# The decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$ in the standard model

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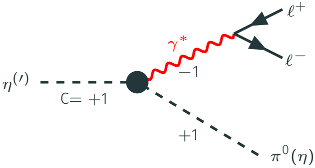
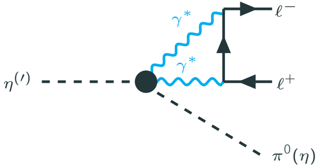
HISKP UNI BONN NRW-FAIR Netzwerk bctp

- Matter-antimatter asymmetry in the universe needs C and CP violation Sakharov 1967
- CP violation in the weak (CKM phase, C and CP violation) and strong ($\theta$ term, P and CP violation) interaction not enough to explain observed asymmetry – search for **additional sources of C and CP violation**
- $\eta$ is C and P eigenstate: ideal test case
- Semileptonic decay channels C and CP violating for $\gamma^*$ intermediate state
- C preserved for $\gamma^*\gamma^*$ intermediate state



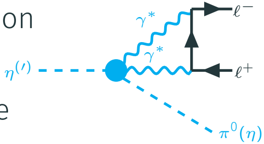
tree-level



one-loop

# Semileptonic $\eta^{(\prime)}$ decays in the standard model

- Calculation of $\eta^{(\prime)} \to \pi^0(\eta)\ell^+\ell^-$ based on $\eta^{(\prime)} \to \pi^0(\eta)\gamma^*\gamma^*$ (similar to $P \to \ell^+\ell^-$)
- Transition form factor has to regularize loop integral

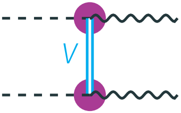



- Several theory calculations Llewellyn Smith 1967; Smith 1968; Cheng 1967 , ... , most recently Escribano and Royo 2020
- Models: effective operators, dispersive reconstruction, vector-meson dominance (VMD), unitarity bounds
- Today: better understanding of $\eta^{(\prime)} \to \pi^0(\eta)\gamma^*\gamma^*$, more experimental input, computational resources
  $\to$Improved calculation HS, Zanke, Korte, Kubis 2023

# Modeling the hadronic part

- In ChPT, $\eta \to \pi^0\gamma\gamma$ is suppressed Ametller et al. 1992, Jetter 1996
  - No $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ tree level diagrams as $\eta^{(\prime)}, \pi^0$ neutral
  - Loop contributions suppressed ($\pi$ loops violate isospin symmetry, $K$ loops by $1/M_K^2$ and combinatorical factors)
- Dominant contribution: counter terms at $\mathcal{O}(p^6)$, estimated by resonance saturation $\to$ **vector-meson dominance**
- Scalar rescattering contributions of moderate size Oset et al. 2003, 2008; Lu and Moussallam 2020
- Similar for $\eta'$ decays Escribano, Gonzàlez-Solís, et al. 2020
- Consider lowest-lying neutral VM as exchange particles: $\rho, \omega, \phi \to$ left-hand cut in the amplitude

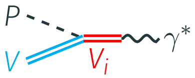
$$\mathcal{M} \sim \sum_{V\in\{\rho,\omega,\phi\}} \mathcal{F}_{V\eta}\,\mathcal{F}_{V\pi^0}\,\mathrm{Prop}_V$$



# Form factor $\mathcal{F}_{VP}$: definition

$$\langle P(p)|j_\mu(0)|V(p_V)\rangle = e\,\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu(p_V)p^\alpha q^\beta \mathcal{F}_{VP}(q^2) \qquad q{=}p_V{-}p$$

- Previous VMD calculations: constant instead of energy-dependent form factors – real photons
- Energy dependence – virtual photons; otherwise, loop integral generally not convergent
- Form factor $\mathcal{F}_{VP}$ modeled via VMD




- Possible combinations of $V$ and $V_i$ determined by isospin and U(3) symmetry

| | $V\pi^0\gamma$ | | | $V\eta^{(\prime)}\gamma$ | | |
|---|---|---|---|---|---|---|
| $V$ | $\rho$ | $\omega$ | $\phi$ | $\rho$ | $\omega$ | $\phi$ |
| $V_i$ | $\omega^{(\prime)}$ | $\rho^{(\prime)}$ | $\rho^{(\prime)}$ | $\rho^{(\prime)}$ | $\omega^{(\prime)}$ | $\phi^{(\prime)}$ |

