Electromagnetic form factors of nucleons from SU(3) chiral effective field theory

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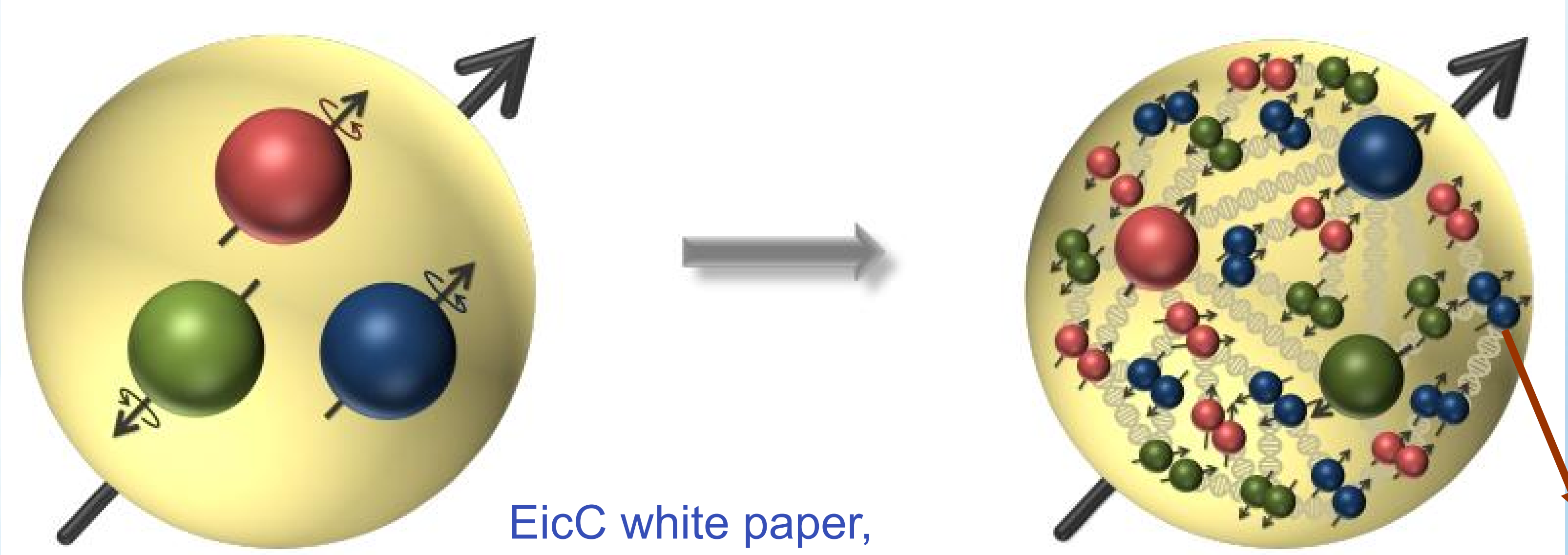
HUNAN UNIVERSITY

Outline

1. **Introduction**
2. **NN scattering**
3. **Application:** EMFFs of nucleons
4. **Summary**

Baryon

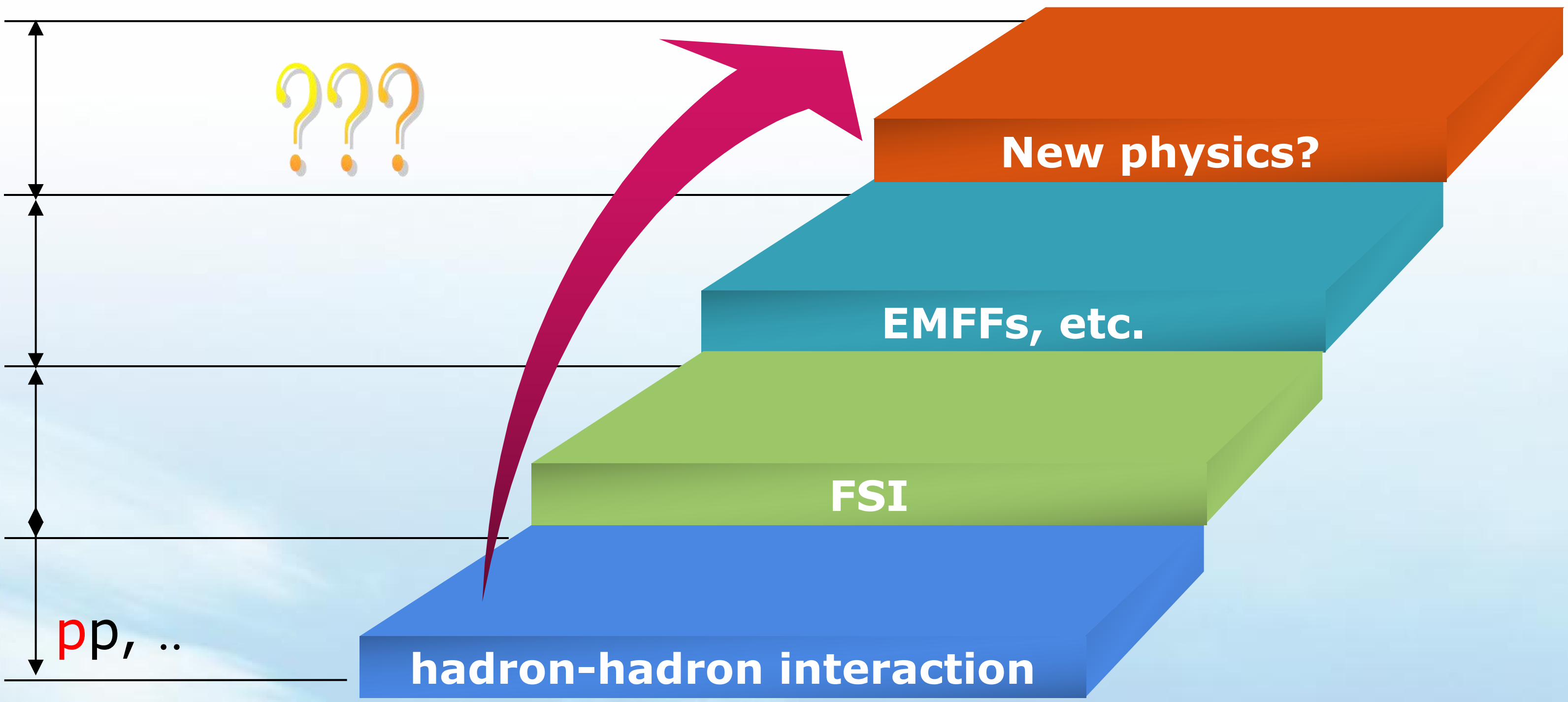
- Baryon inner structure?
- EIC, EicC: 3D structure of proton?
- Mass, spin,radius?
- EMFF: the inner structure of baryons?
- Threshold enhancement? pp, ΛΛ, ΣΣ, $\Lambda_c\Lambda_c$, ΞΞ(?)...



EicC white paper,
Front.Phys.16 (2021) 6, 64701

Strategy

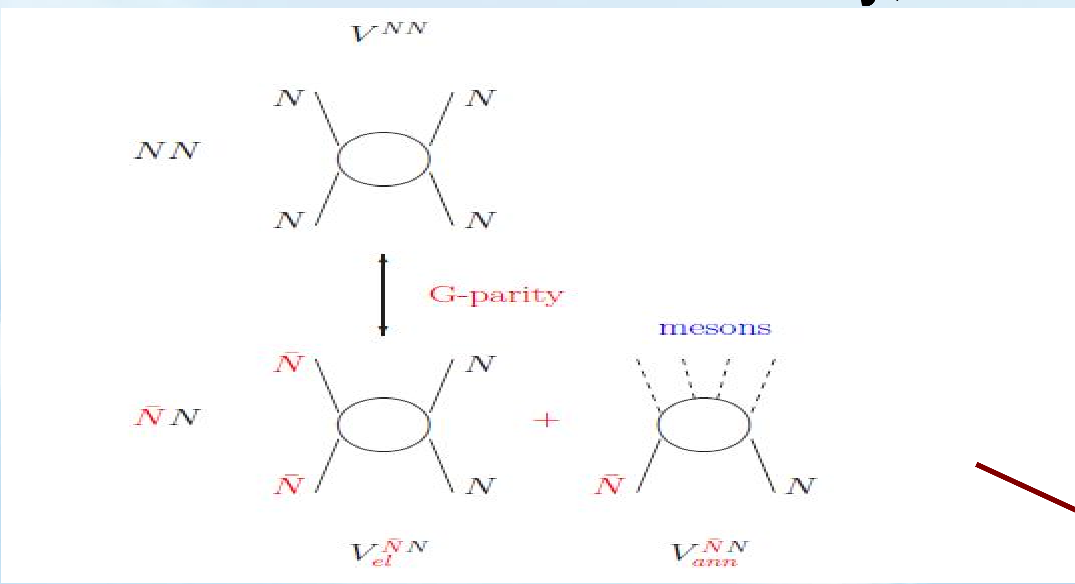
- New insights in strong interactions?



