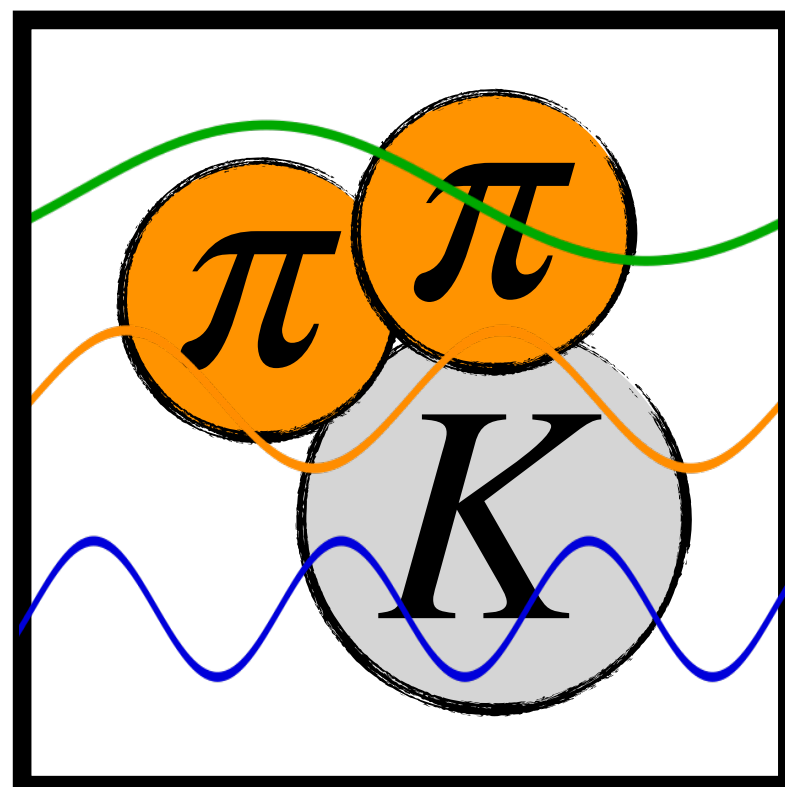


Three-meson scattering amplitudes with physical quark masses

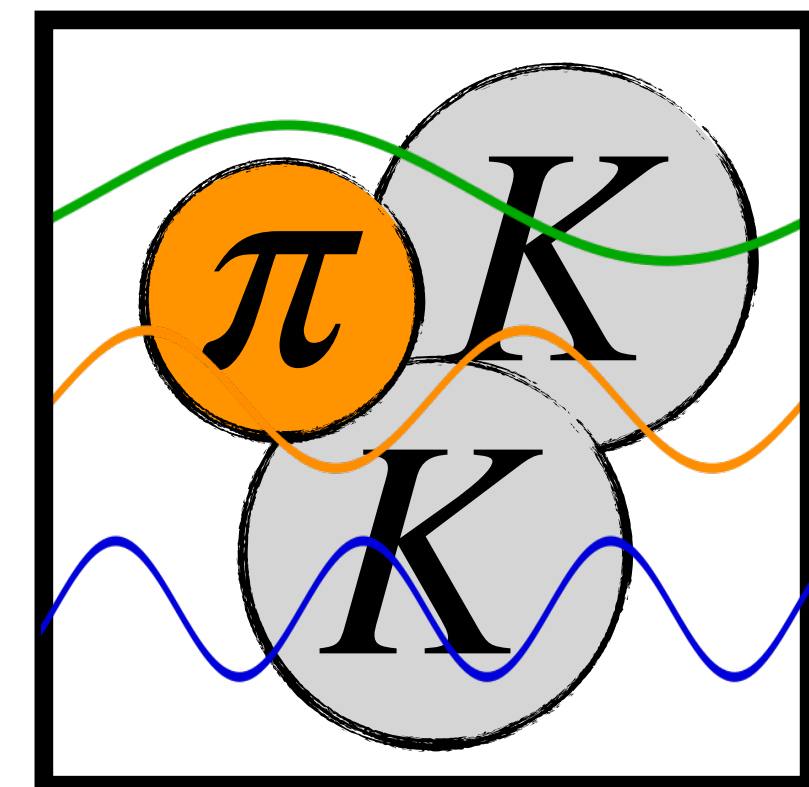
Fernando Romero-López

Uni Bern

fernando.romero-lopez@unibe.ch



u^b



Work in progress with



Zack Draper (UW)



Sebastian Dawid (UW)



Andrew Hanlon (CMU)



Colin Morningstar (CMU)



Ben Hörz (Intel)



Sarah Skinner (CMU)



Steve Sharpe (UW)

The three-hadron frontier

Many reasons to study the three-hadron problem from lattice QCD

- Resonances that decay to three (or more) hadrons
 - ▶ Exotics: $T_{cc} \rightarrow DD^*, DD\pi$
 - ▶ 3π resonances: $\omega(782)$, $h_1(1170)$, $a_1(1260) \dots$
 - ▶ Roper: $N(1440) \rightarrow \Delta\pi \rightarrow N\pi\pi$
- Interest in three-baryon forces: NNN , NNY
- Electroweak processes $K \rightarrow 3\pi$, $K^0 \leftrightarrow 3\pi \leftrightarrow \bar{K}^0$, $\gamma \rightarrow 3\pi$
- ☑ Major developments in the three-particle finite-volume formalism
 - [Hansen, Sharpe, PRD 2014 & 2015], [Hammer, Pang, Rusetsky, JHEP 2017] x 2
 - [Mai, Döring, EPJA 2017] [...]
- ☑ See other related talks at this conference:
 - [M. Hansen (plenary, Tuesday)]
 - [F. Pittler (parallel, Monday), M. Sjö, A. Rusetsky, L. Meng, S. Dawid (parallel, Tuesday)]

Three-meson systems

Important benchmark system: three pseudoscalar mesons at maximal isospin

- Implement formalism and explore its features
- Test fitting strategies to extract three-body K matrix
- Interpret results in combination with EFTs
- Investigate features of scattering amplitudes

$$3\pi^+, \quad 3K^+, \quad \pi^+\pi^+K^+, \quad K^+K^+\pi^+$$

[Blanton ... [FRL](#) ... et al., PRL 2020 & JHEP 2021]

[Draper ... [FRL](#) ... et al., JHEP 2023],

[Fischer ... [FRL](#) ... et al (ETMC), EPJC 2021]

[Alexandrou et al, Brett et al, Culver et al, Mai et al. (GWQCD)]

[Hansen et al (HadSpec)]

Three-meson systems

Important benchmark system: three pseudoscalar mesons at maximal isospin

- Implement formalism and explore its features
- Test fitting strategies to extract three-body K matrix
- Interpret results in combination with EFTs
- Investigate features of scattering amplitudes

$$3\pi^+, \quad 3K^+, \quad \pi^+\pi^+K^+, \quad K^+K^+\pi^+$$

[Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
[Draper ... FRL ... et al., JHEP 2023],
[Fischer ... FRL ... et al (ETMC), EPJC 2021]
[Alexandrou et al, Brett et al, Culver et al, Mai et al. (GWQCD)]
[Hansen et al (HadSpec)]

This work:

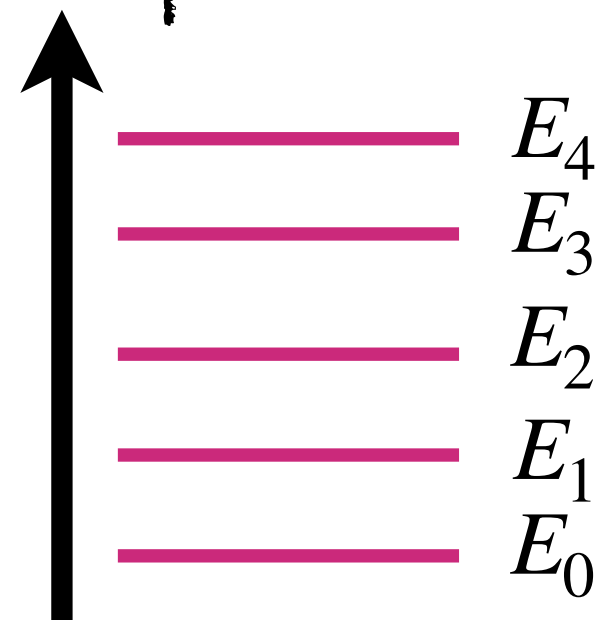
- Extend previous calculations to the physical point
[Blanton ... FRL ... et al., PRL 2020 & JHEP 2021]
[Draper ... FRL ... et al., JHEP 2023]
- Compute physical three-meson scattering amplitudes
[Hansen et al (HadSpec)]

	$(L/a)^3 \times (T/a)$	$M_\pi[\text{MeV}]$	$M_K[\text{MeV}]$	N_{cfg}
N203	$48^3 \times 128$	340	440	771
N200	$48^3 \times 128$	280	460	1712
D200	$64^3 \times 128$	200	480	2000
E250	$96^3 \times 192$	130	500	505

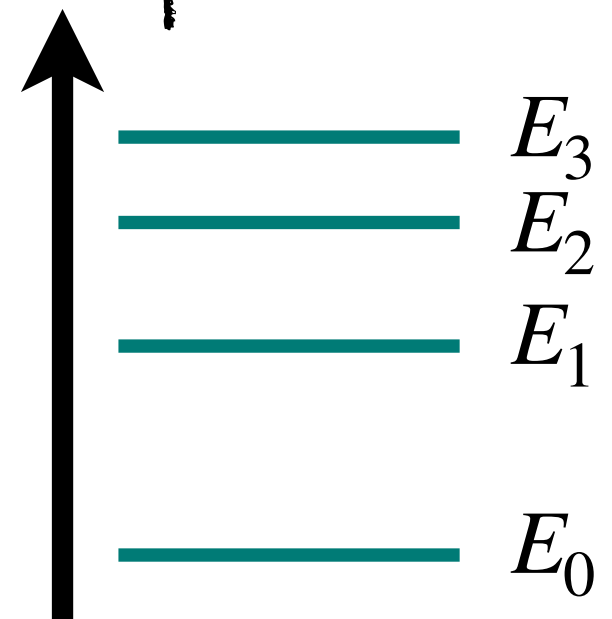
$$a \simeq 0.063 \text{ fm}$$
$$\text{tr } m_q = 2m_{ud} + m_s \simeq \text{const}$$

Formalism

two-meson
spectrum

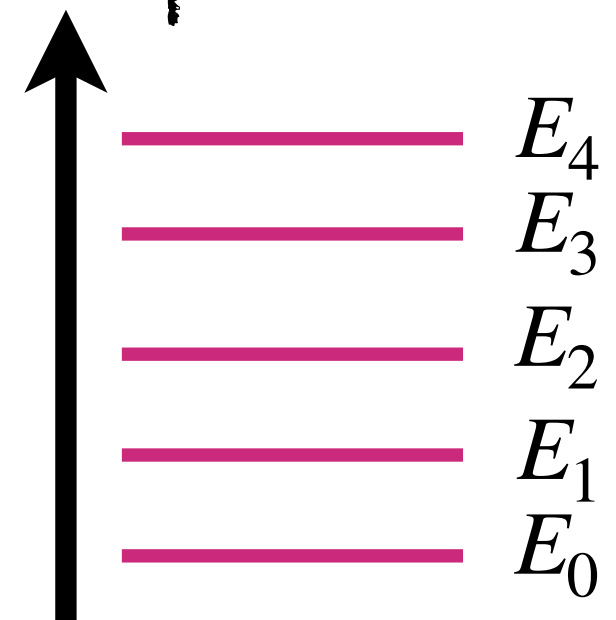


Three-meson
spectrum



Formalism

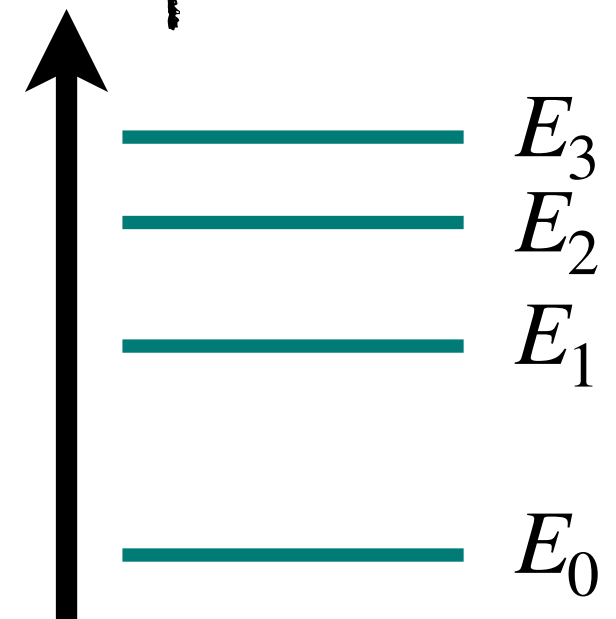
two-meson
spectrum



Quantization
conditions

$$\det_{lm} [\mathcal{K}_2 + F_2^{-1}] = 0$$

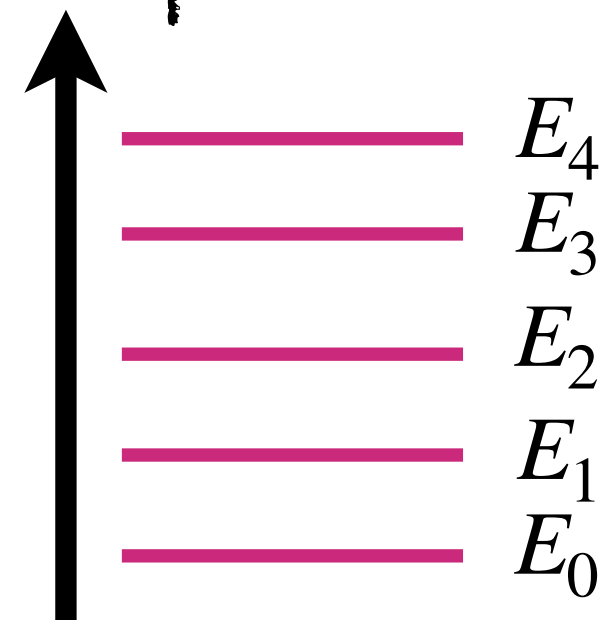
Three-meson
spectrum



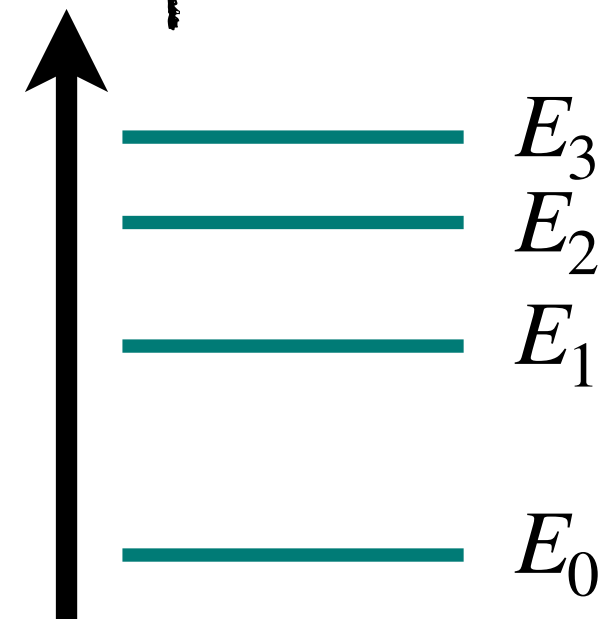
$$\det_{klm} [\mathcal{K}_{\text{df},3} + F_3^{-1}] = 0$$