# Form factor $\mathcal{F}_{VP}$: parametrization

- Coupling for real photons fixed phenomenologically by $V \to P\gamma / P \to V\gamma$ decays, signs by U(3) flavor symmetry

$$\mathcal{F}_{VP}(q^2 = 0) = C_{VP\gamma} \equiv \mathcal{F}_{VP}^{\text{PL}} \qquad \text{point-like (PL)}$$

- Simplest parametrization with $\mathcal{O}(q^{-2})$ asymptotic behavior: **monopole model (MP)**

$$\mathcal{F}_{VP}^{\text{MP}}(q^2) = C_{VP\gamma} M_{V_i}^2 P_{V_i}(q^2)$$

- Expected high energy behavior $\mathcal{O}(q^{-4})$ Cherniyak et al. 1984 $\to$ **dipole model (DP)**, include next-higher multiplet of VM, $\rho' = \rho^0(1450), \omega' = \omega(1420), \phi' = \phi(1680)$

$$\mathcal{F}_{VP}^{\text{DP}}(q^2) = C_{VP\gamma}\left[(1 - \epsilon_{V_i}) M_{V_i}^2 P_{V_i}(q^2) + \epsilon_{V_i} M_{V_i'}^2 P_{V_i'}(q^2)\right],$$

$\epsilon_{V_i}$ tuned such that $\mathcal{O}(q^{-2})$ terms cancel

# Spectral representation of vector meson propagators

- $\omega$ and $\phi$ narrow resonances $\to$ Breit-Wigner (BW) propagators good description

$$P_V^{\mathrm{BW}}(q^2) = \frac{1}{q^2 - M_V^2 + \mathrm{i} M_V \Gamma_V}$$

- Not true for $\rho^{(\prime)}, \omega', \phi' \to$ dispersively improved propagator

$$P_V^{\mathrm{disp}}(q^2) = -\frac{1}{\pi}\int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}x \, \frac{\mathrm{Im}\,[P_V^{\mathrm{BW}}(x)]}{q^2 - x + \mathrm{i}\epsilon}$$

$$\text{with} \quad \mathrm{Im}\,[P_V^{\mathrm{BW}}(x)] = \frac{-\sqrt{x}\,\Gamma_V(x)}{(x - M_V^2)^2 + x\Gamma_V(x)^2}$$

- Energy-dependent widths avoid unphysical imaginary parts

# Strategies to solve the loop integral

- Separation of spinor part in matrix element

$$\mathcal{M} = 16\pi^2\alpha^2\left[\mathcal{M}_{\text{QED}}^{uv}\mathcal{M}_{\text{H}}^{uv} + \mathcal{M}_{\text{QED}}^{u0v}\mathcal{M}_{\text{H}}^{u0v}\right], \quad \mathcal{M}_{\text{QED}}^{uv}=m_\ell \bar{u}_s v_r, \; \mathcal{M}_{\text{QED}}^{u0v}=\bar{u}_s \not{p}_0 v_r$$

- Separation of hadronic part according to coupling constants $\rightarrow$ uncertainty estimation: $\Delta C_V$ dominant

$$\mathcal{M}_{\text{H}}^{u(0)v} = \sum_V C_V \mathcal{M}_V^{u(0)v}, \quad C_V = C_{V\eta^{(\prime)}\gamma} C_{V[\pi^0/\eta]\gamma}$$

- Passarino-Veltman decomposition Passarino, Veltman; ’t Hooft, Veltman 1979 of $\mathcal{M}_V^{u(0)v}$ using package *FeynCalc* Mertig 1990, Shtabovenko 2016, 2020, numerical evaluation with *Collier* Denner and Dittmaier 2003, 2006, 2011

# Observables: branching ratio BR and $\widehat{\text{BR}}$

- $\text{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \frac{\Gamma}{\Gamma_{\eta^{(\prime)}}}$
- $\widehat{\text{BR}}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \frac{\text{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-)}{\text{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)}$

  $\to$ $\Delta C_V$ cancel partially
- Calculation of $\text{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)$ in the same framework

