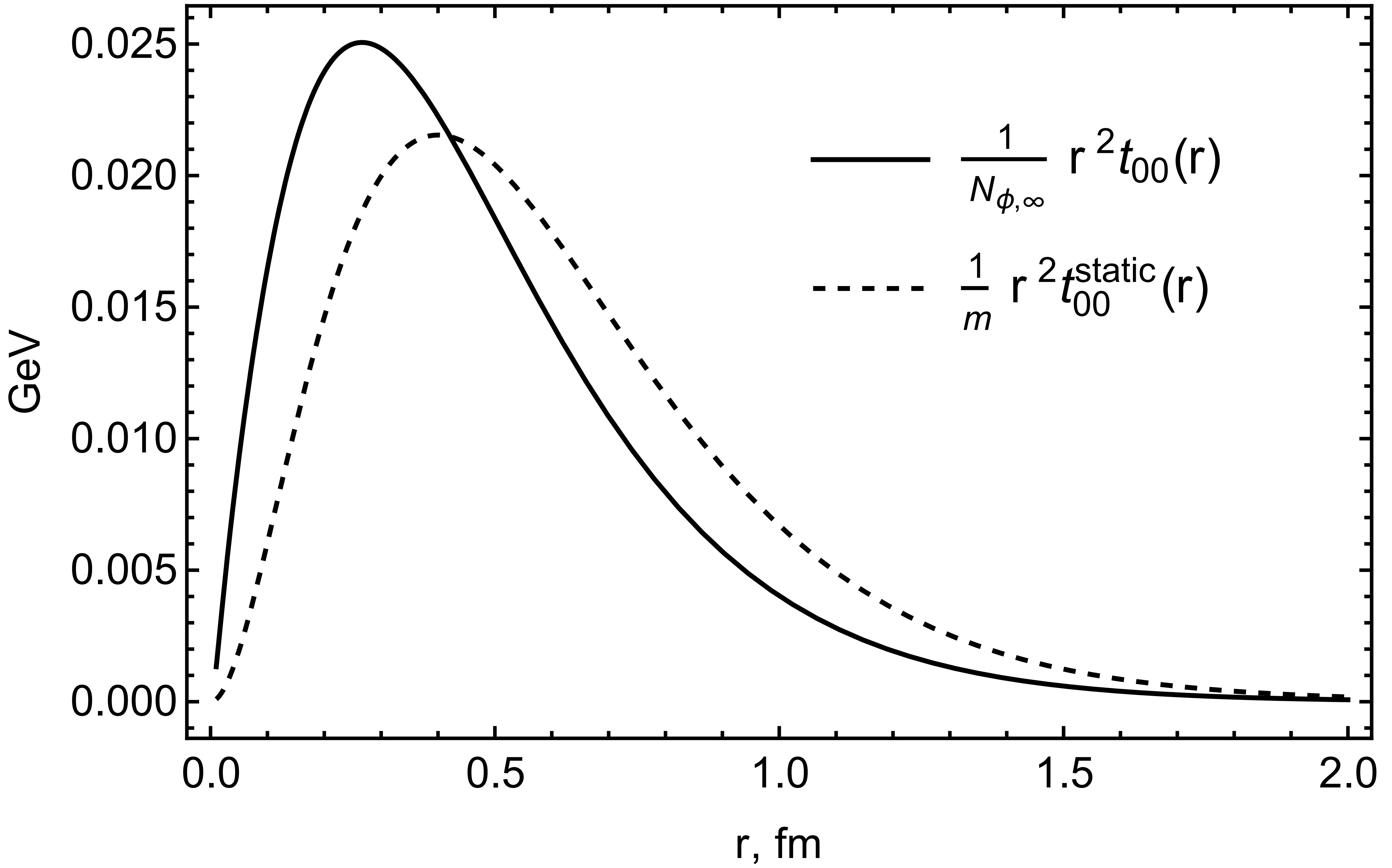
$$\langle r_E^2 \rangle = \frac{\int d^3r r^2 \ t_{00}(r)}{\int d^3r \ t_{00}(r)} = 4A'(0)$$

[G. A. Miller, Phys. Rev. C 99, no.3, 035202 (2019).]

