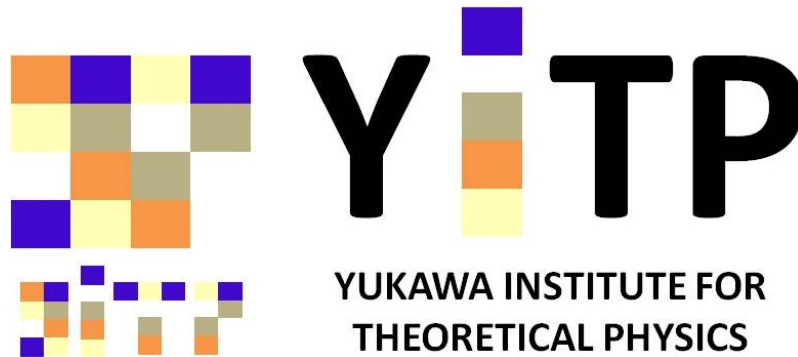


Interactions between two hadrons in lattice QCD

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The 11th International Workshop on Chiral Dynamics (CD2024)
August 26-30, 2024, Ruhr University Bochum, Germany



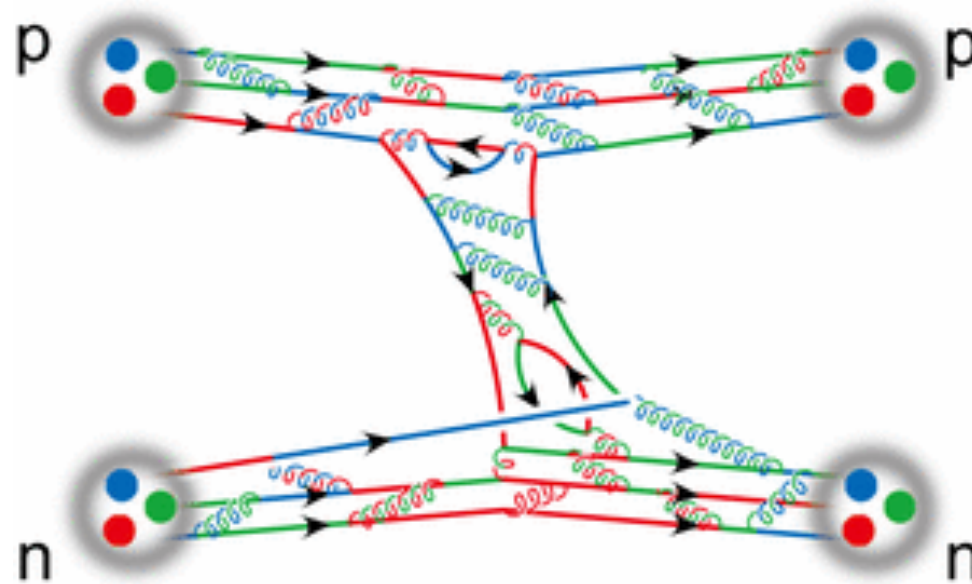
Bochum (Germany)
August 26-30, 2024

www.indico.tp2.rub.de/event/2

11th International Workshop on Chiral Dynamics

CD2024

I. Introduction



Hadron interactions in lattice QCD

Finite volume (FV) method

spectra of two hadrons
in finite box

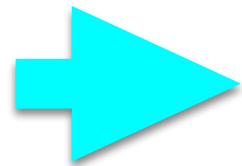


scattering phase shift

Luescher's finite volume formula

HAL QCD method

NBS wave functions



Potential
(Interaction kernel)



scattering
phase shift

Schrodinger equation

It is always better to have two “different” methods for crosschecks.

NN controversy (5 years ago)

FV spectra: both deuteron and dineutron are bound at heavier pion masses.

Potential : Two nucleons are unbound at heavier pion masses.

NN controversy@ heavy pions

Direct method vs HAL QCD potential method

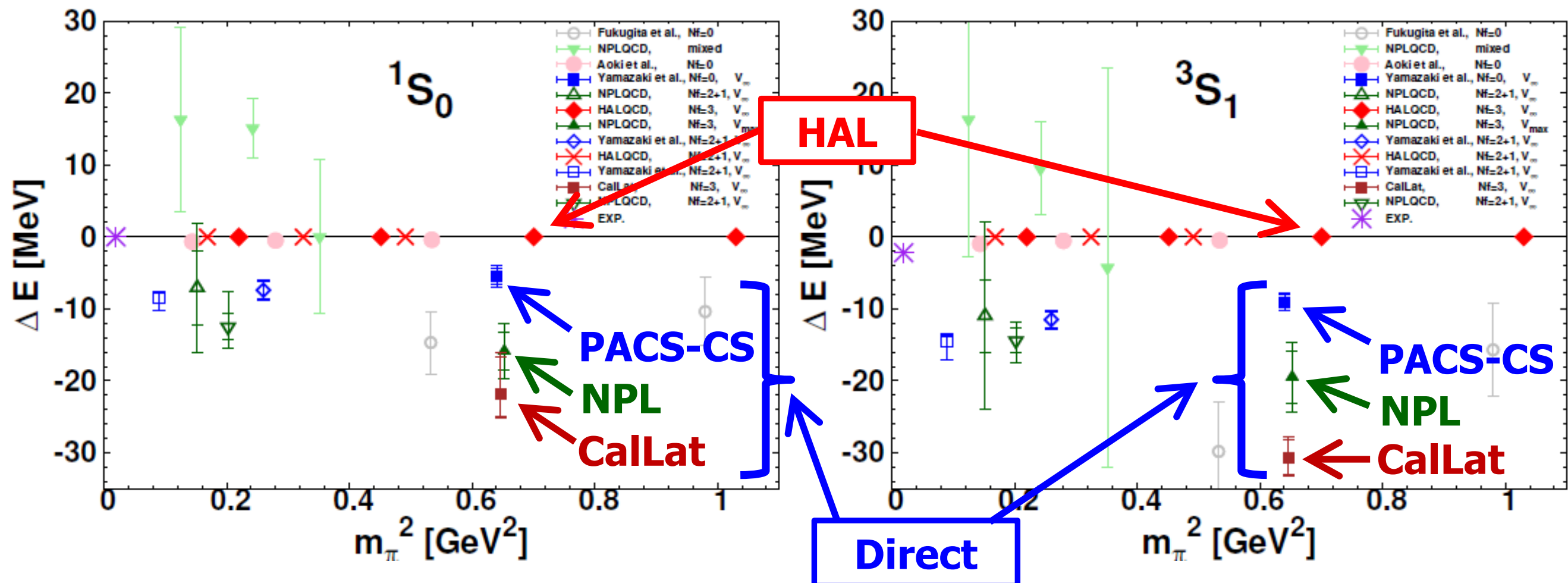
Slide from S. Aoki @ 2019 Santa Fe workshop

Potential method (HAL): unbound

Direct method (PACS-CS/NPL/CalLat): bound

“di-neutron”

“deuteron”



Since then, there appeared more
FV spectra from the community to
resolve this issue.

Walker-loud@lat2023

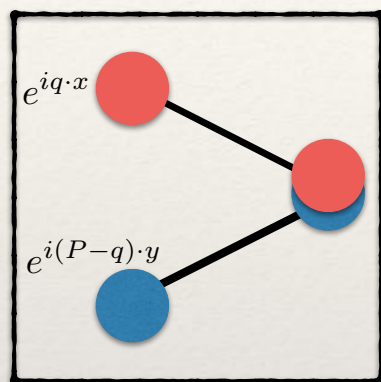
LQCD Results with (deeply) bound di-nucleons

2006	NPLQCD	first dynamical LQCD calculations of NN
2011	NPLQCD	$M\pi \approx 390$ MeV
2012	Yamazaki et al.	$M\pi \approx 510$ MeV
2012	NPLQCD	$M\pi \approx 800$ MeV
2015	Yamazaki et al.	$M\pi \approx 310$ MeV
2015	CalLat	$M\pi \approx 800$ MeV + P,D,F waves
2015	NPLQCD	$M\pi \approx 450$ MeV
2020	NPLQCD	$M\pi \approx 450$ MeV

LQCD Results without bound di-nucleons (or inconclusive)

2012	HAL QCD	$M\pi \approx 710$ MeV
2012	HAL QCD	$M\pi \approx 469 - 1171$ MeV
2019	“Mainz”	$M\pi \approx 960$ MeV
2020	CoSMoN	$M\pi \approx 714$ MeV
2021	NPLQCD	$M\pi \approx 800$ MeV

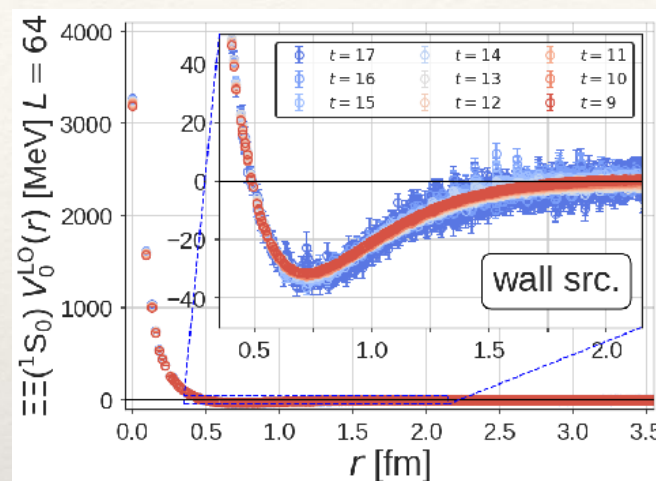
NPLQCD,
Yamazaki et al.,
CalLat (2015)



Compact, hexa-quark
creation operator

Deep bound di-nucleons

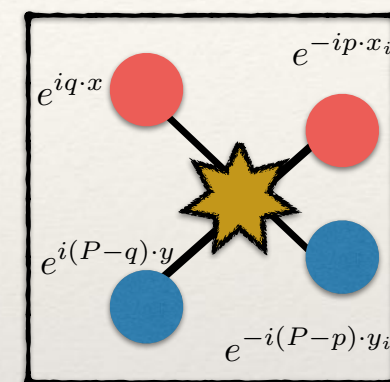
HAL QCD Potential



diffuse - wall source

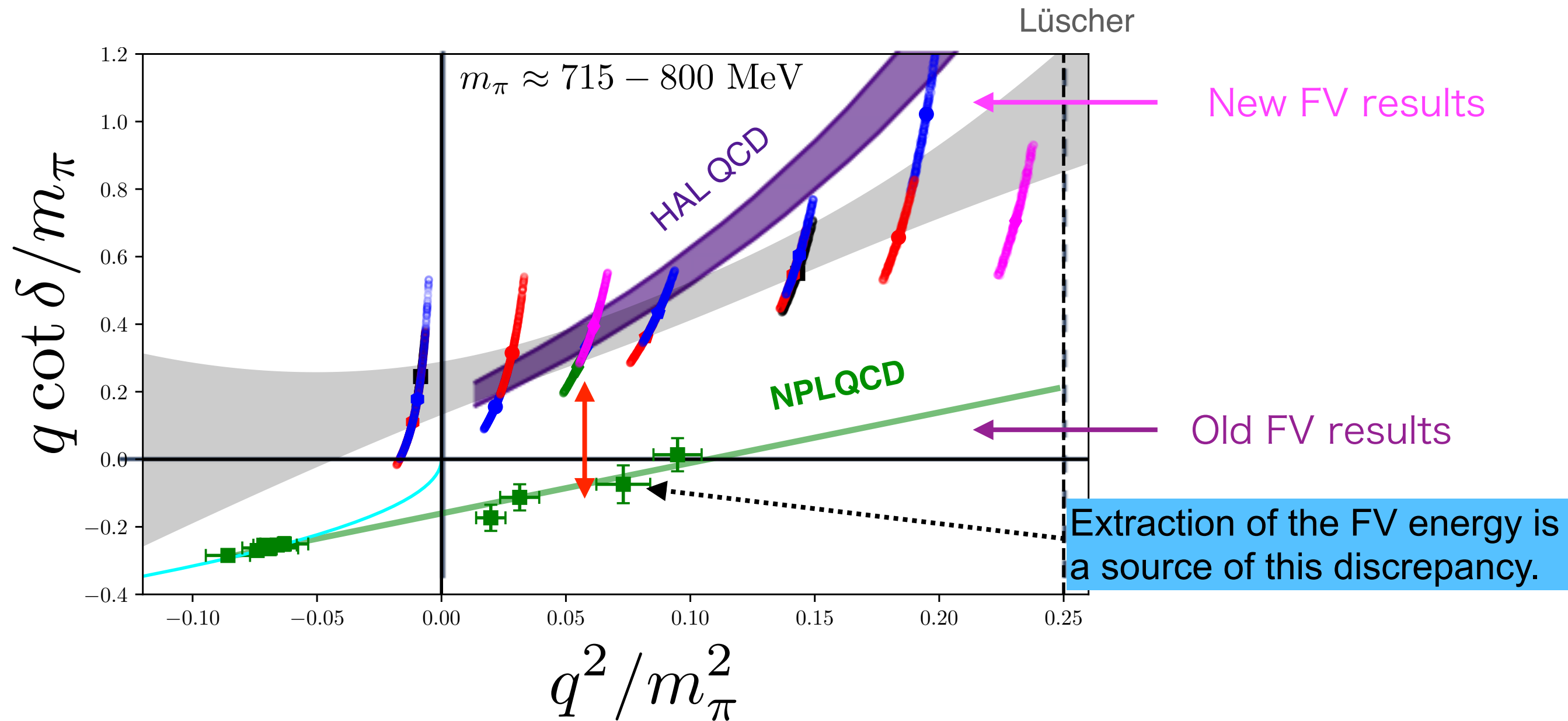
no bound state

“Mainz” (Distillation)
CoSMoN (stochastic LapH)
NPLQCD (sparsened momentum)



momentum-space
creation & annihilation
positive-definite correlation matrix

no bound state



Walker-loud@Lat2023: “I believe the old results are wrong (including those I was involved with)”

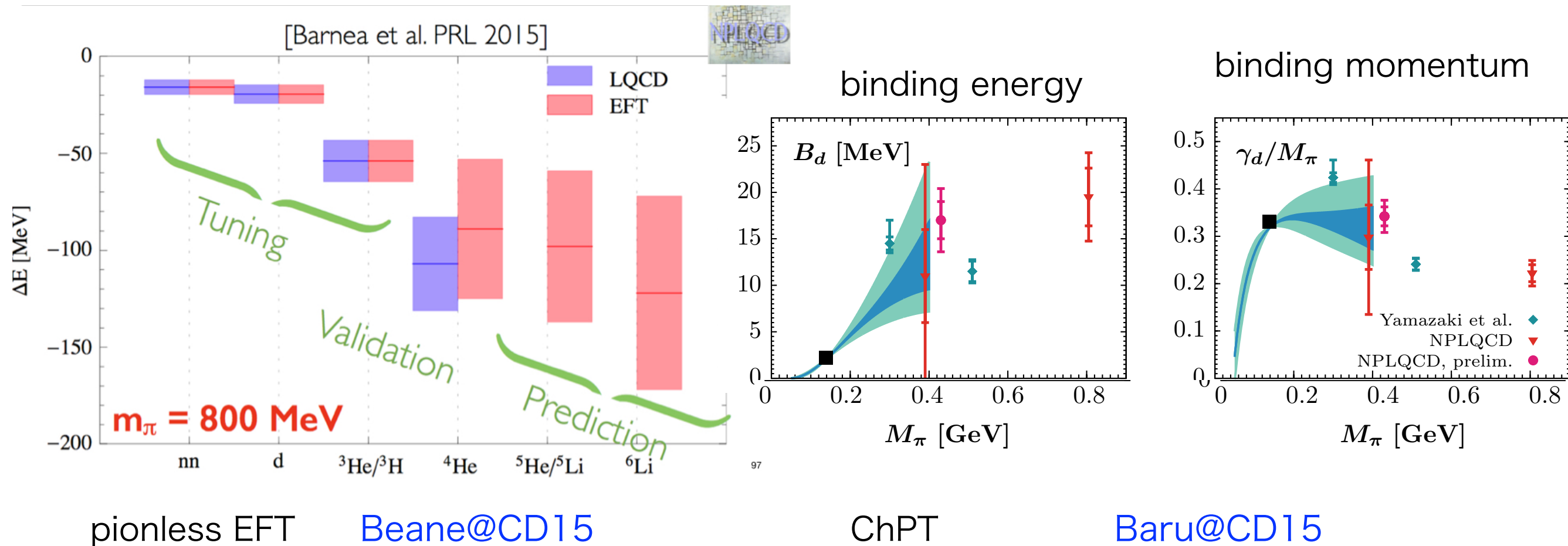
Two nucleons are unbound at heavy pion masses.

c.f. A combination of the finite volume formula and the ERE($k \cot \delta$) was first introduced as “Sanity Check” to investigate this problem.

Iritani et al., PRD96(2017)034521.

Could (chiral) effective theories invalidate old finite volume data ?

Unfortunately, the answer seems “No”.



The chiral EFTs and old lattice data seemed consistent if EFT parameters were fixed by the same lattice data.

It is hard for EFTs to tell whether lattice data are correct or not.

In this talk, I will review recent results on interactions between two hadrons at almost physical pion mass, obtained exclusively by the HAL QCD method, stressing connections with chiral dynamics or experiments.

Contents

I. Introduction

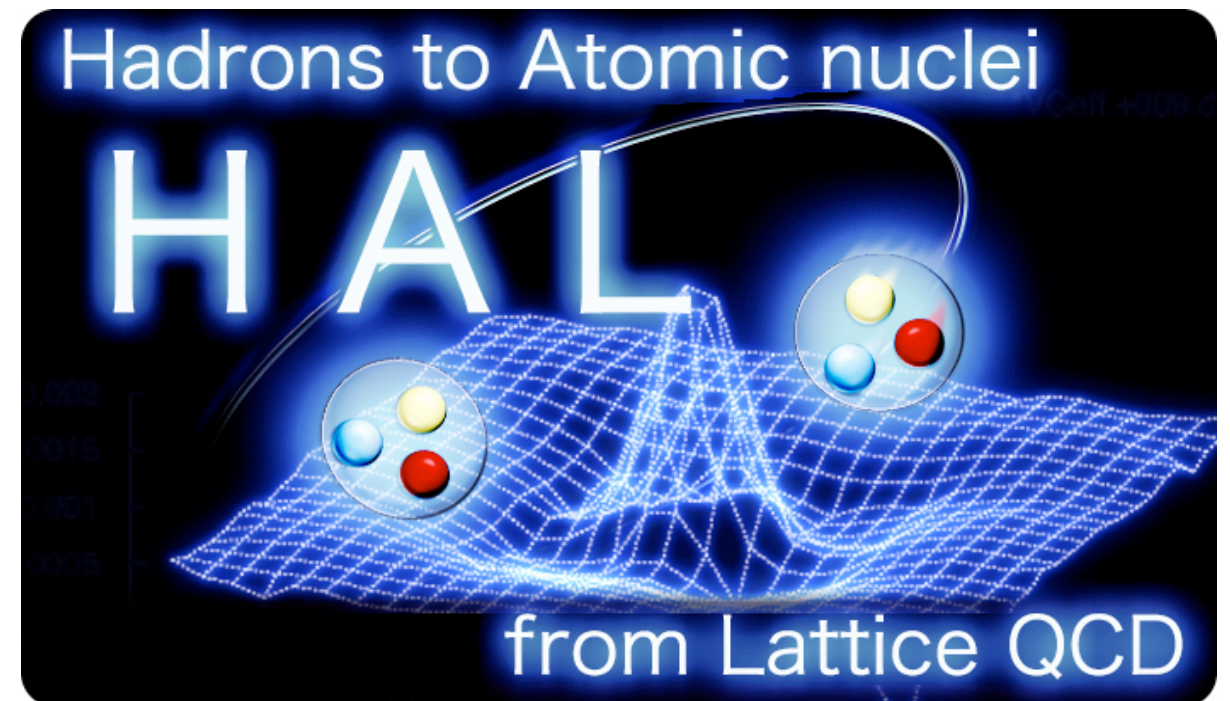
II. $N\Xi$ interactions

III. $N\Omega$ Dibaryon

IV. Tetra quark state T_{cc}

V. $N - \phi$ interaction and 2-pion tail

VI. Summary



Gauge configurations at (almost) physical point

Ishikawa *et al.* (PACS), PoS Lattice2015(2016) 075.

