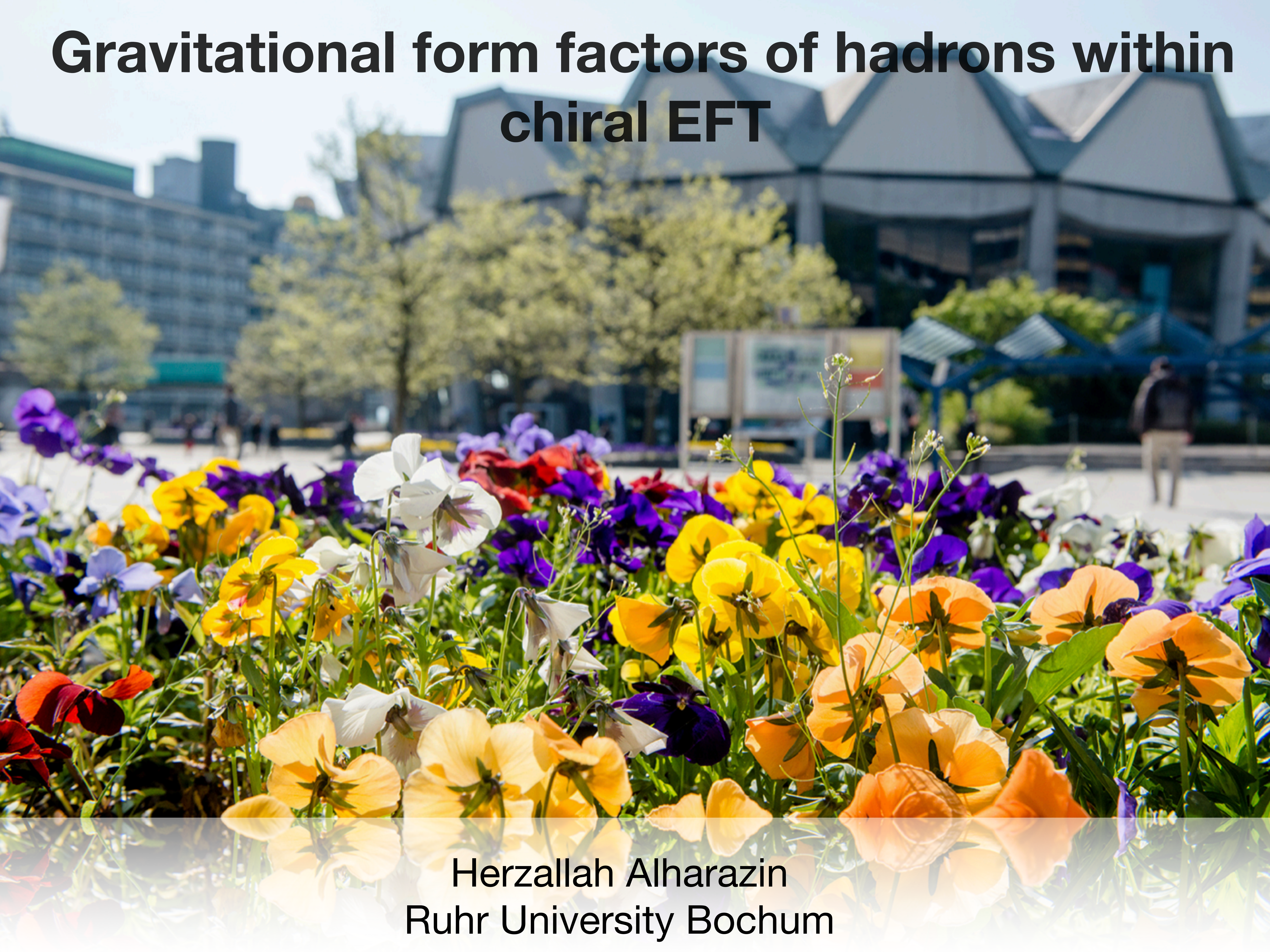
Gravitational form factors of hadrons within chiral EFT



Herzallah Alharazin
Ruhr University Bochum

Part I: Action in Curved Spacetime

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- In the literature, the EChL describing pions, nucleons, and deltas in flat spacetime has been systematically constructed at least up to fourth order in small quantities
- However, terms in the action involving derivatives of the metric fields can also contribute to the $\langle f \,|\, T_{\mu\nu} \,|\, i \rangle$ in flat spacetime, e.g., consider: $S = \int d^4x \sqrt{-g}\, V_{\alpha\beta\lambda} \partial^\alpha g^{\beta\lambda}$. The corresponding EMT is obtained as follows:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \int d^4x \sqrt{-g}\, V_{\alpha\beta\lambda} \partial^\alpha \frac{\delta g^{\beta\lambda}}{\delta g^{\mu\nu}} \Big|_{g=\eta} = -\, \partial^\alpha \left(V_{\alpha\mu\nu} + V_{\alpha\nu\mu} \right)$$

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- Couplings with the Riemann tensor:

$$R^{\mu\nu\alpha\beta}\left(\tilde{c}_x^1\, V^1_{\mu\nu\alpha\beta} + \tilde{c}_y^1\, V^1_{\mu\alpha\nu\beta}\right) + \cdots + R^{\mu\nu\alpha\beta}\left(\tilde{c}_x^m\, V^m_{\mu\nu\alpha\beta} + \tilde{c}_y^n\, V^n_{\mu\alpha\nu\beta}\right)$$

<u>*for*</u> $V^i_{\mu\nu\alpha\beta} \sim \mathcal{O}^{n-2}$

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- Couplings with the Ricci scalar: $\tilde{c}_k^1\, R\, \mathscr{L}^{(n),1} + \cdots + \tilde{c}_k^m\, R\, \mathscr{L}^{(n),m}$

Where $\mathscr{L}^{(n),i}$ *are independent structures that appear in the flat spacetime EChL of order n.*

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- These couplings are essential to absorb divergences and power counting breaking terms

Part I: πN Action in Curved Spacetime

- At order 2:

Phys. Rev. D **102** (2020) no.7, 076023

$$S^{(2)}_{\text{grav}} = \int d^4x \sqrt{-g} \left\{ \frac{c_8}{8} R \bar{\Psi}\Psi + \frac{2c_9}{m} R^{\mu\nu} \left(\bar{\Psi}\, i\gamma_\mu \overleftrightarrow{\nabla}_\nu \Psi \right) \right\}$$

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Corresponding EMT:

$$T_{\mu\nu} = \frac{c_8}{8}\left(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right)\bar{\Psi}\Psi + \frac{ic_9}{m}\left[\partial^2\left(\bar{\Psi}\gamma_\mu \overleftrightarrow{\nabla}_\nu\Psi\right) + \eta_{\mu\nu}\partial^\alpha\partial^\beta\left(\bar{\Psi}\gamma_\alpha\overleftrightarrow{\nabla}_\beta\Psi\right) - \partial^\alpha\partial_\mu\left(\bar{\Psi}\gamma_\alpha\overleftrightarrow{\nabla}_\nu\Psi + \bar{\Psi}\gamma_\nu\overleftrightarrow{\nabla}_\alpha\Psi\right)\right]$$

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- At order 3:

Phys. Rev. D **109** (2024) no.1, 016009

$$S^{(3)}_{\text{grav}} = \int d^4x\sqrt{-g}\left\{ \tilde{d}_{g1}\, R\bar{\Psi}\gamma^\mu\gamma_5 u_\mu\Psi + \tilde{d}_{g2}\, R^{\mu\nu}\bar{\Psi}\gamma_\mu\gamma_5 u_\nu\Psi + \tilde{d}_{g3}R^{\alpha\beta\lambda\mu}\left(\bar{\Psi}\sigma_{\alpha\beta}\gamma_5 u_\lambda \overleftrightarrow{\tilde{\nabla}}_\mu\Psi\right) \right.$$
$$\left. + i\tilde{d}_{g4}\,\nabla_\mu R^{\alpha\beta\lambda\mu}\left(\bar{\Psi}\sigma_{\alpha\beta}\overleftrightarrow{\nabla}_\lambda\Psi\right)\right\}$$

