Interactions between two hadrons in lattice QCD

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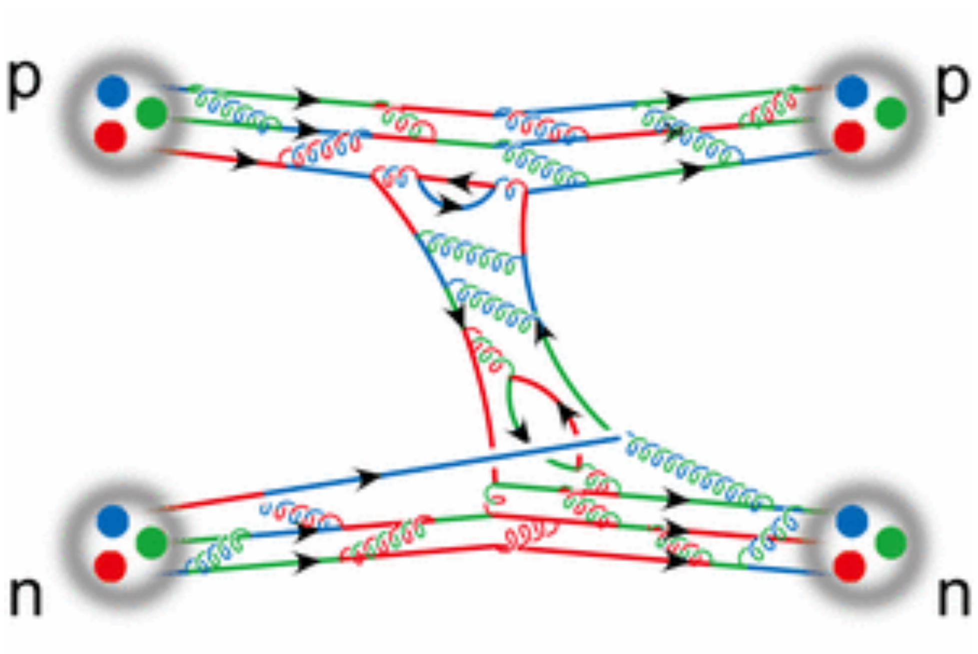
The 11th International Workshop on Chiral Dynamics (CD2024)
August 26-30, 2024, Ruhr University Bochum, Germany

I. Introduction



Hadron interactions in lattice QCD

Finite volume (FV) method

spectra of two hadrons in finite box → scattering phase shift

Luescher's finite volume formula

HAL QCD method

NBS wave functions → Potential (Interaction kernel) → scattering phase shift

Schrodinger equation

It is always better to have two "different" methods ("diversity") for crosschecks.

NN controversy (more than 9 years ago)

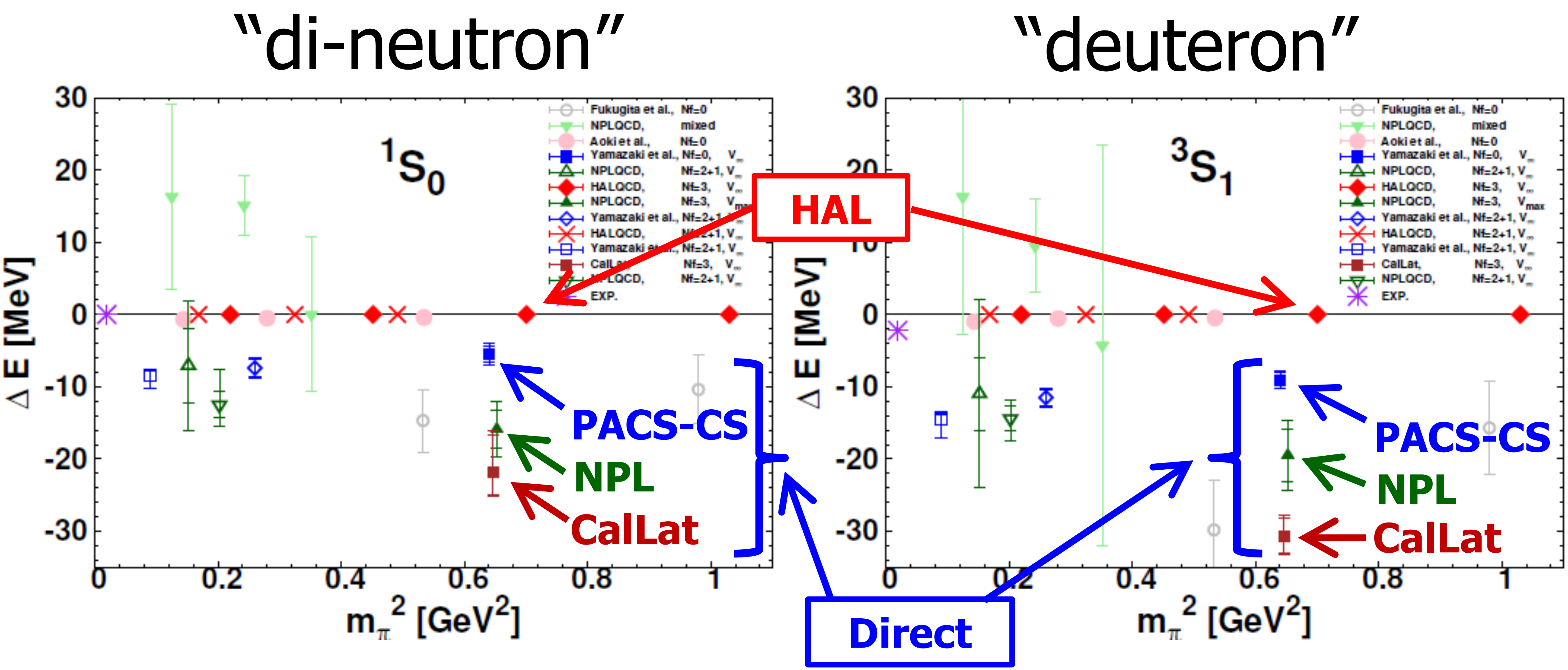
FV spectra: both deuteron and dineutron are bound at heavier pion masses.

Potential : Two nucleons are unbound at heavier pion masses.

NN controversy@ heavy pions
Direct method vs HAL QCD potential method

Slide from S. Aoki @ 2019 Santa Fe workshop

Potential method (HAL): unbound
Direct method (PACS-CS/NPL/CalLat): bound



Since then, there appeared more FV spectra from the community to resolve this issue.

Walker-loud@lat2023

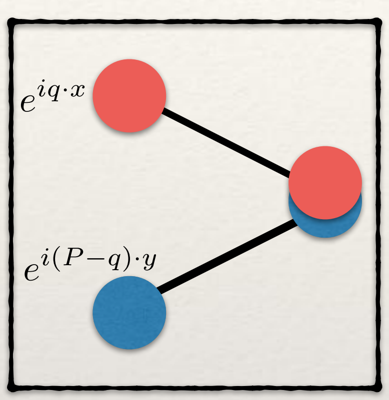
LQCD Results with (deeply) bound di-nucleons

2006 NPLQCD - first dynamical LQCD calculations of NN
2011 NPLQCD $M\pi \simeq 390$ MeV
2012 Yamazaki et al. $M\pi \simeq 510$ MeV
2012 NPLQCD $M\pi \simeq 800$ MeV
2015 Yamazaki et al. $M\pi \simeq 310$ MeV
2015 CallLat $M\pi \simeq 800$ MeV + P,D,F waves
2015 NPLQCD $M\pi \simeq 450$ MeV
2020 NPLQCD $M\pi \simeq 450$ MeV

LQCD Results without bound di-nucleons (or inconclusive)

2012 HAL QCD $M\pi \simeq 710$ MeV
2012 HAL QCD $M\pi \simeq 469$ — 1171 MeV
2019 "Mainz" $M\pi \simeq 960$ MeV
2020 CoSMoN $M\pi \simeq 714$ MeV
2021 NPLQCD $M\pi \simeq 800$ MeV

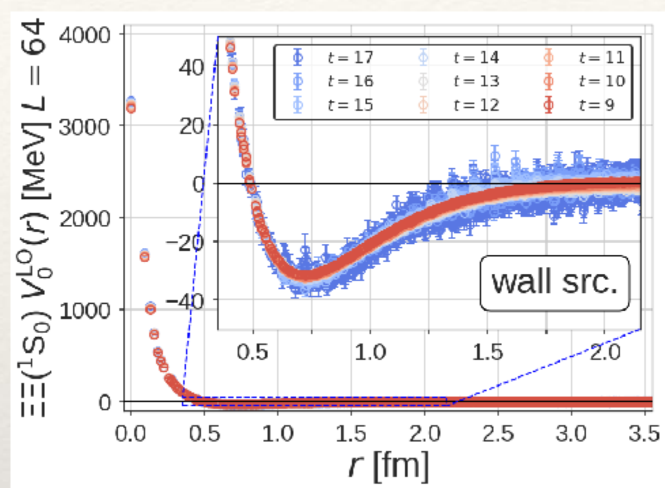
NPLQCD,
Yamazaki et al.,
CallLat (2015)



Compact, hexa-quark creation operator

Deep bound di-nucleons

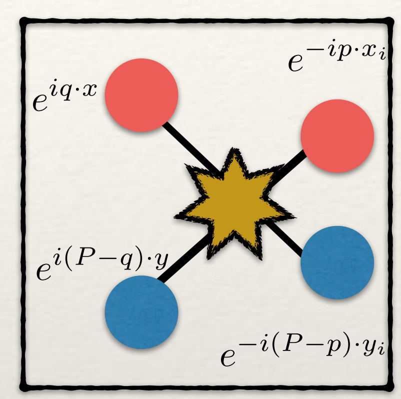
HAL QCD Potential



diffuse - wall source

no bound state

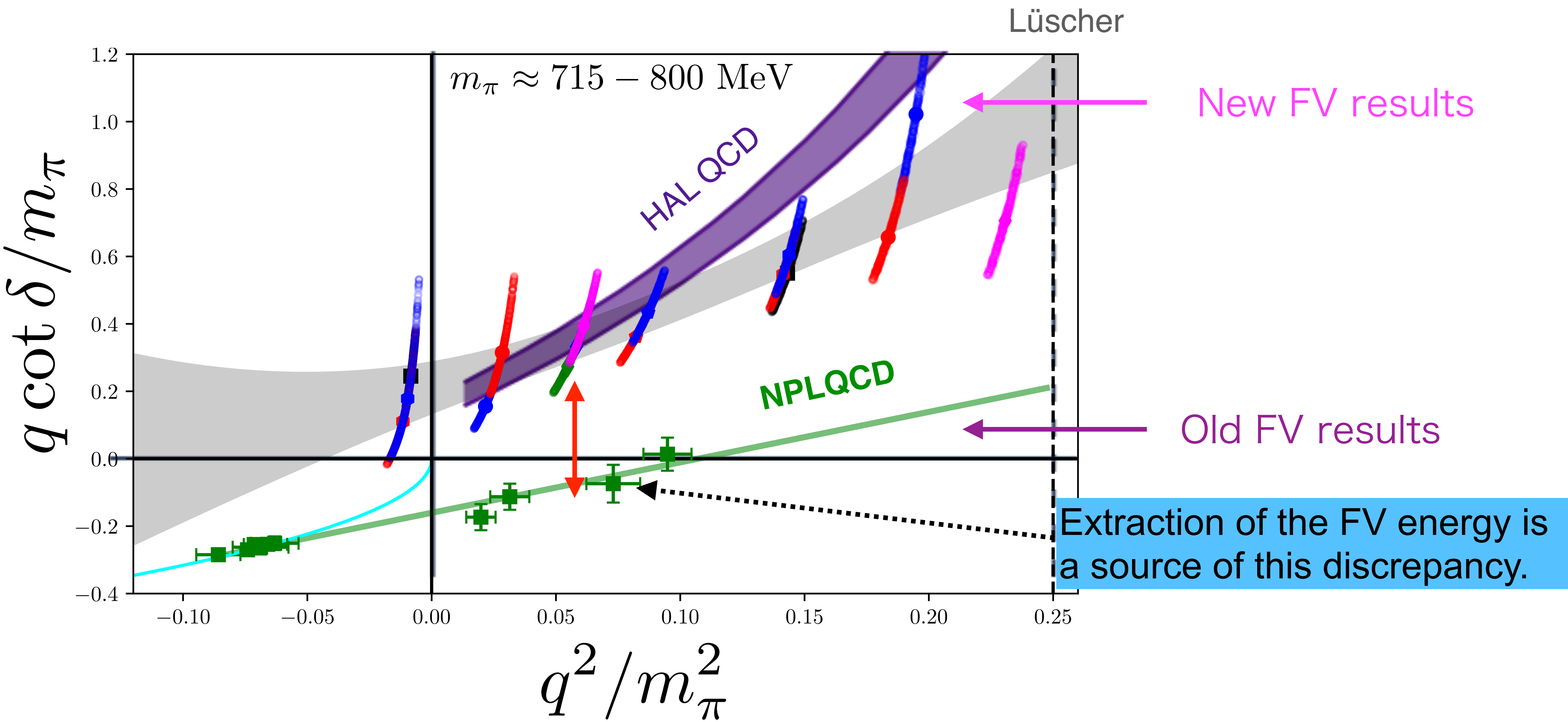
"Mainz" (Distillation)
CoSMoN (stochastic LapH
NPLQCD (sparsened momentum)



momentum-space
creation & annihilation
positive-definite correlation matrix

no bound state

Nicholson @Lat2022 update: Matthur@Lat2024, Morningstar's talk, Green's talk



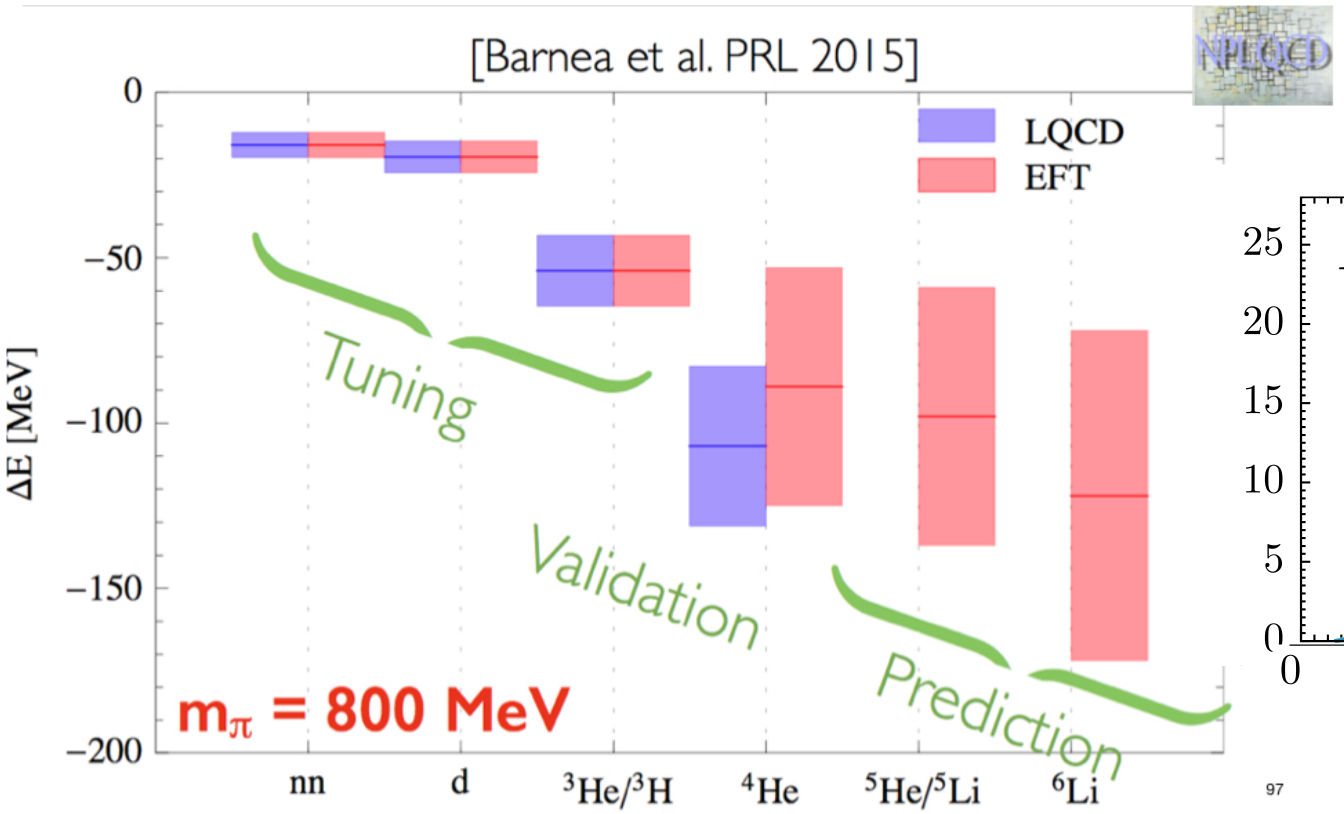
Walker-loud@Lat2023: "I believe the old results are wrong (including those I was involved with)

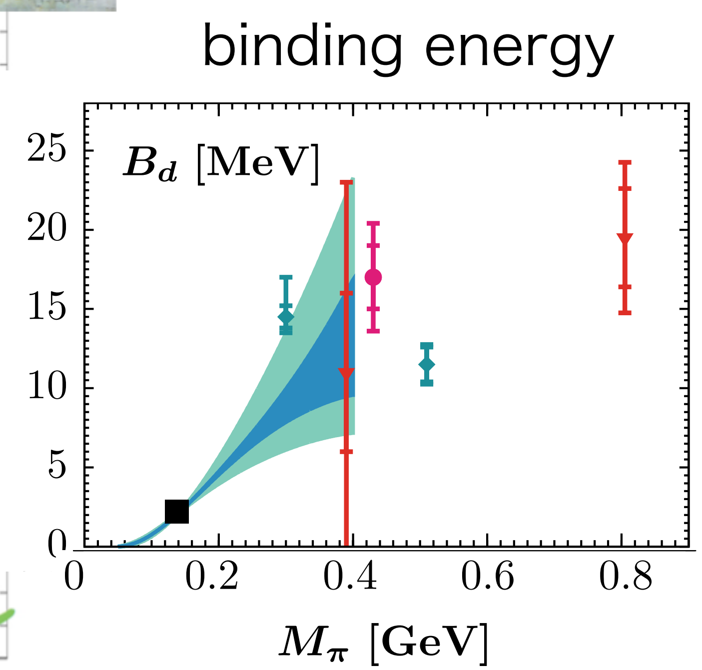
Two nucleons are unbound at heavy pion masses.