Formalism

two-meson
spectrum



Three-meson
spectrum



Quantization
conditions

$$\det_{lm} [\mathcal{K}_2 + F_2^{-1}] = 0$$

$$\det_{klm} [\mathcal{K}_{\text{df},3} + F_3^{-1}] = 0$$

Fit

K-matrices

$\mathcal{K}_2$

$\mathcal{K}_{\text{df},3}$

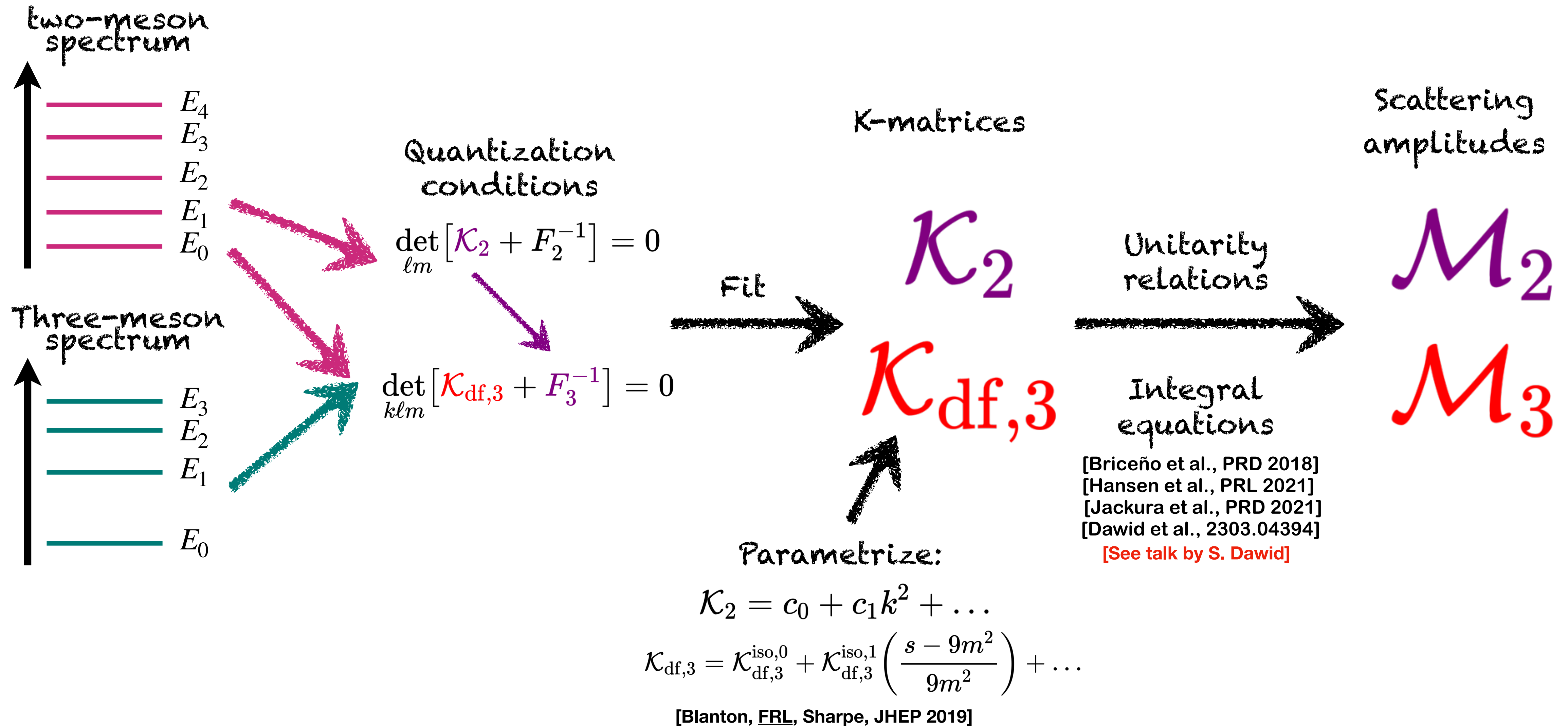
Parametrize:

$$\mathcal{K}_2 = c_0 + c_1 k^2 + \dots$$

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_{\text{df},3}^{\text{iso},0} + \mathcal{K}_{\text{df},3}^{\text{iso},1} \left(\frac{s - 9m^2}{9m^2} \right) + \dots$$

[Blanton, FRL, Sharpe, JHEP 2019]

Formalism



Extracting energies

- Stochastic LapH method, multi-hadron operators

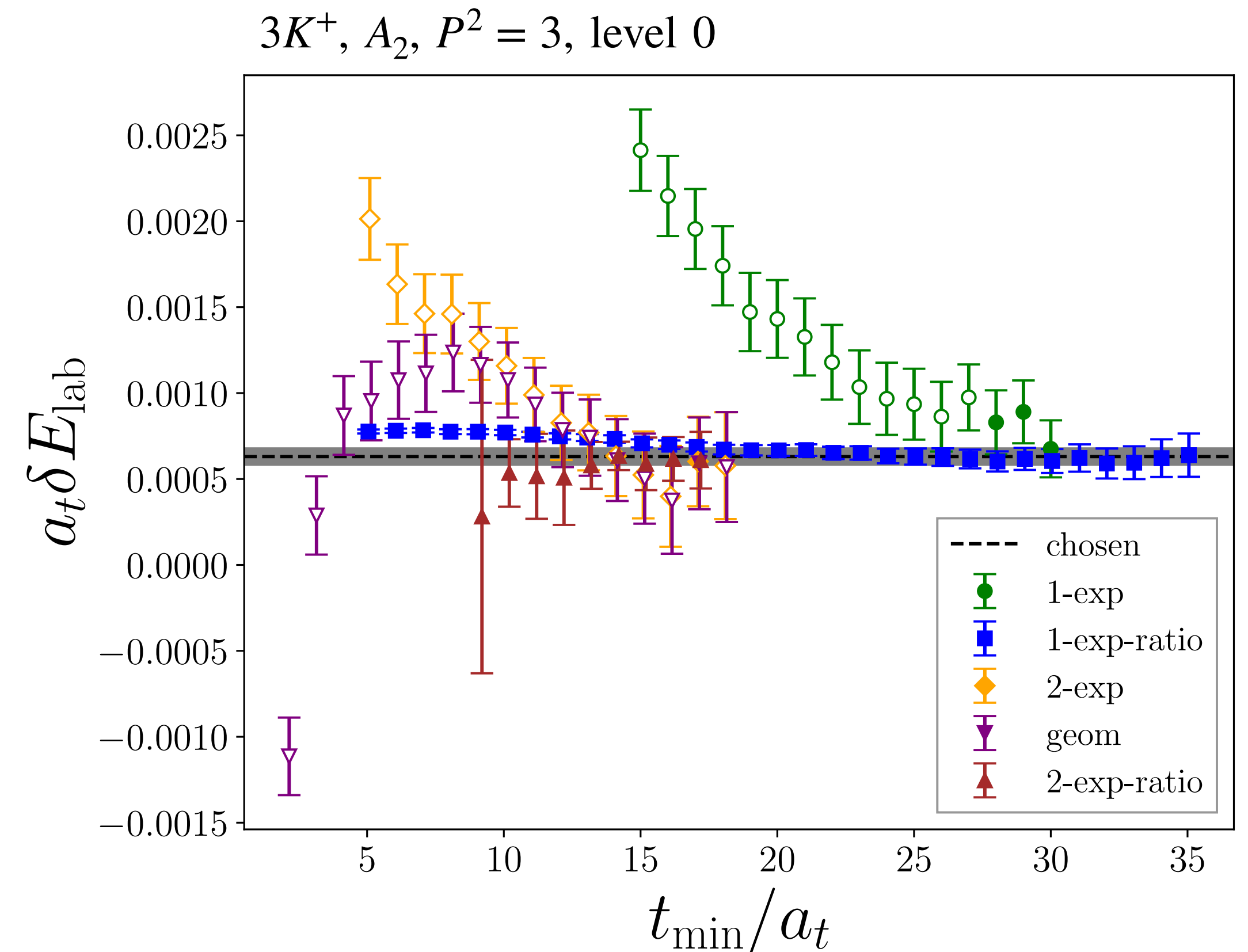
[Morningstar et al, 1104.3870]

- GEVP and look for consistency between methods.

- ▶ Single and double exponential
- ▶ Ratio fit with single and double exponential

- Use ratio fit to benefit from correlated cancellations

$$R_n(t) = \frac{C_{\text{three-meson}}(t)}{C_{\text{meson}}(t)C_{\text{meson}}(t)C_{\text{meson}}(t)}$$



Fitting the spectrum

- Requires a correlated fit for several systems at one. For instance: $\pi\pi K + \pi\pi + \pi K$

- Fit energy shifts in the lab frame (“spectrum method”)

$$\chi^2(\vec{p}) = \sum_{ij} \left(\Delta E_{\text{lab},i} - \Delta E_{\text{lab},i}^{\text{QC}}(\vec{p}) \right) (C^{-1})_{ij} \left(\Delta E_{\text{lab},j} - \Delta E_{\text{lab},j}^{\text{QC}}(\vec{p}) \right)$$

parameters in
K-matrices

covariance
matrix of lab-shifts

“predicted minus measured”
lab-frame energy shifts

- Include several two-meson partial waves:
 - ◆ s and d waves for $\pi\pi$ and KK
 - ◆ s and p waves for πK

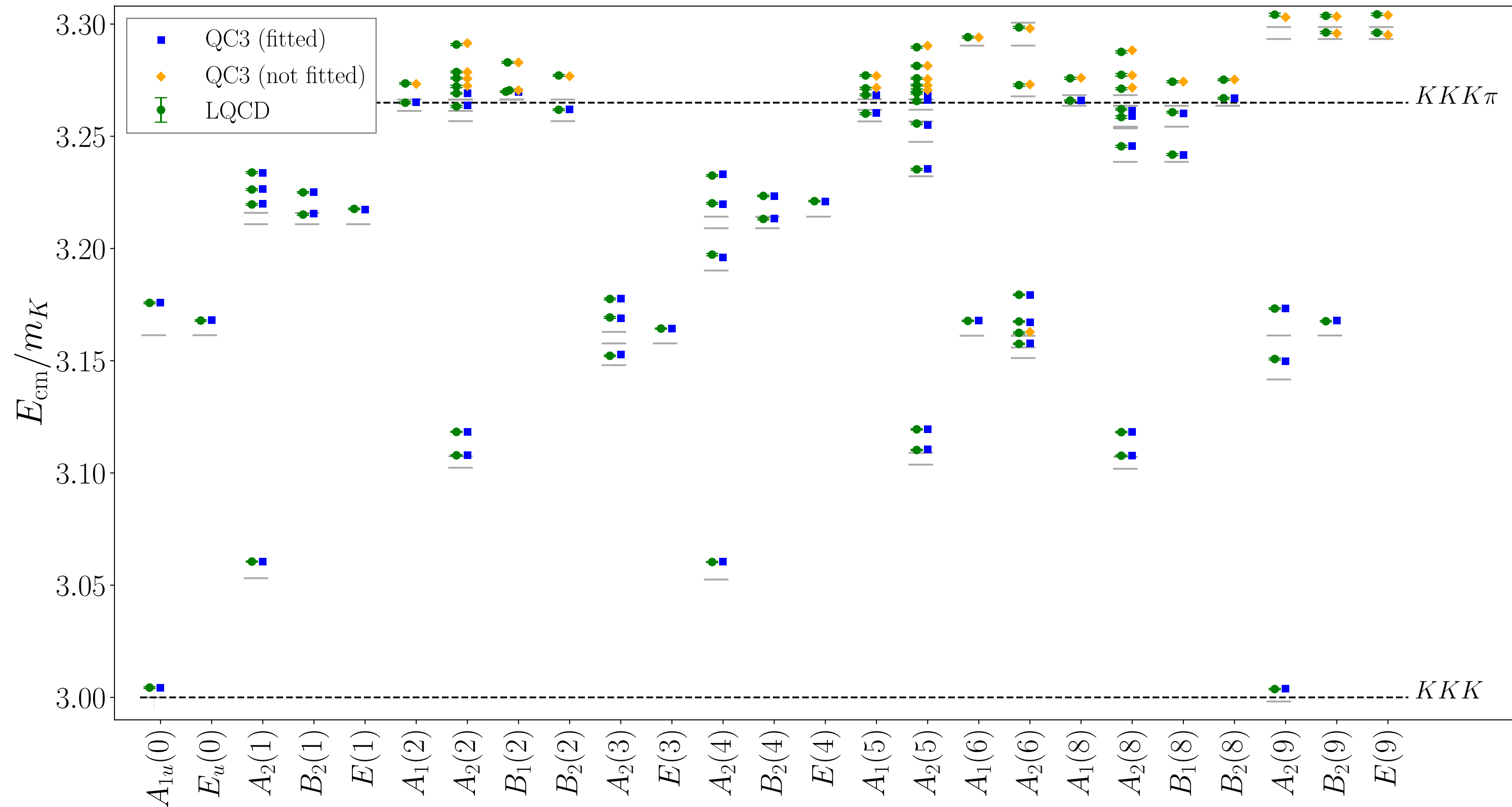
- Threshold expansion in three-body K matrix