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$$T_{\mu\nu} = \tilde{d}_{g1}\left[\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right]\bar{\Psi}\gamma^\alpha\gamma_5 u_\alpha\Psi + \frac{\tilde{d}_{g2}}{2}\left[\partial^2\left(\bar{\Psi}\gamma_\mu\gamma_5 u_\nu\Psi\right) + \eta_{\mu\nu}\partial^\alpha\partial^\beta\left(\bar{\Psi}\gamma_\alpha\gamma_5 u_\beta\Psi\right) - \partial^\alpha\partial_\mu\left(\bar{\Psi}\gamma_\alpha\gamma_5 u_\nu\Psi + \bar{\Psi}\gamma_\nu\gamma_5 u_\alpha\Psi\right)\right] + \frac{1}{2}\partial^\kappa\partial^\beta\left(\mathscr{A}_{\beta\mu\kappa\nu} - \mathscr{A}_{\beta\kappa\mu\nu} - \mathscr{A}_{\mu\nu\kappa\beta} + \mathscr{A}_{\mu\kappa\nu\beta}\right)$$

<u>With</u> $\mathscr{A}_{\mu\lambda\nu\rho} \equiv \tilde{d}_{g3}\ \bar{\Psi}\sigma_{\rho\mu}\gamma_5 u_\lambda \overleftrightarrow{\tilde{D}}_\nu\Psi - i\tilde{d}_{g4}\ \partial_\nu\left(\bar{\Psi}\sigma_{\rho\mu}\overleftrightarrow{D}_\lambda\Psi\right)$

Part I: $\pi N \Delta$ Action in Curved Spacetime

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Eur. Phys. J. C **82** (2022) no.10, 907

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$$S^{(2)}_{\text{grav}} = S^{(2)}_{1,\text{grav}} + S^{(2)}_{2,\text{grav}}$$

$$S^{(2)}_{1,\text{grav}} = \int d^4x \sqrt{-g} \Big\{ h_1 R\, g^{\alpha\beta} \bar{\Psi}^i_\alpha \Psi^i_\beta + h_4 R^{\mu\nu}\, \bar{\Psi}^i_\mu \Psi^i_\nu + 2ih_5 R^{\mu\nu}\, g^{\alpha\beta} \bar{\Psi}^i_\alpha \gamma_\mu \overleftrightarrow{\nabla}_\nu \Psi^i_\beta + ih_6 R^{\mu\nu} g^{\alpha\beta} \left(\bar{\Psi}^i_\alpha \gamma_\mu \overrightarrow{\nabla}_\beta \Psi^i_\nu - \bar{\Psi}^i_\nu \gamma_\mu \overleftarrow{\nabla}_\beta \Psi^i_\alpha \right)$$

$$+ ih_{10} R^{\mu\nu\alpha\beta} \bar{\Psi}^i_\alpha \sigma_{\mu\nu} \Psi^i_\beta + i \left[h_{11}\, R^{\mu\nu\alpha\beta} + h_{12}\, R^{\mu\alpha\nu\beta} \right] \left(\bar{\Psi}^i_\alpha \gamma_\mu \overrightarrow{\nabla}_\nu \Psi^i_\beta - \bar{\Psi}^i_\beta \gamma_\mu \overleftarrow{\nabla}_\nu \Psi^i_\alpha \right) + h_{13} R^{\mu\alpha\nu\beta} \bar{\Psi}^i_\alpha \gamma_\mu \gamma_\nu \Psi^i_\beta \Big\}$$

$$S^{(2)}_{2,\text{grav}} = \int d^4x \sqrt{-g} \Big\{ h_2 R\, \bar{\Psi}^i_\alpha \gamma^\alpha \gamma^\beta \Psi^i_\beta + ih_3 R \left(g^{\alpha\lambda} \bar{\Psi}^i_\alpha \gamma^\beta \overrightarrow{\nabla}_\lambda \Psi^i_\beta - g^{\beta\lambda} \bar{\Psi}^i_\alpha \gamma^\alpha \overleftarrow{\nabla}_\lambda \Psi^i_\beta \right) + ih_7 R^{\mu\nu} \left(\bar{\Psi}^i_\alpha \gamma^\alpha \overrightarrow{\nabla}_\mu \Psi^i_\nu - \bar{\Psi}^i_\nu \gamma^\alpha \overleftarrow{\nabla}_\mu \Psi^i_\alpha \right)$$

$$+ h_8 R^{\mu\nu} \left(\bar{\Psi}^i_\alpha \gamma^\alpha \gamma_\mu \Psi^i_\nu + \bar{\Psi}^i_\nu \gamma_\mu \gamma^\alpha \Psi^i_\alpha \right) + ih_9 R^{\mu\nu} \left(\bar{\Psi}^i_\kappa \gamma^\kappa \gamma^\alpha \gamma_\mu \overrightarrow{\nabla}_\nu \Psi^i_\alpha - \bar{\Psi}^i_\alpha \gamma_\mu \gamma^\alpha \gamma^\kappa \overleftarrow{\nabla}_\nu \Psi^i_\kappa \right)$$

$$+ i \left[h_{14}\, R^{\mu\nu\alpha\beta} + h_{15}\, R^{\mu\alpha\nu\beta} \right] \left(\bar{\Psi}^i_\kappa \gamma^\kappa \gamma_\mu \gamma_\nu \overrightarrow{\nabla}_\alpha \Psi^i_\beta - \bar{\Psi}^i_\beta \gamma_\nu \gamma_\mu \gamma^\kappa \overleftarrow{\nabla}_\alpha \Psi^i_\kappa \right) \Big\}$$

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- At one-loop level and up to fourth order in small quantities, we calculate the following diagrams:

Phys. Rev. D **109** (2024) no.1, 016009

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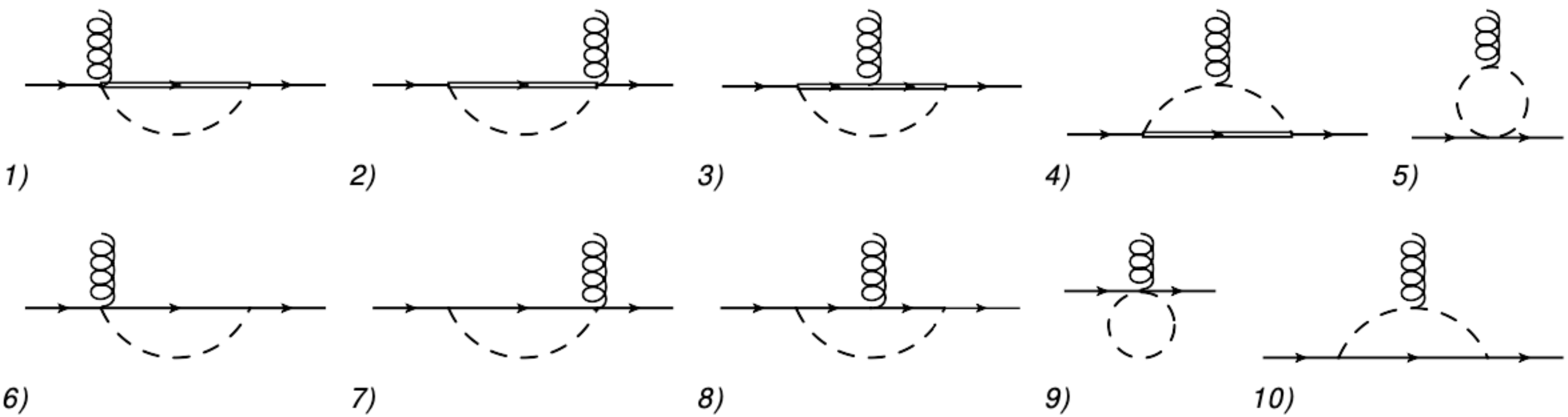