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{64 M_{\eta^{(\prime)}}^3}|\overline{\mathcal{M}}|^2 \mathrm{d}s\,\mathrm{d}\nu \quad \text{and} \quad \mathrm{d}\Gamma_\gamma = \frac{1}{(2\pi)^3}\frac{1}{32 M_{\eta^{(\prime)}}^3}|\overline{\mathcal{M}}_\gamma|^2 \mathrm{d}s_\gamma\,\mathrm{d}t_\gamma$$

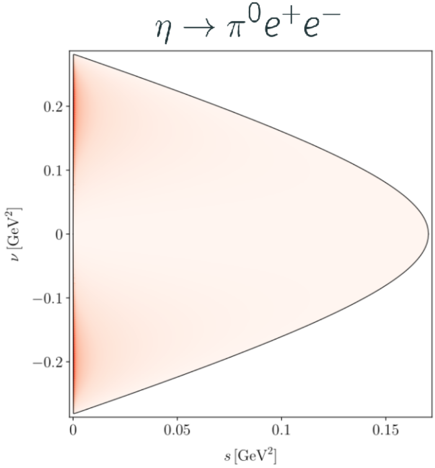
$$\Gamma_{(\gamma)} = C_\rho^2 \Gamma_{\rho,\rho}^{(\gamma)} + C_\omega^2 \Gamma_{\omega,\omega}^{(\gamma)} + C_\phi^2 \Gamma_{\phi,\phi}^{(\gamma)} + C_\rho C_\omega \Gamma_{\rho,\omega}^{(\gamma)} + C_\rho C_\phi \Gamma_{\rho,\phi}^{(\gamma)} + C_\omega C_\phi \Gamma_{\omega,\phi}^{(\gamma)}$$

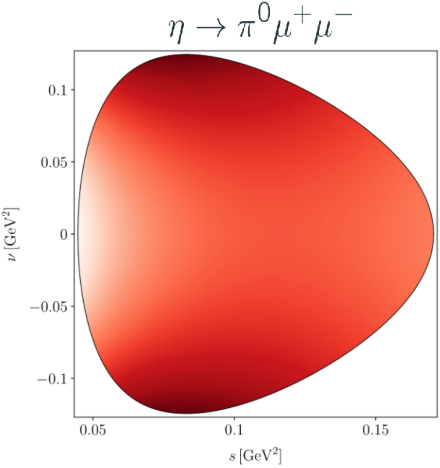
$$s = m_{\ell\ell}^2, \nu = t - u$$

- Numerical integration (phase space and spectral parameters) with *Cuba* library Hahn 2005 (*Cuhre, Vegas*)

  $\to$ numerical uncertainties negligible

# Results: Dalitz plot



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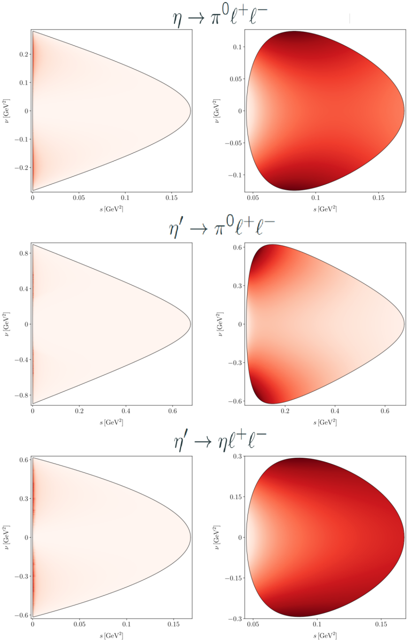
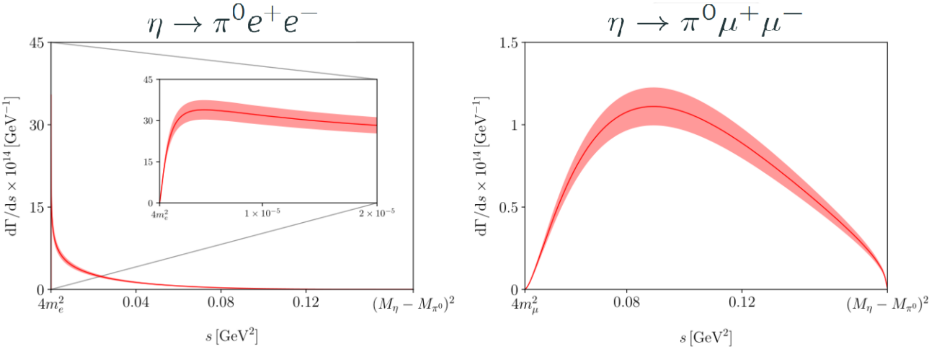