2. NN scattering amplitudes

- **SU(2) NN scattering amplitude**
  - elastic NN scattering:
    - pion(s) exchange: NN Chiral EFT+G-parity
    - LECs of contact term: to be fixed by data

    E.Epelbaum *et.al.*, EPJA51 (2015) , 53
  - annihilation: unitarity, fit to the data

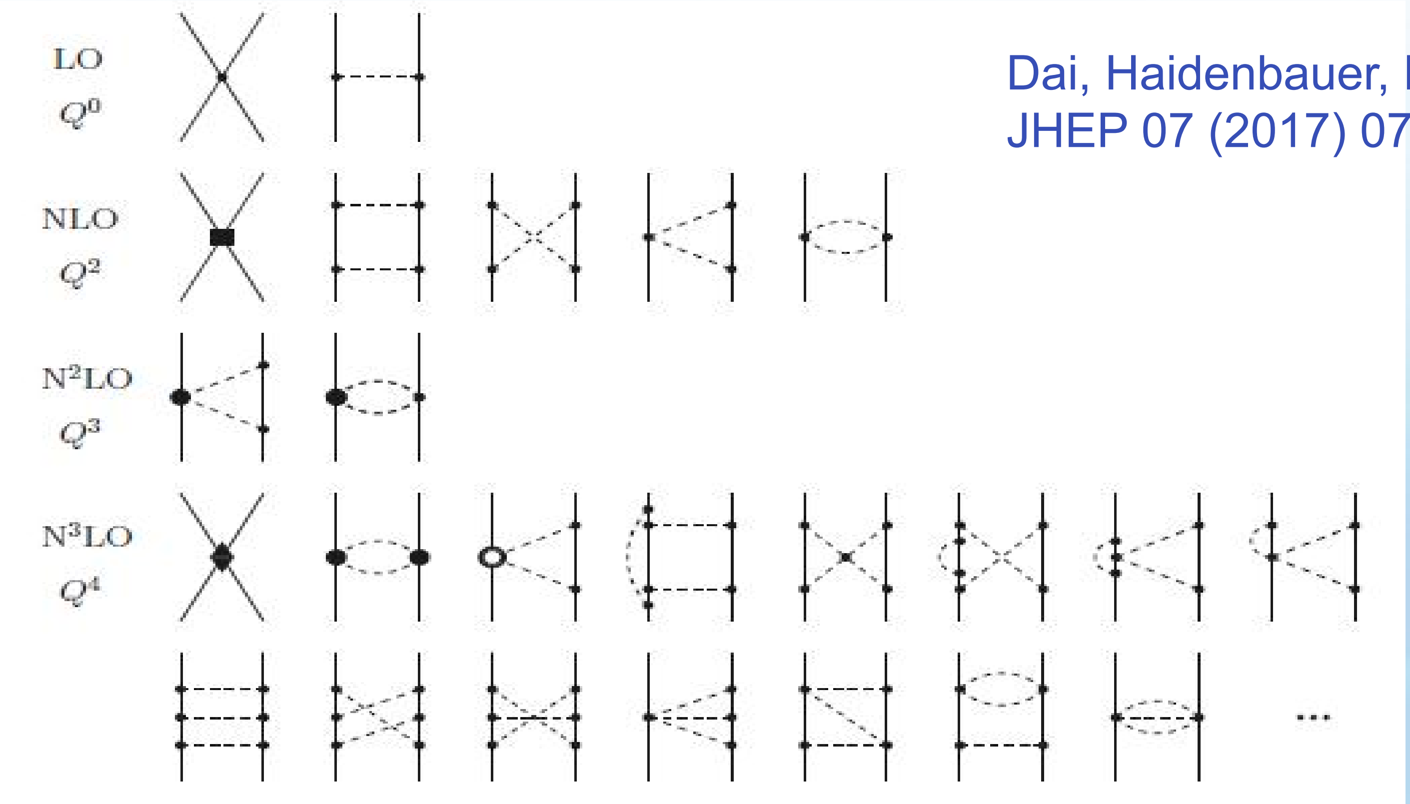


$$V^{NN} = V_{1\pi} + V_{2\pi} + V_{3\pi} + ... + V_{cont}$$
$$V_{el}^{\bar{N}N} = -V_{1\pi} + V_{2\pi} - V_{3\pi} + ... + V_{cont}$$
$$V_{ann}^{\bar{N}N} = \sum_X V^{\bar{N}N \to X}$$

J.Haidenbauer, talk at Bochum

ChEFT

- Up to N³LO, in time ordered ChEFT:
  - only irreducible diagrams contributes
  - Lippmann-Shwinger equation



Dai, Haidenbauer, Meissner, JHEP 07 (2017) 078

ChEFT: potentials

- pion(s) exchange potentials:

$$V_{1\pi}(q) = \left(\frac{g_A}{2F_\pi}\right)^2 \left(1 - \frac{p^2 + p'^2}{2m^2}\right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{\mathbf{q}^2 + M_\pi^2}$$

$$\begin{aligned} V_{2\pi} &= V_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_C + [V_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_S]\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + [V_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_T]\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} \\ &\quad + [V_{LS} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_{LS}]\, i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}), \end{aligned}$$

- Fourier transformation: change it into corordinat space to do regularization

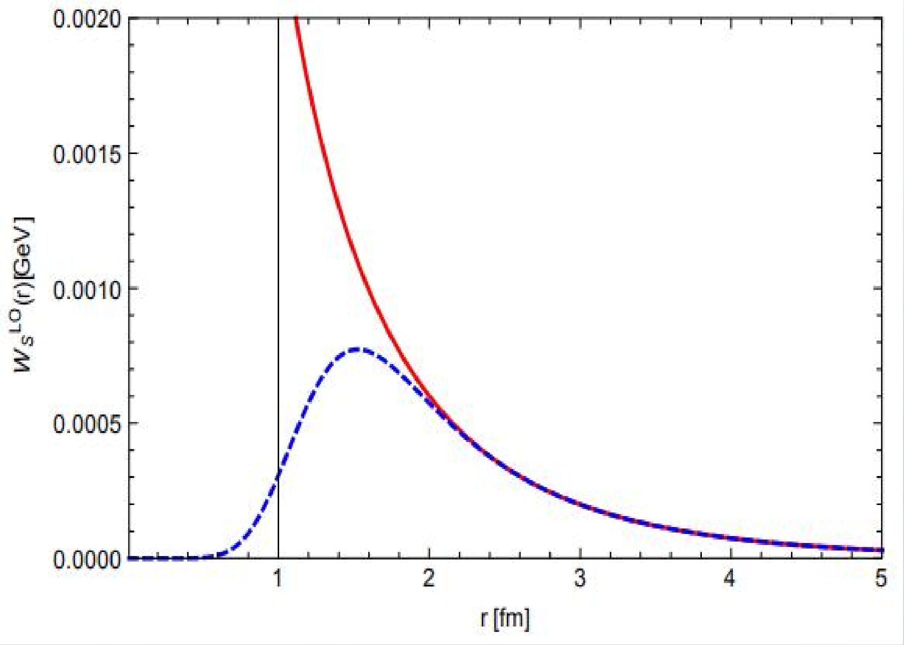
$$V_C(q) = 4\pi \int_0^\infty f(r)\, V_C(r) j_0(qr) r^2 dr\,,$$

$$V_S(q) = 4\pi \int_0^\infty f(r)\, \left(V_S(r) j_0(qr) + \tilde{V}_T(r) j_2(qr)\right) r^2 dr\,,$$

$$V_T(q) = -\frac{12\pi}{q^2} \int_0^\infty f(r)\, \tilde{V}_T(r) j_2(qr) r^2 dr\,,$$

$$V_{SL}(q) = \frac{4\pi}{q} \int_0^\infty f(r)\, V_{LS}(r) j_1(qr) r^3 dr\,.$$

$$f(r) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n .$$



ChEFT: potentials

- Contact terms: short distance

$$V(^1S_0) = \bar{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) + D^1{}_{^1S_0} p^2 p'^2 + D^2{}_{^1S_0}(p^4 + p'^4)\,,$$
$$V(^3S_1) = \bar{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) + D^1{}_{^3S_1} p^2 p'^2 + D^2{}_{^3S_1}(p^4 + p'^4)\,,$$
$$V(^1P_1) = C_{^1P_1}\, p p' + D_{^1P_1}\, p p'(p^2 + p'^2)\,,$$
$$V(^3P_1) = C_{^3P_1}\, p p' + D_{^3P_1}\, p p'(p^2 + p'^2)\,,$$
$$V(^3P_0) = C_{^3P_0}\, p p' + D_{^3P_0}\, p p'(p^2 + p'^2)\,,$$
$$V(^3P_2) = C_{^3P_2}\, p p' + D_{^3P_2}\, p p'(p^2 + p'^2)\,,$$
$$V(^3D_1 - {}^3S_1) = C_{\epsilon_1}\, p'^2 + D^1{}_{\epsilon_1} p^2 p'^2 + D^2{}_{\epsilon_1} p'^4\,,$$
$$V(^3S_1 - {}^3D_1) = C_{\epsilon_1}\, p^2 + D^1{}_{\epsilon_1} p^2 p'^2 + D^2{}_{\epsilon_1} p^4\,,$$