All results in this talk have been obtained on this ensemble.

2+1 flavor gauge configuration on 96^4 lattice

with Iwasaki gauge + NP $O(a)$ improved clover quark

$a \simeq 0.0846$ fm, $m_\pi \simeq 146$ MeV, $m_K \simeq 525$ MeV (almost **physical point**)

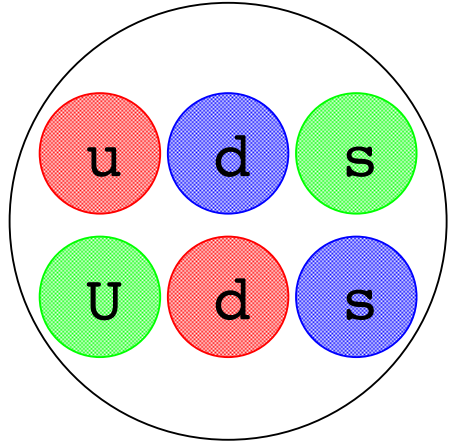
$La \simeq 8.1$ fm

generated by K-computer,
previous supercomputer in
Japan.



II. $\Lambda\Lambda - N\Xi$ interactions

1. $\Lambda\Lambda - N\Xi$ interactions

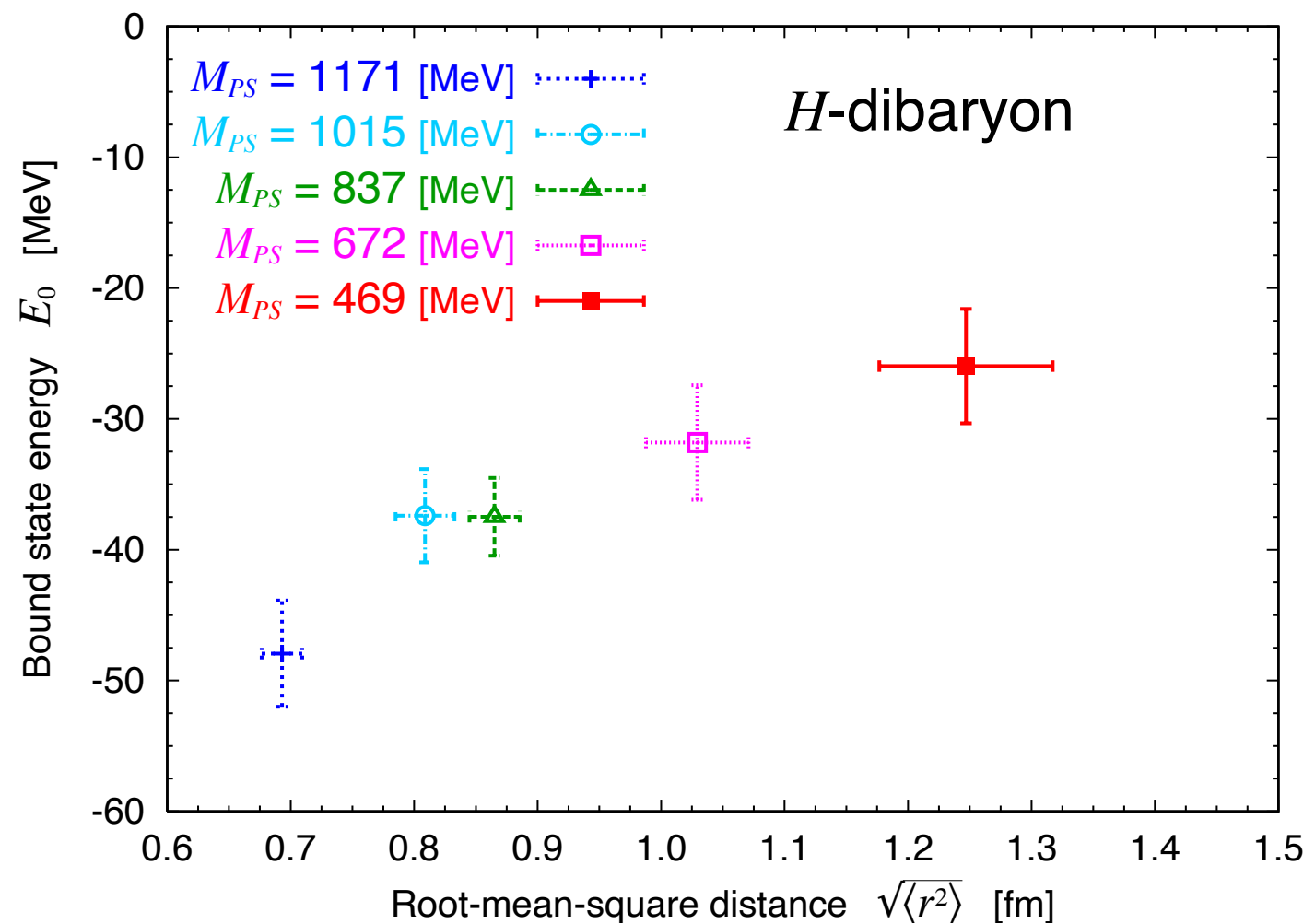
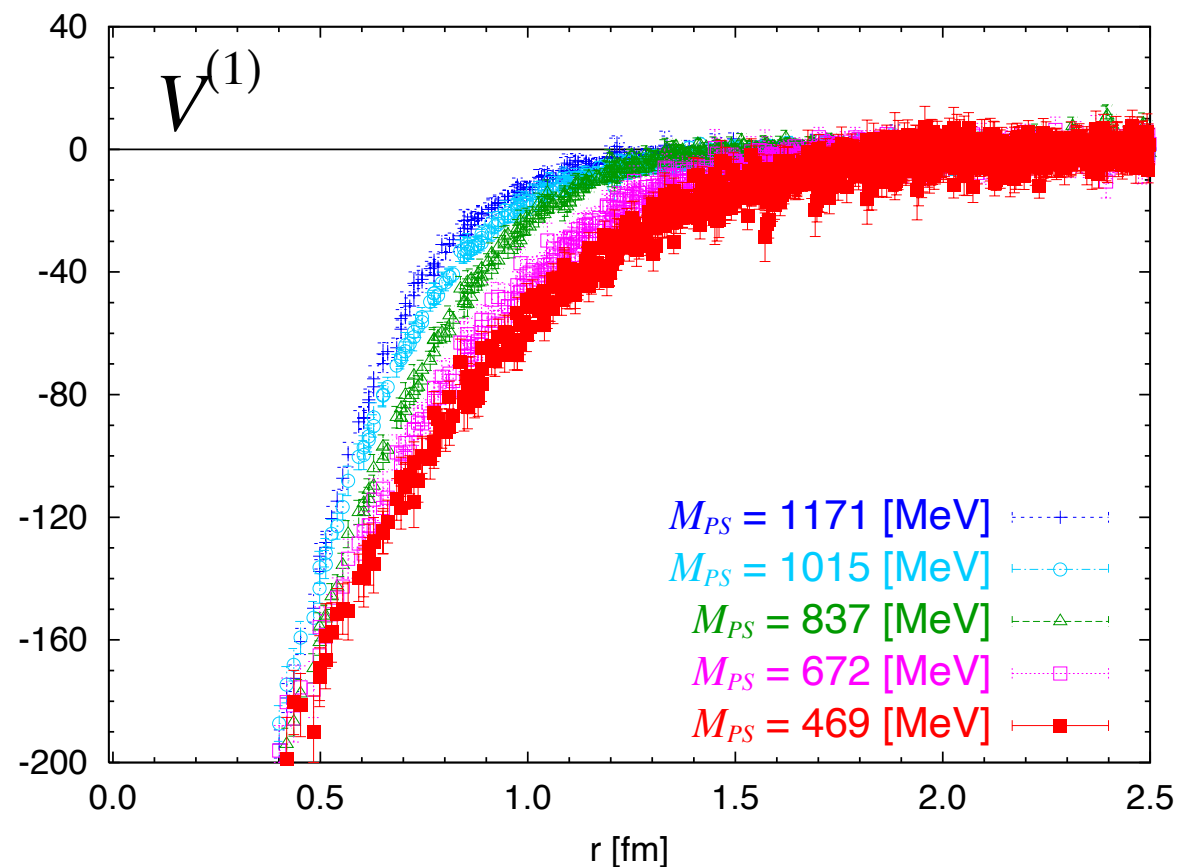


The bound H-dibaryon appears in the flavor SU(3) limit.

$$m_u = m_d = m_s$$

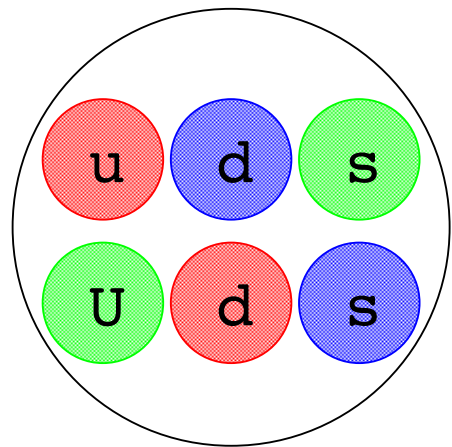
Binding energy vs. size

Potential



(a) Binding energy decreases at lighter M_{ps} .

(b) Size of bound state increases at lighter M_{ps} .



What happen in the real world ?

pion mass becomes lighter
there is flavor SU(3) breaking

SU(3) limit

$\Lambda\Lambda - N\Xi - \Sigma\Sigma$

—



30-40 MeV

H

—

Real world

$\Sigma\Sigma$

—

2386 MeV

H ?

- - - - -

129 MeV

$N\Xi$

—

2257 MeV

H ?

- - - - -

25 MeV

$\Lambda\Lambda$

—

2232 MeV

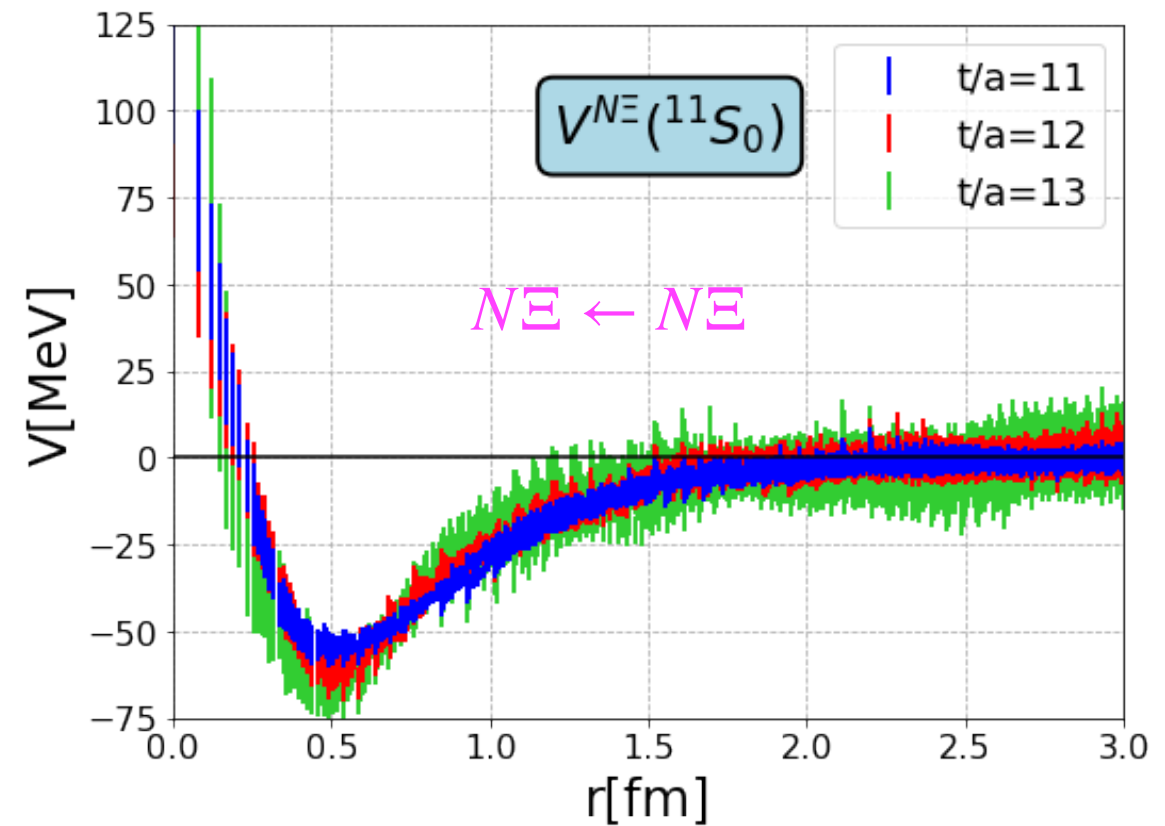
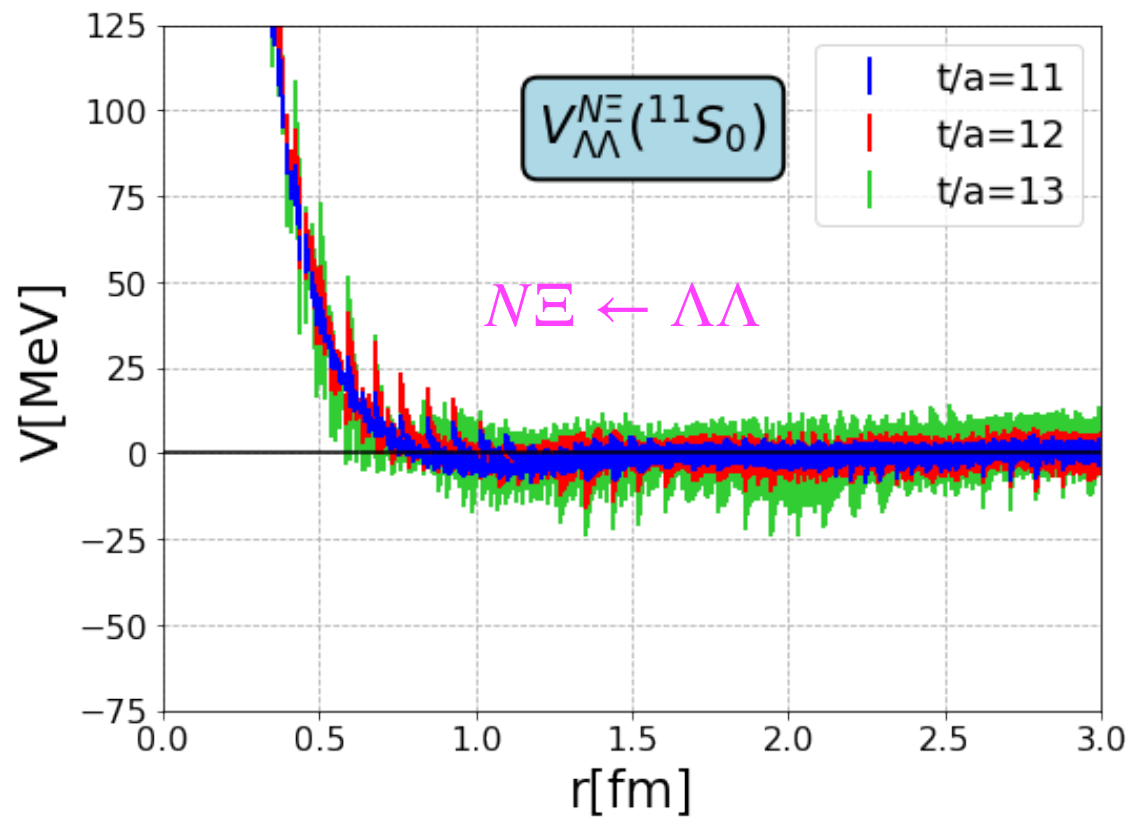
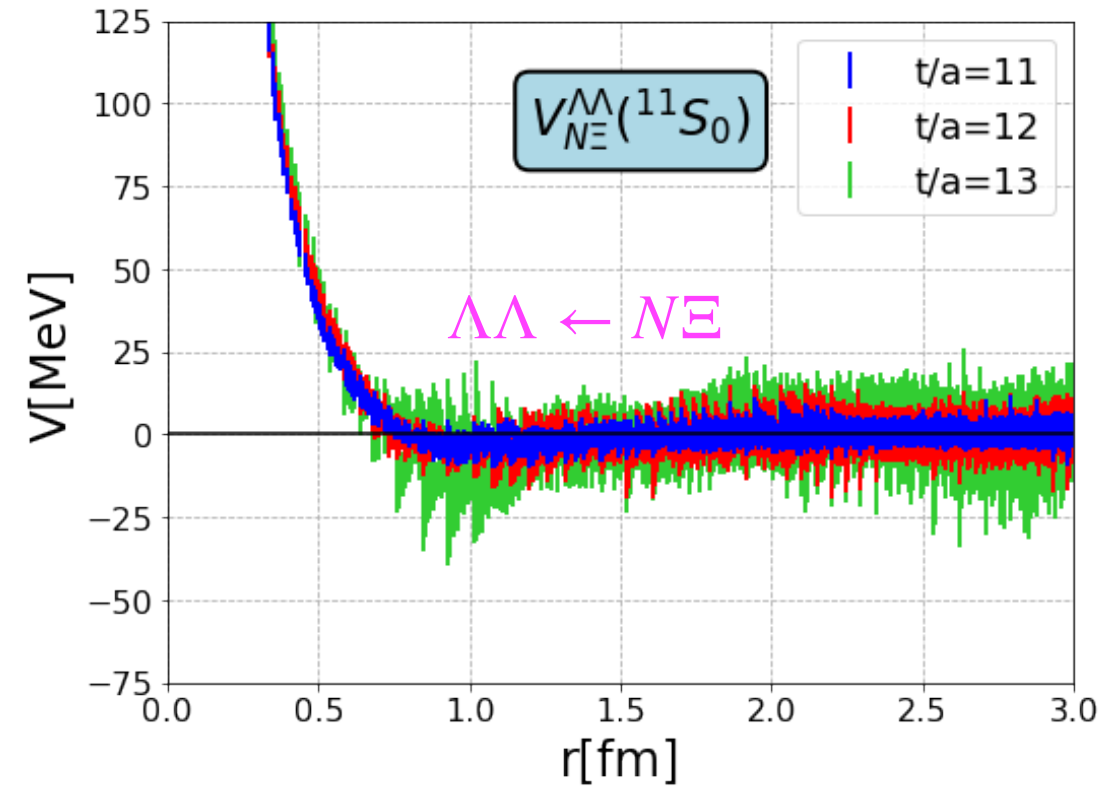
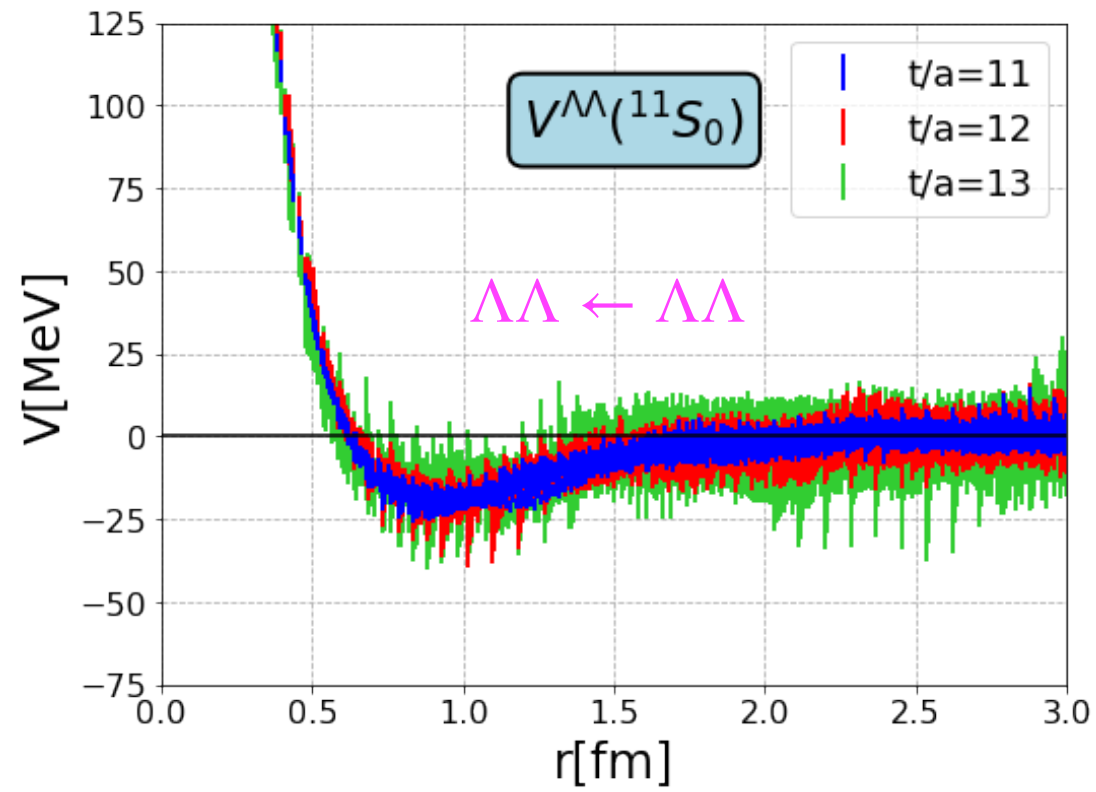
H ?