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \Delta + \mathcal{K}_2 \Delta^2 + \mathcal{K}_A \Delta_A + \mathcal{K}_B \Delta_B,$$

$$\Delta \equiv \frac{s - 9m^2}{9m^2}$$

Functions of Mandelstam
variables

3K spectrum



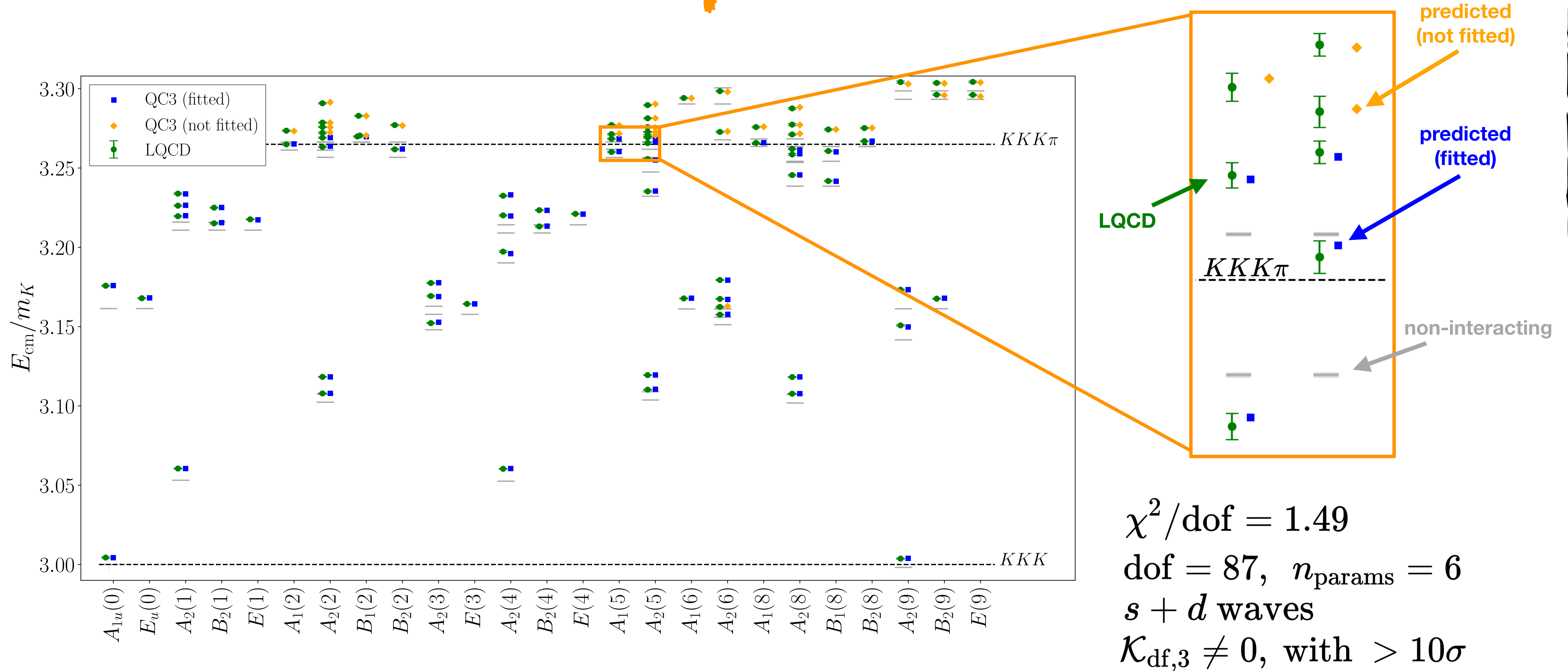
$$\chi^2/\text{dof} = 1.49$$

$$\text{dof} = 87, \quad n_{\text{params}} = 6$$

