Radiative modes $K^+ \to \pi^+ \gamma^* \left[\gamma^{(*)}\right]$ and $K^+ \to \pi^+ \ell^+ \ell^- (\gamma)$ decays

Tomáš Husek
Charles University, Prague and *University of Birmingham*

Bochum, Germany
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FACULTY OF MATHEMATICS AND PHYSICS Charles University
UNIVERSITY OF BIRMINGHAM
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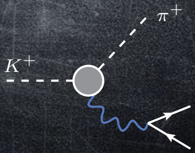
Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

Motivation

Flavor-changing (strangeness-changing) neutral-current weak transitions
$\hookrightarrow$ absent at tree level in Standard Model
$\hookrightarrow$ manifest in radiative non-leptonic kaon decays like $K^+ \to \pi^+ \ell^+ \ell^- (\gamma)$, $\ell = e, \mu$
  $\hookrightarrow$ interesting probe of SM quantum corrections and beyond

Underlying long-distance-dominated radiative modes (transitions) $K^+ \to \pi^+ \gamma^* (\gamma)$ studied before
$\hookrightarrow$ calculated in Chiral Perturbation Theory (ChPT) enriched with electroweak perturbations
$\hookrightarrow$ *Ecker, Pich, de Rafael*, NPB 291 (1987), 303 (1988), at leading order (LO) (at one-loop level)

beyond LO: including the dominant unitarity corrections from $K \to 3\pi$
$\hookrightarrow$ *D'Ambrosio, Ecker, Isidori, Portolés*, JHEP 08 (1998)
$\hookrightarrow$ *Gabbiani*, PRD 59 (1999)



$K^+ \to \pi^+ \gamma^*$ transition

$$\mathcal{M}_\rho \big(K^+(P) \to \pi^+(r) \gamma^*_\rho(k)\big) \equiv i \int \mathrm{d}^4 x \, e^{ikx} \langle \pi(r) | \, T\big[J^{\text{EM}}_\rho(x) \mathcal{L}^{\Delta S=1}(0)\big] \, | K(P) \rangle$$
$$= \frac{e}{2} F(k^2) \big[(P-r)^2 (P+r)_\rho - (P^2 - r^2)(P-r)_\rho\big]$$

Radiative decays $K^+ \to \pi^+ \ell^+ \ell^-$

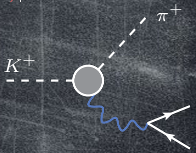
Form-factor parametrization

LO appears at $\mathcal{O}(p^4)$ + unitarity loop correction from $\pi\pi$ rescattering

$\hookrightarrow$ universal parametrization: $F(s) \propto W_+(s/M_K^2)$, $W_+(z) = G_F M_K^2 (a_+ + b_+ z) + W_+^{\pi\pi}(z)$

$\hookrightarrow$ *Ecker et al.*, NPB 291 (1987), *D'Ambrosio et al.*, JHEP 08 (1998)

$$\frac{\mathrm{d}\Gamma_+}{\mathrm{d}z} = \frac{G_F^2 \alpha^2 M_K^5}{3(4\pi)^5} \lambda^{3/2}(z) \sqrt{1 - \frac{4r_\ell^2}{z}} \left(1 + \frac{2r_\ell^2}{z}\right) |W_+(z)|^2$$



LFU

$\hookrightarrow$ a_+ and b_+ should be the same for both (e and μ) channels

$\hookrightarrow$ discrepancy due to new physics via short-distance effects

Moreover, the ratio deviates significantly from the VMD ansatz

$$\text{VMD:} \quad \frac{b_+}{a_+} = \frac{M_K^2}{M_\rho^2} \approx 0.4\,, \qquad \text{exp.:} \quad \frac{b_+}{a_+} \approx 1.25$$

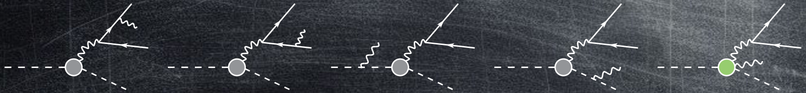
Measurement of quadratic term $c_+ z^2$ may further test the VMD hypothesis