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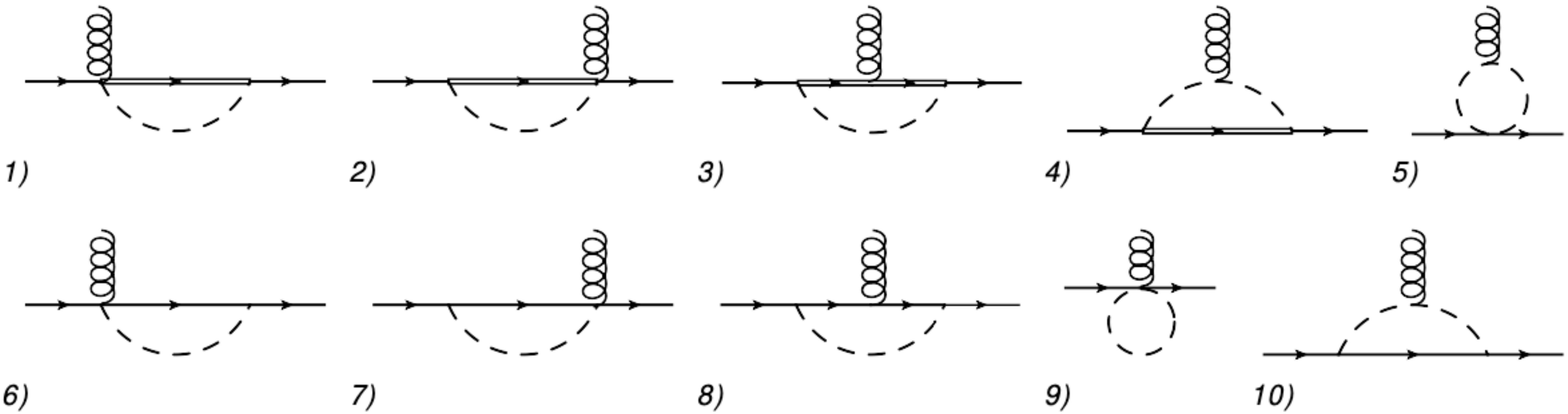

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- To compare the results for the GFFs with and without delta resonances, we fix c_8 from the numerical value of the D-term and substitute some values of the unknown LECs in the theory without Δ resonances, i.e. $x_1, y_1, y_2, \tilde{d}_{g4}$ and c_9, where x_1, y_1 and y_2 parametrize the tree-order contributions of the fourth order

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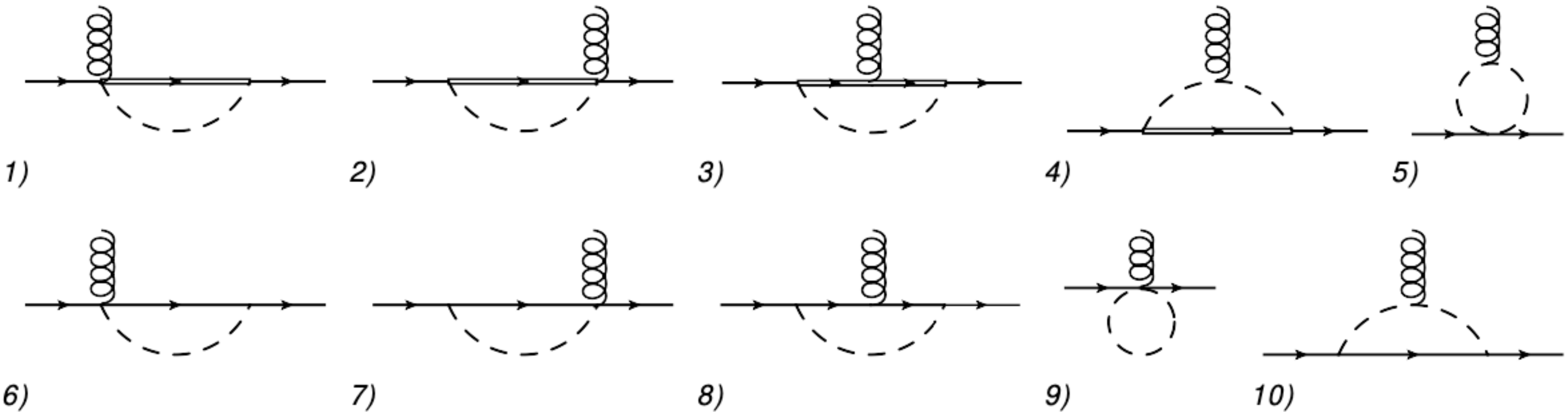

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- The LECs of the effective Lagrangian with explicit deltas are obtained from the condition of matching physical quantities in theories with and without explicit deltas:

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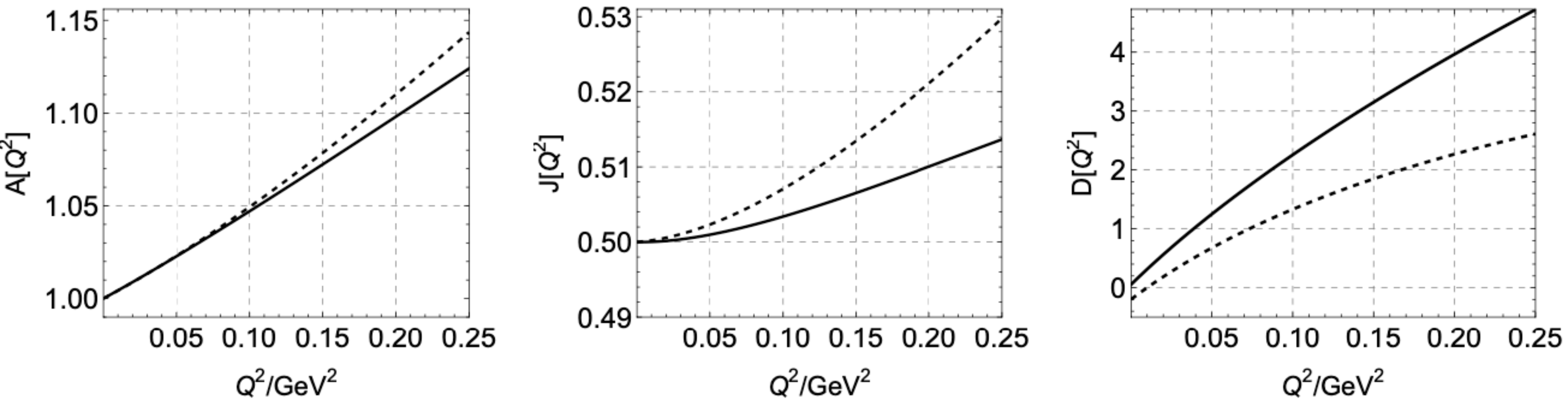

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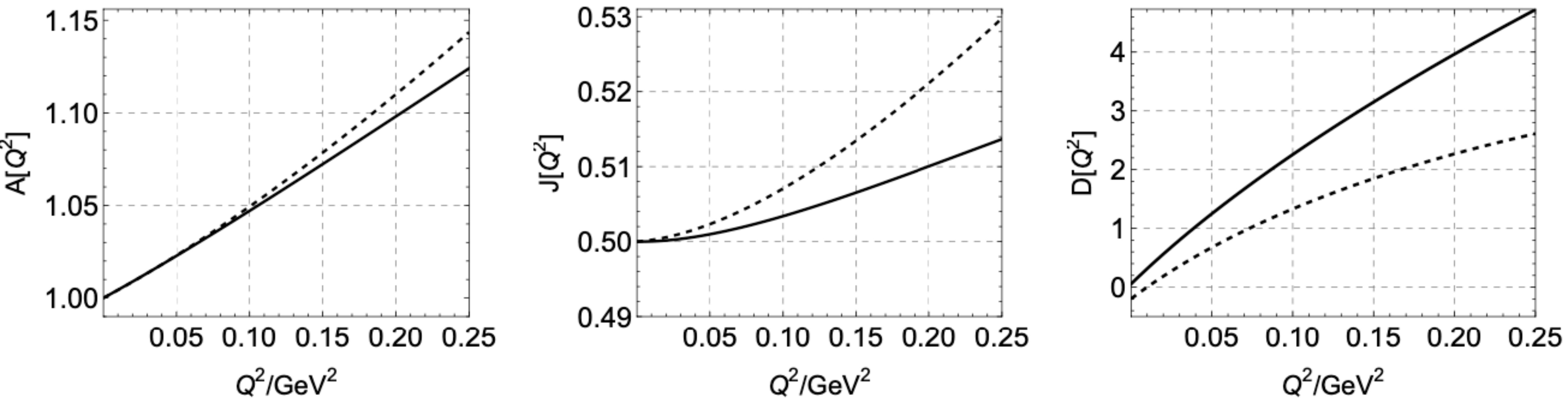

For : $x_1 = y_1 = 0, y_2 = -9.5$ GeV^{-2}, $c_9 = 0.01$ GeV^{-1}, $\tilde{d}_{g4} = -0.2$ GeV^{-3}, $\mathscr{D} = -1$

