Lattice calculation of K→ππ decay and ππ scattering

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Outline

- Introduction
  - Outline of $K \to \pi\pi$ decay & direct CP violation
  - SM prediction desired
- Challenges
  - What confronted lattice QCD for a long time
- Recent status
- Remaining challenges
- Summary & Outlook

About this talk

- This talk mostly focuses on lattice calculation of $K \to \pi\pi$ decay

Also see

- Comprehensive review on $K \to \pi\pi$: Aebischer et al EurPhys.J.C80,705 (2020)
- ChPT including EM/IB correction to $K \to \pi\pi$: Cirigliano et al JHEP,02,032,(2020)

Introduction

$K \to \pi\pi$ & CP violation

CP odd CP even

$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$

ε' direct CPV

ε indirect CPV

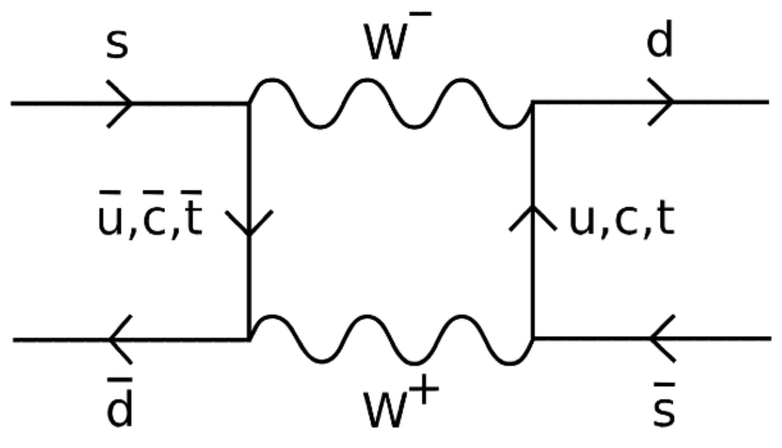
$|\pi\pi\rangle$

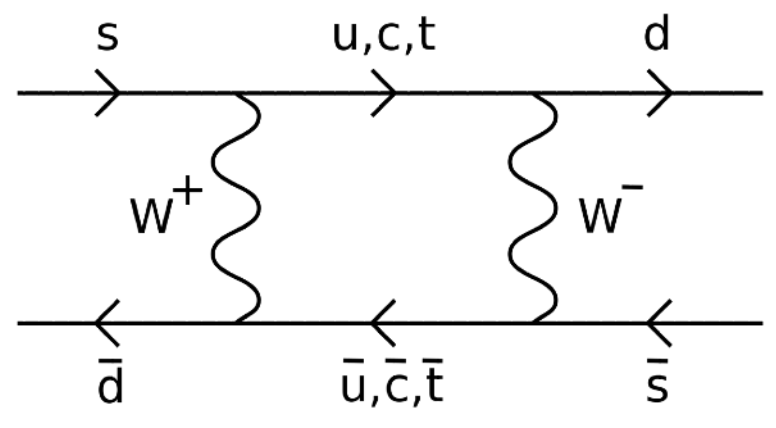
CP even

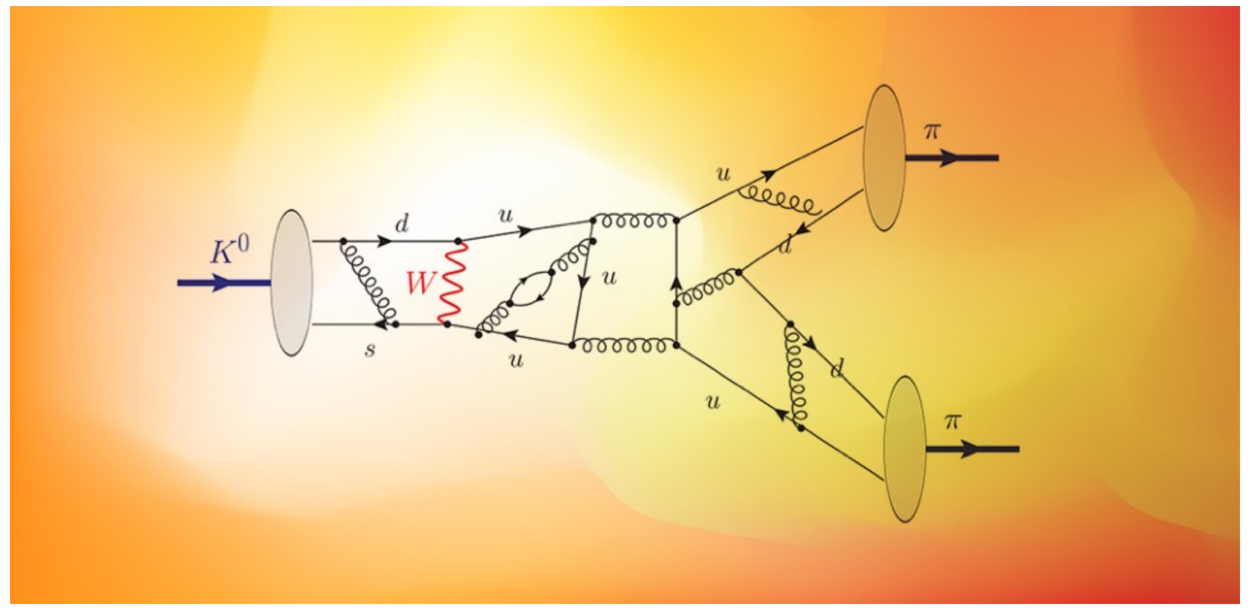
Discovered in 1964

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \Big/ \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = 1 - 6\mathrm{Re}(\varepsilon'/\varepsilon)$$

- $|\varepsilon| = 2.228(11) \times 10^{-3}$ from "odd" mixing b/w K^0 & $\overline{K}^0$
- ε' only found in decays **Discovered in 1999**
  - $\mathrm{Re}(\varepsilon'/\varepsilon)_{exp} = 1.66(23) \times 10^{-3}$ (KTeV & NA48)
  - Consistent with SM?







CP violation & its importance

- CP violation discovered in 1964 – $K_L \to \pi\pi$
- Direct CPV discovered in 1999 – also in $K \to \pi\pi$
- CPV in SM believed to be insufficient to explain the matter-antimatter imbalance in the present universe
- Testing SM via CPV physics can provide a good source for searching BSM
- Lattice QCD capable of testing hadronic sectors K, D, B, …
- Direct CP violation measure ε'/ε in $K \to \pi\pi$ highly demanded

Anticipated significance of ε′

- s → d: most suppressed within SM

$$\mathrm{Re}(\varepsilon'/\varepsilon) \propto \mathrm{Im}\left(V_{td}V_{ts}^*\right)$$

$$V_{CKM} \sim \begin{array}{cc} \begin{array}{ccc} \mathbf{d} & \mathbf{s} & \mathbf{b} \end{array} & \\ \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \boxed{\lambda^3} & \boxed{-\lambda^2} & 1 \end{pmatrix} & \begin{array}{c} \mathbf{u} \\ \mathbf{c} \\ \mathbf{t} \end{array} \end{array}$$