ℓ	a_+	b_+	exp.
e	$-0.587(10)$	$-0.655(44)$	E865
e	$-0.578(16)$	$-0.779(66)$	NA48/2
μ	$-0.575(39)$	$-0.813(145)$	NA48/2
μ	$-0.575(13)$	$-0.722(43)$	NA62(2022)

$\leftarrow$ JHEP 11 (2022) 011

improve precision $\longrightarrow$ radiative corrections, studied earlier: *Kubis and Schmidt*, EPJC 70 (2010)

↪ LO (scalar) QED contributions where the real photon is radiated from lepton (meson) legs



↪ radiation from effective $K^+ \to \pi^+\gamma^*$ vertex ⟶ gauge invariant
  ↪ represented in terms of $F(s)$?

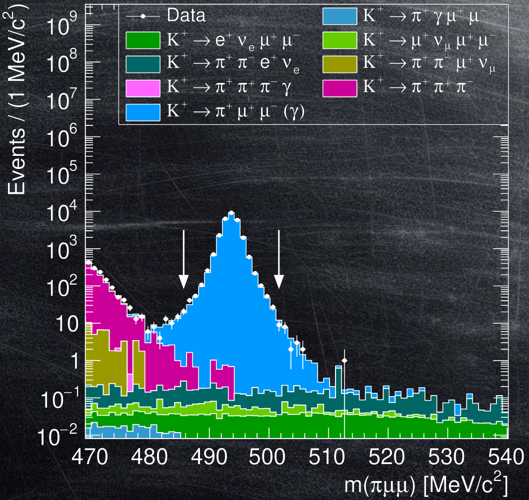
Effective Lagrangian for $K^+ \to \pi^+\gamma^*(\gamma)$ transition

$$\mathcal{L} = \underbrace{ieF(0)D_\mu K^+(w\,\partial_\nu F^{\mu\nu})\pi^-}_{\text{minimal term leading to gauge-invariant amplitude}} - \frac{e^2}{2}\kappa F(0)K^+F_{\mu\nu}F^{\mu\nu}\pi^-$$

$$\mathcal{M}_{\rho\sigma}(K^+(P) \to \pi^+(r)\gamma^*_\rho(k_1)\gamma_\sigma(k_2)) = e^2F(k_1^2)\left\{k_1^2\left(r_\rho\frac{P_\sigma}{P\cdot k_2} - P_\rho\frac{r_\sigma}{r\cdot k_2} + g_{\rho\sigma}\right)\right\} + e^2\tilde{\kappa}F(k_1^2)\left[(k_1\cdot k_2)g_{\rho\sigma} - k_{1\sigma}k_{2\rho}\right]$$

↪ term proportional to $\tilde{\kappa}$
  ↪ mimic the behavior of additional form factors adequately at given order
  ↪ ideally negligible effects in given set-up ⟶ estimate of associated uncertainty

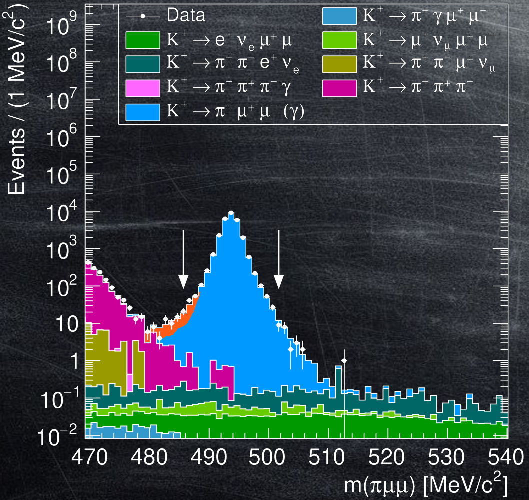
Radiative decays $K^+ \to \pi^+\ell^+\ell^-$
Spectra



(Thanks to *Ľ. Bičian (NA62)*)

Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

Spectra



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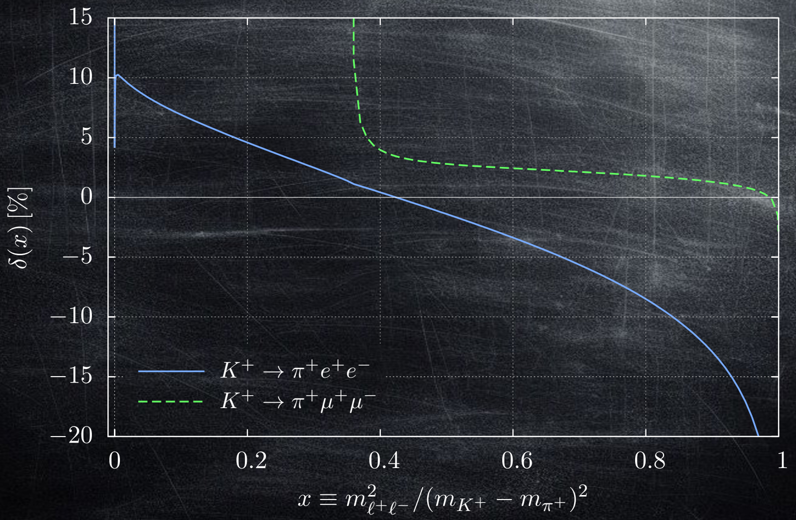
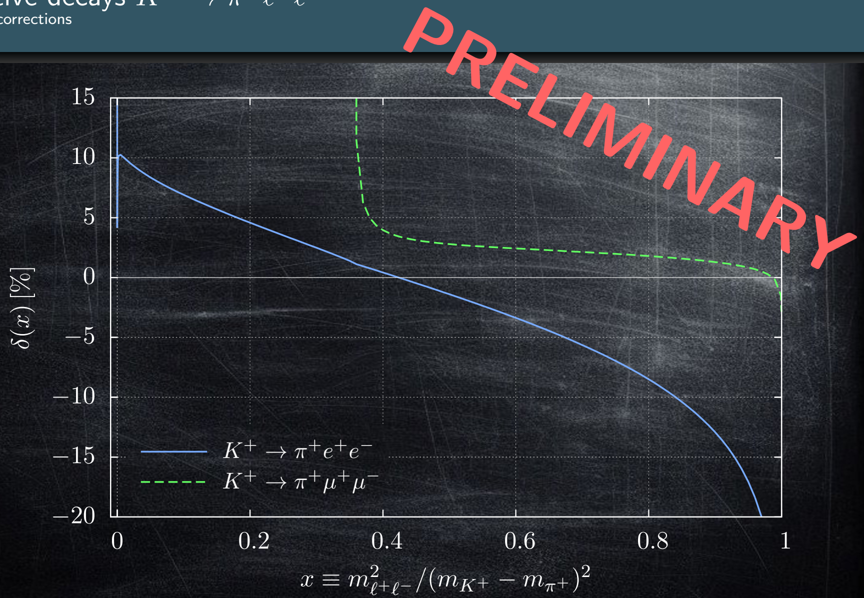
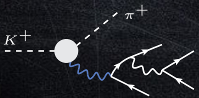

15
10
5
0
−5
−10
−15
−20
$\delta(x)$ [%]
0
0.2
0.4
0.6
0.8
1
$x \equiv m^2_{\ell^+\ell^-}/(m_{K^+} - m_{\pi^+})^2$
$K^+ \to \pi^+e^+e^-$
$K^+ \to \pi^+\mu^+\mu^-$

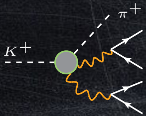
Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

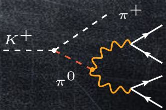
Radiative corrections



$K^+ \to \pi^+ e^+ e^- e^+ e^-$ decay







$K^+ \to \pi^+ e^+ e^- e^+ e^-$
Introduction

The long-distance-dominated $K^+ \to \pi^+\gamma^*$ transition essential also for $K^+ \to \pi^+ 4e$
$\hookrightarrow$ one also needs to consider $K^+ \to \pi^+\gamma^*\gamma^*$ transition