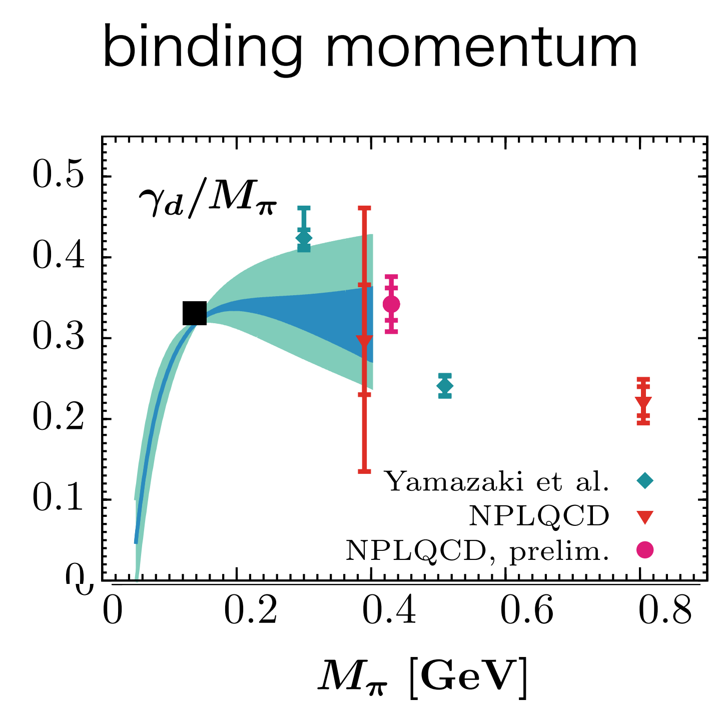
c.f. A combination of the finite volume formula and the ERE($k \cot \delta$) for a bound state was first introduced as "Sanity Check" to this problem. Iritani et al., PRD96(2017)034521.

Could (chiral) effective theories invalidate old finite volume data ?

Unfortunately, the answer seems “No”.







pionless EFT Beane@CD15 ChPT Baru@CD15

The chiral EFTs and old lattice data seemed consistent if EFT parameters were fixed by the same lattice data.

It is hard for EFTs to tell whether lattice data are correct or not.

In this talk, I will review recent results on interactions between two hadrons at almost physical pion mass, obtained exclusively by the HAL QCD method, stressing connections with chiral dynamics or experiments.

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II. $\Lambda\Lambda - N\Xi$ interactions

III. $N\Omega$ Dibaryon

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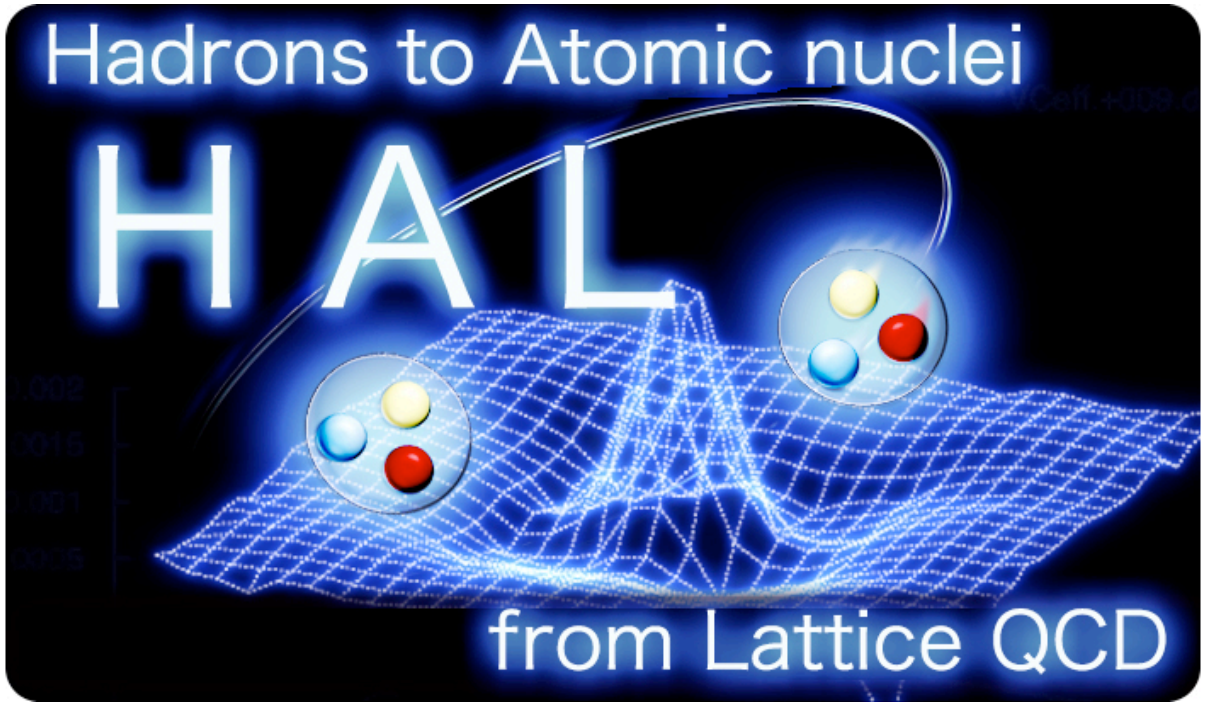
VI. Summary



Gauge configurations at (almost) physical point

Ishikawa *et al.* (PACS), PoS Lattice2015(2016) 075.

All results in this talk have been obtained on this ensemble.

2+1 flavor gauge configuration on 96^4 lattice

with Iwasaki gauge + NP $O(a)$ improved clover quark

$a \simeq 0.0846$ fm, $m_\pi \simeq 146$ MeV, $m_K \simeq 525$ MeV (almost physical point)

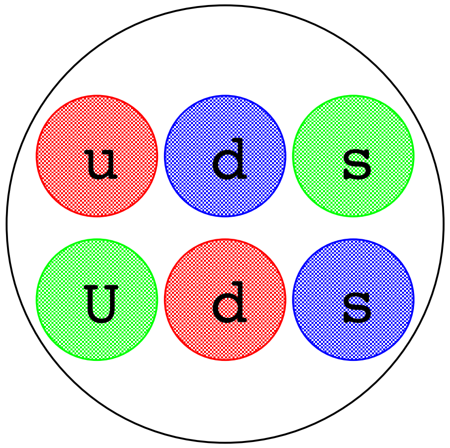
$La \simeq 8.1$ fm

generated by K-computer, previous supercomputer in Japan.

K computer

II. $\Lambda\Lambda - N\Xi$ interactions

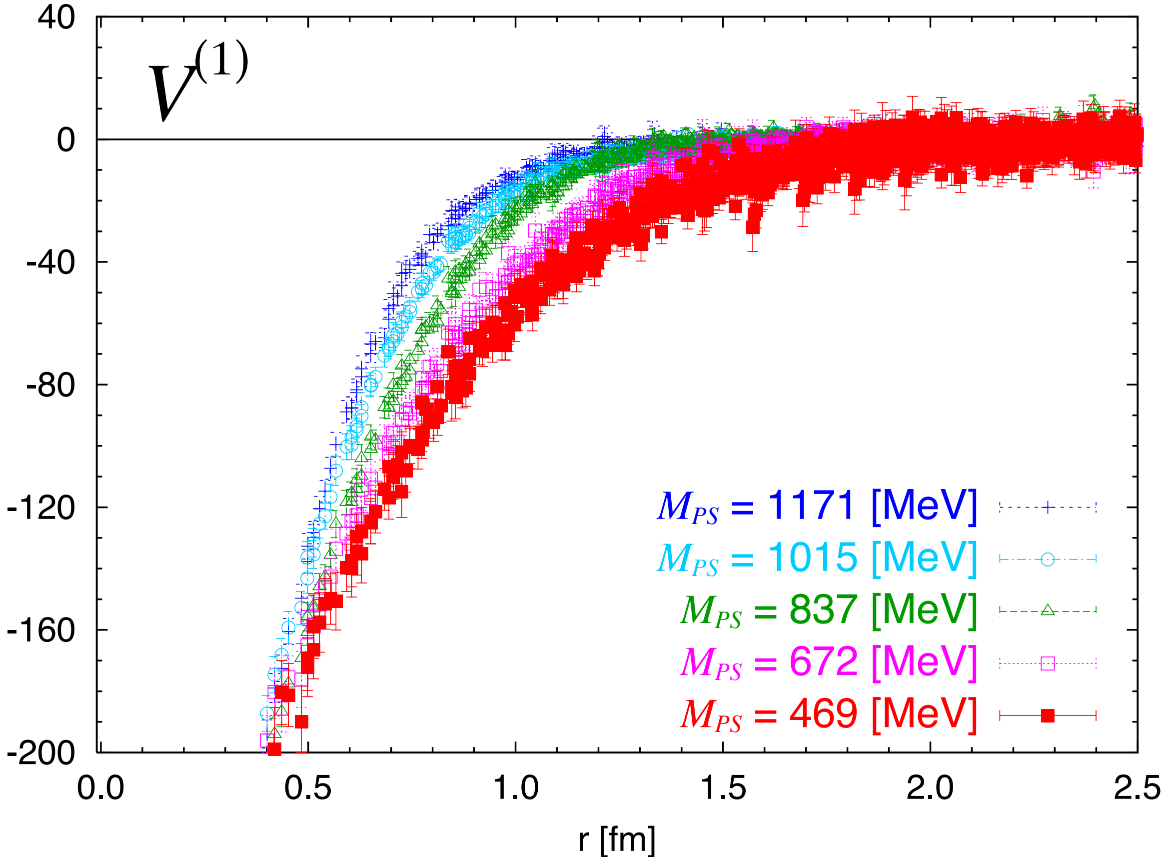
1. $\Lambda\Lambda - N\Xi$ interactions



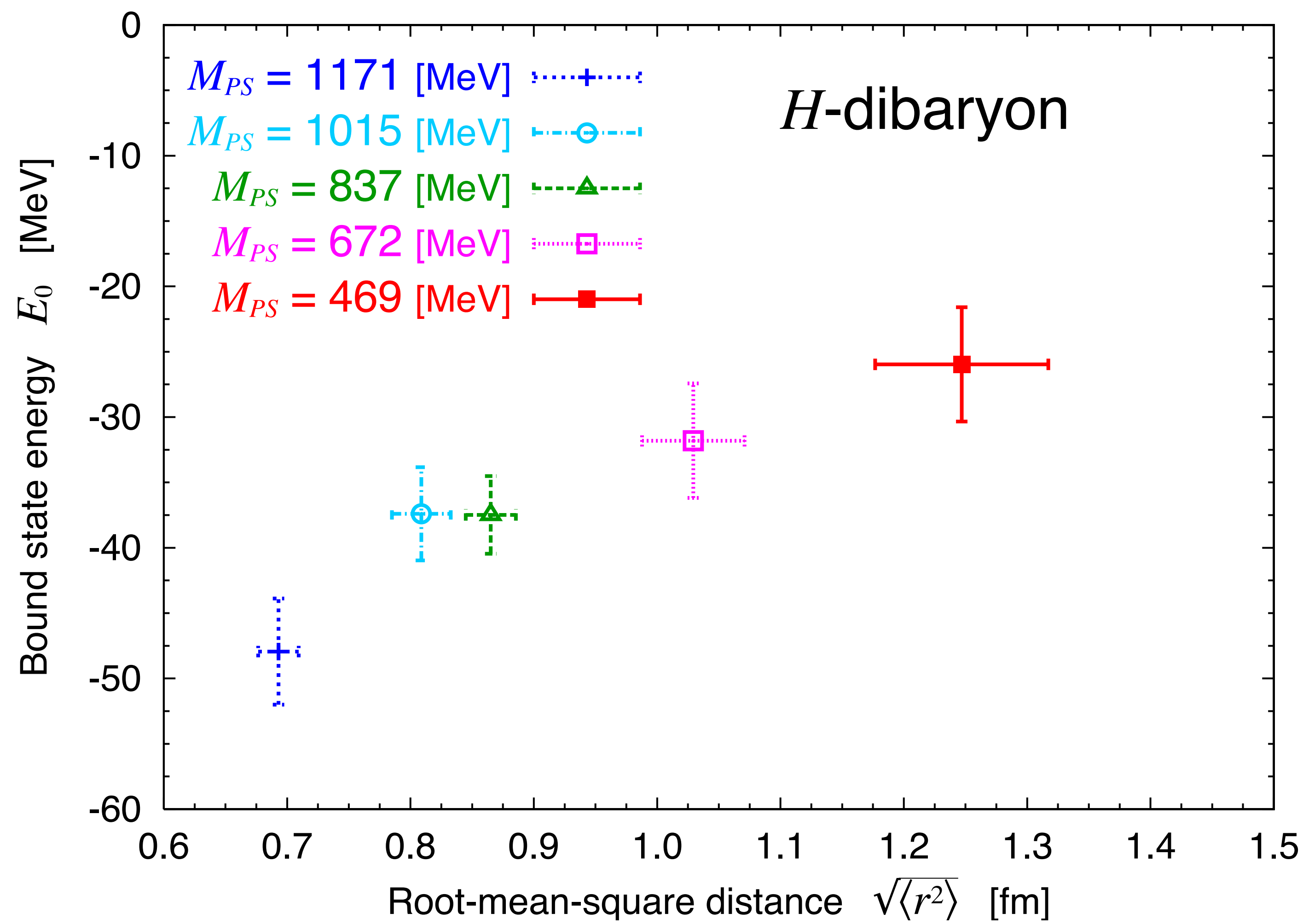
The bound H-dibaryon appears in the flavor SU(3) limit.

$$m_u = m_d = m_s$$

Flavor singlet potential

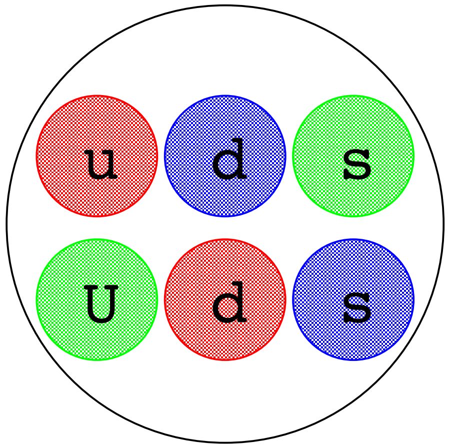


Binding energy vs. size



(a) Binding energy decreases at lighter $M_{\rm ps}$.

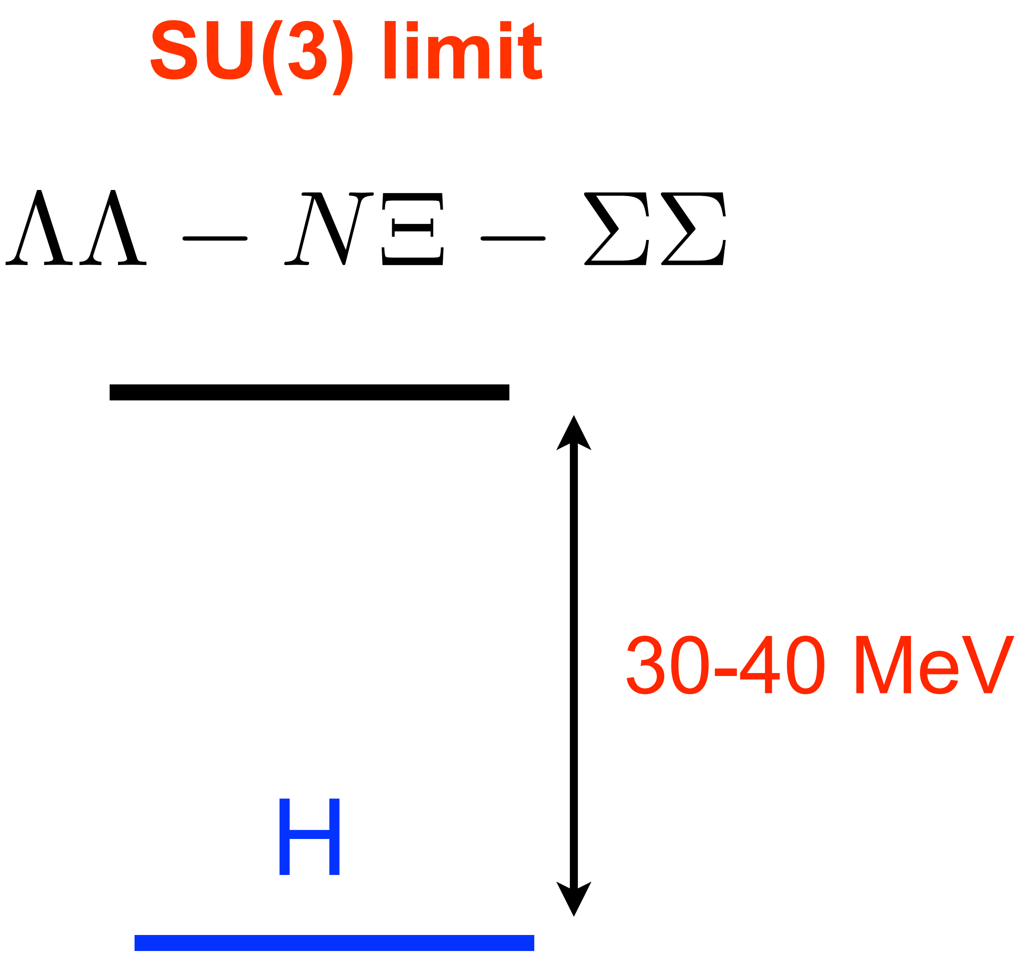
(b) Size of bound state increases at lighter $M_{\rm ps}$.