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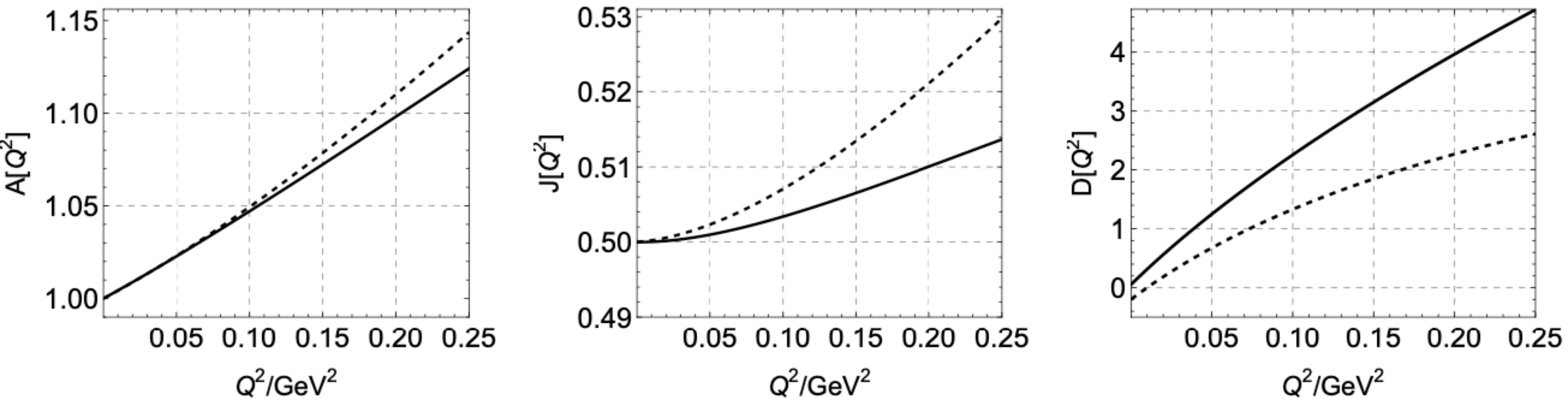

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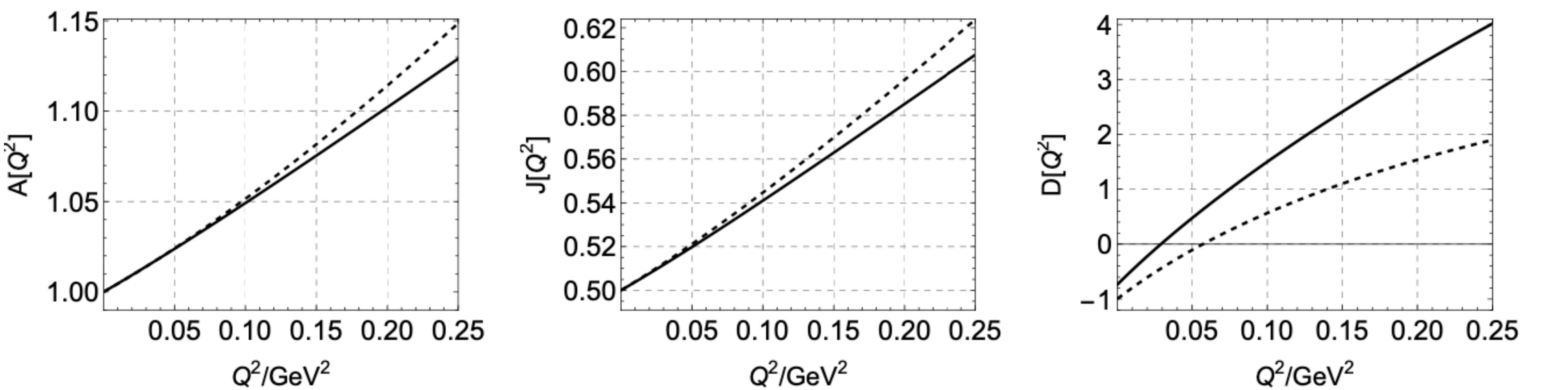

Part II: Nucleon GFFs

<u>*For*</u> : $x_1 = 0\,, y_1 = 3\ \text{GeV}^{-2}, y_2 = -\,10\ \text{GeV}^{-2}, c_9 = 0.05\ \text{GeV}^{-1}, \tilde{d}_{g4} = 0.2\ \text{GeV}^{-3}\,, \mathscr{D} = -\,2$

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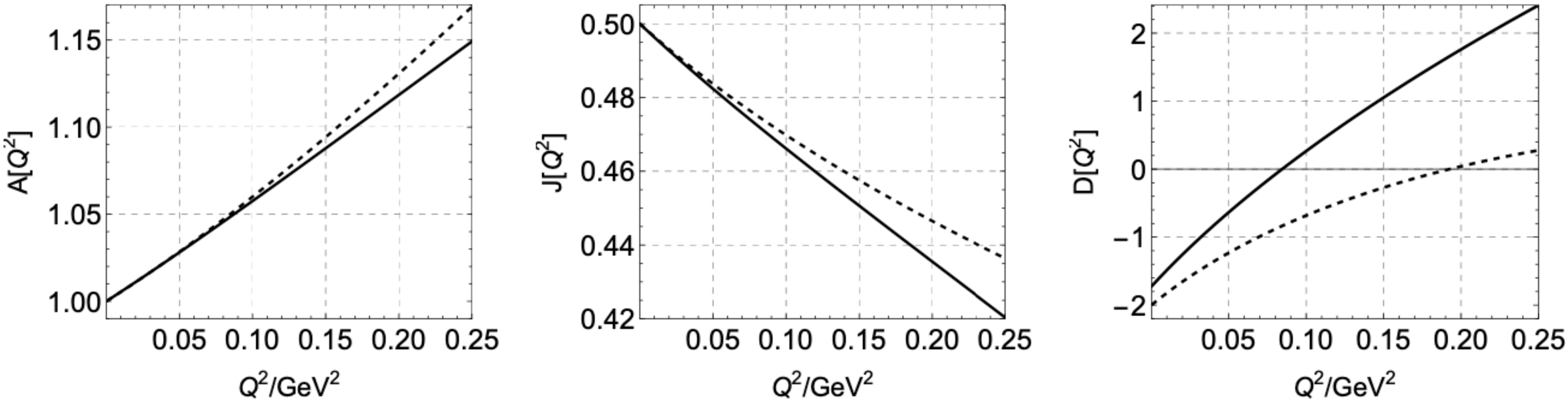

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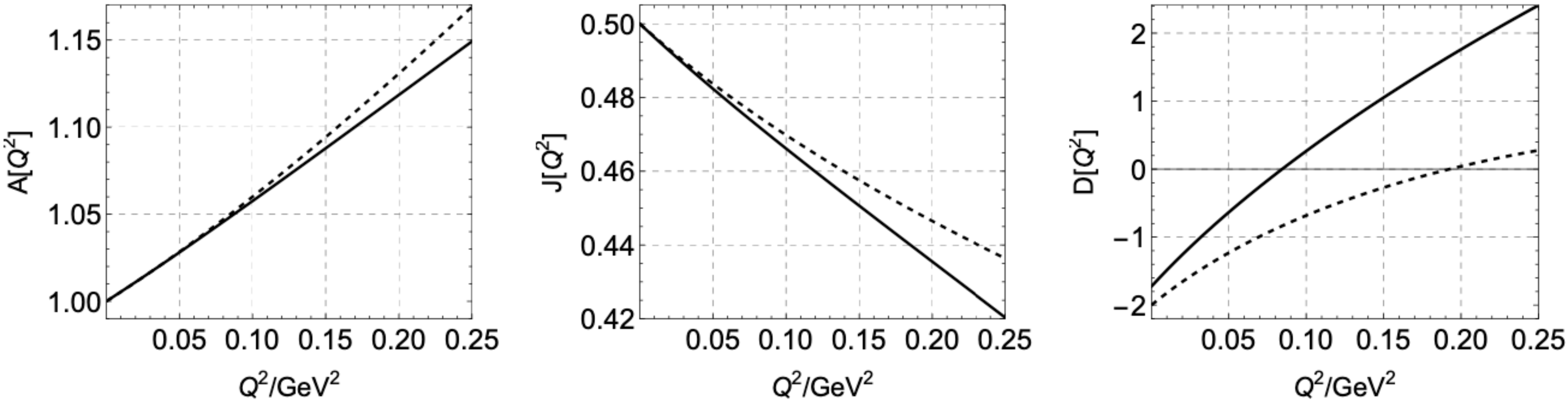