Challenging to observe $K^+ \to \pi^+ 4e$ away from $m_{4e} \simeq M_{\pi^0}$
$\hookrightarrow$ suppressed decay rate $\longrightarrow$ attractive to study possible effects of BSM physics
$\hookrightarrow$ to identify new-physics-scenario contribution $\longrightarrow$ need for (rough) estimate of SM rate
  $\hookrightarrow$ new-physics effects spotted as deviations from such SM predictions

$$B(K^+ \to \pi^+ 4e,\ \text{non-res.}) = 7.2(7) \times 10^{-11}$$

$\longrightarrow$ possible BSM scenarios are being explored

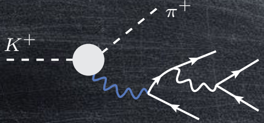
Hostert, Pospelov, PRD 105 (2022)
$\hookrightarrow$ $K \to \pi 4e$ decays proceed via $K \to \pi(X' \to XX)$ intermediate states
  $\hookrightarrow$ cascade of dark-sector particles $X^{(\prime)}$
  $\hookrightarrow$ underlying dynamics potentially significantly enhanced compared to the SM case

$\longrightarrow$ searches in suitable experiments
$\hookrightarrow$ more precise knowledge of SM background essential
  $\hookrightarrow$ ideally at level suited for Monte Carlo (MC) implementation
$\hookrightarrow$ NA62 upper limit: $B(K^+ \to \pi^+ 4e) < 1.4 \times 10^{-8}$
  $\hookrightarrow$ PLB 846 (2023) 138193

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

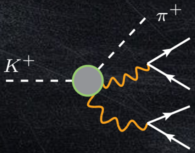
Standard Model prediction: Topologies

One-photon-exchange topology

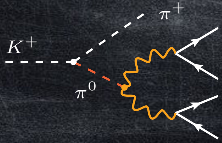


(1)

Two-photon-exchange topology



(2a)



(2b)

TH, PRD 106 (2022)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Two-photon-exchange topology: Matrix element

The two-photon transition of topology (2a) can be written, approximately, as follows:

$$
\begin{aligned}
&\mathcal{M}^{(a)}_{\rho\sigma}\left(K(P) \to \pi(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\right) \\
&\simeq e^2 F(k_1^2)\Big\{(k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\,\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} - (k_1^2 P_\rho - P\cdot k_1 k_{1\rho})\,\frac{(2r+k_2)_\sigma}{2r\cdot k_2 + k_2^2} \\
&\qquad\qquad + \left(k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma}\right) \\
&\qquad\qquad + \widetilde{\kappa}\left[(k_1\cdot k_2) g_{\rho\sigma} - k_{1\sigma}k_{2\rho}\right]\Big\} \\
&+ \{k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\}
\end{aligned}
$$

↪ in this model depends on a single form factor (the same $F(s)$)

↪ useful when measuring $F(s) \longrightarrow$ radiative corrections for the $K^+ \to \pi^+\ell^+\ell^-$ decay

↪ one of the photons on-shell

Soft-photon regime $\longrightarrow$ approximation justified

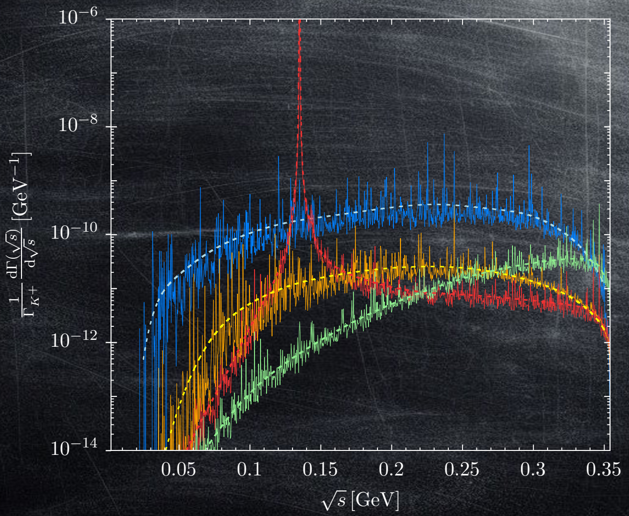
Hard photons $\longrightarrow$ free parameter $|\widetilde{\kappa}| \lesssim 1$ introduced to cover model uncertainty

↪ physical results do not seem to be sensitive to this parameter

For $K^+ \to \pi^+ 4e$, we assume it is good enough (at least) as an order-of-magnitude guess

