Dispersive Determination of the η/η' Transition Form Factors

in collaboration with M. Hoferichter, B.-L. Hoid and B. Kubis

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Introduction: The Muon $g-2$

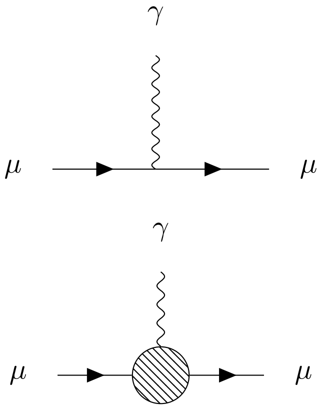
- g-factor: strength of coupling to magnetic field

$$\vec{\mu}_\mu = -g\frac{e}{2m_\mu}\vec{S}$$

- in relativistic QM: $g = 2$
- corrections due to loop effects in Standard Model

$$a_\mu = \frac{g-2}{2} = \frac{\alpha_{\text{em}}}{2\pi} + \mathcal{O}(\alpha_{\text{em}}^2)$$

Schwinger 1948



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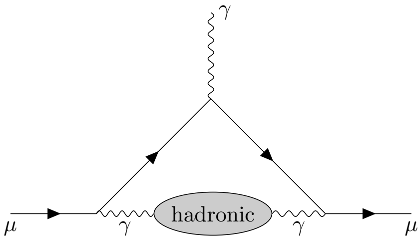
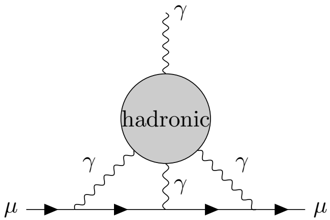
- comp. prediction vs. experiment
- hadronic contr. HVP and HLbL dominate uncertainty





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- comp. prediction vs. experiment
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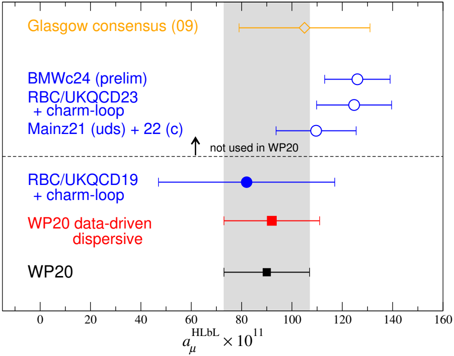
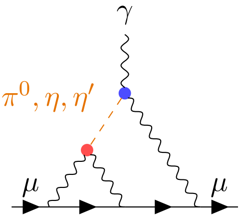
- WP2020: HLbL precision goal $\lesssim 10 \times 10^{-11}$ Aoyama et al., 2020



η/η'-Pole Contribution

- Model-independent dispersive approach to HLbL: relate conts. to observables like form factors Colangelo et al. 2014
- Pseudoscalar pole contribution (π^0 dominant Hoferichter et al. 2018):
  - : Singly-virtual transition form factor (TFF)
  - : Doubly-virtual TFF



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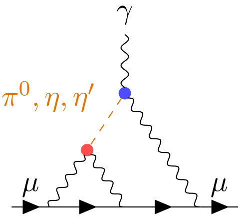
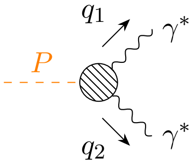




$$i \int \mathrm{d}^4 x \, e^{iq_1 \cdot x} \langle 0 | T\{ j_\mu(x) j_\nu(0) \} | P(q_1 + q_2) \rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

- normalization related to $\Gamma(P \to \gamma\gamma)$ governed by chiral anomaly

Factorization breaking in the η and η' TFFs

- Basic approaches: Application of VMD form factor in the low-energy regime

$$F_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2, q_2^2) \propto \frac{1}{q_1^2 - M_V^2} \times \frac{1}{q_2^2 - M_V^2}$$