- Electrons in the final state: largest contribution for $s = m_{\ell\ell}^2$ very close to lower threshold
  $\to$ important for numerical integration and experimental measurement
- Muons: distribution more centered and spread out




- $\log(s)$ singularity from $\gamma^*\gamma^*$ intermediate state present in $e^+e^-$ channel
- Further away from $\mu^+\mu^-$-channel phase space

# Results: branching ratio

| | | PL / $10^{-9}$ | MP / $10^{-9}$ | DP / $10^{-9}$ | ER / $10^{-9}$ |
|---|---|---|---|---|---|
| $\eta \to \pi^0 e^+ e^-$ | CW | 2.10(23) | 1.35(15) | 1.33(15) | 2.0(1)(1)(1) |
| | VW | 2.06(22) | 1.40(15) | 1.36(15) | |
| $\eta \to \pi^0 \mu^+ \mu^-$ | CW | 1.37(15) | 0.70(8) | 0.66(7) | 1.1(1)(1)(1) |
| | VW | 1.32(14) | 0.71(8) | 0.67(7) | |
| $\eta' \to \pi^0 e^+ e^-$ | CW | 3.82(33) | 3.08(27) | 3.14(27) | 4.5(3)(4)(4) |
| | VW | 3.81(33) | 3.30(28) | 3.30(28) | |
| $\eta' \to \pi^0 \mu^+ \mu^-$ | CW | 2.57(23) | 1.69(15) | 1.68(15) | 1.7(1)(2)(2) |
| | VW | 2.53(23) | 1.81(16) | 1.81(16) | |
| $\eta' \to \eta e^+ e^-$ | CW | 0.53(4) | 0.48(4) | 0.49(4) | 0.43(3)(2)(18) |
| | VW | 0.51(4) | 0.50(4) | 0.50(4) | |
| $\eta' \to \eta \mu^+ \mu^-$ | CW | 0.287(26) | 0.213(18) | 0.207(18) | 0.15(1)(1)(5) |
| | VW | 0.280(25) | 0.225(20) | 0.240(21) | |

ER: Escribano and Royo 2020

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ER: Escribano and Royo 2020

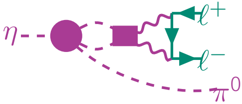
# Comparison to experiment

| | Branching ratio | | |
|---|---|---|---|
| | DP VW | exp.limit | |
| $\eta \to \pi^0 e^+ e^-$ | $1.36(15) \times 10^{-9}$ | $< 7.5 \times 10^{-6}$ | Adlarson et al. 2018 |
| $\eta \to \pi^0 \mu^+ \mu^-$ | $6.7(7) \times 10^{-10}$ | $< 5 \times 10^{-6}$ | Dzhelyadin et al. 1981 |
| $\eta' \to \pi^0 e^+ e^-$ | $3.30(28) \times 10^{-9}$ | $< 1.4 \times 10^{-3}$ | Briere et al. 2000 |
| $\eta' \to \pi^0 \mu^+ \mu^-$ | $1.81(16) \times 10^{-9}$ | $< 6 \times 10^{-5}$ | Dzhelyadin et al. 1981 |
| $\eta' \to \eta e^+ e^-$ | $5.0(4) \times 10^{-10}$ | $< 2.4 \times 10^{-3}$ | Briere et al. 2000 |
| $\eta' \to \eta \mu^+ \mu^-$ | $2.40(21) \times 10^{-10}$ | $< 1.5 \times 10^{-5}$ | Dzhelyadin et al. 1981 |