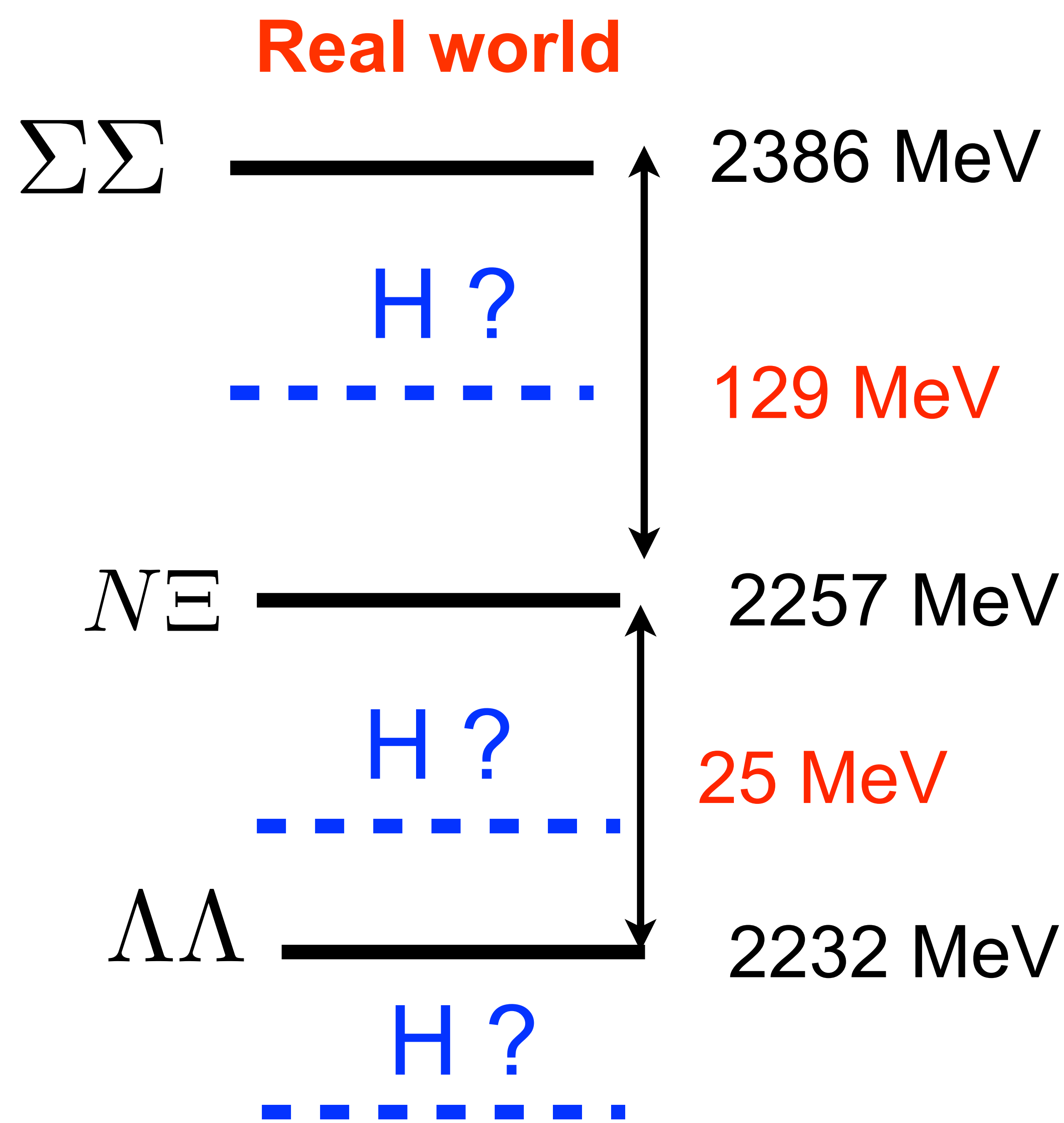


What happen in the real world ?

pion mass becomes lighter

there is flavor SU(3) breaking



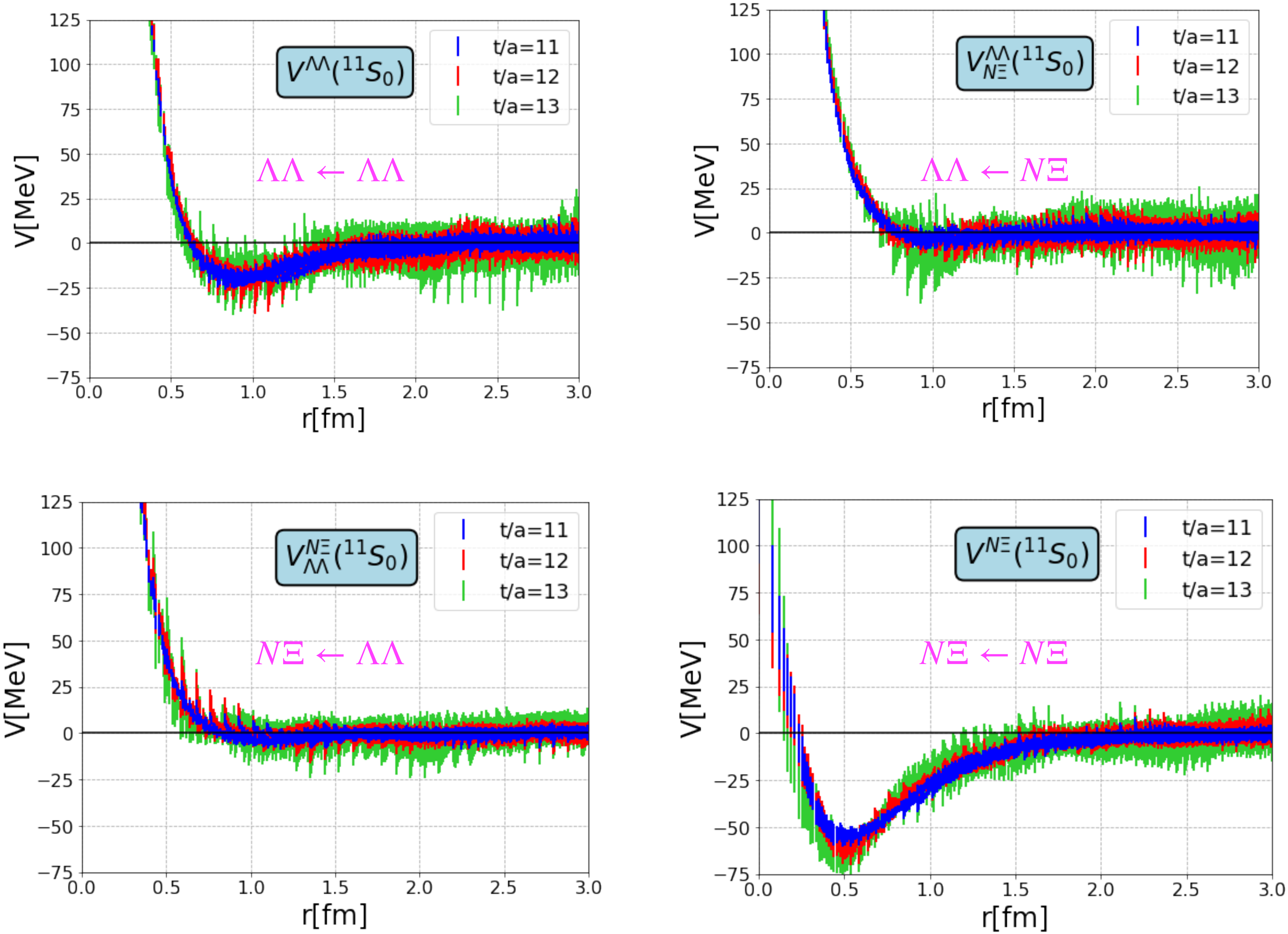


We consider $\Lambda\Lambda - N\Xi$ coupled channel interaction as $\Sigma\Sigma$ threshold is much higher.

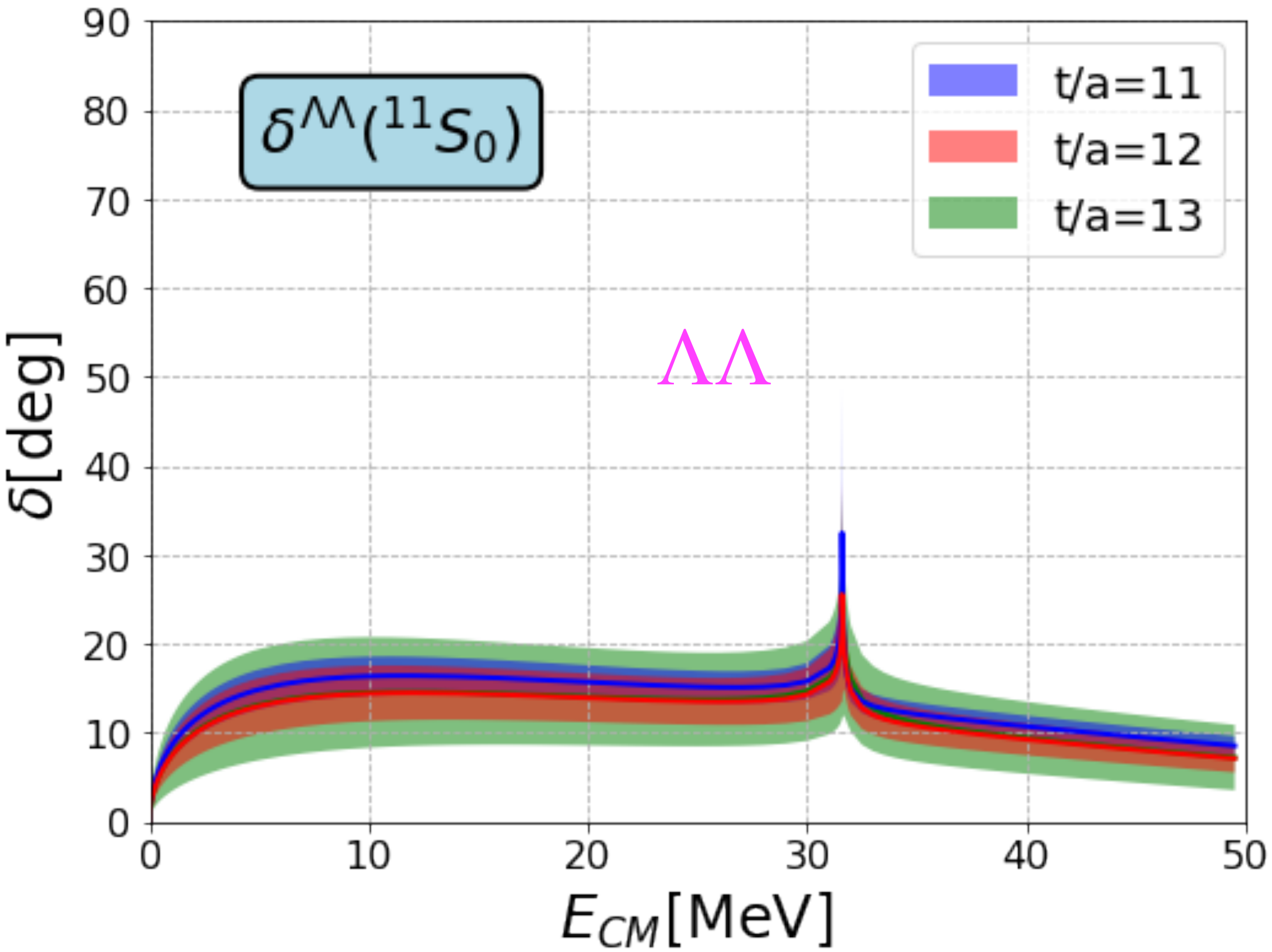
$\Lambda\Lambda - N\Xi$ coupled channel potential

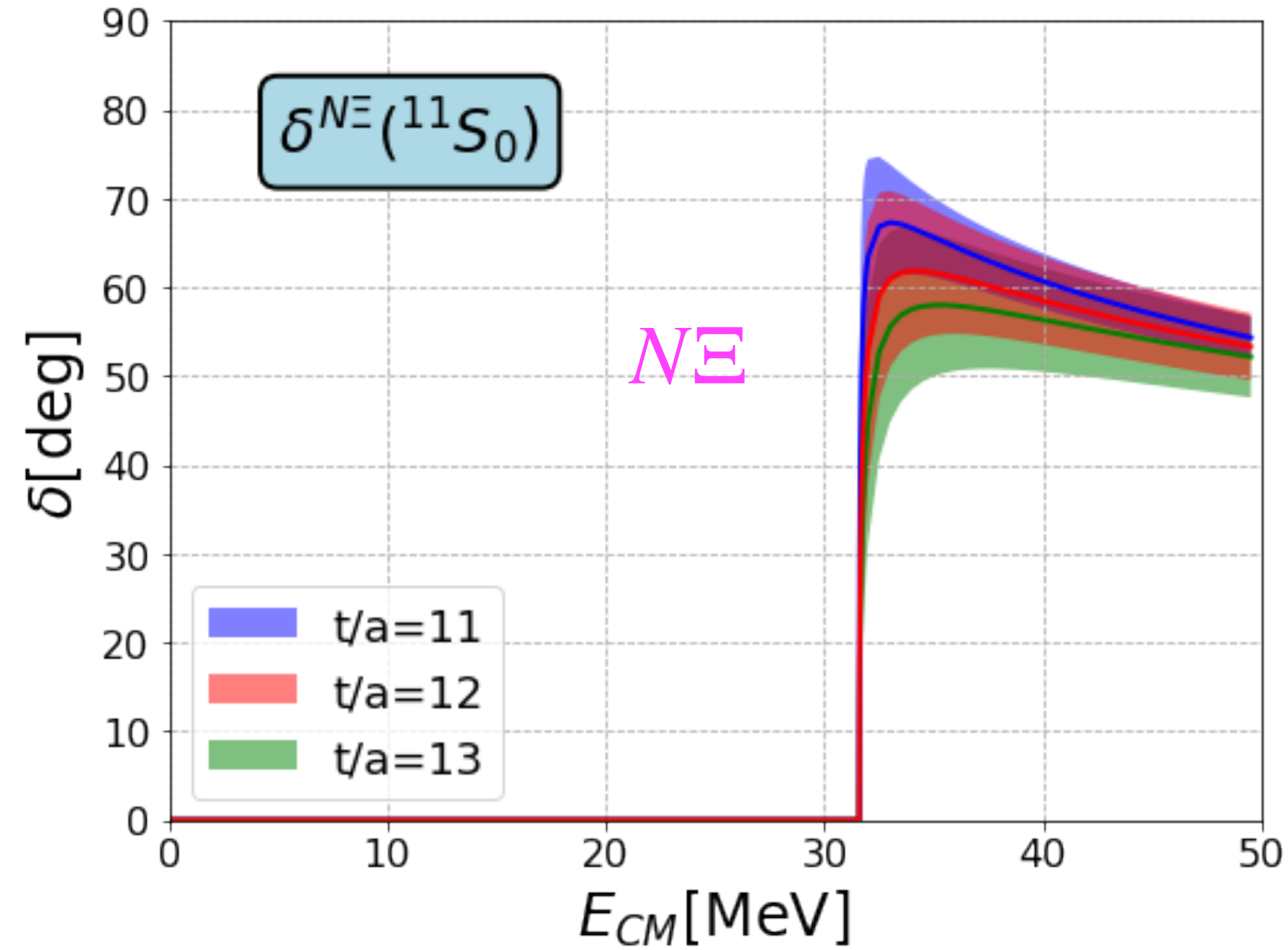
K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737.

$^{2I+1,2s+1}S_J$



scattering phase shifts





(a) No bound state/resonance in $\Lambda\Lambda$.

weak attraction in $\Lambda\Lambda$ potential

(b) No H dibaryon near $\Lambda\Lambda$ threshold.

$$a_0^{\Lambda\Lambda} = 0.8(3)\,\mathrm{fm}$$

(c) Sharpe enhancement and rapid drop near $N\Xi$ threshold.

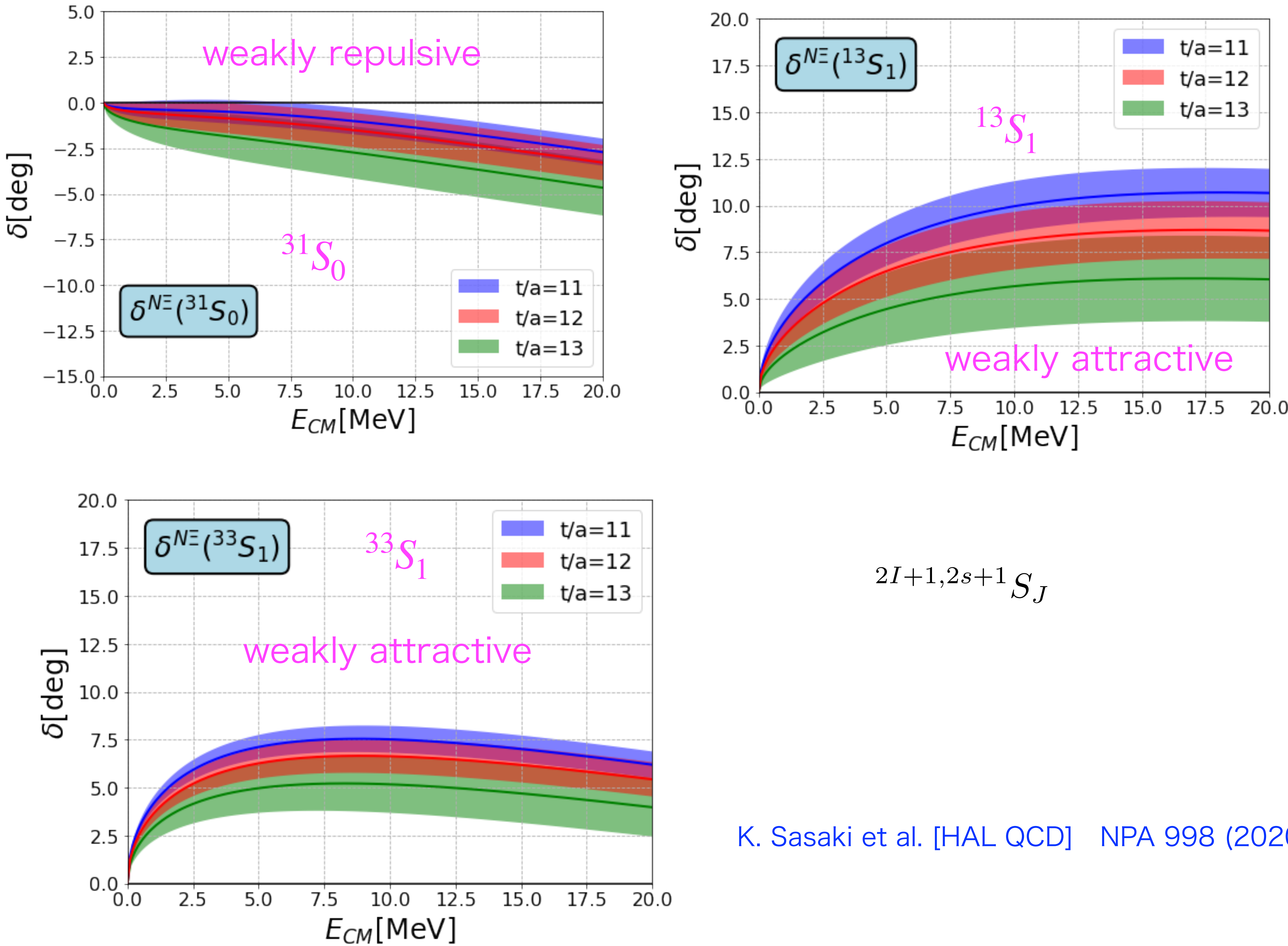
off-diagonal $\Lambda\Lambda \to N\Xi$ potential

