Advances in Quantum Monte Carlo Studies of Nuclear Systems with Chiral Effective Field Theory Interactions

The 11th International Workshop on Chiral Dynamics—August 26-30 2024, Ruhr University Bochum, Germany

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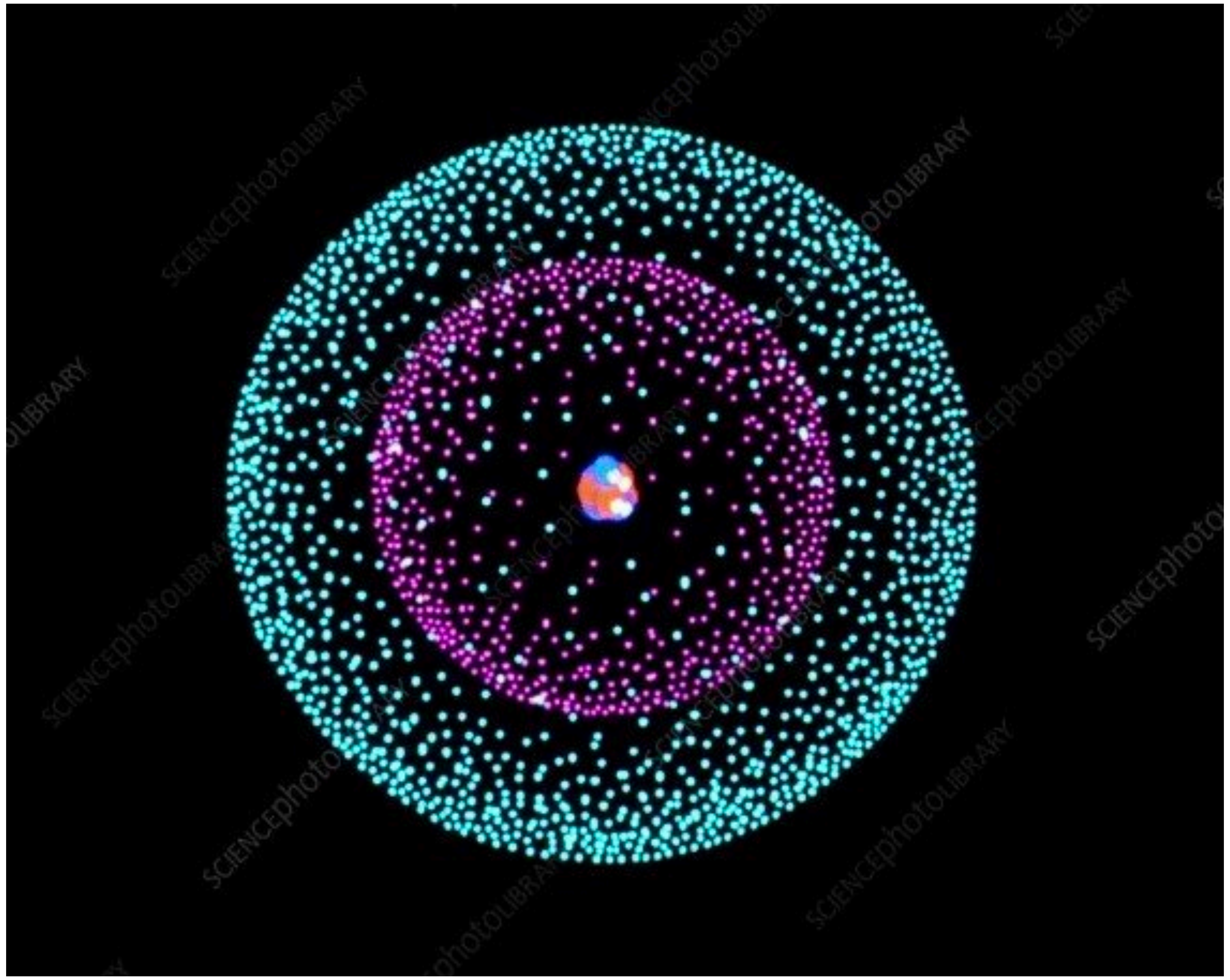
2021 (PI: Lovato), 2022 (PI: Rocco) ALCC and INCITE (PI: Hagen) programs

The Atomic Nucleus



An artist's impression of a beryllium atom. The nucleus, with its four protons and five neutrons, is surrounded by a cloud of electrons.

Credit: https://www.sciencephoto.com/media/2075/view

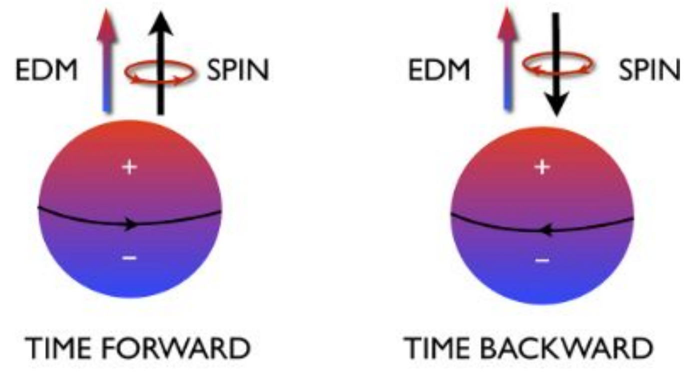
Atomic nuclei are complex quantum many-body systems of strongly interacting fermions (nucleons, p and n), held together by the nuclear force

Nuclei display interesting properties: shell structure, pairing, deformation, strong clustering, etc… that characterize the complexity of matter

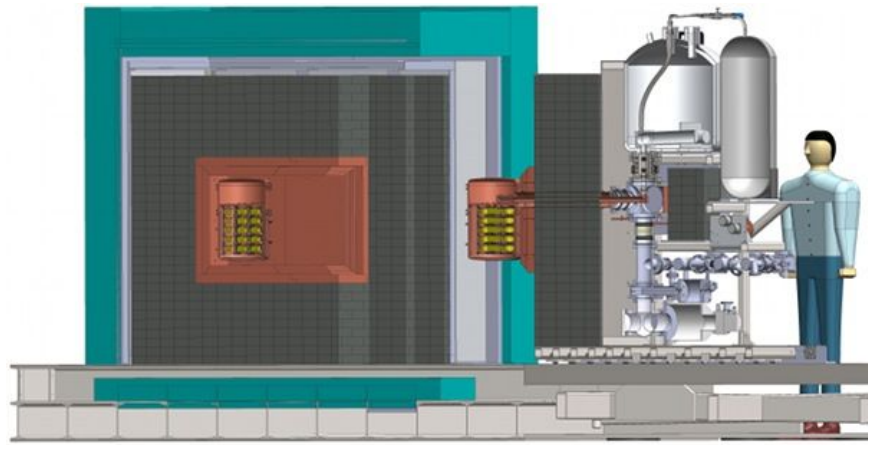
Their structure and scattering by electrons and neutrinos are at the forefront of the nuclear physics and high-energy physics in many worldwide research programs

Understanding Nuclei through Scattering and Electroweak Observables

Ground States' Electroweak Moments, Form Factors, Radii



Neutrinoless Double Beta Decay, Muon-Capture



Accelerator Neutrino Experiments, Lepton-Nucleus XSecs

(ω,q)~0 MeV → ω~few MeVs q~0 MeV → ω~few MeVs q~10^2 MeV → ω~tens of MeVs → ω~10^2 MeV

FRIB

Electromagnetic Decay, Beta Decay, Double Beta Decay & inverse processes

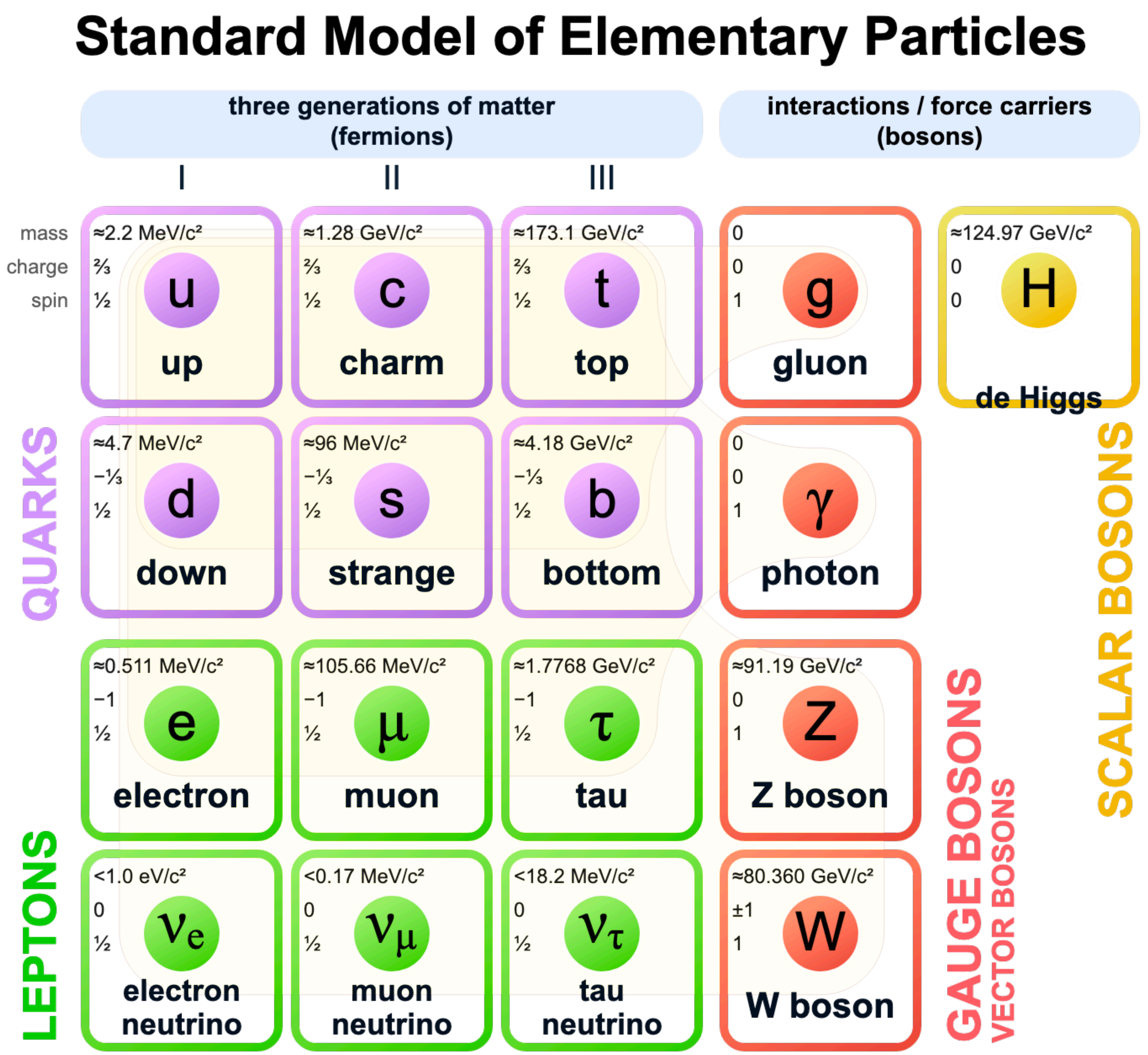
JINA-CEE

Nuclear Rates for Astrophysics

DUNE

Jefferson Lab

Nuclei as a Testbed for Standard Model and Beyond



Nuclei are utilized in precision testing of the SM as well as in the search for physics BSM.

To untangle new physics from many-body nuclear effects, a precise understanding of nuclear structure and dynamics throughout a wide range of energy and momentum transferred is necessary.

This can be done by investigating the role of many-body correlations and currents in selected nuclear electroweak observables at different kinematics.

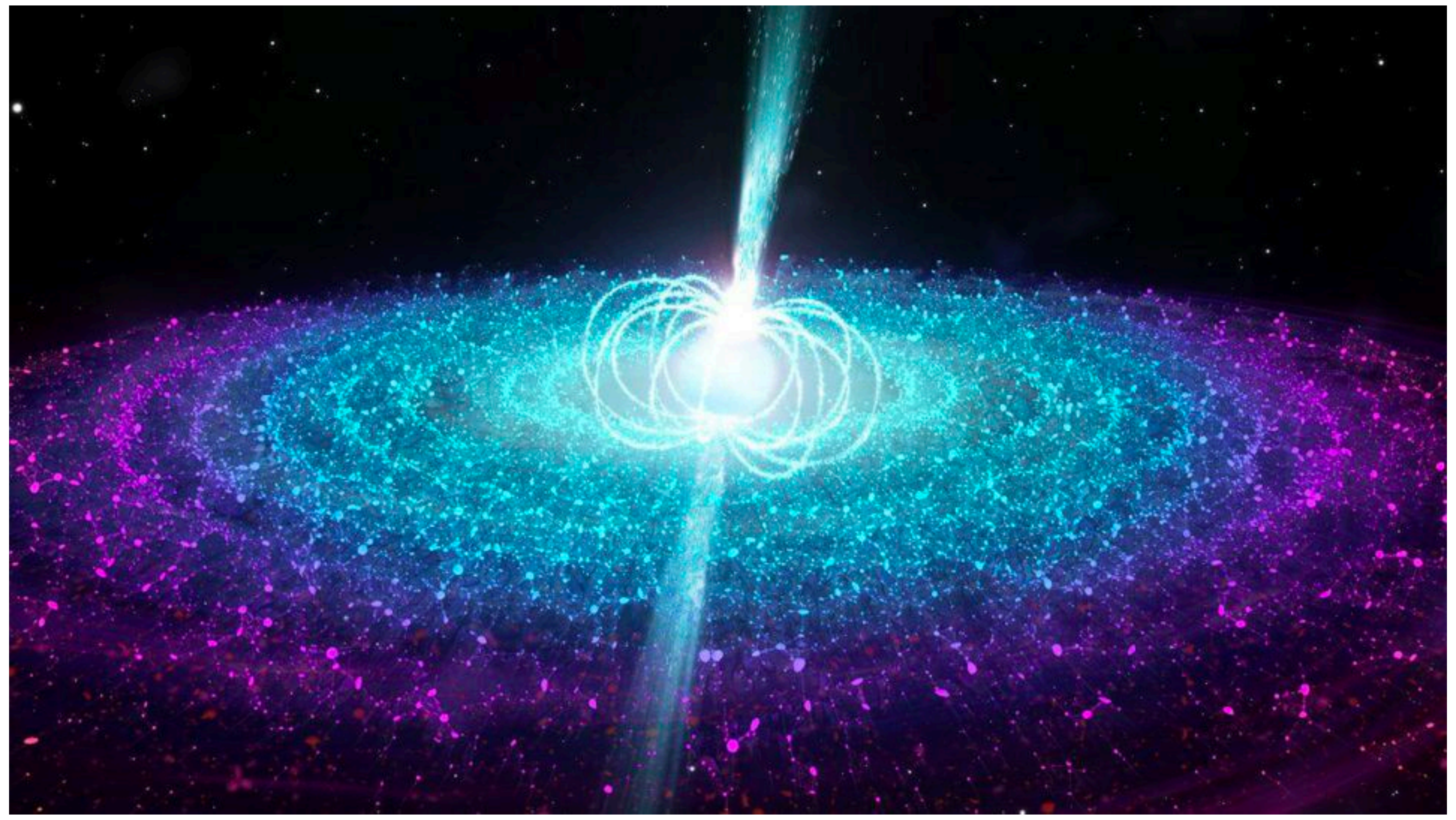
The emphasis will be on light nuclei ($A \leq 12$) that can be treated in Quantum Monte Carlo methods, able to retain many-body effects while delivering precise calculations of the order of a few percents.

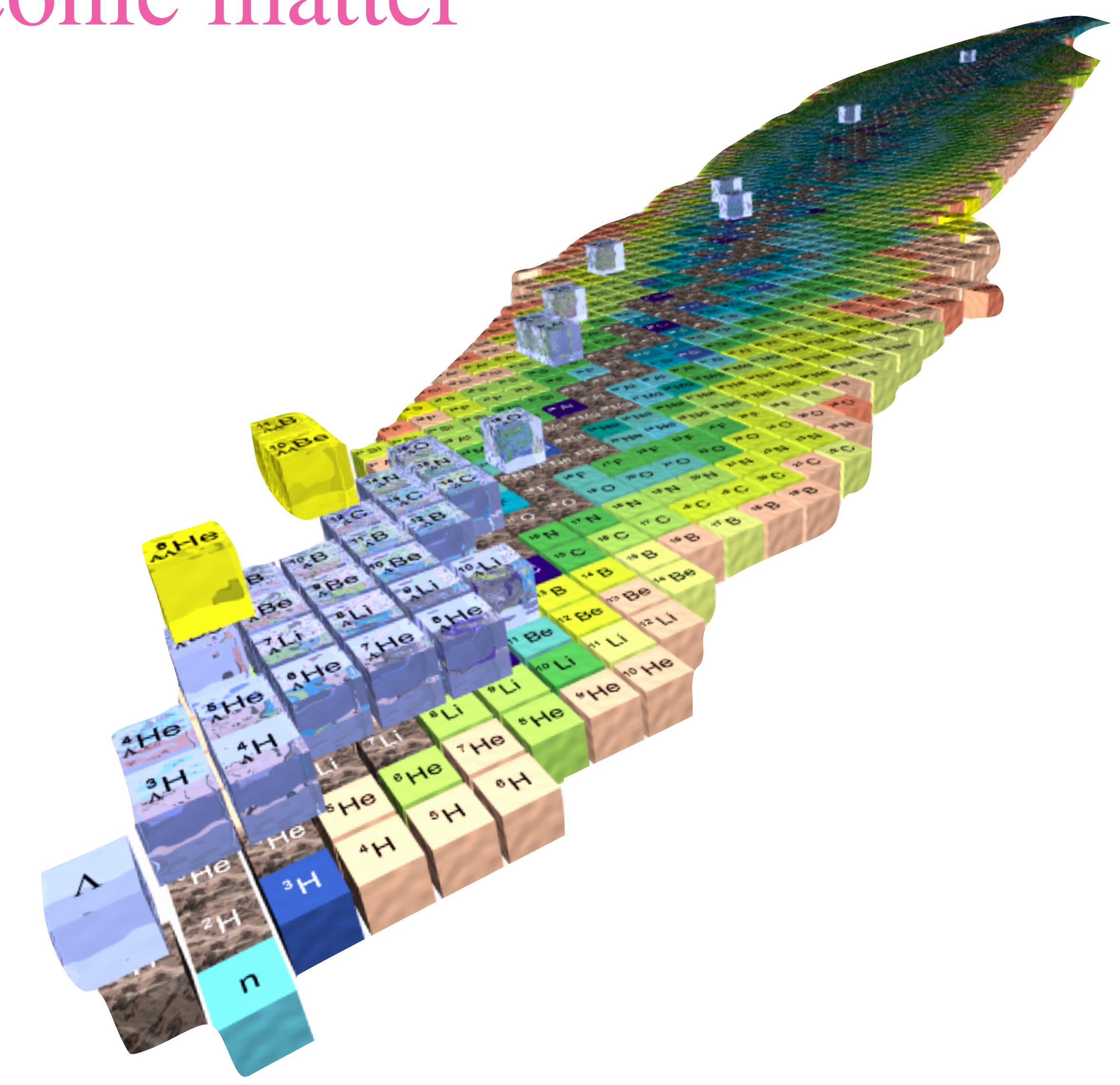
The Role of Quantum Chromodynamics in Nuclear Forces

Question: Where does the nuclear force which binds nucleons together gets its main characteristics, and how it is rooted in the fundamental theory of strong interactions?