↪ numerically negligible (one order of magnitude) compared to the topology (1)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$
Contributions to the branching ratio



[large MC samples generated by A. Shaikhiev, E. Goudzovski] *TH*, PRD 106 (2022)

$K^+ \to \pi^+\gamma^*\gamma^*$ transition

WORK IN PROGRESS

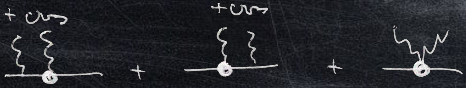
$K^+ \to \pi^+\gamma^*\gamma^*$ transition

$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

$K^+ \to \pi^+\gamma^{(*)}$ transition

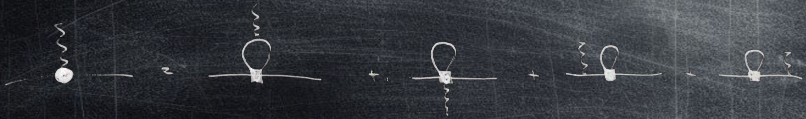
$$= ieF^{(\mathrm{LO})}_{\mathrm{ChPT}}(k^2)\left[k^2 r_\rho - (k\cdot r)k_\rho\right] + e\widetilde{F}(k^2)(M_K^2 + M_\pi^2)(P+r)_\rho$$

$K^+ \to \pi^+\gamma^{(*)}\gamma^{(*)}$ transition



$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

$K^+ \to \pi^+\gamma^{(*)}$ transition



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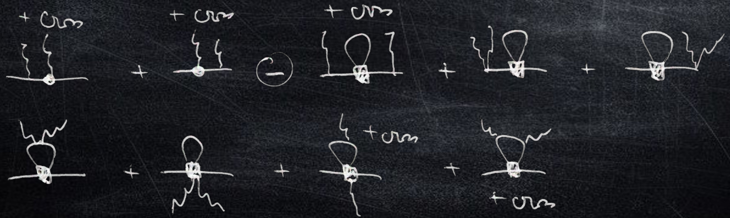
$K^+ \to \pi^+\gamma^{(*)}\gamma^{(*)}$ transition



$K^+ \to \pi^+\gamma^*\gamma^*$ transition at LO in ChPT

$$\mathcal{M}_{\rho\sigma}\left(K^+(P) \to \pi^+(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\right)$$
$$= e^2 F(k_1^2) T^{(0)}_{\rho\sigma} + \left(k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\right)$$
$$+ e^2 C_1 \epsilon_{\rho\sigma(k_1)(k_2)}$$
$$+ e^2 \left[T^{(1)}_{\rho\sigma} A^{(+,1)}_{\gamma^*\gamma^*} - 2T^{(2)}_{\rho\sigma} A^{(+,2)}_{\gamma^*\gamma^*}\right] + 4e^2 (k_1 \cdot k_2) T^{(1*)}_{\rho\sigma} \left[-\hat{c} + \kappa \frac{(k_1+k_2)^2}{2k_1 \cdot k_2}\right] - 4e^2 T^{(2)}_{\rho\sigma} \kappa \frac{(k_1+k_2)^2}{2k_1 \cdot k_2}$$

$$T^{(0)}_{\rho\sigma} = (k_1^2 r_\rho - r \cdot k_1 k_{1\rho}) \frac{(2P-k_2)_\sigma}{2P \cdot k_2 - k_2^2} + (P \leftrightarrow -r) + (k_1^2 g_{\rho\sigma} - k_{1\rho} k_{1\sigma})$$

$$T^{(1)}_{\rho\sigma} = g_{\rho\sigma} - \frac{(k_1 \cdot k_2)\left(k_{1\sigma} k_{2\rho} + k_{1\rho} k_{2\sigma}\right) - k_1^2 k_{2\rho} k_{2\sigma} - k_2^2 k_{1\rho} k_{1\sigma}}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2}, \qquad T^{(1*)}_{\rho\sigma} = g_{\rho\sigma} - \frac{k_{1\sigma} k_{2\rho}}{k_1 \cdot k_2}$$

$$T^{(2)}_{\rho\sigma} = \frac{\left[k_1^2 k_{2\rho} - (k_1 \cdot k_2) k_{1\rho}\right]\left[k_2^2 k_{1\sigma} - (k_1 \cdot k_2) k_{2\sigma}\right]}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2}, \qquad T^{(2*)}_{\rho\sigma} = 0$$

$$A^{(+,i)}_{\gamma^*\gamma^*} = \left[M_K^2 - M_\pi^2 + s\right] \hat{A}^{(i)}_{\gamma^*\gamma^*}(M_\pi^2) - \left[M_K^2 - M_\pi^2 - s\right] \hat{A}^{(i)}_{\gamma^*\gamma^*}(M_K^2)$$