- - - - -

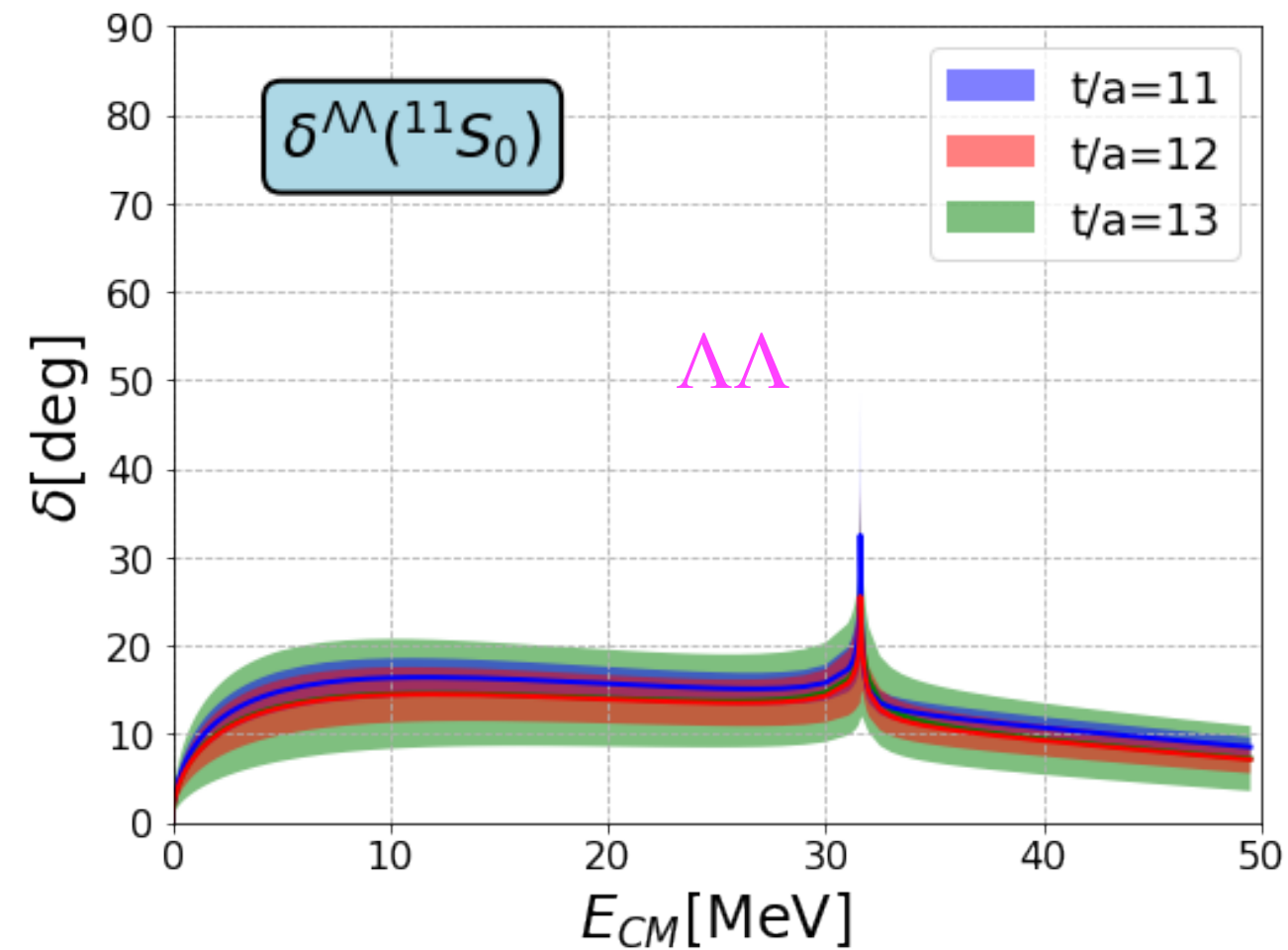
We consider $\Lambda\Lambda - N\Xi$ coupled channel interaction as $\Sigma\Sigma$ threshold is much higher.

$$2I+1, 2s+1 S_J$$

$\Lambda\Lambda - N\Xi$ coupled channel potential



scattering phase shifts



(a) No bound state/resonance in $\Lambda\Lambda$.

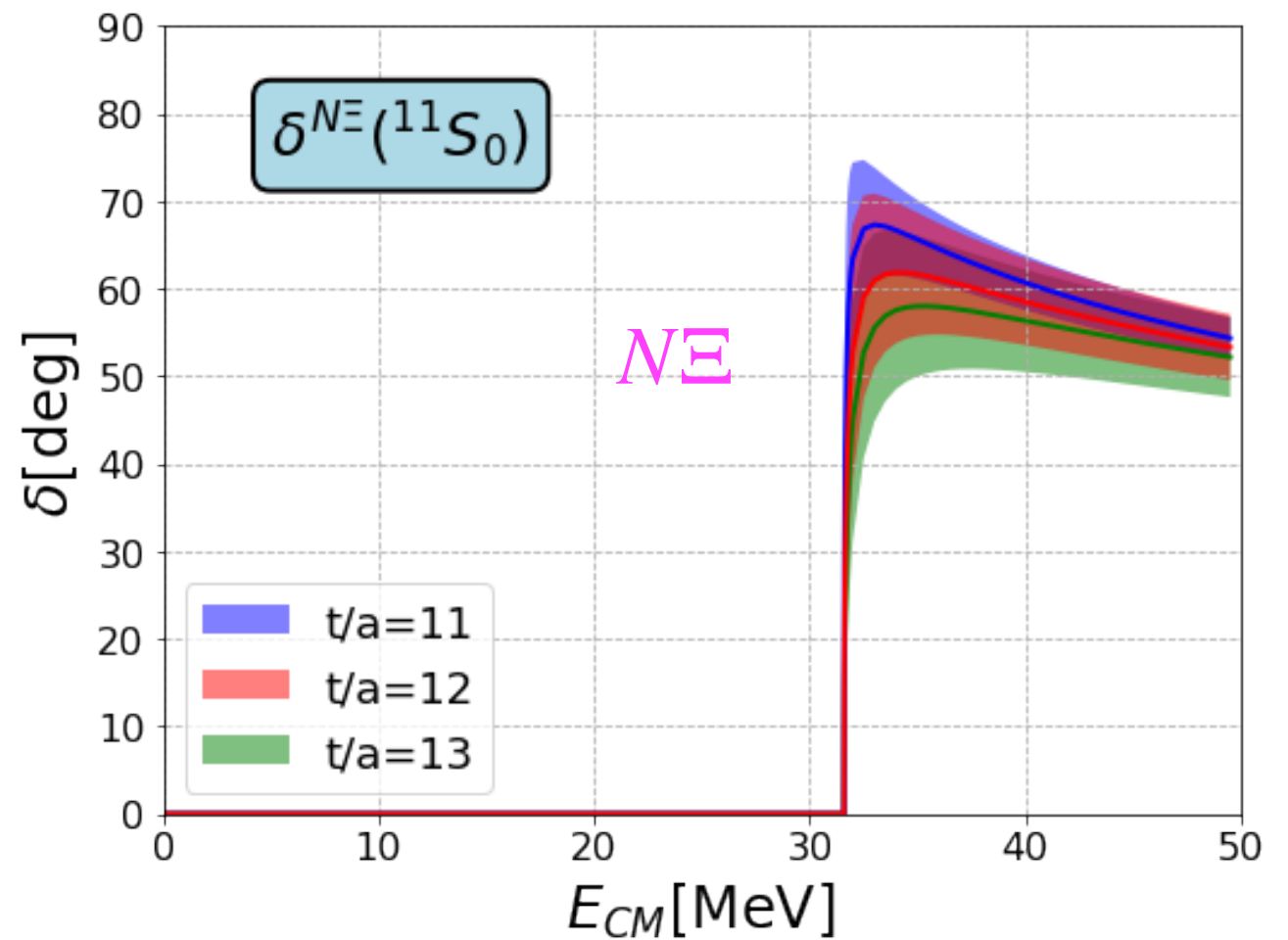
weak attraction in $\Lambda\Lambda$ potential

(b) No H dibaryon near $\Lambda\Lambda$ threshold.

$$a_0^{\Lambda\Lambda} = -0.8(3) \text{ fm}$$

(c) Sharpe enhancement and rapid drop near $N\Xi$ threshold.

off-diagonal $\Lambda\Lambda \rightarrow N\Xi$ potential

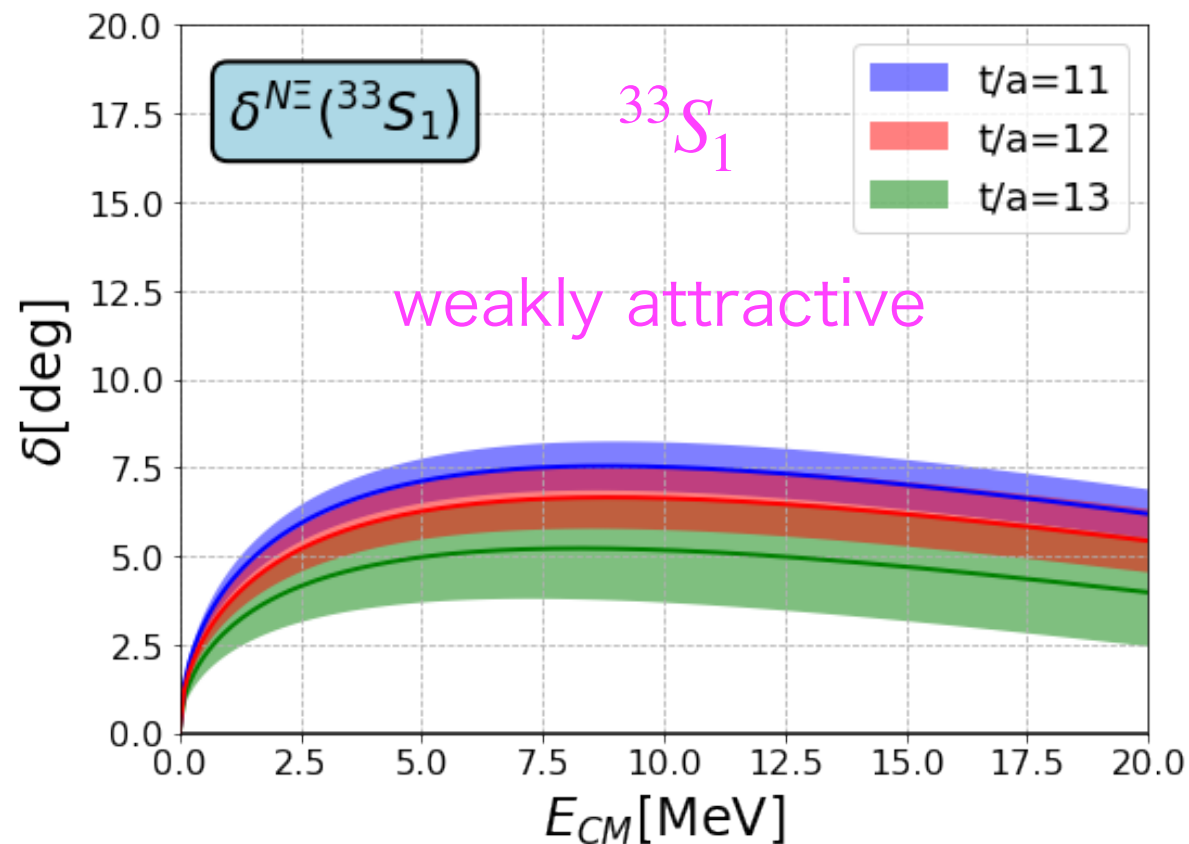
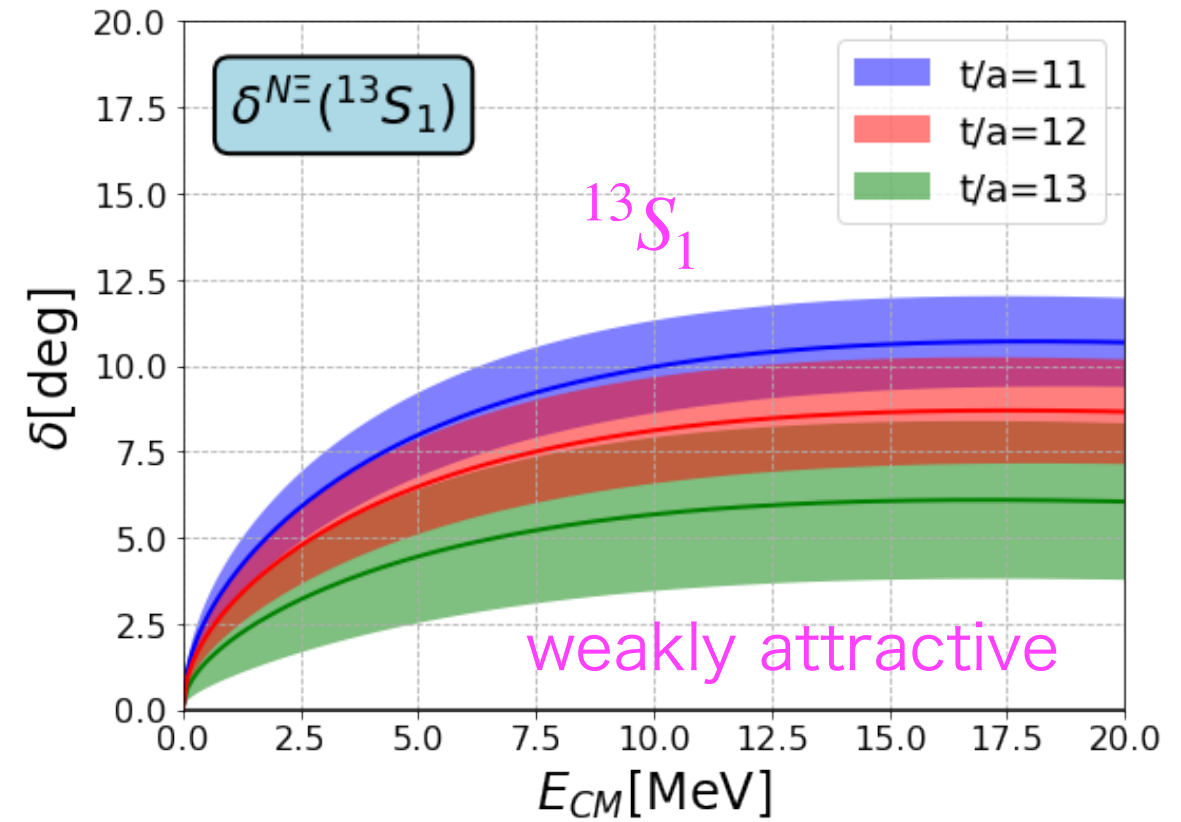
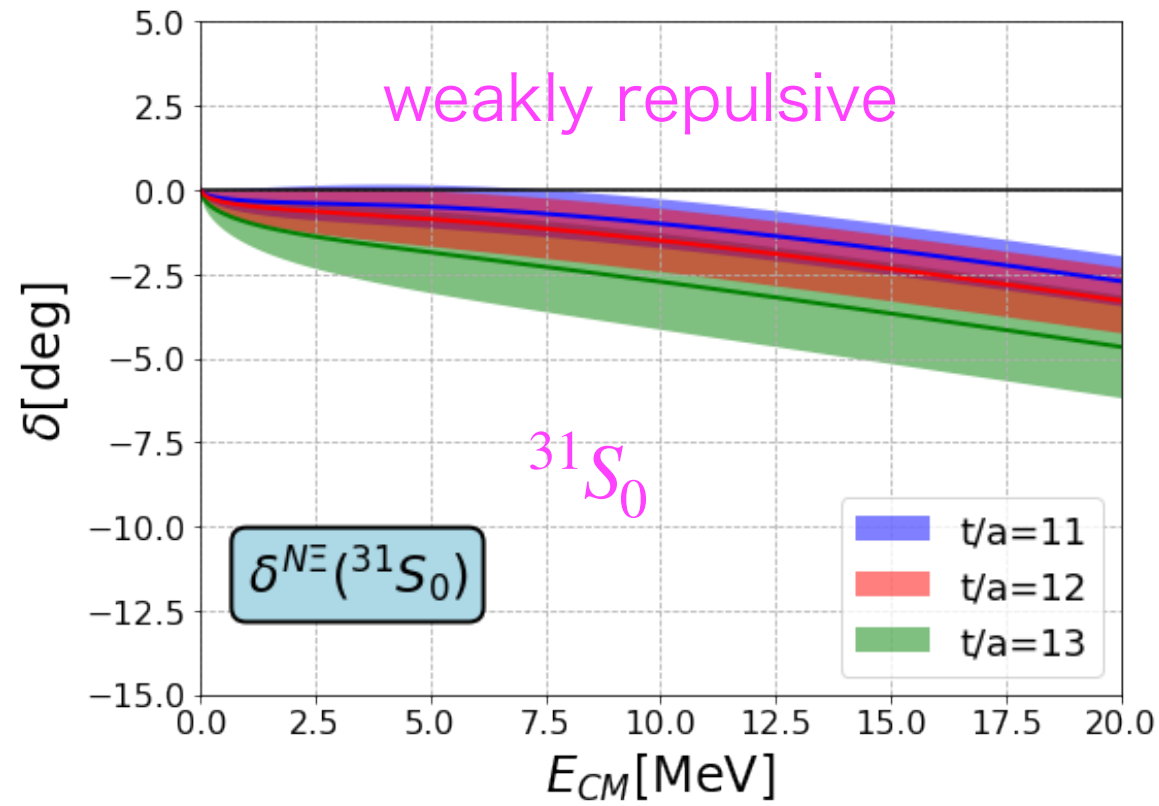