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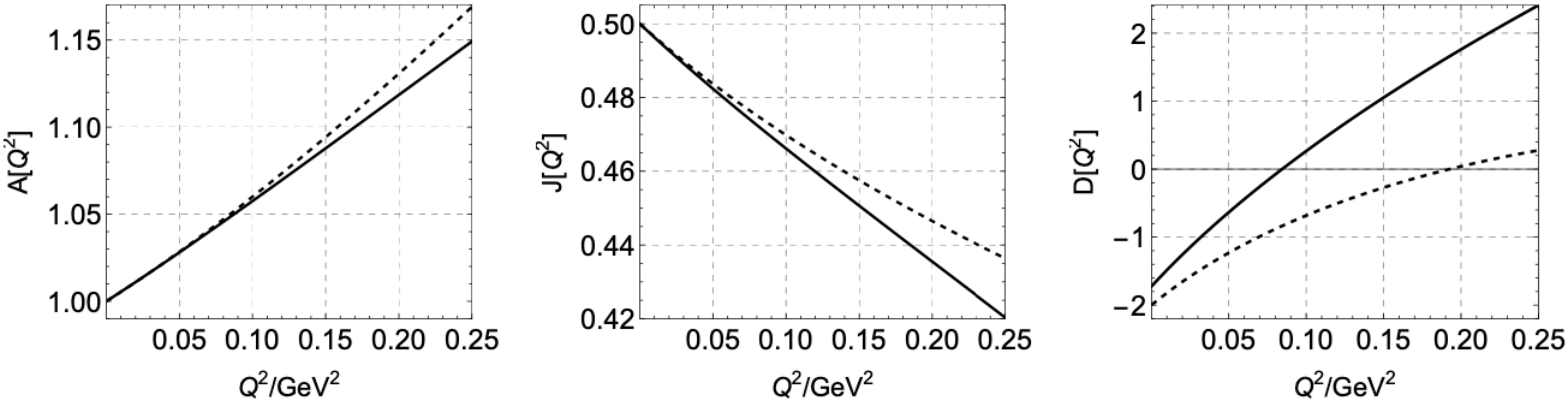

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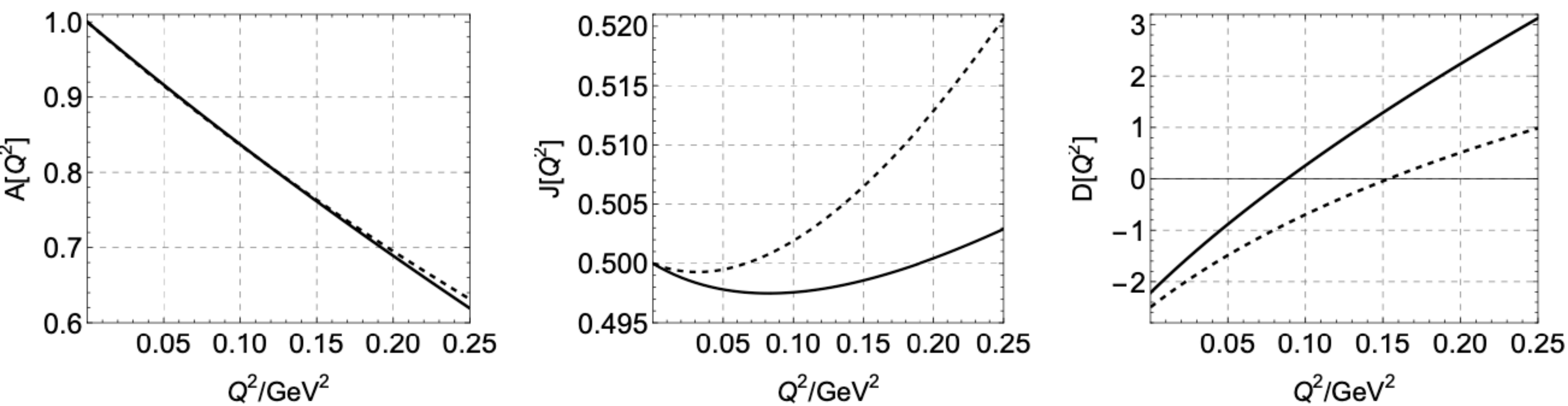

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- This conclusion is supported by considering the non-analytic contributions to the GFFs in the chiral limit, for which all LECs are known and fixed via experiments:

$$A(t) = \frac{g_{\pi N\Delta}^2}{\pi^2 F_\pi^2 m_N^2}\left(\frac{(79\delta + 10m_N)}{5760\delta}t^2 + \frac{(15\delta + 2m_N)}{2304\delta^3}t^3\right)\ln\left(-\frac{t}{m_N^2}\right) + \frac{3g_A^2}{512F_\pi^2 m_N}(-t)^{\frac{3}{2}} - \frac{\left(c_2 m_N - 10g_A^2\right)}{320\pi^2 F_\pi^2 m_N^2}\, t^2 \ln\left(-\frac{t}{m_N^2}\right) + \mathcal{O}(t^{\frac{5}{2}})$$

$$J(t) = \frac{g_{\pi N\Delta}^2\left(4\delta(2\delta + m_N)t^2 + t^3\right)}{2304F_\pi^2\pi^2\delta^3 m_N}\ln\left(-\frac{t}{m_N^2}\right) - \frac{g_A^2}{64\pi^2 F_\pi^2}t\ln\left(-\frac{t}{m_N^2}\right) - \frac{3g_A^2}{512F_\pi^2 m_N}(-t)^{\frac{3}{2}} + \mathcal{O}(t^2)$$

$$D(t) = -\frac{g_{\pi N\Delta}^2(214\delta^3 + 10m_N(t - 14\delta^2) + 5\delta t)}{2880F_\pi^2\pi^2\delta^3}t\ln\left(-\frac{t}{m_N^2}\right) + \frac{3g_A^2 m_N}{128F_\pi^2}\sqrt{-t} - \frac{\left(5g_A^2 + 4\left(c_2 + 5c_3\right)m_N\right)}{160\pi^2 F_\pi^2}\, t\ln\left(\frac{-t}{m_N^2}\right) + \mathcal{O}(t^{\frac{3}{2}})$$

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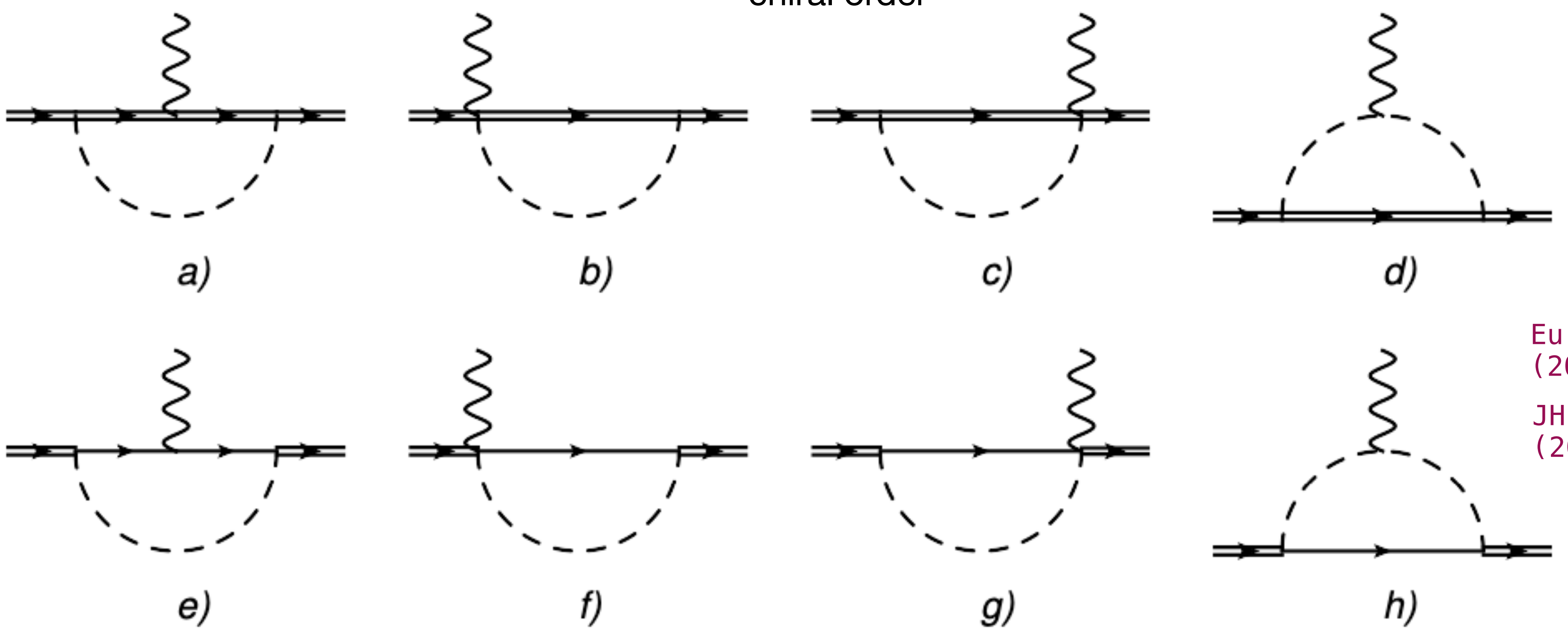