- Non-local regularization

$$f(p', p) = \exp\left(-\frac{p'^m + p^m}{\Lambda^m}\right)$$

- Annihilation terms: short distance physics, around 1 fm or less

$$V_{\rm ann} = V_{\bar{N}N\to X} G_X V_{X\to\bar{N}N}$$

the same form as that of contact terms

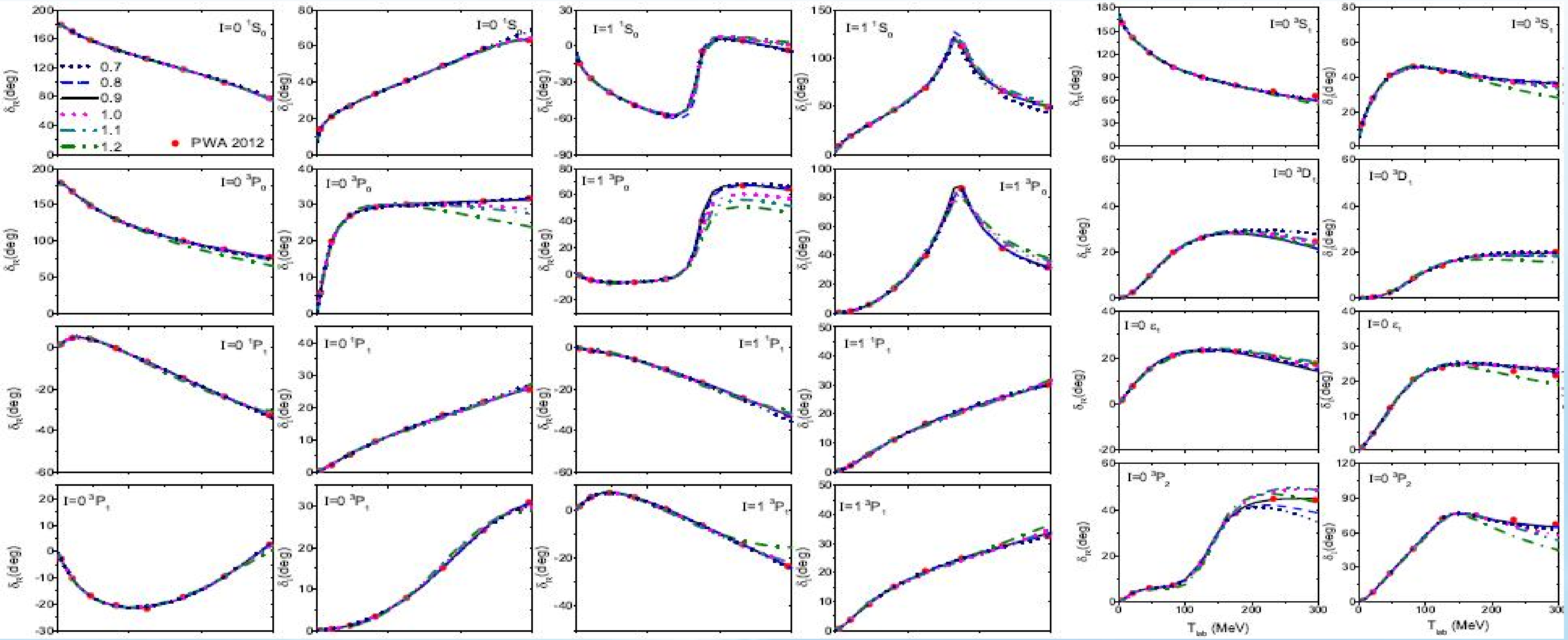
Ignore the transition between annihilation channels

Phase shifts of different cutoff

- LS equation to solve amplitrudes

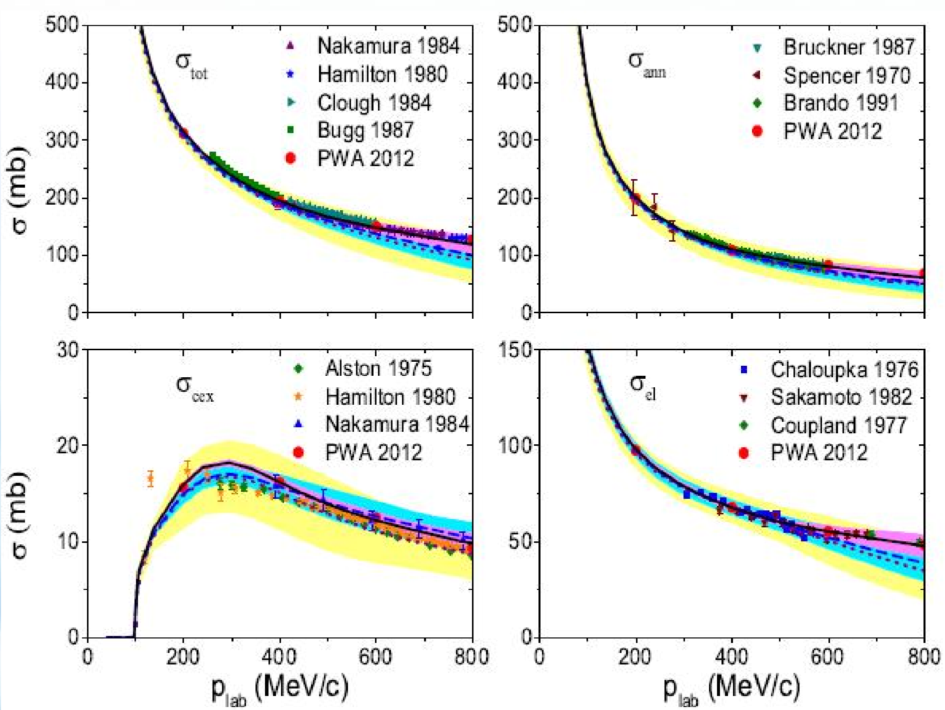
$$T_{L''L'}(p'',p';E_k) = V_{L''L'}(p'',p') + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,p';E_k)$$

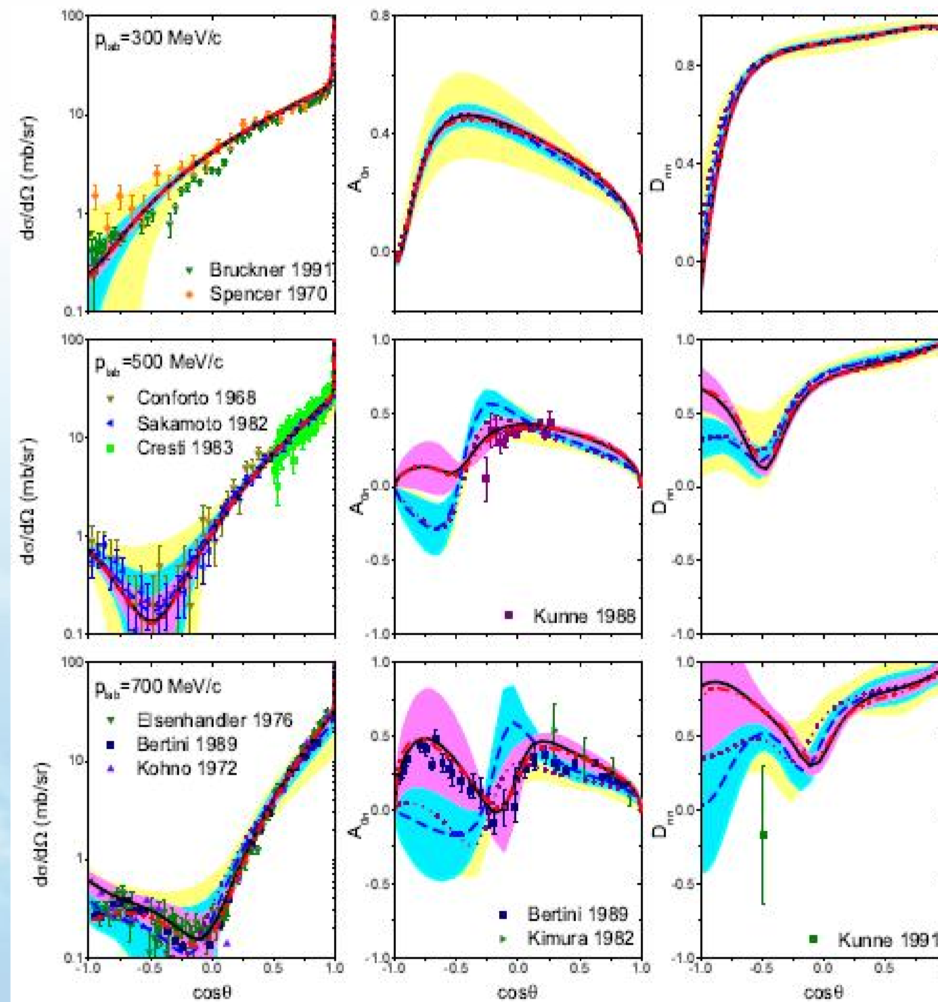
- Lowest partial waves are perfect up to 300 MeV