(d) Sharpe increase near $N\Xi$ threshold.

significant $N\Xi$ attraction

(e) H appears as a virtual state of $N\Xi$ at almost physical point.

scattering phase shifts for other $\Lambda\Lambda - N\Xi$ channels



$$2I+1, 2s+1 S_J$$

2. Comparison with experiments

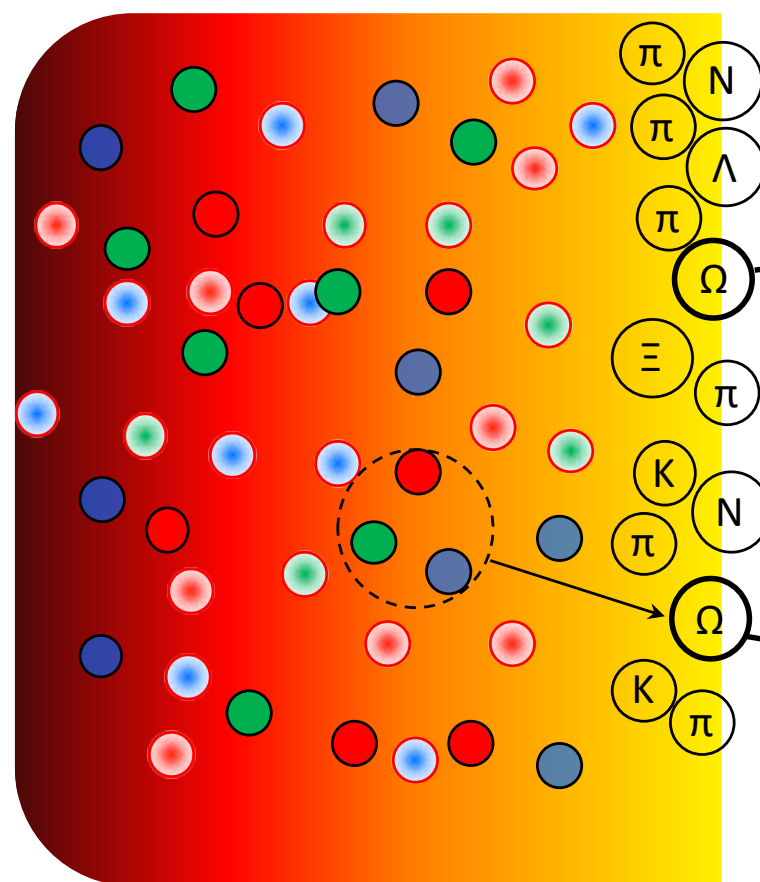
Measurement of two-baryon correlation at RHIC & LHC

Femtoscopy

ALICE Coll., PRL123(2019)112002; PLB797(2019)134622.

two-baryon interaction

two-baryon correlation



$$C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 \\ \text{others} \end{cases}$$

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2) \cdot P}{P^2} P\right)^2}$$

K. Morita et al., NPA967(2017) 586.

$$C_{AB}(Q) = \frac{N_{AB}(Q)}{N_A(Q)N_B(Q)}$$

$N_A(Q), N_B(Q)$ # of each hadron

$N_{AB}(Q)$ # of hadron pairs

theory

$$N_{AB}(Q) = \int \frac{d^3 p_A}{E_A} \frac{d^3 p_B}{E_B} N_{AB}(\mathbf{p}_A, \mathbf{p}_B) \delta(Q - \sqrt{-q^2})$$

$$N_{AB}(\mathbf{p}_A, \mathbf{p}_B) \simeq \int d^4 x d^4 y \overbrace{S_A(x, \mathbf{p}_A) S_B(y, \mathbf{p}_B)}^{\text{source}} \overbrace{|\Psi(x, y, \mathbf{p}_A, \mathbf{p}_B)|^2}^{\text{2-body wave function from the interaction potential}}$$



source

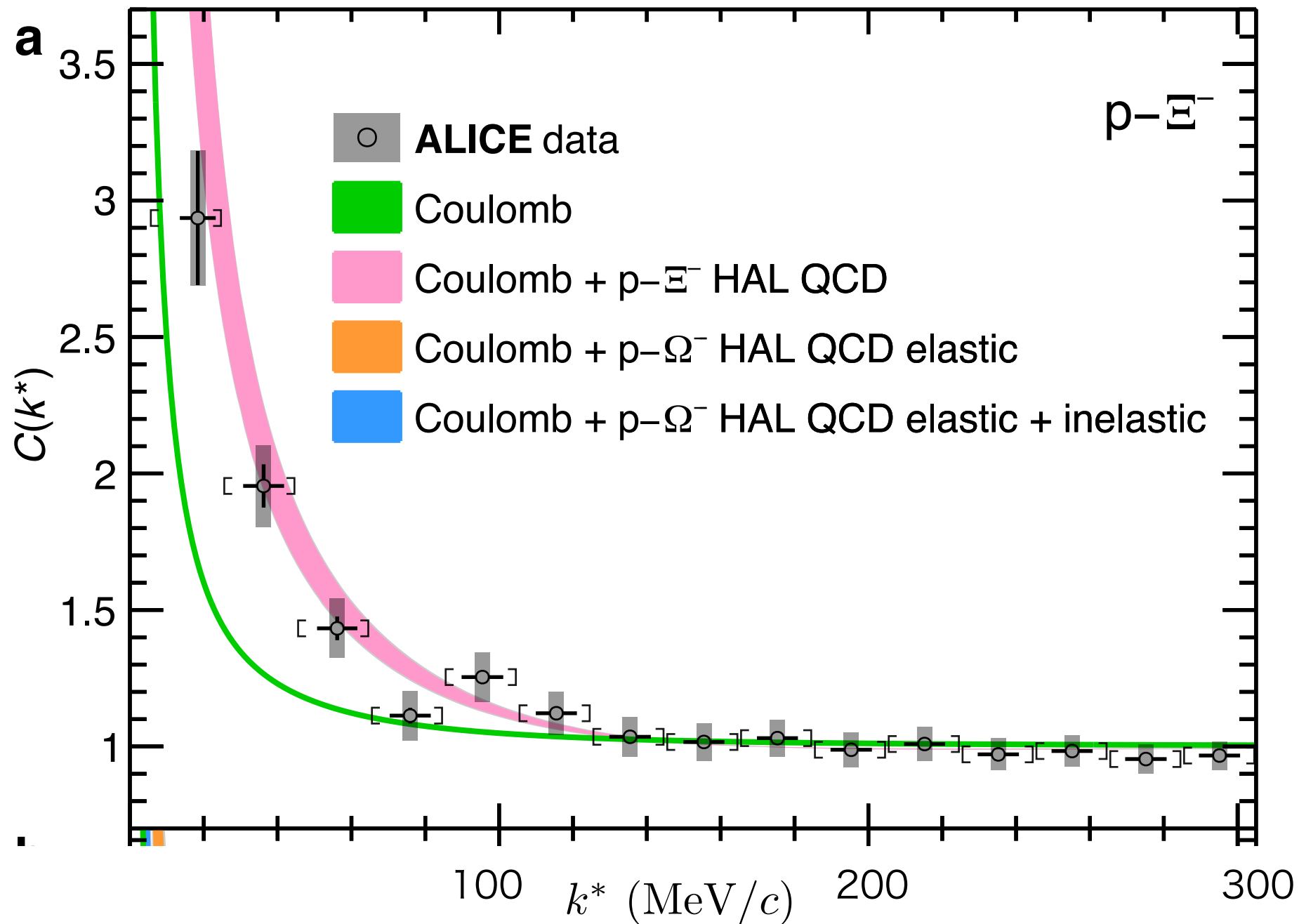


2-body wave function
from
the interaction potential

If the source is approximately known,
one can test hadron interactions using the above formula.

A comparison with data

Alice collaboration, Nature 558 (2020), 232-238.



(a) LHC $p - \Xi^-$ data can not be explained by Coulomb interaction alone.

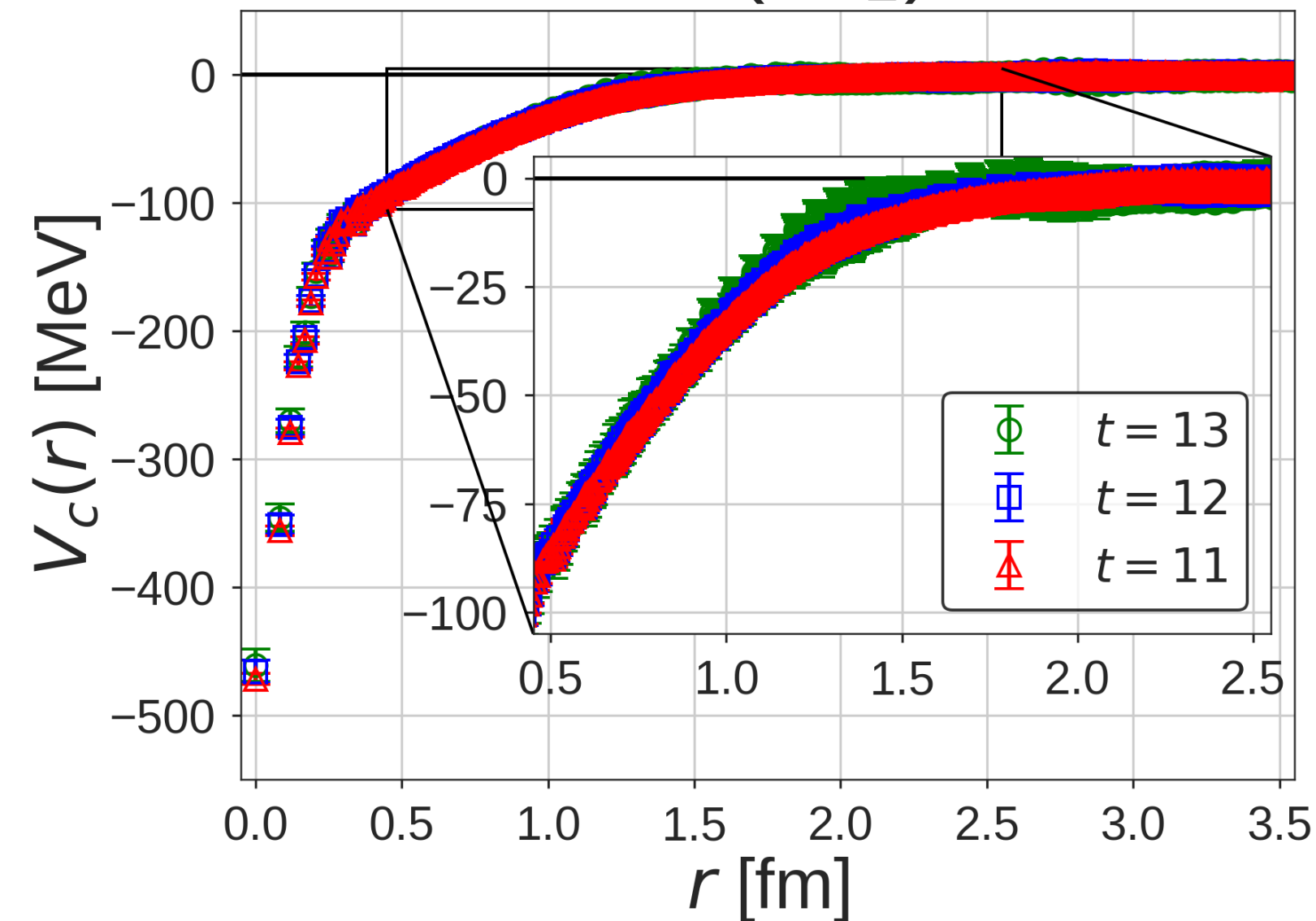
(b) Data are consistent with Coulomb + HAL QCD potential.

III. $N\Omega$ Dibaryon

T. Iritani et al.[HAL QCD], PLB 792 (2019) 284.

$N\Omega$ potential in 5S_2 channel

$N\Omega(^5S_2)$ $S=3, J=3$

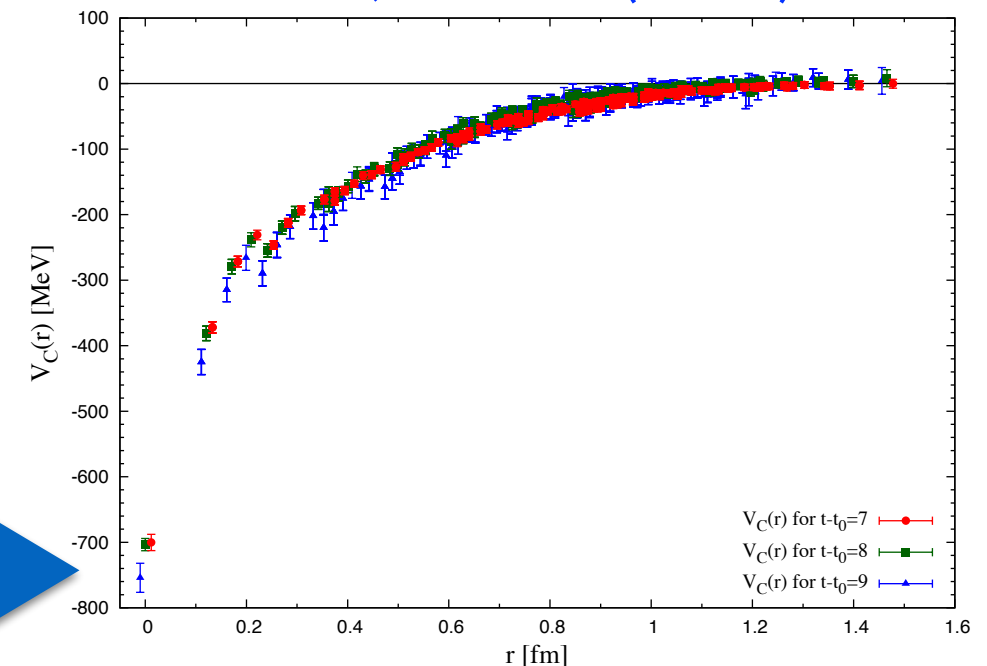


- * we obtain an attractive potential without repulsive core
- * attraction is long range

qualitatively the same at $m_\pi \simeq 875$ MeV



Etminan et al., NPA928(2014)89



B.E. = 18.9(5.0)(+12.1)(-1.8) MeV

Remark

$$m_\pi = 146 \text{ MeV}$$

$$L = \infty$$

$$L = 8.1 \text{ fm}$$

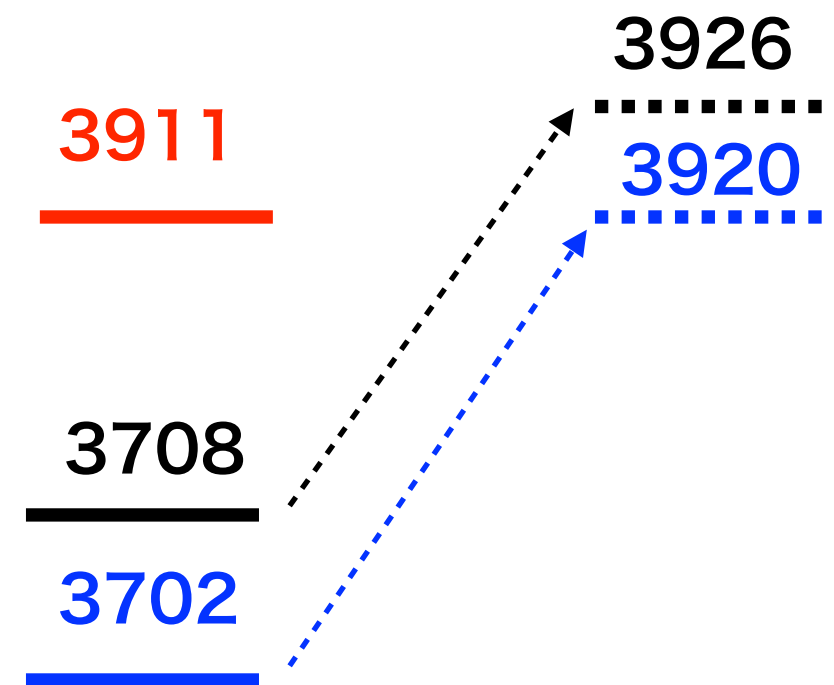
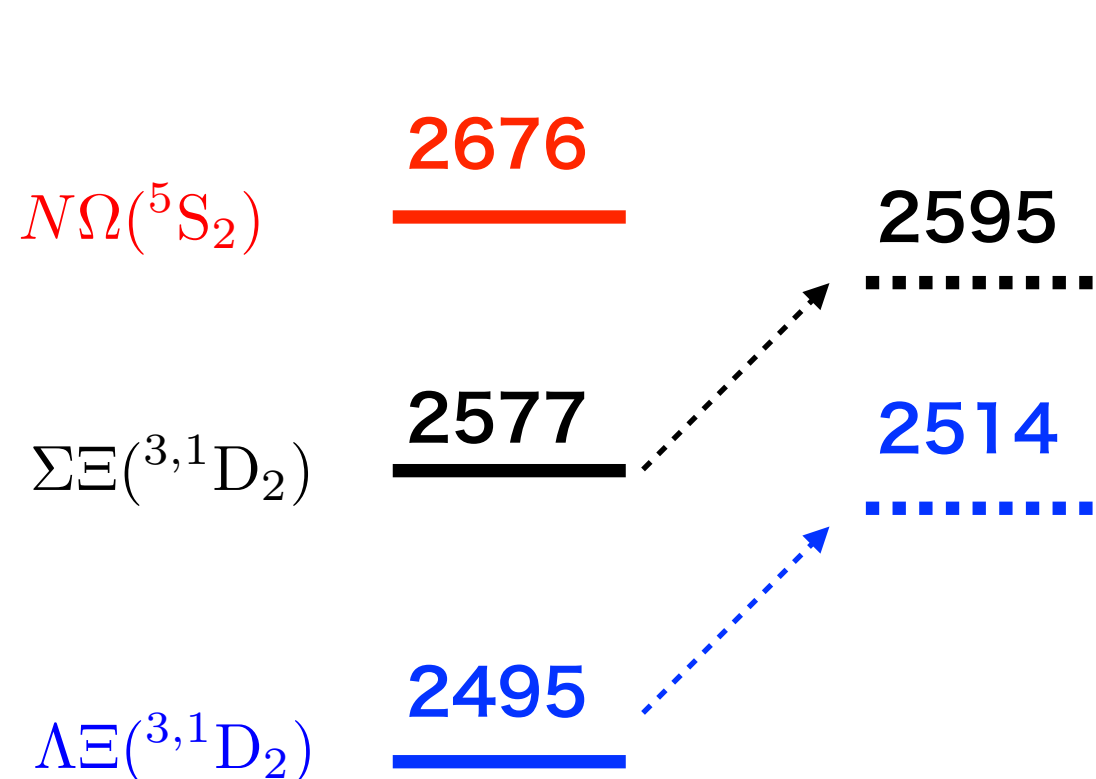
$$p_{\min} = 153 \text{ MeV}$$