Quantum Chromodynamcs

Atomic nuclei and nucleonic matter

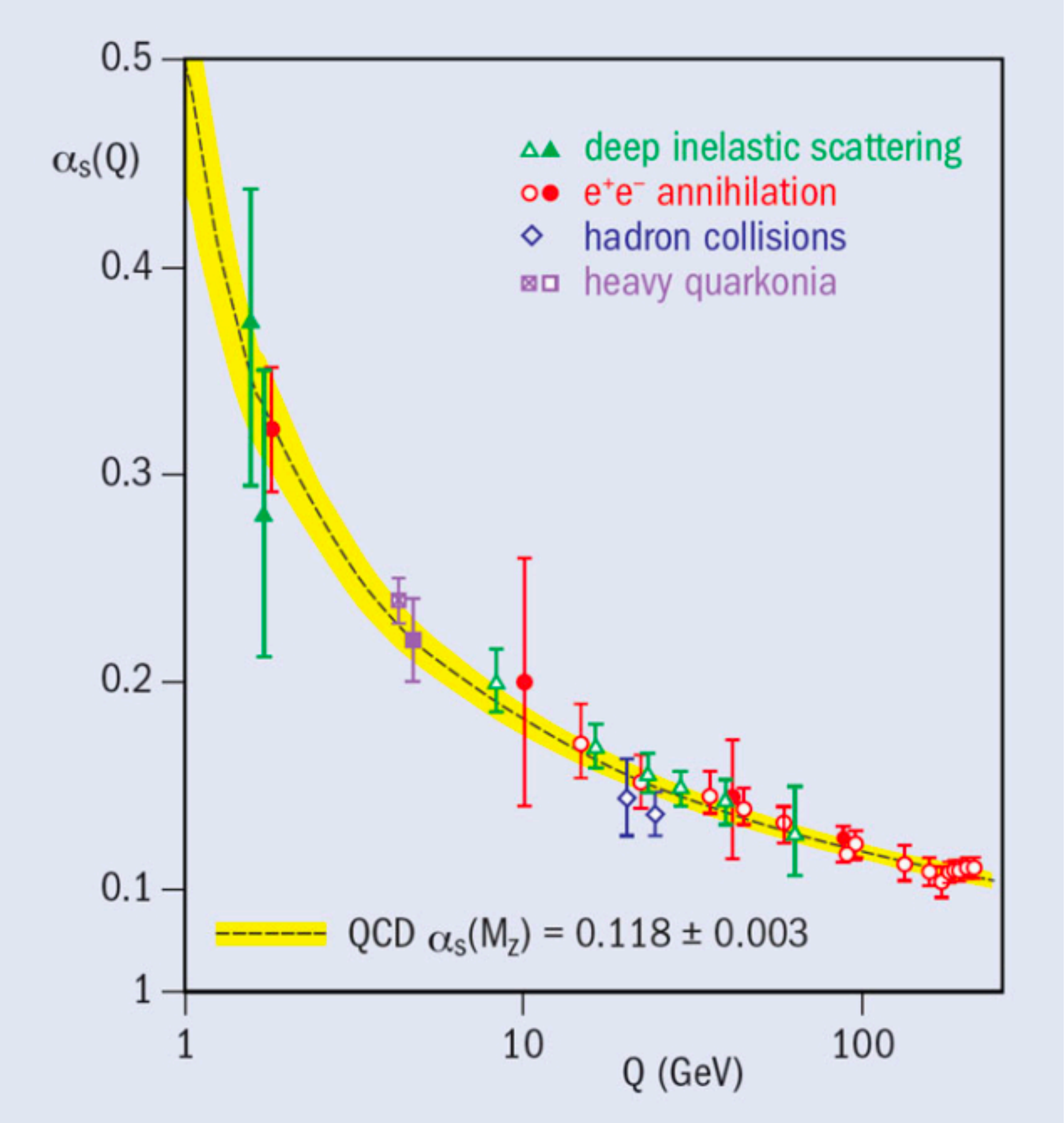




The Role of Quantum Chromodynamics in Nuclear Forces

Question: Where does the nuclear force which binds nucleons together gets its main characteristics, and how it is rooted in the fundamental theory of strong interactions?

This is not a trivial problem due to the nonperturbative nature of QCD at low energy

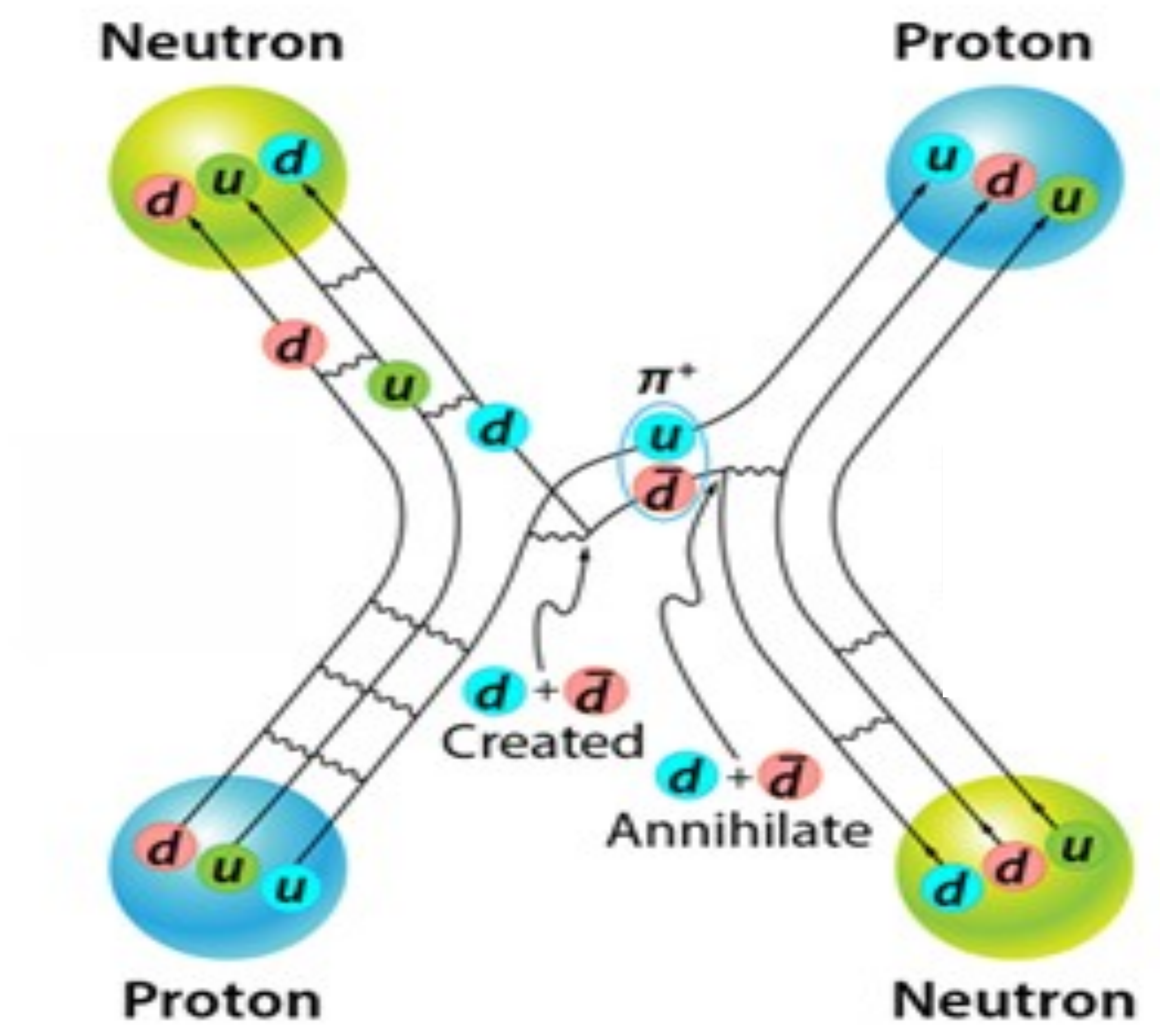


Strong interaction is weak at short distance or high energy (asymptotic freedom) but strong at long-distance or low energy leading to the confinement of quarks into colorless objects the hadron

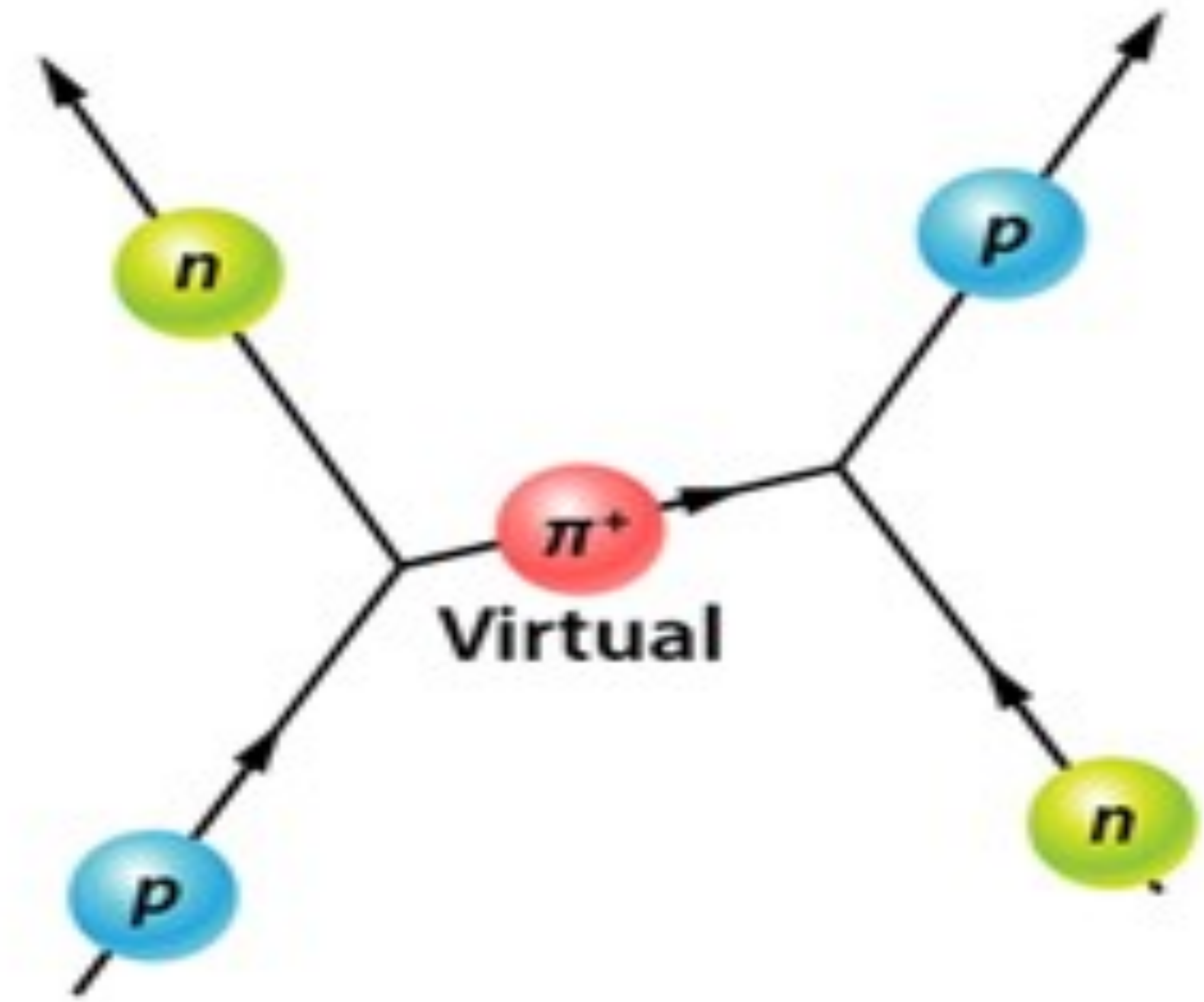
QCD allows for a perturbative analysis at large energies, while it is highly non-perturbative in the low-energy regime

The Role of Quantum Chromodynamics in Nuclear Forces

Question: Where does the nuclear force which binds nucleons together gets its main characteristics, and how it is rooted in the fundamental theory of strong interactions?