Observables

- Cross sections
- Angular distributions

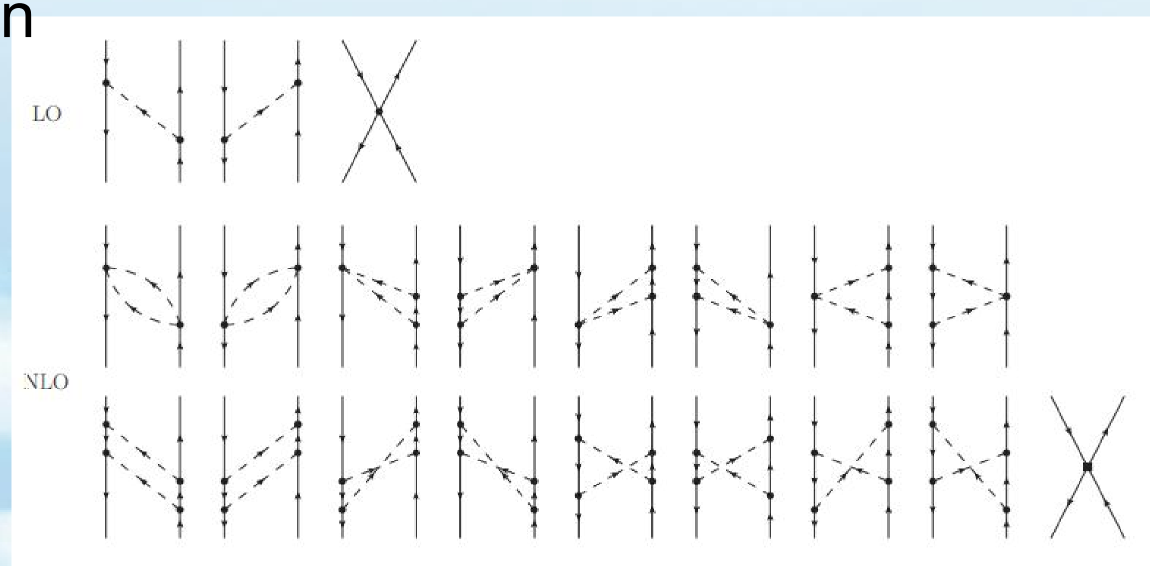




Why SU(3) ChEFT

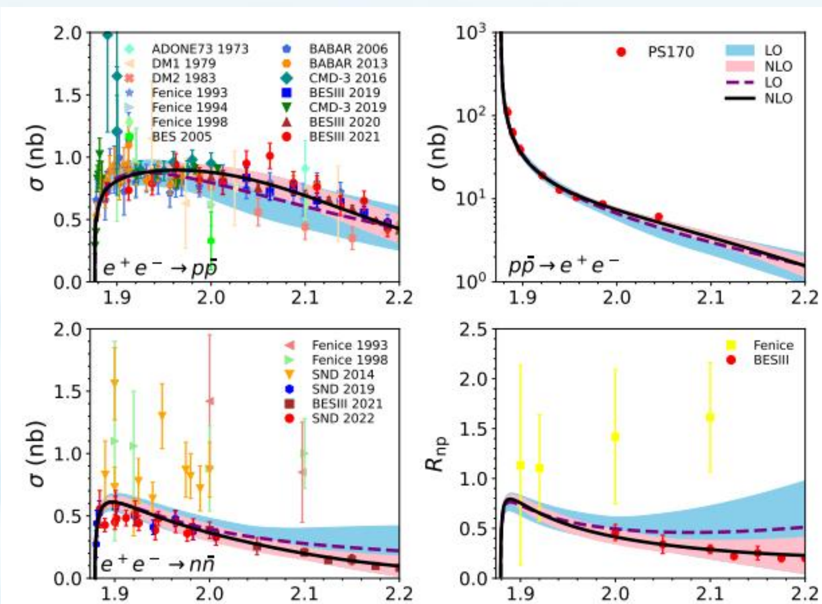
- SU(2): so far, so good, but
  - only pion exchanges
  - only works for nucleons
- SU(3) G-parity transformation is not OK as kaon does not have definitive G-parity
  - Direct calculation of BB scattering
  - Solving LS equation

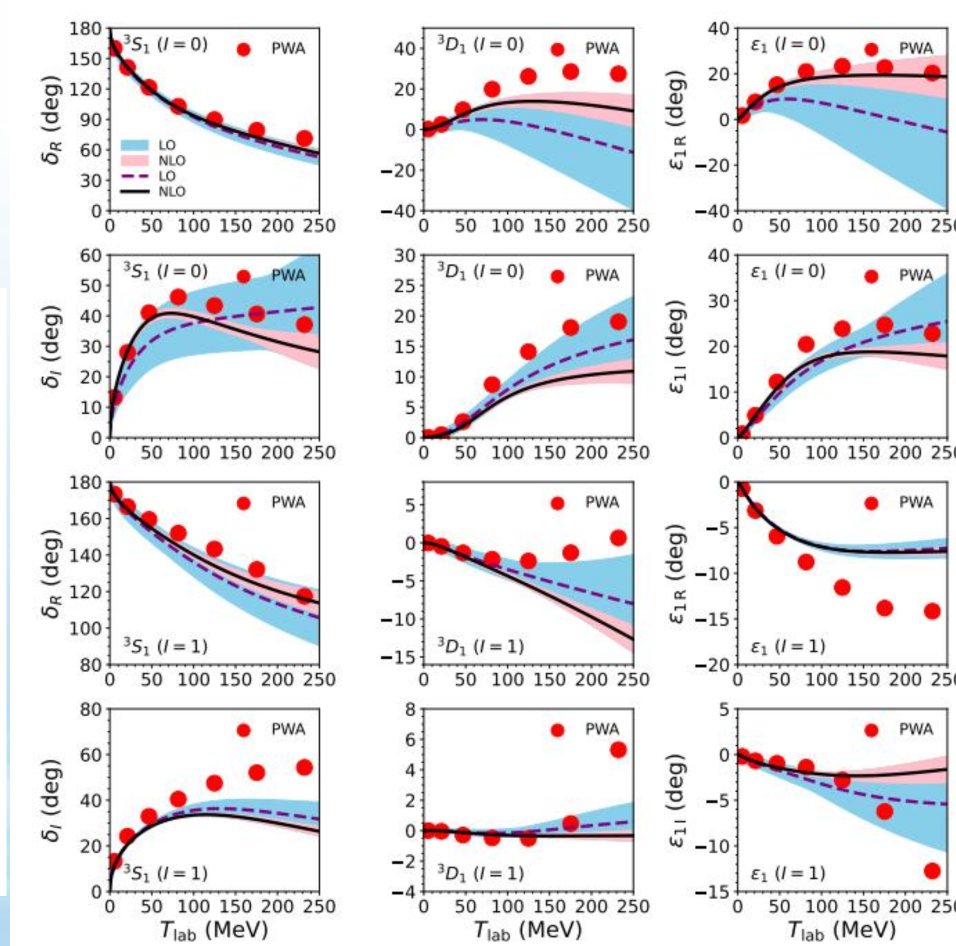
$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$



SU(3) ChEFT

- Fit results
- Phase shifts
- Cross sections
- differential cross sections
- ratios, etc.