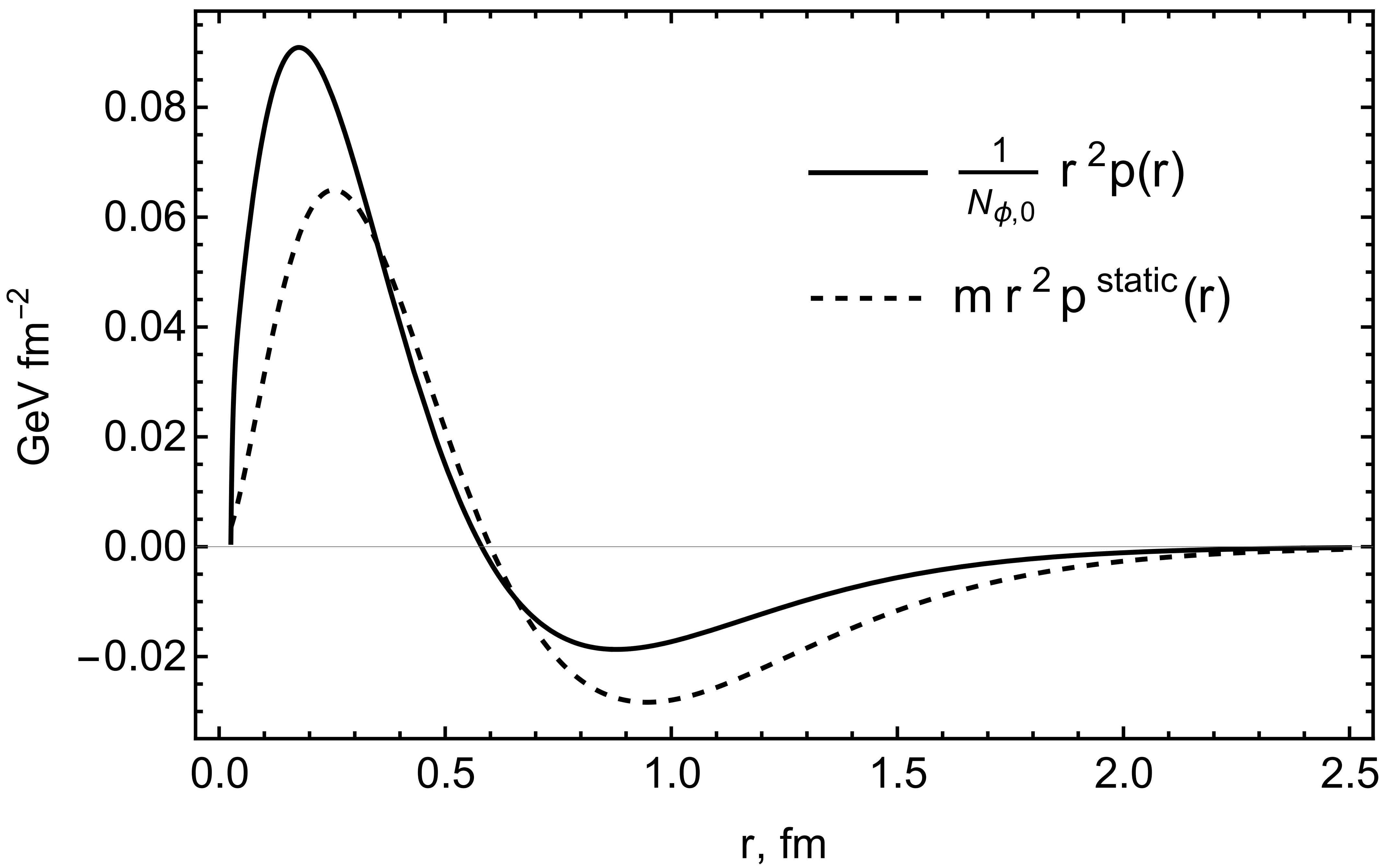
$$\langle r_{mech}^2 \rangle = \frac{\int d^3r r^2 F_n^{static}(r)}{\int d^3r F_n^{static}(r)} = \frac{6D}{\int_0^\infty dt D(t)}$$

[M.V. Polyakov, P. Schweitzer, Int.J.Mod.Phys.A 33 (2018)]

$$\langle r_{mech}^2 \rangle = \frac{\int d^3r r^2 F_n(r)}{\int d^3r F_n(r)} = \frac{6D}{\int_0^\infty dt \int_{-1}^{1} \frac{d\alpha}{2} D\left(t[1-\alpha^2]\right)}$$




*Comparison of energy densities*




*Comparison of pressure distributions*

# Densities from ChPT

[Alharazin, Djukanovic, Gegelia, Polyakov, *Phys.Rev.D* 102 (2020)]