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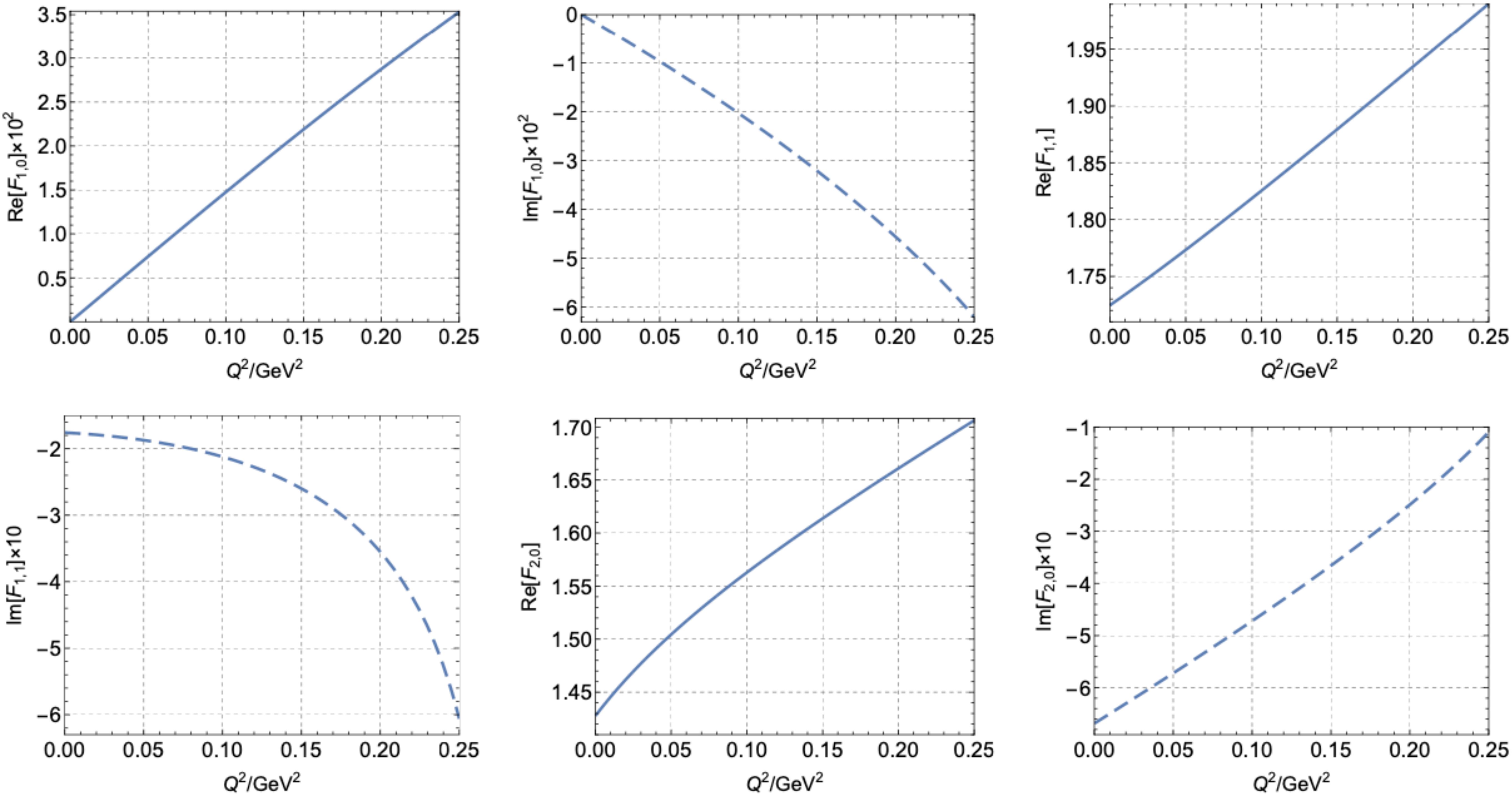
Double, solid and dashed lines correspond to the deltas, nucleons and pions, respectively, while the curly lines represent gravitons.

Part III: Δ GFFs



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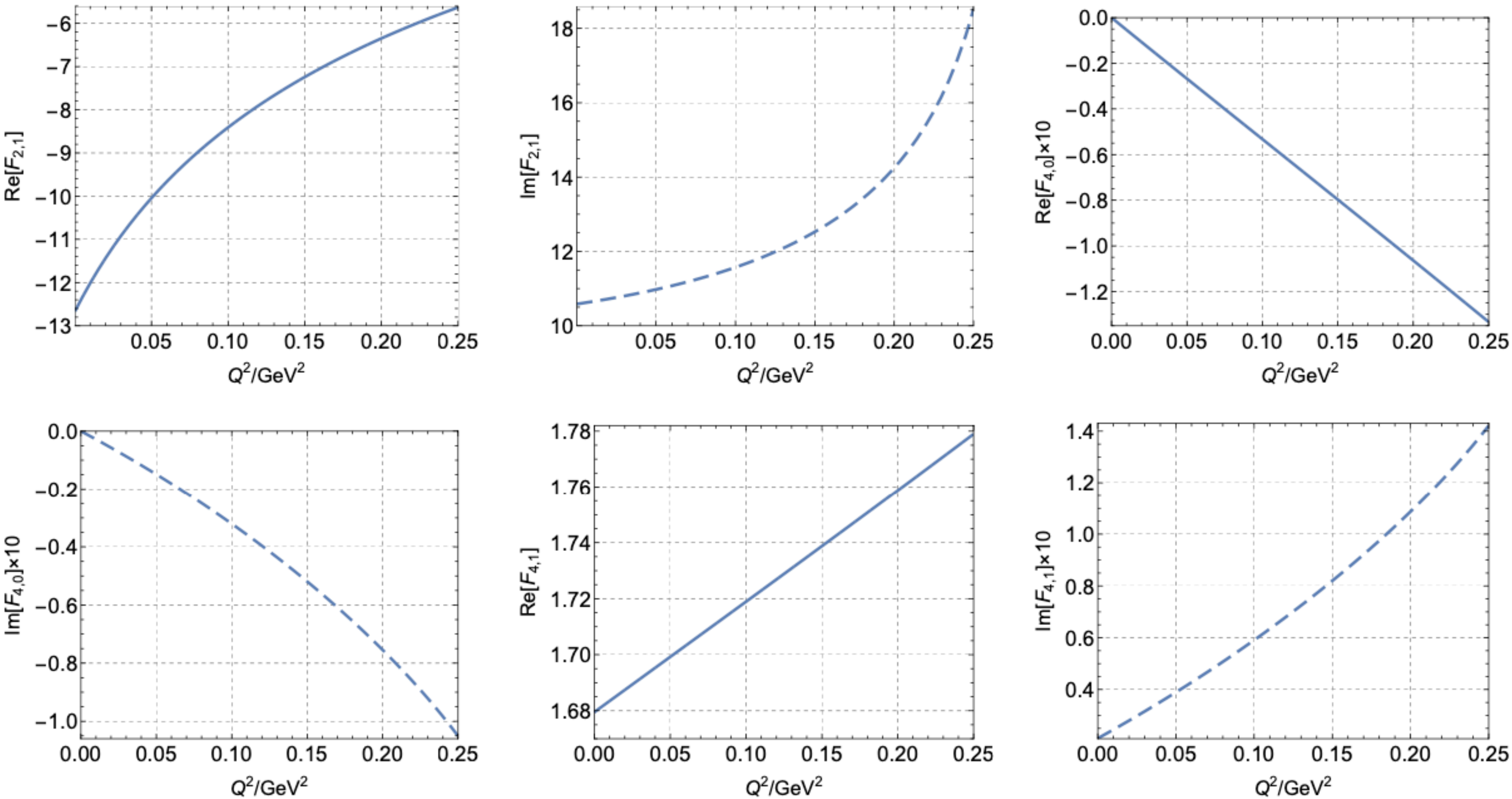