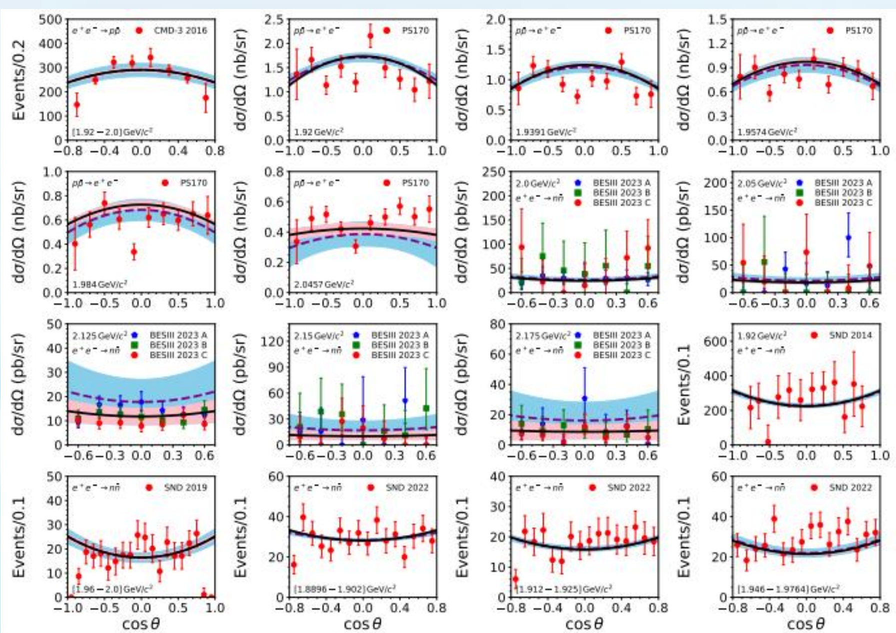


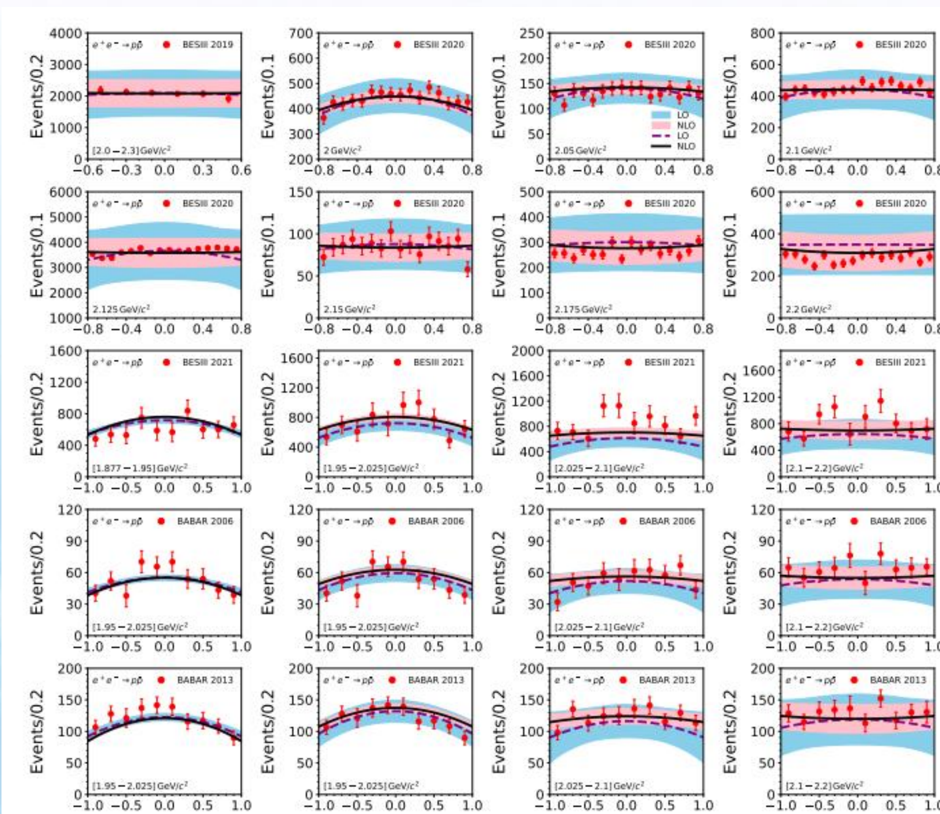
Yang, Guo, Li, Dai*, Haidenbauer, Meissner, JHEP08 (2024) 208

SU(3) ChEFT

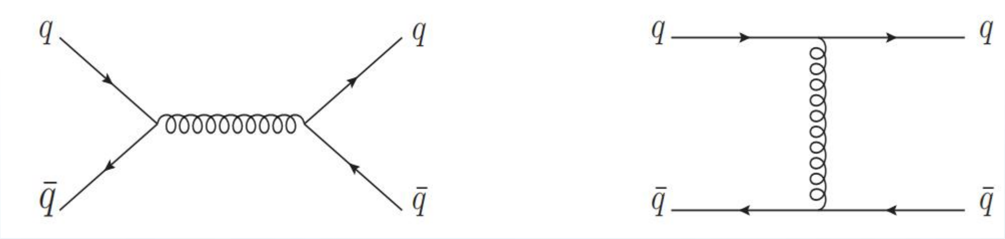
- Angular distributions also help to fix partial wave amplitudes

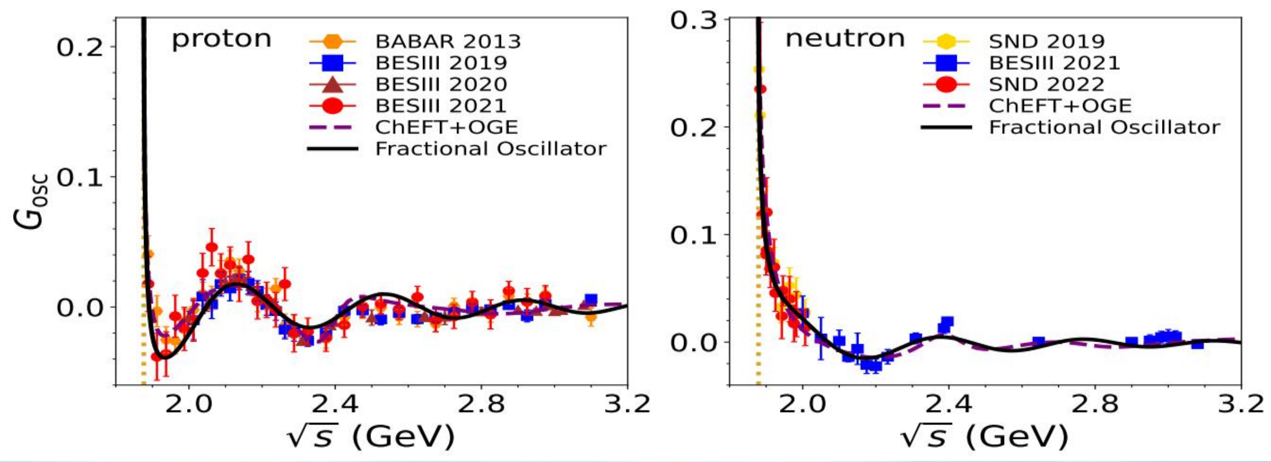
	LO		NLO					
	N	χ^2/N	N	χ^2/N				
Λ (MeV)		850		750	800	850	900	950
Cross Section	105	1.59	154	1.70	1.65	1.58	1.53	1.48
Differential cross section	221	1.31	477	1.59	1.57	1.53	1.49	1.47
R_{np}	1	0.20	7	0.38	0.62	0.99	1.41	1.76
$\|G_E/G_M\|$, $\|G_E\|$ and $\|G_M\|$	13	0.54	44	1.74	1.70	1.60	1.42	1.22
Phase shift	24	0.008	36	0.003	0.004	0.004	0.005	0.006
Scattering length	4	1.41	4	0.86	0.92	0.93	0.87	0.84
total	368	1.28	722	1.53	1.50	1.46	1.42	1.38





ChEFT+OGE?



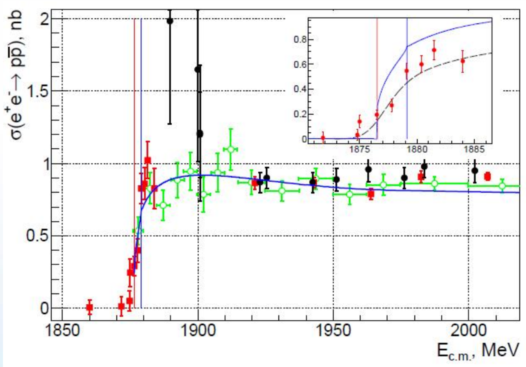


- Consider one-gluon exchange potential in the high energy region
- It can reproduce the fractional oscillations
- An efficient way to describe the strong interaction in both low energy region and high energy region

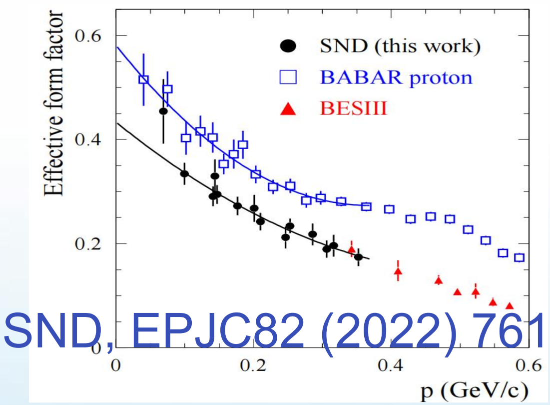
Yang, Guo, Dai*, Haidenbauer, Kang, Meissner, Sci.Bull. 68 (2023) 2729;

3. Application：EMFFs of nucleons

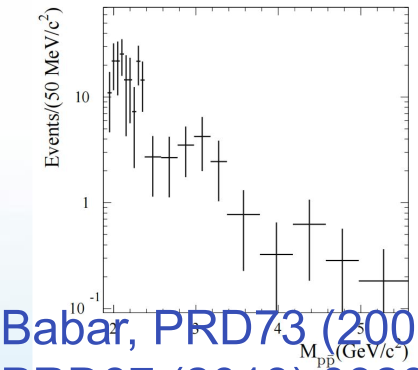
- CMD-3 has excellent measurement in low energy region
- BESIII's high statistics’ measurements on nucleon EMFFs