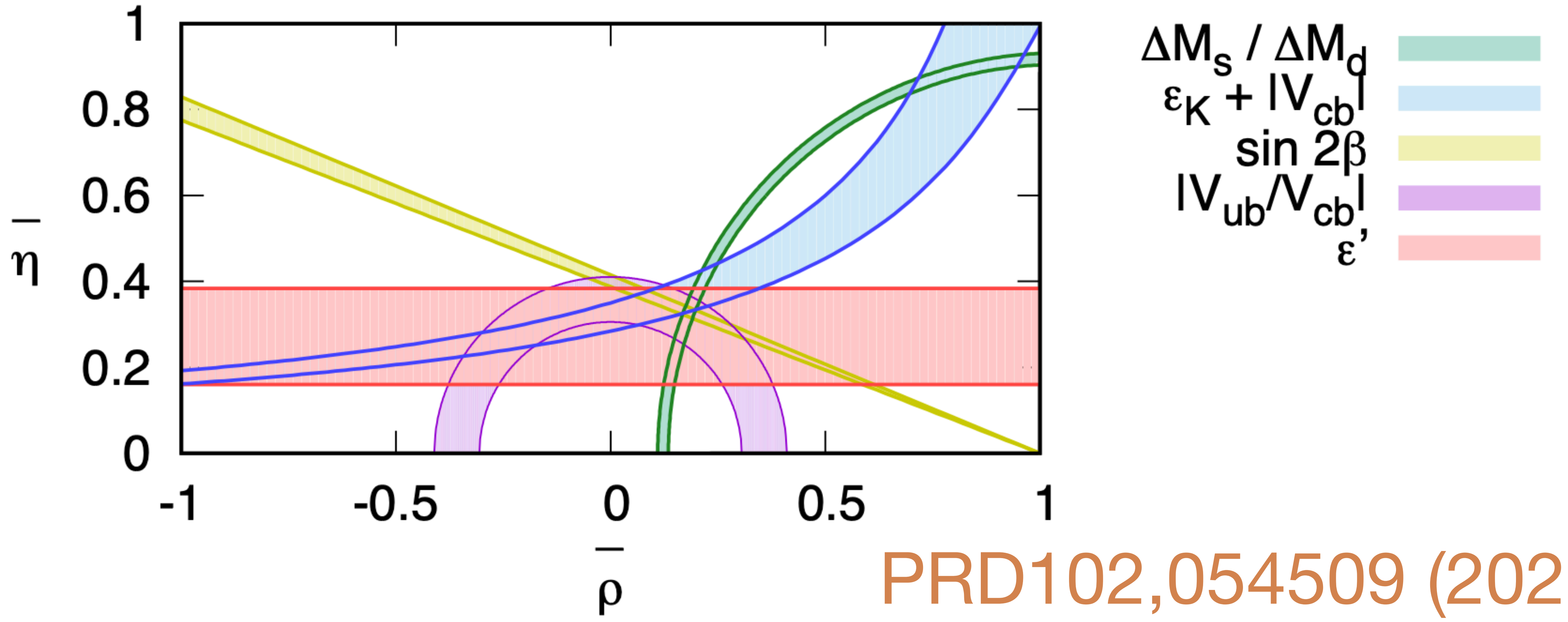
$\lambda \approx 0.23$

$$|V_{td}V_{ts}^*| \sim 5 \times 10^{-4} \quad \ll |V_{td}V_{tb}^*| \sim 1 \times 10^{-2}, \quad |V_{ts}V_{tb}^*| \sim 4 \times 10^{-2}$$

s → d **b → d** **b → s**

- Highly sensitive to BSM

- New constraint on CKM unitarity



PRD102,054509 (2020)

Isospin decay modes & ΔI = 1/2 rule

$$\langle(\pi\pi)_{\mathrm{I}=0}| = \sqrt{1/3}\langle\pi^0\pi^0| + \sqrt{2/3}\langle\pi^+\pi^-|, \quad \langle(\pi\pi)^{\mathrm{I}_3=0}_{\mathrm{I}=2}| = -\sqrt{2/3}\langle\pi^0\pi^0| + \sqrt{1/3}\langle\pi^+\pi^-|$$

- Isospin-definite amplitudes

$$\mathrm{A}_\mathrm{I} = \langle(\pi\pi)_\mathrm{I}|\mathrm{H_W}|\mathrm{K}\rangle \quad \begin{cases} \mathrm{I}=0 \to \Delta\mathrm{I} = 1/2 \\ \mathrm{I}=2 \to \Delta\mathrm{I} = 3/2 \end{cases}$$

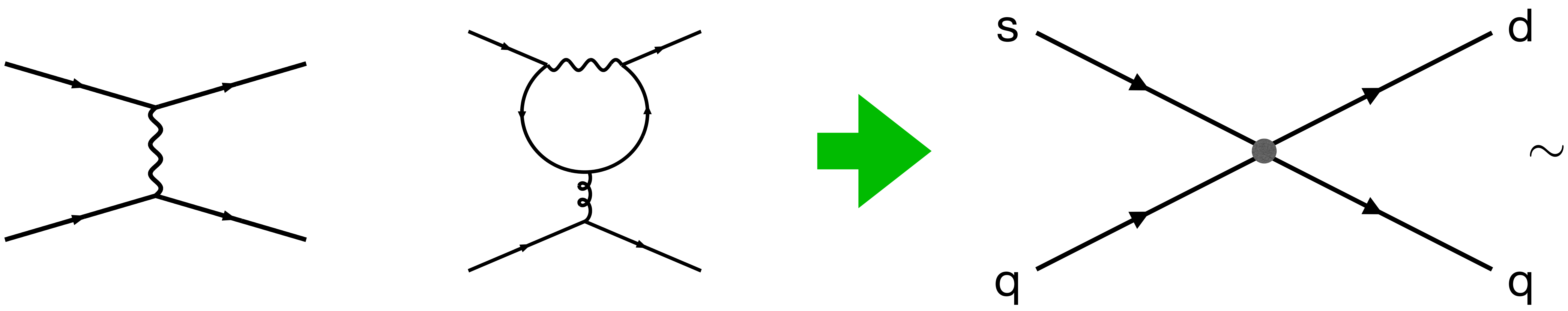
- ΔI = 1/2 rule (experimental fact)

$$\frac{\mathrm{Re\,A_0}}{\mathrm{Re\,A_2}} = 22.45(6)$$: large suppression of ΔI = 3/2 (A_2) mode

  - Factor 2 can be responsible for Wilson coefs from pQCD [Gaillard & Lee, PRL 33,108 (1974)]
  - Remaining factor 10 comes from QCD or BSM?
  - A lot of discussions happening already in 1970s
  - Firm understanding was not established before lattice calculation of matrix elements

Approach to weak decays

- Two typical scales
  - Electroweak scale: m_W = 80 GeV, m_Z = 91 GeV
  - QCD scale: Λ_{QCD} ~ 300 MeV
- Low-energy effective interactions @ QCD scale



$$\sim G_F \propto \frac{g^2}{m_W^2}$$

  - $H_W = \Sigma_i \underbrace{c_i(\mu)}_{\text{Wilson coefficients}} \underbrace{O_i(\mu)}_{\text{Effective operators}}$

Wilson coefficients **Effective operators**

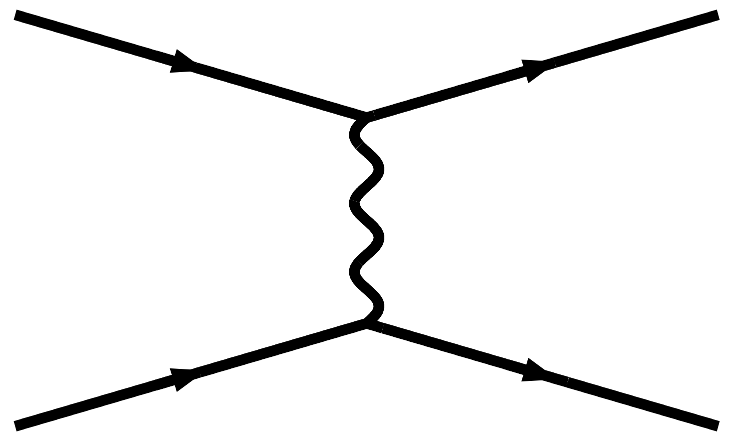
$\Delta S = 1$ effective operators

- $(\bar{s}q)_{V-A}(\bar{q}'q'')_{V\pm A} = \bar{s}\gamma_\mu(1-\gamma_5)q' \cdot \bar{q}'\gamma_\mu(1\pm\gamma_5)q''$
- α, β: color indices