$s + d$ waves

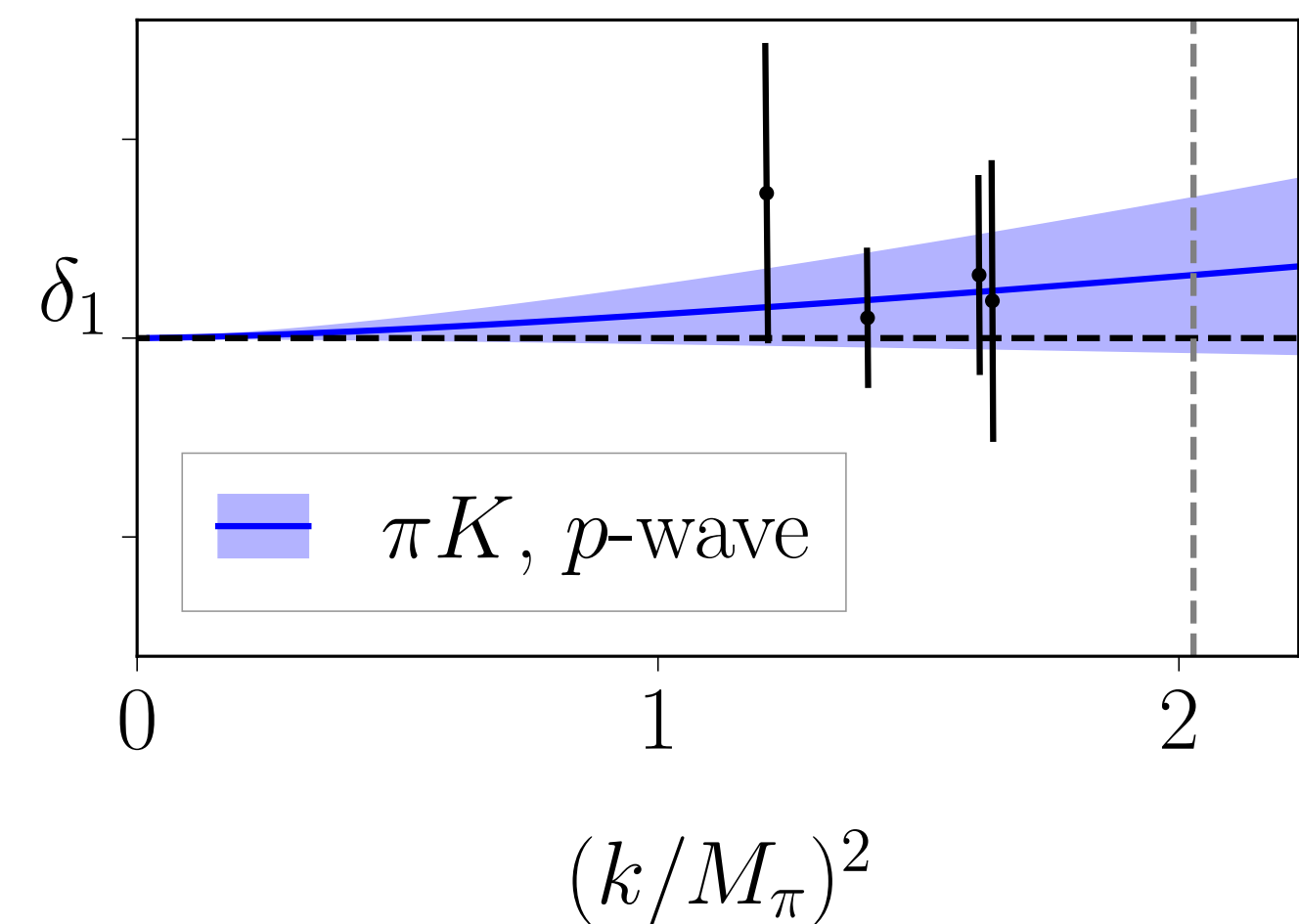
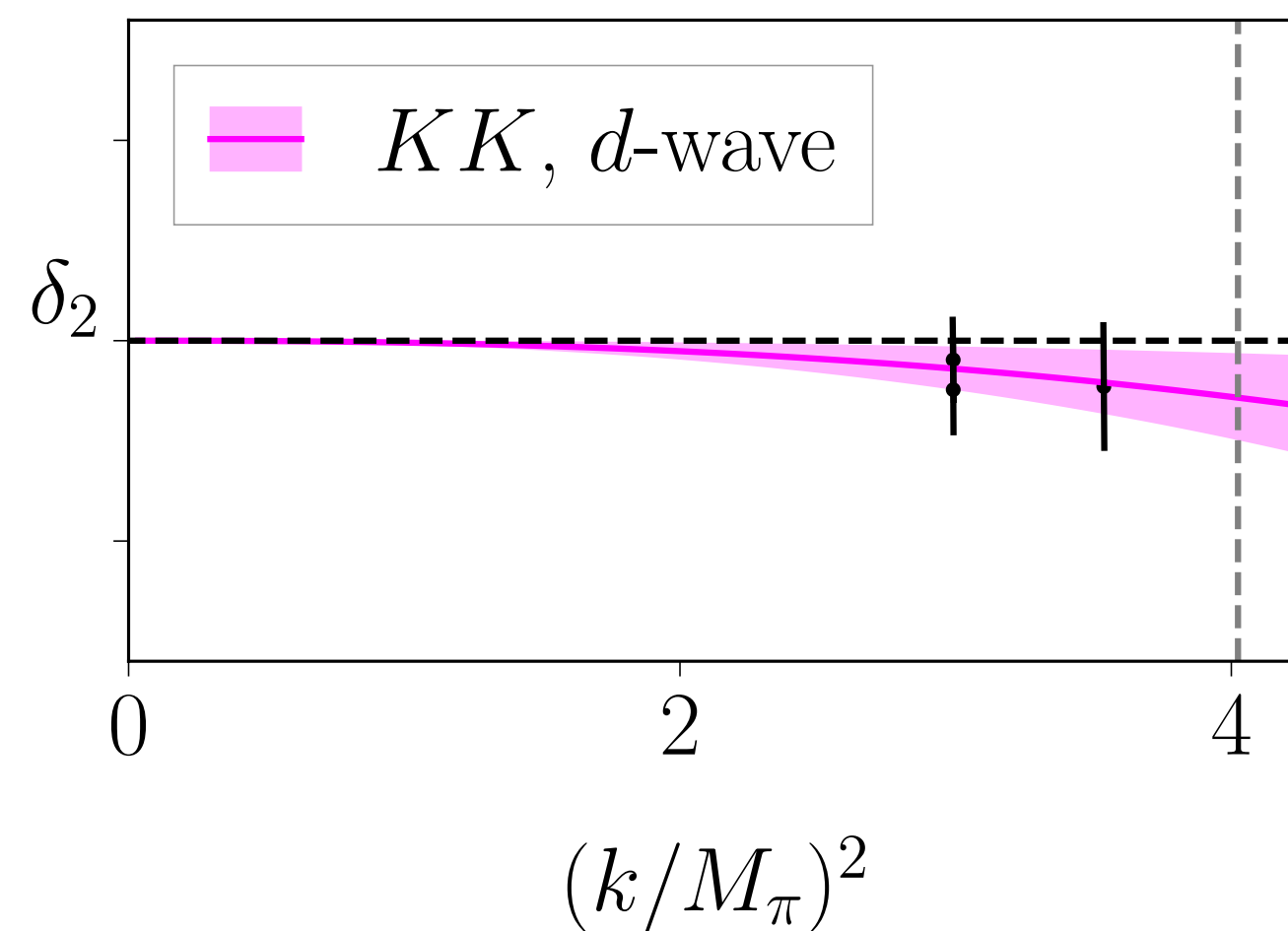
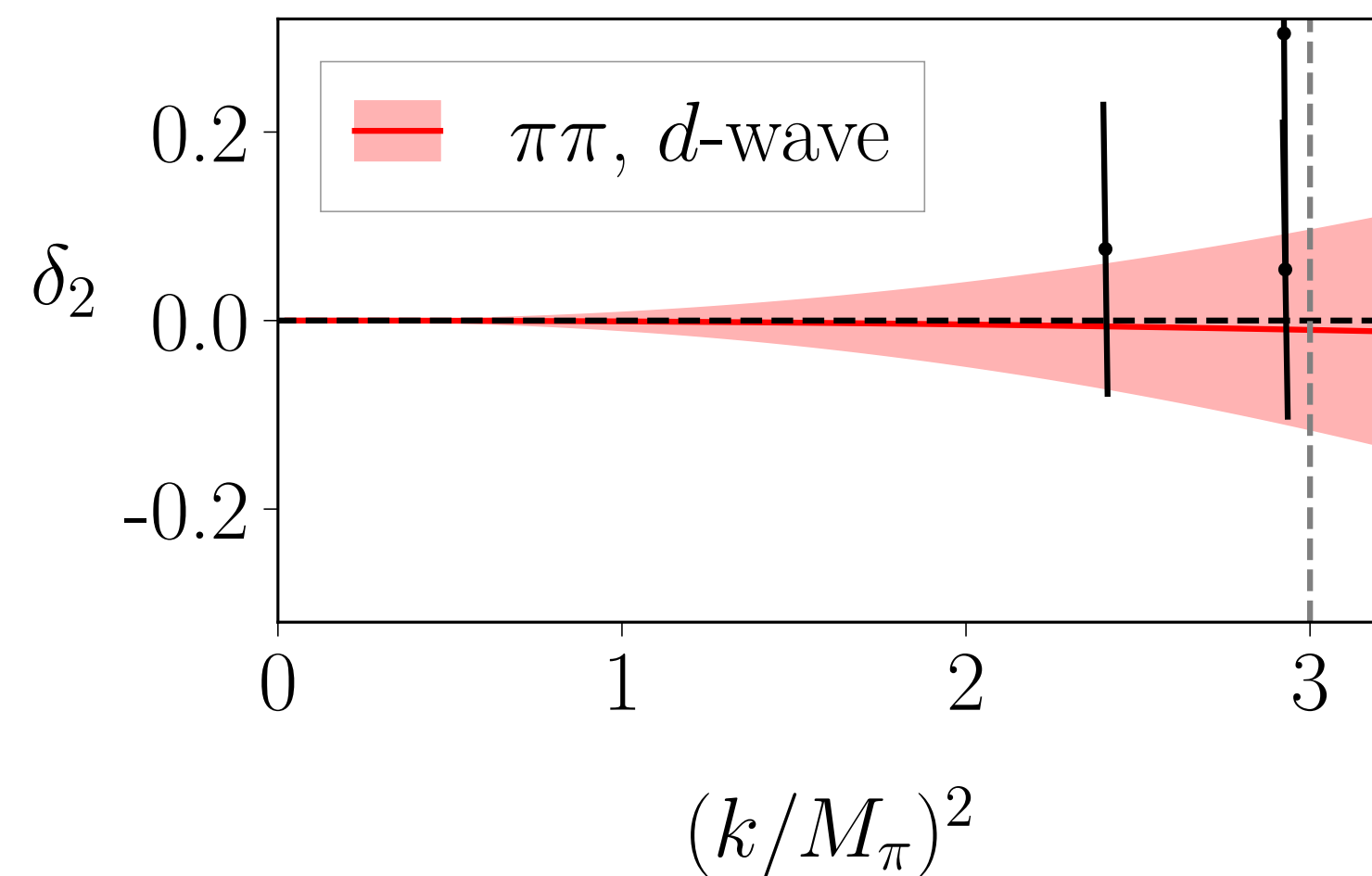
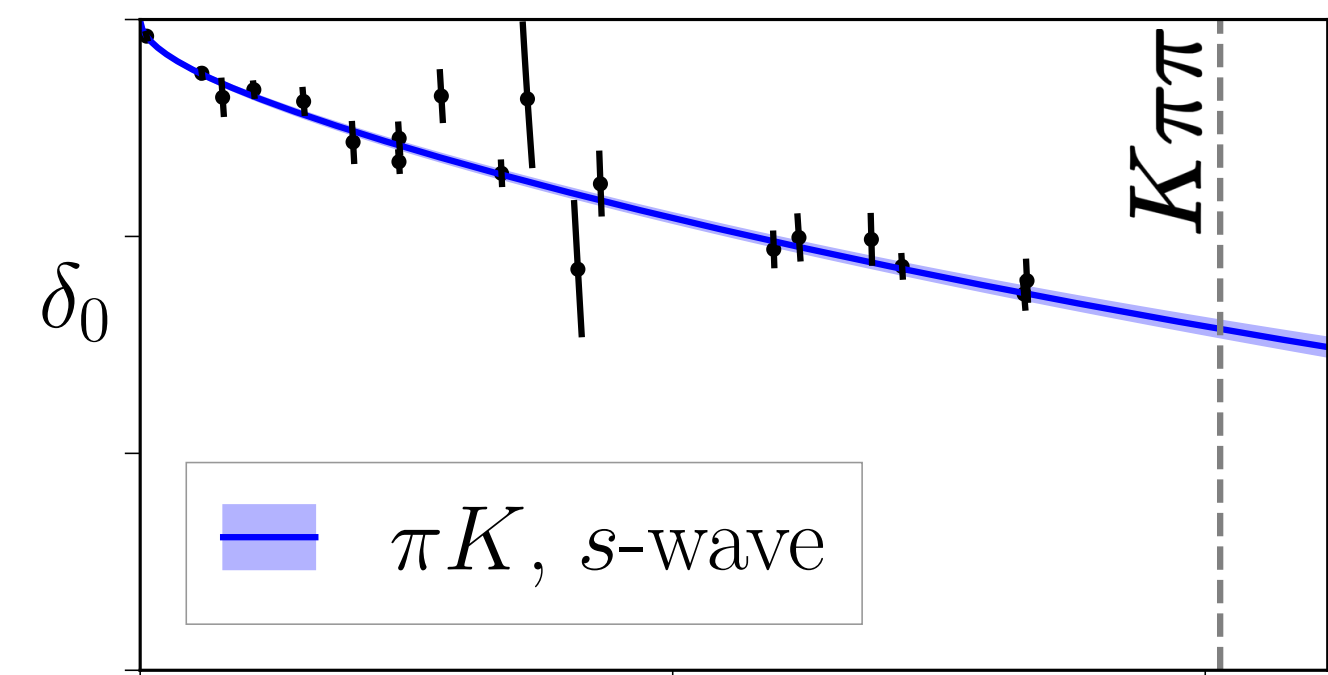
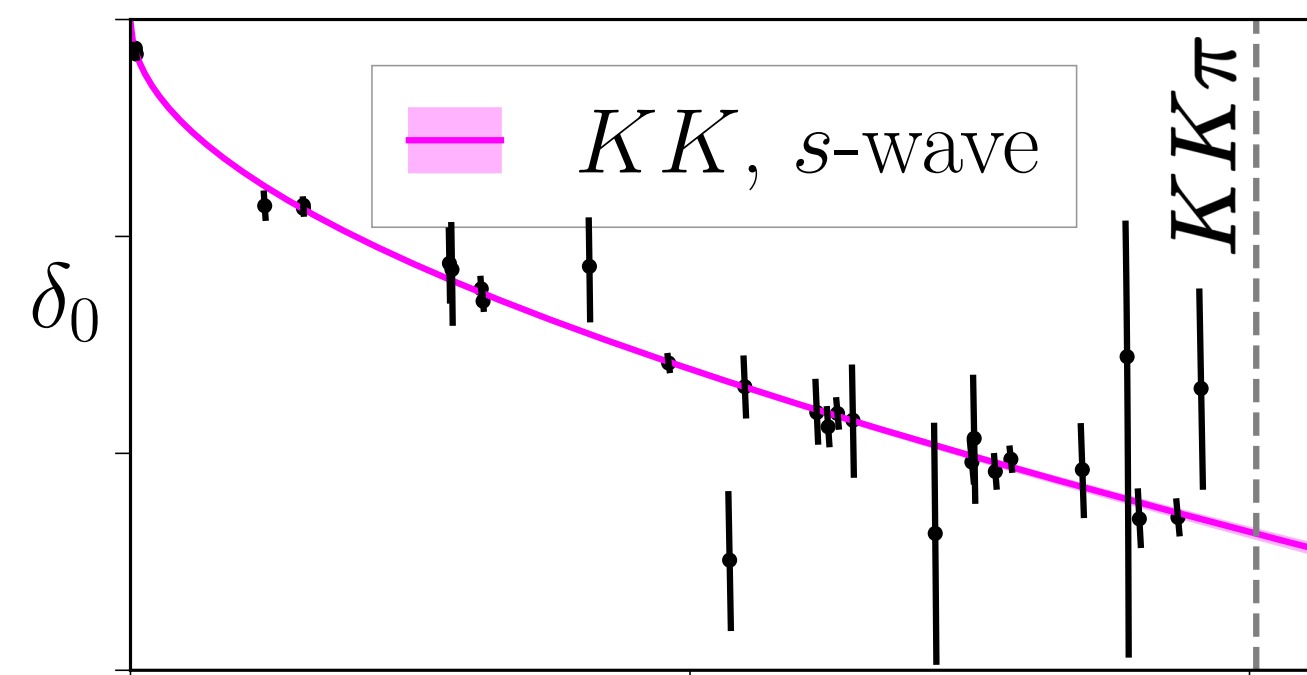
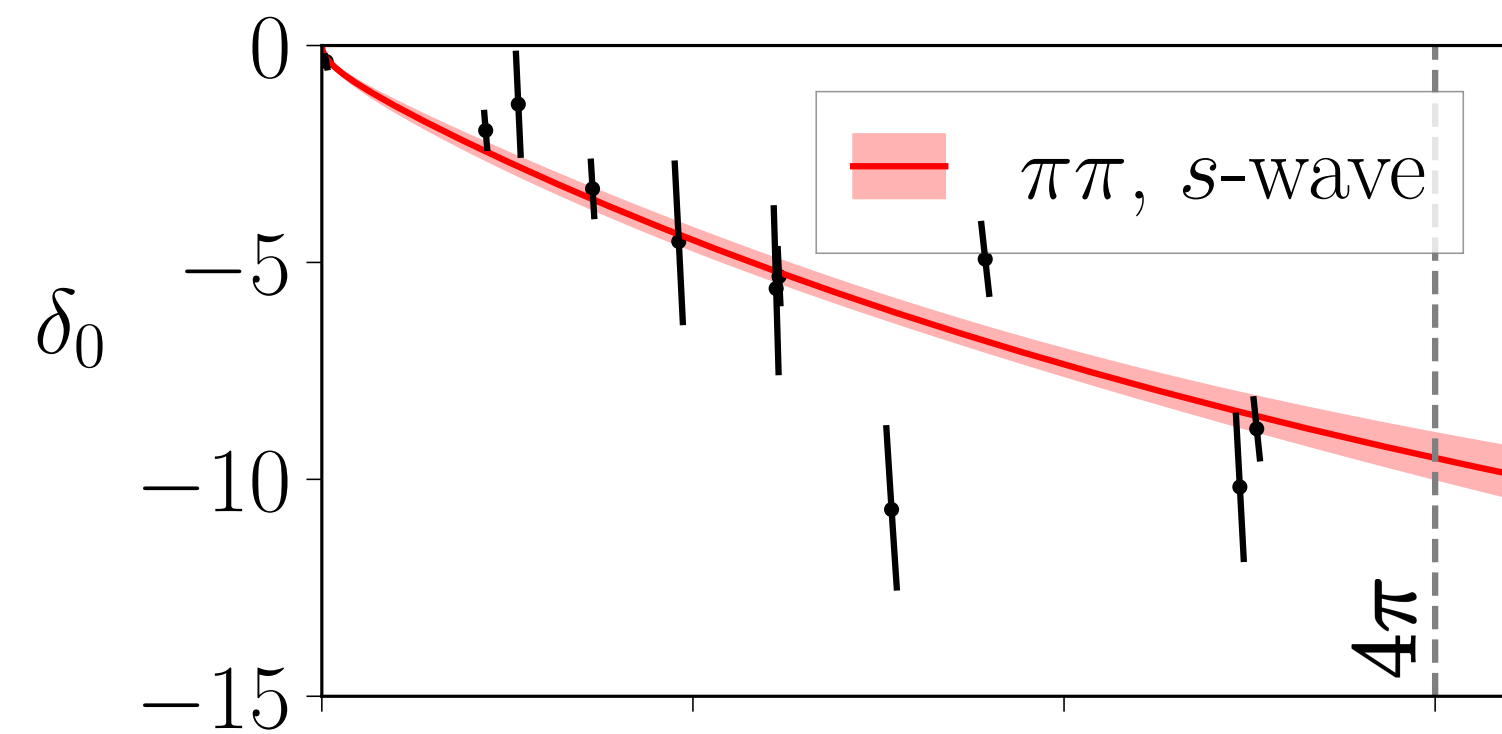
$\mathcal{K}_{\text{df},3} \neq 0$, with $> 10\sigma$