$K_{\mathsf{L}} \to \pi^0\gamma^*\gamma^*$ transition at LO in ChPT

$$\mathcal{M}_{\rho\sigma}\big(K_{\mathsf{L}}(P) \to \pi^0(r)\gamma^*_\rho(k_1)\gamma^*_\sigma(k_2)\big)$$
$$= -2e^2\left[T^{(1)}_{\rho\sigma}A^{(0,1)}_{\gamma^*\gamma^*} - 2T^{(2)}_{\rho\sigma}A^{(0,2)}_{\gamma^*\gamma^*}\right] - 2e^2\kappa M_K^2 T^{(1)}_{\rho\sigma}$$

$$T^{(0)}_{\rho\sigma} = (k_1^2 r_\rho - r\cdot k_1 k_{1\rho})\,\frac{(2P-k_2)_\sigma}{2P\cdot k_2 - k_2^2} + (P \leftrightarrow -r) + \big(k_1^2 g_{\rho\sigma} - k_{1\rho}k_{1\sigma}\big)$$

$$T^{(1)}_{\rho\sigma} = g_{\rho\sigma} - \frac{(k_1\cdot k_2)\big(k_{1\sigma}k_{2\rho} + k_{1\rho}k_{2\sigma}\big) - k_1^2 k_{2\rho}k_{2\sigma} - k_2^2 k_{1\rho}k_{1\sigma}}{(k_1\cdot k_2)^2 - k_1^2k_2^2}\,, \qquad T^{(1*)}_{\rho\sigma} = g_{\rho\sigma} - \frac{k_{1\sigma}k_{2\rho}}{k_1\cdot k_2}$$

$$T^{(2)}_{\rho\sigma} = \frac{\big[k_1^2 k_{2\rho} - (k_1\cdot k_2)k_{1\rho}\big]\big[k_2^2 k_{1\sigma} - (k_1\cdot k_2)k_{2\sigma}\big]}{(k_1\cdot k_2)^2 - k_1^2k_2^2}\,, \qquad T^{(2*)}_{\rho\sigma} = 0$$

$$A^{(0,i)}_{\gamma^*\gamma^*} = \big[s - M_\pi^2\big]\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_\pi^2) + \big[M_K^2 + M_\pi^2 - s\big]\hat{A}^{(i)}_{\gamma^*\gamma^*}(M_K^2)$$

$K \to \pi\gamma^{(*)}\gamma^{(*)}$ transition at LO in ChPT

Comparison with literature

The above results reduce to special cases present in literature:

$K^+ \to \pi^+\gamma\gamma$

↪ *Ecker, Pich, de Rafael*, Nucl. Phys. B **291** (1987), B **303** (1988)

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$K^+ \to \pi^+\gamma\gamma^*$

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$K_\mathsf{L} \to \pi^0\gamma\gamma$

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$K \to \pi\gamma^{(*)}\gamma^{(*)}$ transition at LO in ChPT
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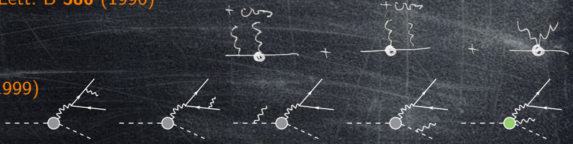