Quantum Chromodynamcs

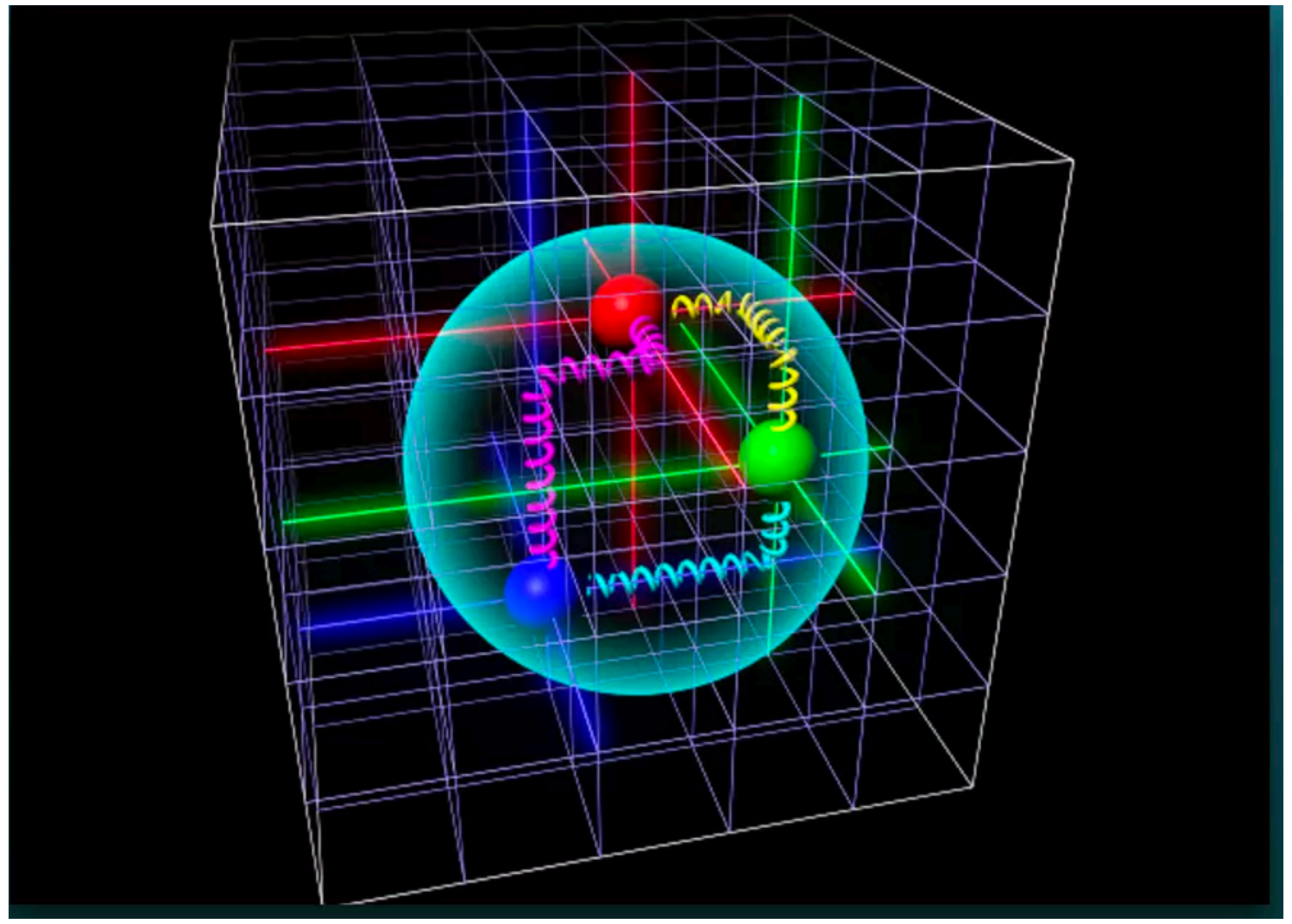


Effective field theories

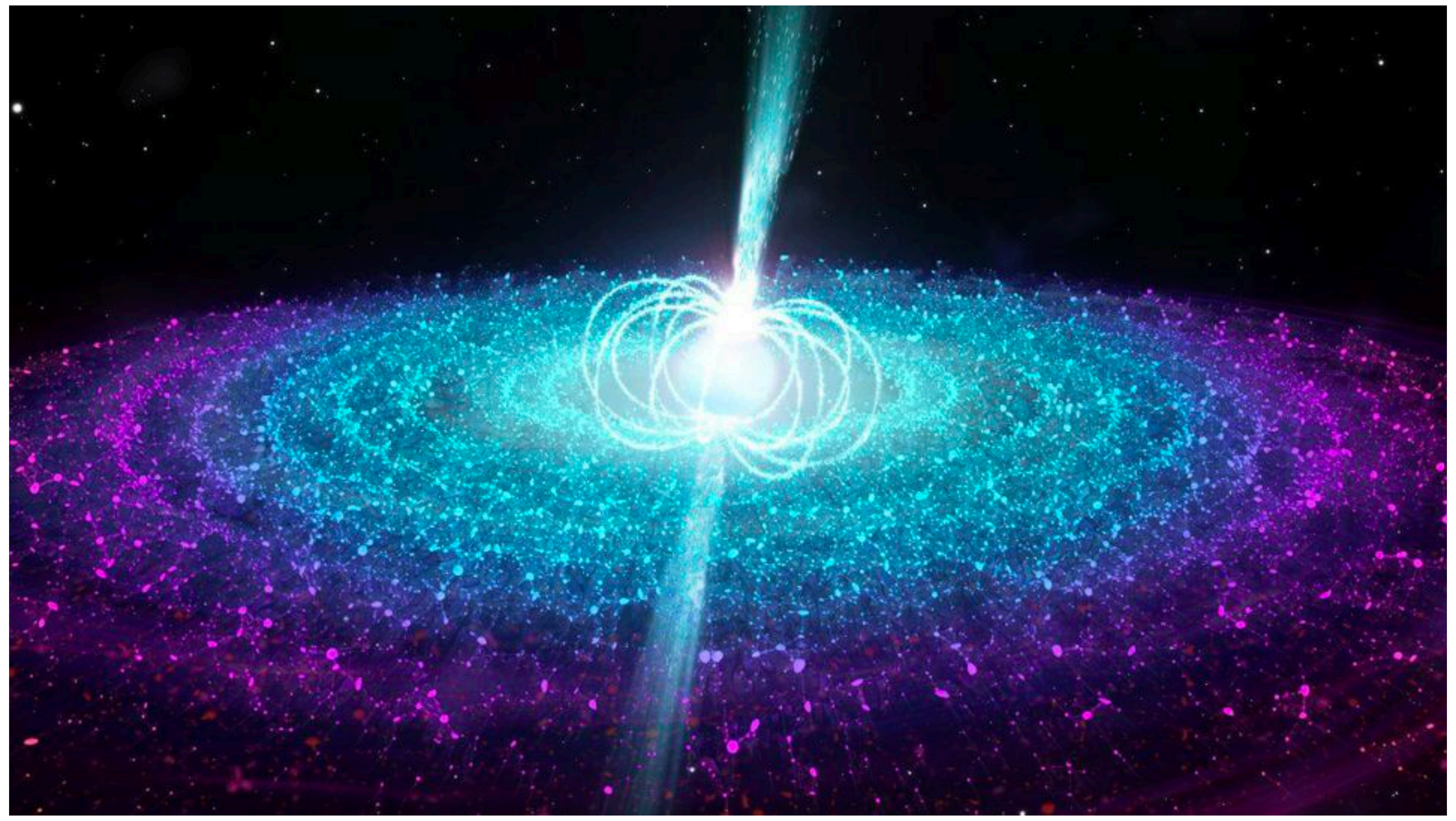
Nuclear few and many-body problem

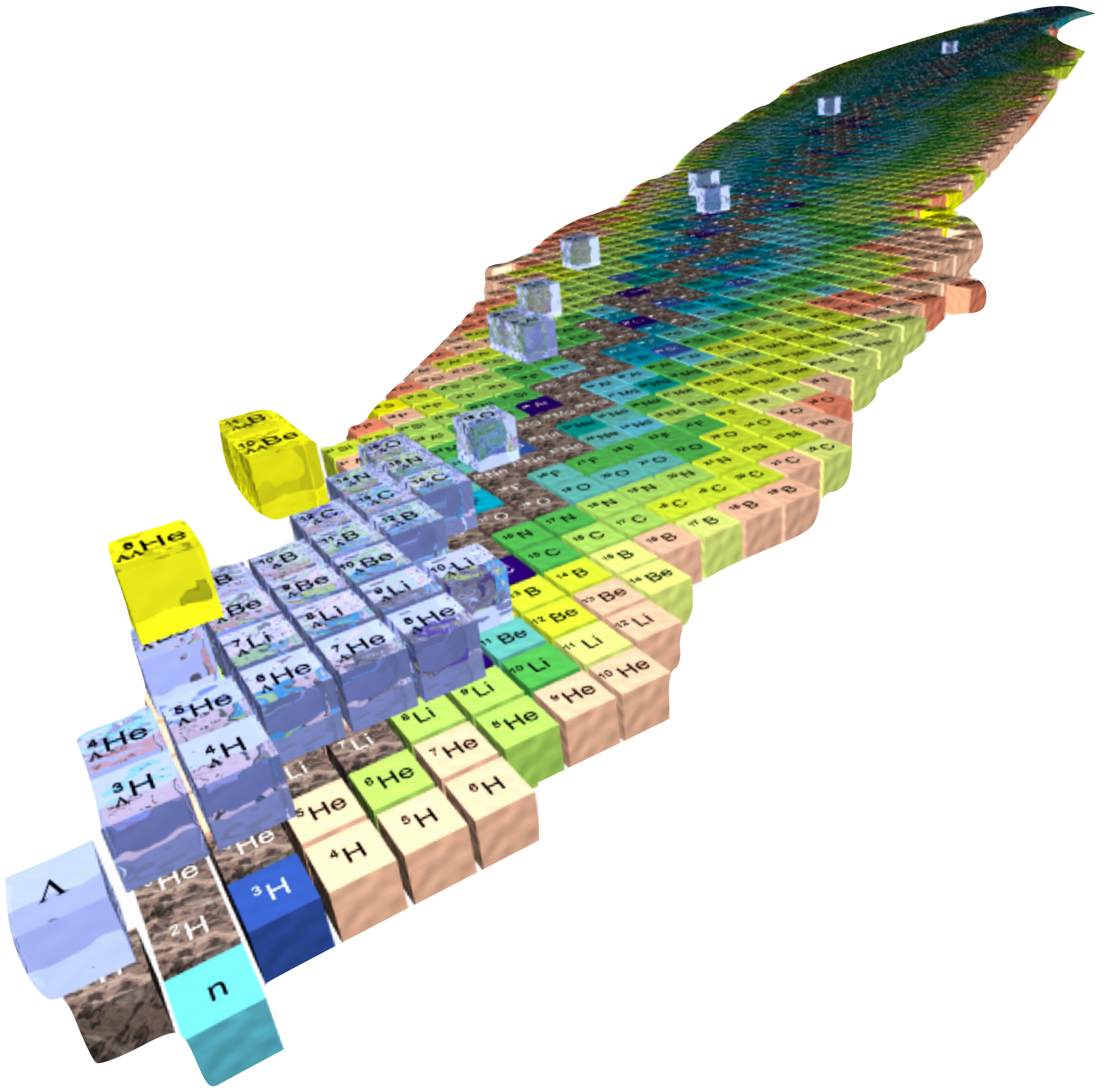
…. Nevertheless Lattice QCD

Lattice Quantum Chromodynamcs

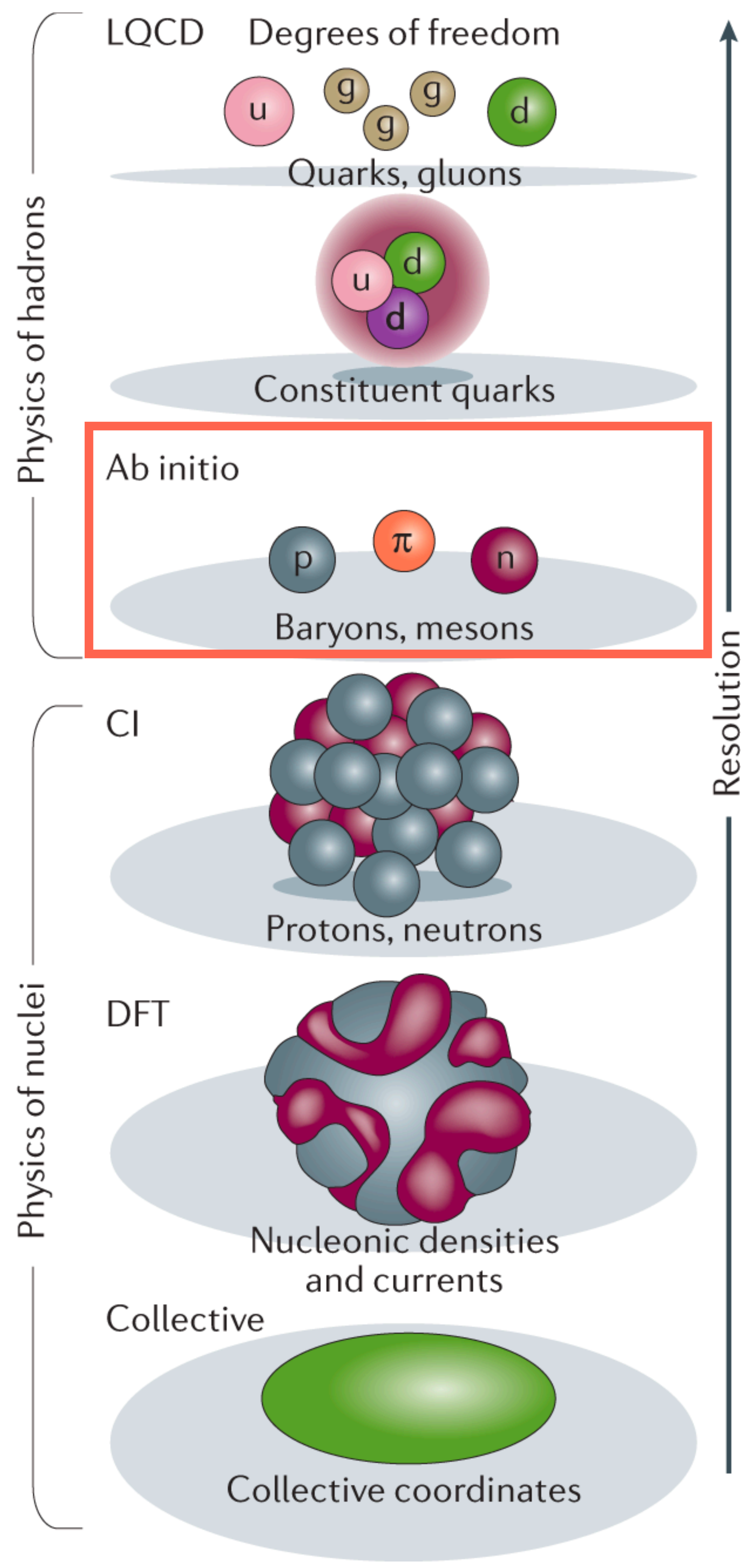


Atomic nuclei and nucleonic matter





The Microscopic Model of Nuclear Theory



Goal: develop a predictive understanding of nuclei in terms of the interactions between individual nucleons and external probes

Definition: *ab-initio* methods seek to solve the non-relativistic Schrödinger equation for all constituent nucleons and the forces between them

Nucleon-nucleon (NN) and 3N scattering data: "thousands" of experimental data available

Spectra, properties, and transition of nuclei: BE, radii, magnetic moments, beta decays rates, weak/radiative captures, electroweak form factors, etc,…

Nucleonic matter equation of state: for ex. EOS neutron matter

Disentangle new physics from nuclear effects: for ex. $0\nu\beta\beta$, BSM with β-decay, EDMs, $\nu - A$ xsec, etc,..

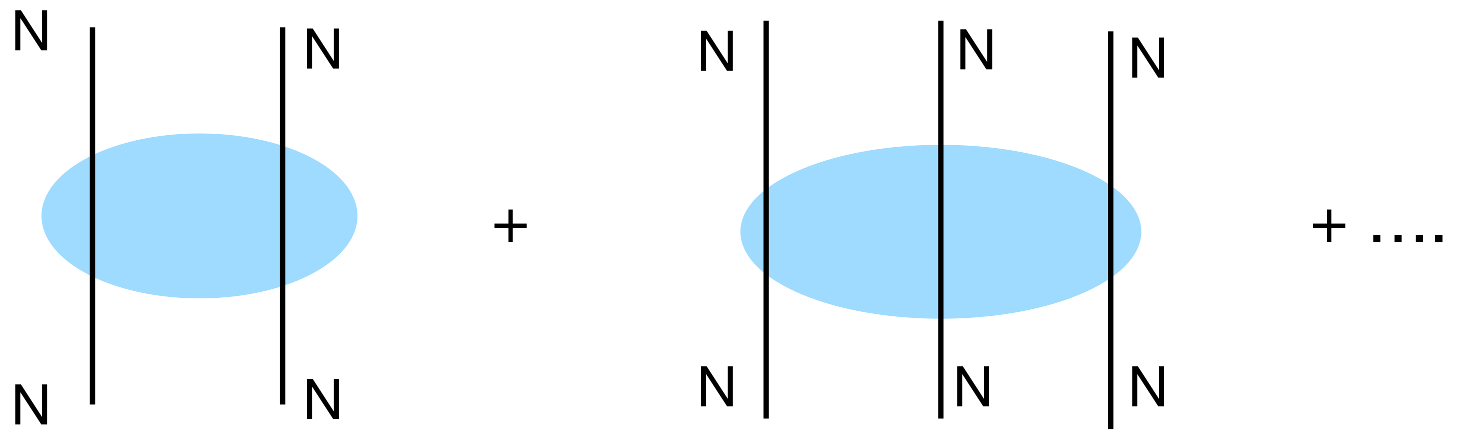
The Microscopic Model of Nuclear Theory

- ***What do we need?***

Two and many-body interactions:

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j=1}^{A} \overbrace{v_{ij}}^{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}} + \ldots..$$

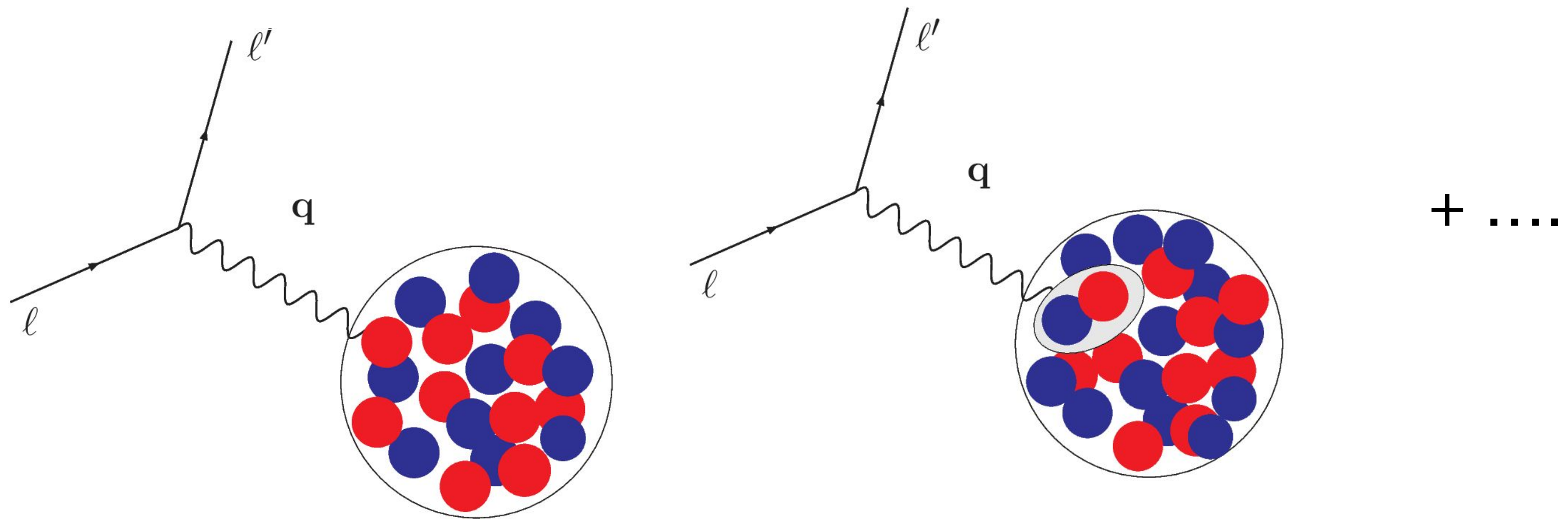
one-body two-body (NN) three-body (3N)



Electroweak current operators:

$$j^{\mathrm{EW}} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} \overbrace{j_{ij}}^{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{j_{ijk}}^{\text{th+exp}} + \ldots.$$

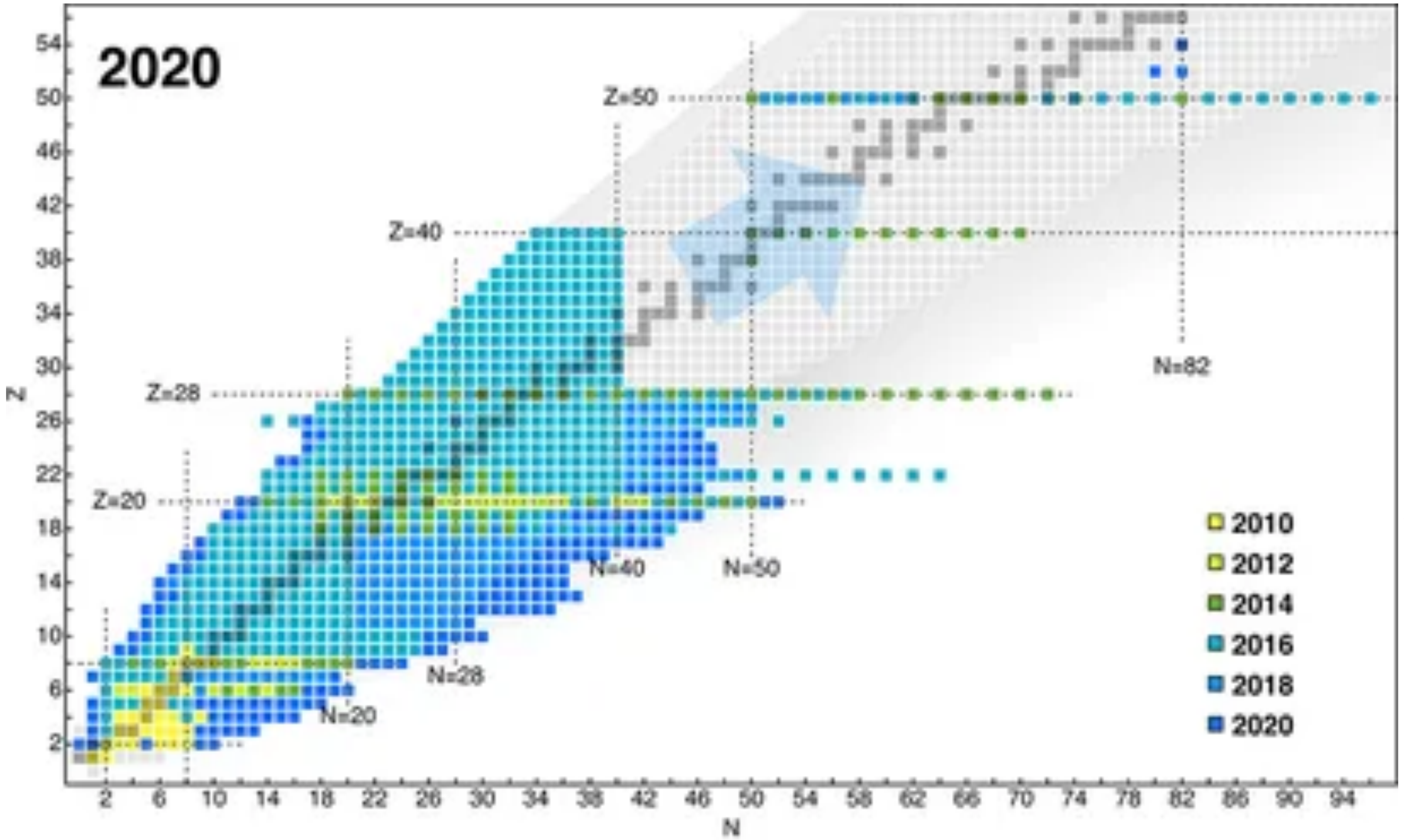
one-body two-body three-body



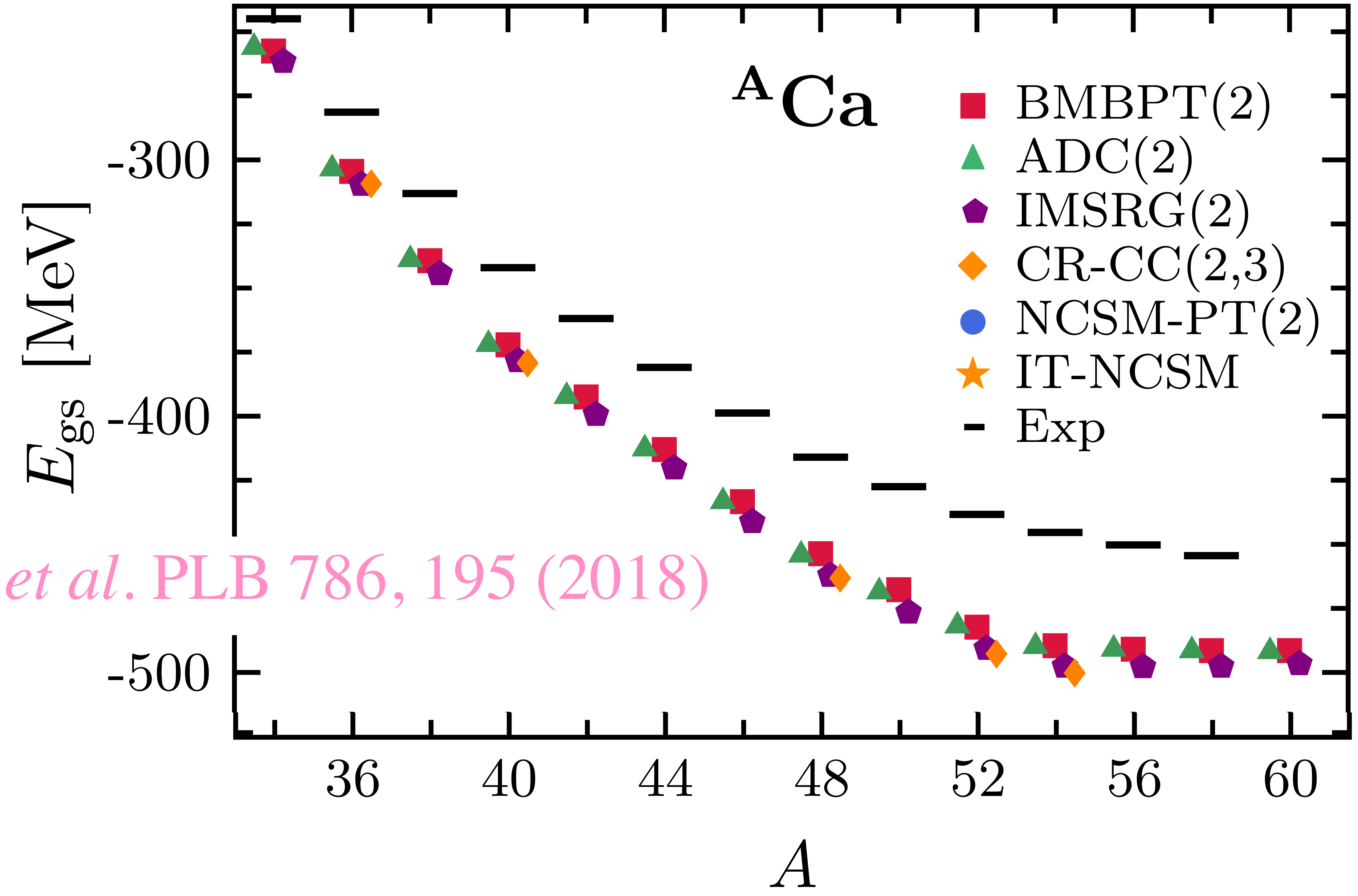
The Microscopic Model of Nuclear Theory

• ***What do we need?***

Ab-initio methods: Several approaches in nuclear physics: QMC, NCSM, CC, …



Hergert Front. Phys., Volume 8 - 2020



Tichai *et al*. PLB 786, 195 (2018)

- Improved and novel many-body frameworks
- Increased computational resources
- Nuclear interactions and currents based on EFTs
- Theoretical uncertainty quantification
- Increased many-body capability, algorithms under control
- Remarkable agreement between different ab initio many-body methods for the structure of nuclei

The Nuclear Many-Body Problem

Many-body Schrödinger equation:

$$H\,\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_A; s_1, s_2, \ldots, s_A; t_1, t_2, \ldots, t_A)$$
$$= E\,\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_A; s_1, s_2, \ldots, s_A; t_1, t_2, \ldots, t_A)$$

where $\mathbf{r}_i$, s_i, and t_i are the nucleon coordinates, spins, and isospins , respectively

Erwin Schrödinger

This corresponds to solve

$2^A \times \binom{A}{Z}$ coupled second-order differential equations in 3A dimensions

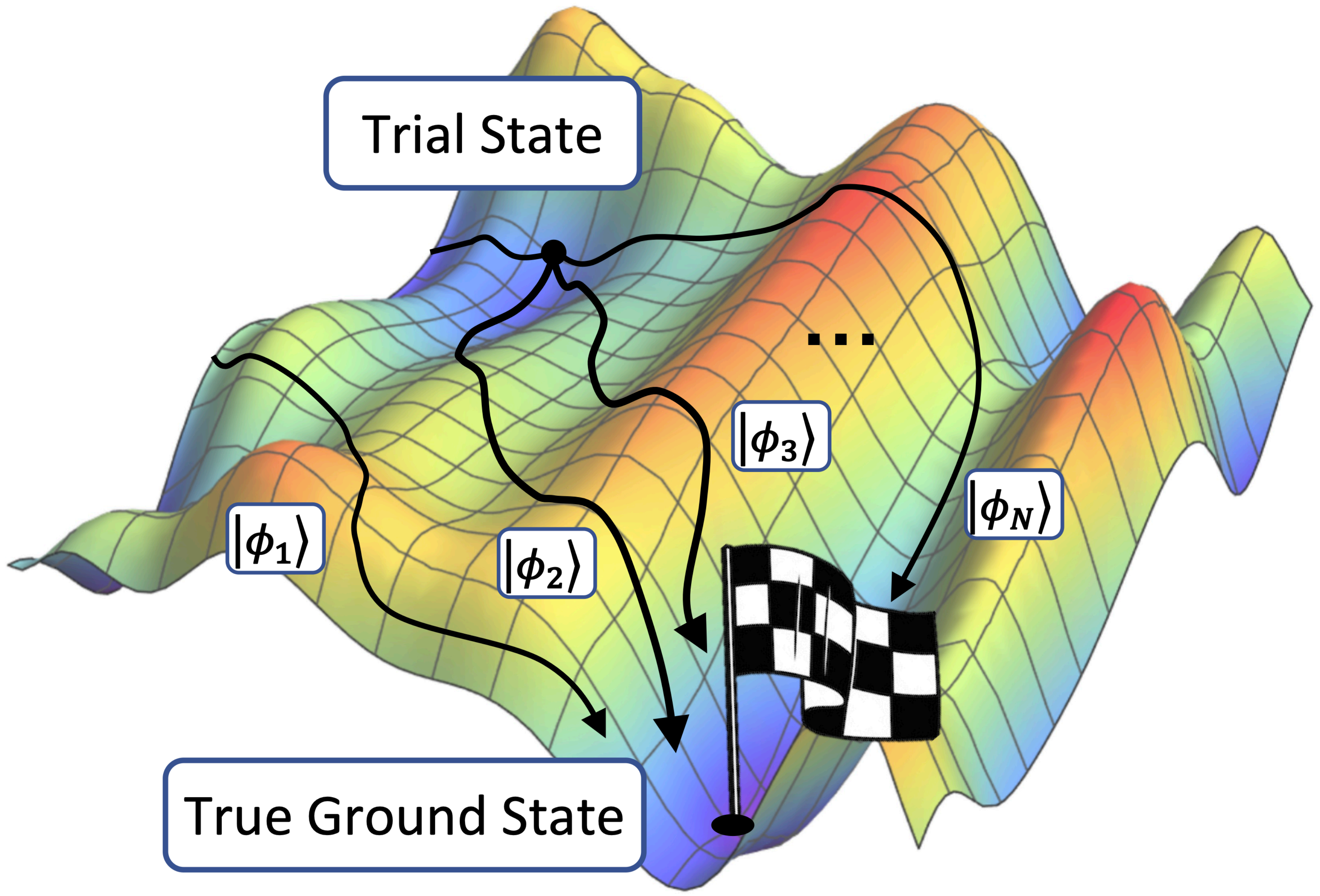
This is a challenging many-body problem!