$$
\begin{aligned}
S_{\pi\mathrm{N}} = & \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,\bar\Psi\, i e^\mu_a\gamma^a\nabla_\mu\Psi - \frac{1}{2}\,\nabla_\mu\bar\Psi\, i e^\mu_a\gamma^a\Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi e^\mu_a\gamma^a\gamma_5 u_\mu\Psi\right. \\
& + c_1\langle\chi_+\rangle\bar\Psi\Psi - \frac{c_2}{8m^2}g^{\mu\alpha}g^{\nu\beta}\langle u_\mu u_\nu\rangle\left(\bar\Psi\left\{\nabla_\alpha,\nabla_\beta\right\}\Psi + \left\{\nabla_\alpha,\nabla_\beta\right\}\bar\Psi\Psi\right) + \frac{c_3}{2}\,g^{\mu\nu}\langle u_\mu u_\nu\rangle\bar\Psi\Psi \\
& + \frac{ic_4}{4}\,\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\,\left[u_\mu,u_\nu\right]\Psi + c_5\bar\Psi\hat\chi_+\Psi + \frac{c_6}{8m}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}F^+_{\mu\nu}\Psi + \frac{c_7}{8m}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\langle F^+_{\mu\nu}\rangle\Psi \\
& \left. + \frac{c_8}{8}\,R\bar\Psi\Psi + \frac{ic_9}{m}\,R^{\mu\nu}\left(\bar\Psi e^a_\mu\gamma_a\nabla_\nu\Psi - \nabla_\nu\bar\Psi e^a_\mu\gamma_a\Psi\right)\right\},
\end{aligned}
$$

**known part**

[Fettes, Meißner, Mojzis, Steininger, Ann. Phys. (N.Y.) 283, 273 (2000)]

**new LECs!**

## Gravitational form factors

$$
\begin{aligned}
A(t) &= 1 - \frac{2c_9}{m_N}\,t + \frac{3g_A^2}{512F^2m_N}(-t)^{\frac{3}{2}} - \frac{\left(c_2m_N - 10g_A^2\right)}{320\pi^2F^2m_N^2}\,t^2\ln\left(\frac{-t}{m_N^2}\right) - \frac{\left(25g_A^2\left(12c_9m_N-7\right)-62c_2m_N\right)}{9600\pi^2F^2m_N^2}t^2 + O(t^{\frac{5}{2}})\,, \\
J(t) &= \frac{1}{2} - \frac{c_9}{m_N}\,t - \frac{g_A^2}{64\pi^2F^2}\,t\ln\left(\frac{-t}{m_N^2}\right) + \frac{g_A^2\left(12c_9m_N-7\right)}{192\pi^2F^2}\,t - \frac{3g_A^2}{512F^2m_N}(-t)^{\frac{3}{2}} + O(t^2)\,, \\
D(t) &= m_Nc_8 + \frac{3g_A^2m_N}{128F^2}\sqrt{-t} - \frac{\left(5g_A^2+4\left(c_2+5c_3\right)m_N\right)}{160\pi^2F^2}\,t\ln\left(\frac{-t}{m_N^2}\right) \\
&+ \frac{\left(5g_A^2\left(40c_9m_N+15c_8m_N+28\right)+94c_2m_N+200c_3m_N\right)}{2400\pi^2F^2}\,t + O(t^{\frac{3}{2}})\,.
\end{aligned}
$$

$$\rho_E(r) = \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{3\left(10g_A^2/m_N + (c_2+10c_3)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right),$$

$$\rho_J(r) = \frac{5g_A^2}{64\pi^3F^2}\frac{1}{r^5} - \frac{9g_A^2}{64\pi^2F^2m_N}\frac{1}{r^6} + O\left(\frac{1}{r^7}\right),$$

$$p(r) = -\frac{3g_A^2}{64\pi^2F^2}\frac{1}{r^6} + \frac{\left(5g_A^2/m_N + 4(c_2+5c_3)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right),$$

$$s(r) = \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{21\left(5g_A^2/m_N + 4(c_2+5c_3)\right)}{128\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right).$$

**Long-range behaviour of densities via the traditional definition**

[Alharazin, Djukanovic, Gegelia, Polyakov, *Phys.Rev.D* 102 (2020)]

$$\rho_E(r) = N_{\phi,R}\left(\frac{27g_A^2}{512F_\pi^2m_N}\frac{1}{r^6} - \frac{g_{\pi N\Delta}^2(79\delta+10m_N)}{45\pi^2F_\pi^2m_N^2\delta}\frac{1}{r^7} + \frac{2(c_2m_N-10g_A^2)}{5\pi^2F_\pi^2m_N^2}\frac{1}{r^7}\right) + \mathcal{O}\left(\frac{1}{r^8}\right),$$

$$\rho_J(r) = N_{\phi,R}\left(\frac{5g_A^2}{16\pi^2m_NF_\pi^2}\frac{1}{r^5} - \frac{81g_A^2}{512F_\pi^2m_N^2}\frac{1}{r^6} - \frac{7g_{\pi N\Delta}^2(2\delta+m_N)}{9F_\pi^2\pi^2\delta^2m_N^2}\frac{1}{r^7}\right) + \mathcal{O}\left(\frac{1}{r^8}\right),$$