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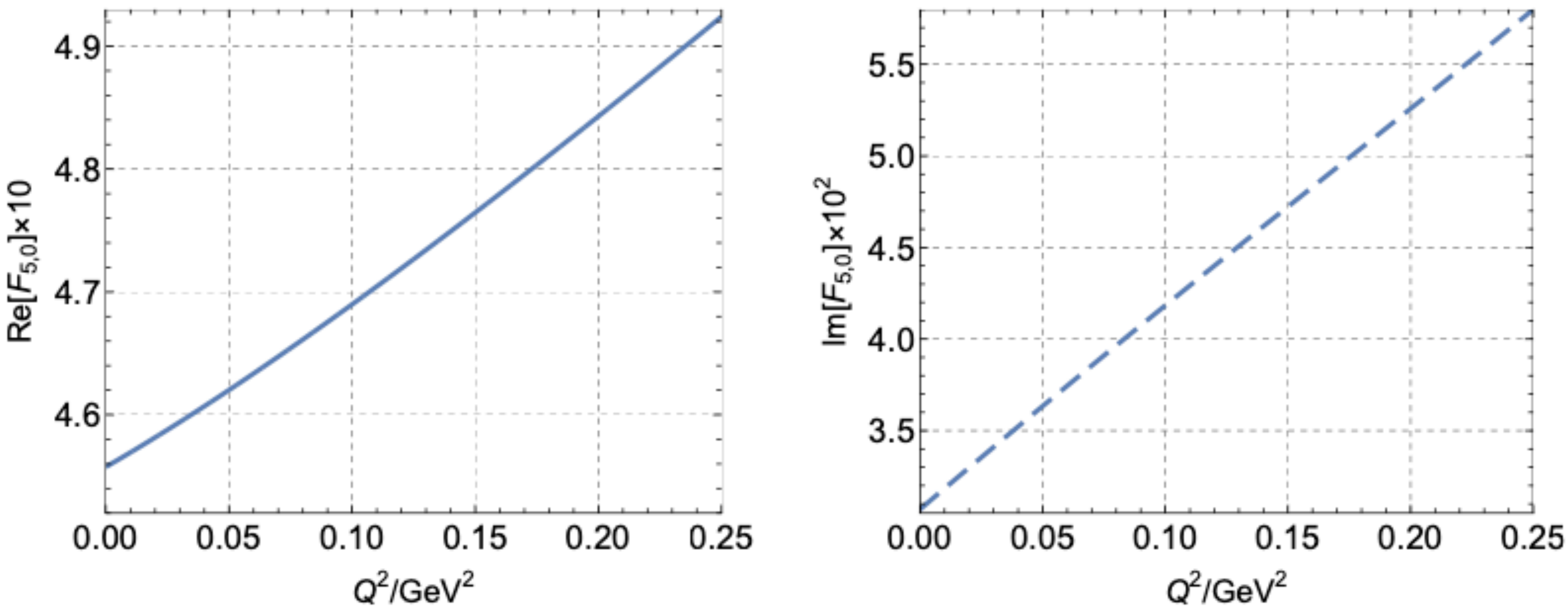

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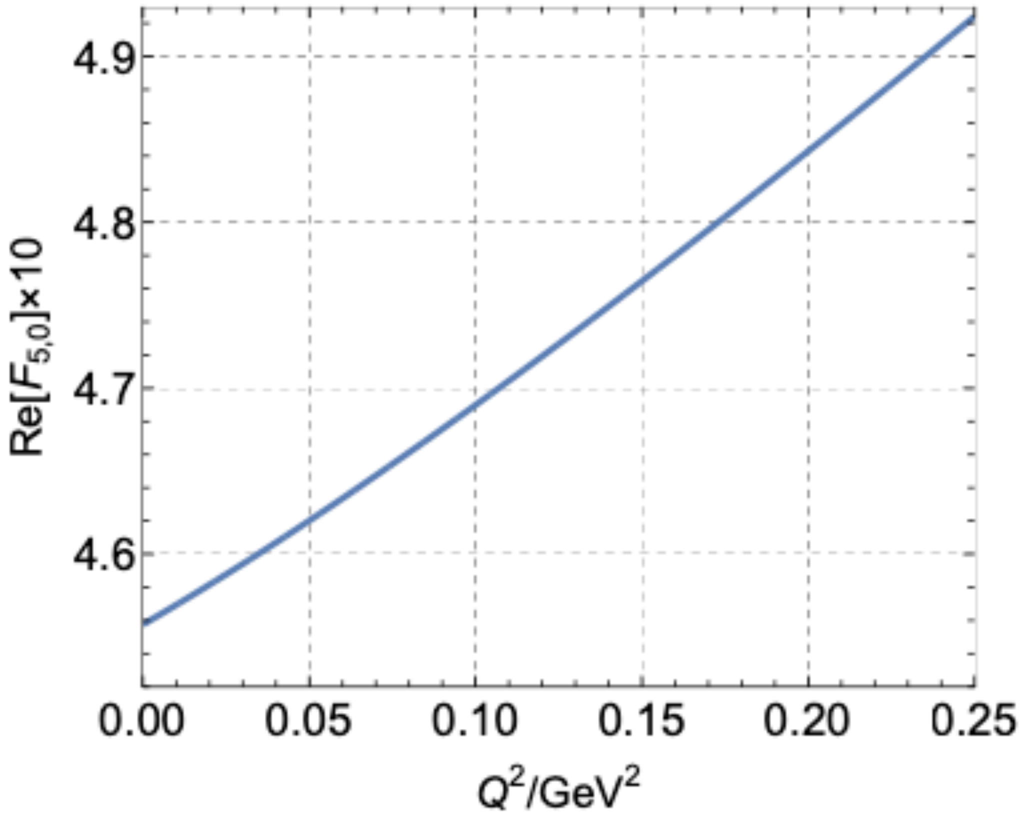