# Normalized branching ratios $\widehat{\text{BR}} = \text{BR}_{\ell^+\ell^-}/\text{BR}_{\gamma\gamma}$

| | $\text{BR}_{\gamma\gamma}$ | | $\widehat{\text{BR}}$ |
|---|---|---|---|
| $\eta \to \pi^0\gamma\gamma$ | $1.18(13) \times 10^{-4}$ | $\eta \to \pi^0 e^+ e^-$ | $1.1531(4) \times 10^{-5}$ |
| | | $\eta \to \pi^0 \mu^+ \mu^-$ | $5.647(5) \times 10^{-6}$ |
| $\eta' \to \pi^0\gamma\gamma$ | $2.81(18) \times 10^{-3}$ | $\eta' \to \pi^0 e^+ e^-$ | $1.18(6) \times 10^{-6}$ |
| | | $\eta' \to \pi^0 \mu^+ \mu^-$ | $6.5(4) \times 10^{-7}$ |
| $\eta' \to \eta\gamma\gamma$ | $1.10(8) \times 10^{-4}$ | $\eta' \to \eta e^+ e^-$ | $4.56(7) \times 10^{-6}$ |
| | | $\eta' \to \eta \mu^+ \mu^-$ | $2.18(4) \times 10^{-6}$ |

# Scalar rescattering corrections




- Helicity suppression $\sim m_\ell$
- Start from $\eta \to \pi^0\gamma\gamma$ Lu and Moussallam 2020

$$\langle\gamma(q_1,\lambda)\gamma(q_2,\lambda')|\,S\,|\eta(P)\pi^0(p_0)\rangle \sim e^{i(\lambda-\lambda')\varphi}L_{\lambda\lambda'}(s,t)$$

- Neglect $D$ and higher waves $\to \mathcal{M}^{\text{scalar}} \sim L_{++} \sim \ell^0_{++}(s)$
- Dispersion relation for $\ell^0_{++}(s)$ with coupled-channel formalism ($\pi^0\eta$ and $K\bar{K}$); subtract VMD contribution

$$|\overline{\mathcal{M}}|^2 = |\overline{\mathcal{M}^{\text{VMD}}}|^2 + |\overline{\mathcal{M}^{\text{resc}}}|^2 + 2\text{Re}\left(\mathcal{M}^{\text{resc}}\mathcal{M}^{\text{VMD}*}\right)$$

| Branching ratio | VMD | rescattering | mixed |
|---|---|---|---|
| $\eta \to \pi^0 e^+e^-$ | $1.36(15)\times 10^{-9}$ | $2.51(9)\times 10^{-13}$ | $4.63(31)\times 10^{-13}$ |
| $\eta \to \pi^0 \mu^+\mu^-$ | $6.7(7)\times 10^{-10}$ | $2.76(11)\times 10^{-11}$ | $-2.63(15)\times 10^{-11}$ |

- BRs of $\mathcal{O}(10^{-2}) - \mathcal{O}(10^{-4})$ of the VMD results, expect results of similar order for $\eta'$ channels
- Negligible compared to coupling constant uncertainties

# Discussion

- Results in the same range as previous calculations
- Higher confidence with results as we took into account
  - photon virtualities $\rightarrow$ energy-dependent form factors
  - correct high-energy behavior of form factors
  - dispersively improved vector meson propagators
  - compared different parameterizations
  - compared different integration algorithms
  - estimated $S$-channel correction to be indeed small
- Prospect of improved experimental precision
  $\rightarrow$ comparison to experimental results in the future?

# Spares

# Matrix element for $\eta(p) \to \pi^0(p_\pi)\ell^+(p_+)\ell^-(p_-)$

$$\mathcal{M} = i\frac{\alpha^2}{\pi^2} \sum_{V\in\{\rho,\omega,\phi\}} \int \mathrm{d}^4k \; g^{\beta\tilde{\beta}} \epsilon_{\mu\nu\alpha\beta}\epsilon_{\tilde{\mu}\tilde{\nu}\tilde{\alpha}\tilde{\beta}} P^\alpha k^\mu (P^{\tilde{\alpha}} k^{\tilde{\mu}} - P^{\tilde{\alpha}} l^{\tilde{\mu}} + k^{\tilde{\alpha}} l^{\tilde{\mu}})$$

$$P_V((P-k)^2)\mathcal{F}_{V\eta}(k^2)\mathcal{F}_{V\pi^0}((l-k)^2)P_\gamma(k^2)P_\gamma((l-k)^2)$$

$$\bar{u}_s \left[ \gamma^{\tilde{\nu}} \frac{\not{k} - \not{p}_+ + m_\ell}{(k-p_+)^2 - m_\ell^2} \gamma^\nu + \gamma^\nu \frac{\not{p}_- - \not{k} + m_\ell}{(p_- - k)^2 - m_\ell^2} \gamma^{\tilde{\nu}} \right] v_r$$