(d) Sharpe increase near $N\Xi$ threshold.

significant $N\Xi$ attraction

(e) H appears as a virtual state of $N\Xi$ at almost physical point.

scattering phase shifts for other $\Lambda\Lambda - N\Xi$ channels



$^{2I+1,2s+1}S_J$

K. Sasaki et al. [HAL QCD] NPA 998 (2020) 121737.

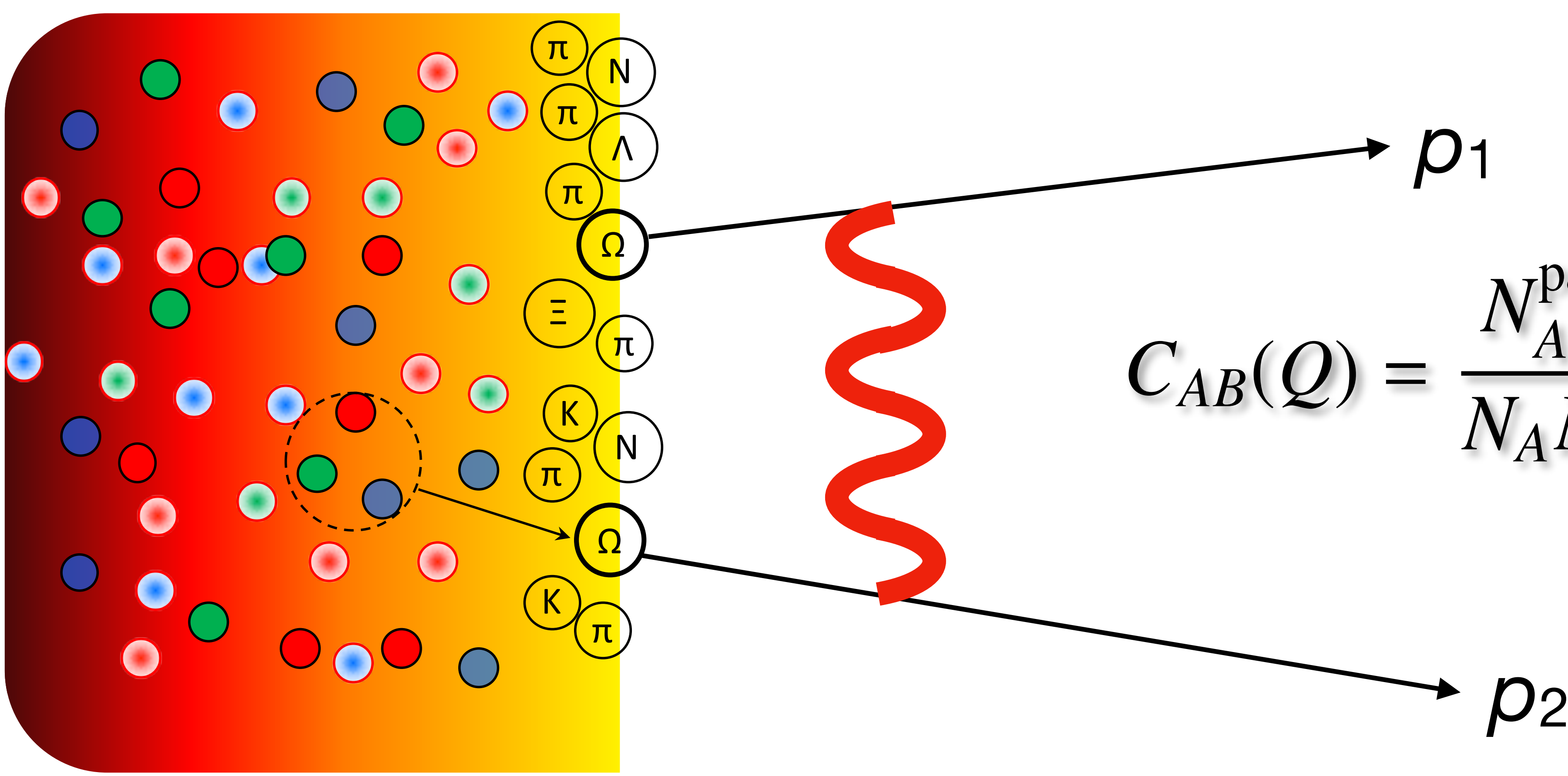
2. Comparison with experiments

Measurements of two-baryon correlation have been made at (RHIC &) LHC

Femtoscopy

ALICE Coll., PRL123(2019)112002; PLB797(2019)134622.

two-baryon interaction ⟺ two-baryon correlation



$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 \\ \text{others} \end{cases}$$

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2}P\right)^2}$$

K. Morita et al., NPA967(2017) 586.

$$C_{AB}(Q) = \frac{N_{AB}(Q)}{N_A(Q)N_B(Q)}$$

$N_A(Q), N_B(Q)$ distribution of each hadron

$N_{AB}(Q)$ distribution of hadron pairs

theory

$$N_{AB}(Q) = \int \frac{d^3p_A}{E_A}\frac{d^3p_B}{E_B} N_{AB}(\mathbf{p}_A, \mathbf{p}_B)\delta(Q - \sqrt{-q^2})$$

$$N_{AB}(\mathbf{p}_A, \mathbf{p}_B) \simeq \int d^4x d^4y S_A(x, \mathbf{p}_A) S_B(y, \mathbf{p}_B) |\Psi(x, y, \mathbf{p}_A, \mathbf{p}_B)|^2$$

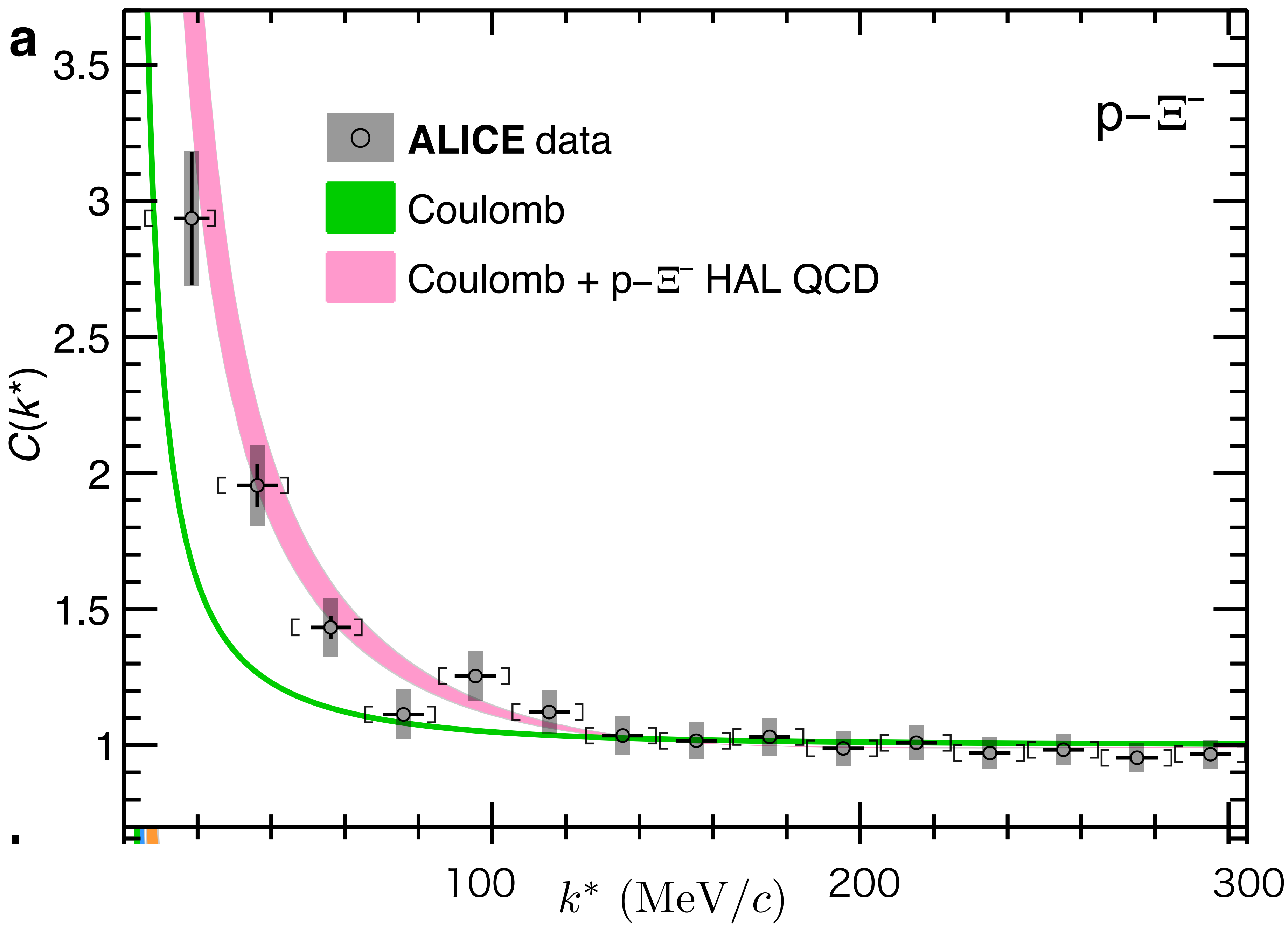
source

2-body wave function from the interaction potential

If the source is approximately known,
one can test hadron interactions using the above formula.

A comparison with data

Allice collaboration, Nature 558 (2020), 232-238.



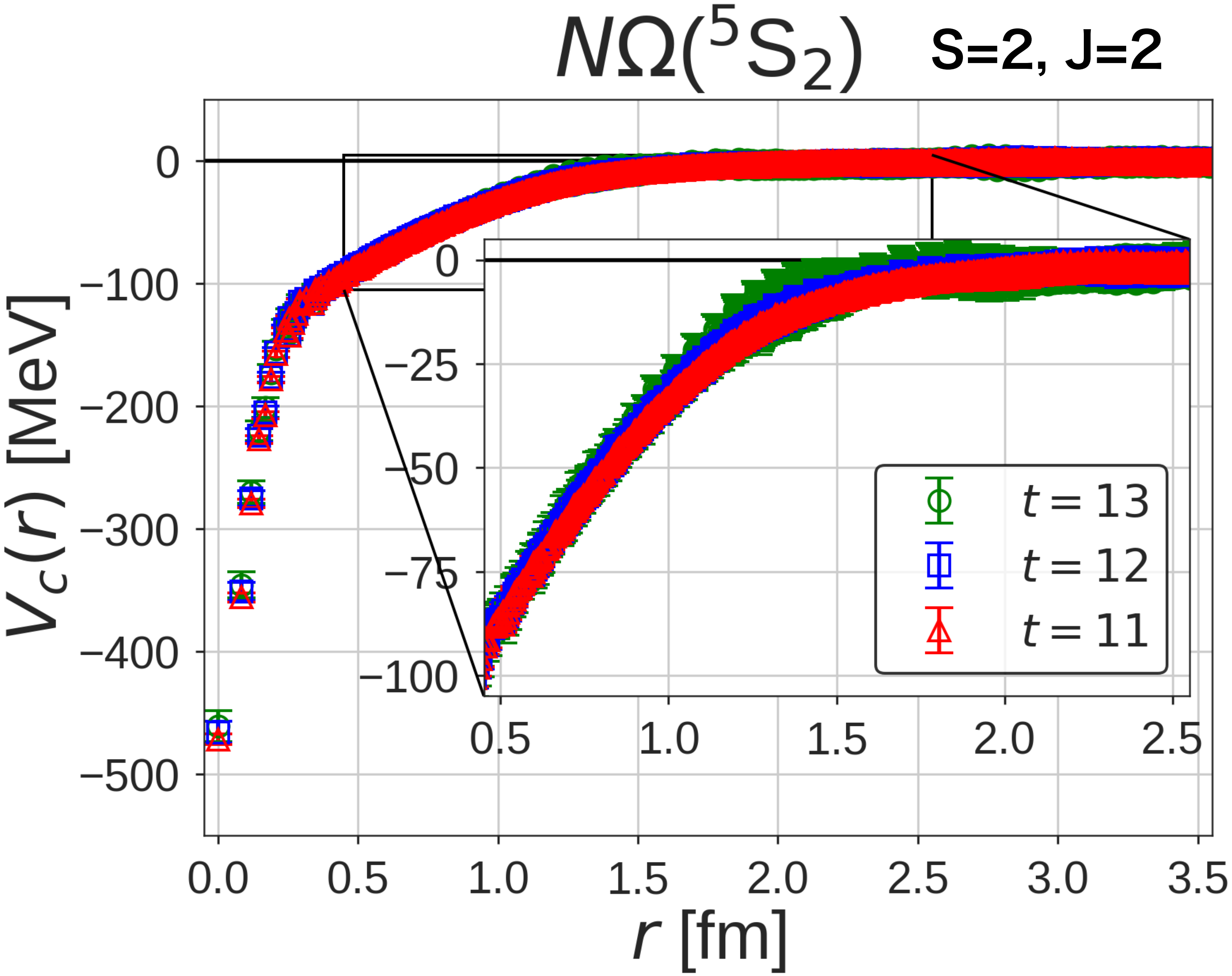
(a) LHC $p - \Xi^-$ data can not be explained by Coulomb interaction alone.

(b) Data are consistent with Coulomb + HAL QCD potential.

III. $N\Omega$ Dibaryon

T. Iritani et al.[HAL QCD], PLB 792 (2019) 284.

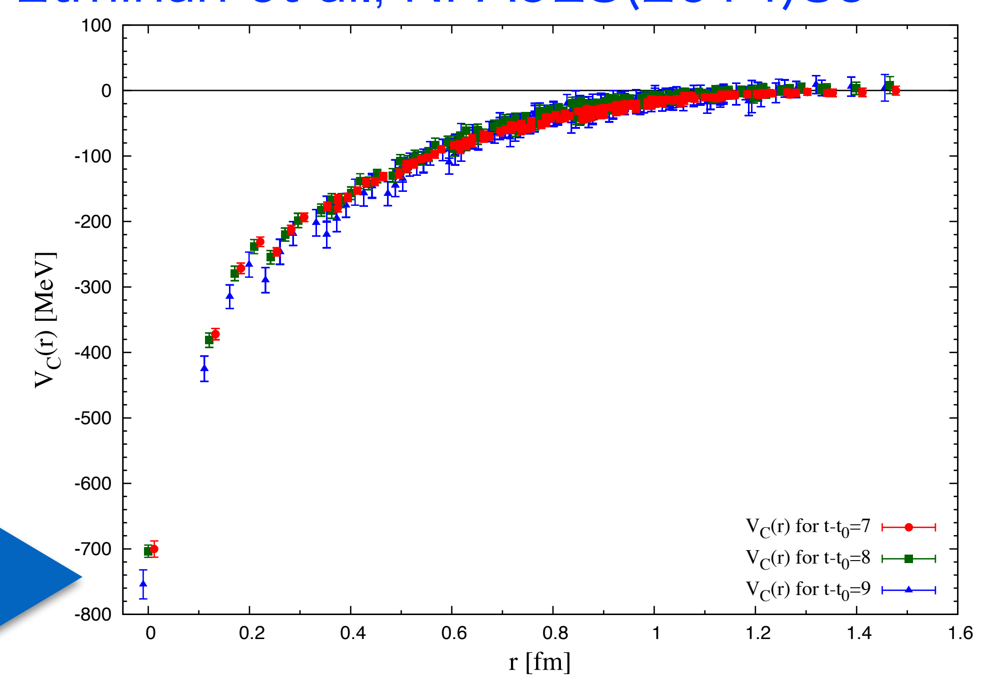
$N\Omega$ potential in $^5\mathrm{S}_2$ channel