$$s(r) = N_{\phi,R,2}\left(\frac{9g_A^2m_N}{32F_\pi^2}\frac{1}{r^6} + \frac{7g_{\pi N\Delta}^2(70m_N-107\delta)}{72\pi^2F_\pi^2\delta}\frac{1}{r^7} - \frac{7\left(5g_A^2+4(c_2+5c_3)m_N\right)}{8\pi^2F_\pi^2}\frac{1}{r^7}\right) + \mathcal{O}\left(\frac{1}{r^8}\right),$$

$$p(r) = -N_{\phi,R,2}\left(\frac{15g_A^2m_N}{256F_\pi^2}\frac{1}{r^6} + \frac{7g_{\pi N\Delta}^2(70m_N-107\delta)}{270\pi^2F_\pi^2\delta}\frac{1}{r^7} - \frac{7\left(5g_A^2+4(c_2+5c_3)m_N\right)}{30\pi^2F_\pi^2}\frac{1}{r^7}\right) + \mathcal{O}\left(\frac{1}{r^8}\right).$$

**Long-range behaviour of densities via the new definition**

[Alharazin, *Phys.Rev.D* 102 (2020)]

- **Form factors contain information about the internal structure of particles**

- **The definition of spatial densities is important for studying the structure of particles**

- **The sharp localisation approach suggests definition of 3D local densities and it is valid for any system independent of mass**

# Thank you for your attention!

# Mechanical properties of hadrons

D-term via the static approximation

$$D \equiv D(0) = -\frac{m}{2}\int d^3r \left(r^i r^j - \frac{1}{3}\delta_{ij}\right) t_{ij}^{static}(r) = m\int d^3r r^2 p_{static}(r) = -\frac{4}{15}m\int d^3r r^2 s_{static}(r)$$

[M.V.Polyakov, Phys. Lett.B 555, 57 (2003)]

D-term via the sharp localisation

$$D = -\frac{4}{15\ N_{\phi,0}}\int d^3r r^2 s(r)$$

$$\partial_i T_{ij}(r) = 0$$

[Laue, Ann. Phys. 340, 524 (1911)]

$$\int d^3r\ p(r) = 0$$

*the von Laue stability condition*

$$\frac{2}{3}s'(r) + p'(r) + \frac{2}{r}s(r) = 0$$

*equilibrium equation*

$$F^i(r) = T^{ij}(r)dS\ n^j = \left(\frac{2}{3}s(r) + p(r)\right) dS^i$$

*the normal forces*

$$\frac{2}{3}s(r) + p(r) > 0$$

*local stability condition*

$$D < 0$$

[Perevalova, Polyakov, Schweitzer, Phys. Rev. D 94, 054024 (2016)]

**Suspicion**

*Local stability condition is applicable only for system with short-range forces*

**Details in**

[Varma and Schweitzer, Phys. Rev. D 102, 014047] (2020)
[Metz, Pasquini, Rodini, Phys. ReV. D 102, 114042] (2020)
[Gegelia and Polyakov, Phys.Lett.B 820, 136572] (2021)
[Varma and Schweitzer, Rev. Mex. Fis. Suppl. 3] (2022)

$$\partial_i T_{ij}(r) = f_j(r)$$

[Landau, Lifshitz, vol. VII ]

$$\int d^3r\ p(r) = -\frac{1}{3}\int d^3r\ r\ f(r)$$

*the modified von Laue stability condition*

$$\frac{2}{3}s'(r) + p'(r) + \frac{2}{r}s(r) = f(r)$$

*equilibrium equation*

$$F^i(r) = (T^{ij}(r) + \sigma(r)\delta^{ij})dS\ n_j = \left(\frac{2}{3}s(r) + p(r) + \sigma(r)\right) dS^i$$

*the normal forces*

$$\frac{2}{3}s(r) + p(r) + \sigma(r) > 0$$

*modified local stability condition*

$$\sigma(r) = \int_r^\infty dx\ f(x)$$

[Panteleeva, Phys.Rev.D 107, 05501 (2023)]

The time-dependent EMT in sharp localisation approach is not conserved $\partial_i T^{ij} = -\partial_0 T^{0j}$

[Alharazin, Sun, Epelbaum, Gegelia, Meißner [*JHEP* 02 163] (2023)]

$$f(r) = -\frac{N_0}{4}\int \frac{d^2n}{4\pi}\frac{d^3q}{(2\pi)^3} D(-\mathbf{q}_\perp^2) e^{-i(\mathbf{q}\cdot\mathbf{r})}\frac{i(\mathbf{q}\cdot\mathbf{r})}{r} q_\parallel^2$$

"external force" for the spin-1/2 system

D-term via the sharp localisation

$$D = \frac{1}{N_{\phi,0}}\int d^3r r^2 (p(r) + \sigma(r))$$