96 for ^{4}He

17,920 for ^{8}Be

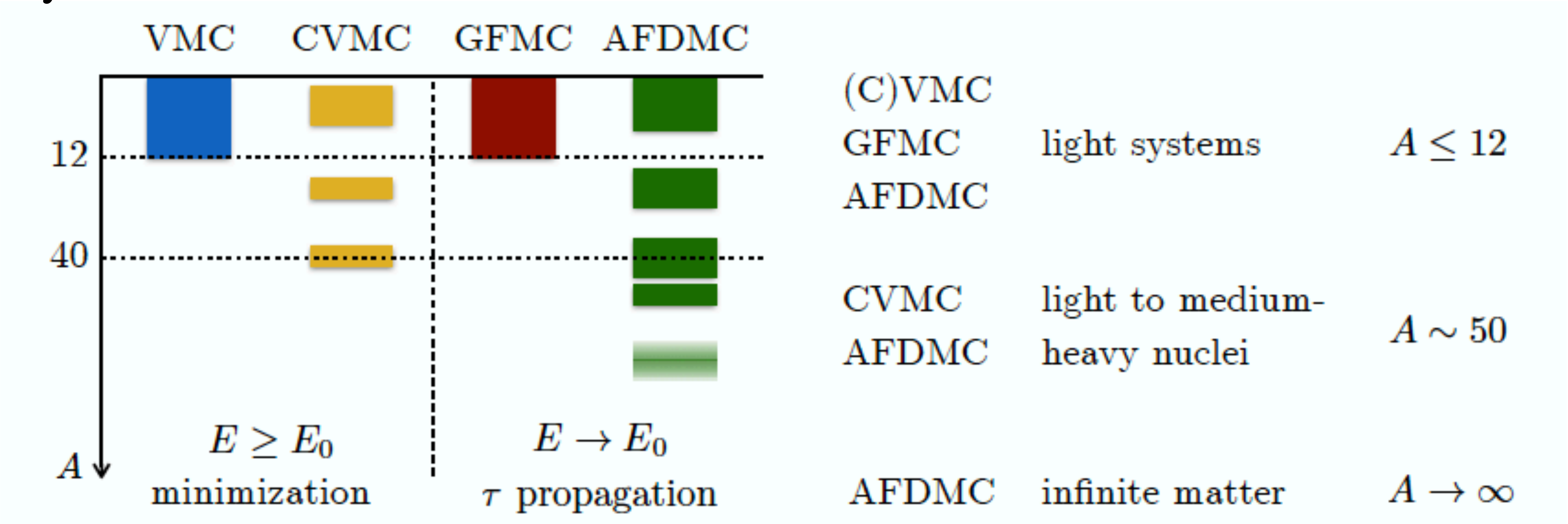
3,784,704 for ^{12}C

Nuclear Quantum Monte Carlo Methods

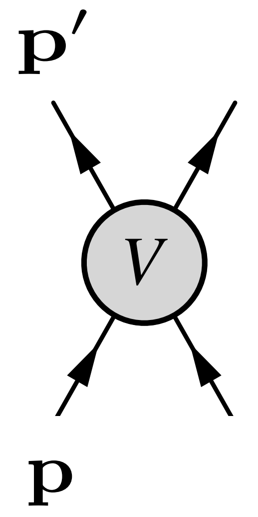


Nuclear Quantum Monte Carlo Methods

Quantum Monte Carlo (QMC) methods: a large family of computational methods whose common aim is the study of complex quantum systems



Work with bare interactions but local r-space representation of the Hamiltonian



$$\mathbf{k} = \mathbf{p}' - \mathbf{p}$$ Local

$$\mathbf{K} = (\mathbf{p}' + \mathbf{p})/2$$ Non-Local

Carlson et al., RMP. 87, 1067 (2015);
Gandolfi, **MP** et. al., Front.in Phys. 8 (2020) 117

Stochastic method: based on recursive sampling of a probability density, statistical errors quantifiable and systematically improvable

Variational Monte Carlo Method

In variational Monte Carlo, one minimize the expectation value of *H*: $E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$

One assumes a suitable form for the trial wave function (involves variational parameters):

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} \left(1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}\right) \right] |\Psi_J\rangle \qquad |\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

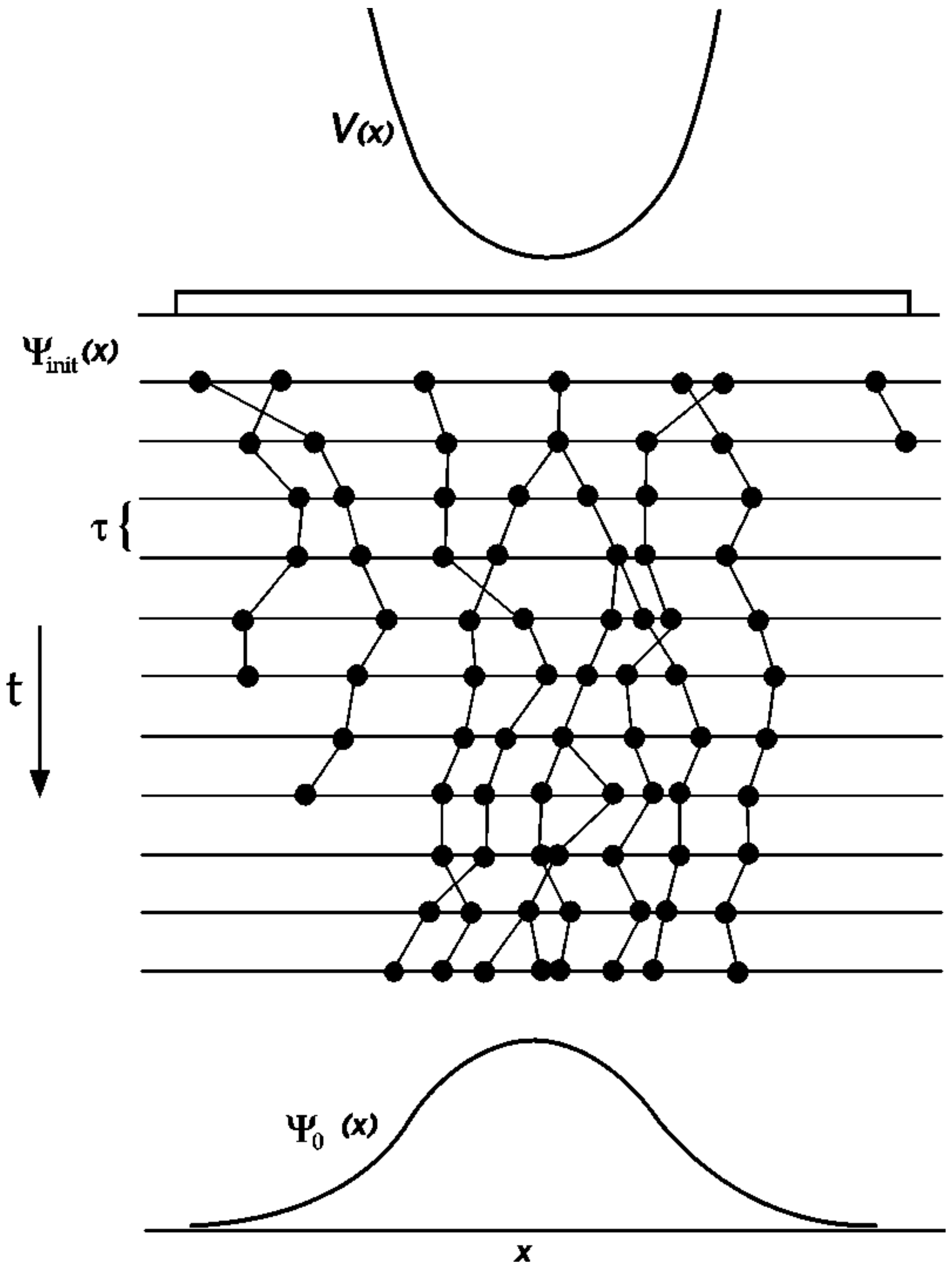
$$|\Phi_d(1100)\rangle = \mathcal{A}|\uparrow p \uparrow n\rangle \; ; \; |\Phi_\alpha(0000)\rangle = \mathcal{A}|\uparrow p \downarrow p \uparrow n \downarrow n\rangle$$

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O^p_{ij} \; ; \; U_{ijk} = -\epsilon V_{ijk}(\tilde{r}_{ij}, \tilde{r}_{jk}, \tilde{r}_{ki})$$

Functions $f_c(r_{ij})$ and $u_p(r_{ij})$ obtained with coupled differential equations with v_{ij}

R.B. Wiringa, PRC 43, 1585 (1991)

Diffusion Monte Carlo Method



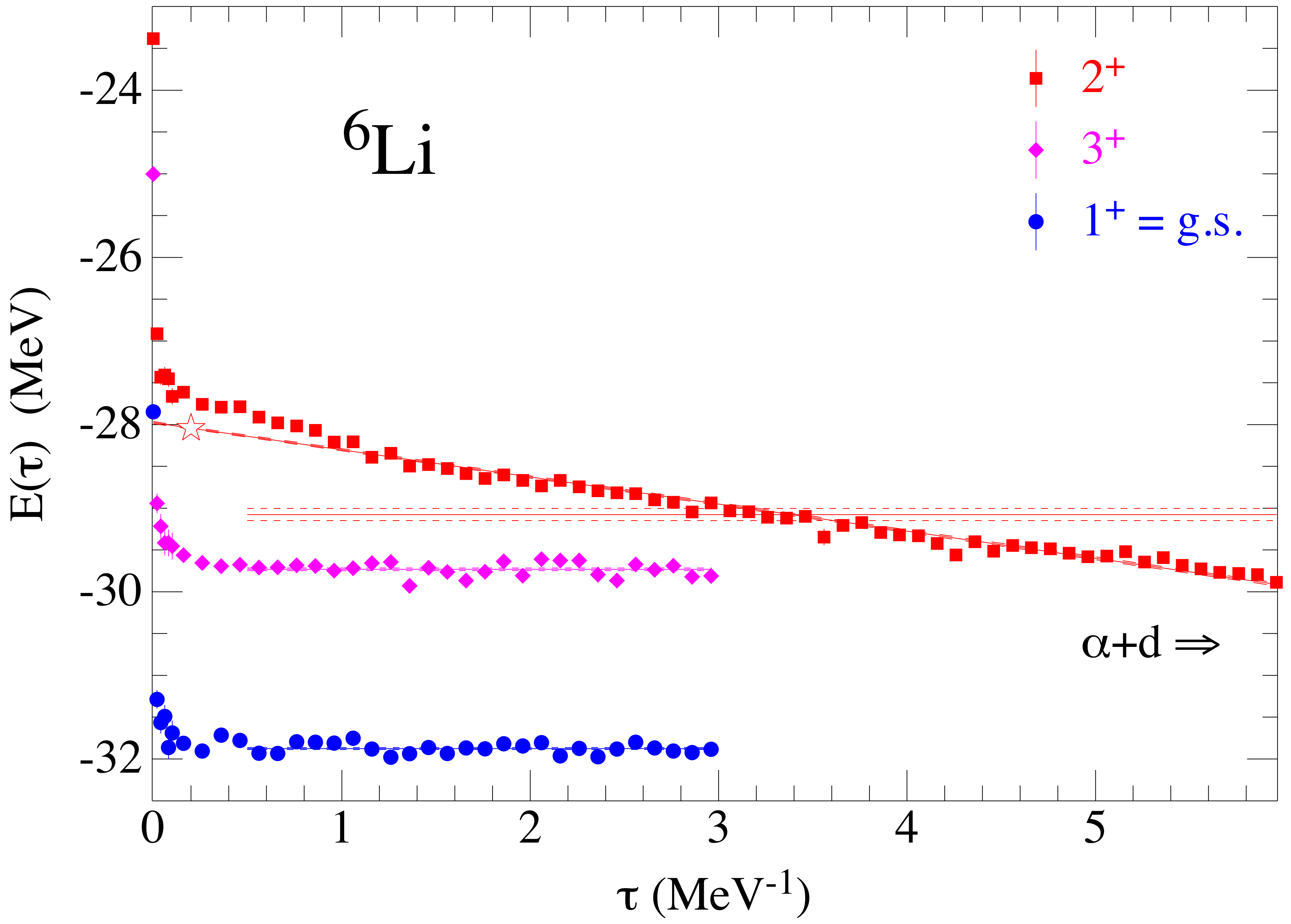
The DMC method (ex. GFMC or AFDMC) overcomes the limitations of VMC by using a projection technique to determine the true ground-state

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$\lim_{\tau\to\infty} |\Psi(\tau)\rangle = \lim_{\tau\to\infty} e^{-(H-E_0)\,\tau}\,|\Psi_T\rangle = c_0|\Psi_0\rangle \qquad |\Psi(\tau=0)\rangle = |\Psi_T\rangle$$

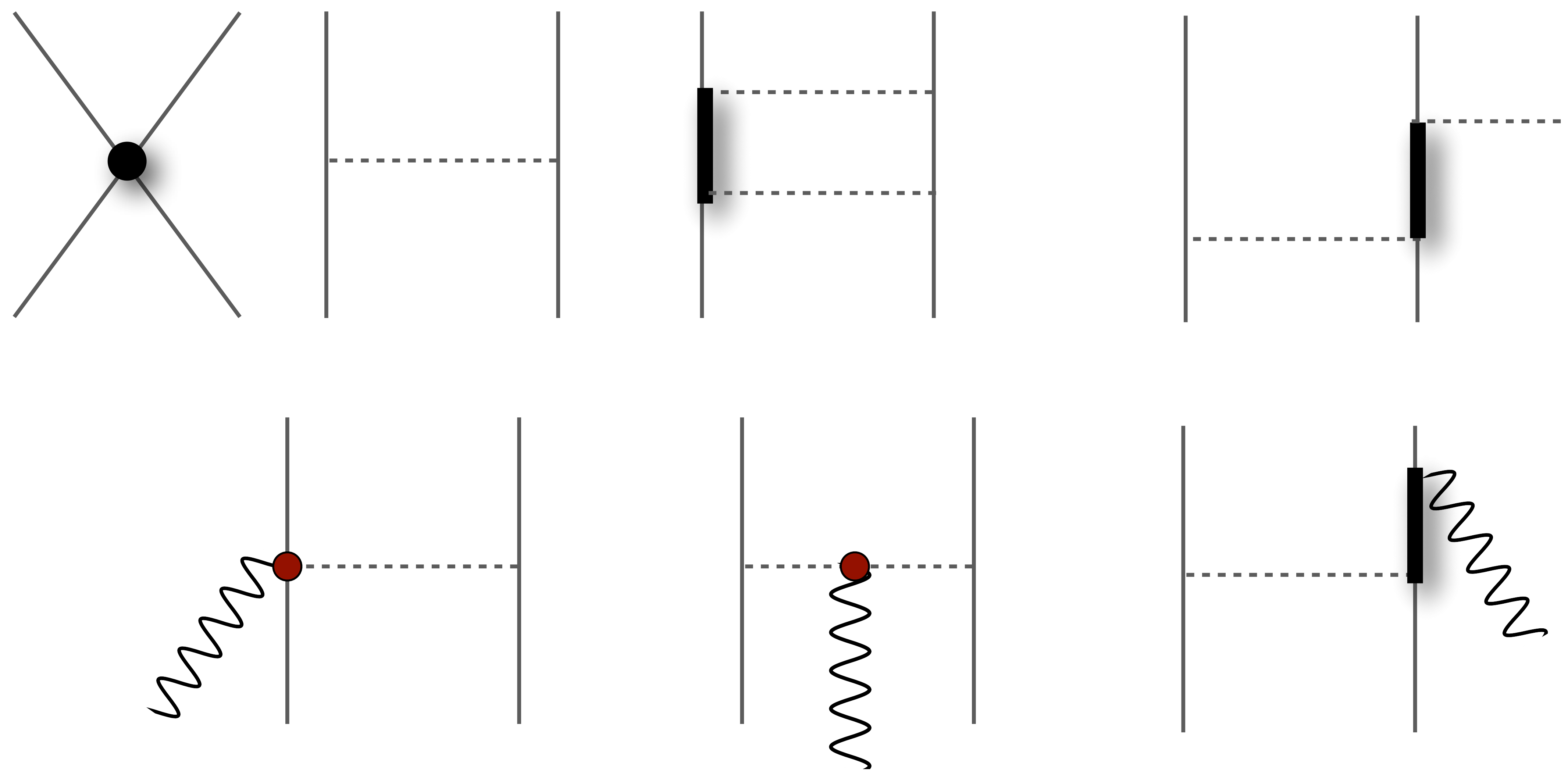
GFMC propagated energy vs imaginary time propagation for first 3 states in 6Li

- g.s (1^+) & 3^+ stable after $\tau = 0.2\,\mathrm{MeV}^{-1}$
- 2^+ (a broad resonance) never stable-decaying to separate $\alpha + d$
- $E(\tau = 0.2)$ is best GFMC estimate of resonance energy

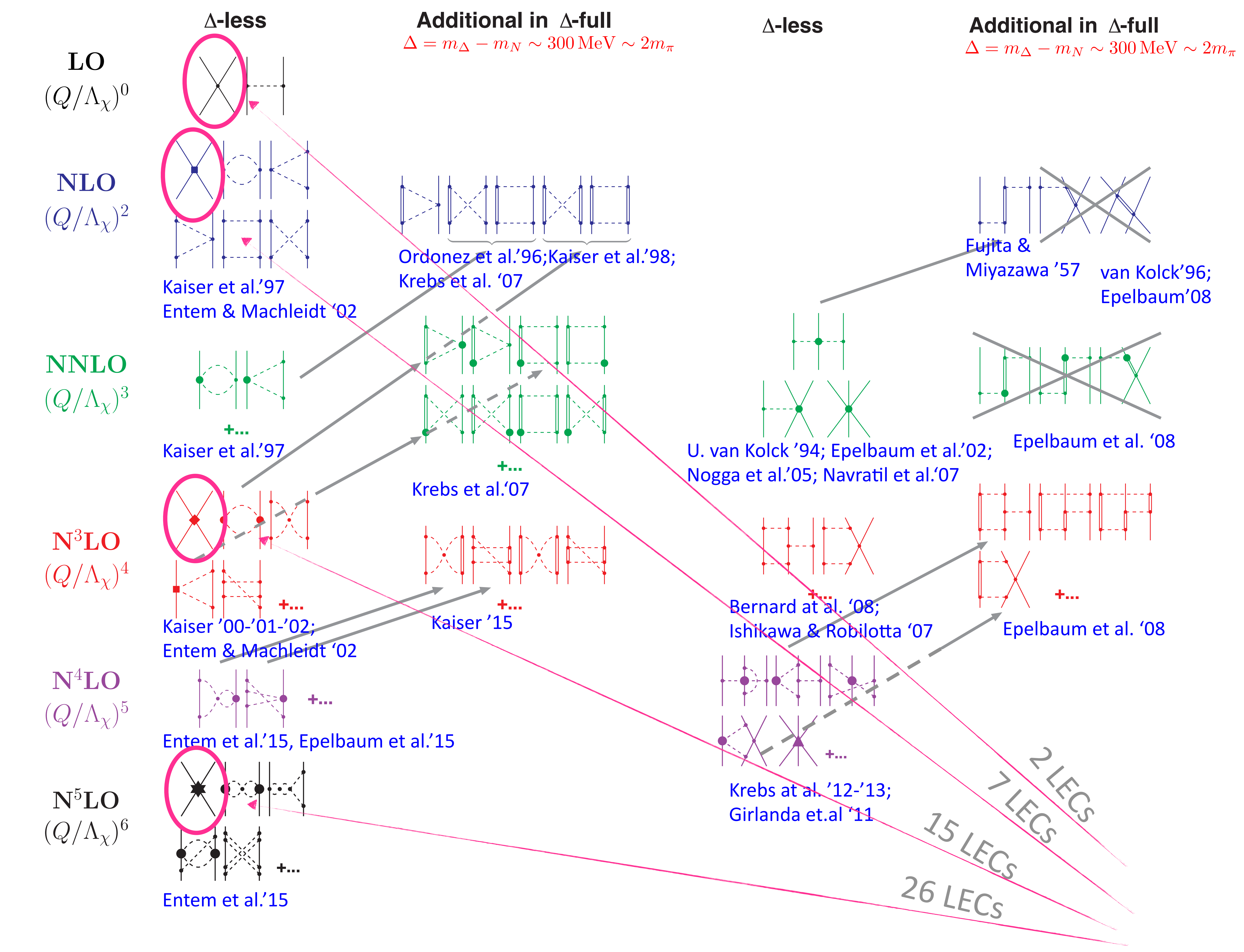


J. Carlson et al., RMP. 87, 1067 (2015); S. Gandolfi, **MP** et. al., Front.in Phys. 8 (2020) 11

Hamiltonian and electroweak currents



State-of-the art Chiral EFT interactions



Advantages:

- Consistent description of two- and many-body interactions and currents
- Different processes described on the same footing: piN, NN, electroweak
- UQ due to the truncation in the chiral expansion
- Scheme can be systematically improved