* we obtain an attractive potential without repulsive core

* attraction is long range

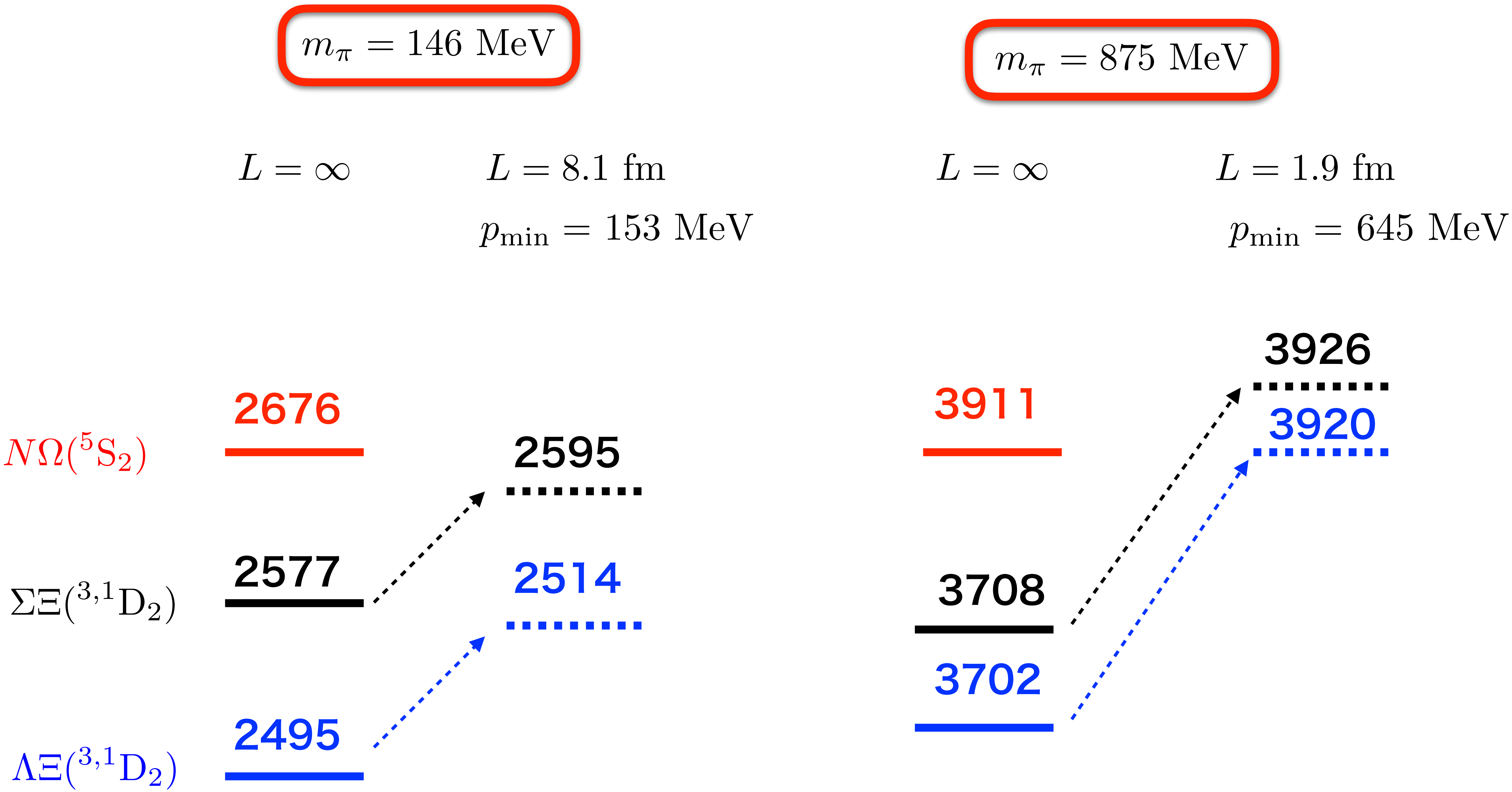
Etminan et al., NPA928(2014)89



qualitatively the same at $m_\pi \simeq 875$ MeV

B.E. = 18.9(5.0)(+12.1)(-1.8) MeV

Remark

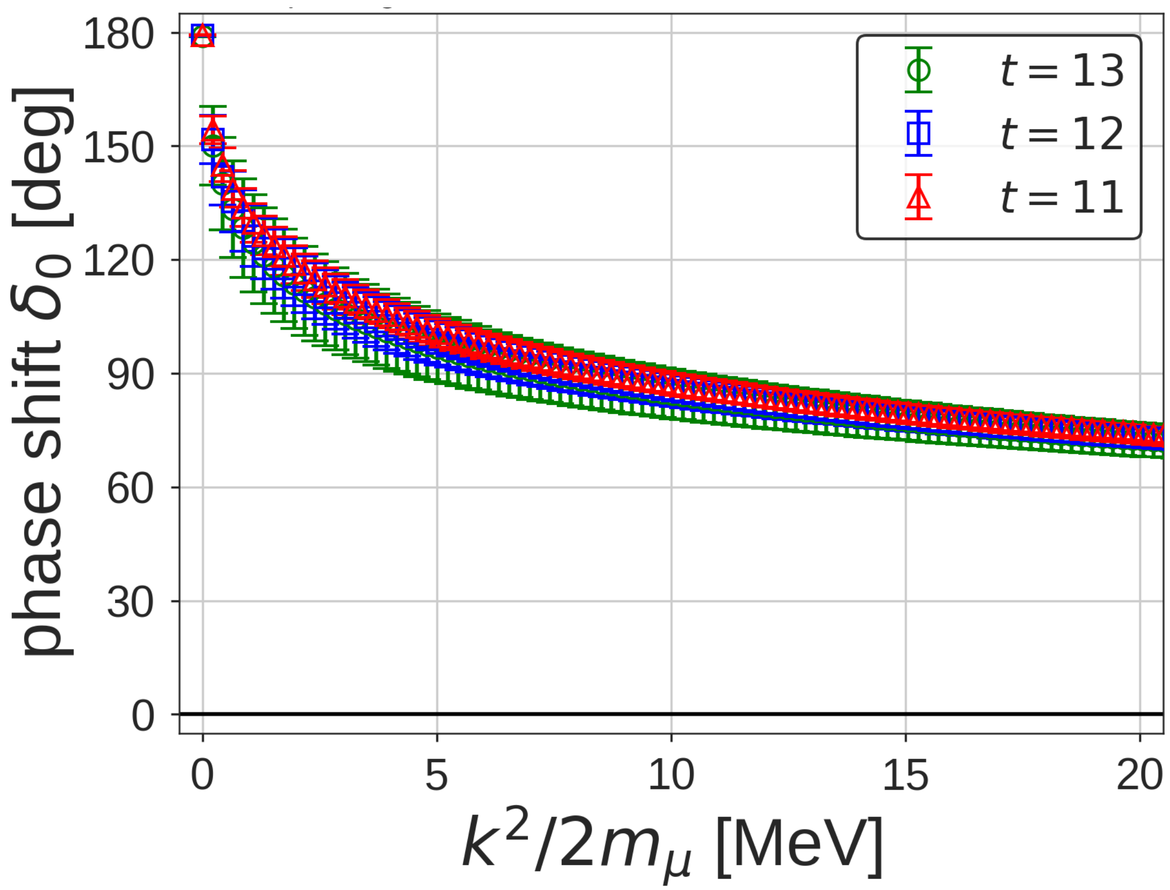


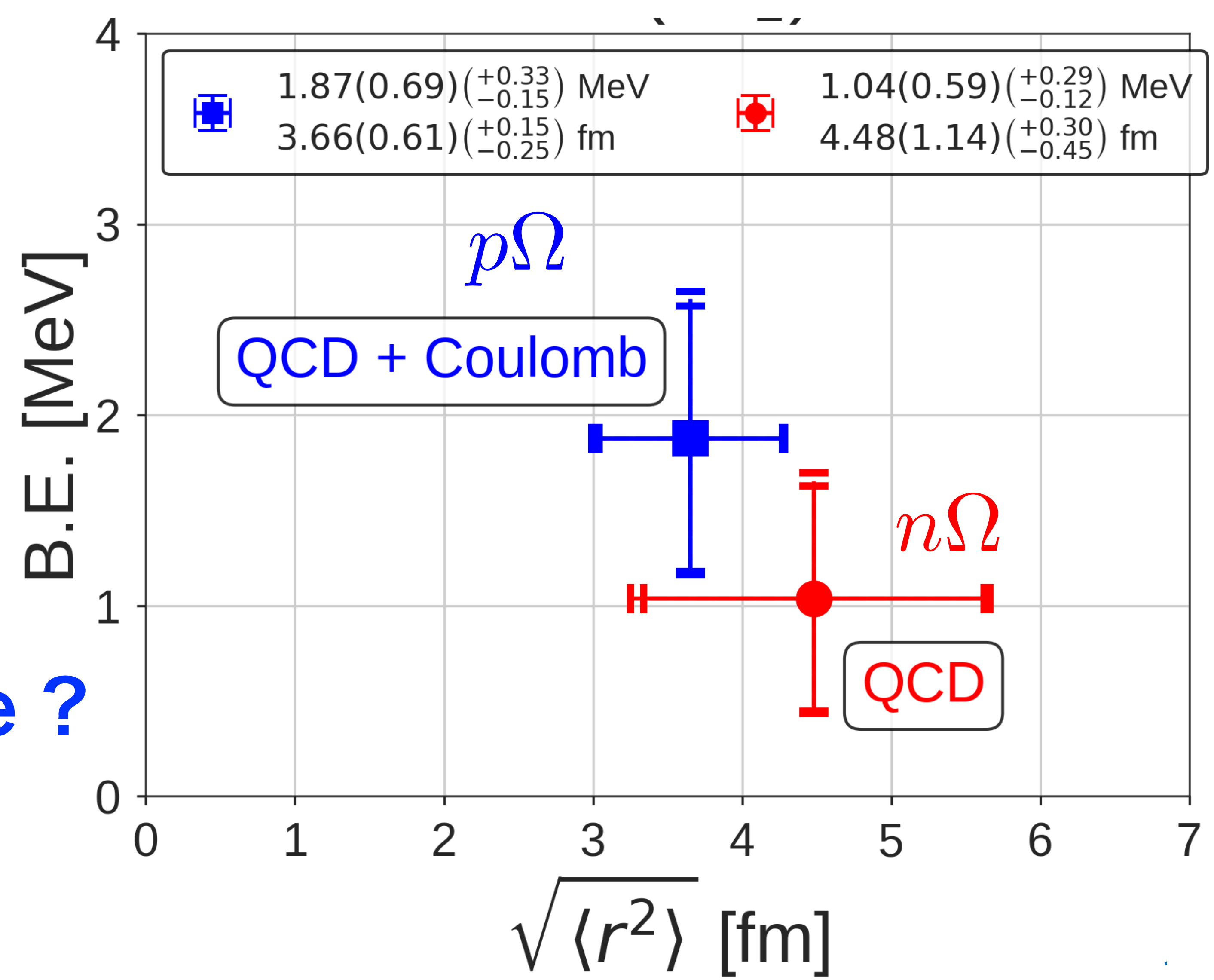
* Only single channel analysis is made.

* We assume small couplings to D-waves, supported by weak t-dep.

* We need coupled channel analysis in the future

Phase shift and binding energy

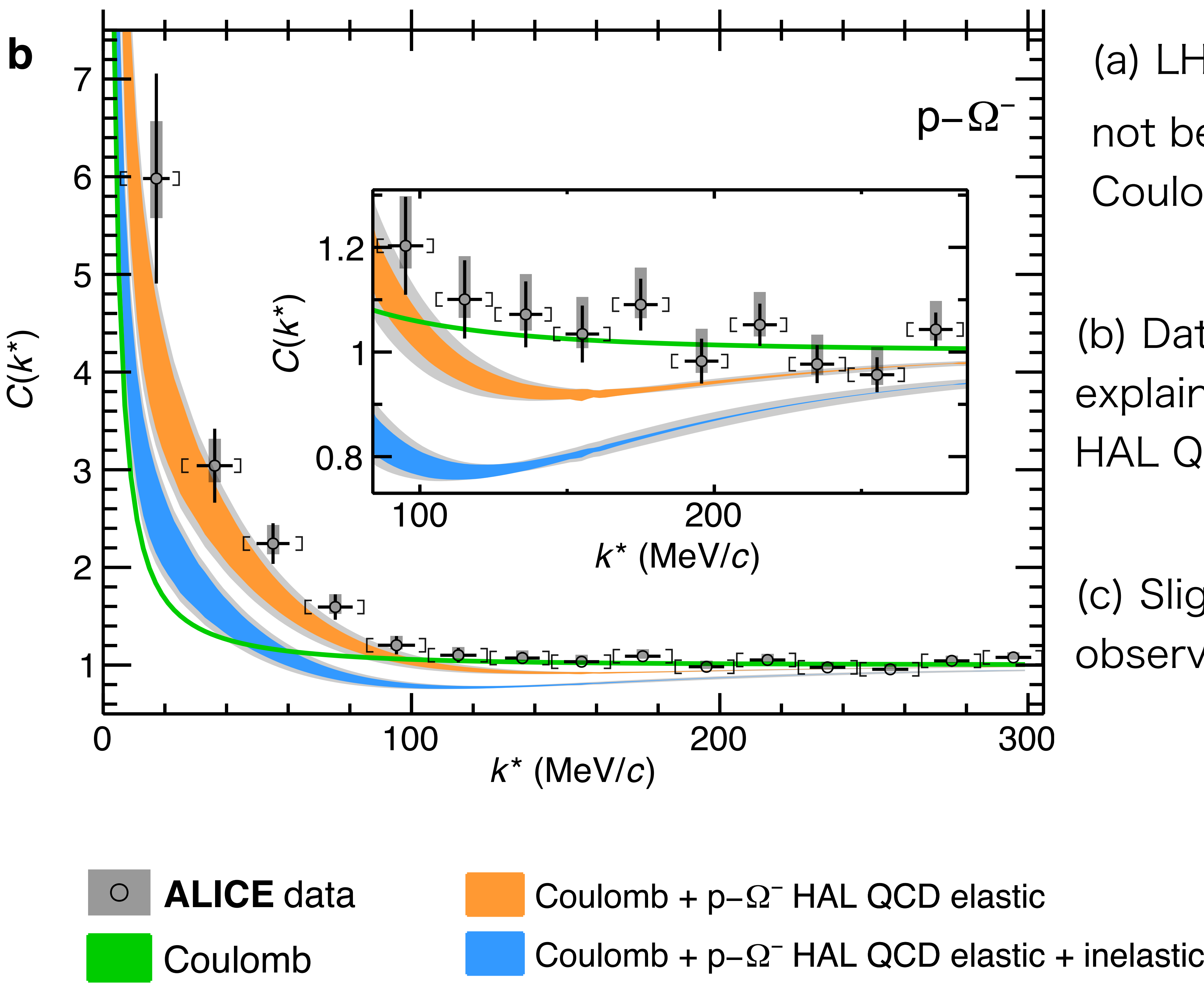




New dibaryon resonance ?

Femtoscopy

ALICE collaboration, Nature 558 (2020), 232-238.



(a) LHC $p-\Omega^-$ data can not be explained by Coulomb interaction alone.

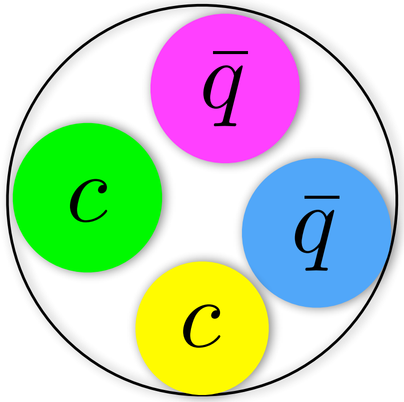
(b) Data are better explained by Coulomb + HAL QCD elastic interaction.

(c) Slight deviations are observed around 100 MeV.

IV. Tetra quark state T_{cc}

Y. Lyu et al.[HAL QCD], PRL 131 (2023) 161901.

Heavy tetra-quark states T_{cc}

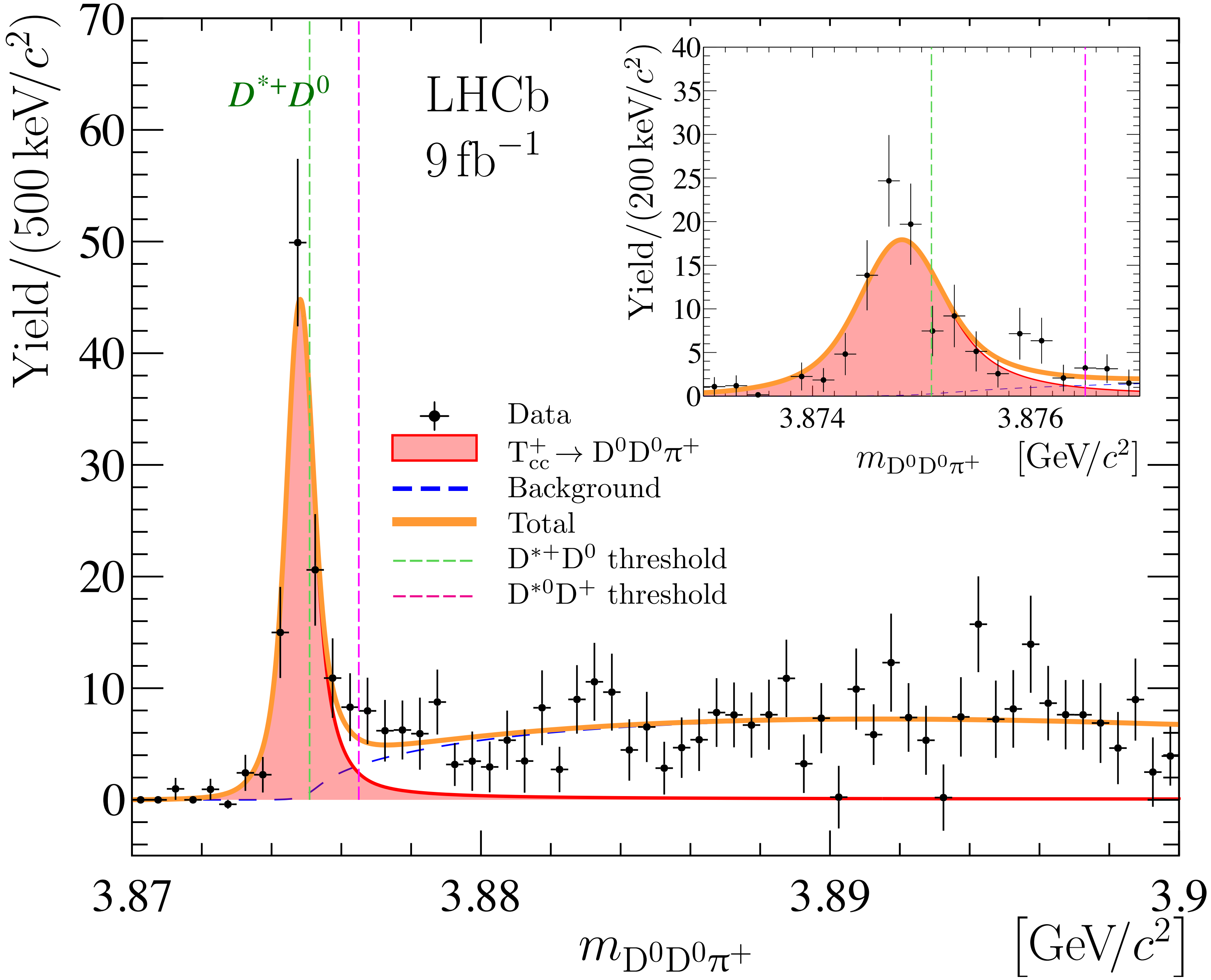


$\bar{q}$: light anti-quark

genuine tetra-quark states

$T_{cc}(cc\bar{u}\bar{d})$ observation by LHCb.