$$m_\pi = 875 \text{ MeV}$$

$$L = \infty$$

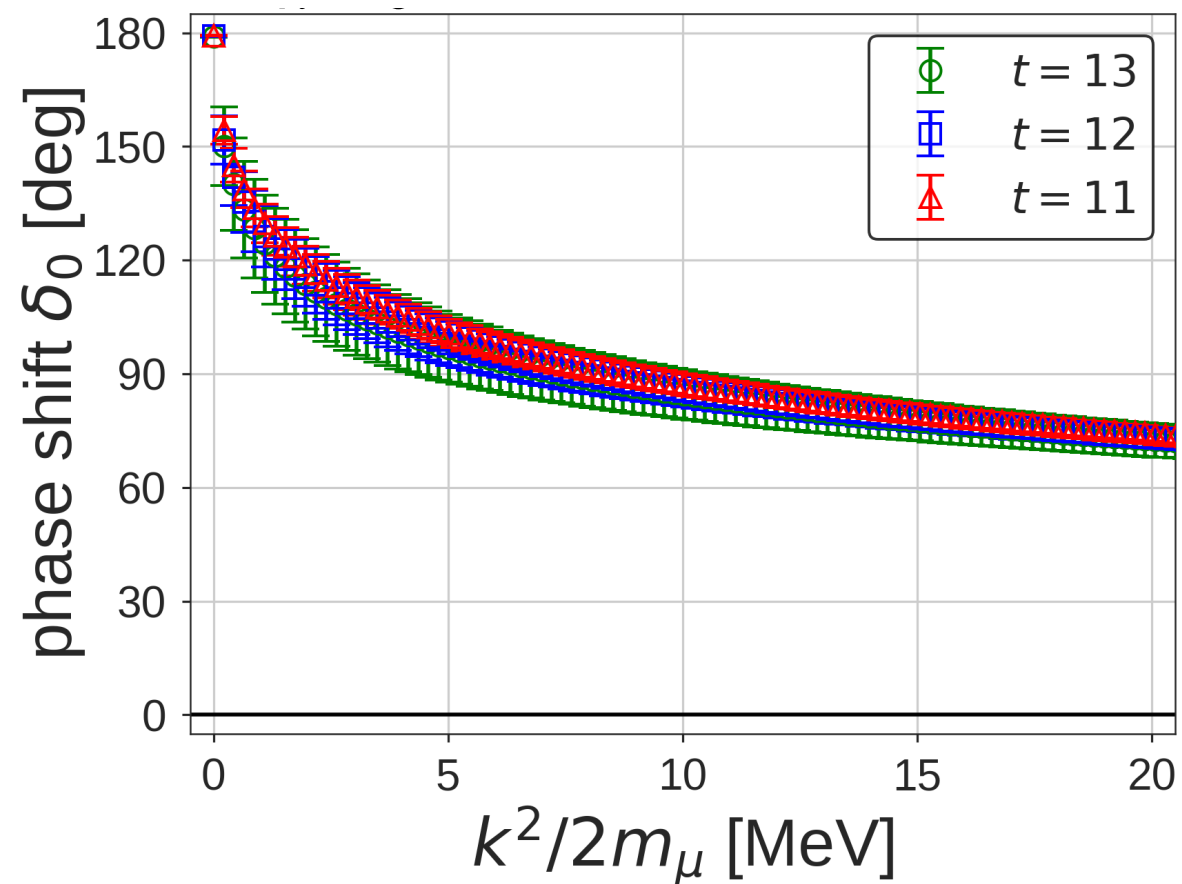
$$L = 1.9 \text{ fm}$$

$$p_{\min} = 645 \text{ MeV}$$

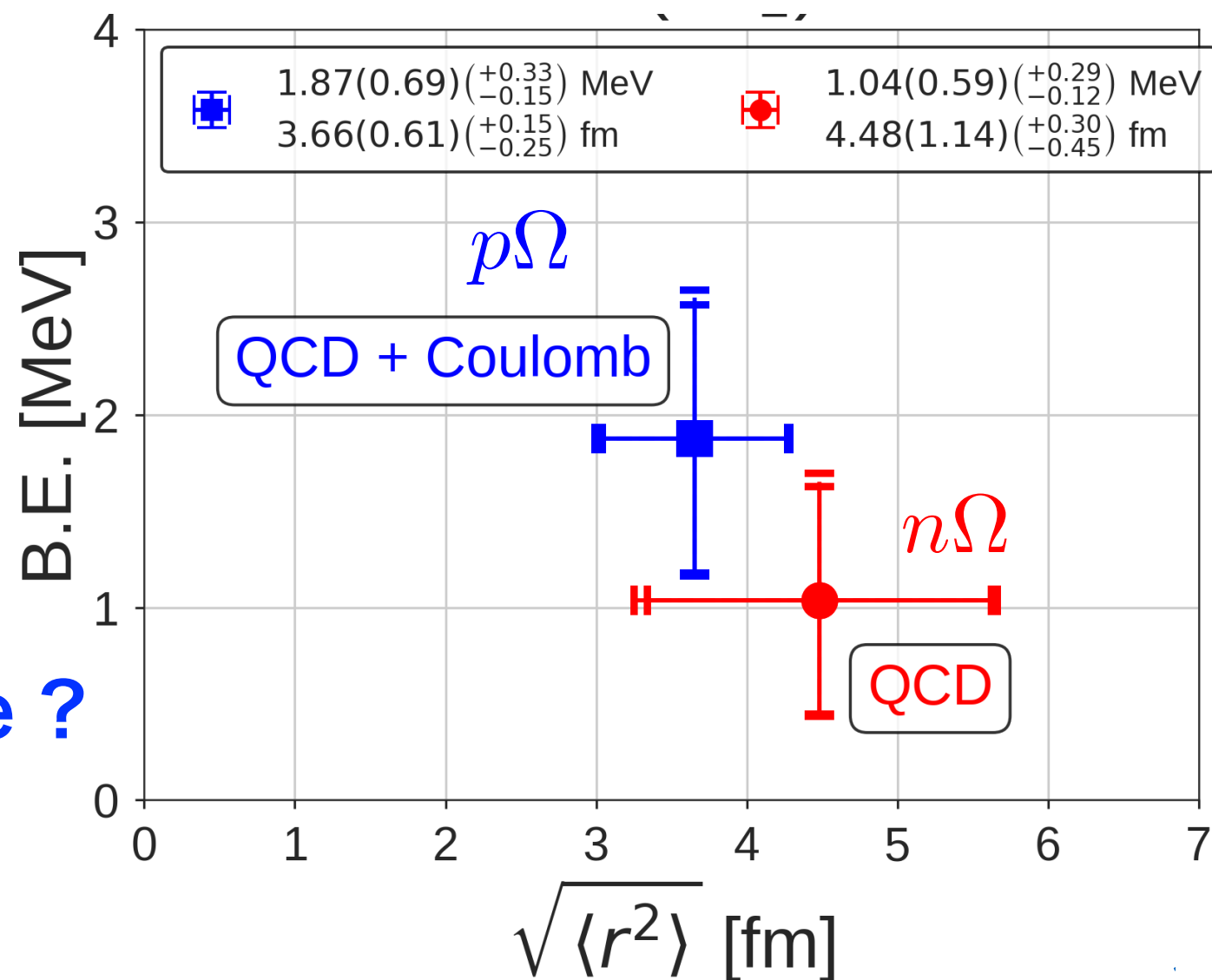


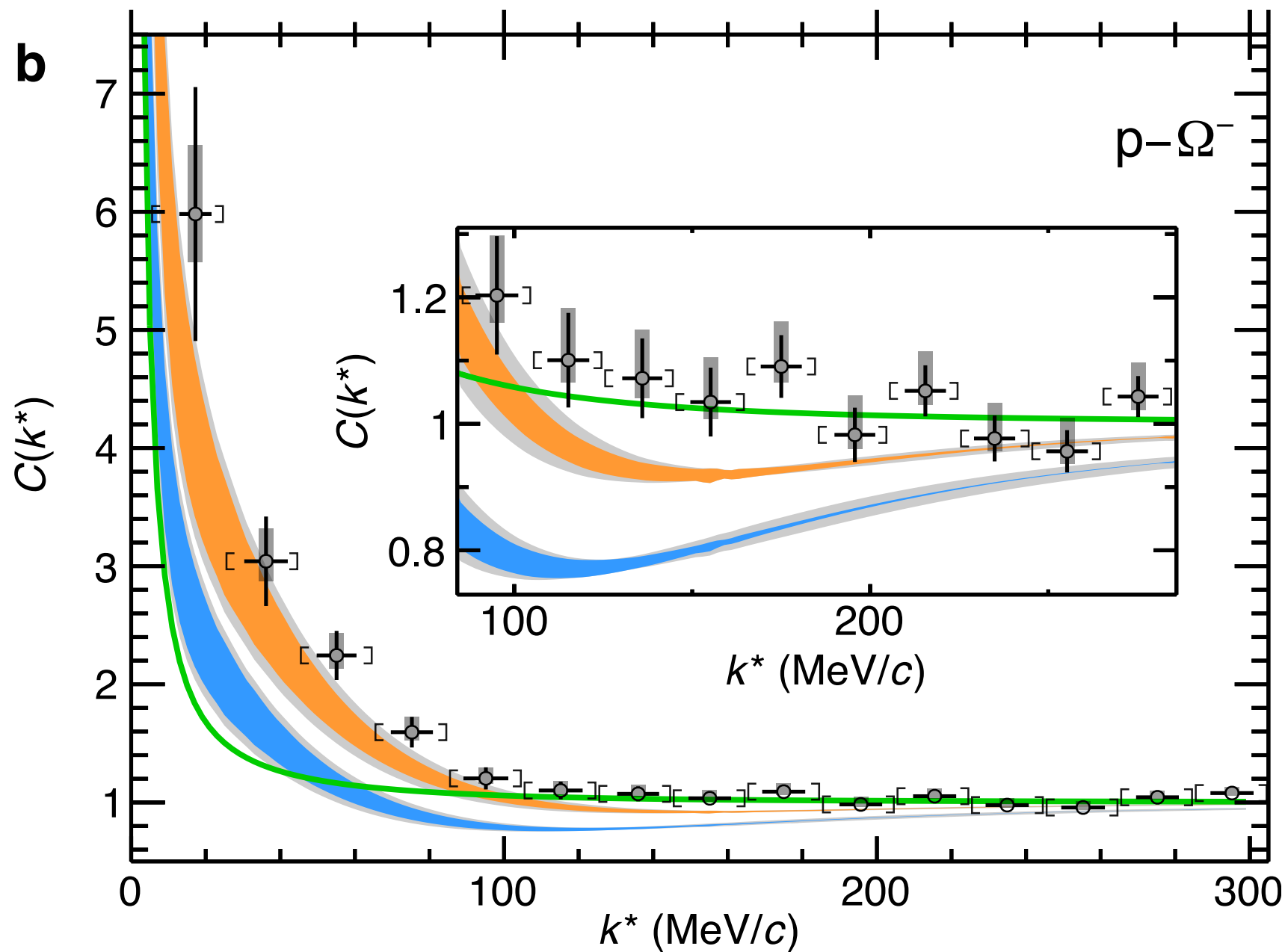
- * Only single channel analysis is made.
- * We assume small couplings to D-waves, supported by weak t-dep.
- * We need coupled channel analysis in the future

Phase shift and binding energy



New dibaryon resonance ?

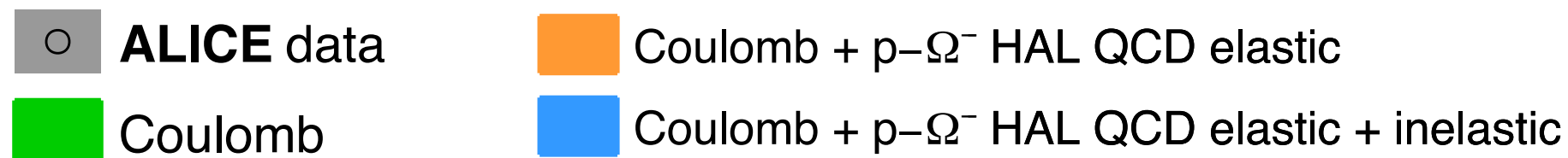




(a) LHC $p - \Omega^-$ data can not be explained by Coulomb interaction alone.

(b) Data are better explained by Coulomb + HAL QCD elastic interaction.

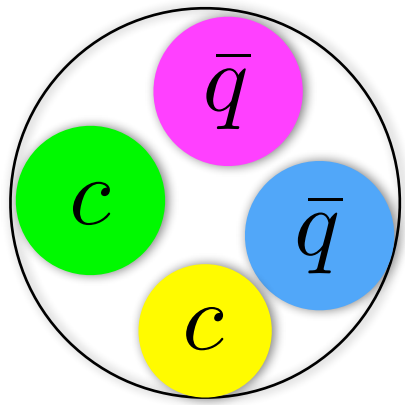
(c) Slight deviations are observed around 100 MeV.



IV. Tetra quark state T_{cc}

Y. Lyu et al.[HAL QCD], PRL 131 (2023) 161901.

Heavy tetra-quark states T_{cc}



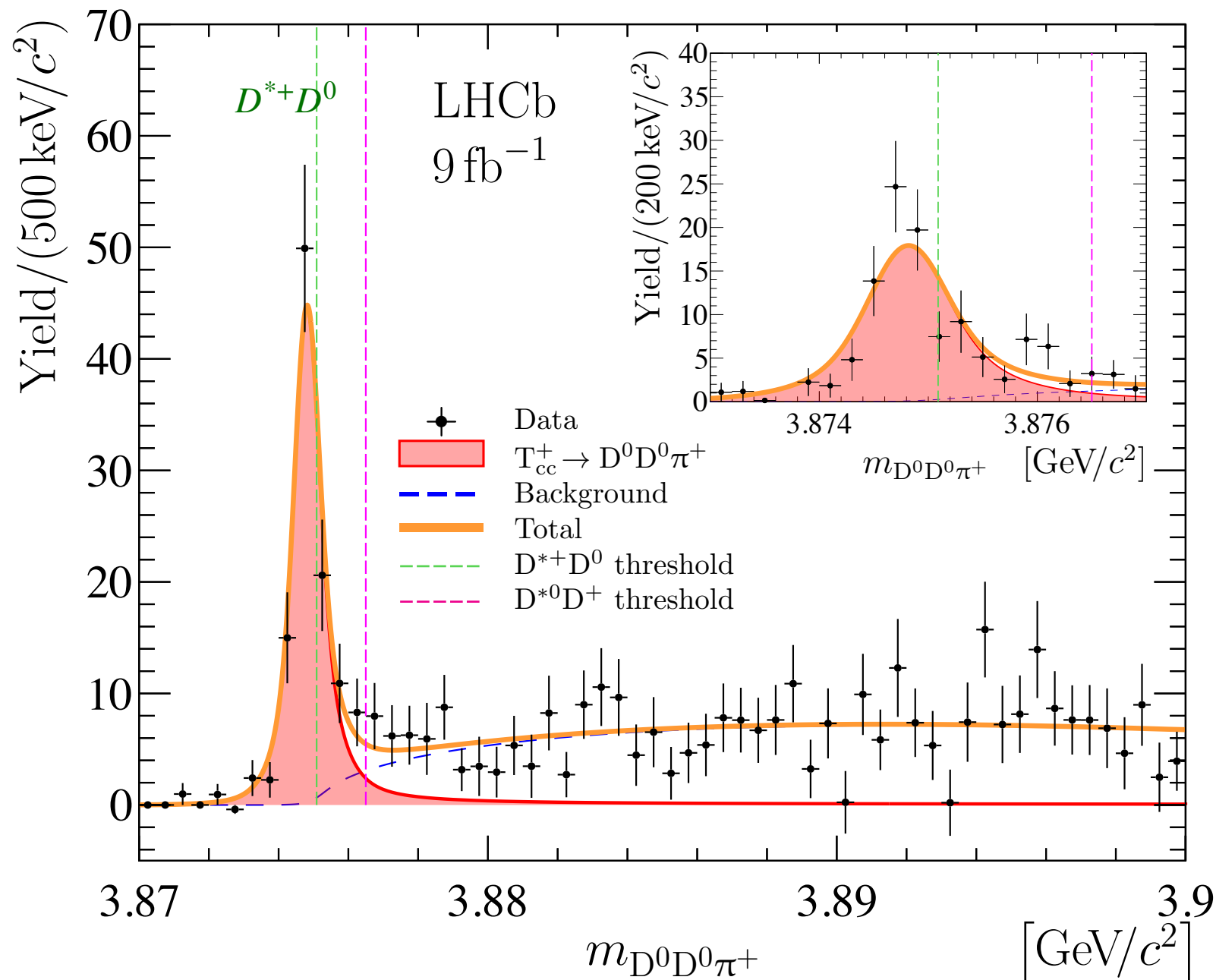
$\bar{q}$: light anti-quark

genuine tetra-quark states

$T_{cc}(cc\bar{u}\bar{d})$

observation by LHCb.