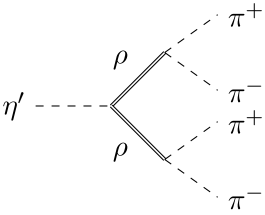
- For high energies ($|q_1^2|, |q_2^2| \to \infty$) pQCD predicts Walsh, Zerwas 1972

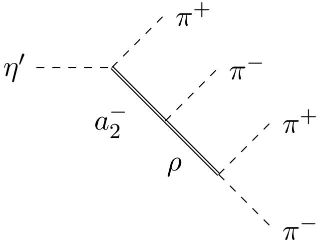
$$F_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2, q_2^2) \propto \frac{1}{q_1^2 + q_2^2}$$

- No factorization in the singly-virtual TFFs present
- Model-independent description of intermediate energy regime with factorization breaking of paramount importance for control over uncertainties
- Exp. study (BaBar 2018) showed for $|q_1^2| = |q_2^2| \in [6.5,\, 45]$GeV^2 VMD factorization is breaking down

Formalism for doubly-virtual representations

- Start from $\eta' \to 2(\pi^+\pi^-)$ amplitude
  - describe decay via two rho resonances by hidden local symmetry (HLS) model Guo, Kubis, Wirzba 2012
  - left-hand-cut contribution due to a_2 exchange by phenomenological Lagrangian models





Final-state interaction

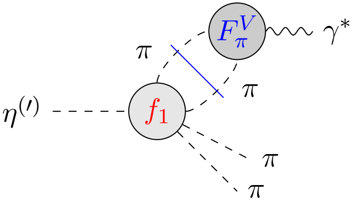
- in HLS amplitude: introduce pair-wise pion rescattering by replacing ρ propagators by Omnès functions
- in a_2 exchange amplitude $\Rightarrow$ inhomogenous Omnès problem

Towards a TFF representation

1st step

- unitarity condition:

$$\mathrm{Im}\, M(\eta^{(\prime)} \to \pi^+\pi^-\gamma^*)$$
$$\sim \int \mathrm{d}\Phi_2\, M(\eta^{(\prime)} \to 2(\pi^+\pi^-)) M(\pi^+\pi^- \to \gamma^*)$$

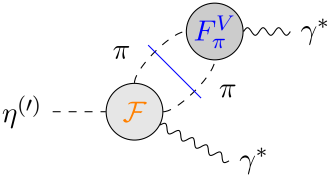


- fix subtraction constants from fit to pion spectra in real photon decays $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ KLOE 2012, BESIII 2017
- a_2 induced LHC leads to curvature effect

2nd step

- apply another (unsubtracted) dispersion relation

$\Rightarrow$ double-spectral representation of isovector doubly-virtual TFF



Putting the pieces together

Construct TFF from four ingredients:

$$F_{\eta^{(\prime)}\gamma^*\gamma^*} = F^{(I=1)}_{\eta^{(\prime)}\gamma^*\gamma^*} + F^{(I=0)}_{\eta^{(\prime)}\gamma^*\gamma^*} + F^{\text{eff}}_{\eta^{(\prime)}\gamma^*\gamma^*} + F^{\text{asym}}_{\eta^{(\prime)}\gamma^*\gamma^*}$$

Isospin 1

- Dispersive piece: offers low-energy description
- reproduces low-energy cuts and singularities
  - additionally, left-hand cut contribution