Aaij et al. (LHCb Collaboration), Nature Phys. (2022)

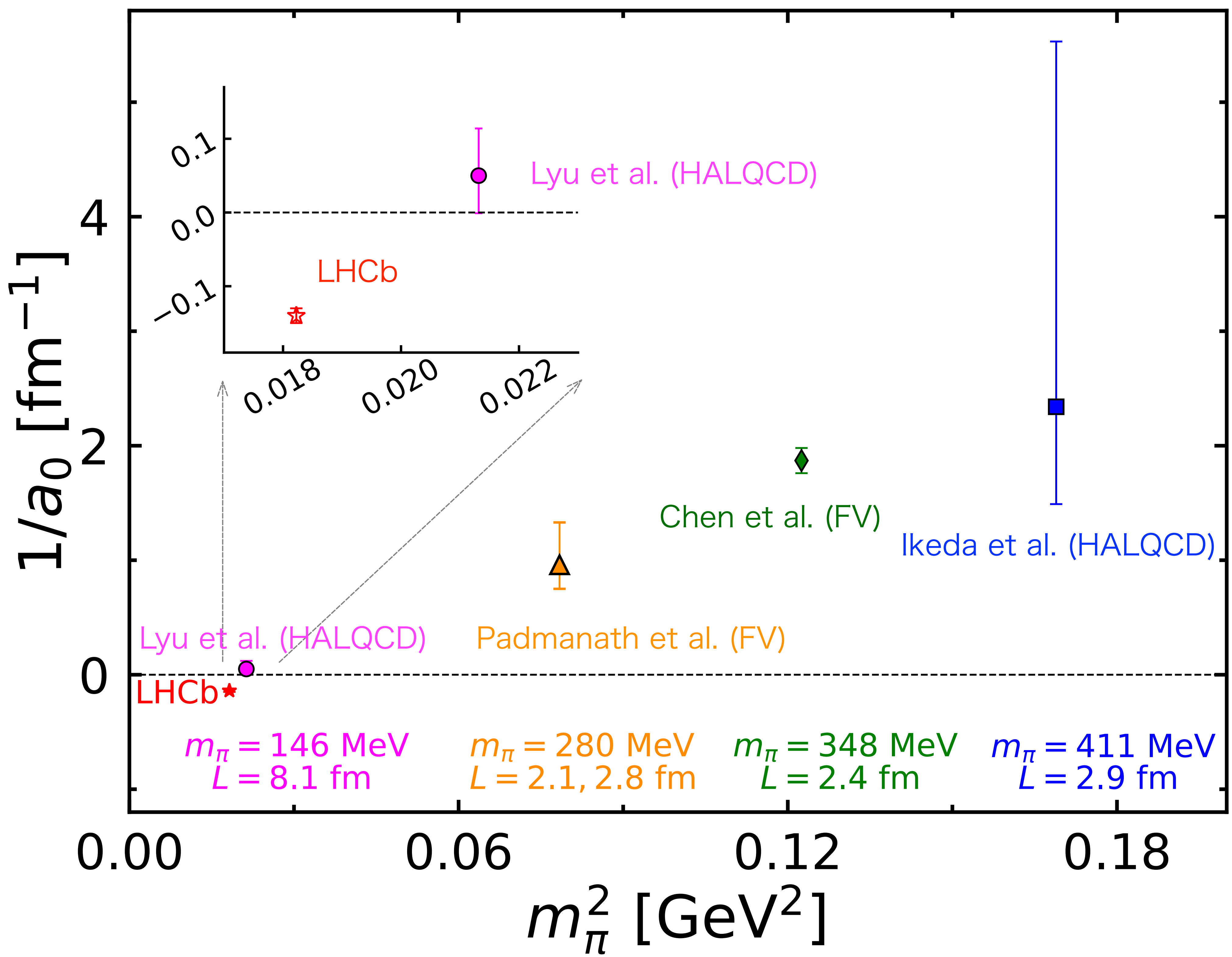


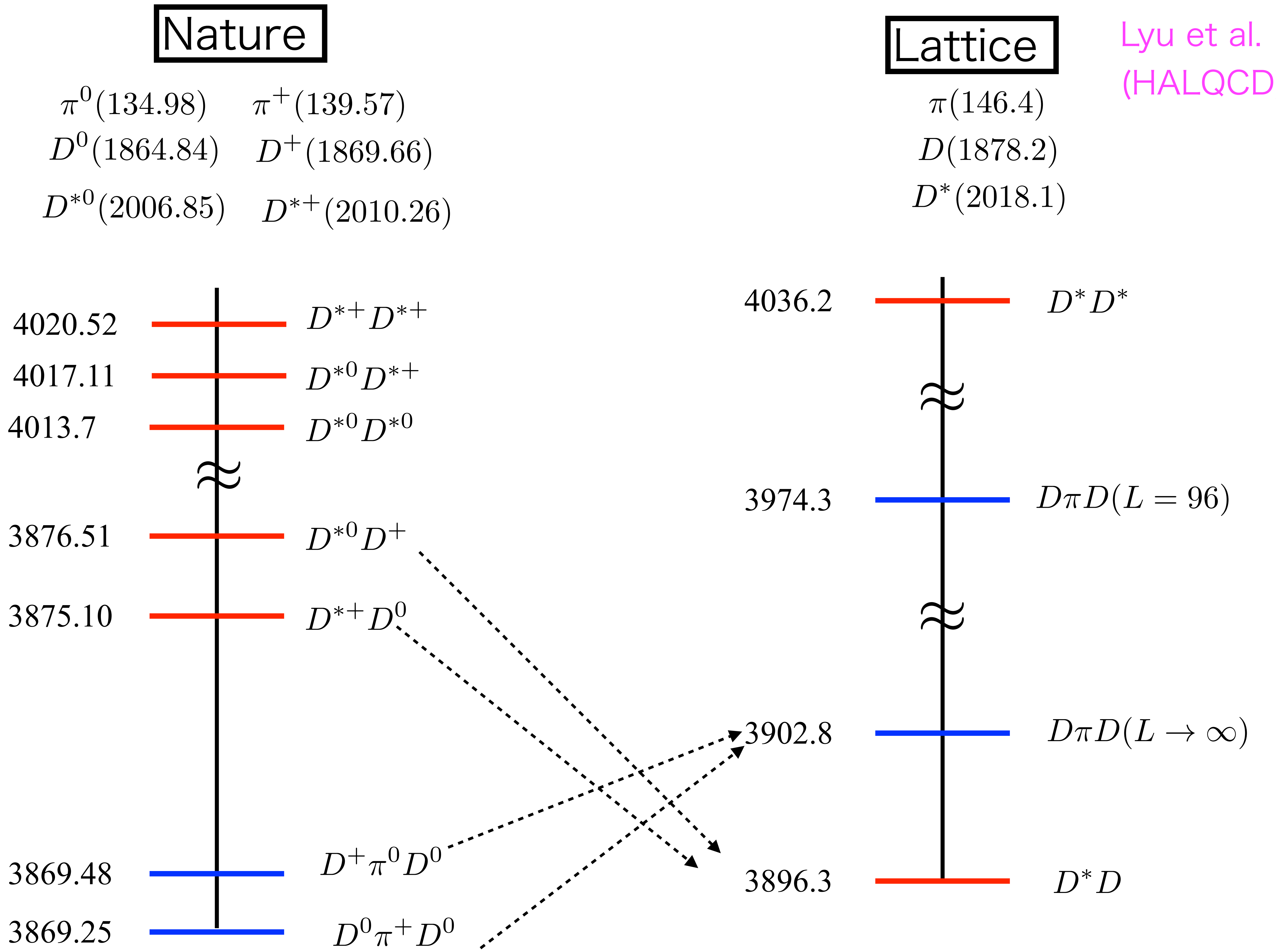
360 keV below $D^{*+}D^0$ threshold

$$(I, J^P) = (0, 1^+)$$

(Latest) Lattice QCD results

inverse scattering length   significant pion mass dependence

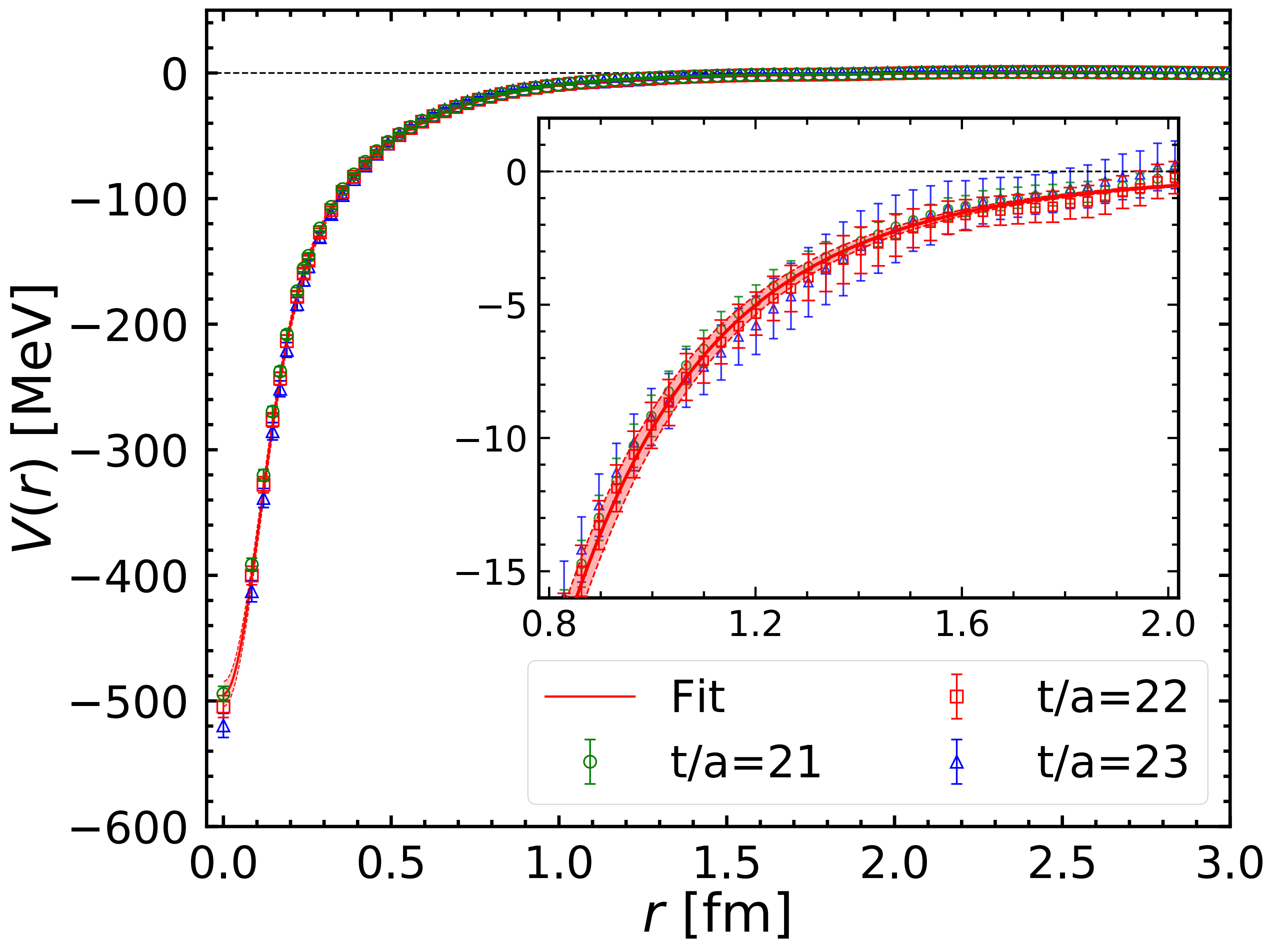




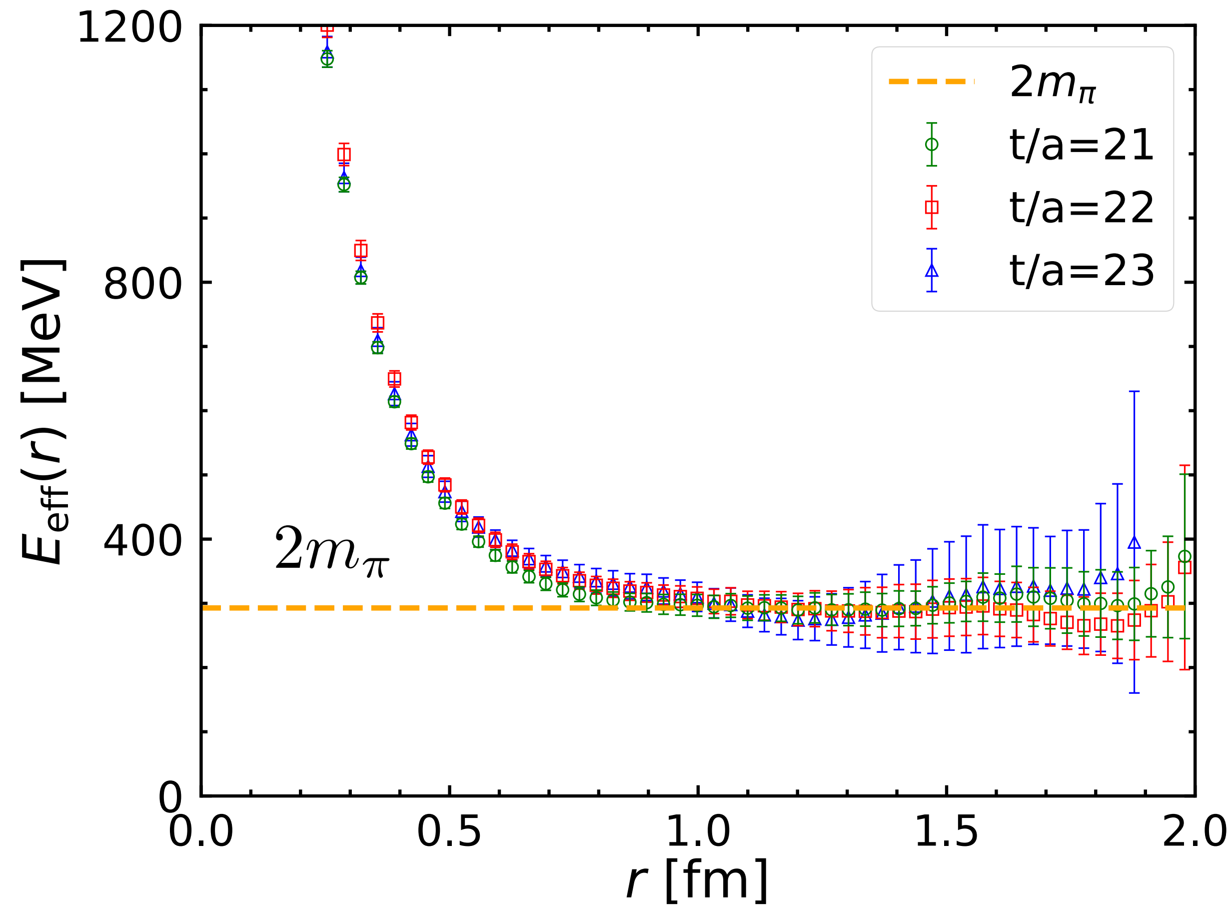
Nature
$\pi^0(134.98)$ $\pi^+(139.57)$
$D^0(1864.84)$ $D^+(1869.66)$
$D^{*0}(2006.85)$ $D^{*+}(2010.26)$
4020.52 $D^{*+}D^{*+}$
4017.11 $D^{*0}D^{*+}$
4013.7 $D^{*0}D^{*0}$
3876.51 $D^{*0}D^+$
3875.10 $D^{*+}D^0$
3869.48 $D^+\pi^0D^0$
3869.25 $D^0\pi^+D^0$
Lattice
Lyu et al. (HALQCD)
$\pi(146.4)$
$D(1878.2)$
$D^*(2018.1)$
4036.2 D^*D^*
3974.3 $D\pi D(L=96)$
3902.8 $D\pi D(L\to\infty)$
3896.3 D^*D

D^*D Potentials $m_\pi \simeq 146$ MeV

D^*D potential



Effective energy in space



attractive at all distances with long range tail

$$E_{\rm eff}(r) = -\frac{\ln[-V(r) r^2/a_3]}{r}$$

consistent with Yukawa^2 at large r

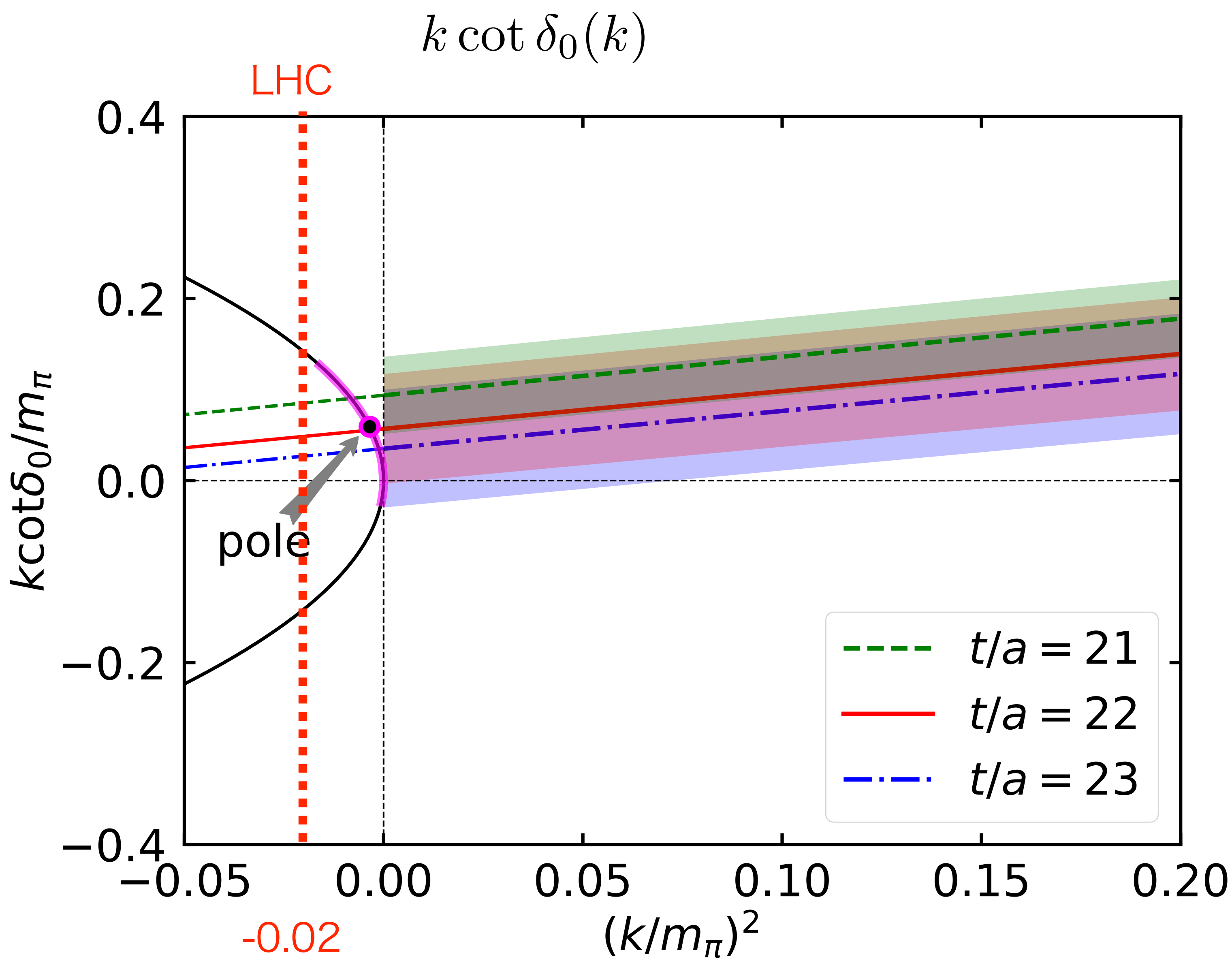
2-pion rather than 1-pion exchange dominates at long distance.

$$V_{\rm fit}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$$

Caution: 1-pion exchange is NOT a dominant contribution for T_{cc} .