Aaij et al. (LHCb Collaboration),
Nature Phys. (2022)



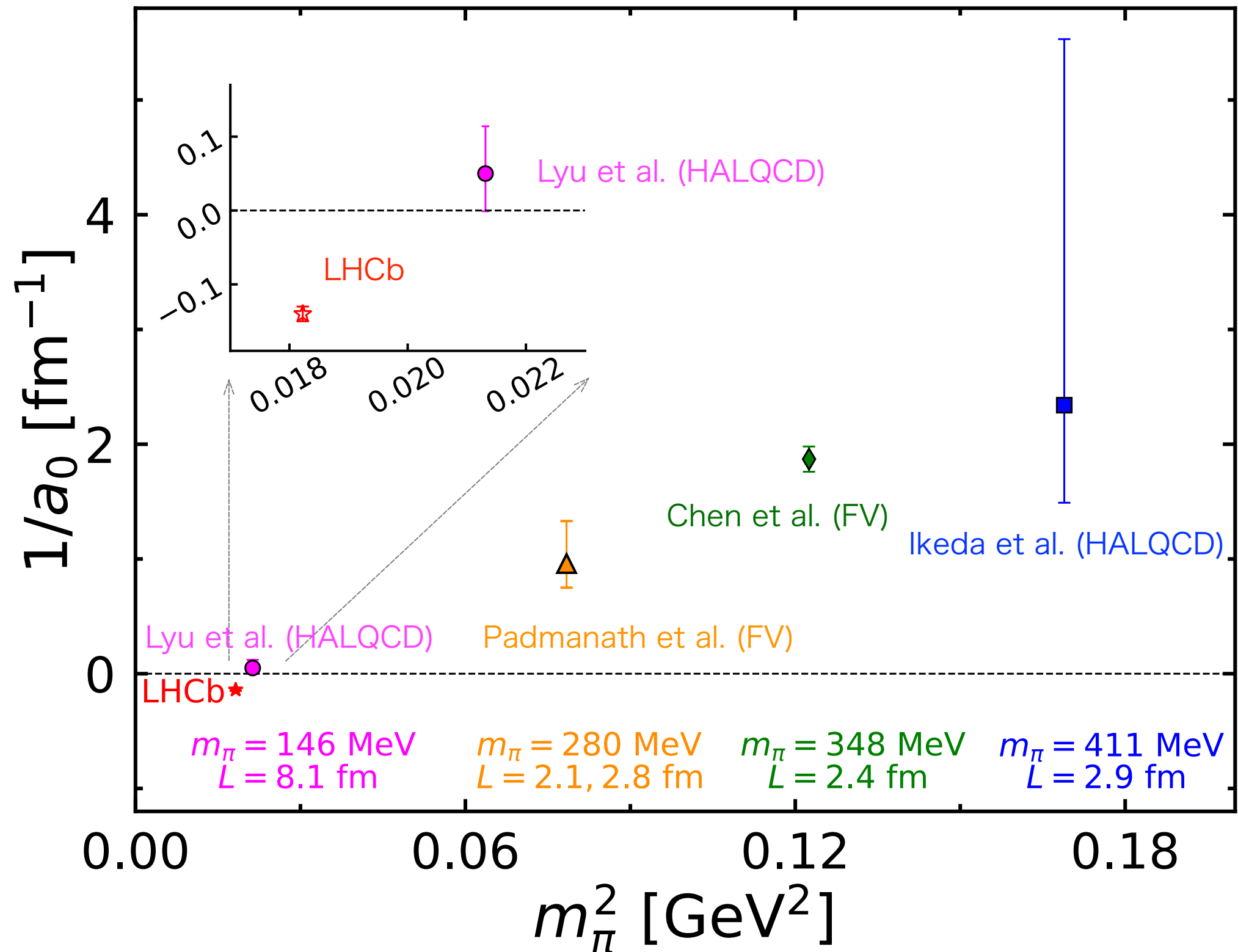
360 keV below $D^{*+}D^0$ threshold

$$(I, J^P) = (0, 1^+)$$

(Latest) Lattice QCD results

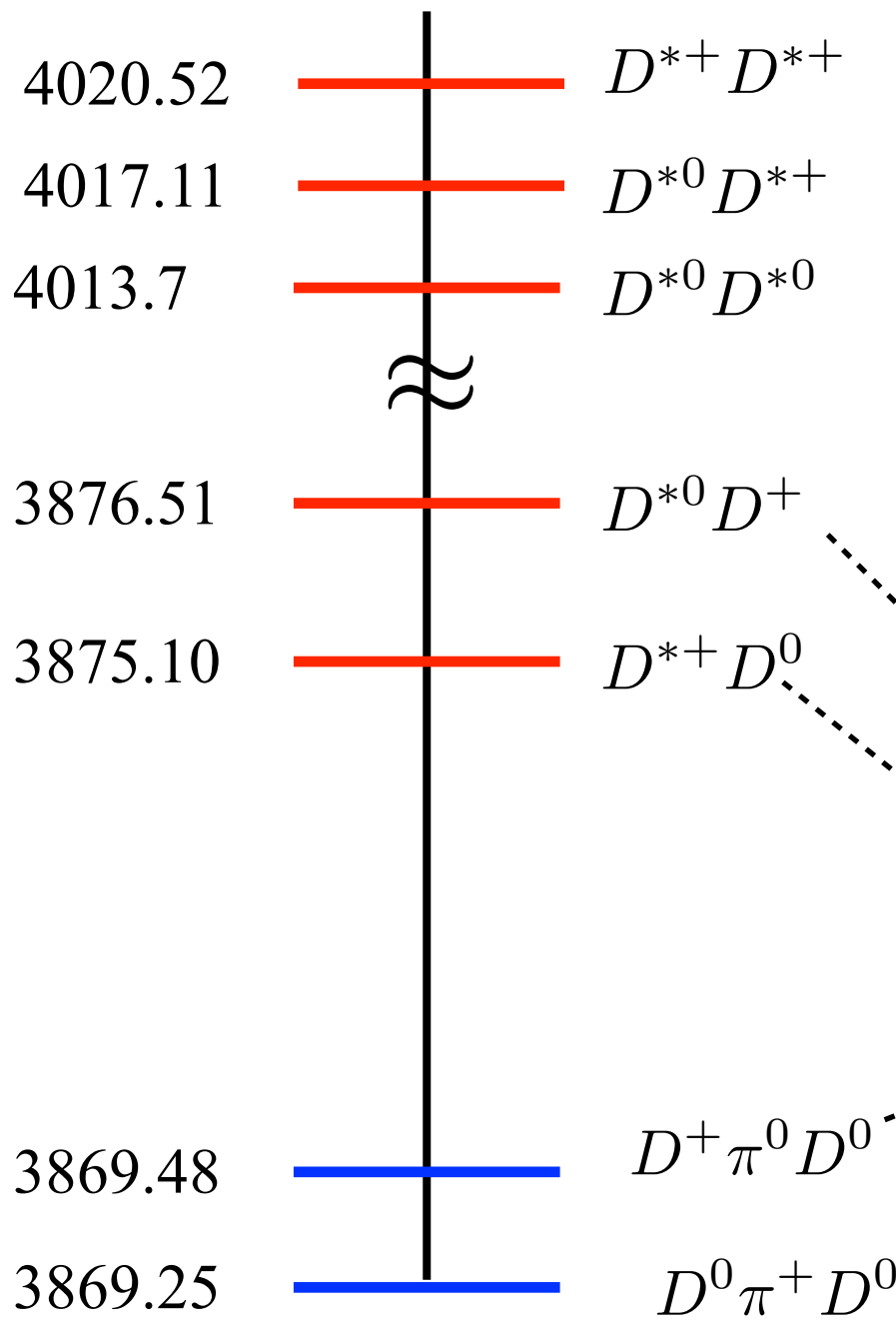
inverse scattering length

significant pion mass dependence



Nature

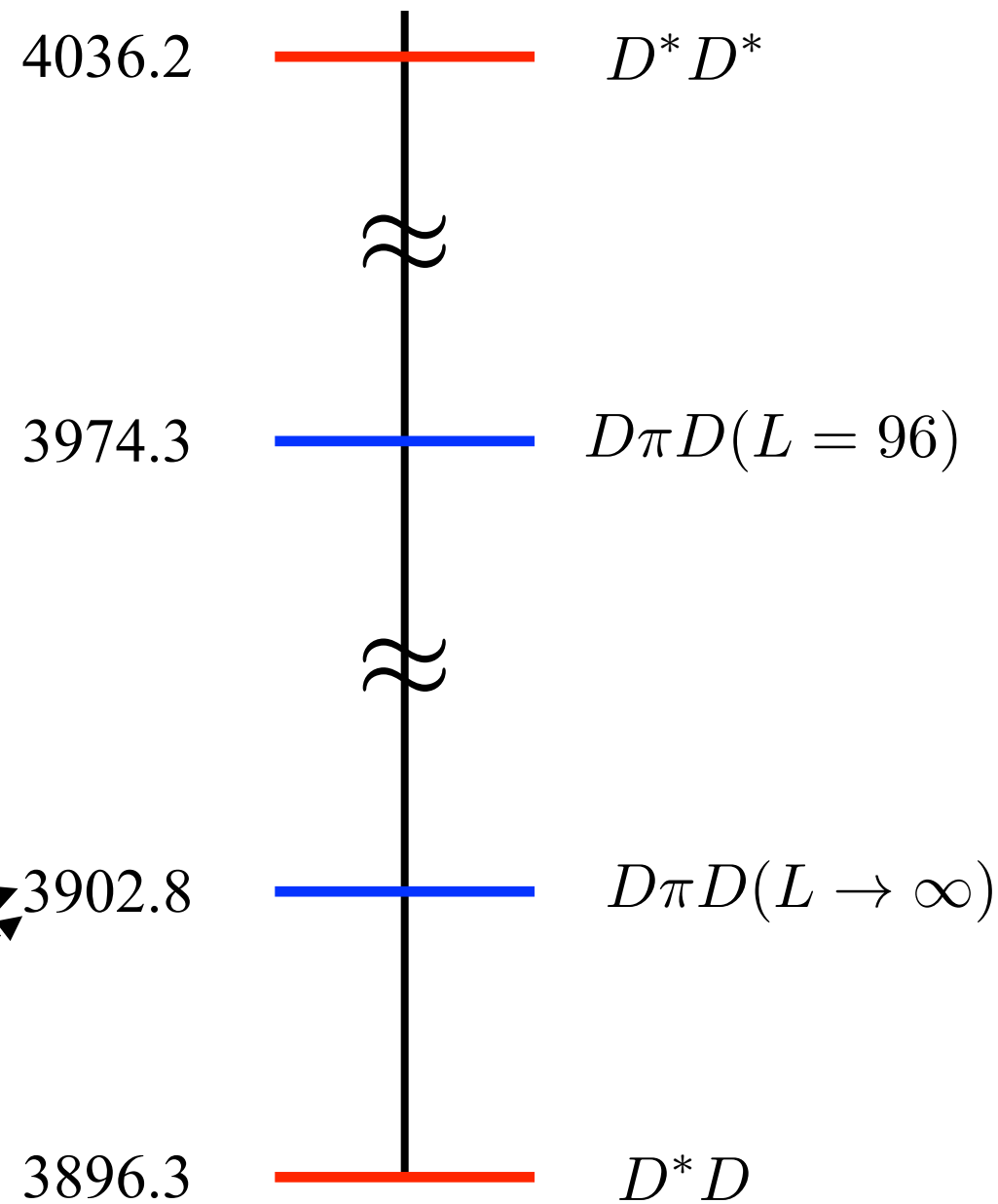
$\pi^0(134.98)$ $\pi^+(139.57)$
 $D^0(1864.84)$ $D^+(1869.66)$
 $D^{*0}(2006.85)$ $D^{*+}(2010.26)$



Lattice

Lyu et al.
(HALQCD)

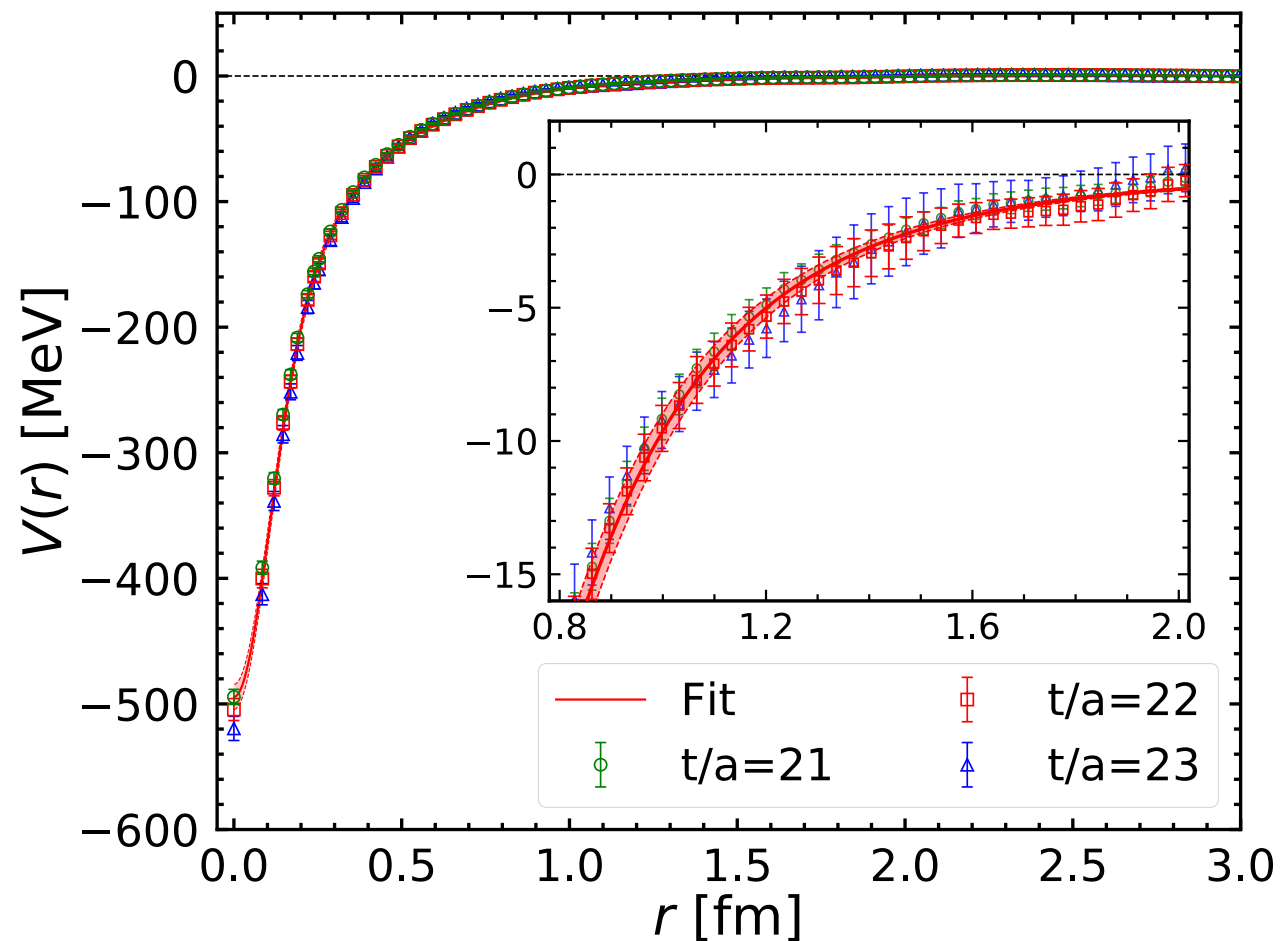
$\pi(146.4)$
 $D(1878.2)$
 $D^*(2018.1)$



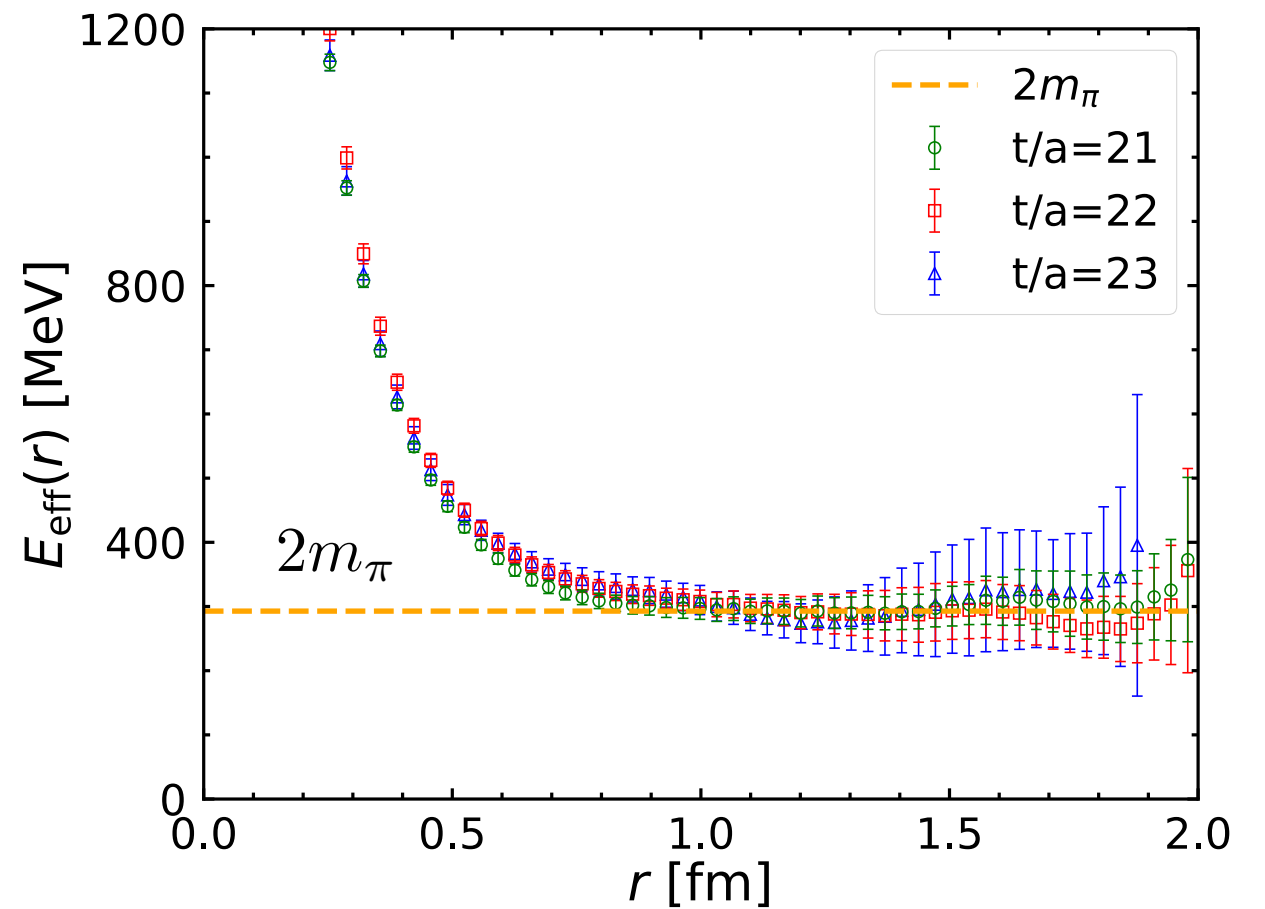
D^*D Potentials

$$m_\pi \simeq 146 \text{ MeV}$$

D^*D potential



Effective energy in space



attractive at all distances with long range tail

consistent with Yukawa² at large r

2-pion rather than 1-pion exchange dominates at long distance.