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\bar{u}_\beta d_\alpha)_{\mathrm{V-A}}\,, \\
Q_2 &= (\bar{s}u)_{\mathrm{V-A}}\,(\bar{u}d)_{\mathrm{V-A}}\,, \\
Q_3 &= (\bar{s}d)_{\mathrm{V-A}}\sum_q(\bar{q}q)_{\mathrm{V-A}}\,, \\
Q_4 &= (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q(\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}\,, \\
Q_5 &= (\bar{s}d)_{\mathrm{V-A}}\sum_q(\bar{q}q)_{\mathrm{V+A}}\,, \\
Q_6 &= (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q(\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}\,, \\
Q_7 &= \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}q)_{\mathrm{V+A}}\,, \\
Q_8 &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V+A}}\,, \\
Q_9 &= \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}q)_{\mathrm{V-A}}\,, \\
Q_{10} &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}\,,
\end{aligned}
$$

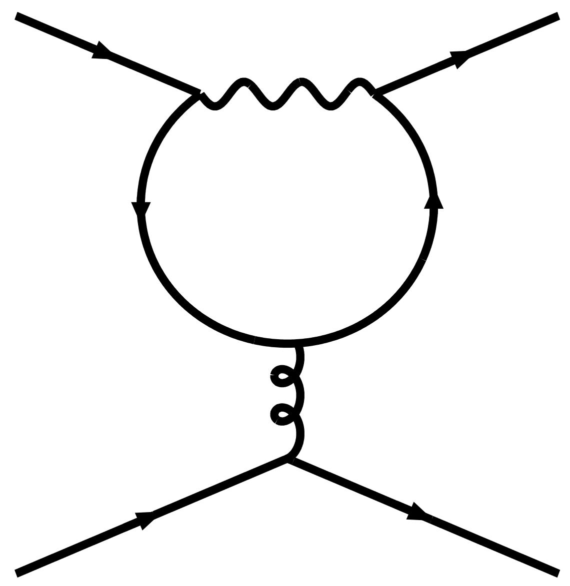
Current-current operators

- $Q_1^c = (\bar{s}_\alpha c_\beta)_{\mathrm{V-A}}(\bar{c}_\beta d_\alpha)_{\mathrm{V-A}}$ & $Q_2^c = (\bar{s}c)_{\mathrm{V-A}}(\bar{c}d)_{\mathrm{V-A}}$ enter when $n_f \geq 4$



QCD penguin operators

- sum over q runs for all active quarks



EW penguin operators

$K \to \pi\pi$ Amplitude and ε'

ππ phase shifts at m_K

$$\varepsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right] \qquad (\omega = \mathrm{Re}A_2/\mathrm{Re}A_0)$$

Renormalization matrix

$$A_I = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \sum_{i,j} \underbrace{[z_i(\mu) + \tau y_i(\mu)]}_{\text{Wilson coefs. pQCD}} \underbrace{Z_{ij}(\mu)}_{\text{LQCD (+pQCD)}} \underbrace{\left\langle (\pi\pi)_I \middle| Q_j^{lat} \middle| K \right\rangle}_{\text{LQCD}}$$

- Lüscher & Lellouch-Lüscher's formalisms play a key role in finite-volume calculation
- A_2 already reached a sufficient precision RBC/UKQCD PRD91 (2015) 074502
  - $\mathrm{Re}\,A_2 = 1.50(4)_{stat}(14)_{sys} \times 10^{-8}$ GeV, $\mathrm{Im}\,A_2 = -6.99(20)_{stat}(84)_{sys} \times 10^{-13}$ GeV

    cf: $(\mathrm{Re}\,A_2)_{exp} = 1.479(4) \times 10^{-8}$ GeV
- A_0 still challenging because of many difficulties

Challenges

what confronted LQCD for a long time

Challenges confronted for past few decades

- Computational cost/Statistics
  - disconnected diagrams
  - challenge enhanced due to the other difficulties

- Charm-loop effects
  - expected significant
  - directly on lattice? → am_c not small on current lattices
    ↕ window problem
  - absorb into WCs? → NLO pQCD at $\mu = m_c$ not ideal

next slides

- Chiral symmetry
  - 10 four-quark operators
  - strongly desired to prevent mixing with other operators
  - domain wall fermions preferable and used by RBC/UKQCD

- Two-pion final state on the euclidean lattice
  - only $E \approx 2m_\pi \approx 280$ MeV state extracted in a straightforward manner
  - (in the rest frame) $E = m_K \approx 500$ MeV state needed

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⁕ These are why very few lattice collaborations have computed ε′ → focusing on the work by RBC/UKQCD

ππ system in finite volume

- Maiani-Testa no-go theorem (1990)
  - Not possible to extract scattering/decay amplitude above the kinematic threshold from Euclidean correlation functions

$\pi\pi$ system in finite volume

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ππ system in finite volume

- Maiani-Testa no-go theorem (1990)
  - Not possible to extract scattering/decay amplitude above the kinematic threshold from Euclidean correlation functions **in infinite volume**
- Finite-Volume methods
  - Lüscher-Wolf: variational method (GEVP) to control excited states (1990)
  - Lüscher: prescription to determine scattering phase shift (1991)
  - Lellouch-Lüscher: normalization of two-hadron state in finite volume essential for decays (2000)
- Theory done a long time ago, now computers and lattice computational techniques getting developed to apply to physics!

Realizing on-shell kinematics

- The lightest ππ state with “2 stationary pions” in Euclidean rest frame
  - $E_{\pi\pi,0} \approx 280$ MeV → off-shell
  - need $| E_{\pi\pi} = m_K \approx 500 \text{ MeV} \rangle$ state

- Possible approaches

💡 Finite volume → two-pion spectrum not continuous

- Moving frame (Ishizuka et al 2018)

e.g. $\sqrt{m_K^2 + p_{tot}^2} = m_\pi + \sqrt{m_\pi^2 + p_{tot}^2}$

- Analyze correlation functions taking multiple states into account (GEVP, led by MT)
- Manipulate boundary conditions → pions anti-periodic → must move → 500 MeV ground state possible (G-parity BC led by C. Kelly)
  - ✳ For A_2 imposing anti-periodic BC on d quark, was enough to make relevant pions moving (RBC/UKQCD 2012/2015).