Isospin 0

- Small; Description of narrow low-energy resonances

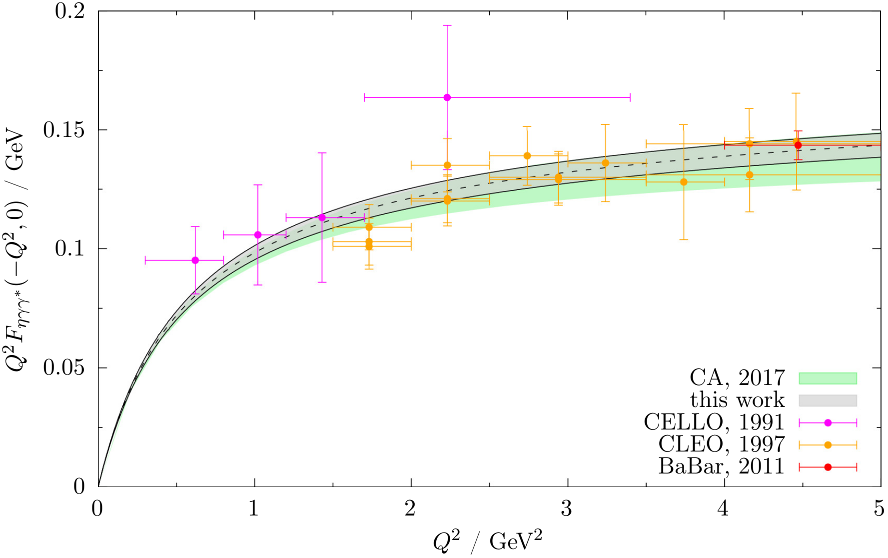
Effective Pole Term

- Parameterize higher intermediate states
- Full saturation of normalization sum rule
- Describe high-energy singly-virtual data

pQCD piece

- Induces leading-twist behavior of TFF ($\mathcal{O}(1/Q^2)$ asymptotics)

η Transition Form Factor



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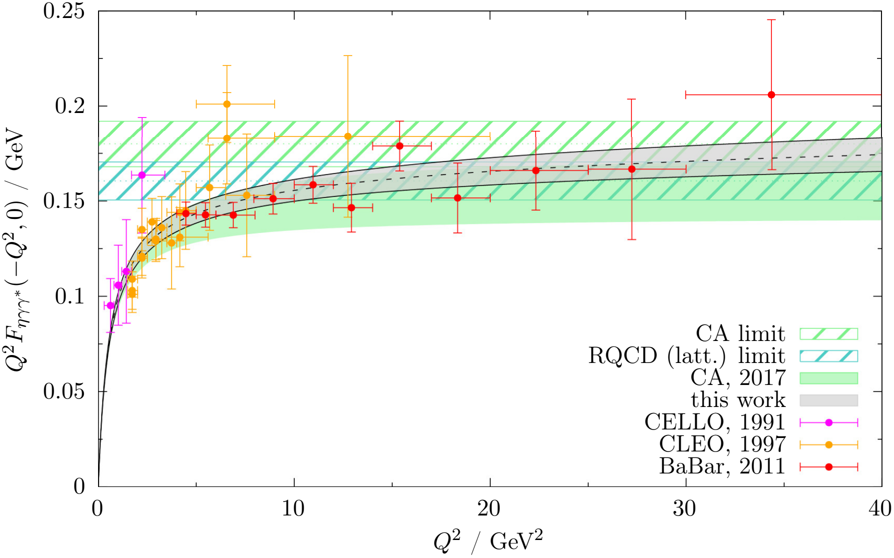

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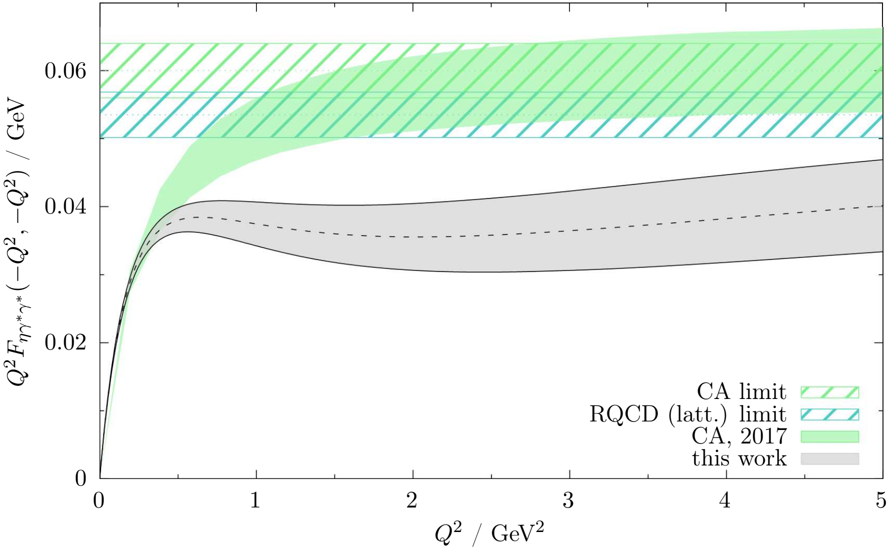