Disadvantages:

- Increase in number of diagrams at higher orders; When do we stop in the chiral expansion? Convergence, power counting, etc….
- Consistency between strong- and electroweak sector very challenging to achieve
- More LECs appearing at higher orders; challenging optimization problem

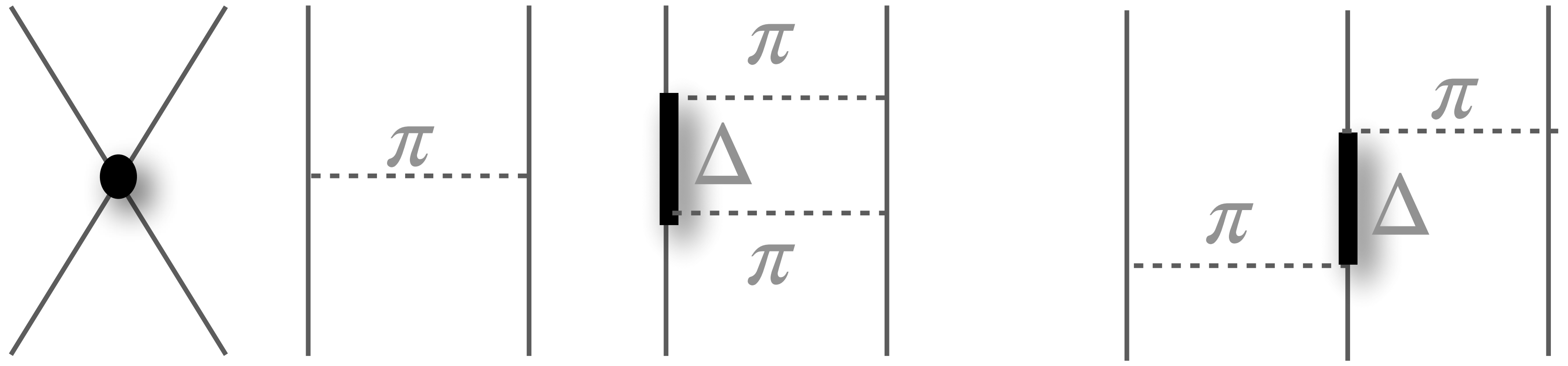
Many-body Nuclear Interactions

Many-body Nuclear Hamiltonian

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j=1}^{A} v_{ij} + \sum_{i<j<k=1}^{A} V_{ijk}$$

one-body two-body (NN) three-body (3N)

- Accurate understanding of the interactions/ correlations between nucleons in pairs, triplets,..(v_{ij} and V_{ijk} are the two- and three-nucleon forces)
- Operators constrained by experimental data; fitted parameters encode underlying QCD dynamics



long-range $r \sim m_\pi^{-1}$: pion-exchange
intermediate range $r \sim (2m_\pi)^{-1}$: ex. two-pion exchange
short-range: ex. contact terms

In our Quantum Monte Carlo calculations we use:

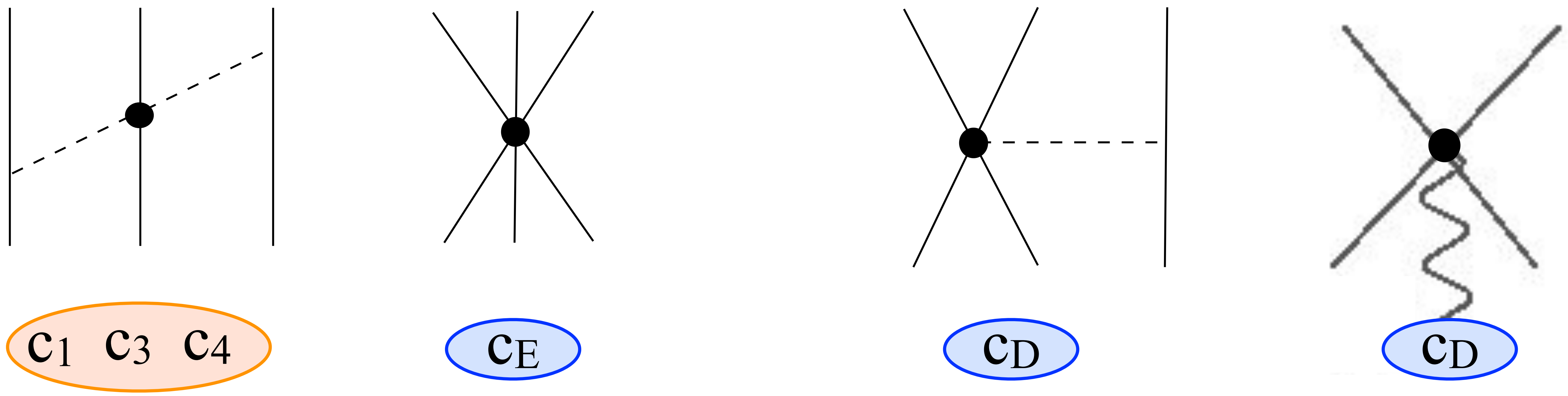
- AV18+UIX; AV18+IL7 phenomenological models

Wiringa, Stoks, Schiavilla PRC **51**, 38 (1995); J. Carlson et al. NP **A401**, 59 (1983); S. Pieper et al. PRC **64**, 014001 (2001)

- chiral $\pi N\Delta$ N3LO+N2LO Norfolk models

MP et al. PRC 91, 024003 2015; PRC 94, 054007 2016; MP *et al.* PRL **120**, 052503 (2018); A. Baroni, MP et al.PRC 98, 044003 (2018)

Three-body LECs and N3LO-CT



$$\mathbf{j}_{5,a}^{\mathrm{N3LO}}(\mathbf{q};\mathrm{CT}) = z_0 \mathcal{O}_{ij}(\mathbf{q})$$

$$z_0 \propto (c_D + \text{ known LECs})$$

The NV2+3s model fits c_D using *strong interaction data only*

$E_0(^3\mathrm{H}) = -8.482\,\mathrm{MeV}$

$^2a_{nd} = (0.645 \pm 0.010)\,\mathrm{fm}$

The NV2+3s* model fits c_D with *strong and weak interaction data*

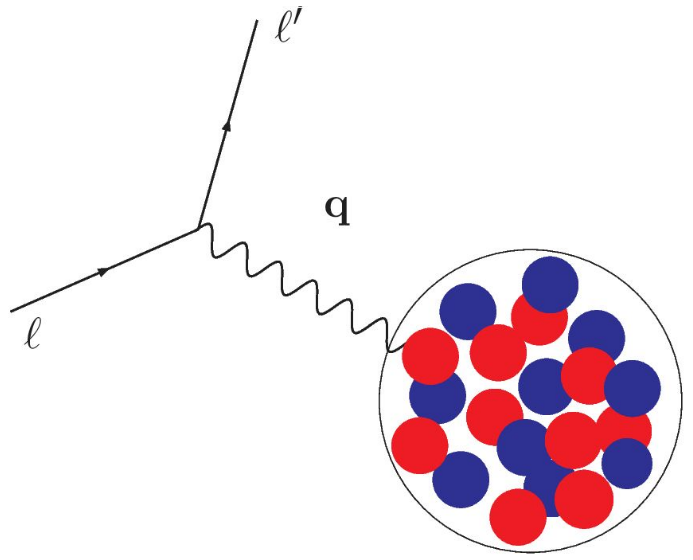
$E_0(^3\mathrm{H}) = -8.482\,\mathrm{MeV}$

GT m.e. in $^3\mathrm{H}$ β-decay

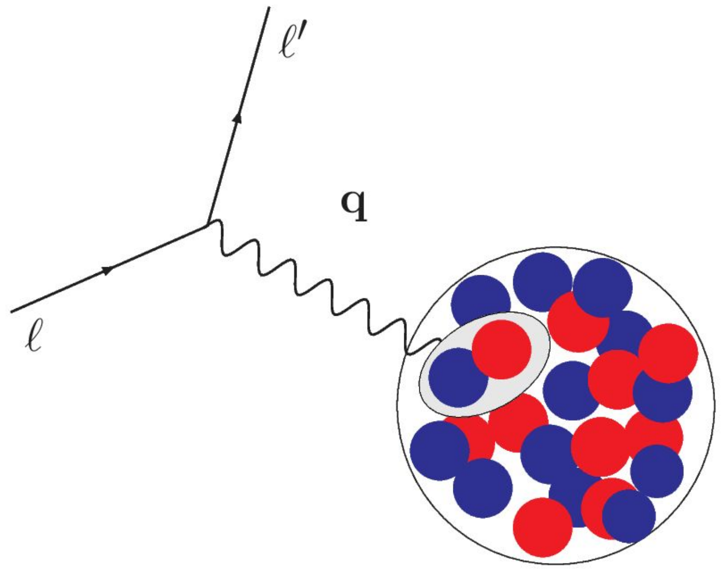
A. Baroni, **MP** et al. PRC 98, 044003 (2018)

Many-body Nuclear Electroweak Currents

Electroweak structure and reactions:



one-body



two-body

- Electroweak form factors
- Magnetic moments and radii
- Electroweak Response functions
- Radiative/weak captures
- G.T. matrix elements involved in beta decays
- ……..

- Accurate understanding of the electroweak interactions of external probes with nucleons, correlated nucleon-pairs,…
- Two-body currents are a manifestation of two-body correlations
- Electromagnetic two-body currents are required to satisfy current conservation

$$\mathbf{q}\cdot\mathbf{j} = [H,\rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Nuclear charge operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij}$$

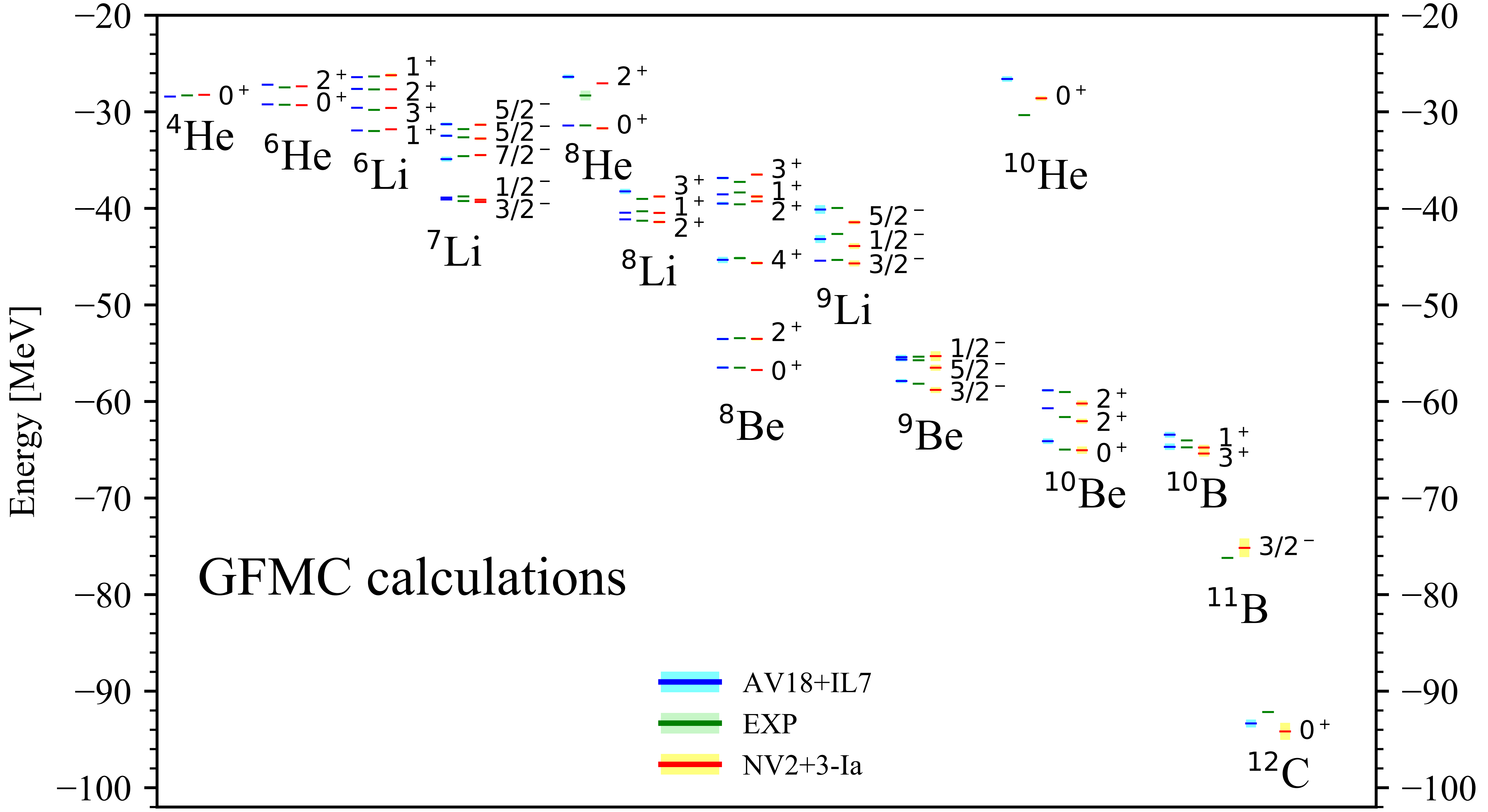
Nuclear vector/axial operator

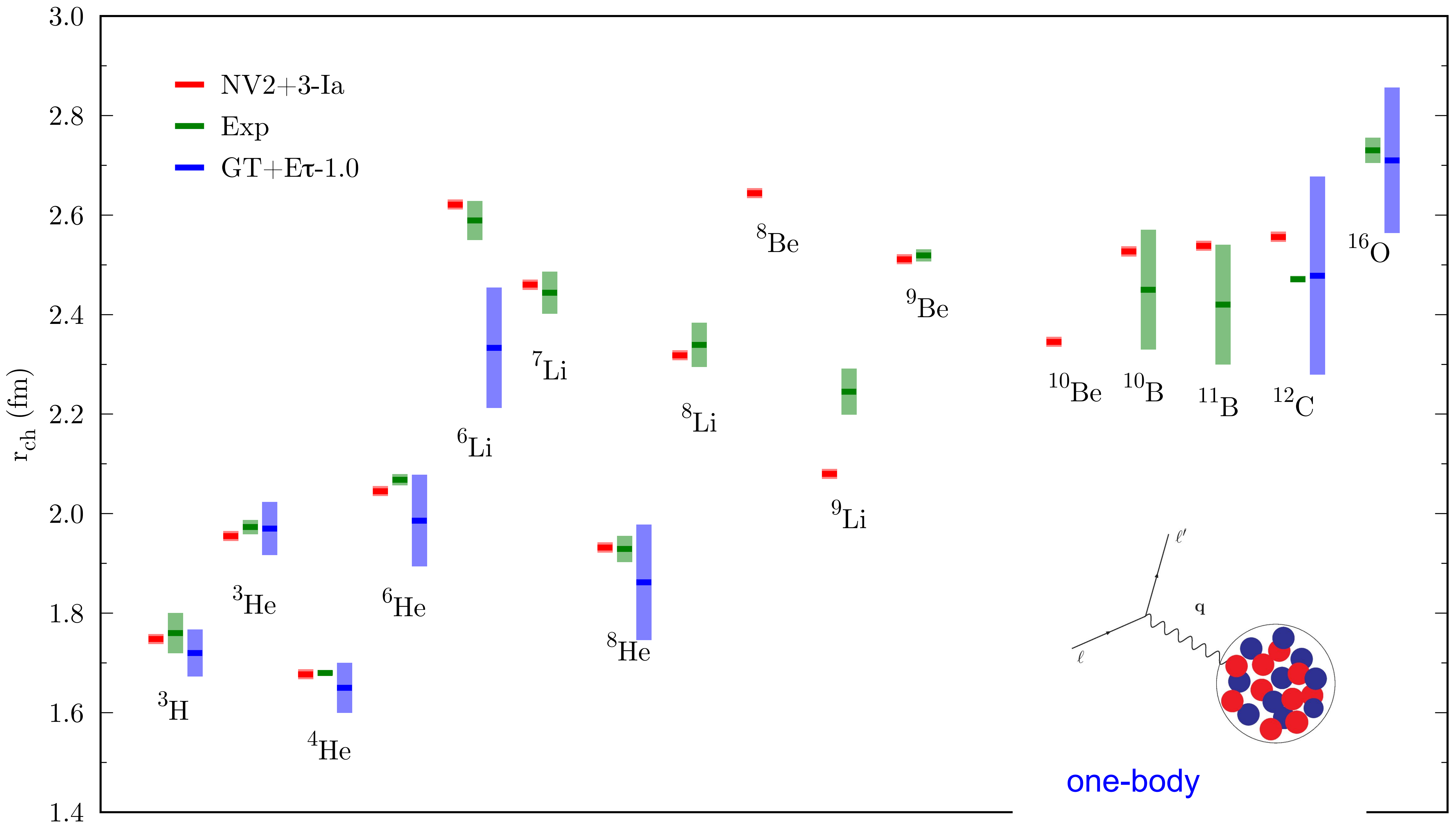
$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij}$$

- Meson exchange currents: Schiavilla et al., PRC 45, 2628 (1992), Marcucci *et al.* PRC **72**, 014001 (2005), Marcucci et al., PRC 78, 065501 (2008),…
- Chiral EFT currents: Park *et al.* NPA **596**, 515 (1996); Pastore *et al.* PRC **78**, 064002 (2008), PRC **80**, 034004 (2009); Piarulli *et al.* PRC **87**, 014006 (2013), Baroni *et al.* PRC **93**, 015501 (2016); Phillips et al. PRC **72**, 014006 (2005), Kölling *et al.* PRC **80**, 045502 (2009), PRC **84,** 054008, PRC **86**, 047001 (2012); Krebs et al., Ann. Phys. **378**, 317 (2017),….,

Selected Properties in Light Nuclei

Understanding Light Nuclei Through Binding Energy and Charge Radii Calculations





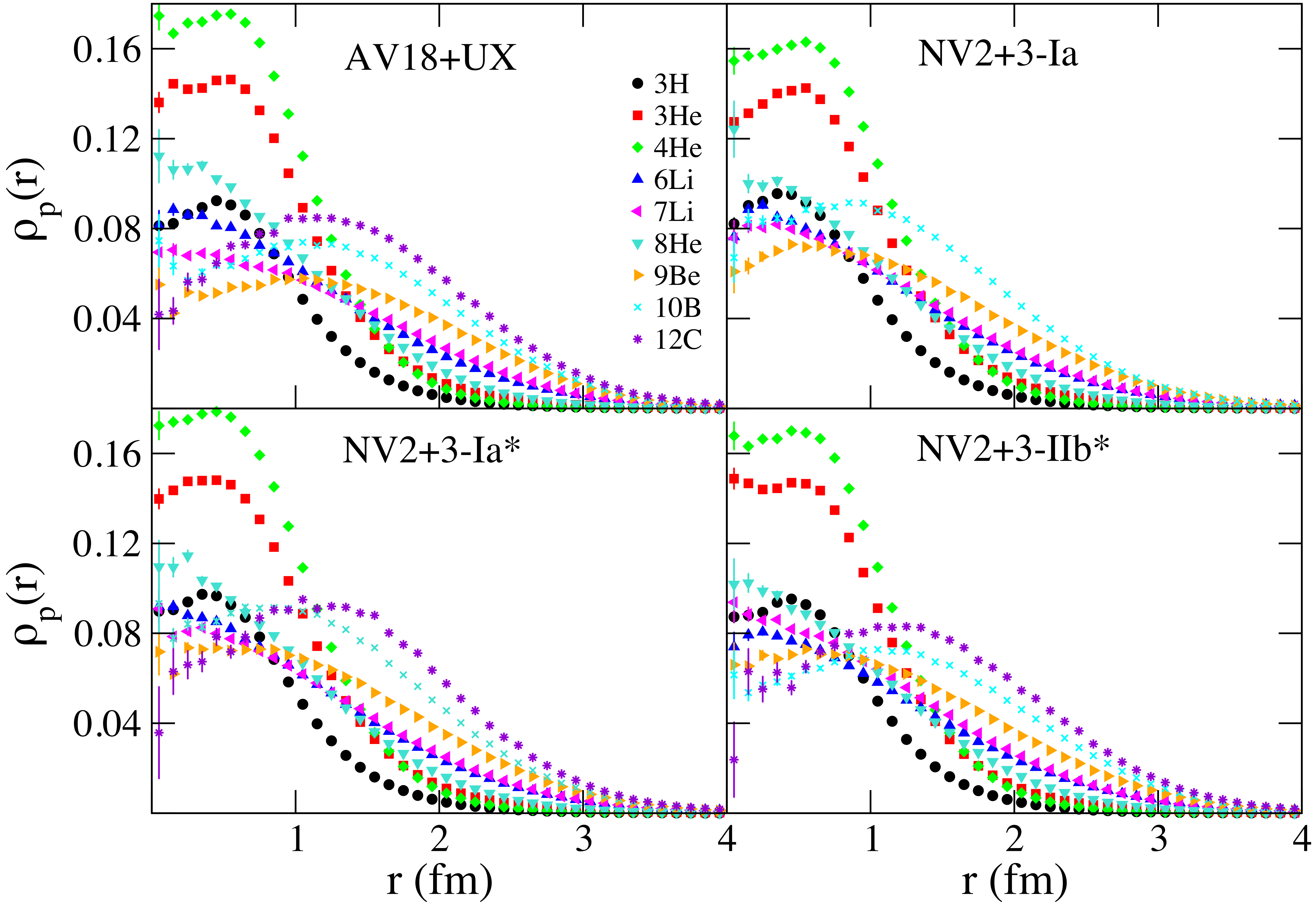
$$\langle r_{\rm ch}^2 \rangle = \langle r_{\rm pt}^2 \rangle + \langle R_p^2 \rangle + \frac{A-Z}{Z}\langle R_n^2 \rangle + \frac{3\hbar^2}{4M_p^2c^2} + \langle r_{\rm so}^2 \rangle$$