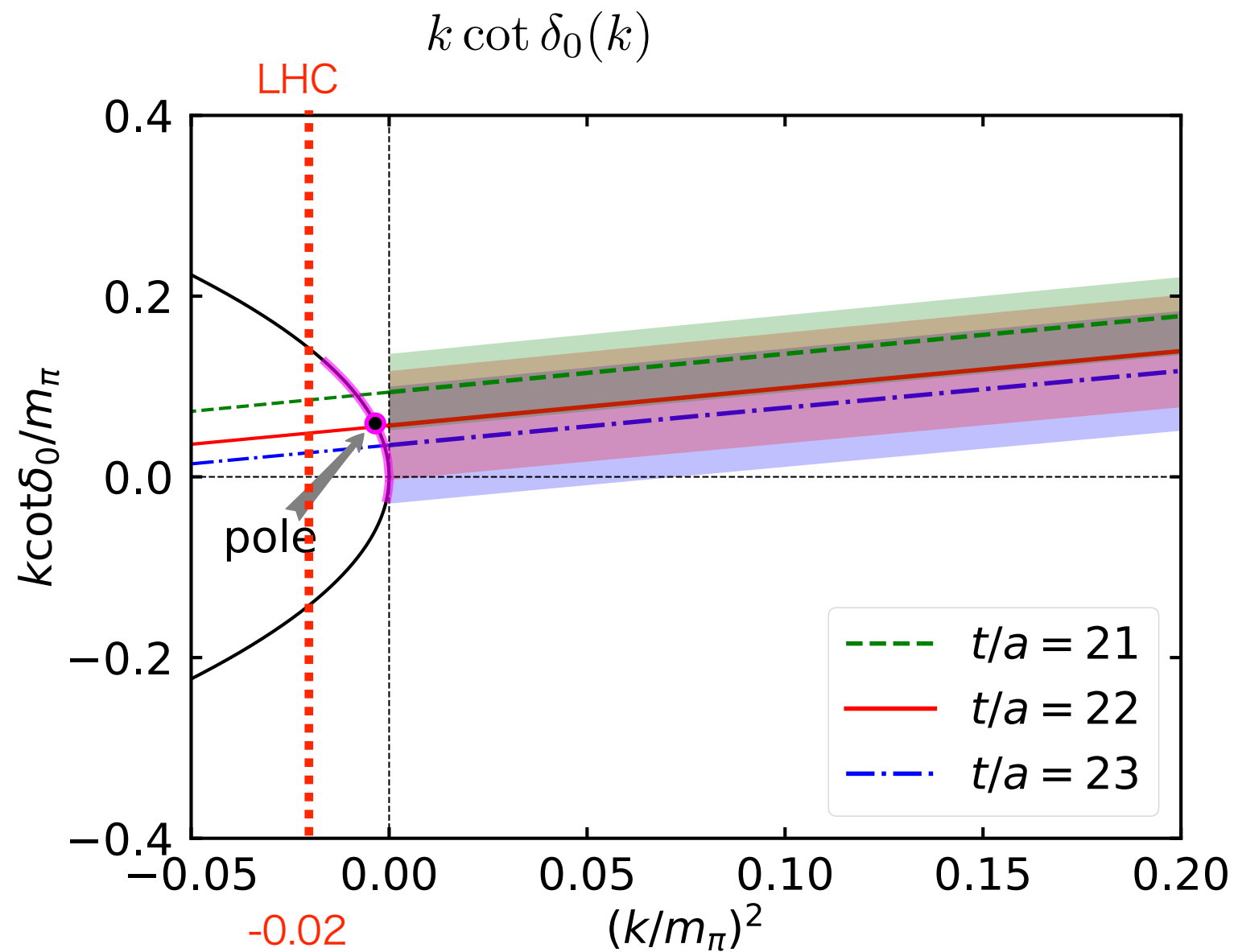
$$E_{\text{eff}}(r) = -\frac{\ln[-V(r)r^2/a_3]}{r}$$

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$$

Caution: 1-pion exchange is NOT a dominant contribution for T_{cc} .

Scattering phase shift

$$m_\pi \simeq 146 \text{ MeV}$$



$$\frac{1}{a_0} [\text{fm}^{-1}] = 0.05(5) \left({}^{+2}_{-2} \right)$$

one shallow “virtual” state

$$\kappa_{\text{pole}} = -8(8) \left({}^{+3}_{-5} \right) \text{ MeV}$$

$$E_{\text{pole}} = -59 \left({}^{+53}_{-99} \right) \left({}^{+2}_{-67} \right) \text{ keV}$$

The virtual pole appears above the left-hand cut (LHC) in this lattice setup.

Issue on the LHC [Aoki @ Lat2024](#)

See also [Du et al. PRL 131 \(2023\) 131903.](#)

[Meng et al. PRD 109 \(2024\) L071506.](#)

[Collins et al. PRD 109 \(2024\) 094509.](#)

“chiral” correction to the potential

2-Gauss + Yukawa² $V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$

$m_\pi : 146.4 \text{ MeV} \rightarrow 135 \text{ MeV}$

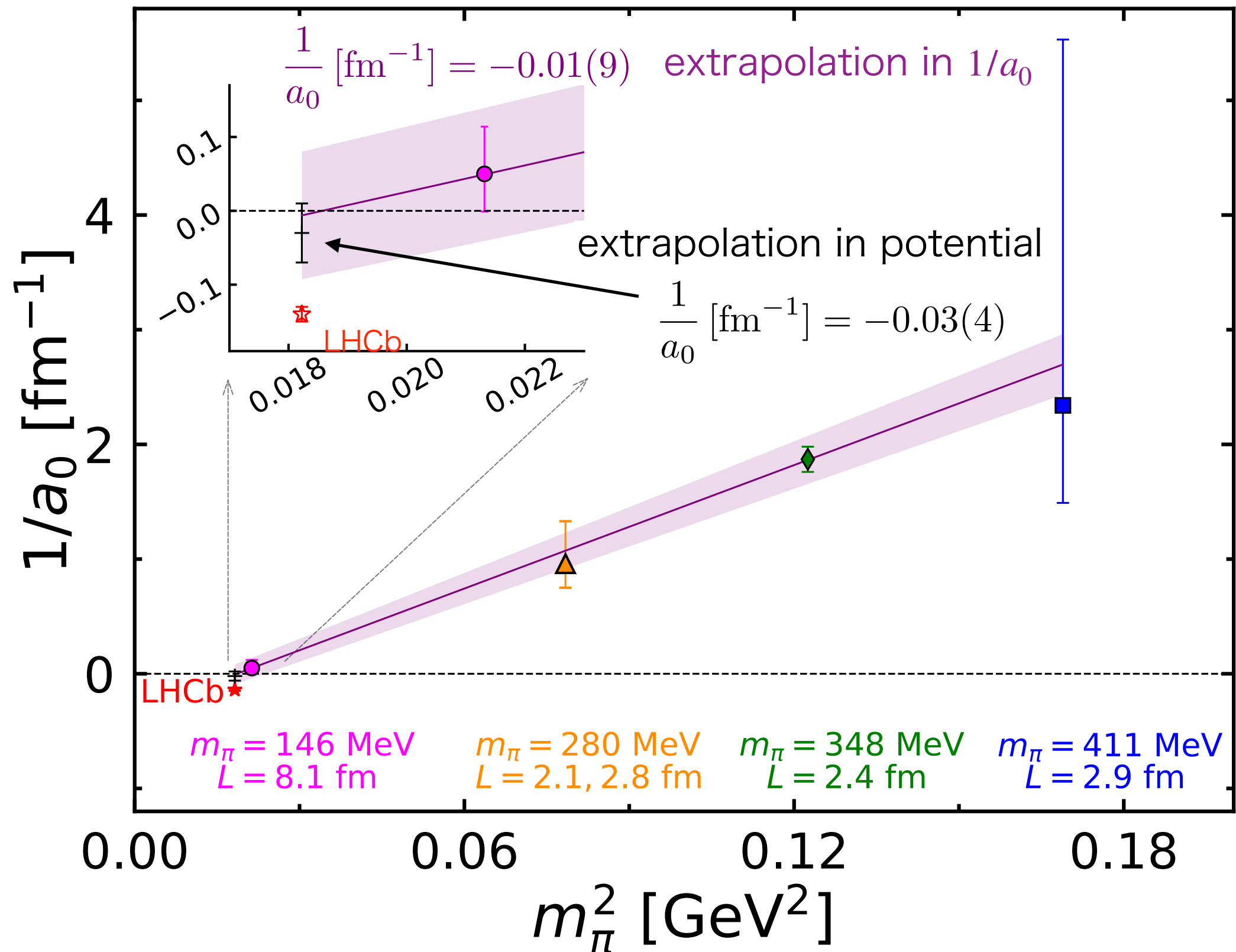
ERE $k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^4)$

m_π [MeV]	146.4	135.0
$1/a_0$ [fm ⁻¹]	$0.05(5) \begin{pmatrix} +2 \\ -2 \end{pmatrix}$	$-0.03(4)$
r_{eff} [fm]	$1.12(3) \begin{pmatrix} +3 \\ -8 \end{pmatrix}$	$1.12(3)$
κ_{pole} [MeV]	$-8(8) \begin{pmatrix} +3 \\ -5 \end{pmatrix}$	$+5(8)$
E_{pole} [keV]	$-59 \begin{pmatrix} +53 \\ -99 \end{pmatrix} \begin{pmatrix} +2 \\ -67 \end{pmatrix}$	$-45 \begin{pmatrix} +41 \\ -78 \end{pmatrix}$

one shallow “bound” state appears at “physical” pion mass

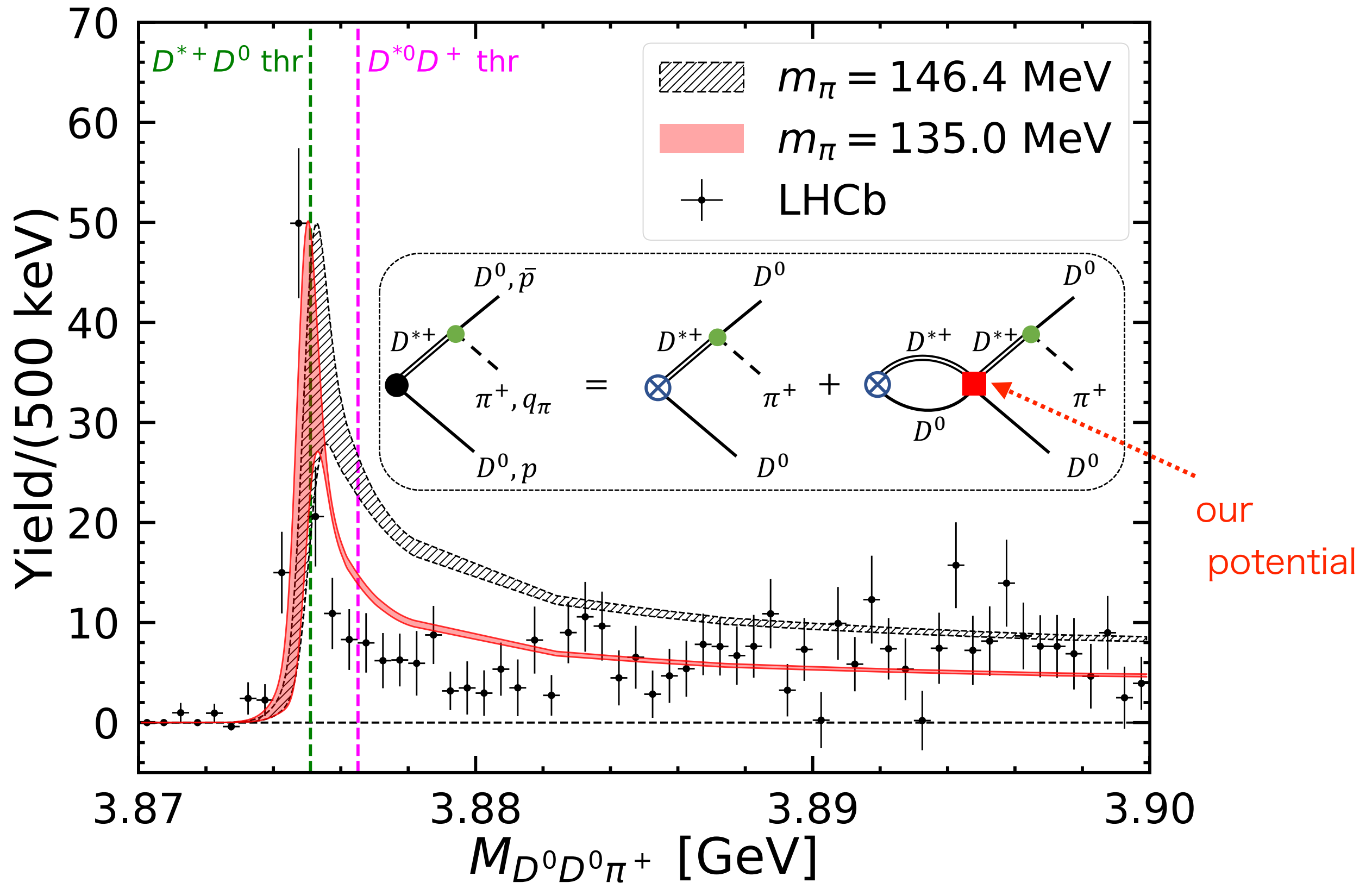
$$E_{\text{pole}} = -45 \begin{pmatrix} +41 \\ -78 \end{pmatrix} \text{ keV}$$

chiral extrapolation of $1/a_0$ linear in m_π^2



Two chiral “extrapolations” are consistent.

$D^0 D^0 \pi^+$ mass spectrum



Our potential at “physical” pion mass explains LHCb data better.

V. $N - \phi$ interaction and 2-pion tail

Y. Lyu et al.[HAL QCD], PRD 106 (2022) 074507.

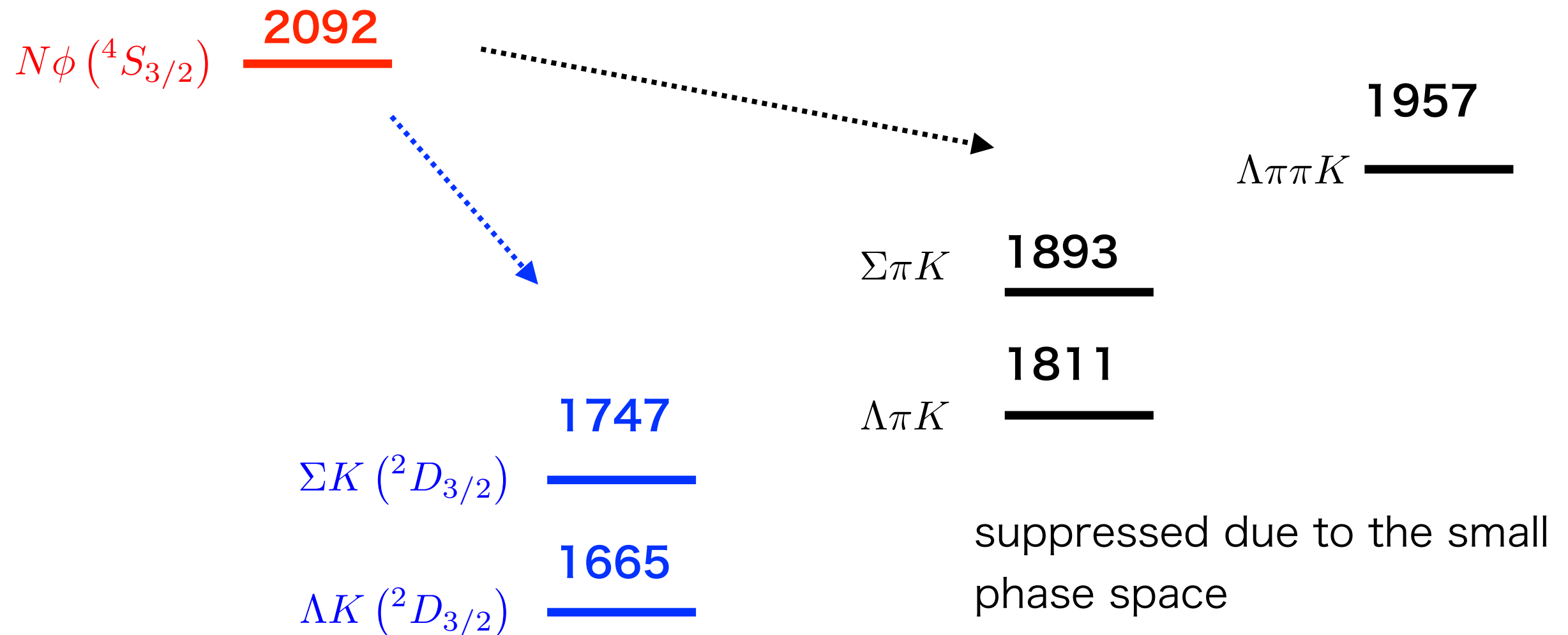
E. Chizzali et al., arXiv:2212.12690.

$N - \phi$ interaction

Our lattice setup

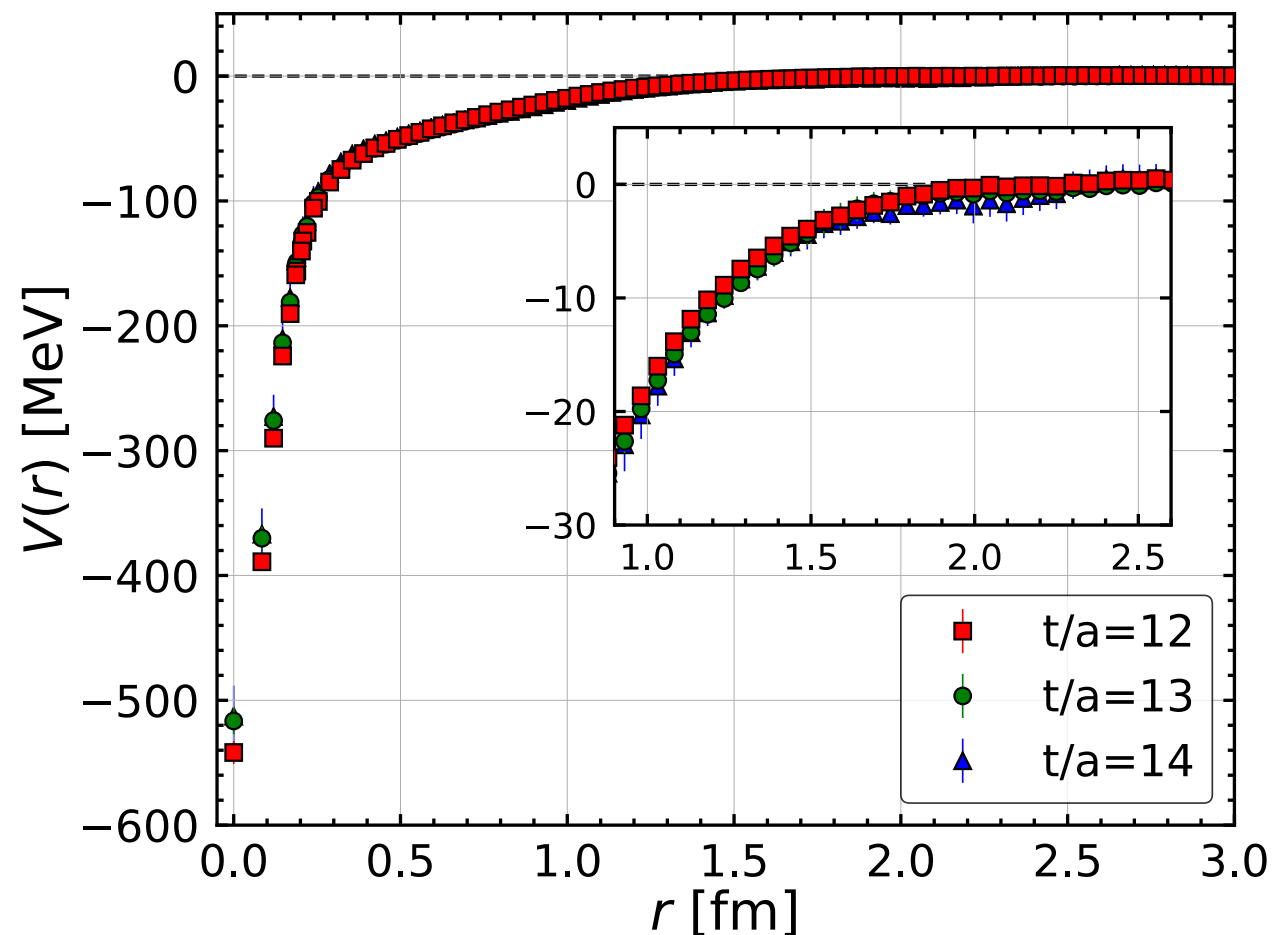
$$m_\pi \simeq 146 \text{ MeV}, \quad m_K \simeq 525 \text{ MeV}, \quad m_\phi \simeq 1048 \text{ MeV}$$

$$m_N \simeq 954 \text{ MeV}, \quad m_\Lambda \simeq 1140 \text{ MeV}, \quad m_\Sigma \simeq 1222 \text{ MeV}$$