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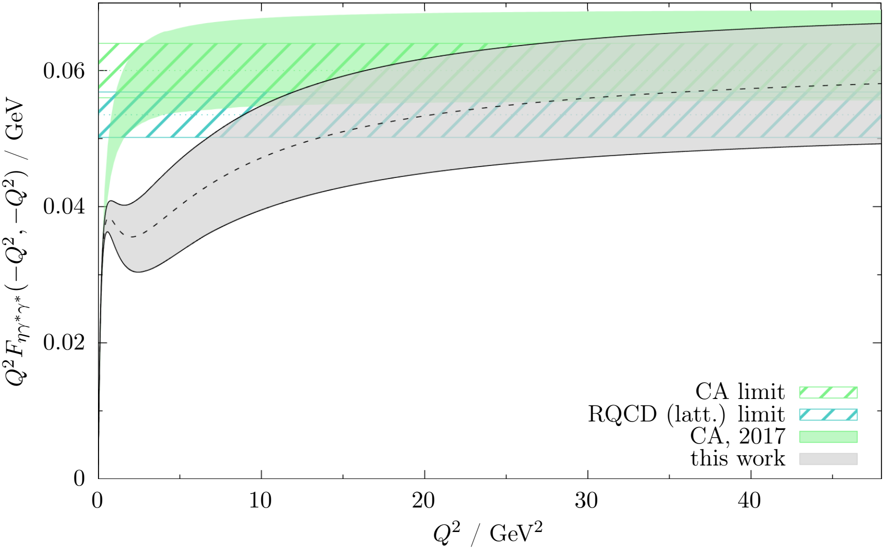
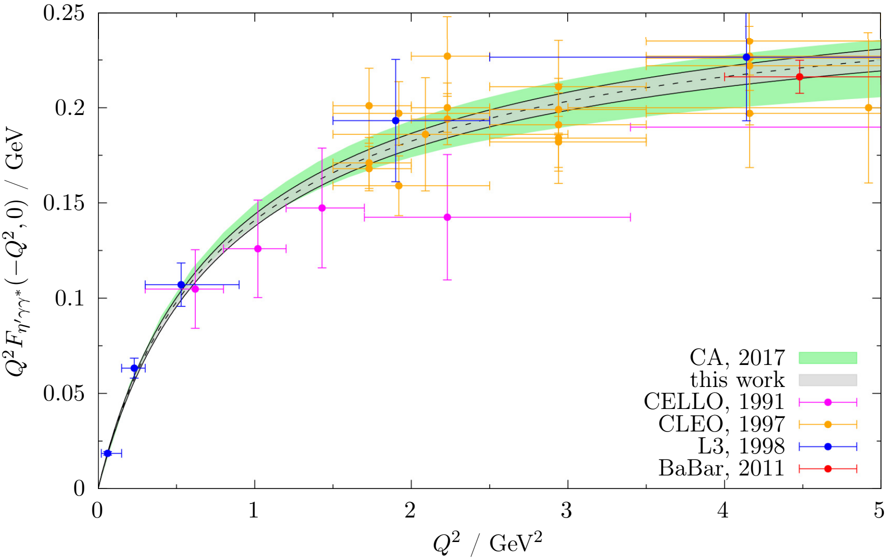
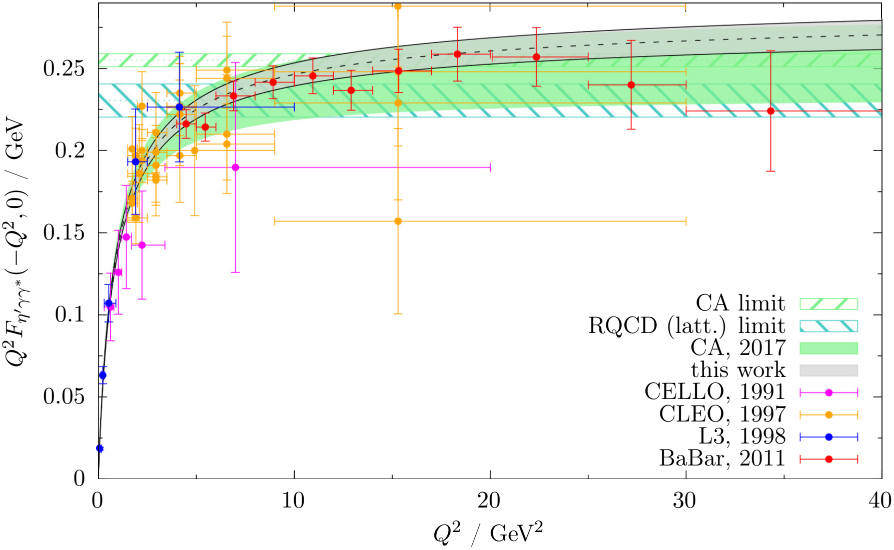