MP et al. Phys.Rev.C 91 (2015) 2, 024003

Gandolfi, **MP**, et al. Front.in Phys. 8 (2020) 11

Probing Nuclear Structure: Insights from Single-Nucleon Densities

In QMC methods, single-nucleon densities are calculated as: $\rho_N(r) = \frac{1}{4\pi r^2}\left\langle \Psi \left| \sum_i P_{N_i} \delta\left(r - |\mathbf{r}_i - \mathbf{R}_{\rm cm}|\right) \right| \Psi \right\rangle$



For symmetric nuclei N = Z nuclei, proton and neutron densities are the same

s-shell nuclei (A ≤ 4) exhibit large peaks at small separation, while the p-shell nuclei (A ≥ 6) are much reduced at small r and more spread out

Densities are not observables but single-nucleon density can be related to longitudinal (charge) form factor physical quantity experimentally accessible via electron-nucleus scattering processes

MP et al. Phys.Rev.C 107 (2023) 1, 014314

Analyzing Charge Form Factors in Light Nuclei

$$F_L(q) = \frac{1}{Z} \frac{G_E^p(Q_{\rm el}^2)\tilde{\rho}_p(q) + G_E^n(Q_{\rm el}^2)\tilde{\rho}_n(q)}{\sqrt{1 + Q_{\rm el}^2/(4m_N^2)}}$$

$\tilde{\rho}_N(q)$: the Fourier transform of $\rho_N(r)$

$$Q_{\rm el}^2 = q^2 - \omega_{\rm el}^2 \qquad \omega_{\rm el} = \sqrt{q^2 + m_A^2} - m_A$$

$G_E^N(Q^2)$: electric nucleonic form factor

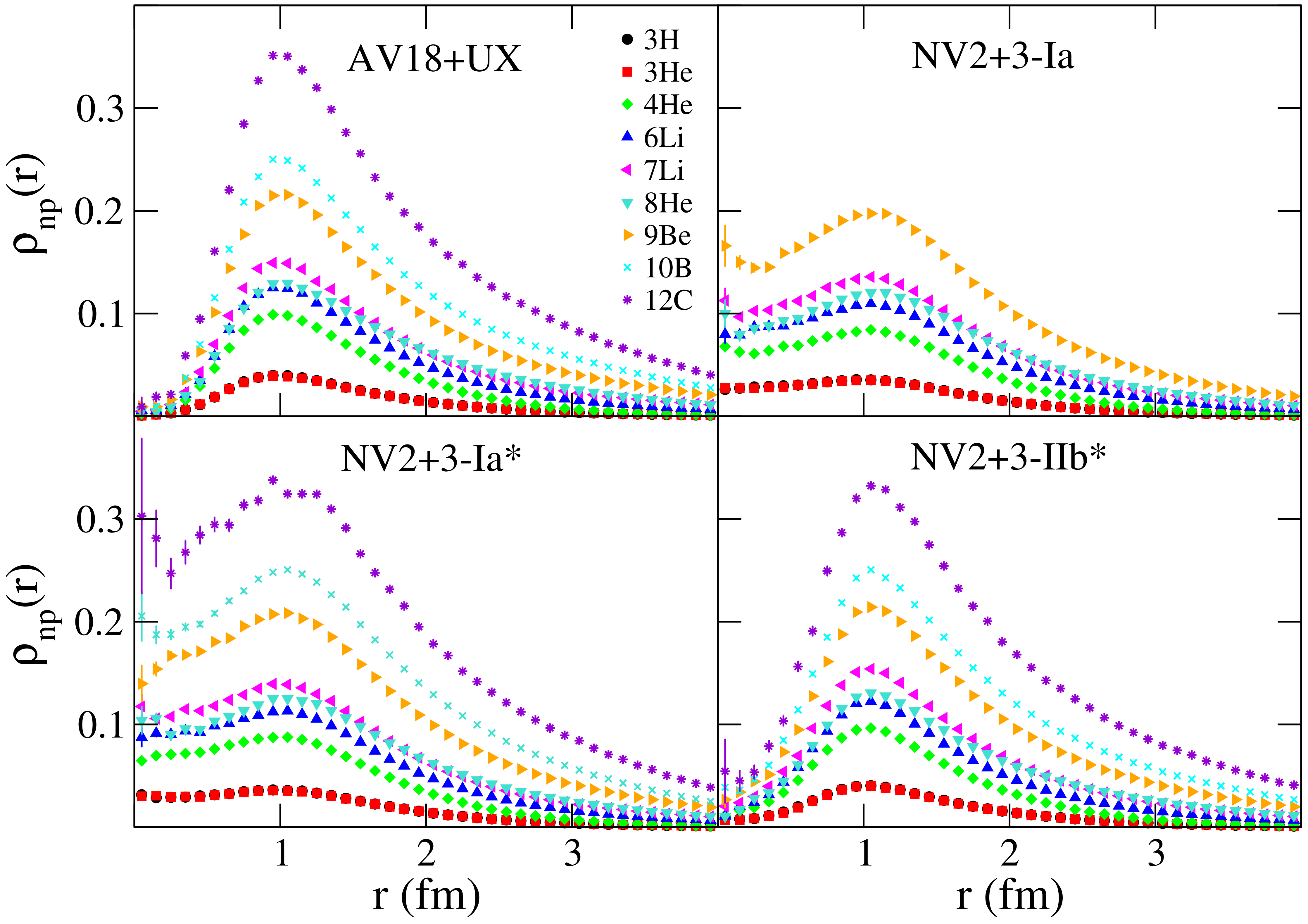
Inclusion of the 1b relativistic corrections and 2b corrections is in progress!

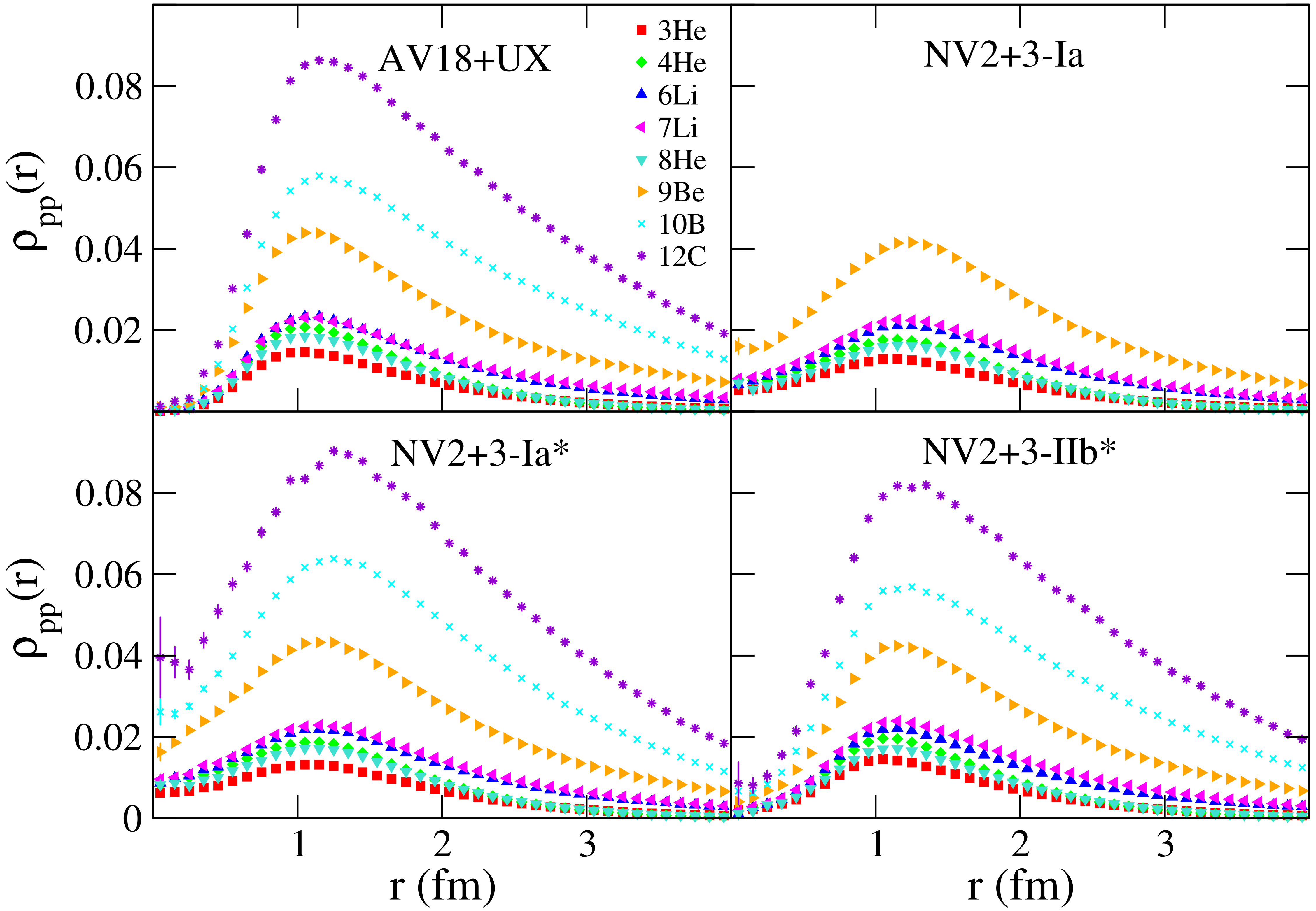
Gandolfi, **MP**, et al. Front.in Phys. 8 (2020) 11

Probing Nuclear Structure: Insights from Two-Nucleon Densities

In QMC methods, two-nucleon densities are calculated as:

$$\rho_{NN}(r) = \frac{1}{4\pi r^2} \langle \Psi | \sum_{i<j} \mathcal{P}_{N_i} P_{N_j} \delta(r - |\mathbf{r_i} - \mathbf{r_j}|) | \mathbf{\Psi} \rangle$$





Within a fixed interaction model, $\rho_{\mathrm{NN}}(r)$ at $r \lesssim 1.5$ fm for various nuclei exhibit a similar behavior: cooperation of the short-range repulsion and the intermediate-range tensor attraction of the NN interaction

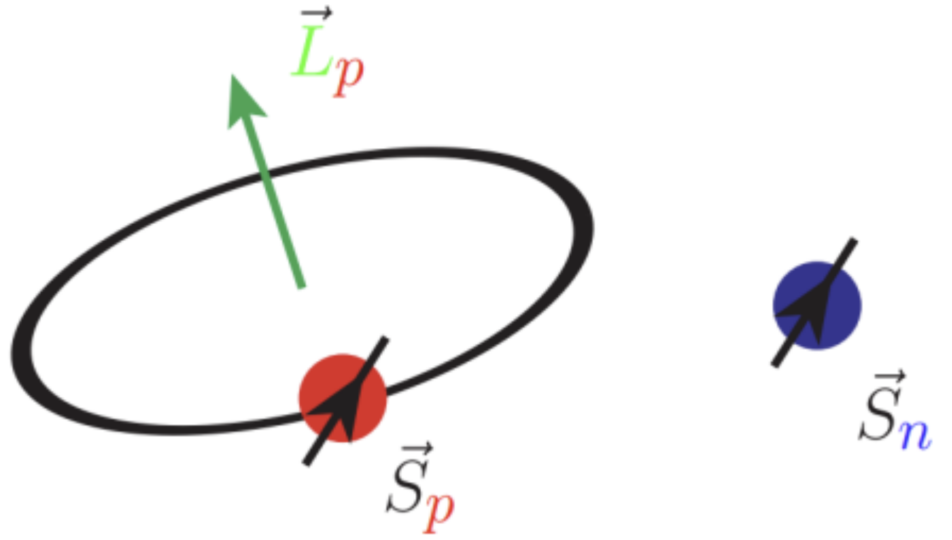
Tables and figures that tabulate the 2N densities (including pair distributions in different combinations of ST) are available online

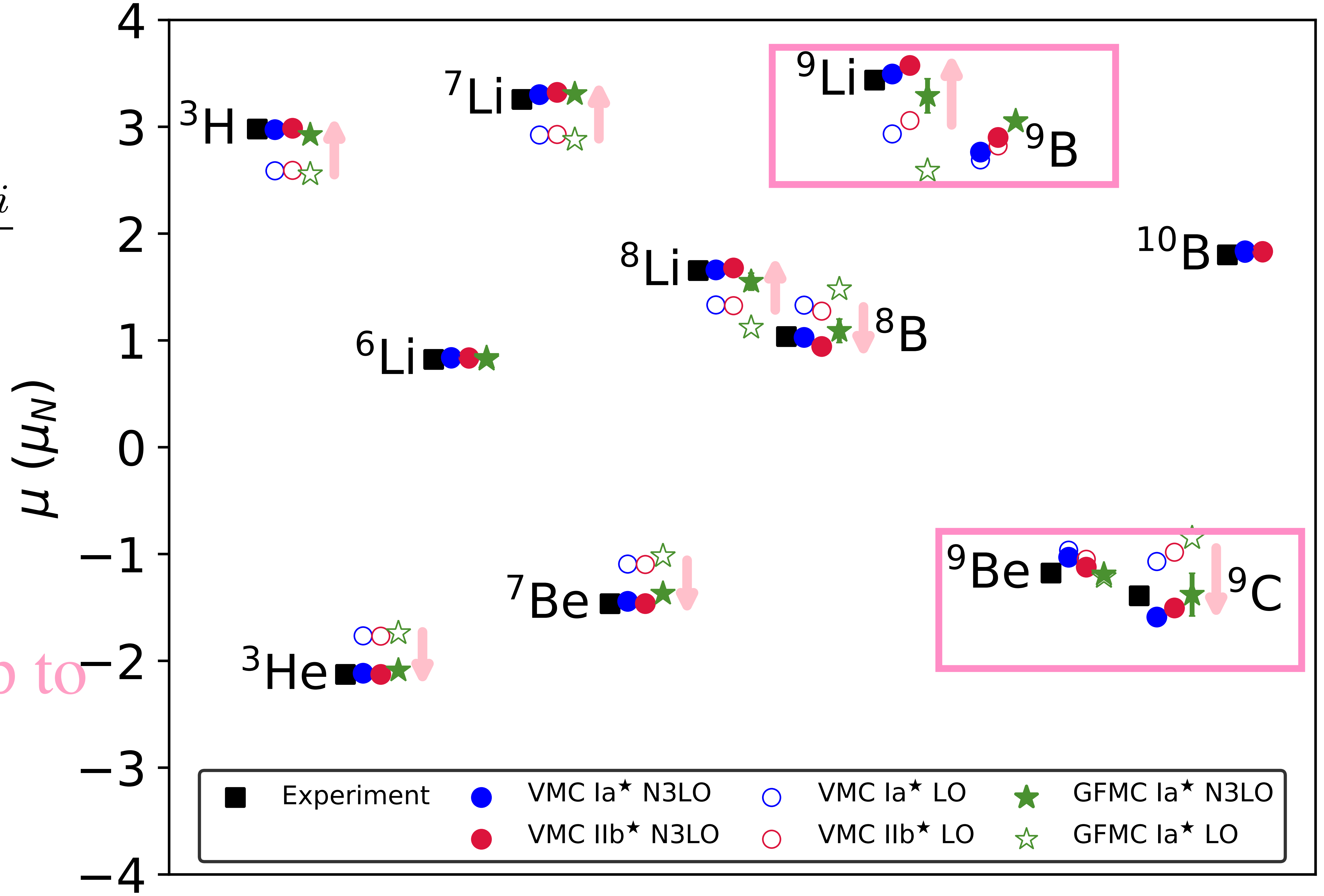
MP et al. Phys.Rev.C 107 (2023) 1, 014314

Impact of Two-Body Currents on Magnetic Moments

One-body picture

$$\mu^{LO} = \sum_i \left(L_{i,z} + g_p S_{i,z}\right) \frac{1+\tau_{3,i}}{2} + g_n S_{i,z} \frac{1-\tau_{3,i}}{2}$$

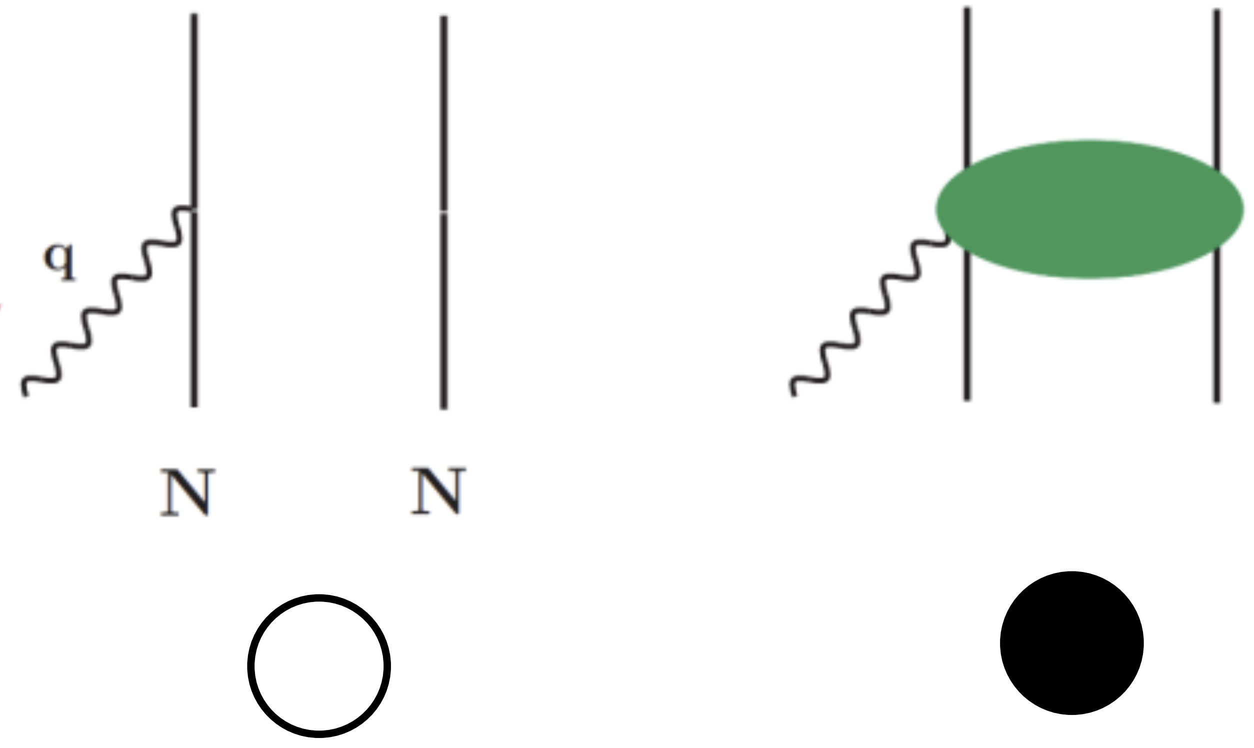


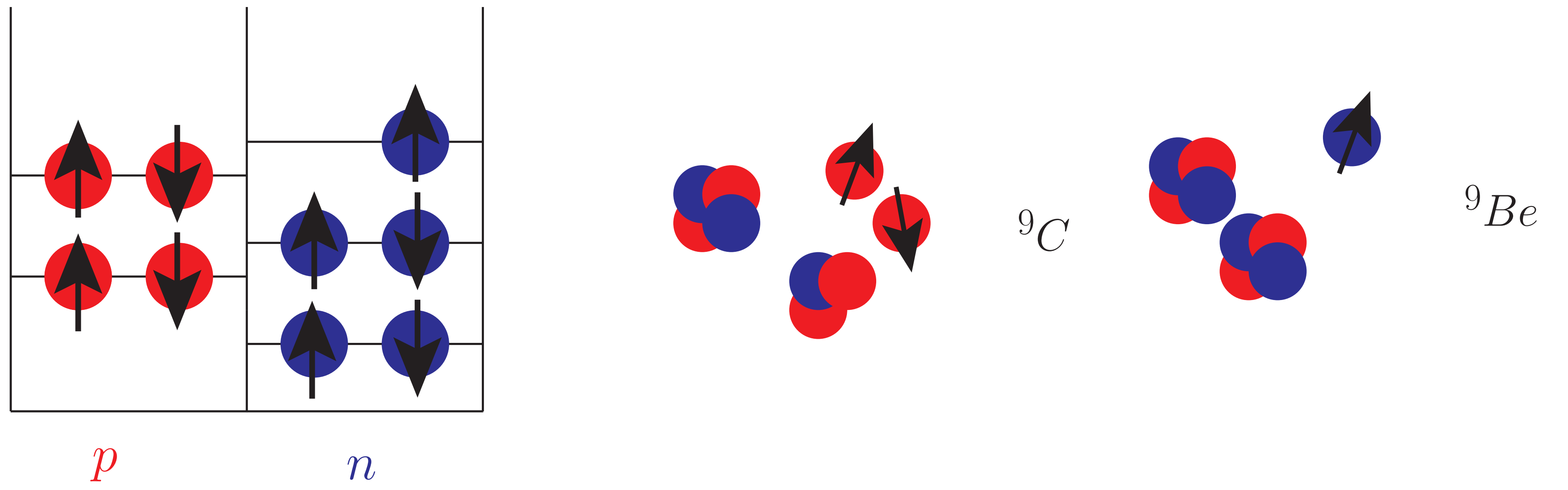


Two-body currents can play a large role (up to 33%) in describing magnetic moments