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$K_L \to \pi^0\gamma\gamma^*$
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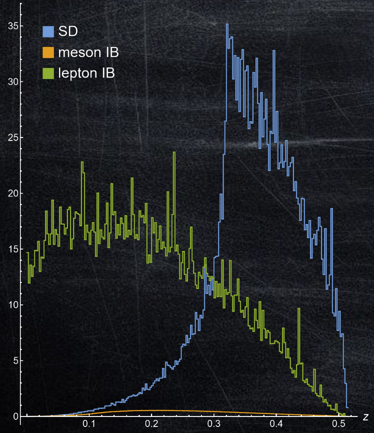
$K^+ \to \pi^+ e^+ e^- \gamma$

$E_\gamma^* > 30\,\text{MeV}$

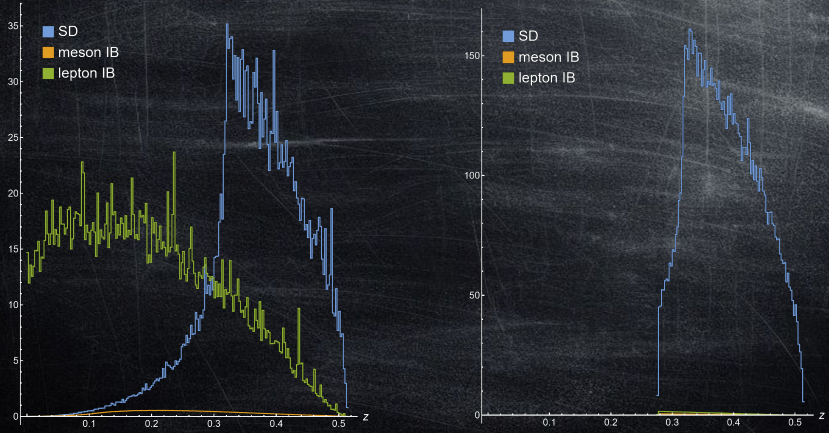


$K^+ \to \pi^+ e^+ e^- \gamma$

$E^*_\gamma > 30\,\text{MeV}$

"NA48 cuts" in $e^+e^-\gamma$ rest frame [PLB 659 (2008) 493]

$$\hookrightarrow z \equiv \frac{m^2_{ee\gamma}}{M_K^2} > 0.277\,, \quad \sphericalangle(e^+, e^-) < \text{min}(\sphericalangle(e^\pm, \gamma))$$



$K^+ \to \pi^+\mu^+\mu^-\gamma$

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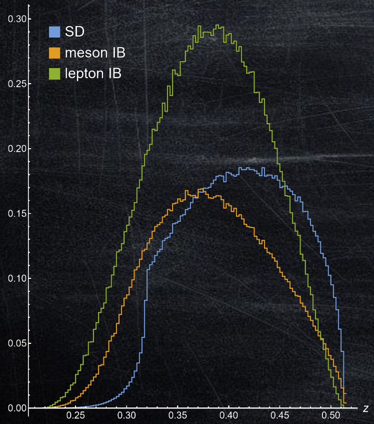

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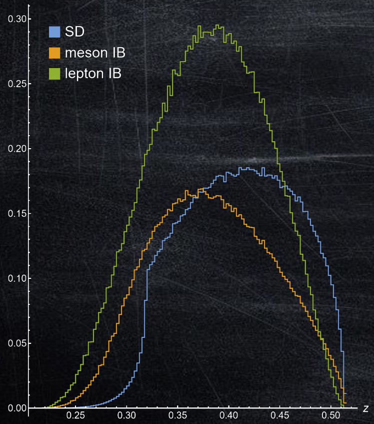
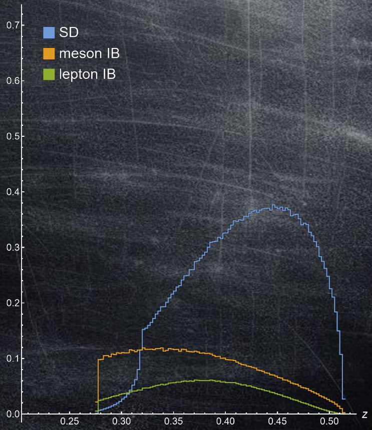
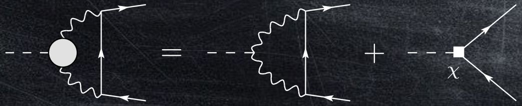


"NA48 cuts"