Recent status

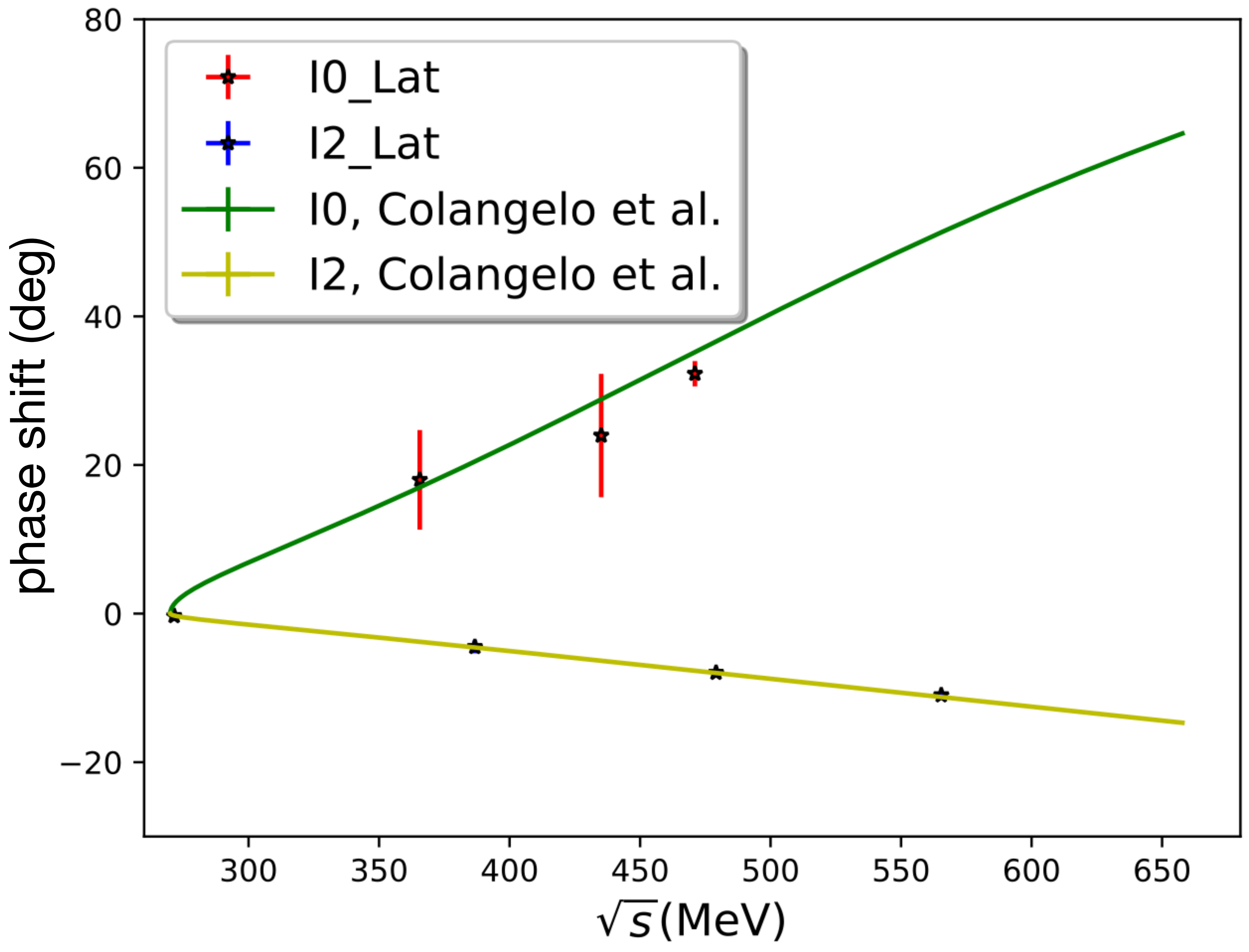
ππ phase shifts

Lattice results consistent with dispersive approach

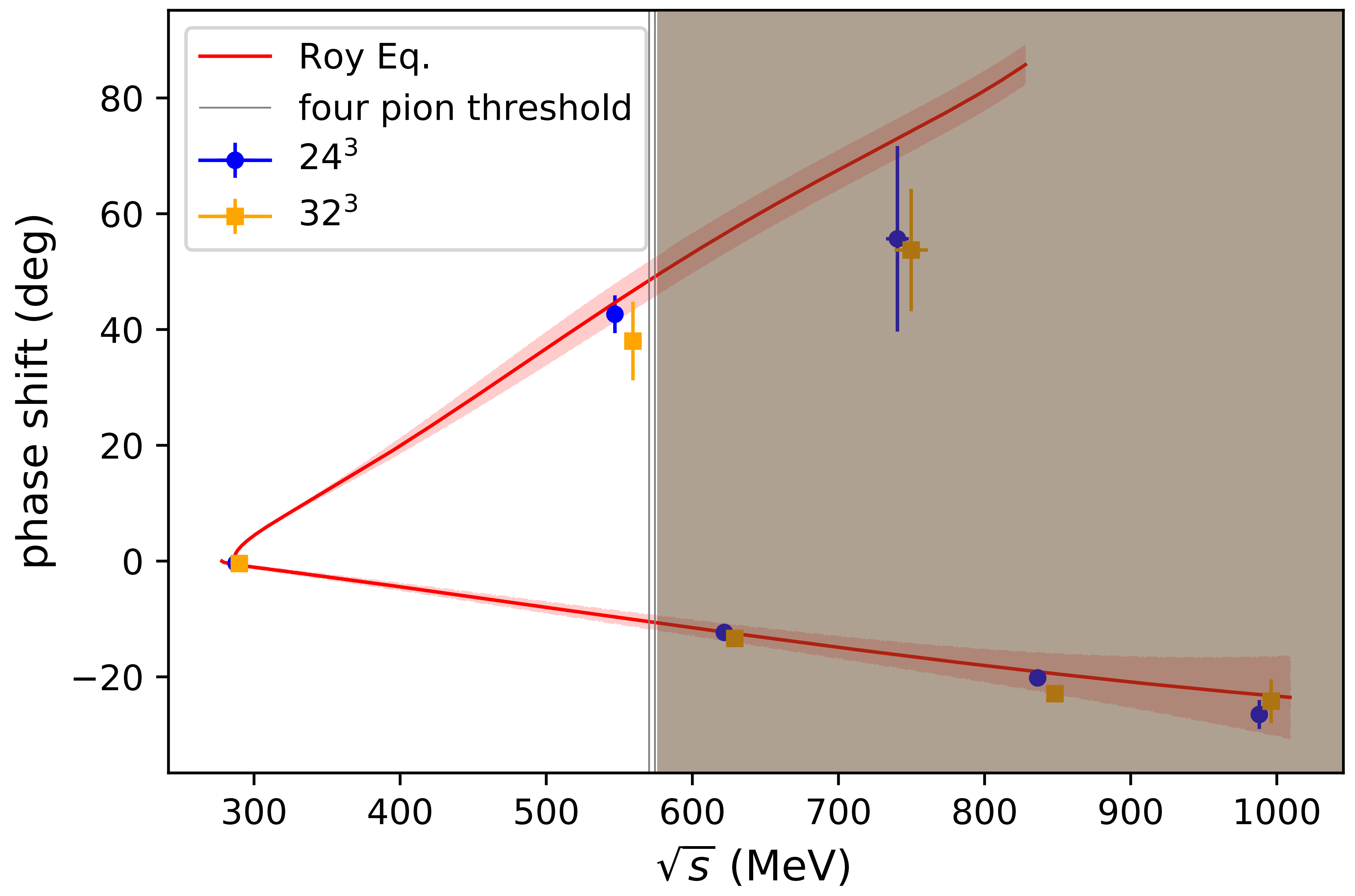
Colangelo et al, NPB603,125 (2001)