^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = [α, ^{3}He(^{3}H), $pp(nn)$] → Large MEC

^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = [$\alpha, \alpha, n(p)$]





Chambers-Wall, **MP** et al. e-Print: 2407.03487, e-Print: 2407.04744

Magnetic Structure: Two-body Currents

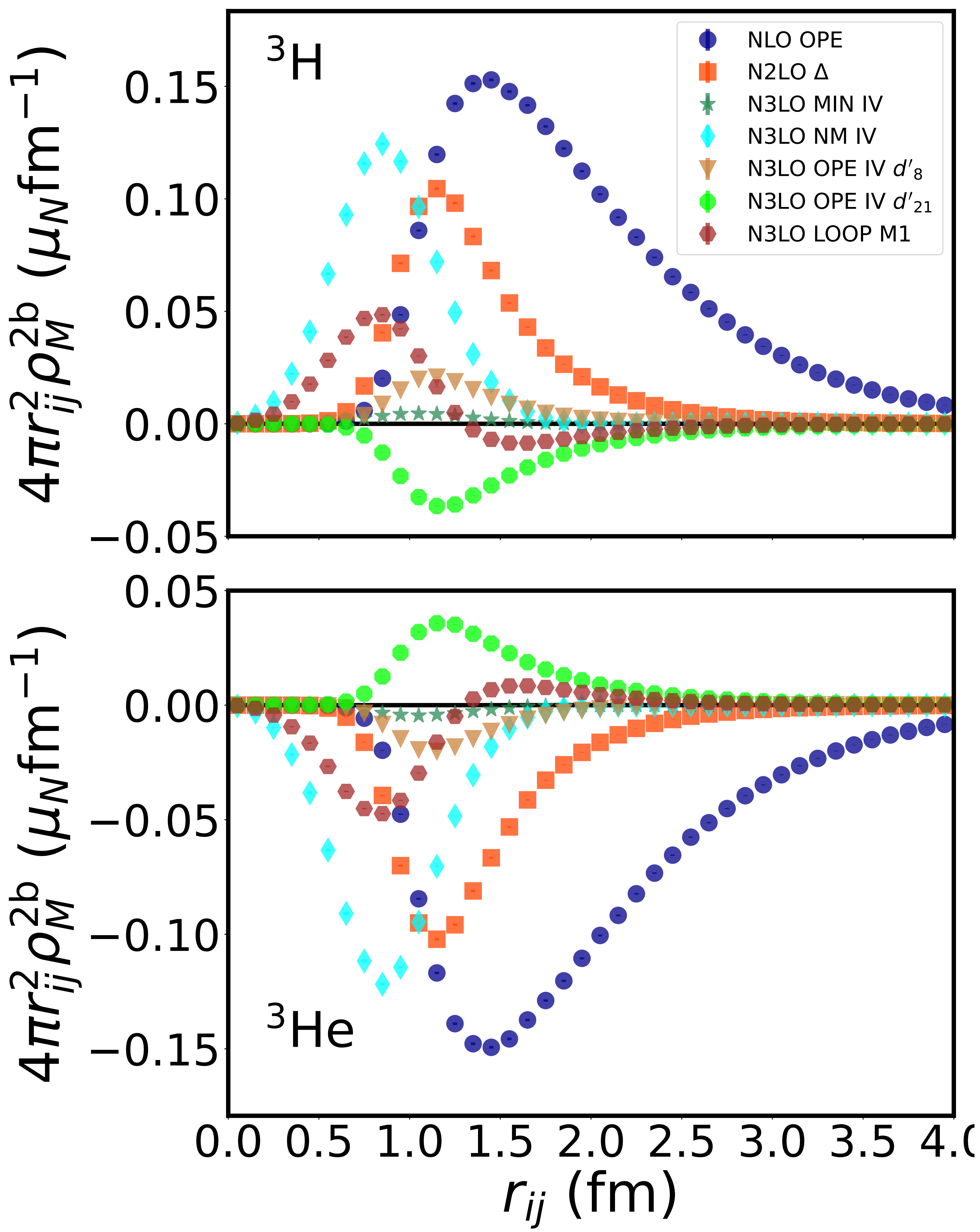
Non-minimal (NM) contact term should naively be suppressed by Q^3

It is in fact order $\sim Q^{1.5}$, resulting in larger-than-expected N3LO contribution

Summed contributions agree with data, but power counting is not converging order-by-order

$$\mu^{2b} = \int dr_{ij}\, 4\pi\, r_{ij}^2\, \rho_M^{2b}(r_{ij})$$

NV2+3-IIb*



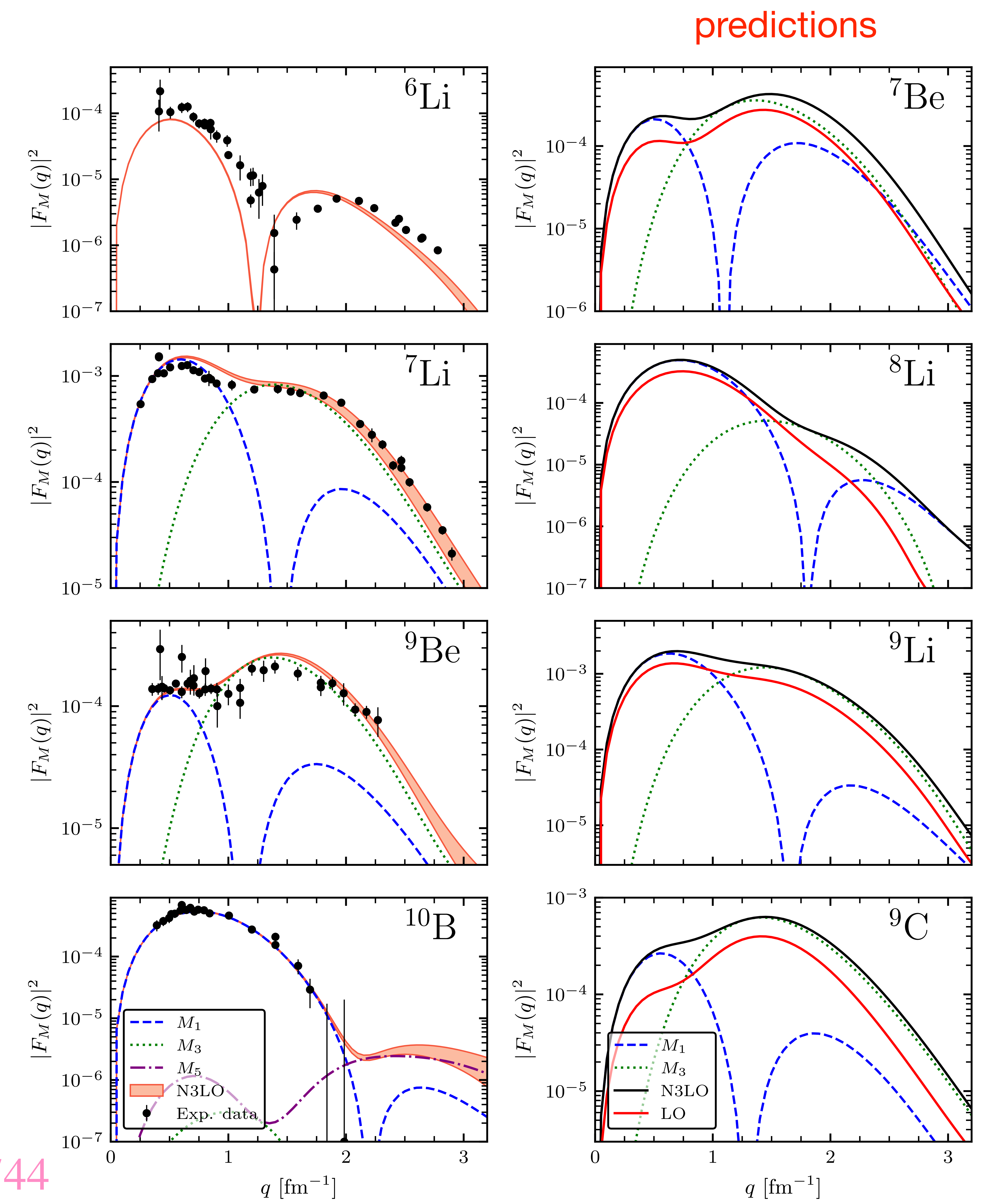
Chambers-Wall, **MP** et al. e-Print: 2407.03487, e-Print: 2407.04744

Magnetic Structure: Form Factors

NV2+3-IIb* is able to capture the shape of magnetic form factor

In some cases, good quantitative agreement at large momentum transfer

Two-body effects ~20% to 50% at large momentum transfer in various nuclei



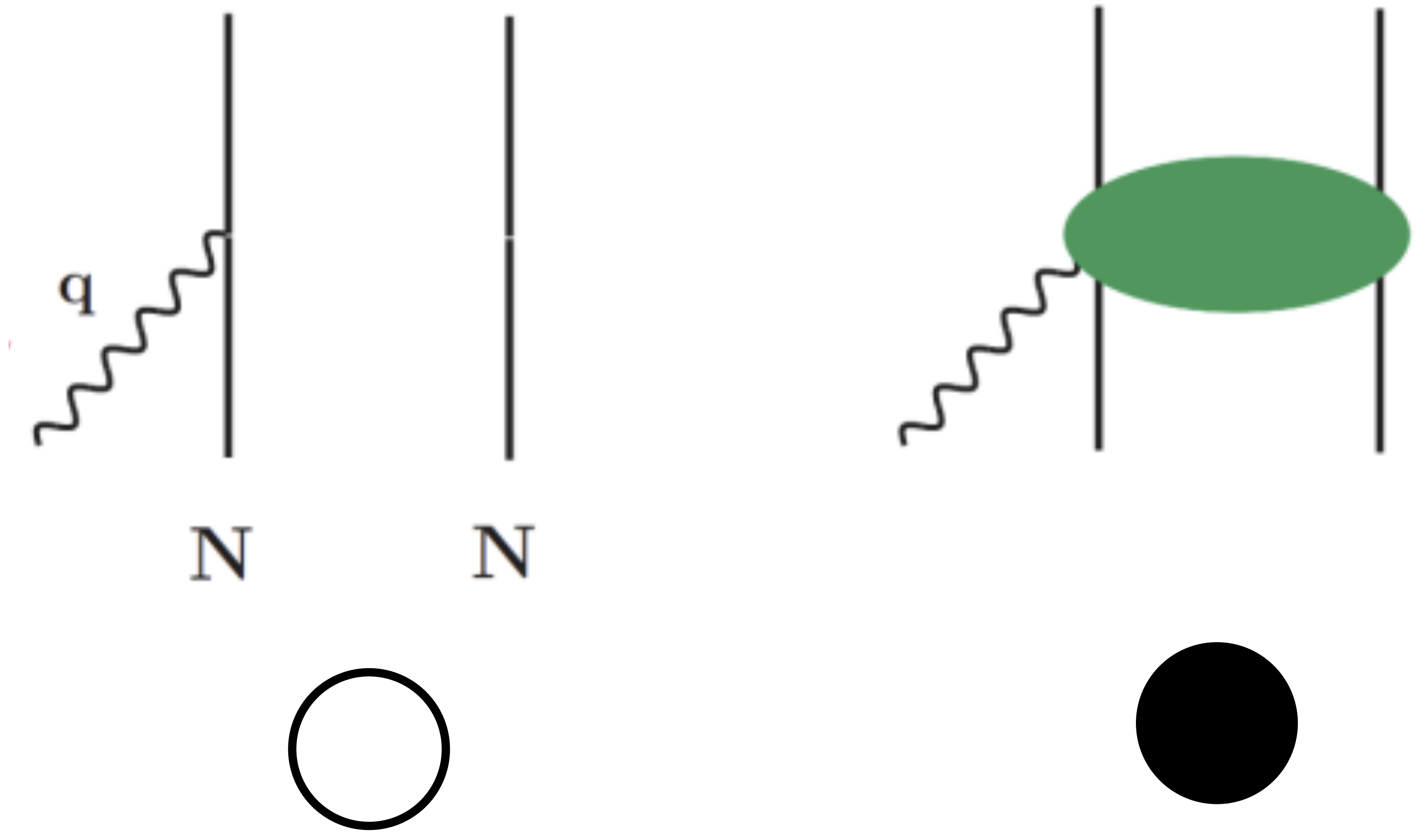
Chambers-Wall, **MP** et al. e-Print: 2407.03487, e-Print: 2407.04744

Insights from Gamow-Teller Matrix Elements in Beta Decay

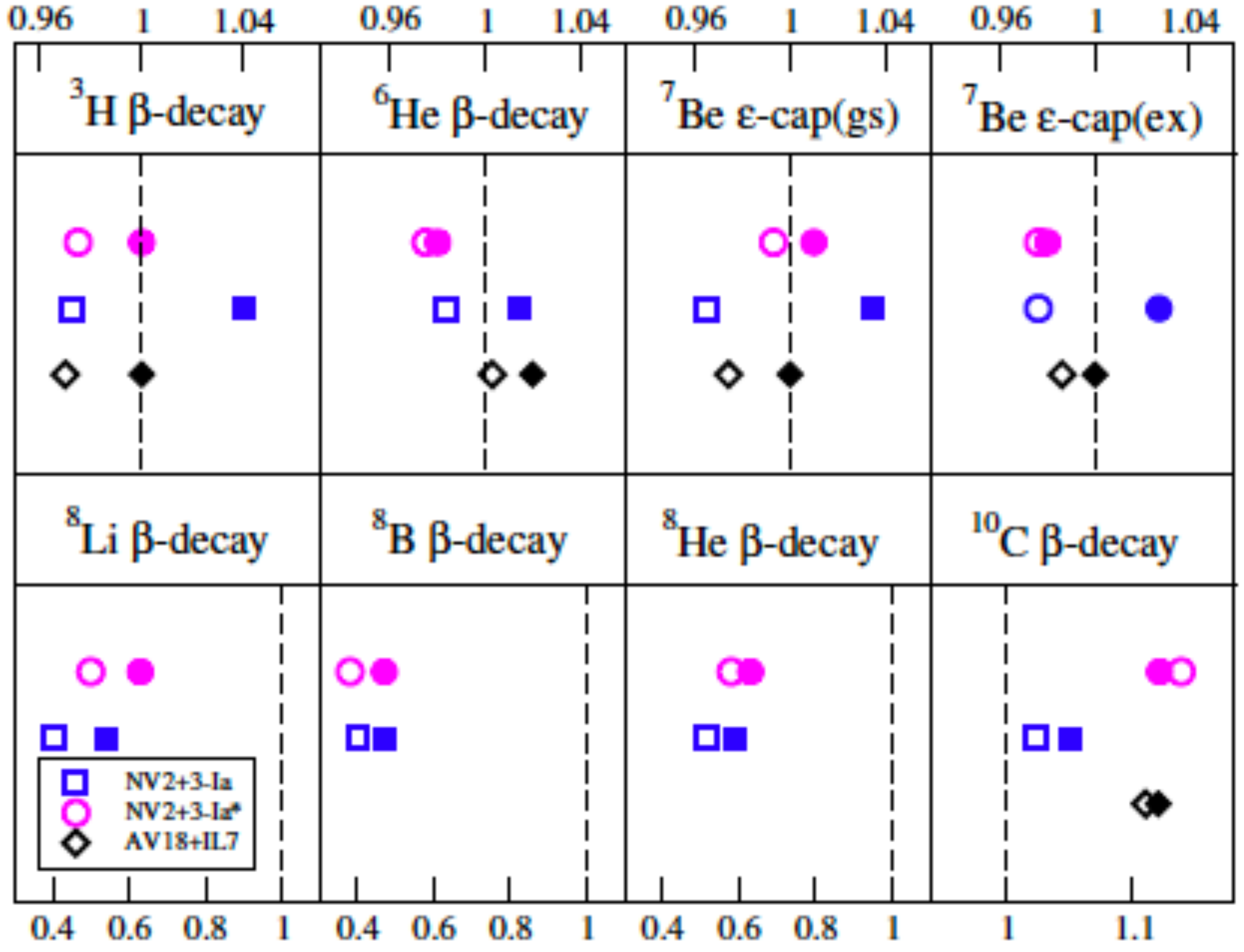
Calculations with NV2+3-Ia* and NV2+3-Ia compared to AV18+IL7 (◊) and exp (dashes)

Correlations provide bulk of quenching

Two-body almost always enhances



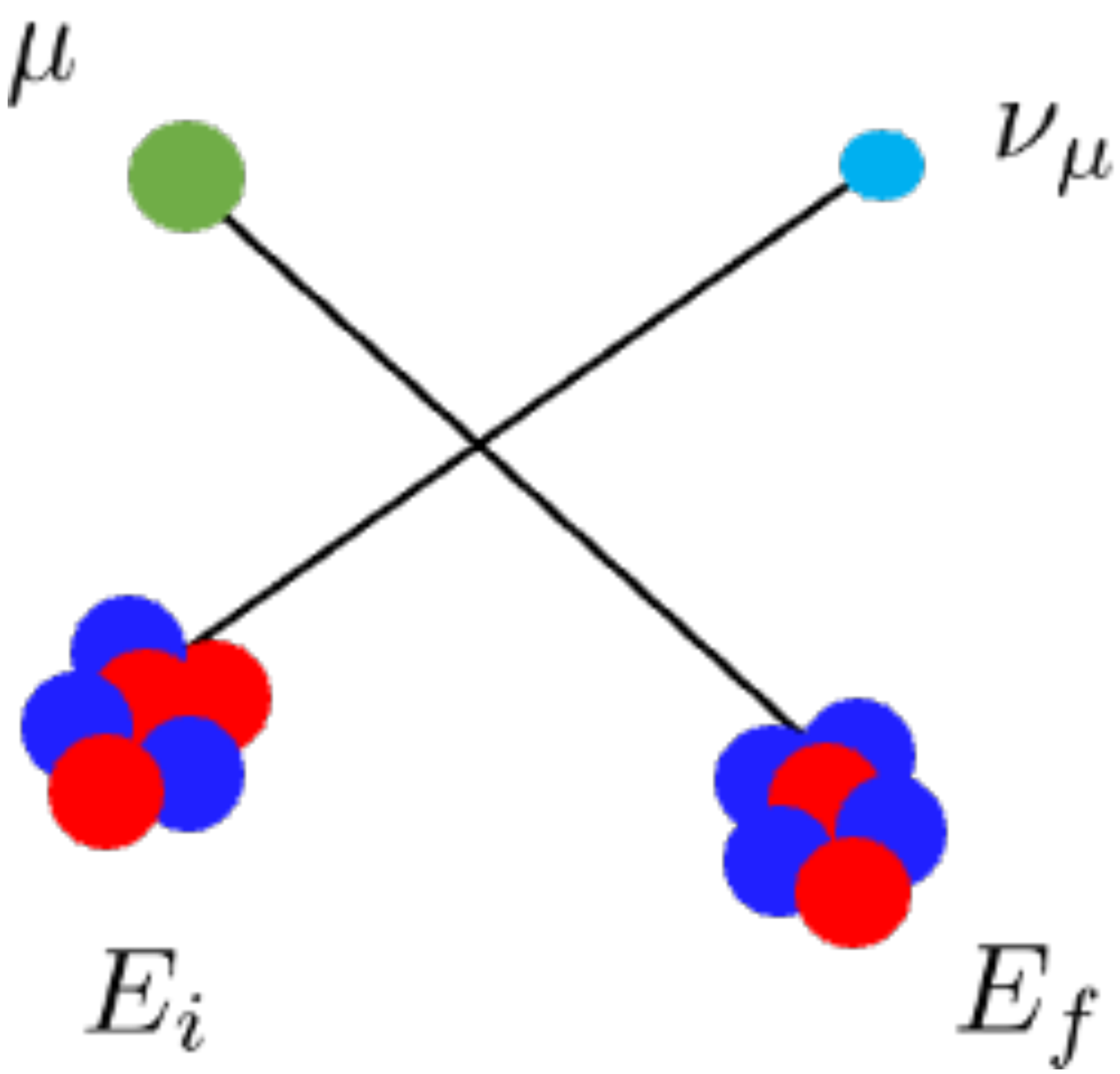
Baroni, **MP** *et al*. PRC **93**, 015501 (2016)



King, **MP** *et al*. PRC 102, 025501 (2020)

Partial Muon Capture in Light Nuclei

$$\Gamma \propto \sum_{\alpha\beta} |M_{\alpha\beta}|^2$$



Validation of vector and axial currents at q ~ 100 MeV

Same momentum transfer as neutrinoless double beta decay

Partial Muon Capture Rates

QMC rate for ^{3}He(1/2$^+$;1/2) → ^{3}H(1/2$^+$;1/2)

