Studying electroweak few-body observables in chiral effective field theory

The 11th International Workshop on Chiral Dynamics
Ruhr University Bochum, Germany
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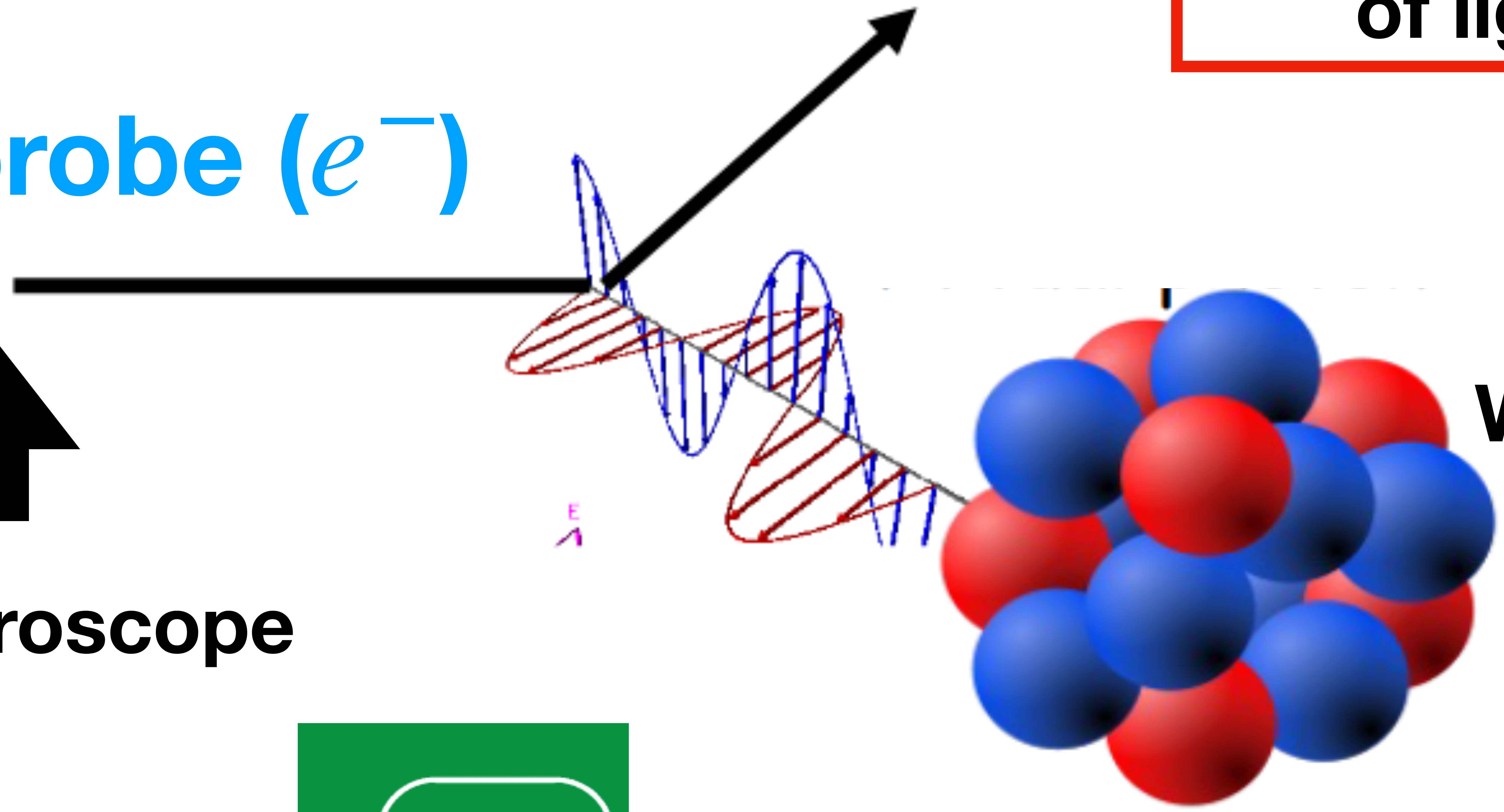
OLD DOMINION UNIVERSITY®

Jefferson Lab

Studying the nature of the strong interaction

Magnetic form factors of light nuclei

External probe (e^-)