Inelastic region

Lüscher’s formalism not strictly valid

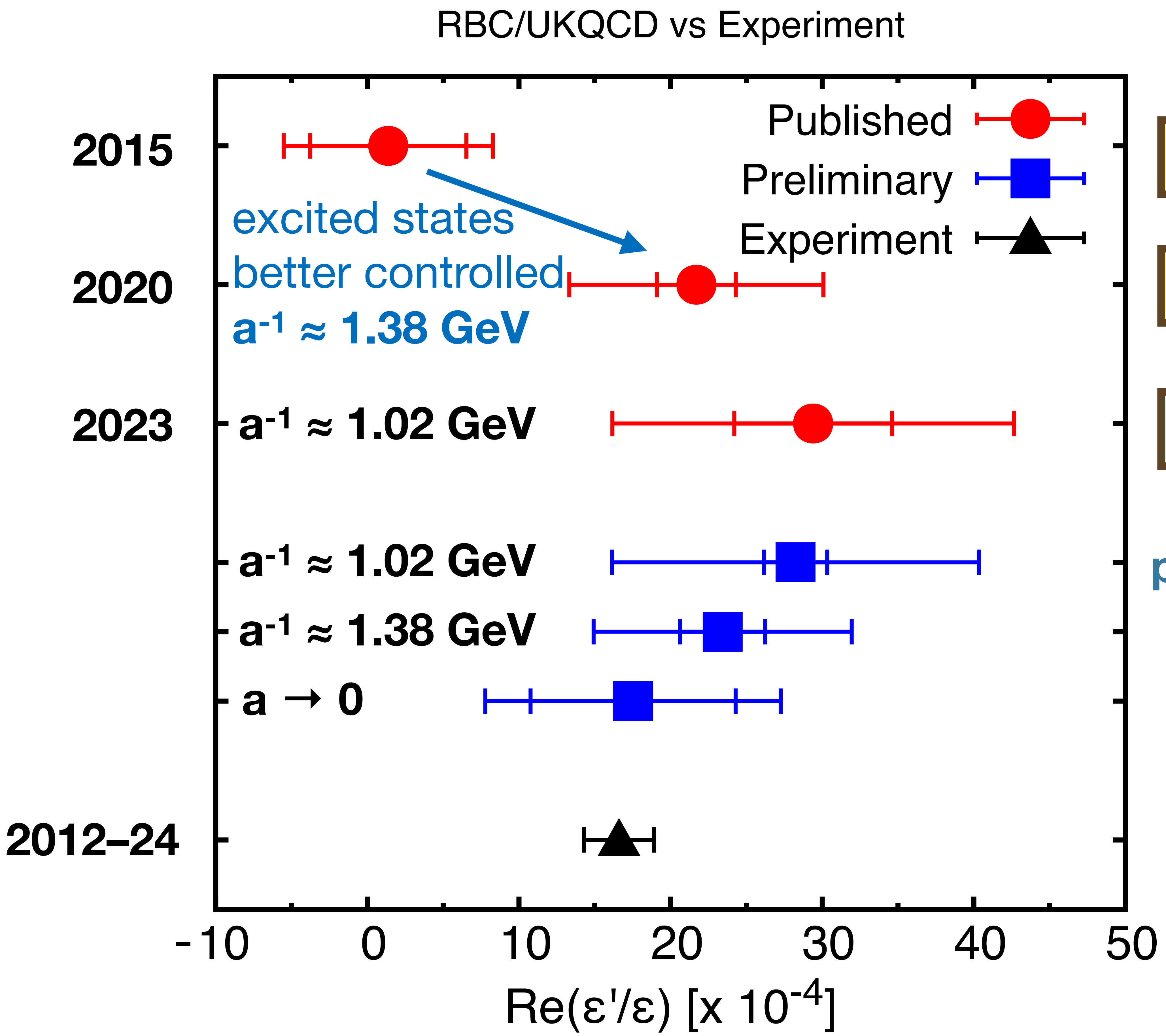


RBC/UKQCD GPBC, PRD104,114506 (2021)



RBC/UKQCD PBC, PRD108,039902 (2023)

Current status of ε′/ε



PRL 115,212001

PRD 102,054509

PRD 102,094517

preliminary results with multiple lattice spacings

- G-parity Boundary Conditions (GPBC)
  - $a^{-1} \approx$ 1.38 GeV
  - efforts started by early 2000s
  - continuing calculation on finer lattice(s) C. Kelly's talk at Lattice24

- Periodic Boundary Conditions (PBC)
  - newer project
  - important for introducing EM/IB effects
  - Led by MT (see talk at Lattice 24)

- **$a^{-1} \approx$ 1.38 GeV almost done, wrapping up**
- **starting calculation at $a^{-1} \approx$ 1.73 GeV**

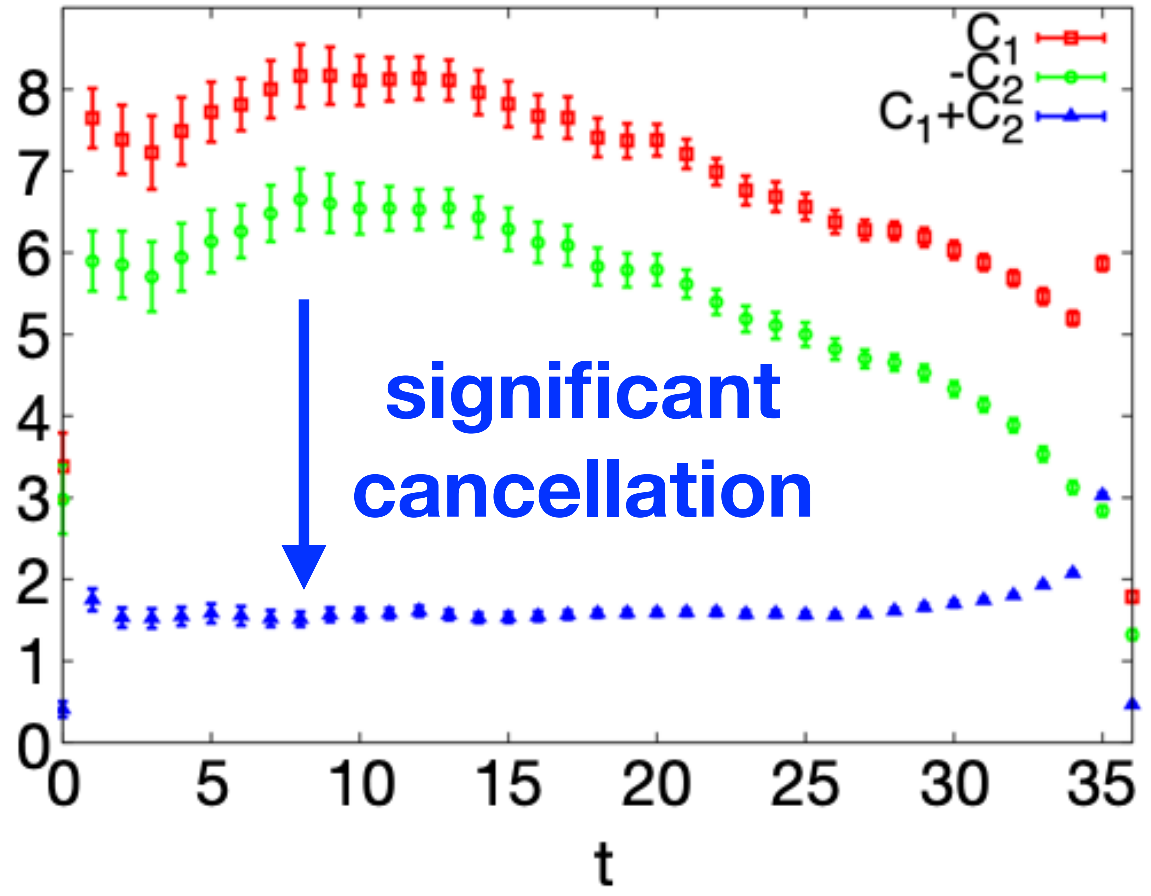
✷ Result from another group, Ishizuka et al 2018: Re(ε′/ε) = (19 ± 57) x 10^{-4} (calculated at unphysical m_π, m_K)

The ΔI = 1/2 rule

- Experimental fact

$$\frac{\text{Re } A_0}{\text{Re } A_2} = 22.45(6)$$: large suppression of ΔI = 3/2 (A_2) mode

- Significant suppression of ReA_2 (2012/2015)
  - C_1, C_2 contributions of different color structure to $K \to \pi\pi$ correlation function most significant to ReA_2
  - Naïvely $C_1 = -3C_2$ based on color counting
  - Significant cancellation at physical m_π observed



RBC/UKQCD, PRD91,074502 (2015)

- Lattice confirmation of the ΔI = 1/2 rule with the result for A_0 (2020)

$$\frac{\text{Re}A_0}{\text{Re}A_2} = 19.9(2.3)_{\text{stat}}(4.4)_{\text{sys}}$$

Remaining Challenges

to accomplish the experimental precision

Systematic errors in 2020

- Systematic errors on Im A_0

Finite lattice spacing	12%
Wilson coefficients/charm-loop effects	12%
Lelloch-Lüscher FV correction	1.5%
Residual FV correction	7%
Parametric error	6%
Off-shellness	5%
Renormalization	4%
Missing G_1 operator	3%
TOTAL	21%

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Improvement desired (Finite lattice spacing)