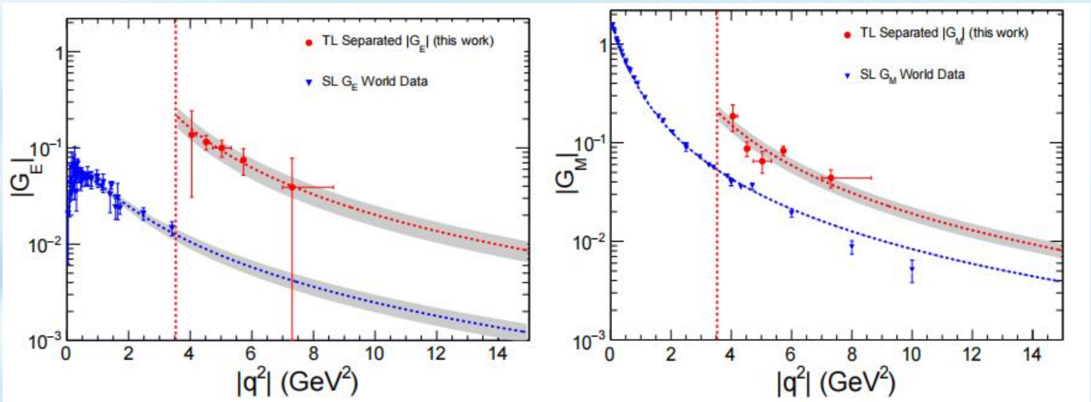
CMD-3, PLB05 (2019) 032



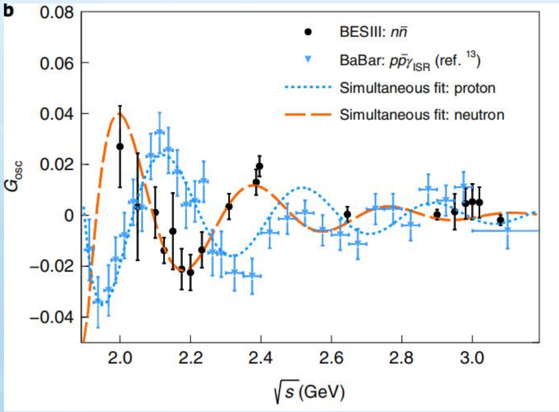
SND, EPJC82 (2022) 761



Babar, PRD73 (2006) 012005
PRD87 (2013) 092005



BESIII: PRL 130 (2023) 15, 151905



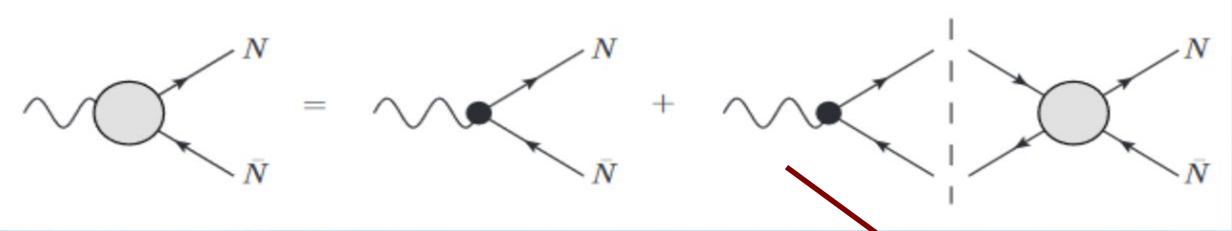
BESIII: PRD 99 (2019) 092002;
PRL 124 (2020) 4, 042001,
Nature Phys.17 (2021) 1200

FSI

- To analyze ee→NN, we need to consider FSI
- Distorted-wave Born approximation (DWBA):

$$f_{L'}(k;E_k) = f^0_{L'}(k) + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} f^0_L(p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,k;E_k)$$

$$f_0^{\bar{N}N} = G_M + \frac{M_N}{\sqrt{s}} G_E, \quad f_2^{\bar{N}N} = \frac{1}{2}\left(G_M - \frac{2M_N}{\sqrt{s}} G_E\right)$$



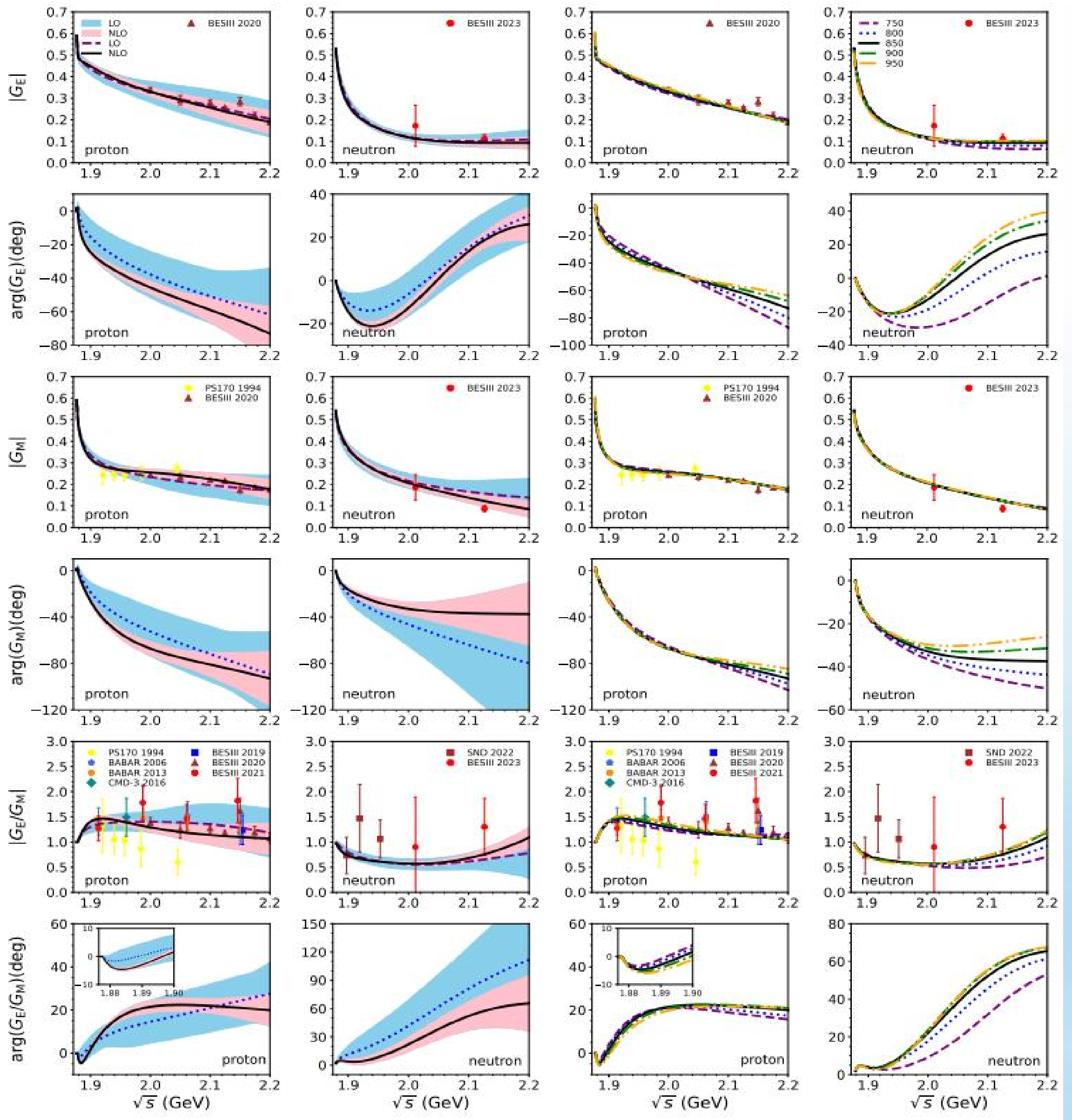
SU(3) ChEFT: Yang, Guo, Dai*, Haidenbauer, Kang, Meissner, Sci.Bull. 68 (2023) 2729;

- Vector meson dominance: $^3S_1$-$^3D_1$

SU(2)ChEFT: J.Haidenbauer, X.-W. Kang, U.-G. Meißner, NPA 929 (2014) , PRD91 (2015) 074003.

Individual EMFFs of nucleons

- Modulus: $|G_E|=|G_M|$at threshold, and will restore in 2.2 GeV
- Phases:
  - An overall phase is unobservable
  - relative phase chenges rapidly near threshold