η' Transition Form Factor



η' Transition Form Factor



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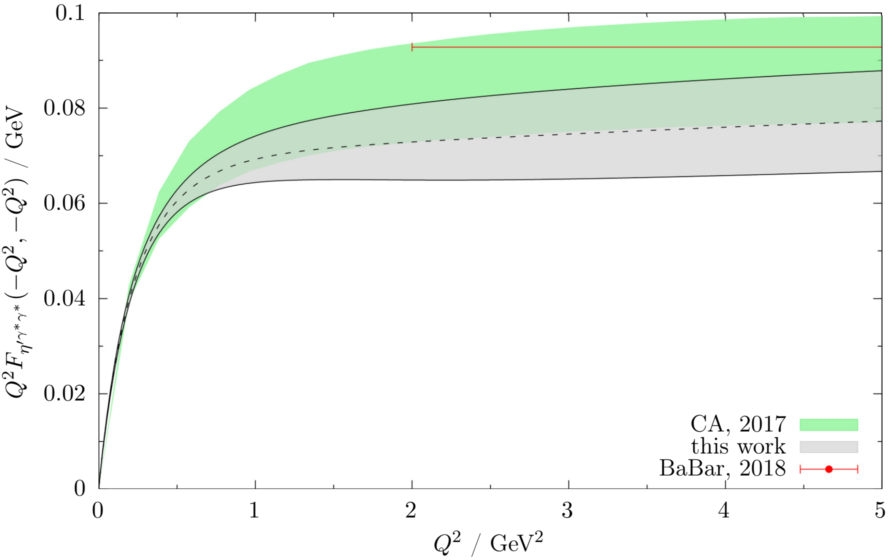