What we want to study

The microscope

Nucleus

Jefferson Lab

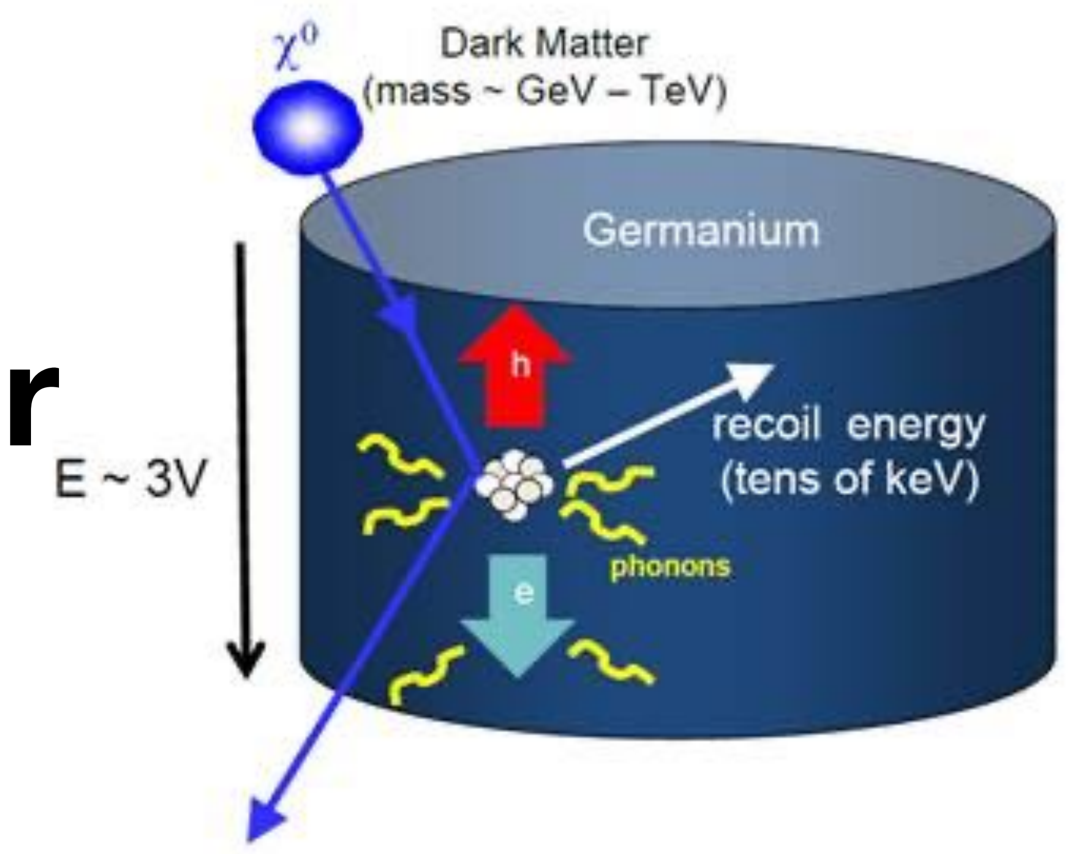
mami MAINZER MIKROTRON

EIC

Fundamental physics with nuclei

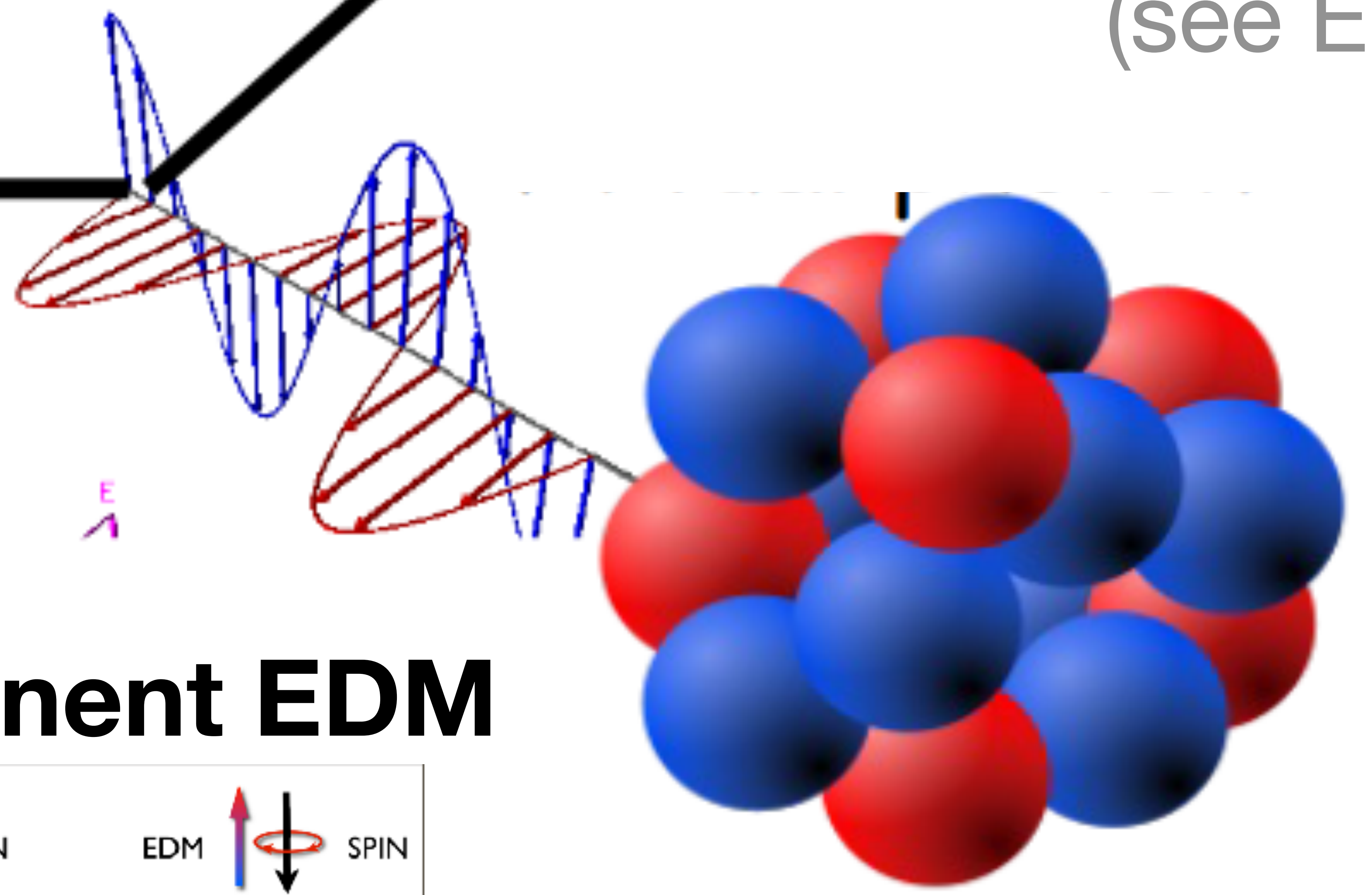
What we want to study

Dark matter



(see Elena Filandri's talk)