Scattering phase shift

$m_\pi \simeq 146$ MeV



$$\frac{1}{a_0}\,[\mathrm{fm}^{-1}] = 0.05(5)(^{+2}_{-2})$$

one shallow "virtual" state

$$\kappa_{\mathrm{pole}} = -8(8)\left(^{+3}_{-5}\right)\,\mathrm{MeV}$$

$$E_{\mathrm{pole}} = -59\left(^{+53}_{-99}\right)\left(^{+2}_{-67}\right)\,\mathrm{keV}$$

The virtual pole appears above the left-hand cut (LHC) in this lattice setup.

Issue on the LHC Aoki @ Lat2024

See also Du et al. PRL 131 (2023) 131903.
Meng et al. PRD 109 (2024) L071506.
Collins et al. PRD 109 (2024) 094509.

“chiral” correction to the potential

2-Gauss + Yukawa^2

$$V_{\rm fit}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r}\right)^2$$

$$m_\pi : 146.4\,{\rm MeV} \to 135\,{\rm MeV}$$

ERE

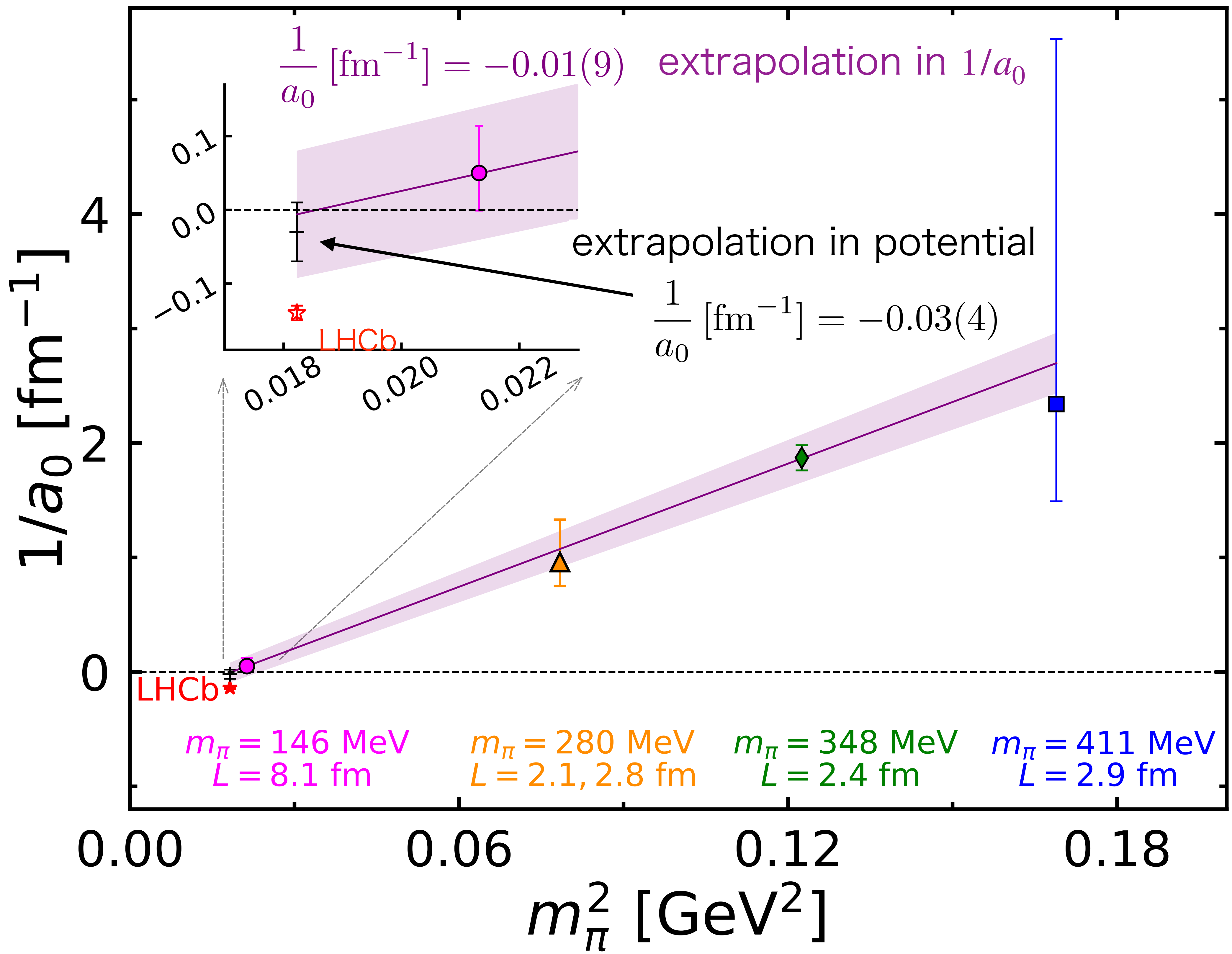
$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + O(k^4)$$

m_π [MeV]	146.4	135.0
$1/a_0$ [fm^{-1}]	$0.05(5)\ \left(^{+2}_{-2}\right)$	$-0.03(4)$
$r_{\rm eff}$ [fm]	$1.12(3)\ \left(^{+3}_{-8}\right)$	$1.12(3)$
$\kappa_{\rm pole}$ [MeV]	$-8(8)\ \left(^{+3}_{-5}\right)$	$+5(8)$
$E_{\rm pole}$ [keV]	$-59\ \left(^{+53}_{-99}\right)\ \left(^{+2}_{-67}\right)$	$-45\ \left(^{+41}_{-78}\right)$

one shallow “bound” state appears at “physical” pion mass

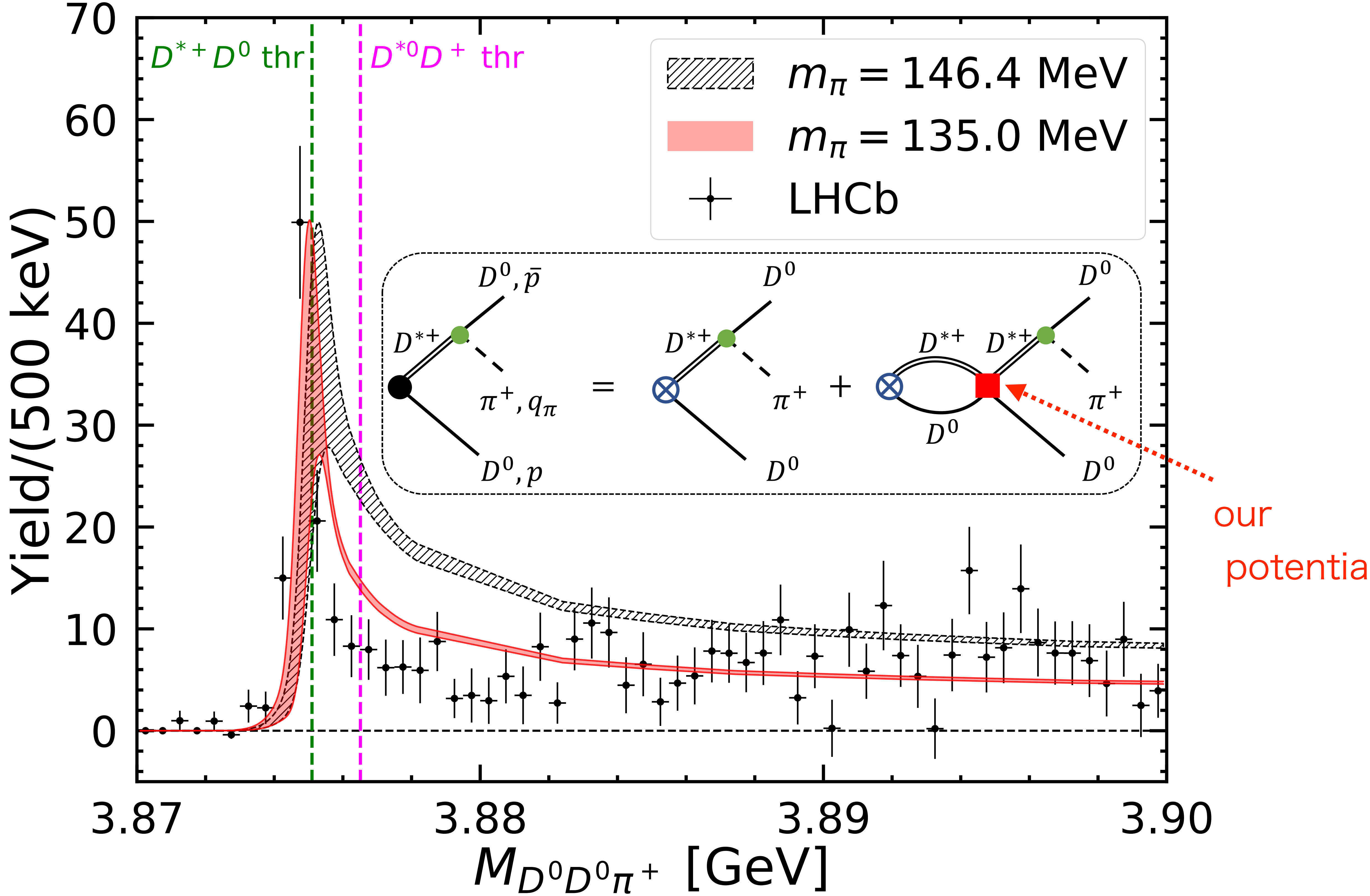
$$E_{\rm pole} = -45\ \left(^{+41}_{-78}\right)\ {\rm keV}$$

chiral extrapolation of $1/a_0$ linear in m_π^2



Two chiral “extrapolations” are consistent.

$D^0D^0\pi^+$ mass spectrum



Our potential at “physical” pion mass explains LHCb data better.

V. $N-\phi$ interaction and 2-pion tail

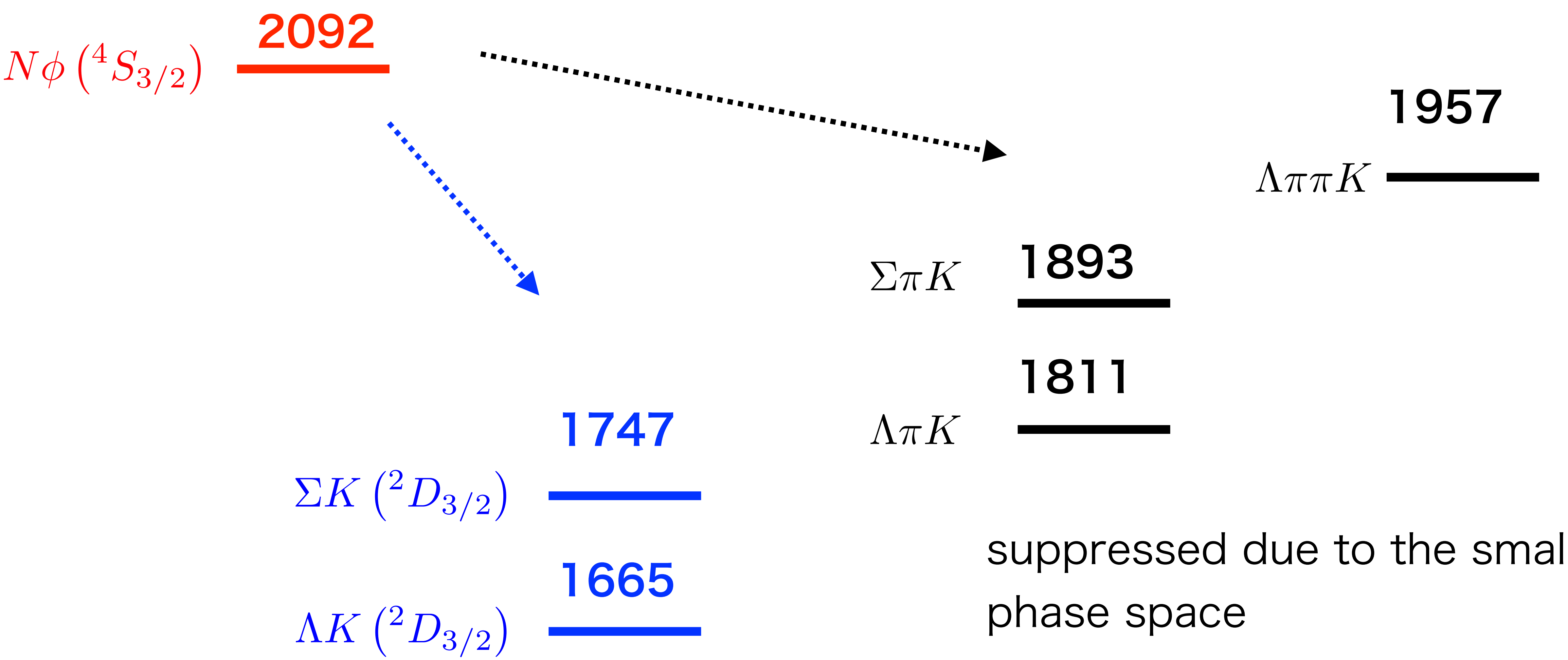
Y. Lyu et al.[HAL QCD], PRD 106 (2022) 074507.

E. Chizzali et al., arXiv:2212.12690.

$N-\phi$ interaction

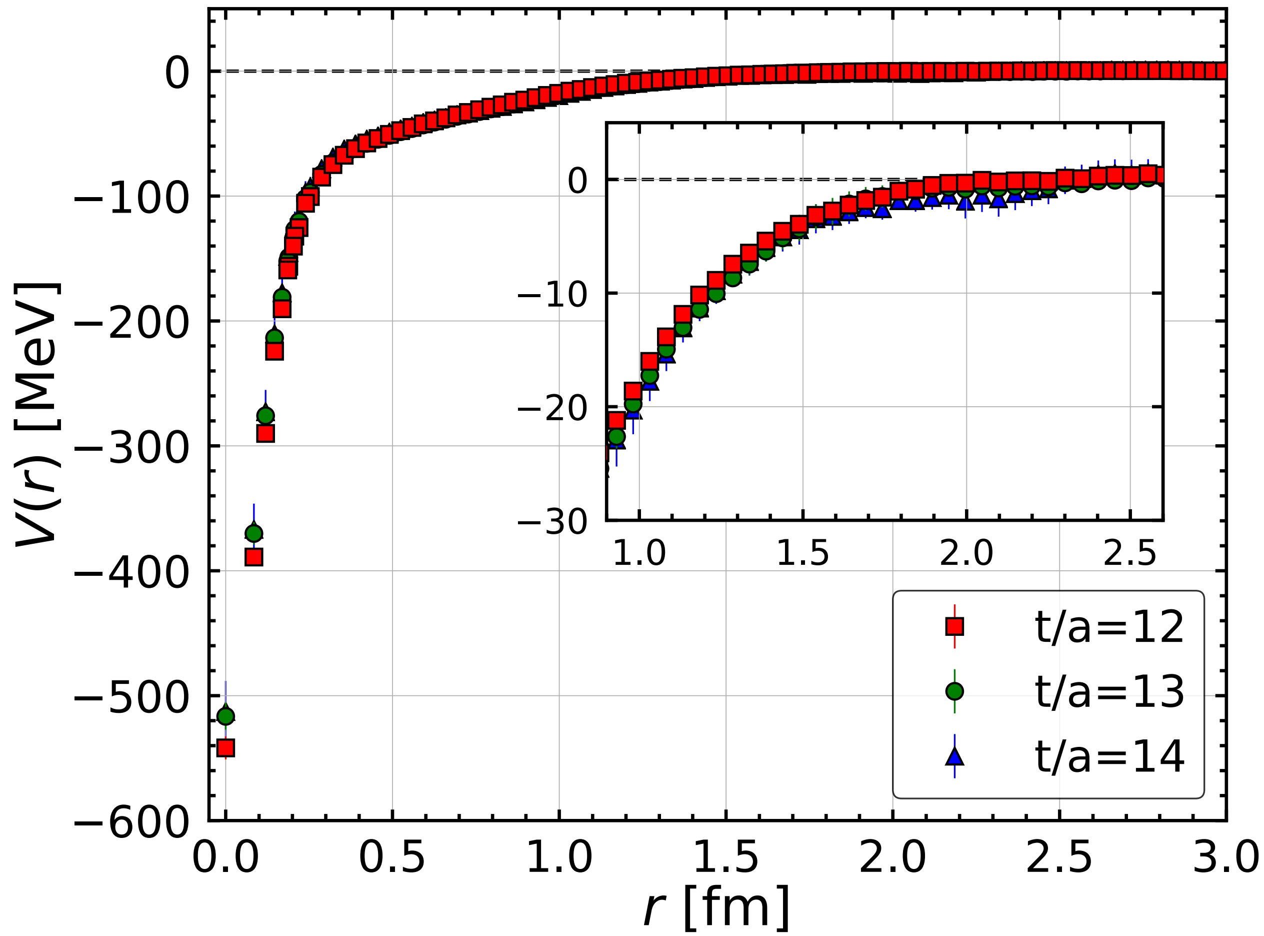
Our lattice setup $m_\pi \simeq 146$ MeV, $m_K \simeq 525$ MeV, $m_\phi \simeq 1048$ MeV