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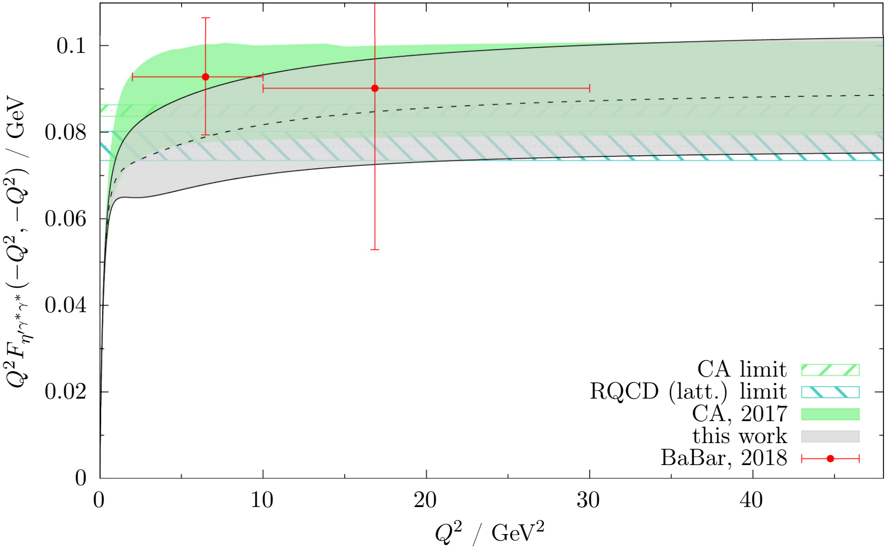
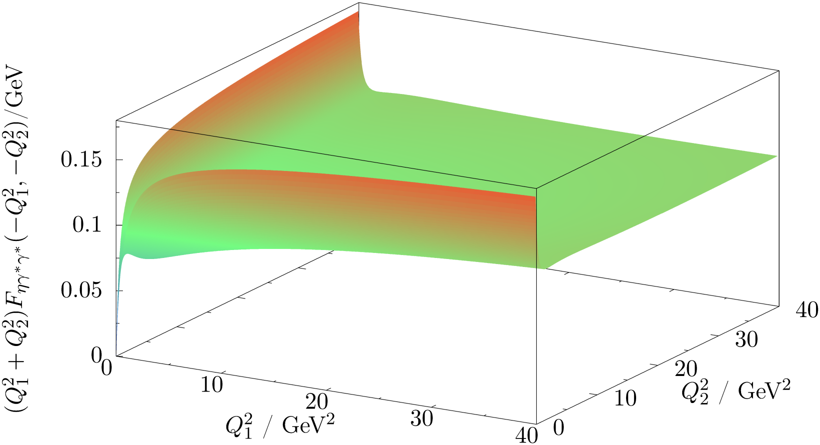
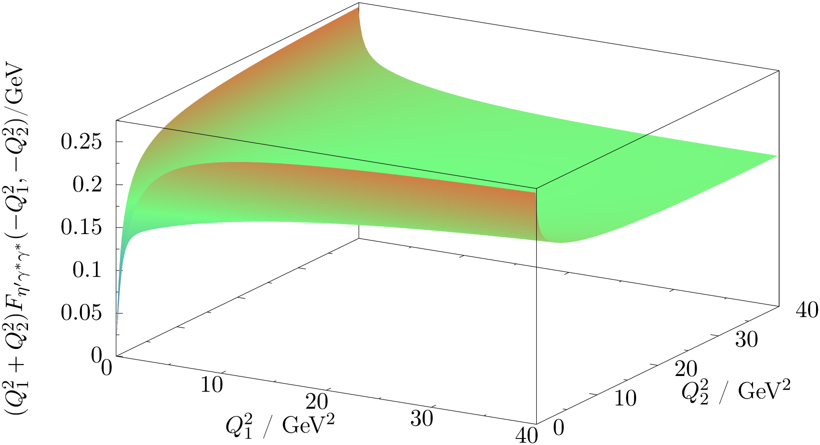