3K spectrum

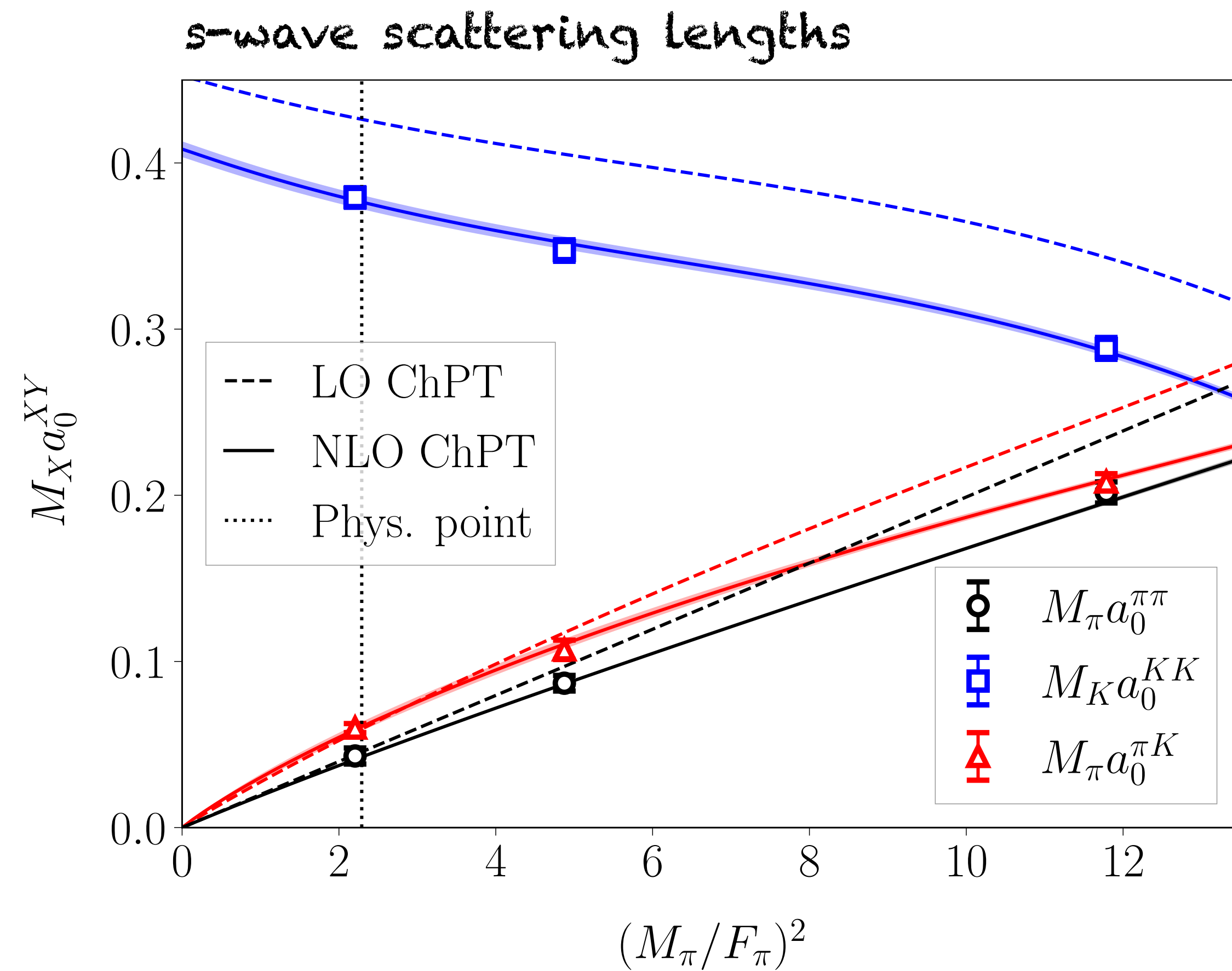


Two-body phase shift

○ Required input for three-meson calculations

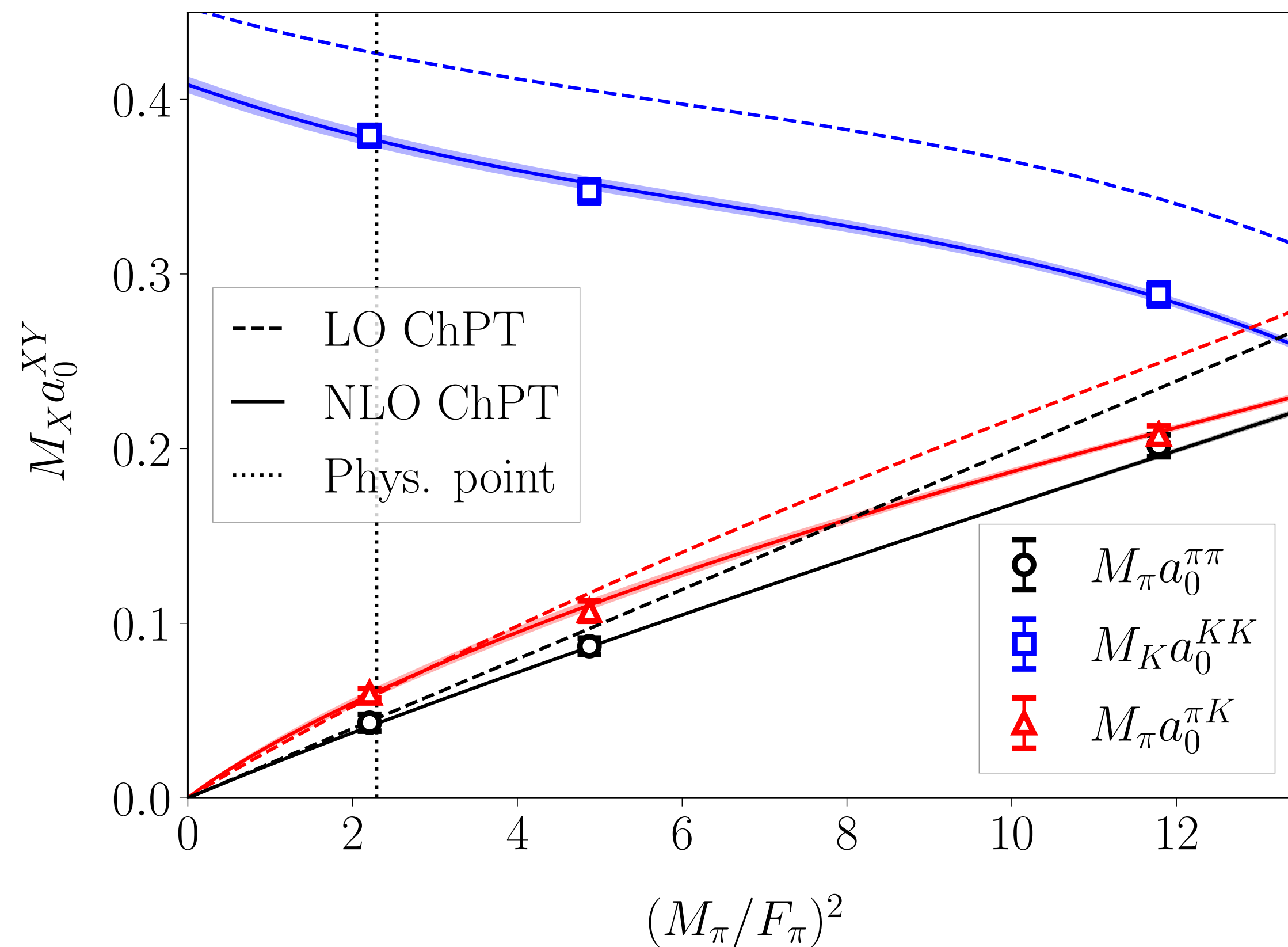


Scattering lengths

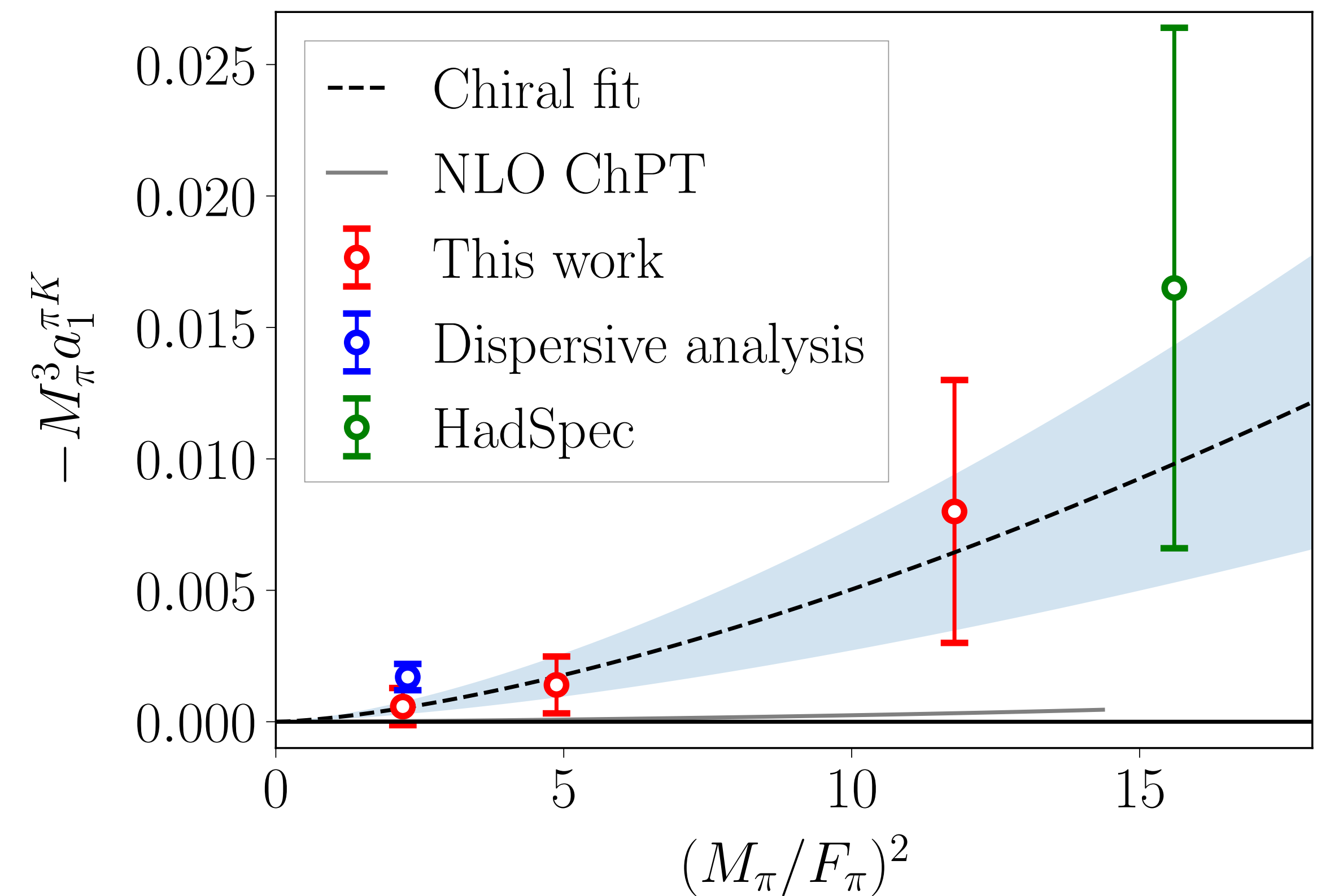


Scattering Lengths

s-wave scattering lengths



p-wave πK scattering lengths



Dispersive results: [Pelaez, Rodas, 2010.11222]

Three-pion K matrix

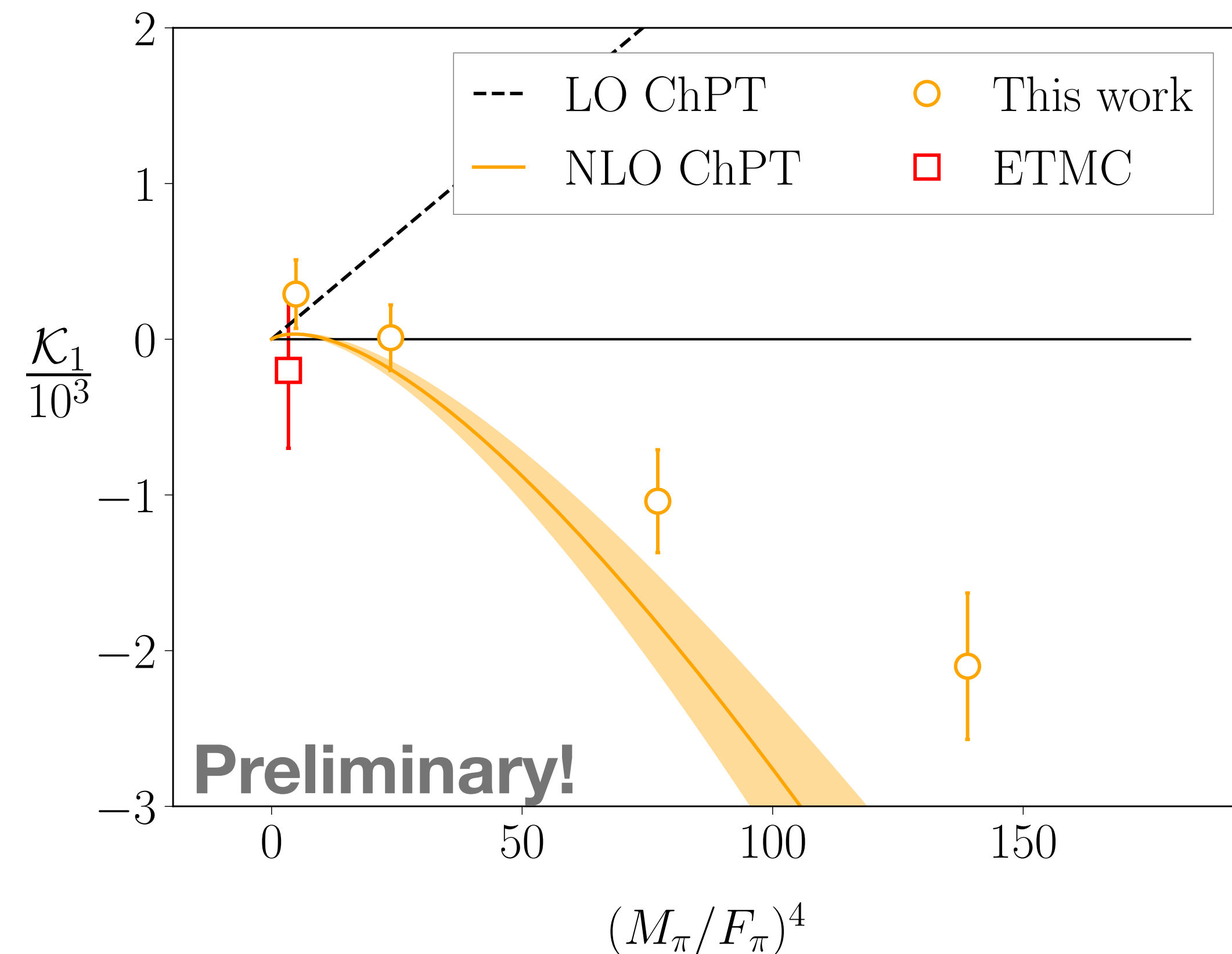
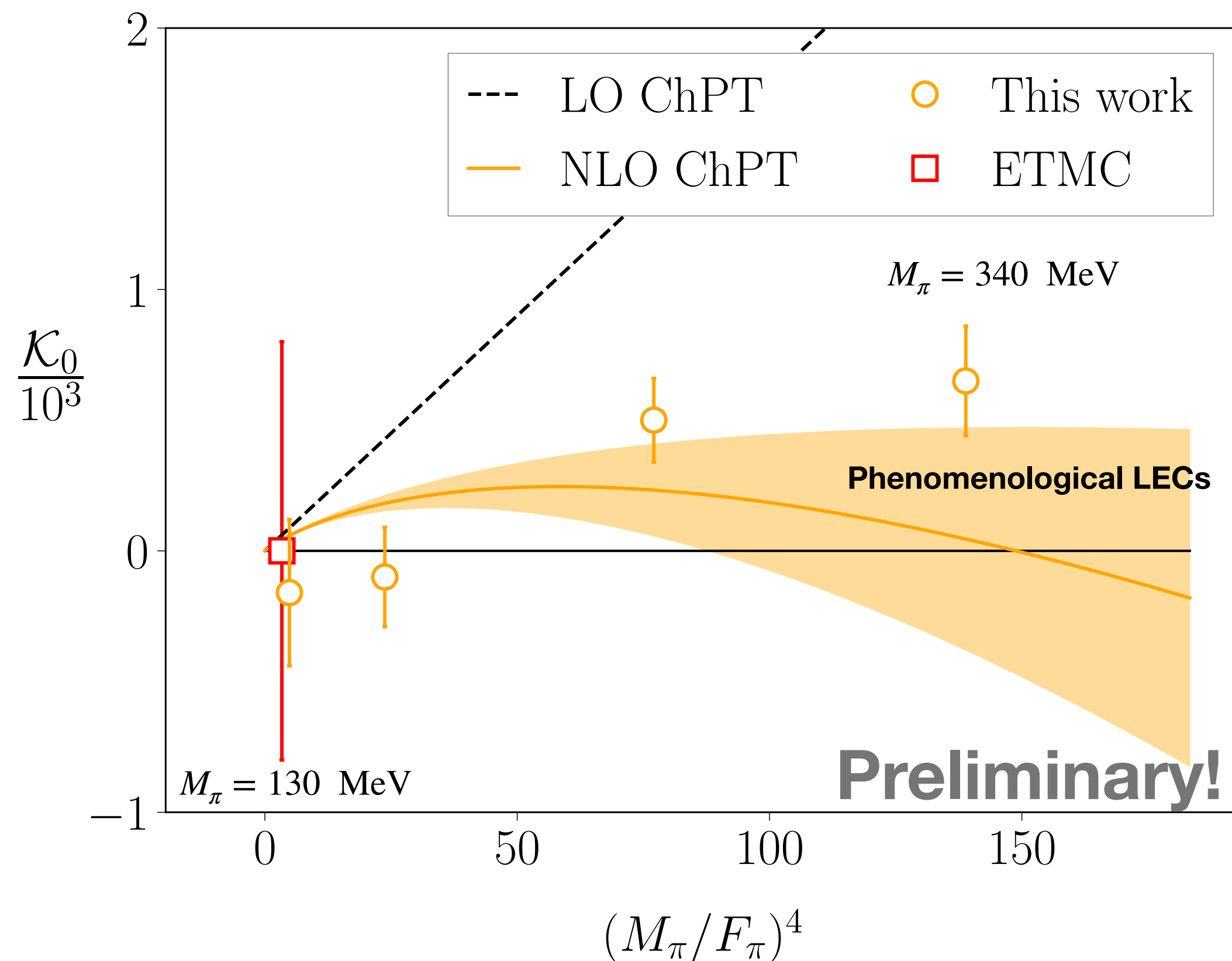
Compare to chiral perturbation theory

NLO ChPT: [Baeza-Ballesteros, Bijmens, Husek, [FRL](#), Sharpe, Sjö, JHEP 2023] [\[See talk by M. Sjö\]](#)

ETMC: [Fischer, Kostrzewa, Liu, [FRL](#), Ueding, Urbach, EPJC 2021]

This work: [Dawid, Draper, Hanlon, Hörz, Morningstar, [FRL](#), Sharpe, Skinner, JHEP 2023 + on-going work]

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_0 + \mathcal{K}_1 \left(\frac{s - 9M_\pi^2}{9M_\pi^2} \right) + \dots$$

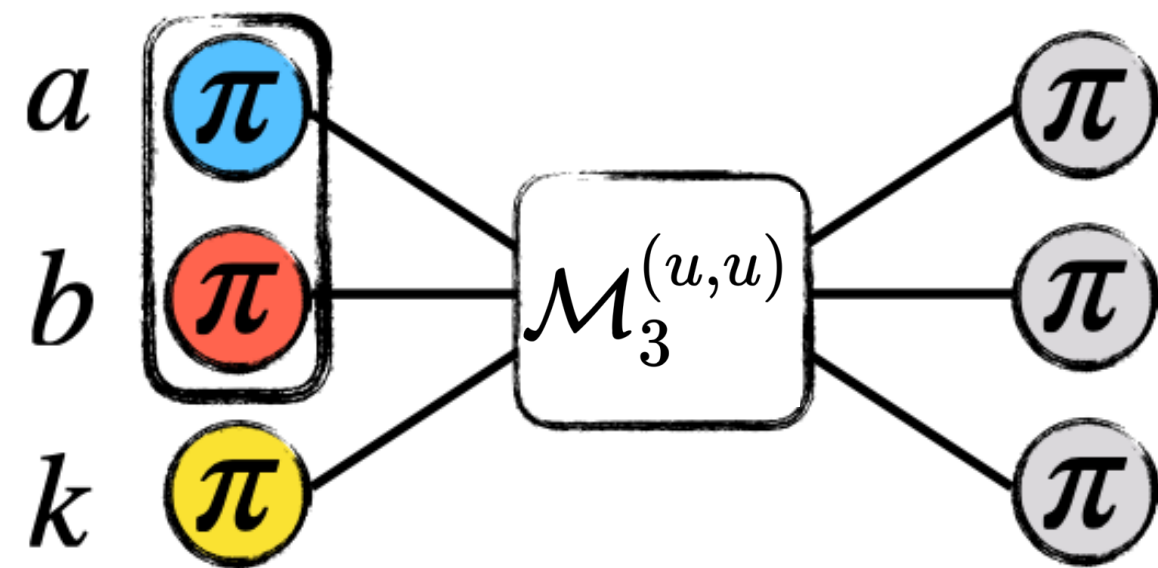


Scattering amplitudes

- Physical amplitudes can be obtained after solving integral equations:

$$\mathcal{M}_3^{(u,u)}$$

“Unsymmetrized amplitude”