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Improvement desired

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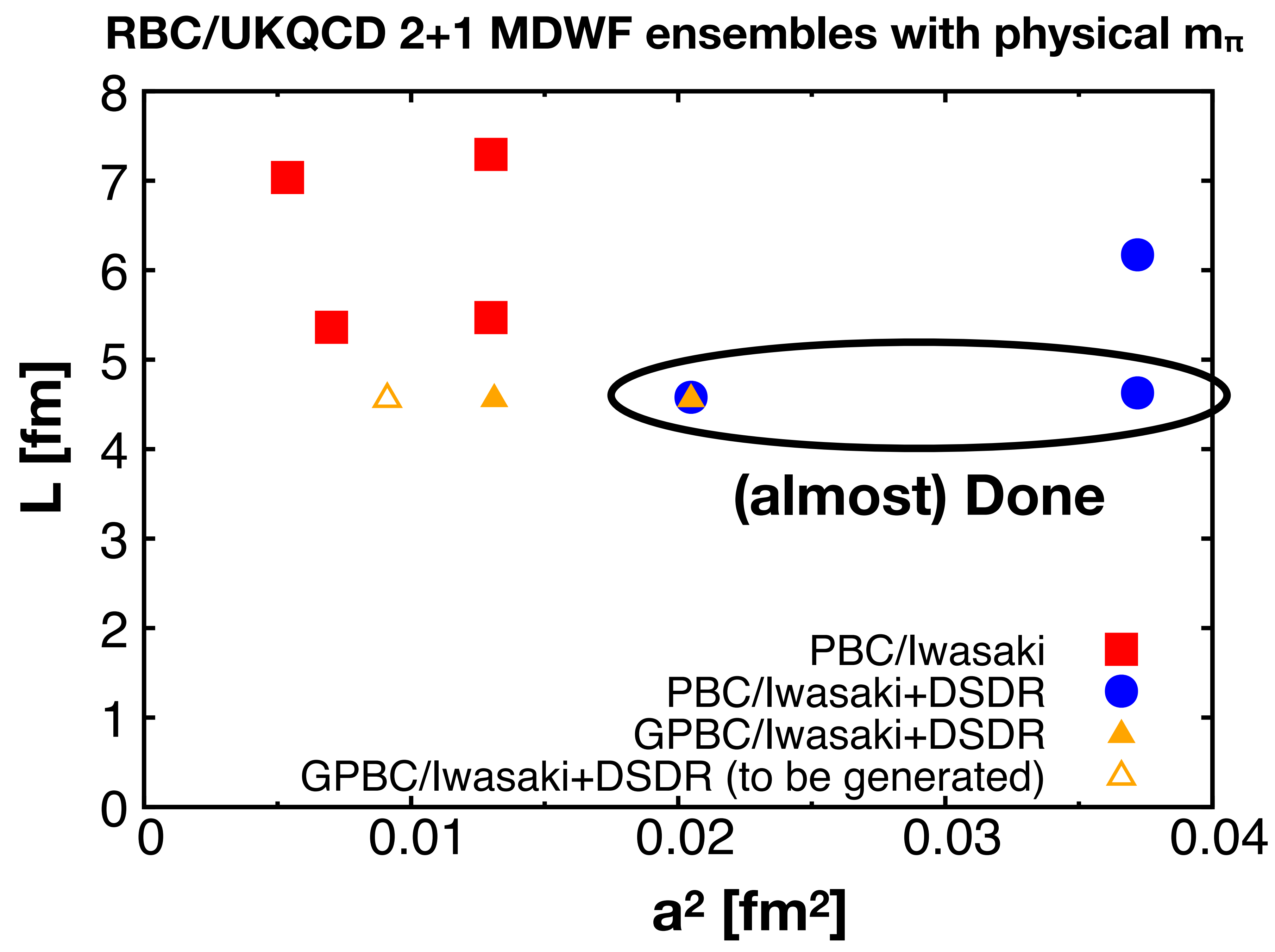
- In addition

Hope to compute near future

  - ε' could be significantly affected by EM/IB effects ($\Delta I = 1/2$ rule → 25%)

Finite lattice spacing error

- Can be resolved by taking **continuum limit**
  - Results with different lattice spacings needed
- G-parity BC
  - $32^3 \times 64$, $a^{-1} \approx 1.4$ GeV: Done (2020)
  - Ensemble generation speed-up algorithm (Lat23, C. Kelly)
  - $40^3 \times 64$, $a^{-1} \approx 1.7$ GeV: Calculation on-going
  - $48^3 \times ??$, $a^{-1} \approx 2.1$ GeV: in the future as needed
- Ensembles already generated for PBC



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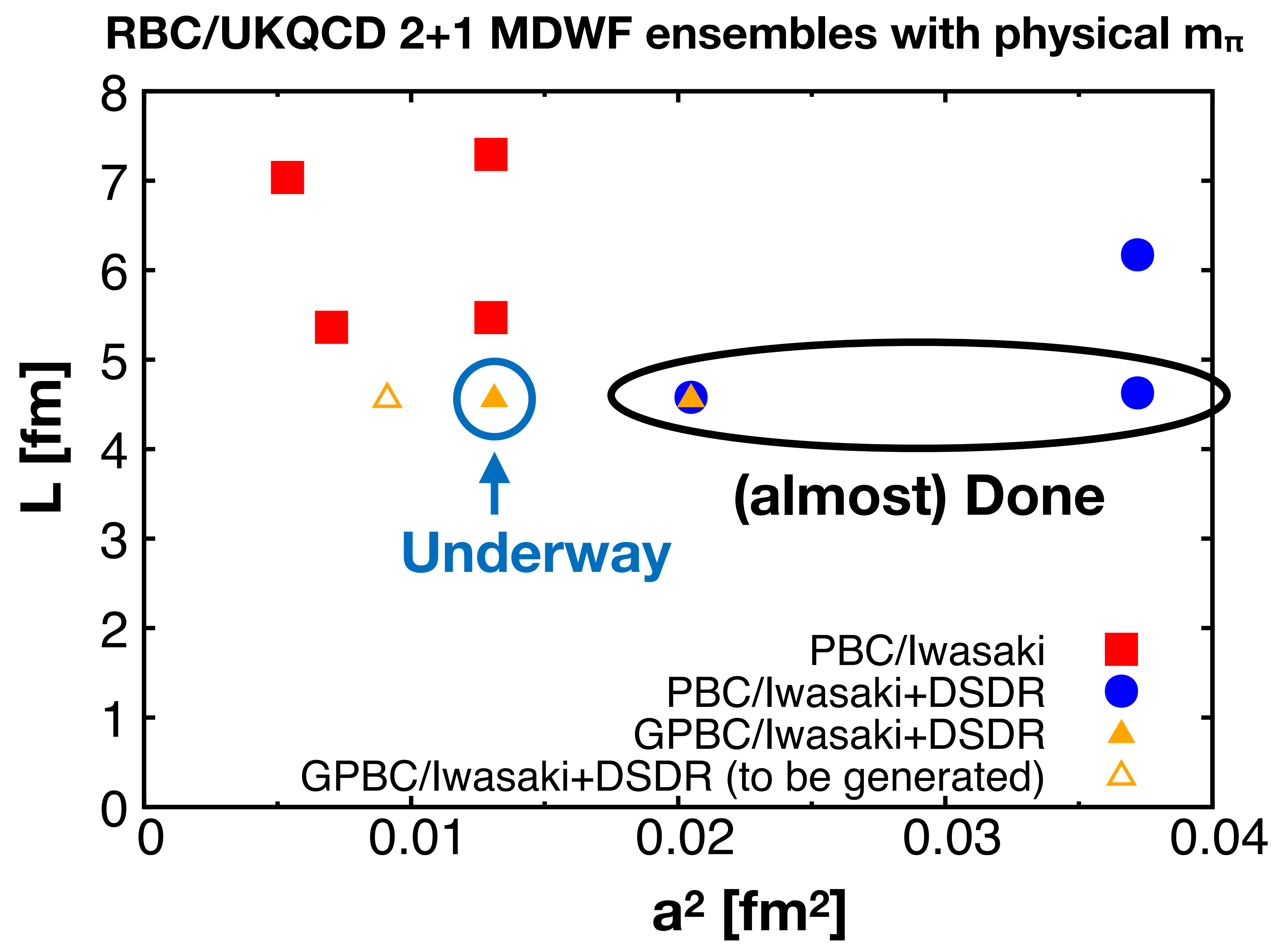