External probes

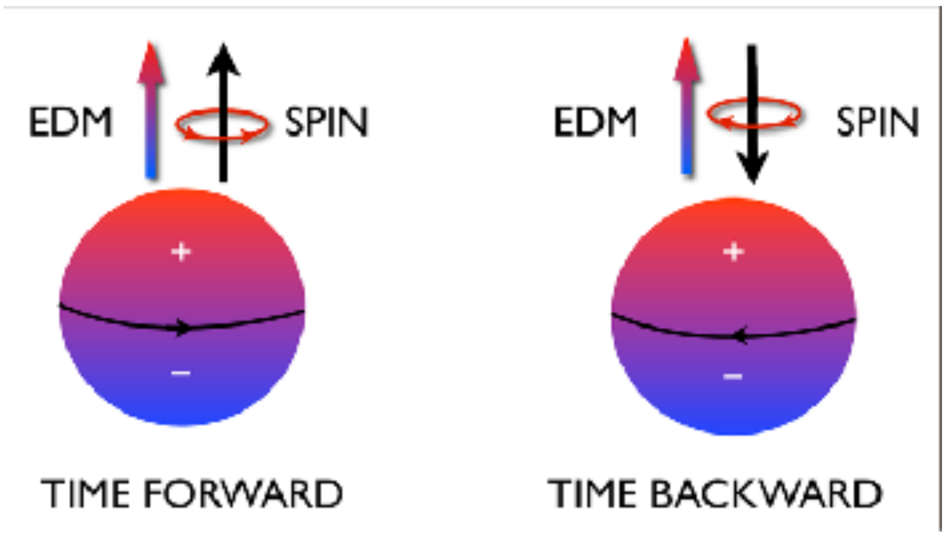


Neutrino Physics

DUNE

T2K

Permanent EDM



Nucleus

The microscope

An accurate understanding of nuclear structure and dynamics is needed to extract new physics from nuclear effects