Oscillation

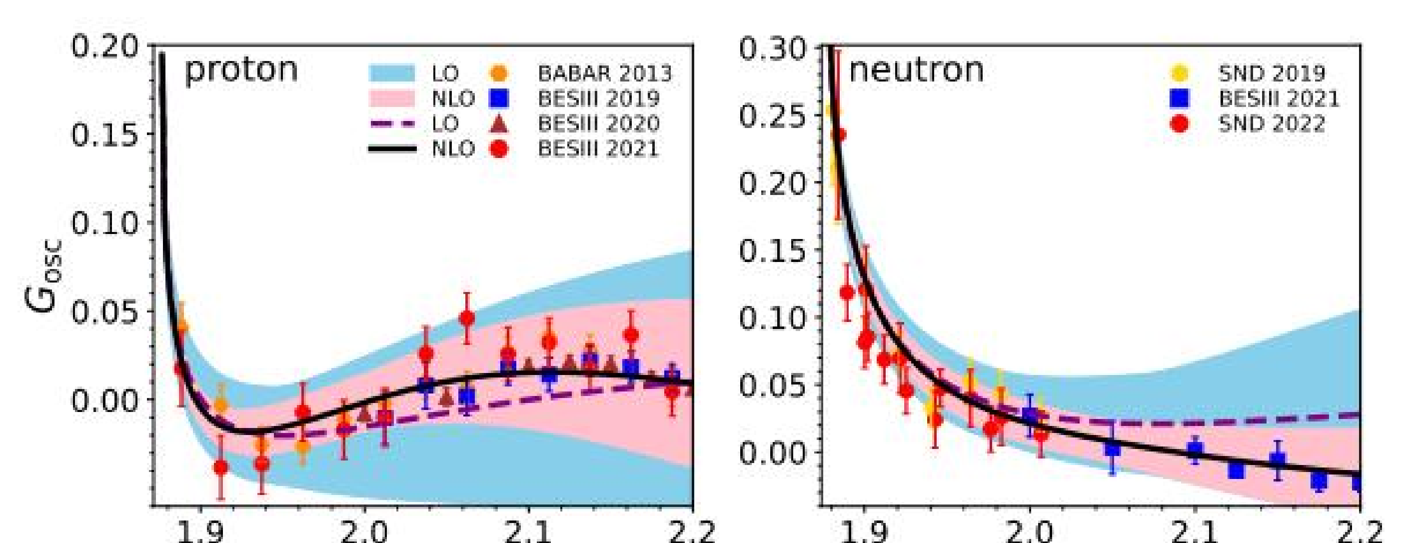
- Effective EMFFs

$$|G_{\rm eff}(s)| = \sqrt{\frac{\sigma_{e^+e^-\to \bar{N}N}(s)}{\frac{4\pi\alpha^2\beta}{3s}C(s)[1+\frac{2M_N^2}{s}]}}$$

- Subtracted form factors: oscillation

A. Bianconi & E. Tomasi-Gustafsson, PRL114 (2015) 232301; PRC103 (2021) 035203

$$G_{\rm osc}(s) = |G_{\rm eff}| - G_D(s), \quad G_D^p(s) = \frac{\mathcal{A}_p}{(1+s/m_a^2)[1-s/q_0^2]^2}, \quad G_D^n(s) = \frac{\mathcal{A}_n}{[1-s/q_0^2]^2}$$



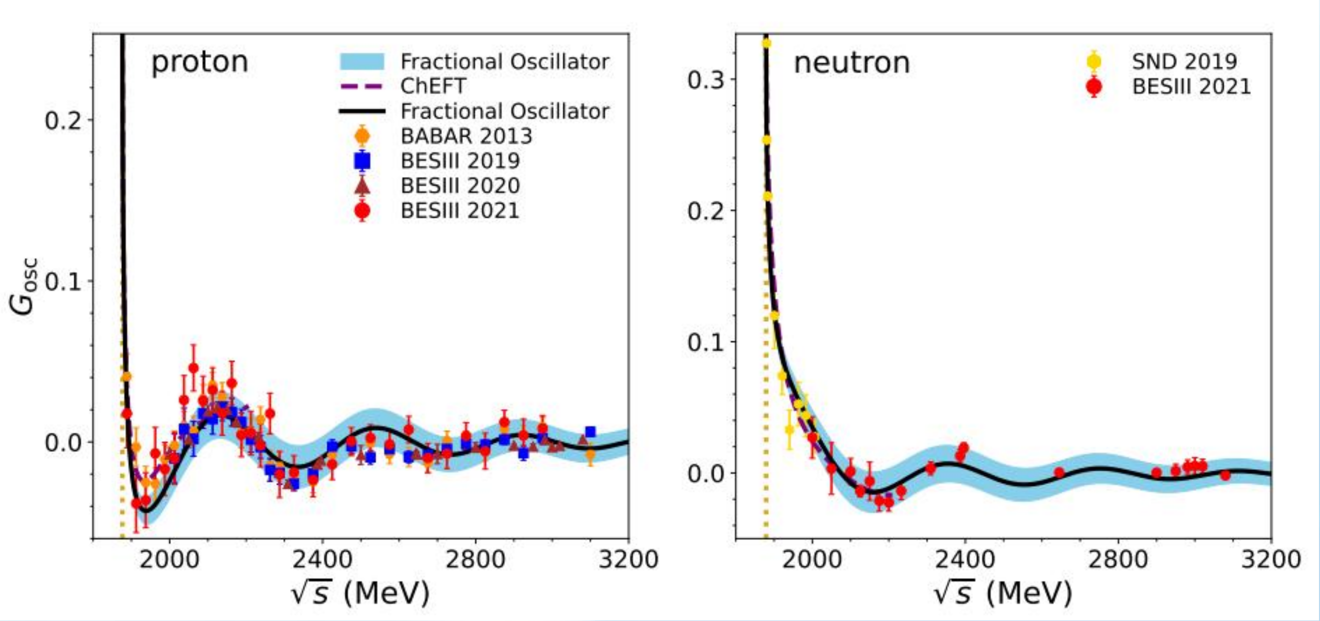
Oscillation

- We propose a fractional oscillation model

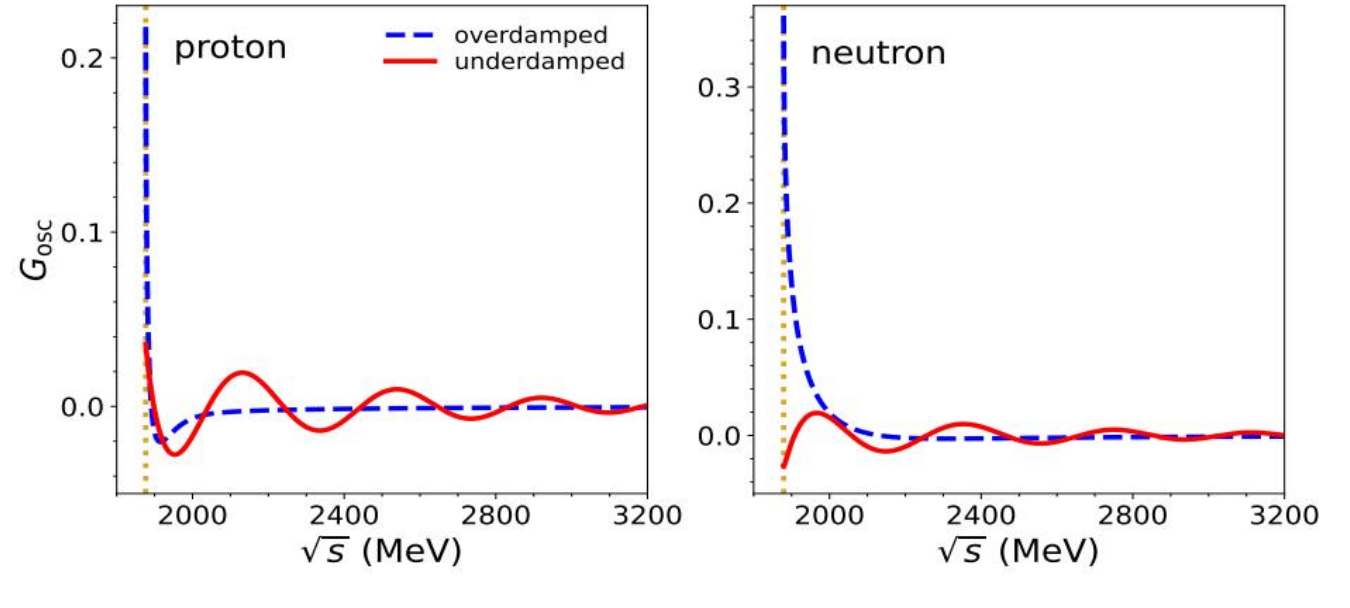
$$G^N_{\rm osc}(\tilde{p}) = G^N_{{\rm osc},1}(\tilde{p}) + G^N_{{\rm osc},2}(\tilde{p}),$$

$$G^N_{{\rm osc},j}(\tilde{p}) = G^{0,N}_{{\rm osc},j} - \frac{\omega_j^2}{\Gamma(\alpha_j^N)} \int_0^{\tilde{p}+p_0^N} (\tilde{p} + p_0^N - t)^{\alpha_j^N - 1} G^N_{{\rm osc},j}(t) dt$$

- Oscillation behavior of SFFs



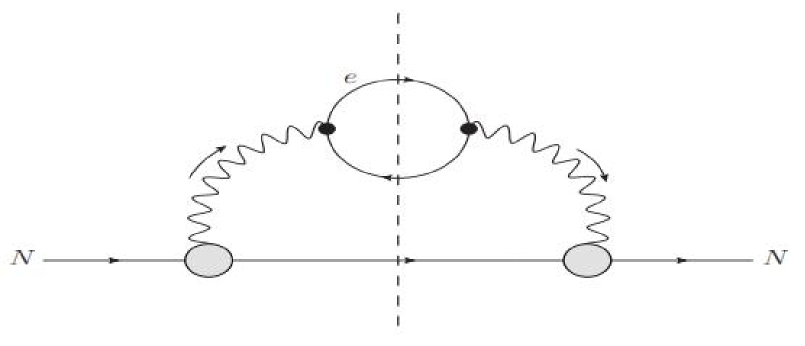
Oscillation



- The ‘overdamped’ oscillator dominates near the threshold. It reveals the enhancement near threshold.
- The ‘underdamped’ oscillator dominates in the high energy region. The proton’s and neutron’s has a ‘phase delay’.
- Other dynamics?