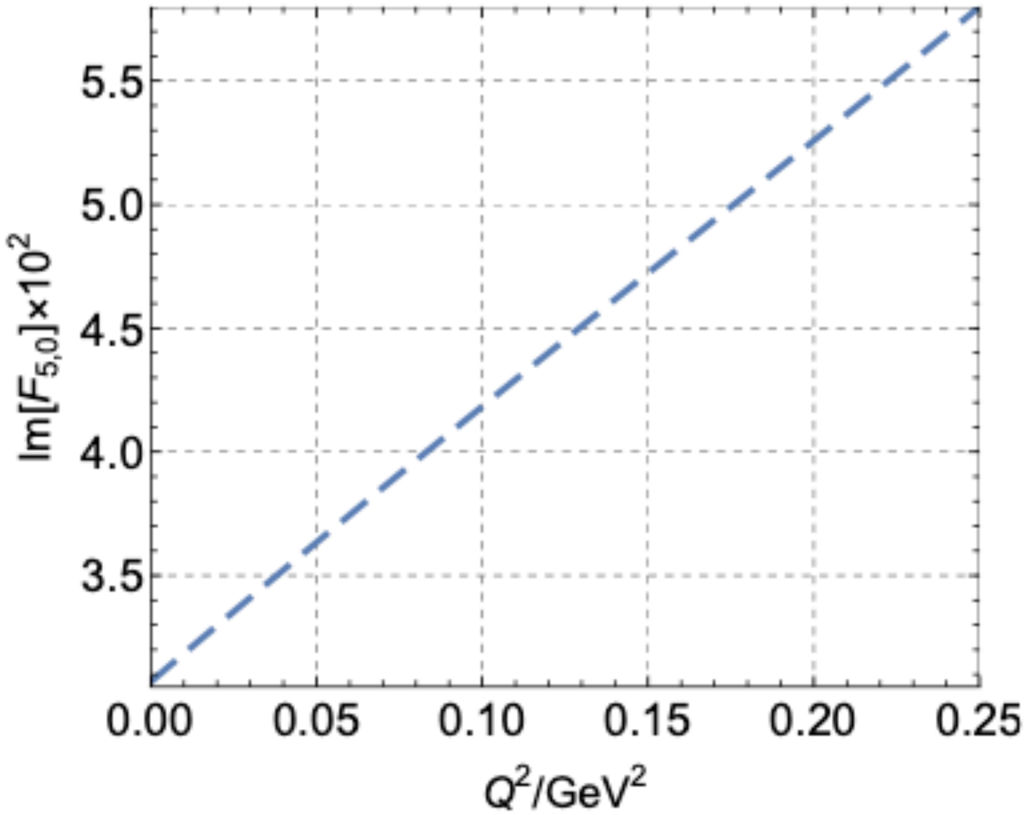

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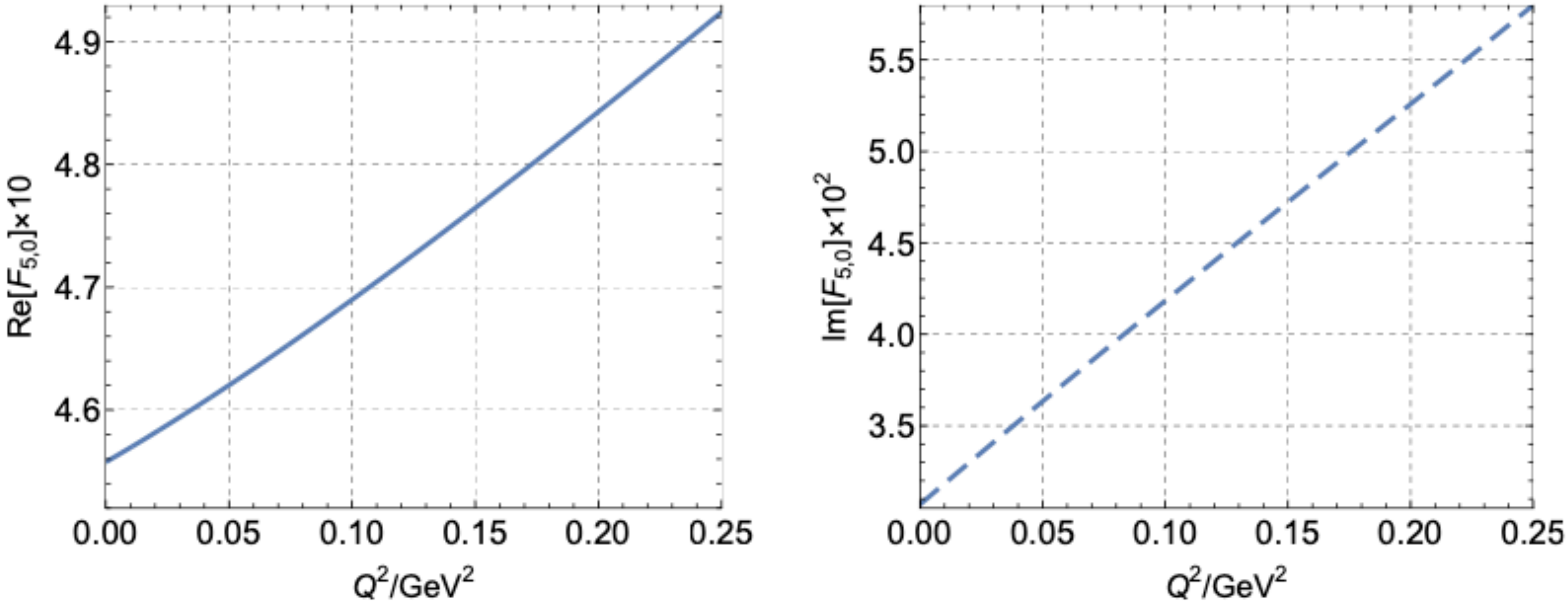

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$$F_{1,0}(t) = -\frac{25 g_1^2}{9216 F^2 m_\Delta}\, t\sqrt{-t}\,, F_{1,1}(t) = -\frac{5 g_1^2 m_\Delta}{1536 F^2}\sqrt{-t}\,, F_{2,0}(t) = \frac{5 g_1^2 m_\Delta}{768 F^2}\sqrt{-t}\,, F_{2,1}(t) = \frac{5 g_1^2 m_\Delta^3}{384 F^2}\,\frac{\sqrt{-t}}{t}$$

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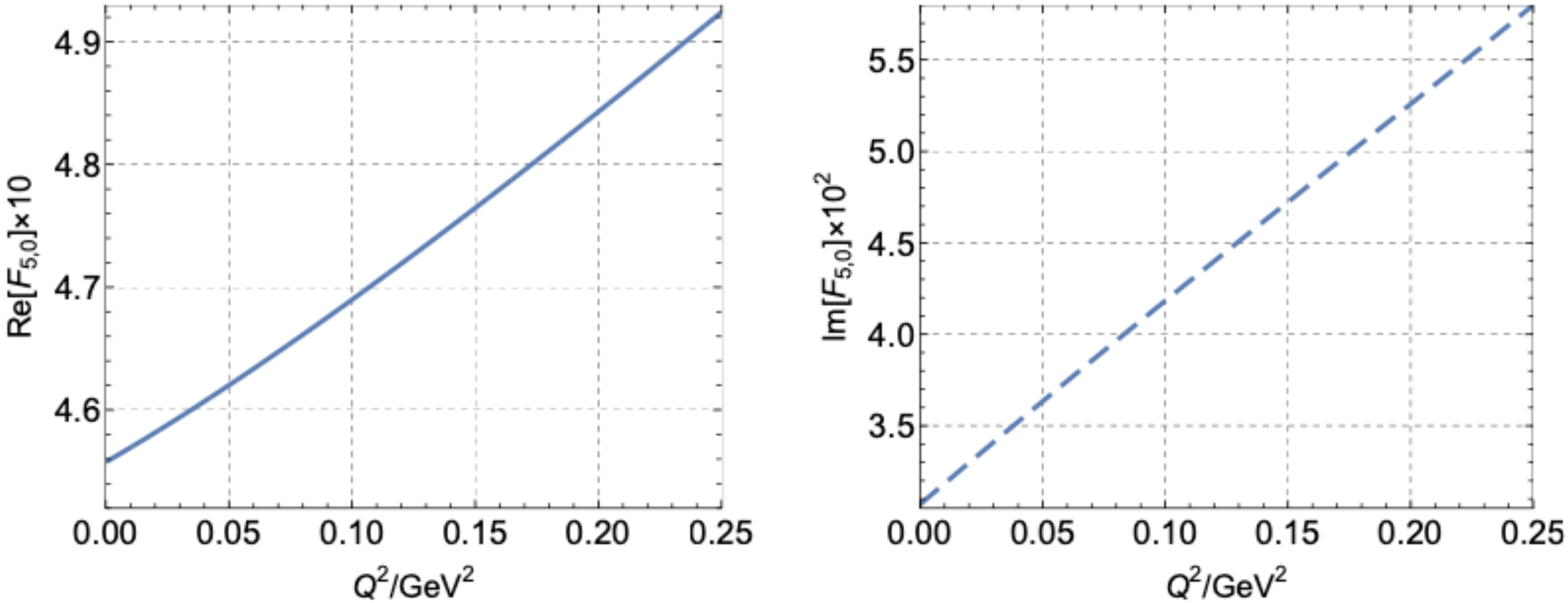

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JHEP **03** (2024), 007

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For more details, see the talk by Bao-Dong Sun

Part V: transition GFFs of the OPGP

- The amplitude of the OPGP has the following general form

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Phys. Rev. D **109** (2024) no.1, 016009

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$\tilde{\Delta} = p_f + q - p_i\,, P = p_f - q + a\ p_i\,, \Lambda = p_f + q + b\ p_i\,,$ with

$$a = -1 - \frac{2M_\pi^2 - 2t}{m_N^2 - s + \tilde{t}}\,, b = \frac{-m_N^2 + s + \tilde{t}}{m_N^2 - s + \tilde{t}}$$

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- Moreover, we write $\gamma^\mu \gamma_5$ and $\gamma^\lambda \tilde{\Delta}_\lambda \gamma_5$ in terms of structures that contain $\epsilon^{\alpha\beta\gamma\kappa}$

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Part V: transition GFFs of the OPGP

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- In general, one can define P and Λ differently, such that the parametrization for $O^{\mu\nu}$ remains the same, as long as they are linearly independent and orthogonal to $\tilde{\Delta}$

Comment

V. Burkert "DVCS and the Gravitational Structure of the Proton 9/22/2018“ :

„It can be shown that any massless spin-2 field would give rise to a force indistinguishable from gravitation, because a massless spin-2 field would couple to the stress–energy tensor in the same way that gravitational interactions do…. Except that the ”spin-2” field from DVCS is many orders of magnitude stronger than gravitation. ”

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For more details, see the talk by Iuliia Panteleeva

Thank you very much for listening!