↓

Comprehensive quantitative and predictable theory description of all nuclear structure and reaction

The ingredients

- **The interaction**

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V_{ij} + \sum_{i,j,k} V_{ijk} + \cdots$$

- **A way of solving the few- and many-body problem**
    - **Hyperspherical Harmonic method** [Front. Phys. 8, 69 (2020)]
    - **Neural Network Quantum States** (a small advertisement)
    - **Monte Carlo Methods** (see Maria's talk)
    - **Many more…**

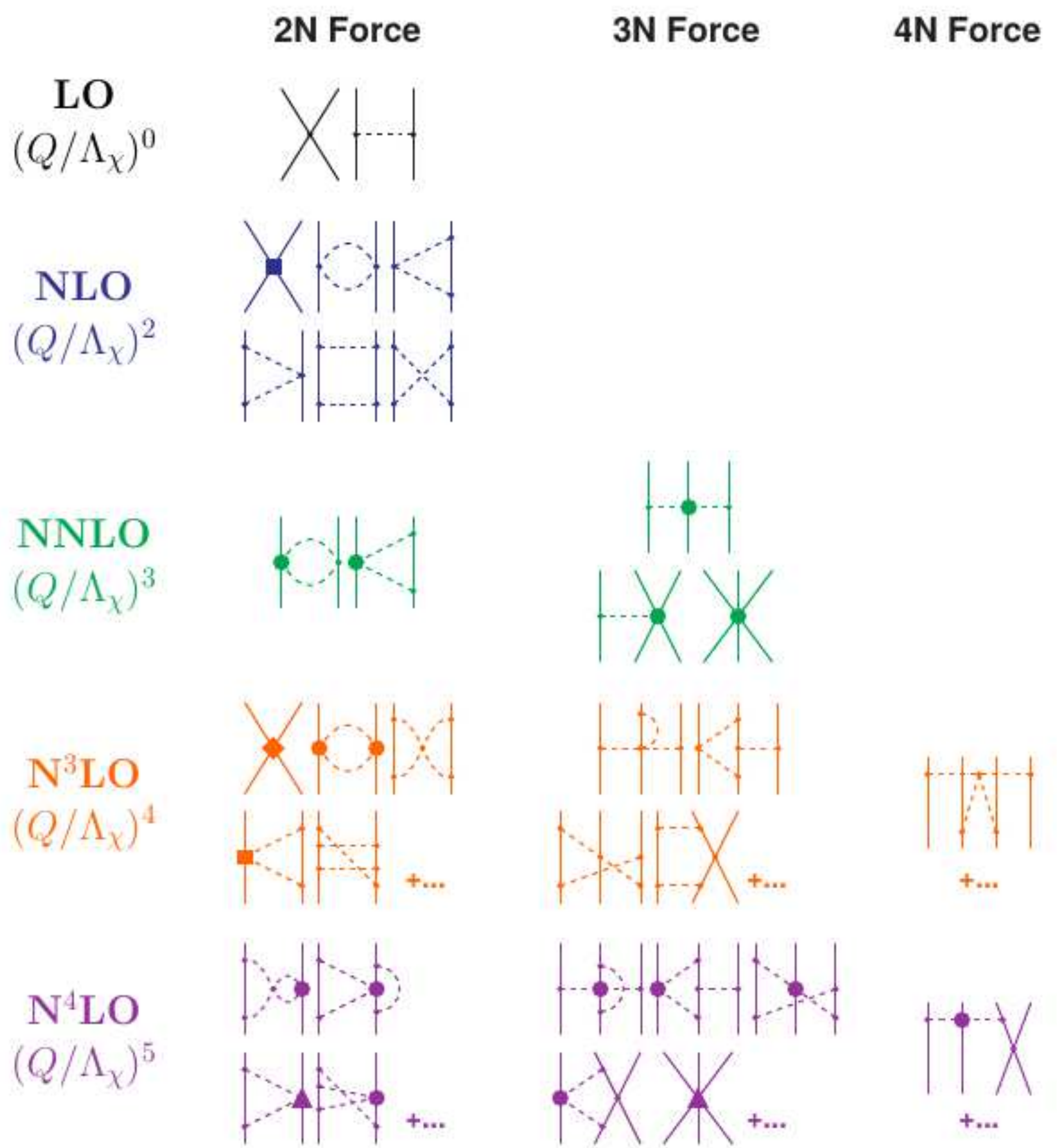
Chiral effective field theory

- Only Nucleons and Pions (and Deltas) as degrees of freedom ($\Lambda_{QCD} \sim 1$ GeV)

- Direct connection with QCD: chiral symmetry (+ discrete symmetries + Lorentz invariance)

- Low Energy Constants (LECs): fitted on experimental data

- Organize the interaction as a power expansion Q/Λ_{QCD}



Phys. Rev. C **96**, 024004 (2017)
Phys. Rev. Lett. **115**, 122301 (2015)

Why chiral EFT?