- Choice of spectator is fixed
- Needs “symmetrization”

$$\mathcal{M}_3 = \mathcal{S} \left[\mathcal{M}_3^{(u,u)} \right]$$

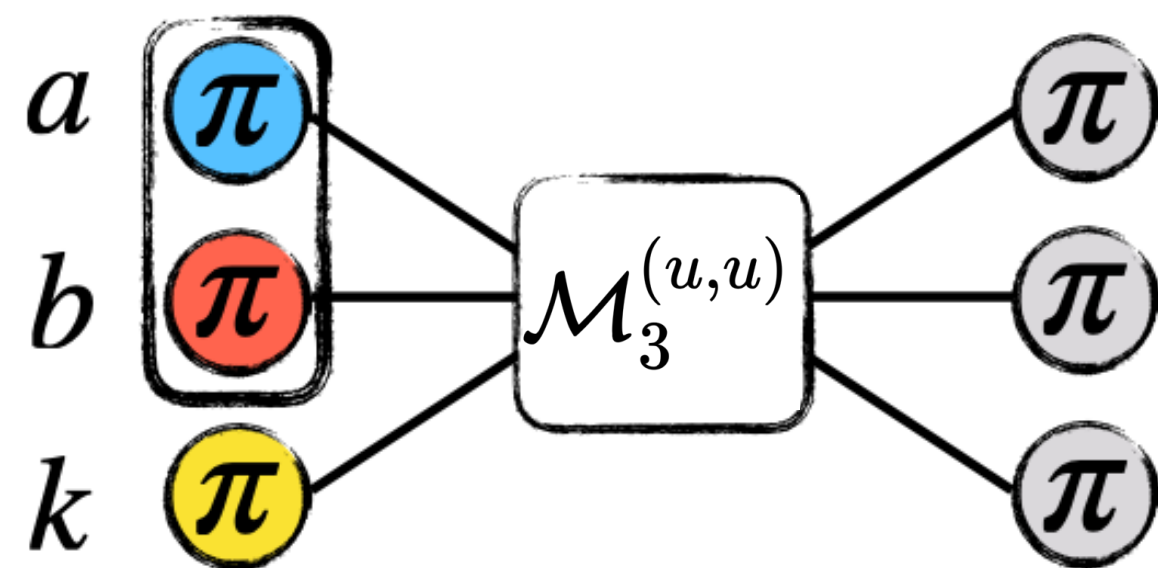
Scattering amplitudes

Physical amplitudes can be obtained after solving integral equations:

Partial-wave projection
[Jackura, Briceño, 2312.00625]

$$\mathcal{M}_3^{(u,u)} = \mathcal{D}^{(u,u)} + \mathcal{M}_{\text{df},3}^{(u,u)}$$

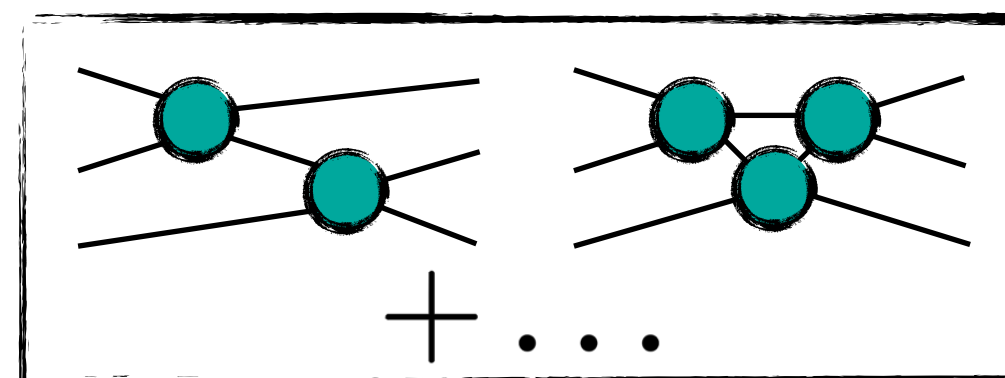
“Unsymmetrized amplitude”



- Choice of spectator is fixed
- Needs “symmetrization”

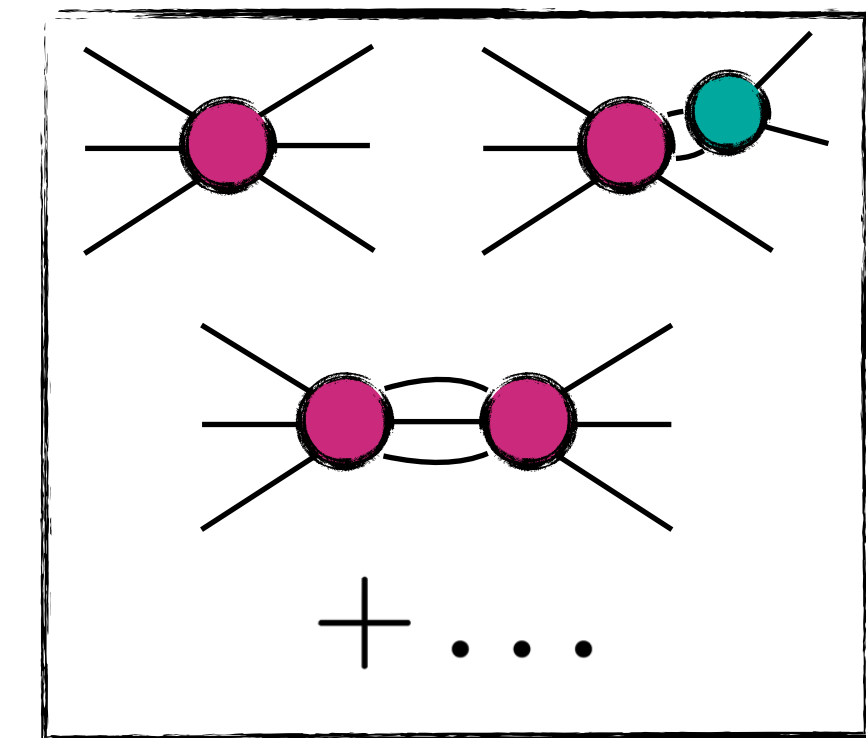
$$\mathcal{M}_3 = \mathcal{S} \left[\mathcal{M}_3^{(u,u)} \right]$$

“ladder amplitude”



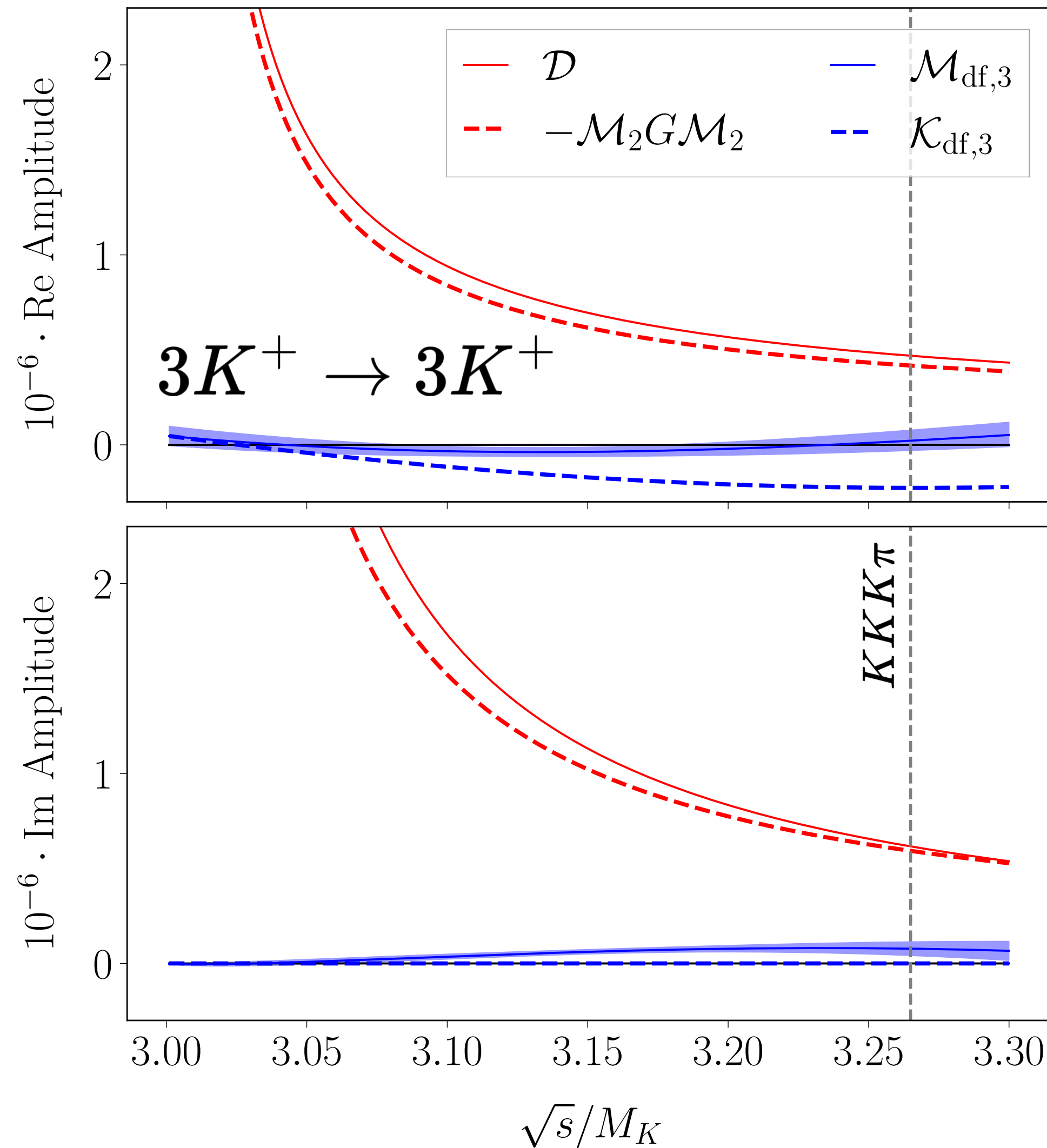
$$\mathcal{D}^{(u,u)} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G \mathcal{D}^{(u,u)}$$

“divergence-free amplitude”



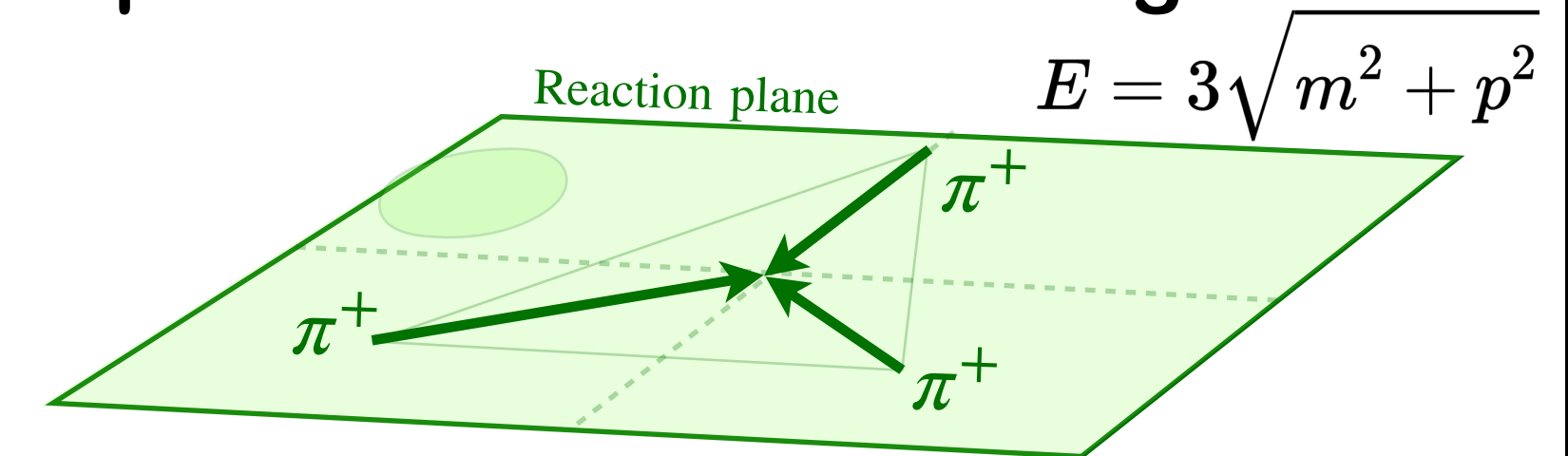
At least one
three-body interaction

Three-kaon $J^P=0^-$ amplitude



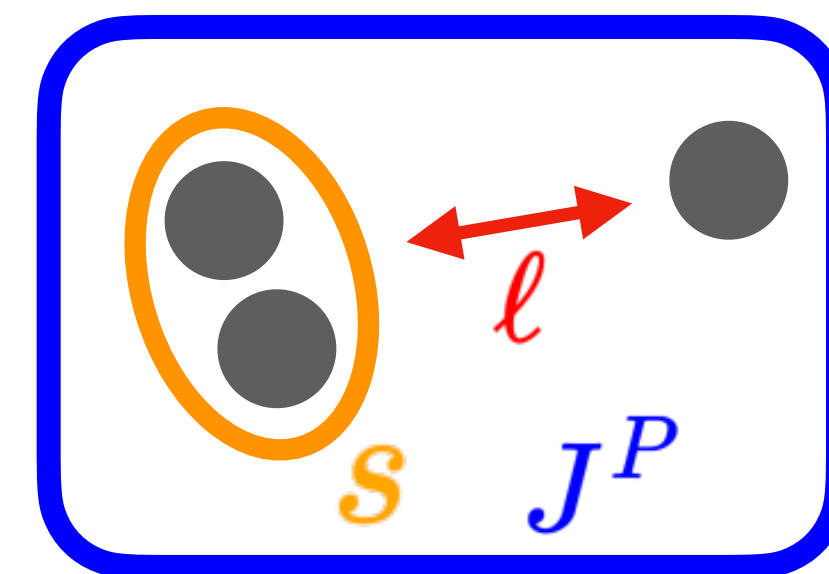
$$\mathcal{M}_3 = \mathcal{D} + \mathcal{M}_{\text{df},3}$$

Equilateral kinematic configuration



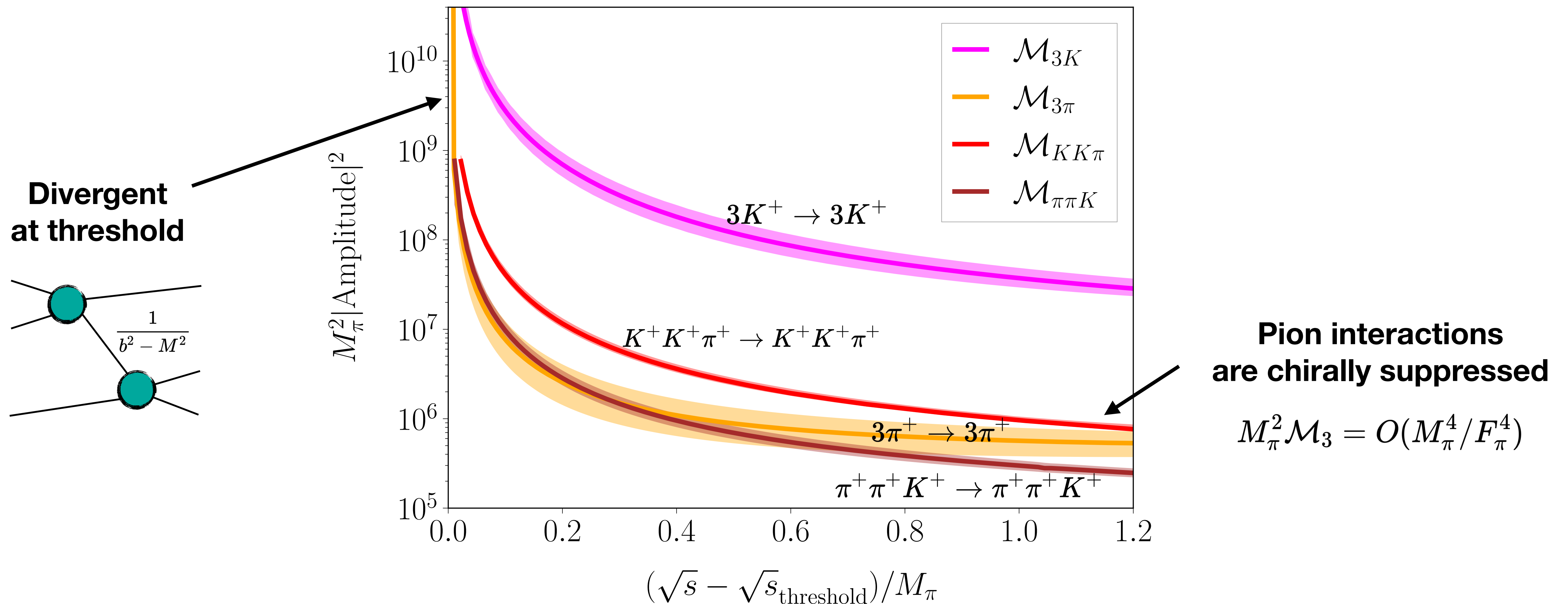
► Partial-wave projected to $J^P = 0^-$

[Jackura, Briceño, 2312.00625]