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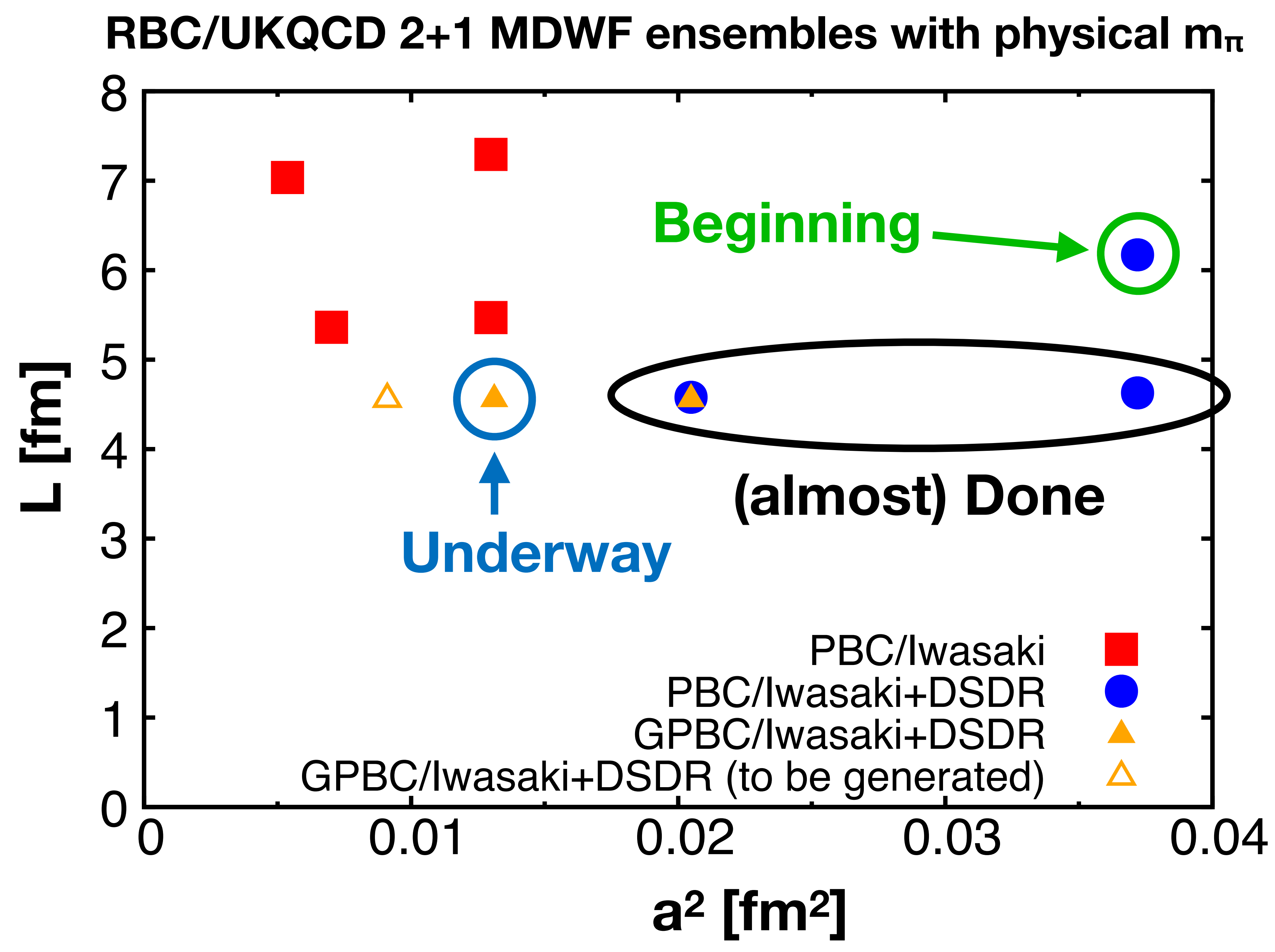

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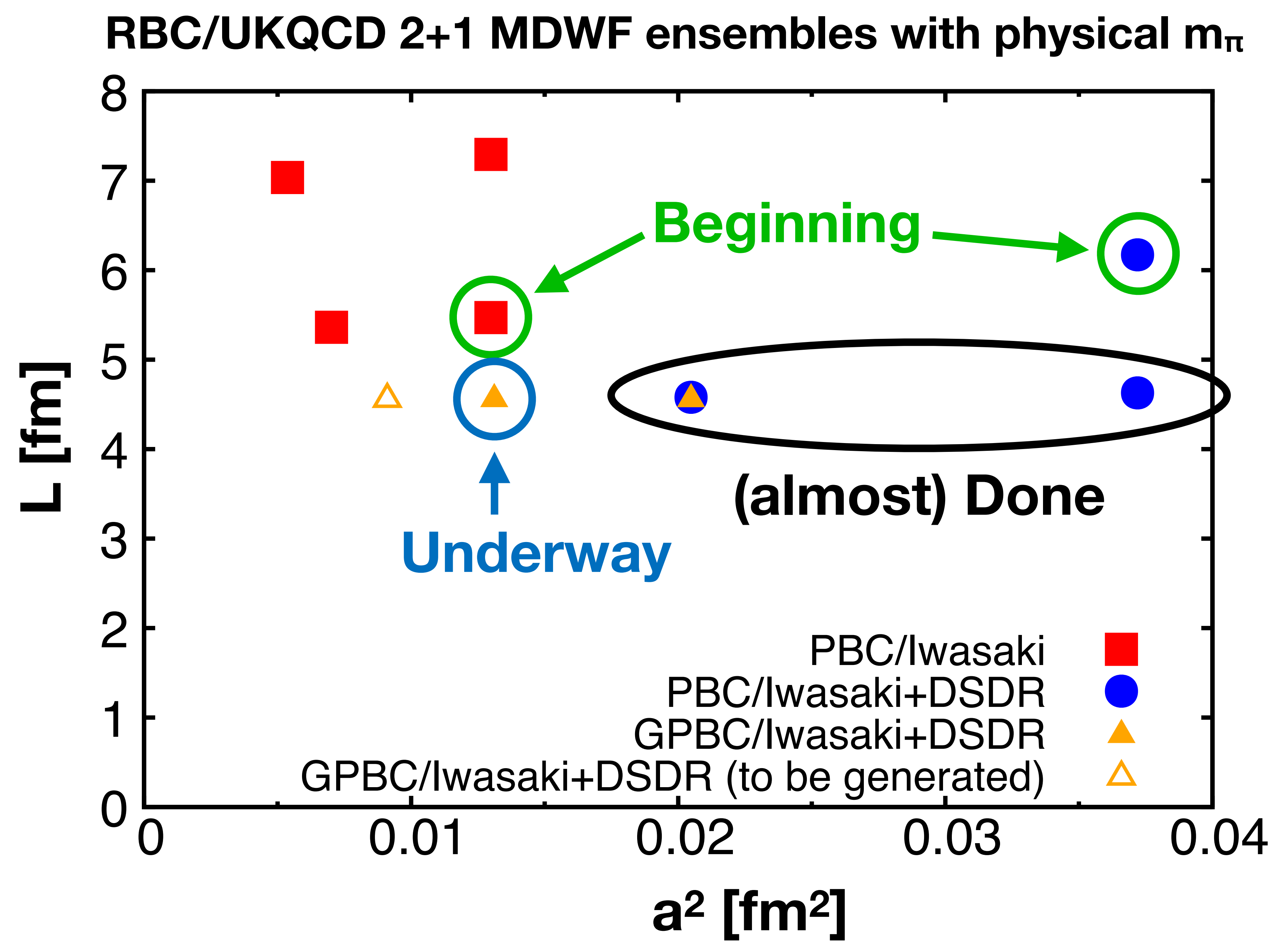