$l = p_+ + p_-$

$P_V$ vector-meson propagator

$P_\gamma$ photon propagator

*t*-channel    *u*-channel

# Energy-dependent widths

- $\rho$ meson

$$\Gamma_\rho(q^2) = \theta(q^2 - 4M_{\pi^\pm}^2)\frac{\gamma_{\rho\to\pi^+\pi^-}(q^2)}{\gamma_{\rho\to\pi^+\pi^-}(M_\rho^2)}f(q^2)\Gamma_\rho$$

$$\text{with } \gamma_{\rho\to\pi^+\pi^-}(q^2) = \frac{(q^2 - 4M_{\pi^\pm}^2)^{3/2}}{q^2}$$

$$\text{barrier factors } f(q^2) = \frac{\sqrt{q^2}}{M_\rho}\frac{M_\rho^2 - 4M_{\pi^\pm}^2 + 4p_R^2}{q^2 - 4M_{\pi^\pm}^2 + 4p_R^2}$$

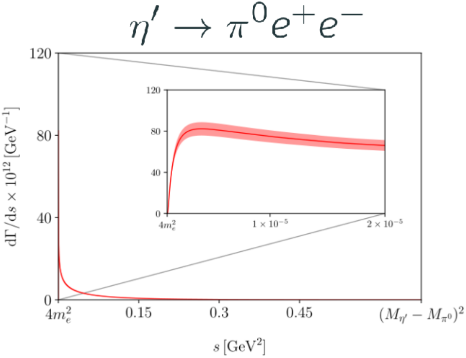
Adolph et al. 2017; Hippel and Quigg 1972

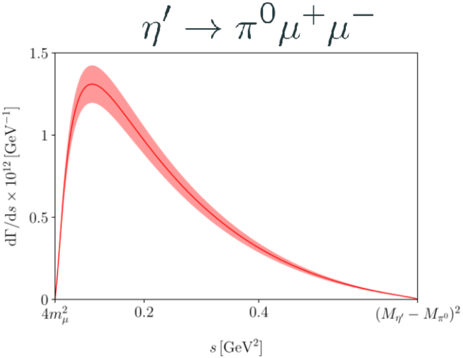
- $\rho', \omega', \phi'$ mesons

$$\Gamma_{V'}(q^2) = \theta(q^2 - (M_V + M_P)^2)\frac{\gamma_{V'\to VP}(q^2)}{\gamma_{V'\to VP}(M_{V'}^2)}\Gamma_{V'}$$

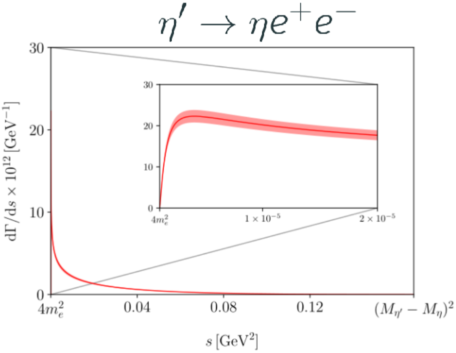
$$\text{with } \gamma_{V'\to VP}(q^2) = \frac{\lambda(q^2, M_V^2, M_P^2)^{3/2}}{(q^2)^{3/2}}$$

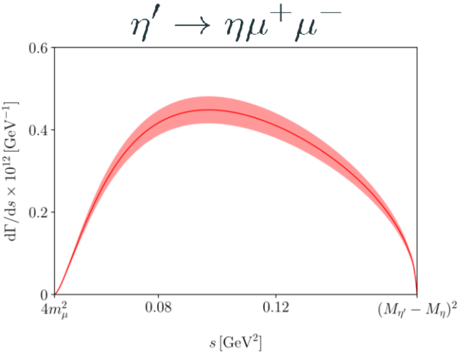
# Results: singly-differential decay width