Rare electromagnetic decay $\pi^0 \to e^+e^-$



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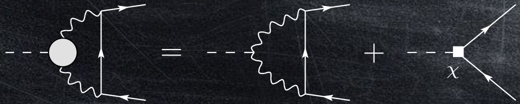


See also talk by M. Kovaľ on Aug 29, 15:40
$\hookrightarrow$ *"New $\pi^0 \to e^+e^-$ result from NA62"*

$\pi^0 \to e^+e^-$ update

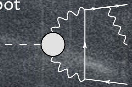
Short review paper on radiative corrections

Neutral-pion decay into an electron-positron pair: A review and update

$\hookrightarrow$ Phys. Rev. D 110 (2024) 033004

$\hookrightarrow$ accompanies the latest branching-ratio measurement at NA62 (preliminary results)

$\hookrightarrow$ revises, discusses, and summarizes radiative correction for $\pi^0 \to e^+e^-$ in one spot

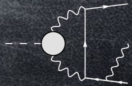


$\hookrightarrow$ describes relation between the theoretical and experimental observables

$\hookrightarrow$ $B(\pi^0 \to e^+e^-(\gamma), x > x_{\text{cut}}) = [1 + \delta(x_{\text{cut}})] B(\pi^0 \to e^+e^-)$

$\hookrightarrow$ updates the critical NLO QED correction $\delta_+(x_{\text{cut}})$ that relates these

$\hookrightarrow$ $\delta_+(0.95) = [-6.06(7)_\xi - 0.08\widetilde{\chi}]\% = -6.1(2)\%$



$\hookrightarrow$ calculates the overall correction $\delta = [10.67(7)_\xi - 0.25\widetilde{\chi}]\% = 10.7(1)_\xi(2)_\chi\%$

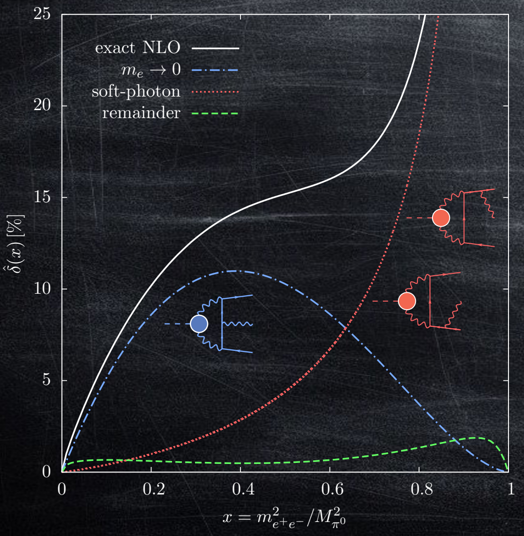
$\hookrightarrow$ derives other related useful experimental quantities / ratios

$\hookrightarrow$ discusses the latest measurements (KTeV, NA62)

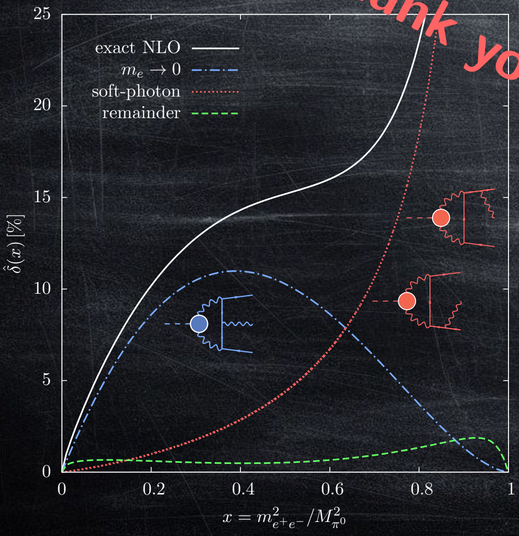
$\hookrightarrow$ properties of the $\pi^0 \to e^+e^-(\gamma)$ amplitude in various limits

$\hookrightarrow$ discusses its relation with the Dalitz-decay radiative corrections

Properties of $\pi^0 \to e^+e^-(\gamma)$ amplitude in various limits



Properties of $\pi^0 \to e^+e^-(\gamma)$ amplitude in various limits



Thank you for listening!