- **Consistent treatment of interactions with external probes and BSM physics**

→ Electric dipole moments of light nuclei

- **Reliable estimate of the uncertainties generated by the theory**

→ Bayesian analysis of muon capture on deuteron

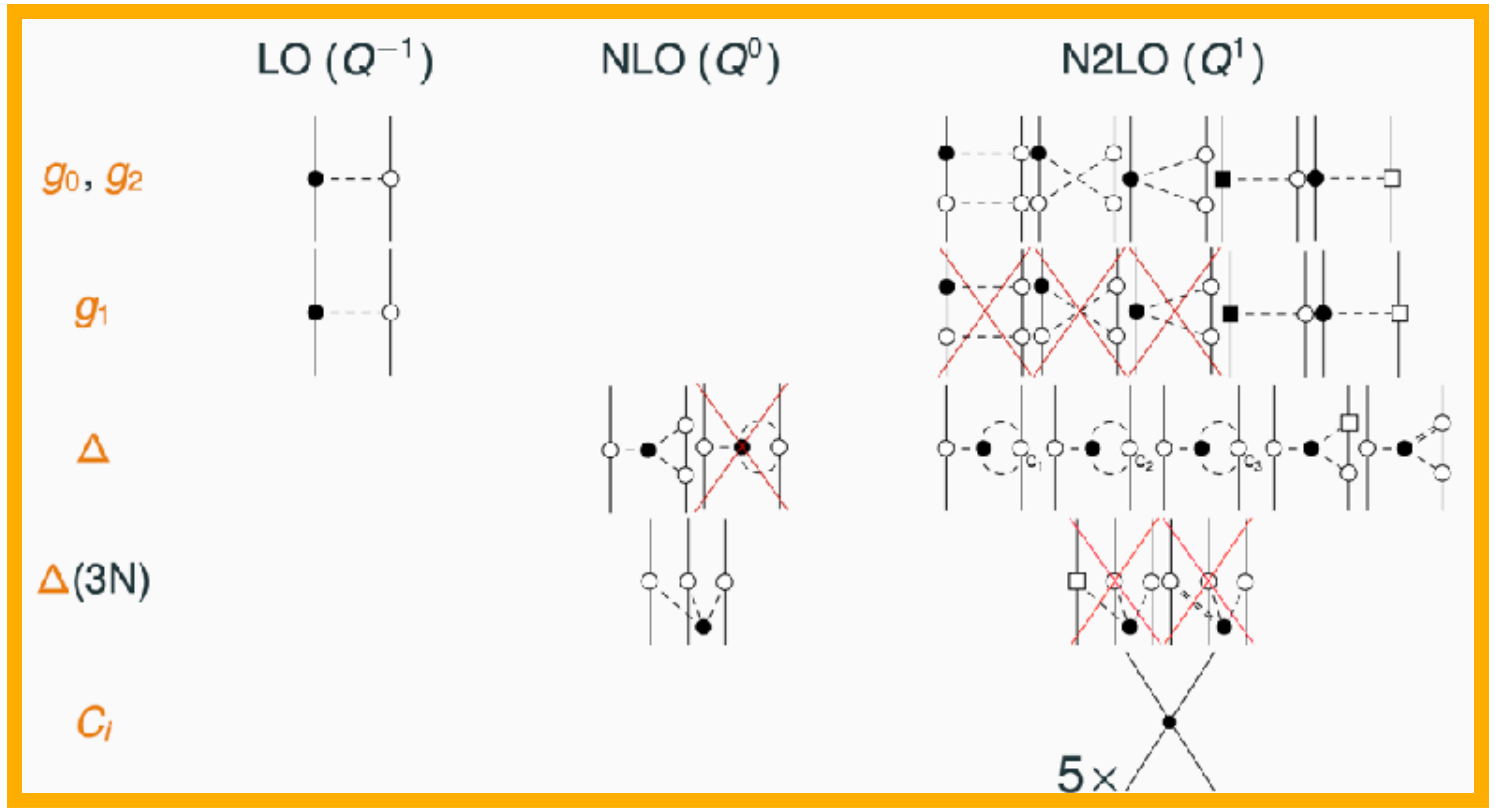
The electric dipole moment

Fundamental theory (SM,BSM,…) with time reversal violation