Three-meson amplitudes

Lattice QCD predictions for physical three-meson scattering amplitudes



Conclusion & Outlook

- We have studied systems of two and three-mesons at maximal isospin at the physical point
 - ▶ Important benchmark system for three-hadron spectroscopy
- Constraints on two-meson scattering amplitudes at the physical point
 - ▶ Some evidence for d waves (KK) and p waves (πK)
 - ▶ Chiral dependence of scattering lengths
- Solving the integral equations, we have determined physical three-meson scattering amplitudes
 - ▶ Dominated by two-meson rescattering
 - ▶ Evidence for “contact” three-hadron force ($3K$)
- Next steps involve other three-pion isospin channels, $DD\pi$ systems and systems with baryons

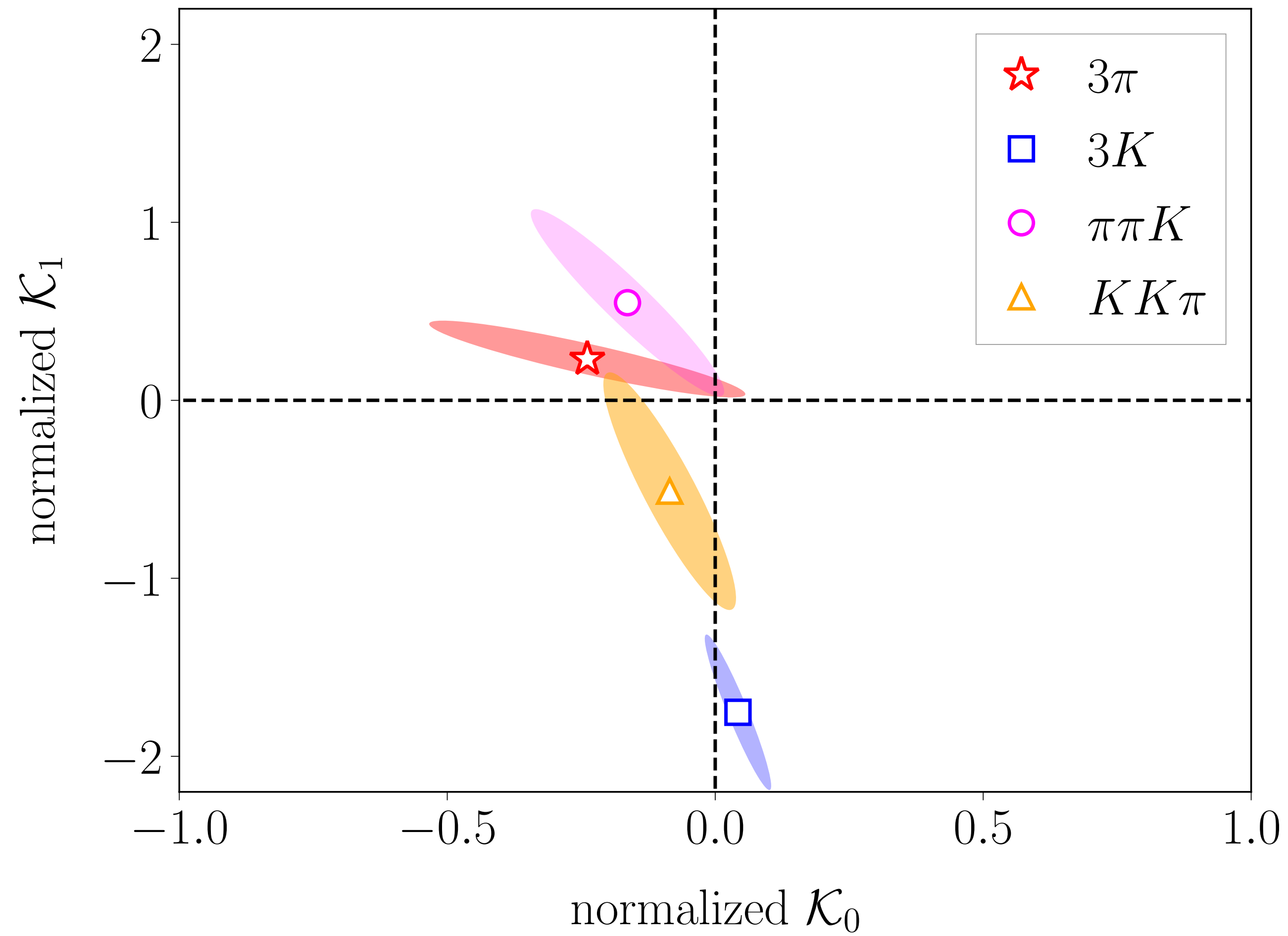
Conclusion & Outlook

- We have studied systems of two and three-mesons at maximal isospin at the physical point
 - ▶ Important benchmark system for three-hadron spectroscopy
- Constraints on two-meson scattering amplitudes at the physical point
 - ▶ Some evidence for d waves (KK) and p waves (πK)
 - ▶ Chiral dependence of scattering lengths
- Solving the integral equations, we have determined physical three-meson scattering amplitudes
 - ▶ Dominated by two-meson rescattering
 - ▶ Evidence for “contact” three-hadron force ($3K$)
- Next steps involve other three-pion isospin channels, $DD\pi$ systems and systems with baryons

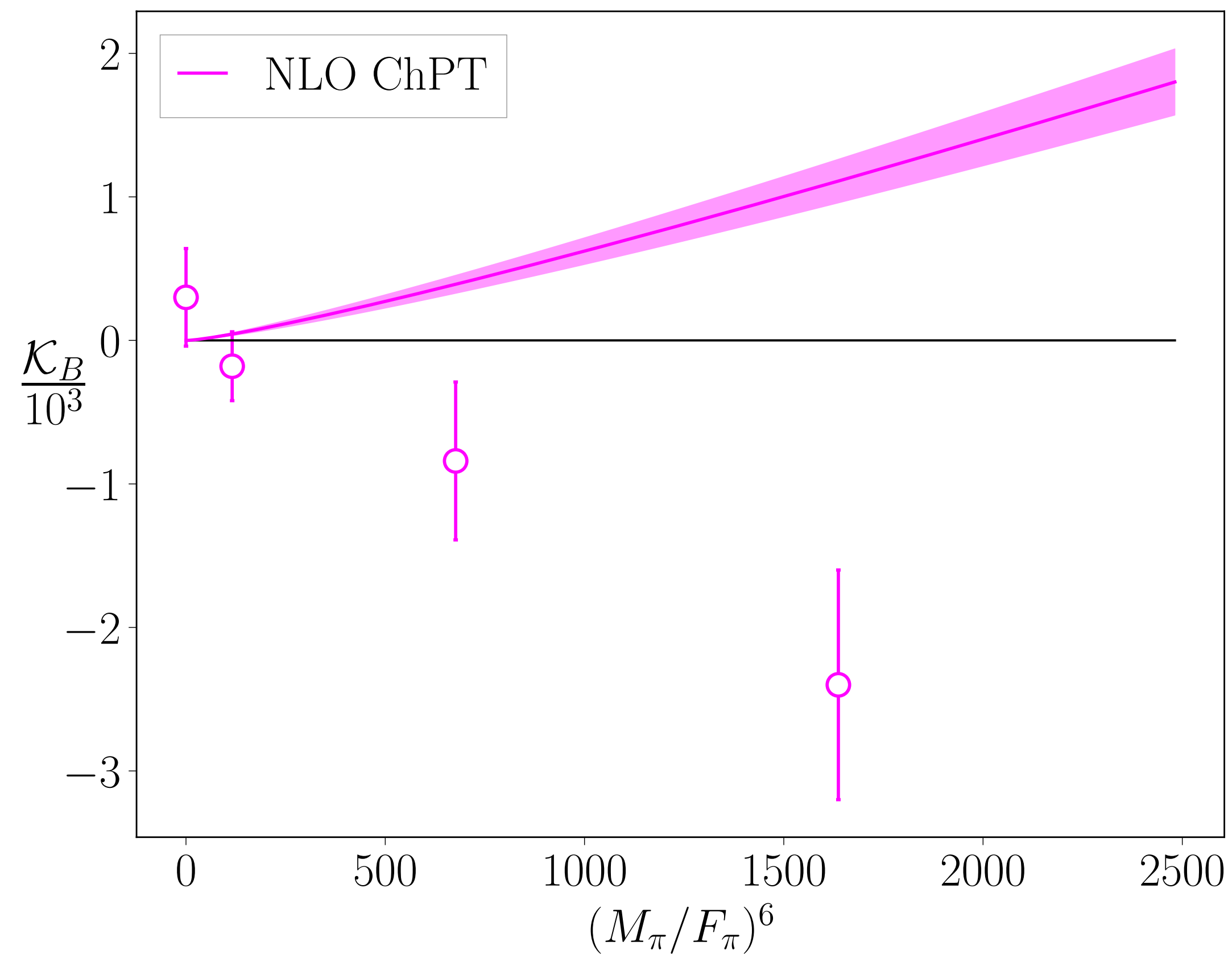
Thanks!