In Q_1^2-Q_2^2-Plane



- $1/Q_i^2$ behavior in entire domain of space-like virtualities

In Q_1^2-Q_2^2-Plane



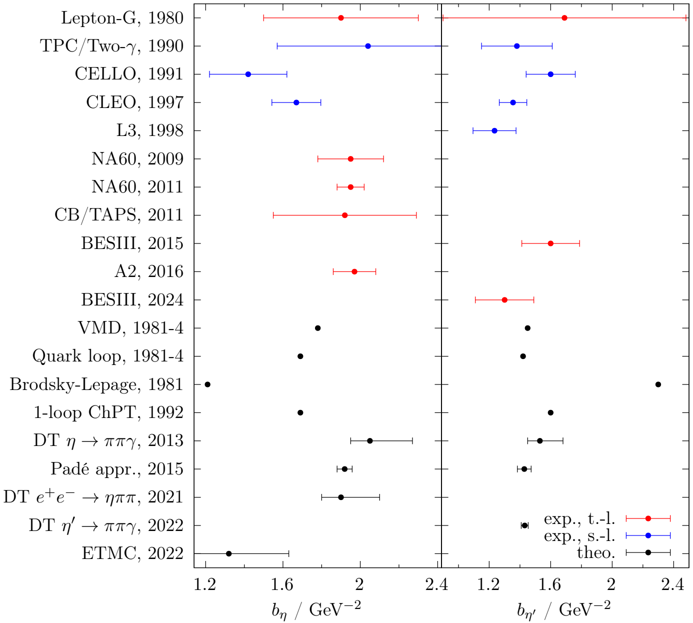
- $1/Q_i^2$ behavior in entire domain of space-like virtualities

Slope Parameters

- TFF symmetric in arguments and analytic

$$\Rightarrow \ b_{\eta^{(\prime)}} := \frac{1}{F_{\eta^{(\prime)}\gamma\gamma}} \frac{\mathrm{d}F_{\eta^{(\prime)}\gamma\gamma^*}(q^2,0)}{\mathrm{d}q^2}\bigg|_{q^2=0} = -\frac{1}{F_{\eta^{(\prime)}\gamma\gamma}} \frac{\mathrm{d}F_{\eta^{(\prime)}\gamma\gamma^*}(-Q^2,0)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Slope Parameters



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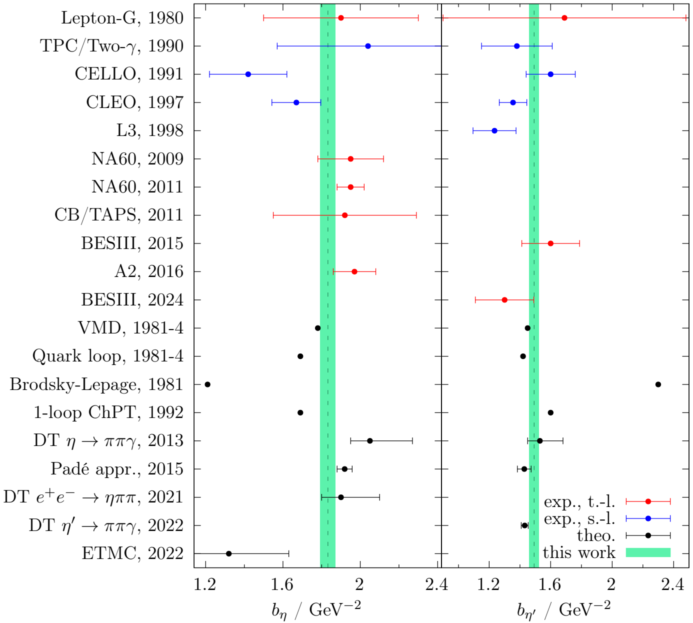
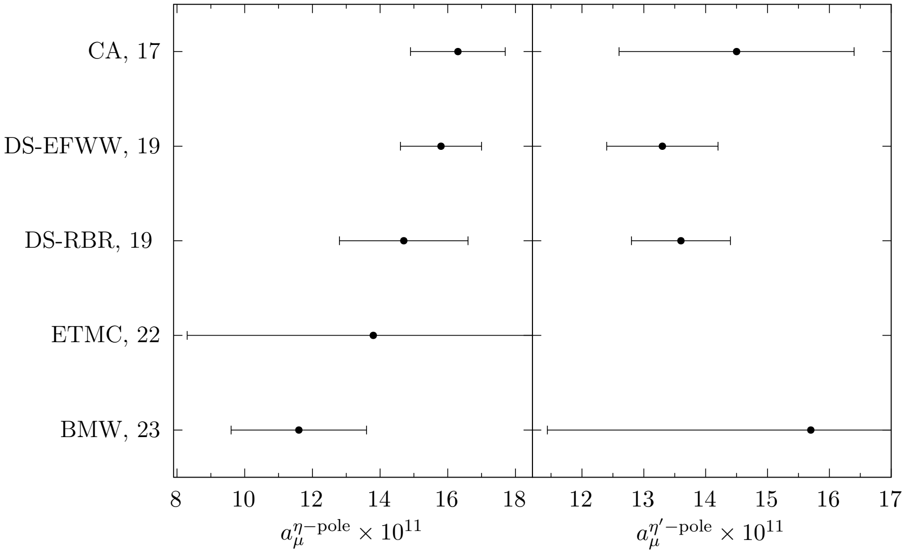