$\Gamma_{VMC} = 1512\ s^{-1} \pm 32\ s^{-1}$

$\Gamma_{GFMC} = 1476\ s^{-1} \pm 43\ s^{-1}$

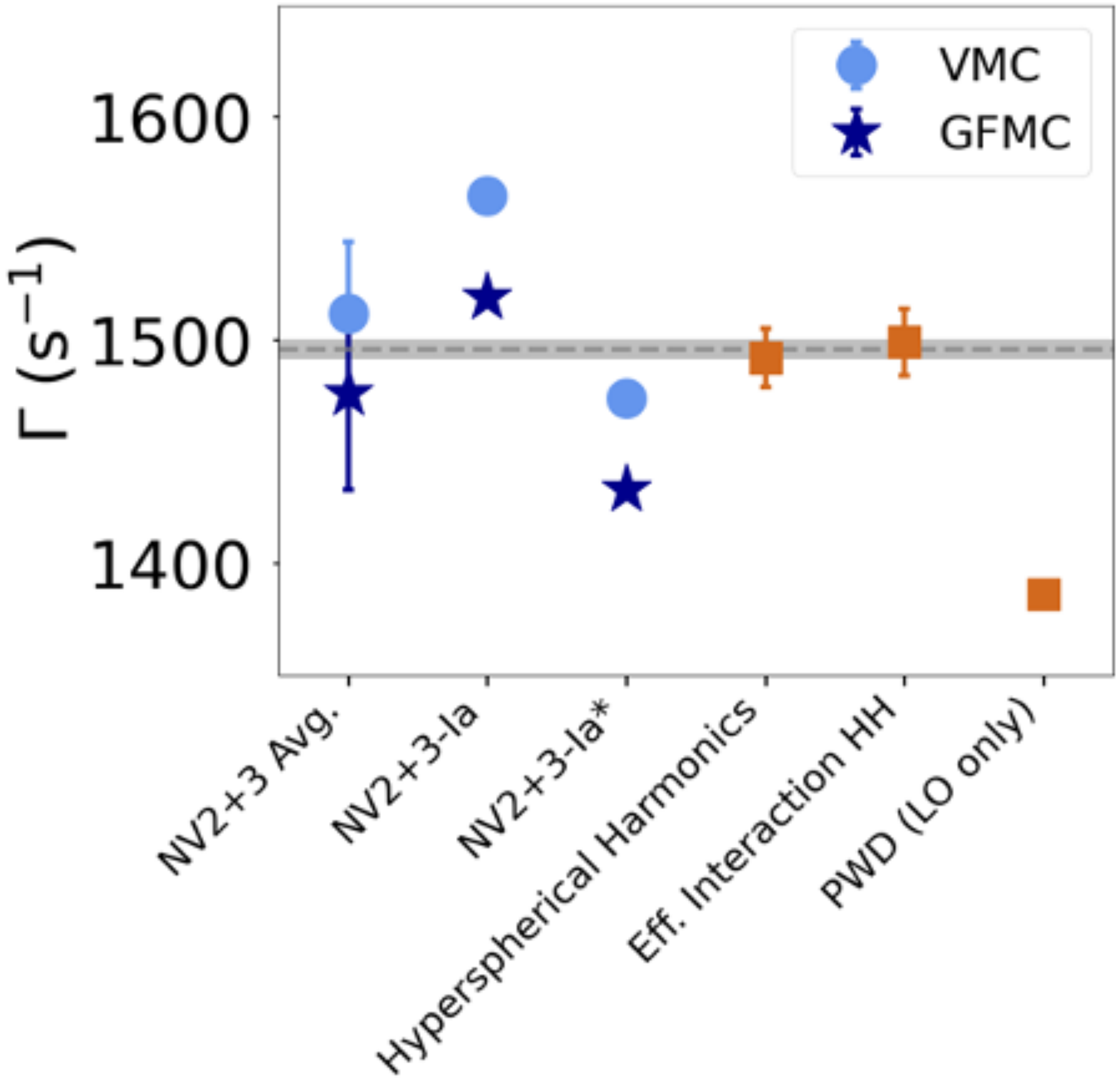
$\Gamma_{expt} = 1496.0\ s^{-1} \pm 4.0\ s^{-1}$

[Ackerbauer et al. Phys. Lett. B417 (1998)]

The inclusion of 2b electroweak currents increase the rate by about 9% to 16%.

uncertainty estimates:

Cutoff: 8 s^{-1} (0.5%)
Energy range of fit: 11 s^{-1} (0.7%)
Three-body fit: 27 s^{-1} (1.8%)
Systematic: 9 s^{-1} (0.6%)



King, **MP** et al. PRC 105 (2022) 4, L042501

Partial Muon Capture Rates

QMC rate for $^6Li(1^+;0) \rightarrow {}^6He(0^+;1)$

$\Gamma_{VMC} = 1243\ s^{-1} \pm 59\ s^{-1}$

$\Gamma_{GFMC} = 1056\ s^{-1} \pm 180\ s^{-1}$

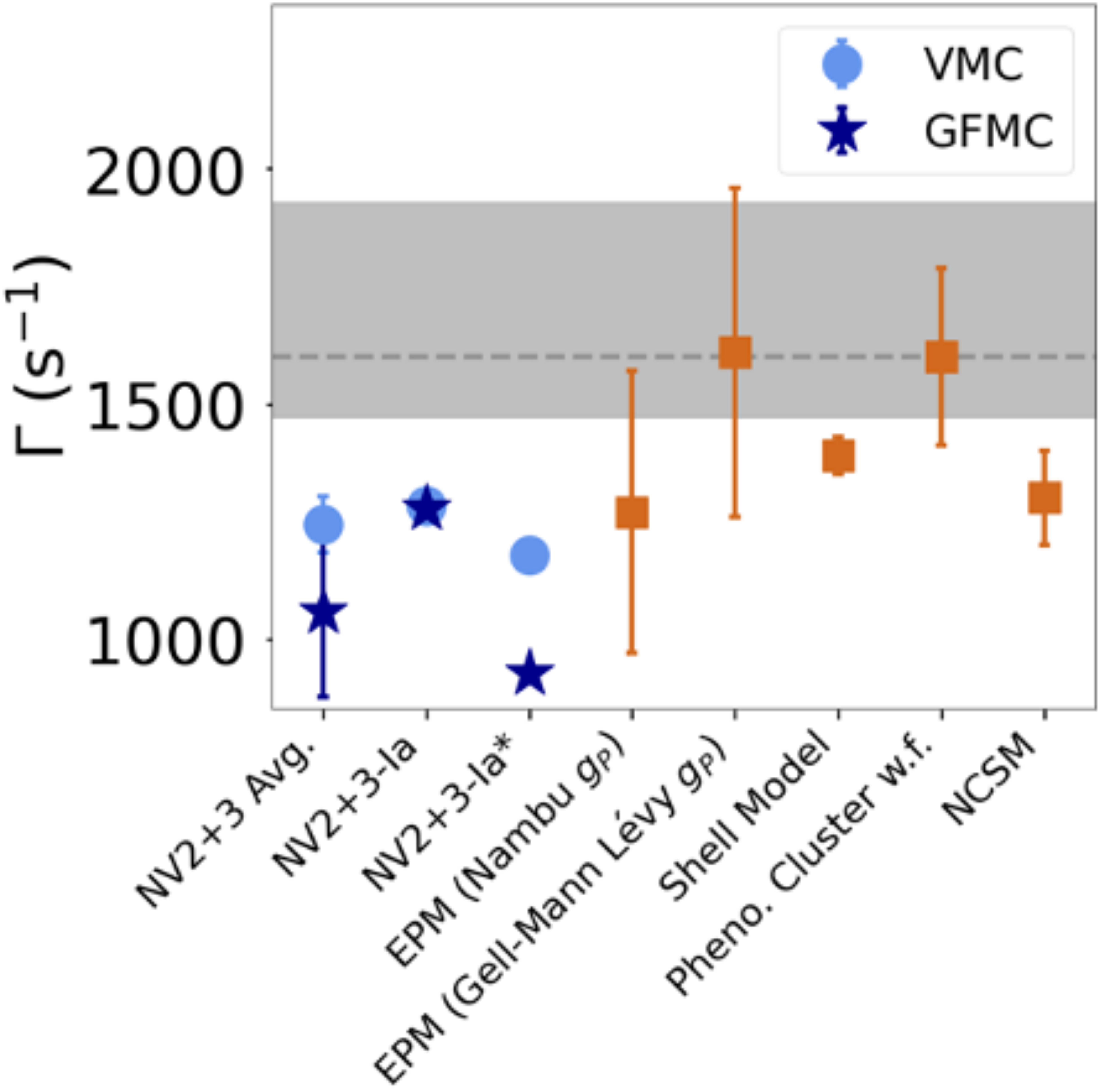
$\Gamma_{expt} = 1600\ s^{-1}\ +300/-129\ s^{-1}$

Deutsch et al. Phys. Lett. B26 (1968)

The inclusion of 2b electroweak currents increase the rate by about 3% to 7%.

uncertainty estimates:

Cutoff: 36 s^{-1} (2.9%)
Energy range of fit: 36 s^{-1} (2.9%)
Three-body fit: 30 s^{-1} (2.4%)
Systematic: 8 s^{-1} (0.6%)



King, **MP** et al. PRC 105 (2022) 4, L042501

Working on Theoretical UQ

Next-Generation χEFT Interactions

We are interested in calibrating the next generation of EFT nucleon-nucleon interactions.

These models should have robust uncertainty quantification:

- Parametric uncertainty
- Truncation uncertainty

Incorporating these uncertainties into our model calibration we aim to better understand and predict nuclear phenomena based on microscopic interactions of nucleons.

Bayes' Theorem

For a model calibration problem in a Bayesian approach, we have

$$\underbrace{\mathrm{pr}(\vec{\mathrm{a}}\,|\,\vec{\mathrm{y}},\mathrm{I})}_{\text{Posterior}} \propto \underbrace{\mathrm{pr}(\vec{\mathrm{y}}\,|\,\vec{\mathrm{a}})}_{\text{Likelihood}}\;\underbrace{\mathrm{pr}(\vec{\mathrm{a}}\,|\,\mathrm{I})}_{\text{Prior}}$$

The likelihood can be formulated as it is in standard model fitting for uncorrelated data

$$\mathrm{pr}(\vec{\mathrm{y}}\,|\,\vec{\mathrm{a}}) \sim e^{-\sum_i \left(y_{\text{exp}}^{(i)} - y_{\text{th}}^{(i)}(\vec{a})\right)^2/2\sigma_i^2} = e^{-\chi^2/2}$$

What the prior does for us is encode any previous information that we may know.

- Ex: LECs are natural, i.e., order 1 $\rightarrow$ $\mathrm{pr}(\vec{\mathrm{a}}\,|\,\mathrm{I}) \sim \mathcal{N}\left(\vec{0}, \Sigma_{\text{pr}}\right)$

Modeling the Model

Since our model is a perturbative series, we can write an observable as such (BUQEYE framework)*:

$$y_{\mathrm{th}}(x) = y_{\mathrm{ref}}(x) \sum_{n=0}^{\infty} c_n(x) Q^n(x), \quad Q \equiv \frac{\max[p_{soft}, p]}{\Lambda_b},$$

where $y_{\mathrm{ref}}(x)$ sets a reference scale for the observable y_{th}, Λ_b is the EFT breakdown scale, and c_n are the natural coefficients.

This series follows the truncation scheme of the EFT:

$$y_{\mathrm{th}}(x) = y_{\mathrm{ref}}(x) \sum_{n=0}^{k} c_n(x) Q^n(x) + y_{\mathrm{ref}}(x) \sum_{n=k+1}^{\infty} c_n(x) Q^n(x) = y_{\mathrm{th}}^{(k)}(x) + \delta y_{\mathrm{th}}^{(k)}(x).$$

*R. J. Furnstahl et. al. Phys. Rev. C **92**, 024005

Truncation Errors

From the neglected terms, we have

$$\delta y_{\mathrm{th}}^{(k)}(x) = y_{\mathrm{ref}}(x) \sum_{n=k+1}^{\infty} c_n(x) Q^n(x).$$

Under the assumption that the truncation error is uncorrelated across orders, this is a geometric series in Q, so we can find*

$$\delta y_{\mathrm{th}}^{(k)}(x) = \frac{y_{\mathrm{ref}}\, \bar{c}\, Q^{(k+1)}}{1 - Q},$$

Where we assume that $c_n \,|\, \bar{c} \sim \mathcal{N}\left(0, \bar{c}^2\right)$.

*J. A. Melendez et. al. Phys. Rev. C **100**, 044001

Additional Parameters

In this process, we have introduced two new parameters: $\bar{c}$ and Λ_b.

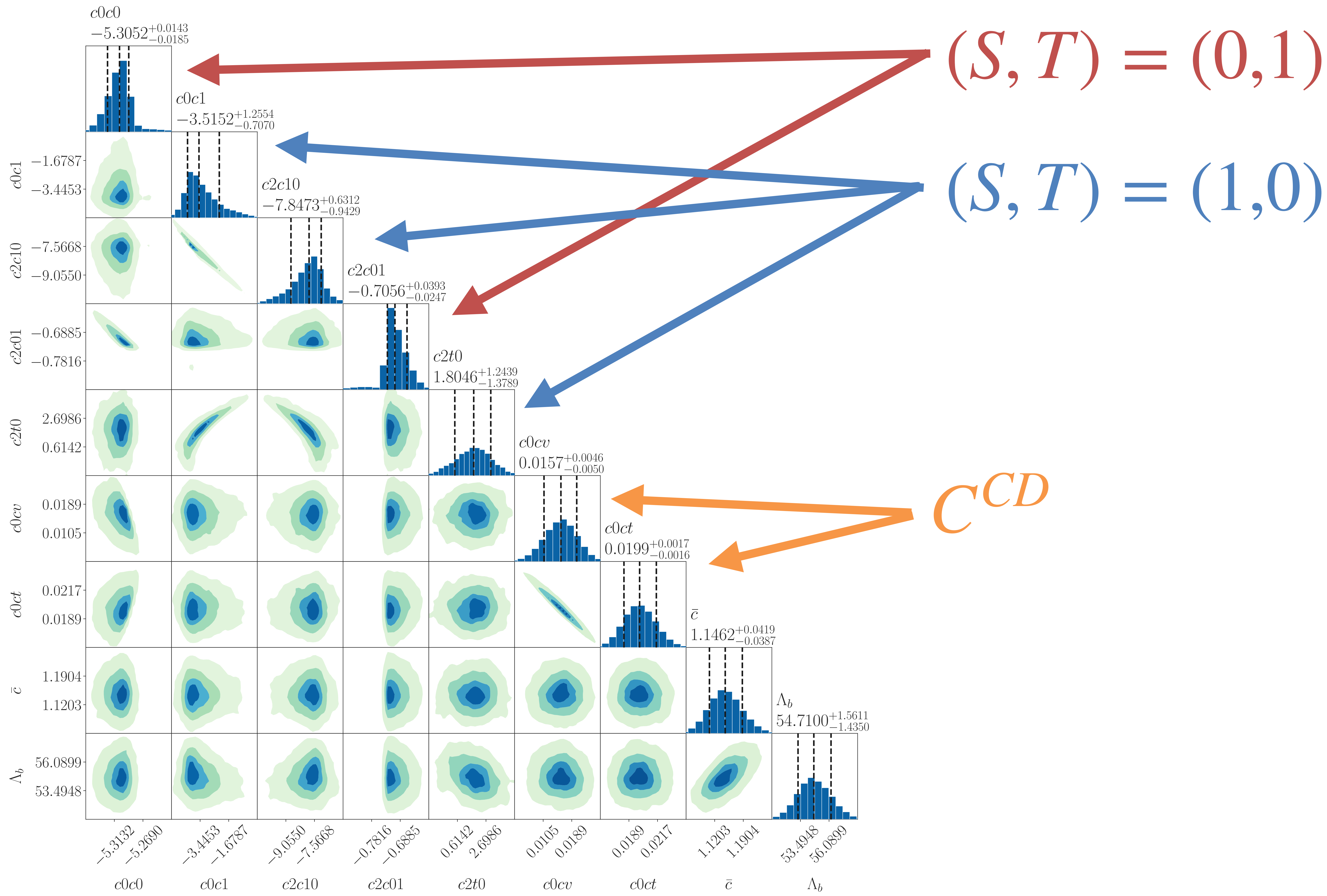
This changes the posterior we need to find:

$$\underbrace{\mathrm{pr}(\vec{a}, \bar{c}^2, \Lambda_b \,|\, \vec{y}_{\mathrm{exp}}, I)}_{\text{Total posterior}} \propto \underbrace{\mathrm{pr}(\vec{y}_{\mathrm{exp}} \,|\, \vec{a}, \Sigma, I)}_{\text{Likelihood for } \vec{a}} \; \underbrace{\mathrm{pr}(\vec{a} \,|\, I)}_{\text{Prior for } \vec{a}} \; \underbrace{\mathrm{pr}(\bar{c}^2 \,|\, \Lambda_b, \vec{a}, I)}_{\text{Posterior for } \bar{c}^2} \; \underbrace{\mathrm{pr}(\Lambda_b \,|\, \vec{a}, I)}_{\text{Posterior for } \Lambda_b} \; .$$

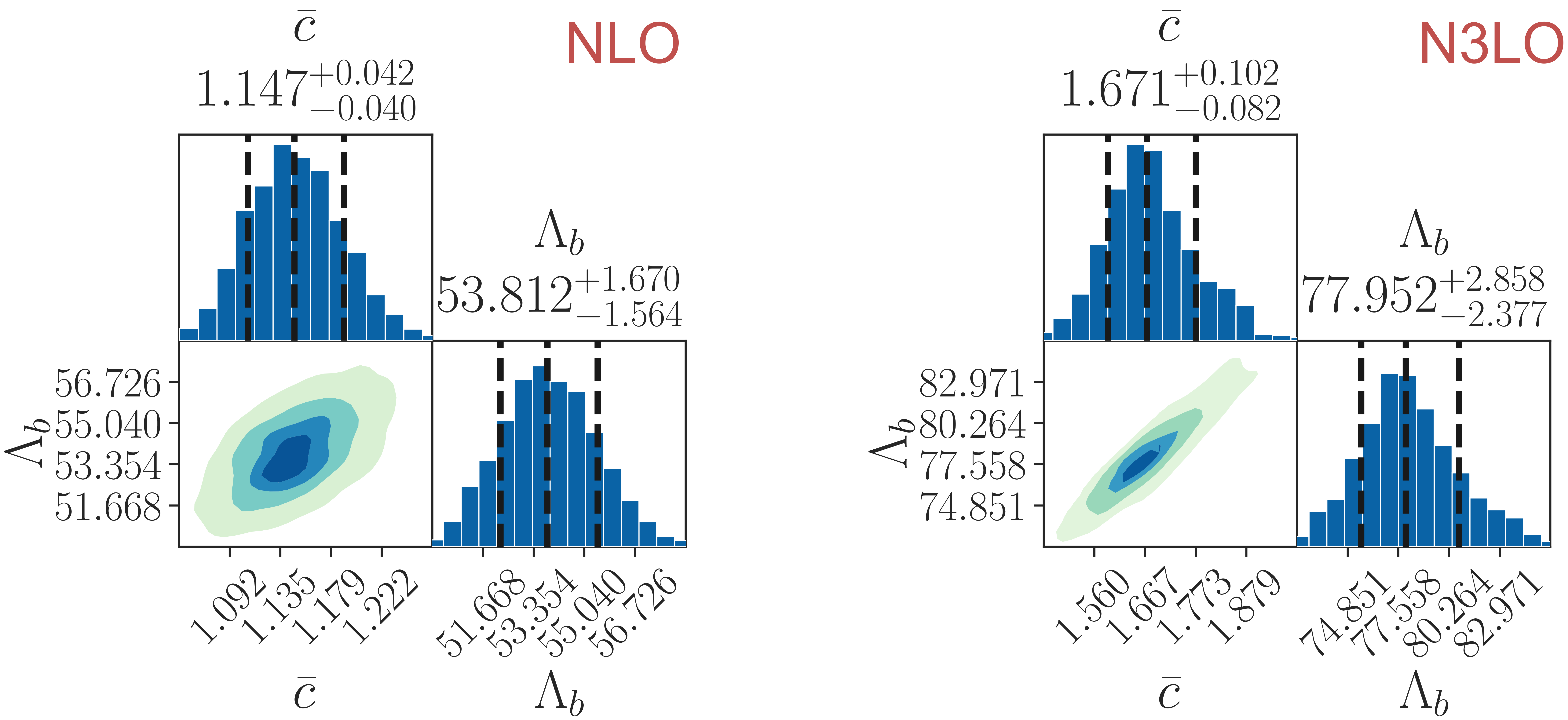
We can find a closed form of $\mathrm{pr}(\bar{c}^2 \,|\, \Lambda_b, \vec{a}, I)$ and $\mathrm{pr}(\Lambda_b \,|\, \vec{a}, I)$.

*J. A. Melendez et. al. Phys. Rev. C **100**, 044001

Posterior Distributions: Pionless Interaction



$\bar{c}$ and Λ_b Posteriors: Pionless Interaction



Bub, **MP** et al. e-Print: 2408.02480 [nucl-th]

Progress/Needs:

- *(Progress):* Tremendous progress in ab-initio theory: i) algorithms and interactions; ii) increased algorithm efficiency; iii) computational resources; iv) successful algorithm benchmarks; v) advent of EFTs and UQ
- *(Progress):* Microscopic description of nuclei represent a powerful tool to elucidate the role of two-body effects in nuclear interactions and currents: two- and many-body effects can be sizable and improve the agreement of theory with experiment
- *(Progress):* Possibility to perform consistent calculations for nuclei and infinite matter, connecting nuclei observables to astrophysical quantities and observations (not discussed in this talk!)

- *(Needs):* New protocols to build realistic nuclear interactions: improvements in the formulation of the 3NFs; which observables to use? In which mass range? Uncertainty quantification?
- *(Needs):* Developments in ab-initio methods to go to larger and larger nuclei (effort also within QMC to go above A>12)
- *(Needs):* Developments in ab-initio methods to tackle connect bound states with continuum states (effort also within QMC)