$m_N \simeq 954$ MeV, $m_\Lambda \simeq 1140$ MeV, $m_\Sigma \simeq 1222$ MeV



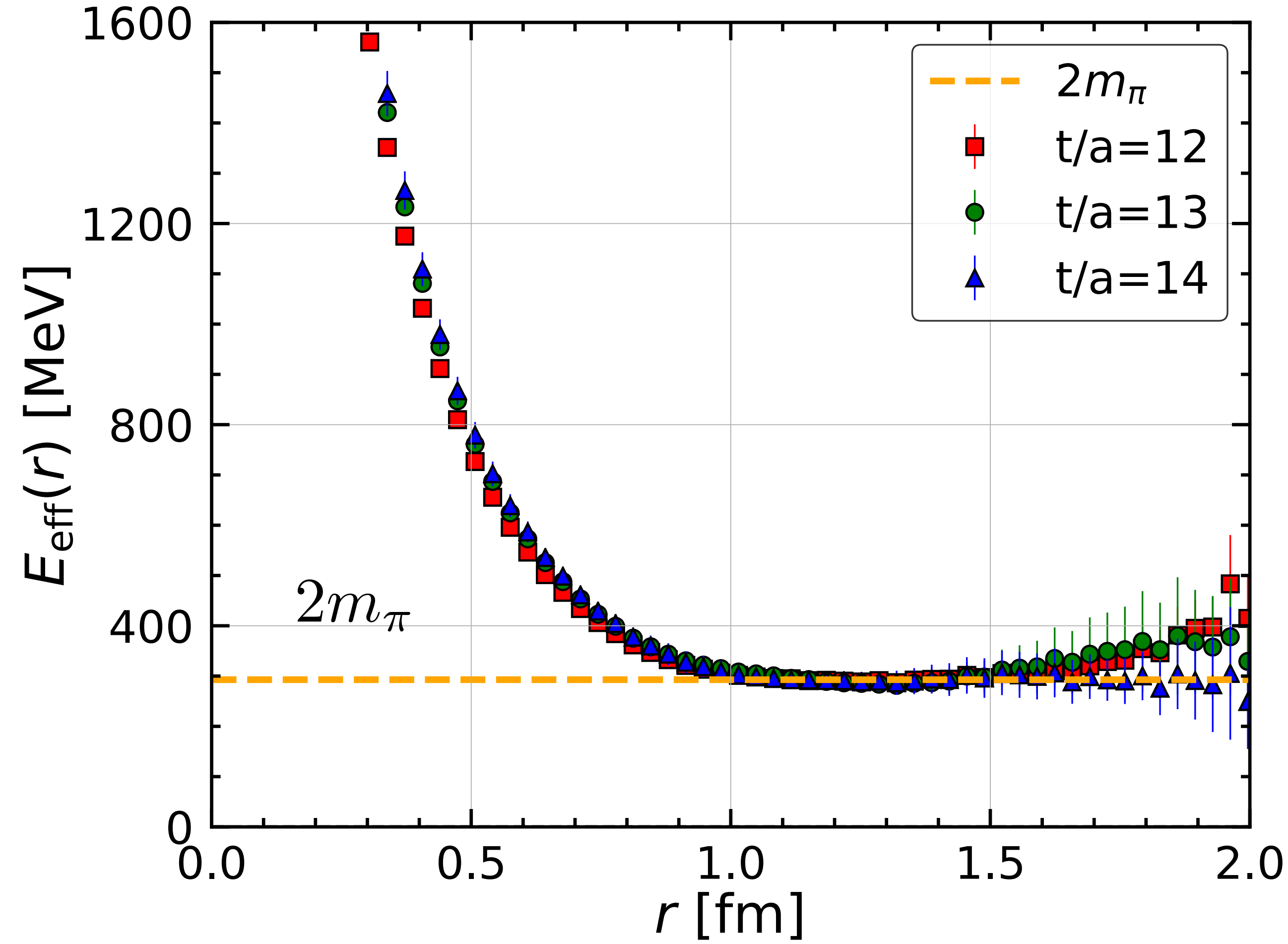
kinematically suppressed at low energy
due to D-wave nature

$N\phi$ Potentials



attractive at all distances

Effective energy in space



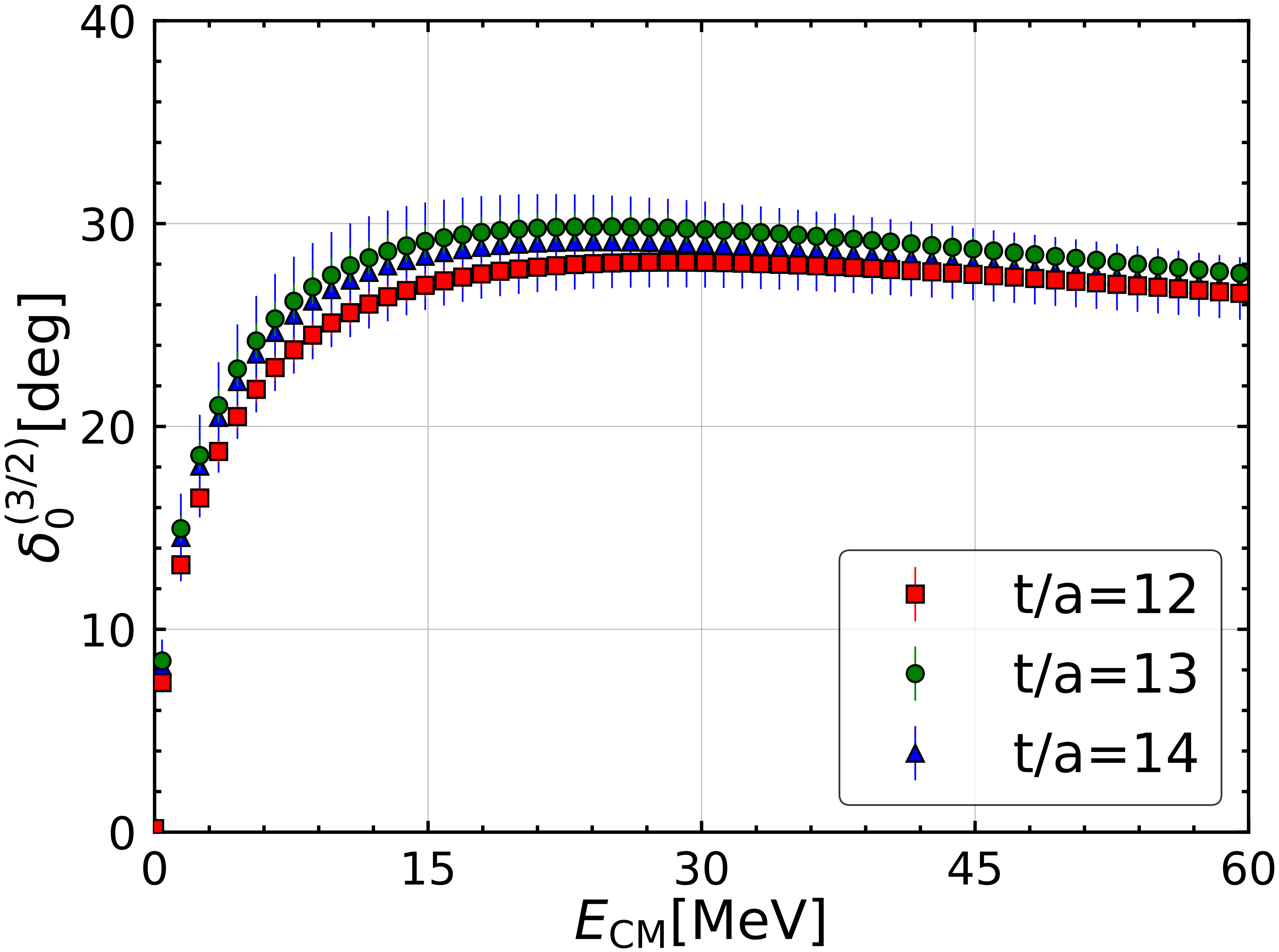
$$E_{\text{eff}}(r) = -\frac{\ln[-V(r)r^2/a_3]}{r}$$

consistent with Yukawa[2] at large r

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r}\right)^2$$

TPE (two pion exchange) seems universal !?

Scattering phase shift



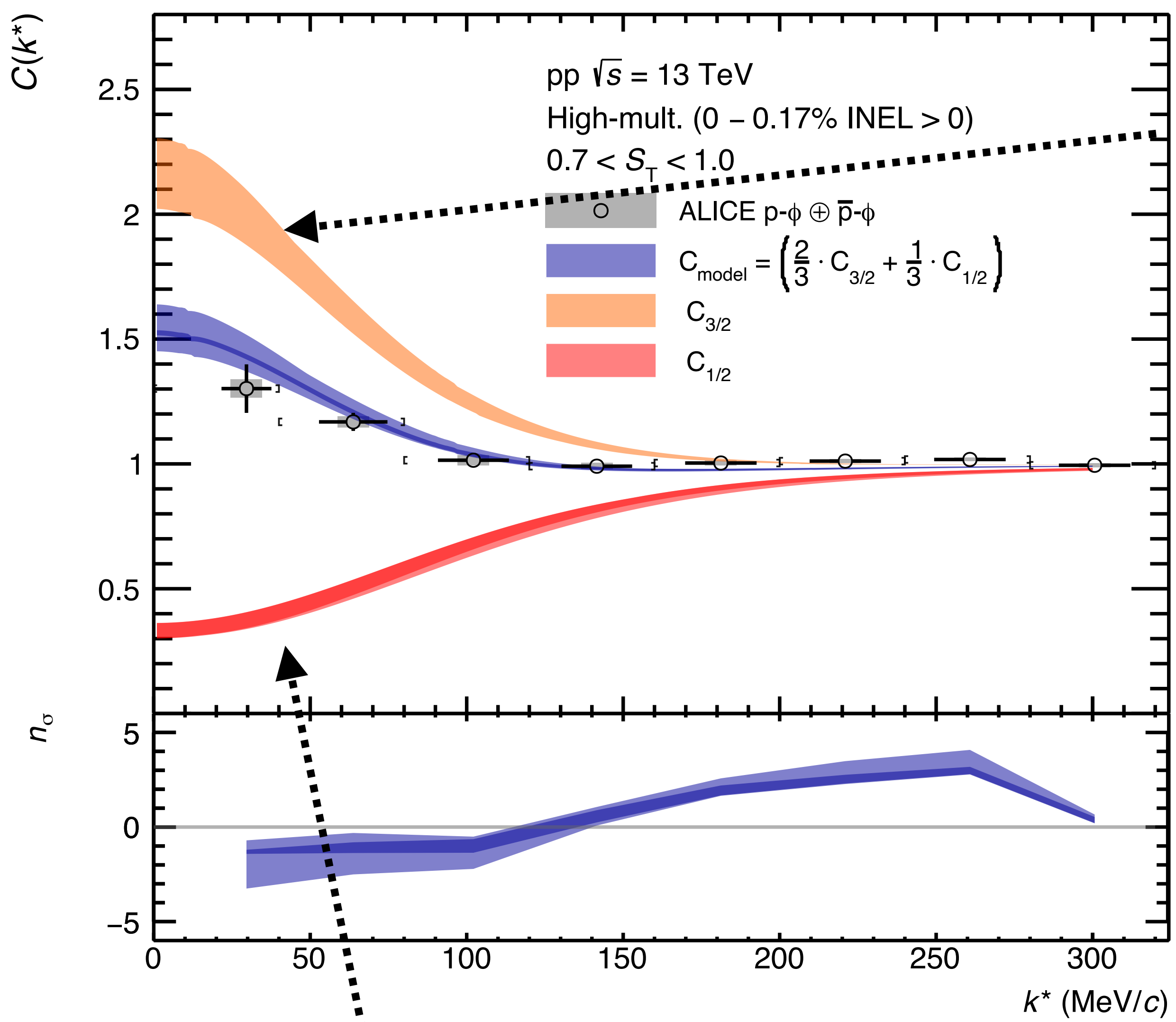
$$k \cot \delta_0^{(3/2)}(k) = \frac{1}{a_0^{(3/2)}} + \frac{1}{2} r_{\rm eff}^{(3/2)} k^2 + O(k^4)$$

$$a_0^{(3/2)} = 1.43(23) \left({}^{+36}_{-06}\right) \text{ fm} \longrightarrow a_0^{(3/2)} \simeq 1.25(23)\,\text{fm}$$

$$m_\pi = 146.4\,\text{MeV} \qquad m_\pi = 138.0\,\text{MeV}$$

Lattice data + correlation@LHC

E. Chizzali et al., arXiv:2212.12690.



Lattice $N\phi(^4S_{3/2})$ potential

$+$

experimental data of $N\phi(^4S_{3/2})$ and $N\phi(^2S_{1/2})$

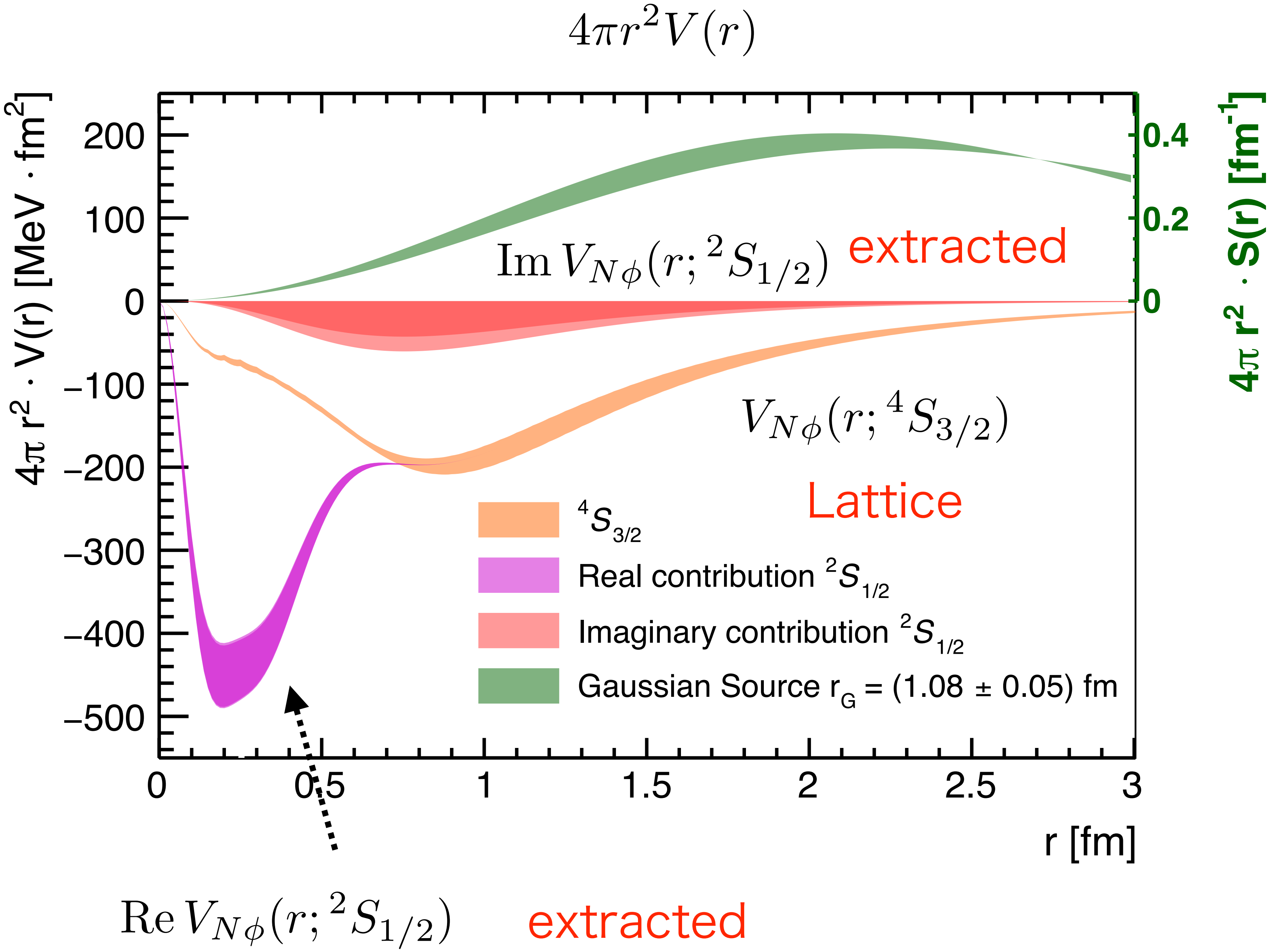
Potential for $N\phi(^2S_{1/2})$

$$V_{N\phi}(r;{}^2S_{1/2}) = \beta \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r}\right)^2$$

$$+ \quad i\gamma(1 - e^{-(r/b_3)^2})^2 \frac{e^{-2m_K r}}{m_K r^2}$$

2 parameters β, γ

2nd order K exchange



The attraction at short distances in $^2S_{1/2}$ (from correlation) is stronger than in $^4S_{3/2}$ (lattice).

From the potential in ${}^2S_{1/2}$, one obtain

$$\mathrm{Re}\, a_0^{(1/2)} = -1.47 \left(^{+0.44}_{-0.37}\right) \left(^{+0.14}_{-0.17}\right) \ \mathrm{fm}$$

$$\mathrm{Im}\, a_0^{(1/2)} = 0.00 \left(^{+0.26}_{-0.00}\right) \left(^{+0.15}_{-0.00}\right) \ \mathrm{fm}$$

→ One $N\phi$ bound state

Estimated binding energy

$$E_B = 14.7 \sim 56.6\,\mathrm{MeV}$$

A direct lattice calculation of the potential in ${}^2S_{1/2}$ is called for.

A coupled channel analysis among $N\phi$, ΣK and ΛK in ${}^2S_{1/2}$ is necessary.

VI. Summary

- No deeply bound deuteron/dineutron at heavy pions. A cross-check by the FV method and the HAL QCD method is crucial. The community could be mislead if the HAL QCD method were absent. ("Diversity" is important.)
- H-dibaryon seems a virtual state near the $N\Xi$ threshold in real-world.
- Lattice $N\Xi$ interactions are compared with LHC data.
- $N\Omega$ dibaryon is predicted at physical point and $p^+\Omega^-_{sss}$ interactions is compared with LHC data.
- Tetra quark T_{cc} and $N-\phi$ interaction are investigated. Both show a 2-pion exchange tail at long distance. Results are compared with experiments.
- (challenge) Local potential have problem for channels including quark annihilations. A use of separable potentials ?

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More results at physical point will be expected. Stay tuned.

Thank you for your attention !