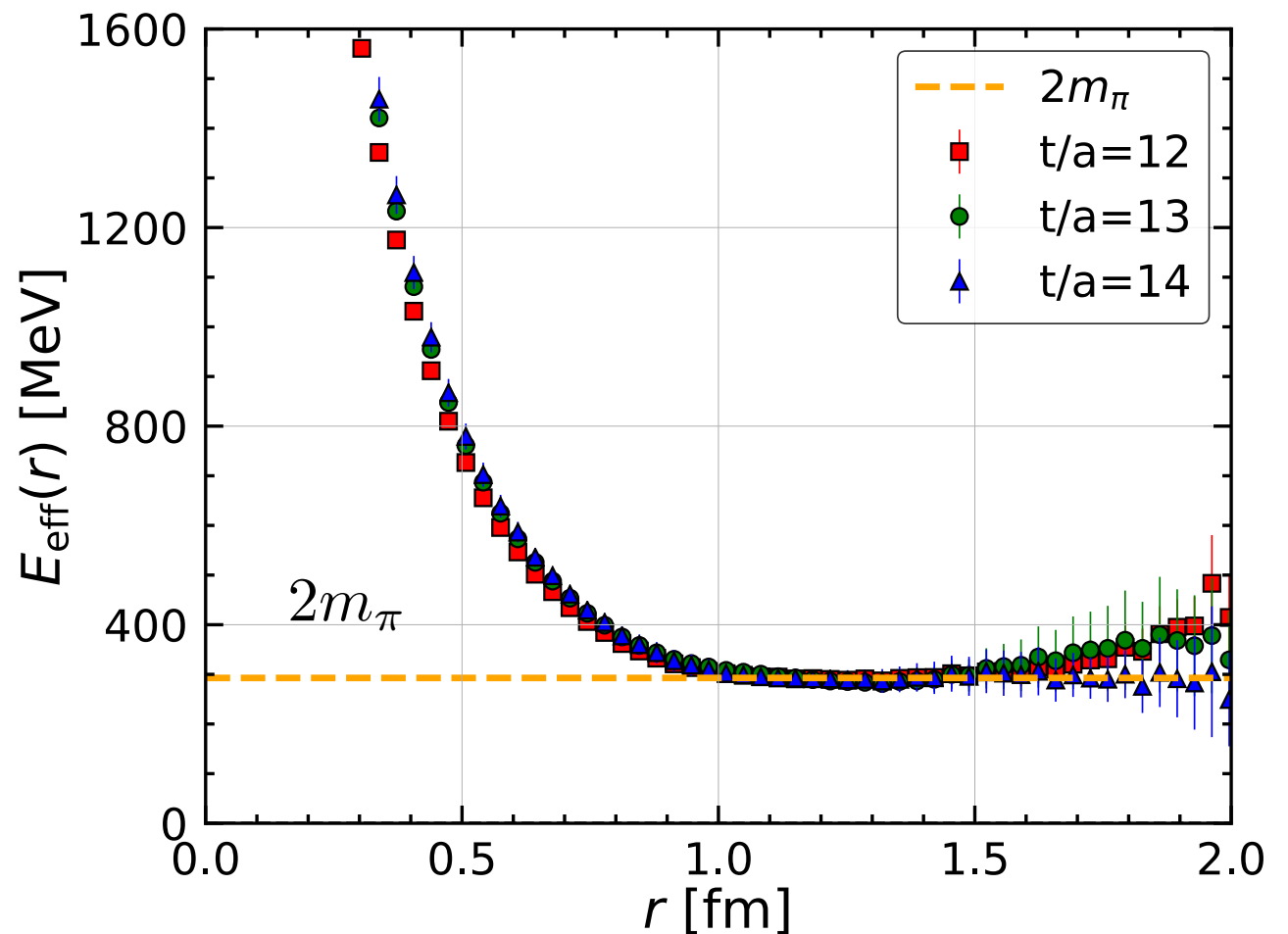
kinematically suppressed at low energy
due to D-wave nature

$N\phi$ Potentials



attractive at all distances

Effective energy in space



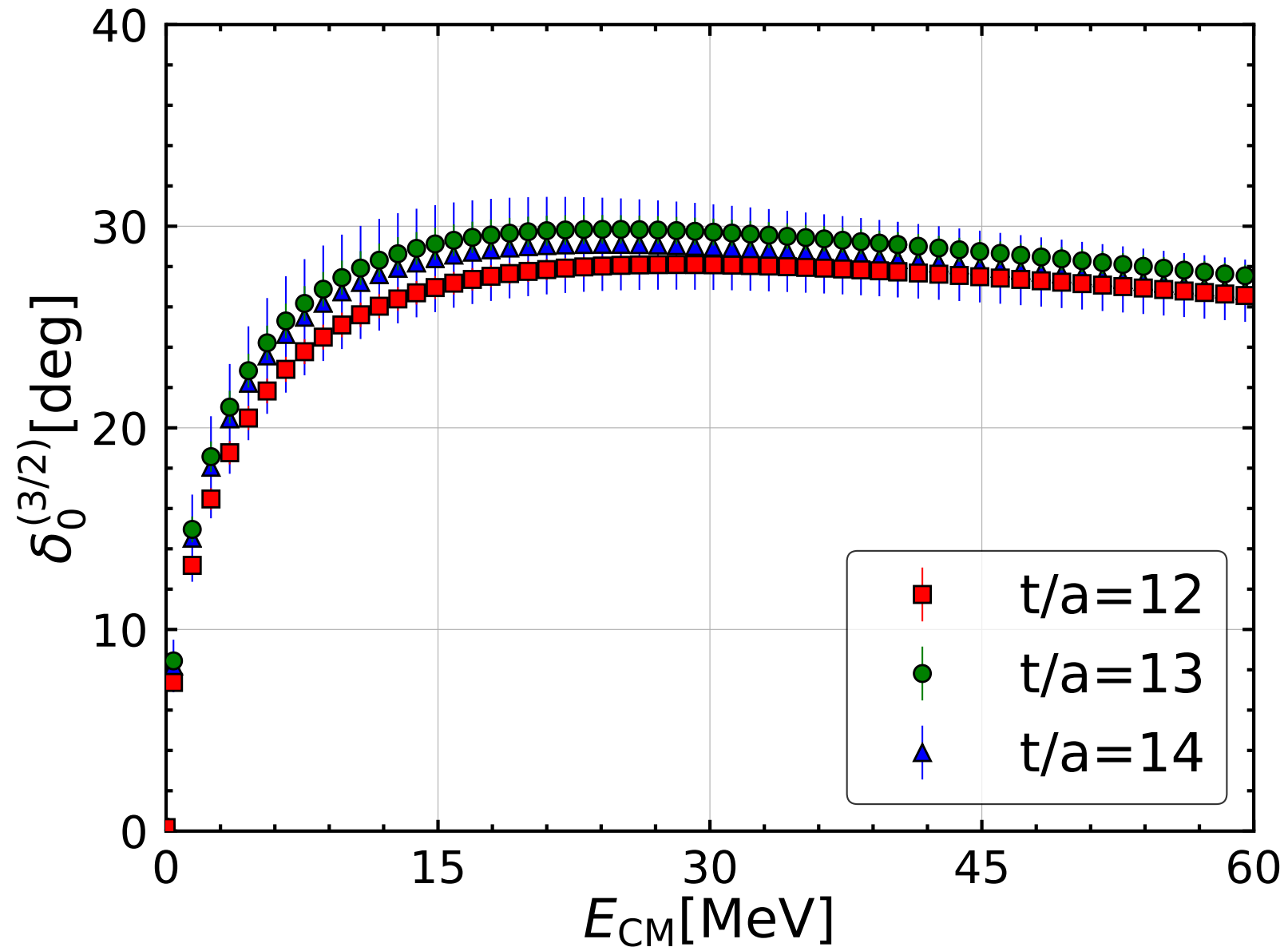
$$E_{\text{eff}}(r) = -\frac{\ln[-V(r)r^2/a_3]}{r}$$

consistent with **Yukawa**² at large r

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$$

TPE (two pion exchange) seems universal !?

Scattering phase shift



$$k \cot \delta_0^{(3/2)}(k) = \frac{1}{a_0^{(3/2)}} + \frac{1}{2} r_{\text{eff}}^{(3/2)} k^2 + O(k^4)$$

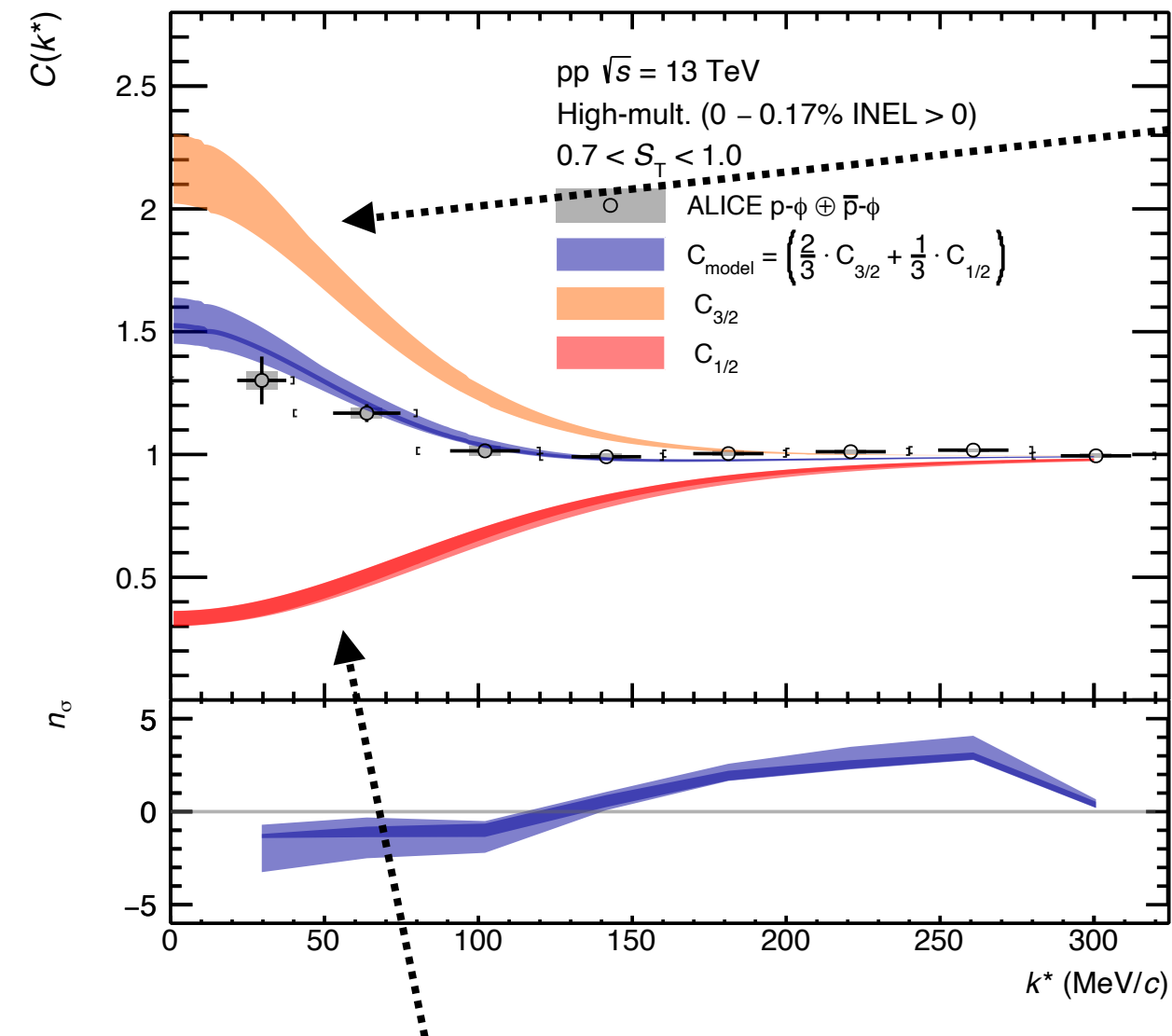
$$a_0^{(3/2)} = 1.43(23) \left({}^{+36}_{-06} \right) \text{ fm} \longrightarrow a_0^{(3/2)} \simeq 1.25(23) \text{ fm}$$

$$m_\pi = 146.4 \text{ MeV}$$

$$m_\pi = 138.0 \text{ MeV}$$

Lattice data + correlation@LHC

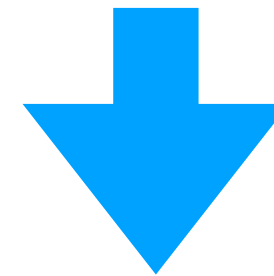
E. Chizzali et al., arXiv:2212.12690.



Lattice $N\phi(^4S_{3/2})$ potential

+

experimental data of $N\phi(^4S_{3/2})$ and $N\phi(^2S_{1/2})$



Potential for $N\phi(^2S_{1/2})$

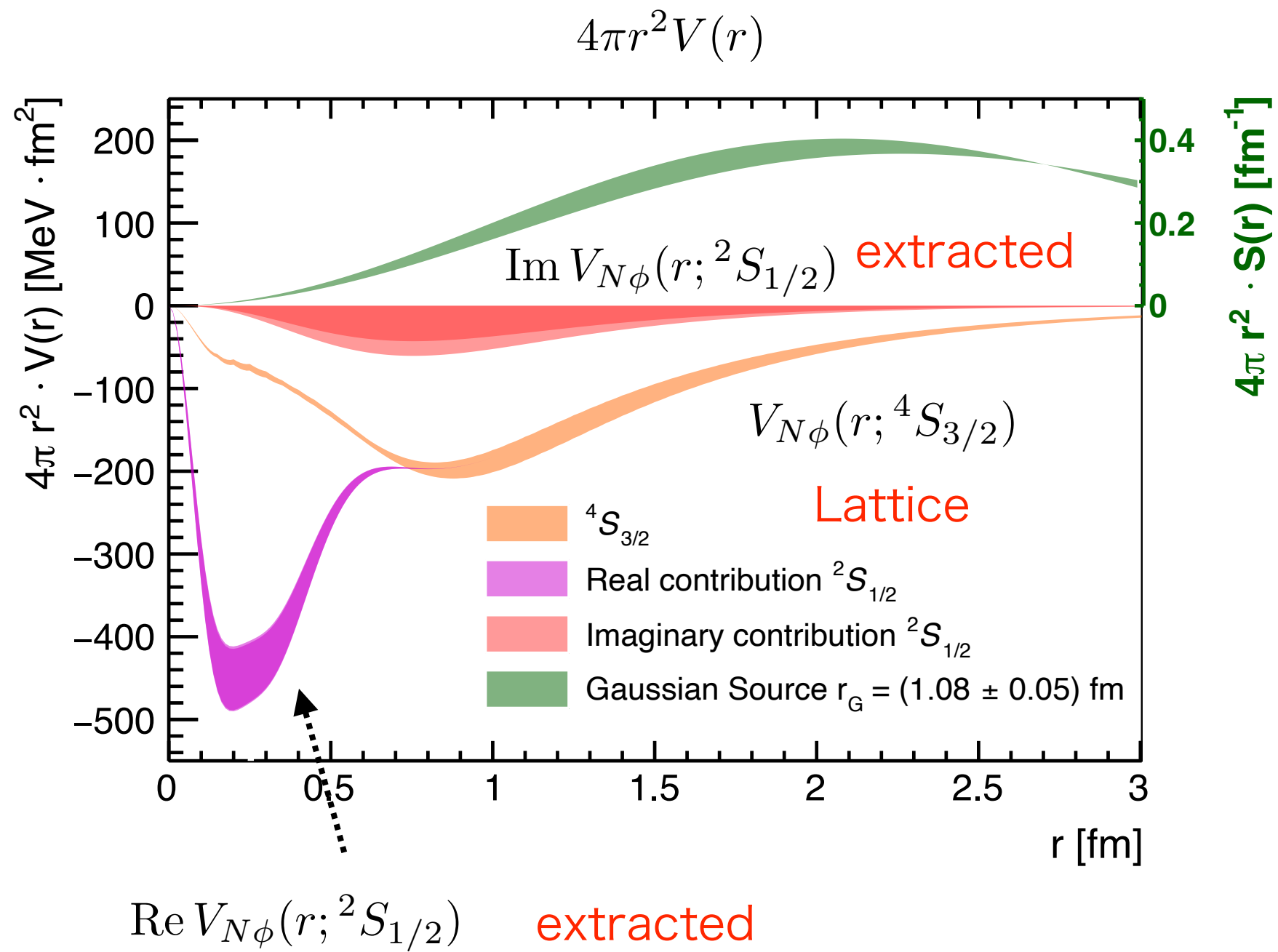
$$V_{N\phi}(r; ^2S_{1/2}) = \beta \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 (1 - e^{-(r/b_3)^2})^2 \left(\frac{e^{-m_\pi r}}{r} \right)^2$$

+

$$i\gamma (1 - e^{-(r/b_3)^2})^2 \frac{e^{-2m_K r}}{m_K r}$$

2 parameters β, γ

decay through 2K



The attraction at short distances in ${}^2S_{1/2}$ (from correlation) is stronger than in ${}^4S_{3/2}$ (lattice).

From the potential in $^2S_{1/2}$, one obtain

$$\text{Re } a_0^{(1/2)} = -1.47 \left({}^{+0.44}_{-0.37} \right) \left({}^{+0.14}_{-0.17} \right) \text{ fm} \longrightarrow \text{One } N\phi \text{ bound state}$$

$$\text{Im } a_0^{(1/2)} = 0.00 \left({}^{+0.26}_{-0.00} \right) \left({}^{+0.15}_{-0.00} \right) \text{ fm}$$

Estimated binding energy

$$E_B = 14.7 \sim 56.6 \text{ MeV}$$

A direct lattice calculation of the potential in $^2S_{1/2}$ is called for.

A coupled channel analysis among $N\phi$, ΣK and ΛK in $^2S_{1/2}$ is necessary.

VI. Summary

- No deeply bound deuteron/dineutron at heavy pions. A cross-check by the FV method and the HAL QCD method is crucial. The community could be mislead if the HAL QCD method were absent. (“Diversity” is important.)
- H-dibaryon seems a virtual state near the $N\Xi$ threshold in real-world.
- Lattice $N\Xi$ interactions are compared with LHC data.
- $N\Omega$ dibaryon is predicted at physical point and $p^+\Omega_{sss}^-$ interactions is compared with LHC data.
- Tetra quark T_{cc} and $N - \phi$ interaction are investigated. Both show a 2-pion exchange tail at long distance. Results are compared with experiments.
- (challenge) Local potential have problem for channels including quark annihilations. A use of separable potentials ?

Meng@hadron2023, Murakami@Lat2024

More results at physical point will be expected. Stay tuned.

Thank you for your attention !