Pole Contributions



Pole Contributions

Blinded results

rel. err of $a_\mu^{\eta^{(\prime)}-\text{pole}}$	disp.	norm	BL	asym.	total
η / %	2.1				
η' / %	1.1				

disp.: dispersive uncertainty

- Variation of cutoffs
- Different representations of pion vector form factor

Pole Contributions

Blinded results

rel. err of $a_\mu^{\eta^{(\prime)}-\text{pole}}$	disp.	norm	BL	asym.	total
η / %	2.1	3.8			
η' / %	1.1	3.6			

norm: normalization uncertainty

- Uncertainty on TFF normalization $F_{\eta^{(\prime)}\gamma\gamma}$
- for η: $1.7\,\%$, for η': $1.6\,\%$ from PDG fit

Pole Contributions

Blinded results

rel. err of $a_\mu^{\eta^{(\prime)}-\mathrm{pole}}$	disp.	norm	BL	asym.	total
η / %	2.1	3.8	1.6		
η' / %	1.1	3.6	1.0		

BL: 'Brodsky-Lepage' uncertainty

- Uncertainty on high-energy singly-virtual TFF
- Dependent on the available exp. data CELLO, CLEO, L3, BaBar

Pole Contributions

Blinded results

rel. err of $a_\mu^{\eta^{(\prime)}-\text{pole}}$	disp.	norm	BL	asym.	total
η / %	2.1	3.8	1.6	2.3	
η' / %	1.1	3.6	1.0	3.6	

asym.: asymptotic uncertainty

- Variation of matching point to pQCD piece
- Variation of (doubly-virtual) asymptotic limits

Pole Contributions

Blinded results

rel. err of $a_\mu^{\eta^{(\prime)}-\text{pole}}$	disp.	norm	BL	asym.	total
η / %	2.1	3.8	1.6	2.3	5.2
η' / %	1.1	3.6	1.0	3.6	5.3

- Unblinding is imminent

Conclusions and Outlook

Dispersive reconstruction of η/η' transition form factors

- Incorporated all of the lowest-lying singularities
- Non-factorizing effects included via a_2-exchange model
- Matched to perturbative QCD for asymptotic piece
- Analogous analysis to dispersive π^0 TFF Hoferichter et al., 2018

Data-driven determination of $a_\mu^{\eta^{(\prime)}-\text{pole}}$

- Carefully estimated improvable uncertainties
- WP 2020 precision goal of $\lesssim 10\,\%$ reached Aoyama et al., 2020
- normalization from experimental input of $\Gamma(\eta^{(\prime)} \to \gamma\gamma)$
- dispersive inputs may be consolidated with additional data for $\eta \to \pi^+\pi^-\gamma$, $e^+e^- \to \eta^{(\prime)}\pi^+\pi^-$, ...
- Additional singly-virtual TFF data in the future?