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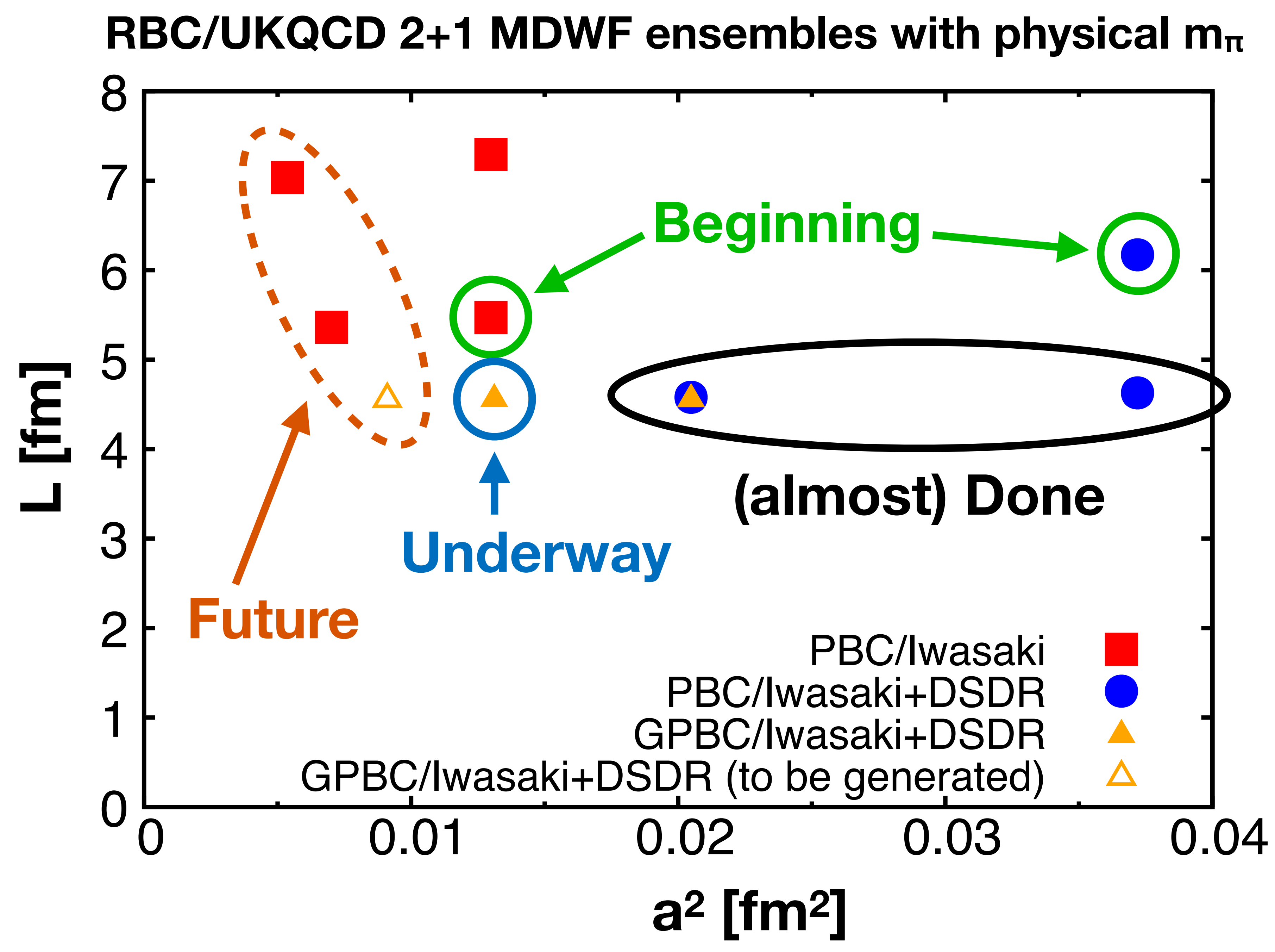
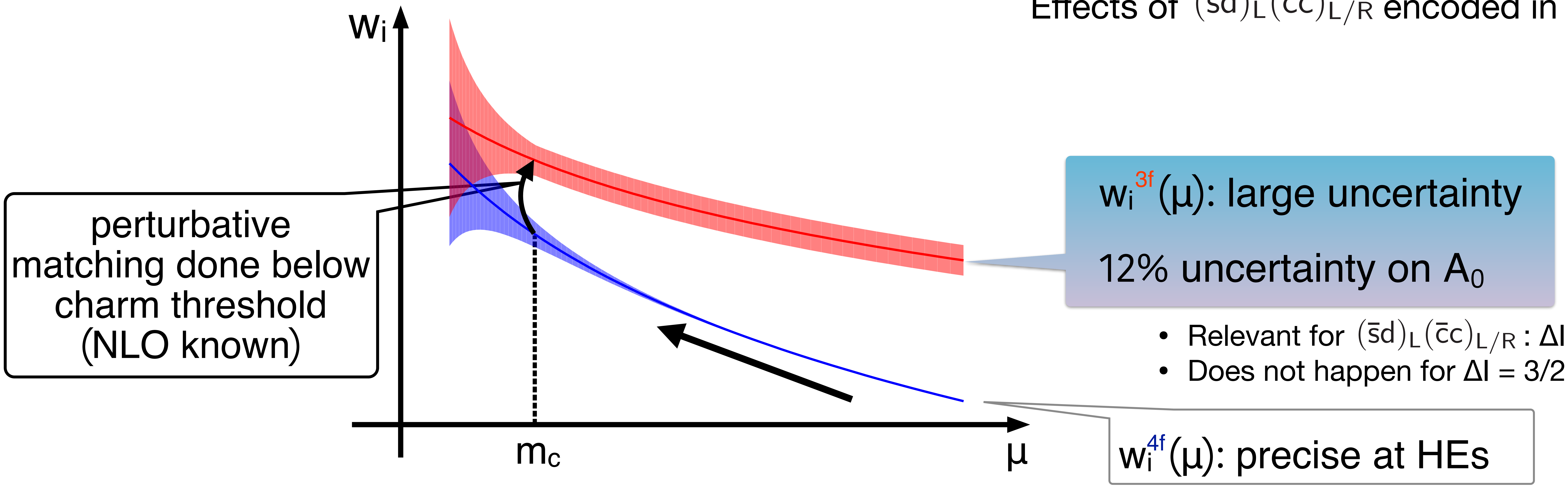


Wilson coefs/charm-loop effects

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$

Effects of $(\bar{s}d)_L(\bar{c}c)_{L/R}$ encoded in w_i^{3f} or O_i^{4f}



- Relevant for $(\bar{s}d)_L(\bar{c}c)_{L/R}$: $\Delta I = 1/2$
- Does not happen for $\Delta I = 3/2$

- Possible resolutions
  - NNLO matching only partially done [Cerda-Sevilla et al. Acta Phys.Polon.B 4 (2018) 1087-1096]
  - Finding another approach potentially incorporated in the next RBC/UKQCD paper

EM/IB effects

- Usually O(1%) but …

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right] = -\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left[1 - \frac{1}{\omega}\frac{\mathrm{Im}A_2}{\mathrm{Im}A_0}\right]$$

$(\omega = \mathrm{Re}A_2/\mathrm{Re}A_0)$

Even small correction to this can amplify ε' $(1/\omega \approx 22.5)$

Ciligriano et al, JHEP 02, 032 (2020)
NLO ChPT + large N_c
(example estimation)

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}} - \frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}(1-\hat{\Omega}_{\mathrm{eff}})\right]$$

$$\boxed{\hat{\Omega}_{\mathrm{eff}} = 0.170\left(\begin{smallmatrix}+91\\-90\end{smallmatrix}\right)}$$

- Developing approaches to introduce QED/IB effects on the lattice
  - Extension of Lüscher's formalism for treatment of ππ state in a finite box
  - Coulomb correction to $\pi^+\pi^+$ scattering [Christ et al, PRD106 (2022), 1, 014508]
  - Contribution of transverse radiation contribution getting understood
  - PBC appear necessary to introduce these effects

Summary & Outlook

Summary & Outlook

- Determination of ε' & understanding $\Delta I = 1/2$ rule: long-time desire in high energy physics to discover new physics beyond the SM
- Lattice calculation of ε' finally became possible
- $\Delta I = 1/2$ rule well reproduced by lattice calculation at physical m_π
- Main sources of systematic errors on ε'
  - Finite lattice spacing - *Calculations on finer lattices underway, first continuum extrapolation this year*
  - Wilson coefficients - *New idea being tested and possibly incorporated in the upcoming paper*
  - QED/IB effects - *Theoretical approach being developed [Christ et al, PRD106, 014508 (2022)]*
- Precision can compete with experiment in the near future
  - Could attract a big attention from lattice, particle phenomenology & experiment!