# Results: singly-differential decay width



# Ratio of branching ratios $\widehat{\text{BR}} = \text{BR}_{\ell^+\ell^-}/\text{BR}_{\gamma\gamma}$

| | $\text{BR}_{\gamma\gamma}$ | | $\widehat{\text{BR}}$ |
|---|---|---|---|
| $\eta \to \pi^0\gamma\gamma$ | $1.18(13) \times 10^{-4}$ | $\eta \to \pi^0 e^+e^-$ | $1.1531(4) \times 10^{-5}$ |
| | | $\eta \to \pi^0 \mu^+\mu^-$ | $5.647(5) \times 10^{-6}$ |
| $\eta' \to \pi^0\gamma\gamma$ | $2.81(18) \times 10^{-3}$ | $\eta' \to \pi^0 e^+e^-$ | $1.18(6) \times 10^{-6}$ |
| | | $\eta' \to \pi^0 \mu^+\mu^-$ | $6.5(4) \times 10^{-7}$ |
| $\eta' \to \eta\gamma\gamma$ | $1.10(8) \times 10^{-4}$ | $\eta' \to \eta e^+e^-$ | $4.56(7) \times 10^{-6}$ |
| | | $\eta' \to \eta \mu^+\mu^-$ | $2.18(4) \times 10^{-6}$ |

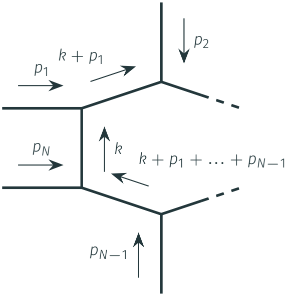
- Tension in $\text{BR}(\eta \to \pi^0\gamma\gamma)$, experimentally and theoretically – our result is compatible with latest KLOE-2 result $\text{BR}(\eta \to \pi^0\gamma\gamma) = (1.21 \pm 0.13 \pm 0.28) \times 10^{-4}$ Gauzzi 2022
- $\text{BR}(\eta' \to [\pi^0/\eta]\gamma\gamma)$ compatible with PDG results Zyla et al. 2020

# Coupling constants

| | $C_\rho$/ GeV$^{-2}$ | $C_\omega$/ GeV$^{-2}$ | $C_\phi$/ GeV$^{-2}$ |
|---|---|---|---|
| $\eta \to \pi^0 \ell^+ \ell^-$ | 1.16(11) | 1.05(5) | 0.0936(20) |
| $\eta' \to \pi^0 \ell^+ \ell^-$ | 0.95(8) | 0.937(26) | −0.0965(25) |
| $\eta' \to \eta \ell^+ \ell^-$ | 2.05(8) | 0.180(9) | −0.492(10) |

# Passarino-Veltman reduction: general loop integral

$$T^N_{\mu_1 \dots \mu_M}(p_1, \dots, p_N, m_0, \dots, m_{N-1}) =$$




$$= \frac{(2\pi)^{4-D}}{i\pi^2} \int d^D k$$

$$\frac{k_{\mu_1} \dots k_{\mu_M}}{(k^2 - m_0^2)[(k + p_1)^2 - m_1^2] \dots [(k + p_1 + \dots + p_N)^2 - m_{N-1}^2]}$$

# Passarino-Veltman reduction: scalar integrals

- Define
  - $A_0(m_0)$
  - $B_0(p, m_0, m_1)$
  - $C_0(p_1, p_2, m_0, m_1, m_2)$
  - $D_0(p_1, p_2, p_3, m_0, m_1, m_2, m_3)$

  as integrals without loop momentum in the numerator
- Can reduce any integral of shape $T$ to those four scalar integrals
- Calculate coefficients
- General solutions to $A_0, B_0, C_0, D_0$ are known 't Hooft and Veltman 1979
- Numerical evaluation can be tedious; use e.g. *Collier* Denner and Dittmaier 2003, 2006, 2011

# Classes of symmetry violations

CPT conserved in the Standard Model, but not individually
Different classes of violations:

- C, P, T violated (e.g. weak interaction $\rightarrow$ direct/indirect CP violation in e.g. kaon decays)
- T=CP even, C and P odd (e.g. w.i. without CKM phase)
- T=CP odd, P odd: TOPO (e.g. QCD $\theta$ term $\rightarrow$ neutron EDM)
- T=CP odd, P even ($\rightarrow$ C odd): TOPE