Lin, Hammer, Meißner, PRL128 (2022) 052002
Cao, J.P. Dai, Lenske, PRD 105 (2022) 7, L071503, etc
Qian, Liu, Cao, Liu, PRD 107 (2023) 9, L091502;
Yan, Chen, Xie, PRD 107 (2023) 7, 076008

Underlying physics?



overdamped oscillator
octupole?

underdamped oscillator
quadrupole?

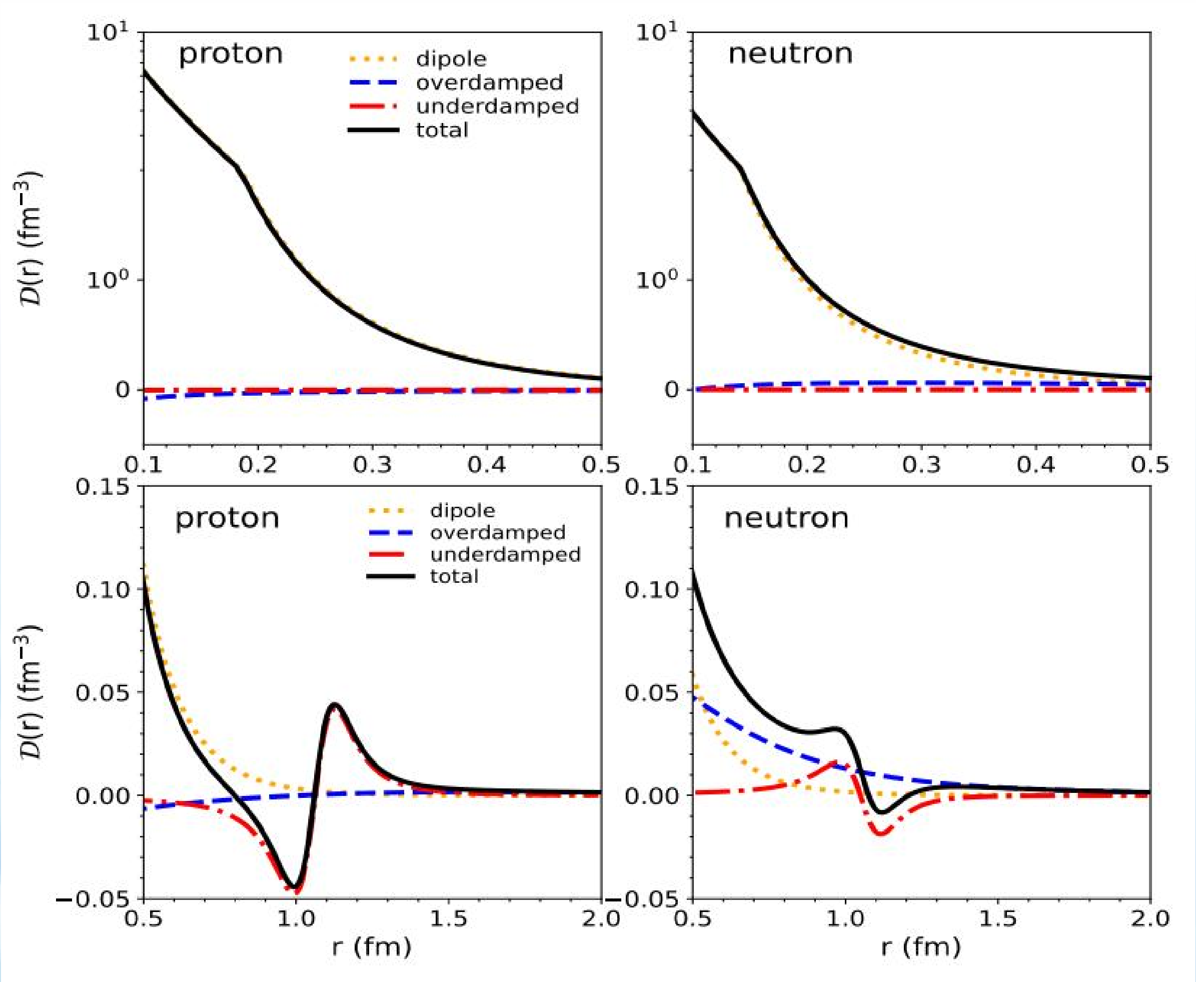
- Vacuum polarization around the nucleon

$1 < \alpha_j^N < 2$ $\alpha_1^p = 1.23$ $\alpha_1^n = 1.04$ $\alpha_2^p = \alpha_2^n = 1.87$

- Two limits of fractional oscillators： 1 for diffusion and 2 for wave equations of motions.

- Distributions of higher order polarized charges.

Underlying physics?



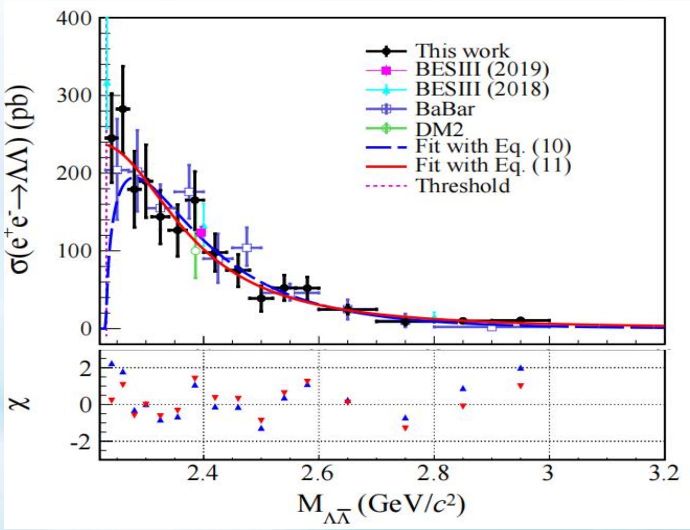
- Proton: valence quarks of uud; Neutron: udd
- negative polarization electric charges for the proton, when not very faraway from the nucleon.
- positive polarization for the neutron
- It explains the phase difference!

SU(3) ChEFT!

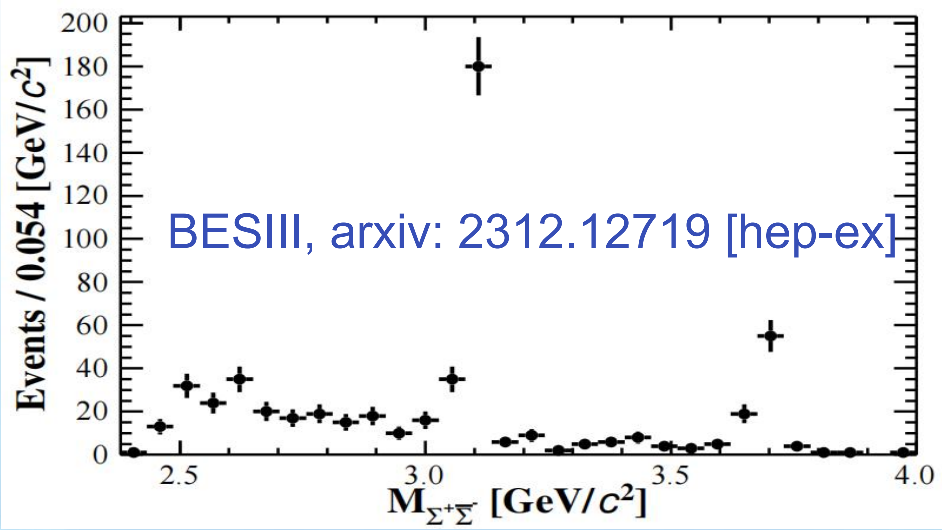
- SU(3) gives more information in pp, ΣΣ, ΛΛ coupled channel scattering

Juelich model: Haidenbauer et.al., NPA562 (1993) 317; Haidenbauer, Meissner, Dai, PRD103 (2021) 014028.

- More data in BB scattering: pp → ΣΣ, ΛΛ, etc.



BESIII, PRD107 (2023) 7, 072005



For BESIII's YN scattering data, See Jielei Zhang's Talk

- An overall description of the EMFFs of the Octet?
- Some processes of BB scattering are done

4. Summary

NN Amplitude

SU(2) ChEFT works well at P_{Lab}<300 MeV up to N^3LO. For SU(3) one, we calculate NN scattering with other Baryons included. Need more measurements on hyperons.

EMFFs of N

We study the EMFFs of nucleons within SU(3) ChEFT. A fractional oscillation model is proposed, polarized charge density distributions.

EMFFs of Y

YY amplitude are calculated based on Juelich model. The EMFFs are predicted. SU(3) ChEFT is necessary to improve the analysis.

Prospects?

BESIII's new data for SU(3) ChEFT? ChEFT + OGE to study NN scatterings? EMFFs of hyperons and other interesting physics within SU(3) ChEFT?

Thank You For your patience !