Back-up

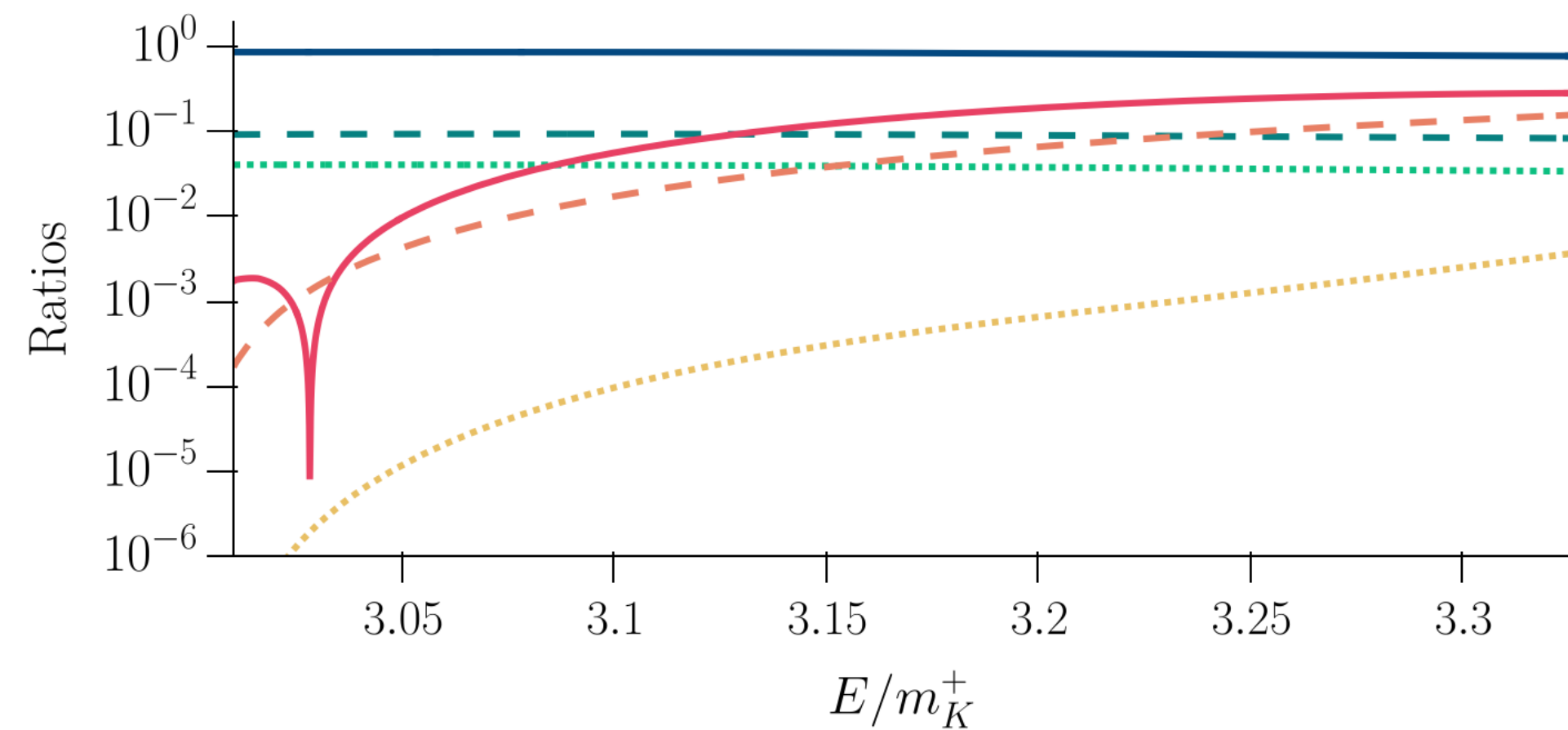
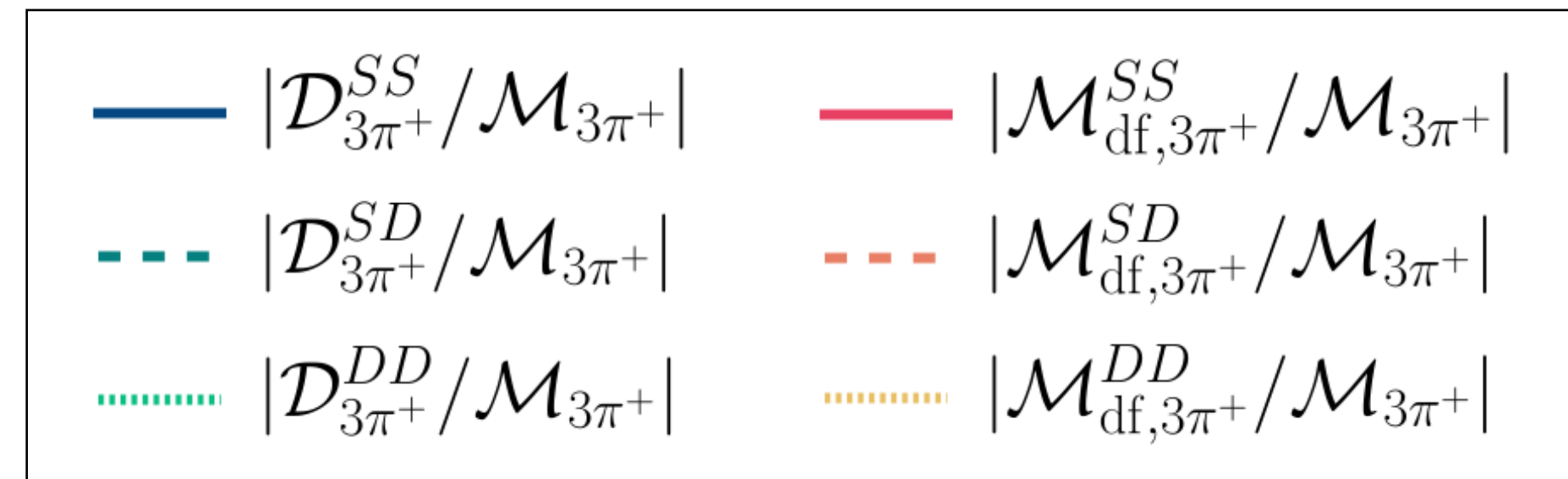
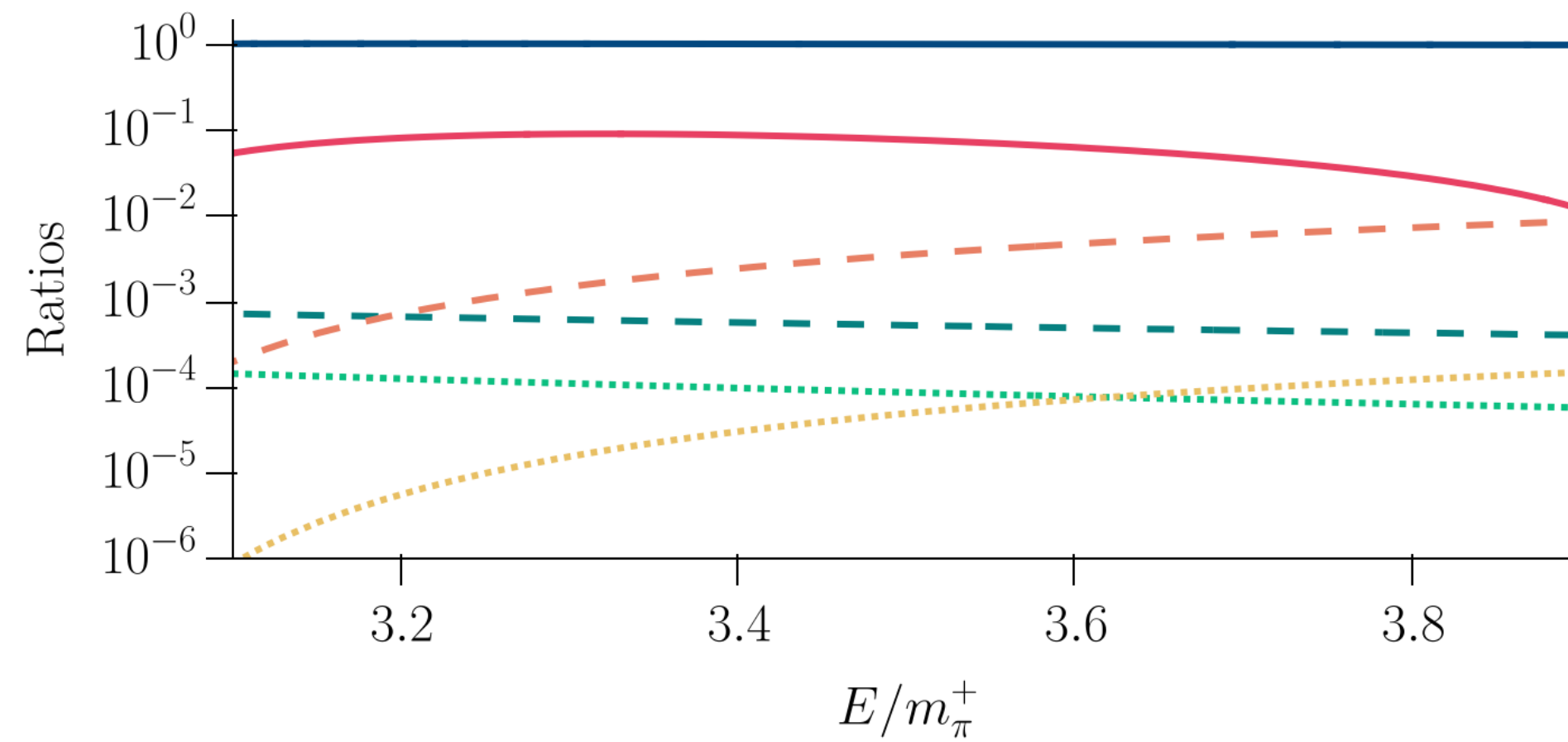
Significance of $K_d f_3$



Comparison to ChPT

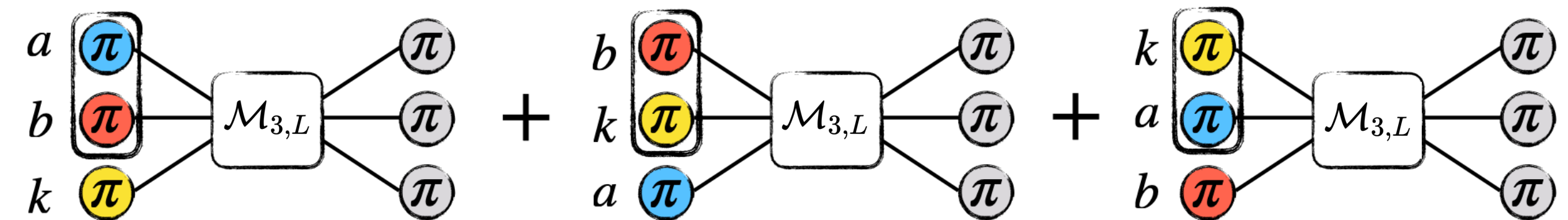


Three-meson amplitudes



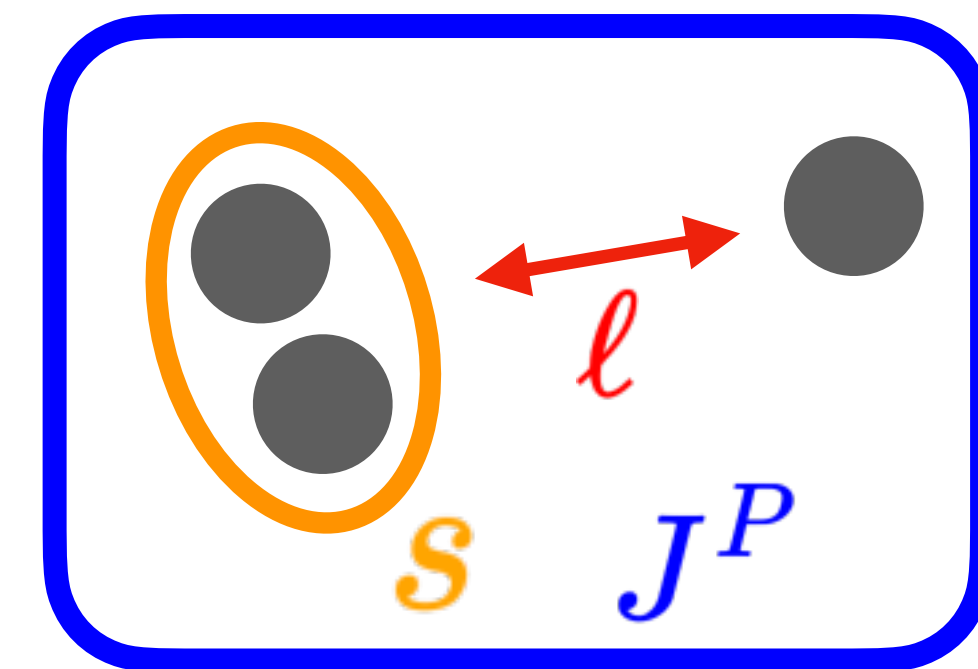
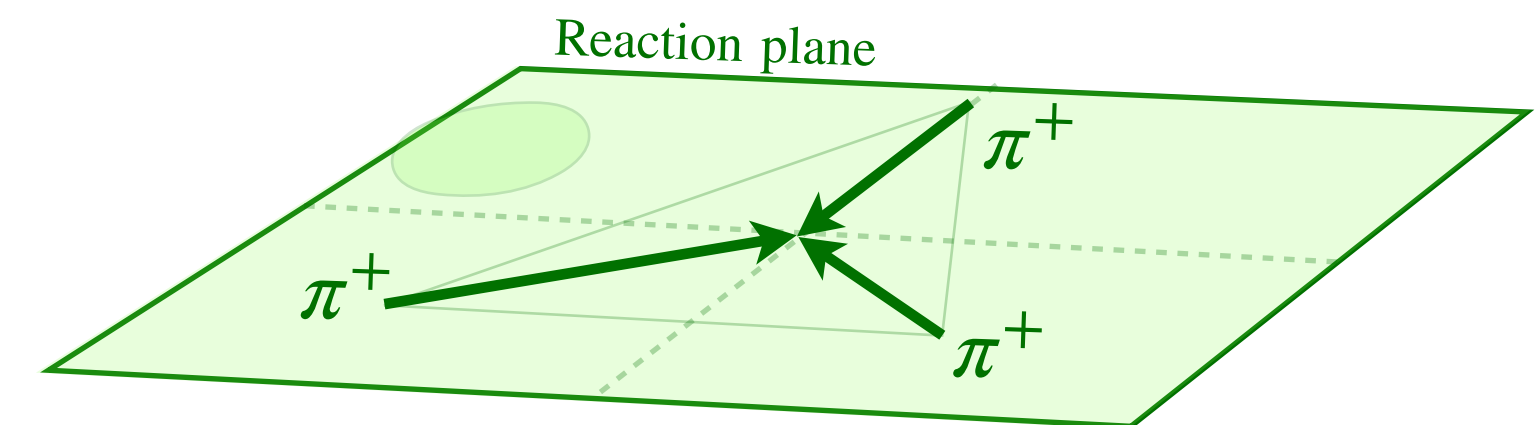
Kinematic configuration

- Symmetrize (each particle gets a turn to be the spectator)

$$\mathcal{M}_{3,L}(P) \equiv \mathcal{S} \left[\mathcal{M}_{3,L}^{(u,u)}(P) \right] =$$


- Equilateral kinematic configuration $E = 3\sqrt{m^2 + p^2}$

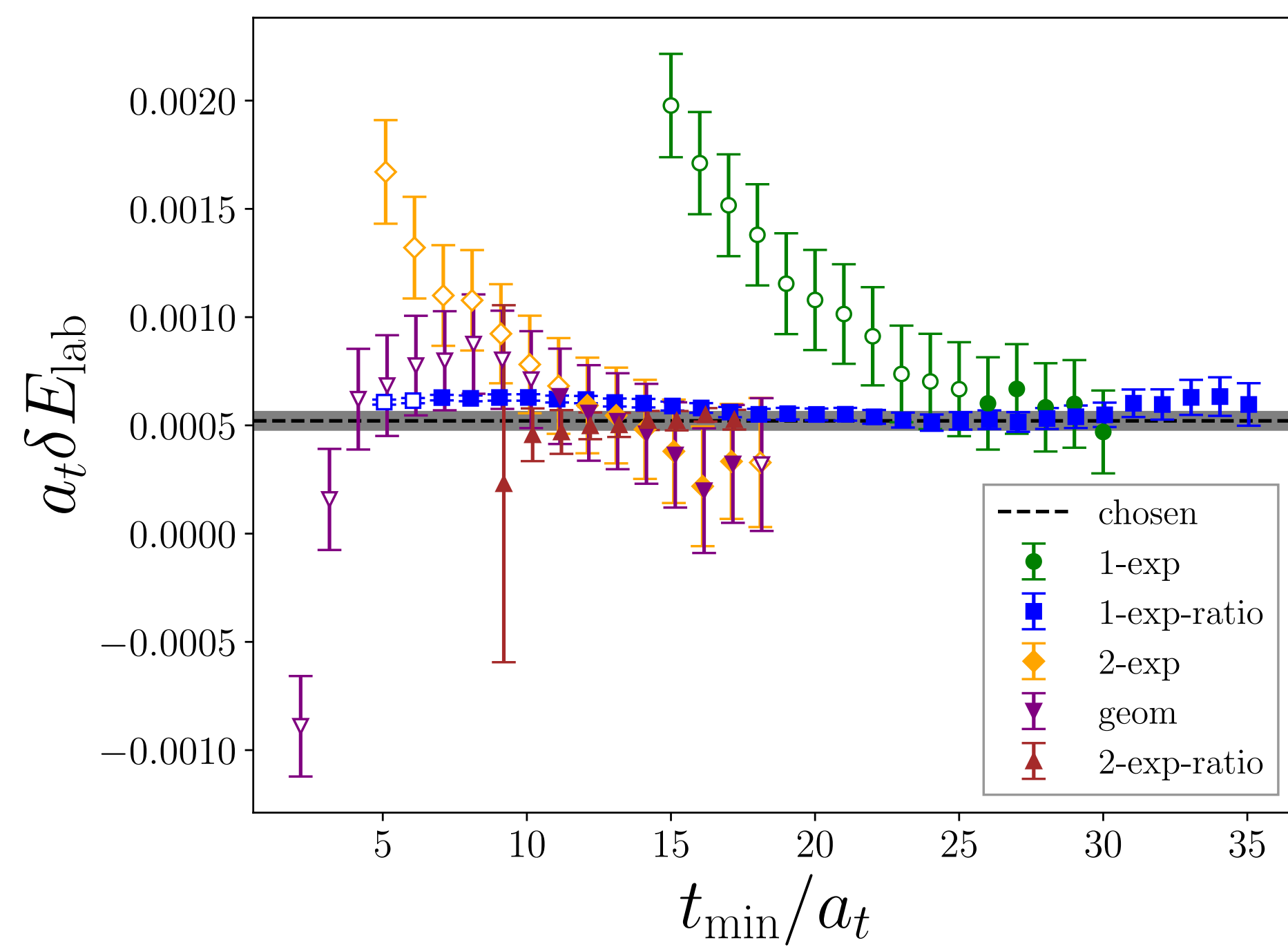
$$\mathcal{M}_3^{J=0}(E) = 9 \left[\mathcal{M}_{3,00;00}^{(u,u)J=0}(E) + \frac{5}{4} \mathcal{M}_{3,22;22}^{(u,u)J=0}(E) \right. \\ \left. - \frac{\sqrt{5}}{2} \left(\mathcal{M}_{3,22;00}^{(u,u)J=0}(E) + \mathcal{M}_{3,00;22}^{(u,u)J=0}(E) \right) \right],$$



Partial-wave projection
[Jackura, Briceño, 2312.00625]

Other $t_{\min}$ plot

$3K^+, B_2, P^2 = 1, \text{ level } 0$



$2K^+, A_2, P^2 = 2, \text{ level } 3$