LECs

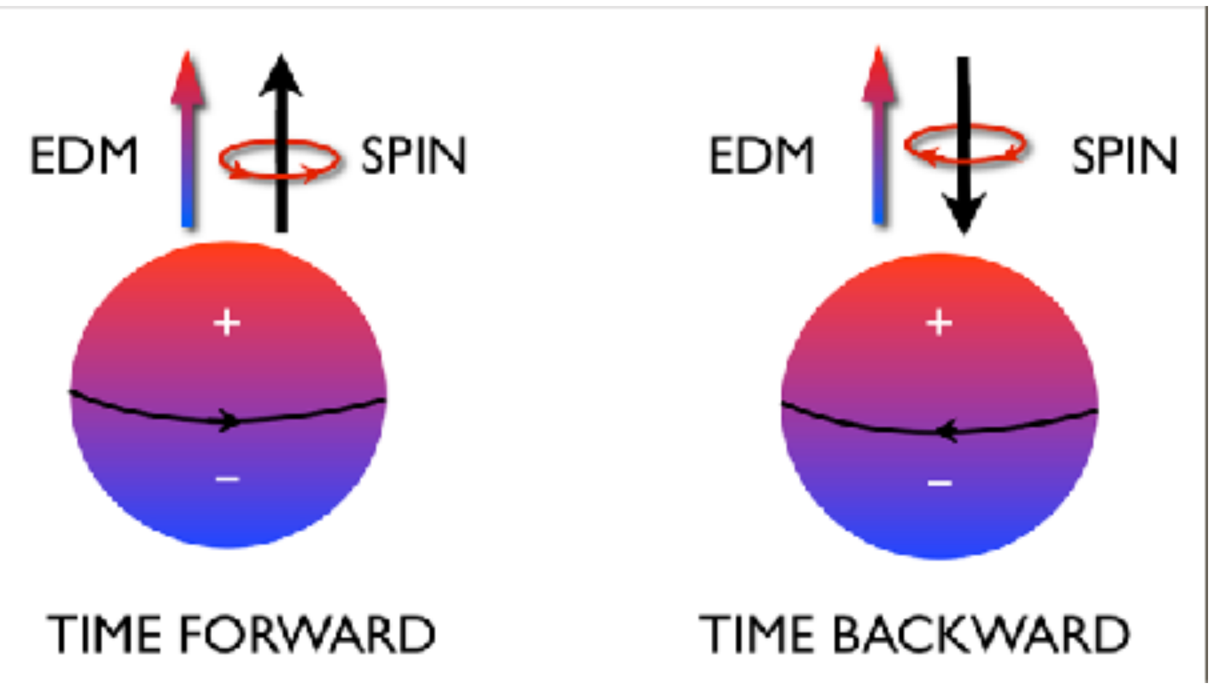
$$d_A = d_n A_n + d_p A_p + g_0 A_0 + g_1 A_1 + \dots$$

Nuclear dynamics



Nuclear potential + ab-initio

[AG et al., PRC 101, 024004 (2020)]
[Front. Phys. 8, 218 (2020)]
(see Lukas talk on Thursday for updates)



Experiment

Muon capture on deuteron

[AG et al., Phys. Rev. C 109, 035502 (2024)]

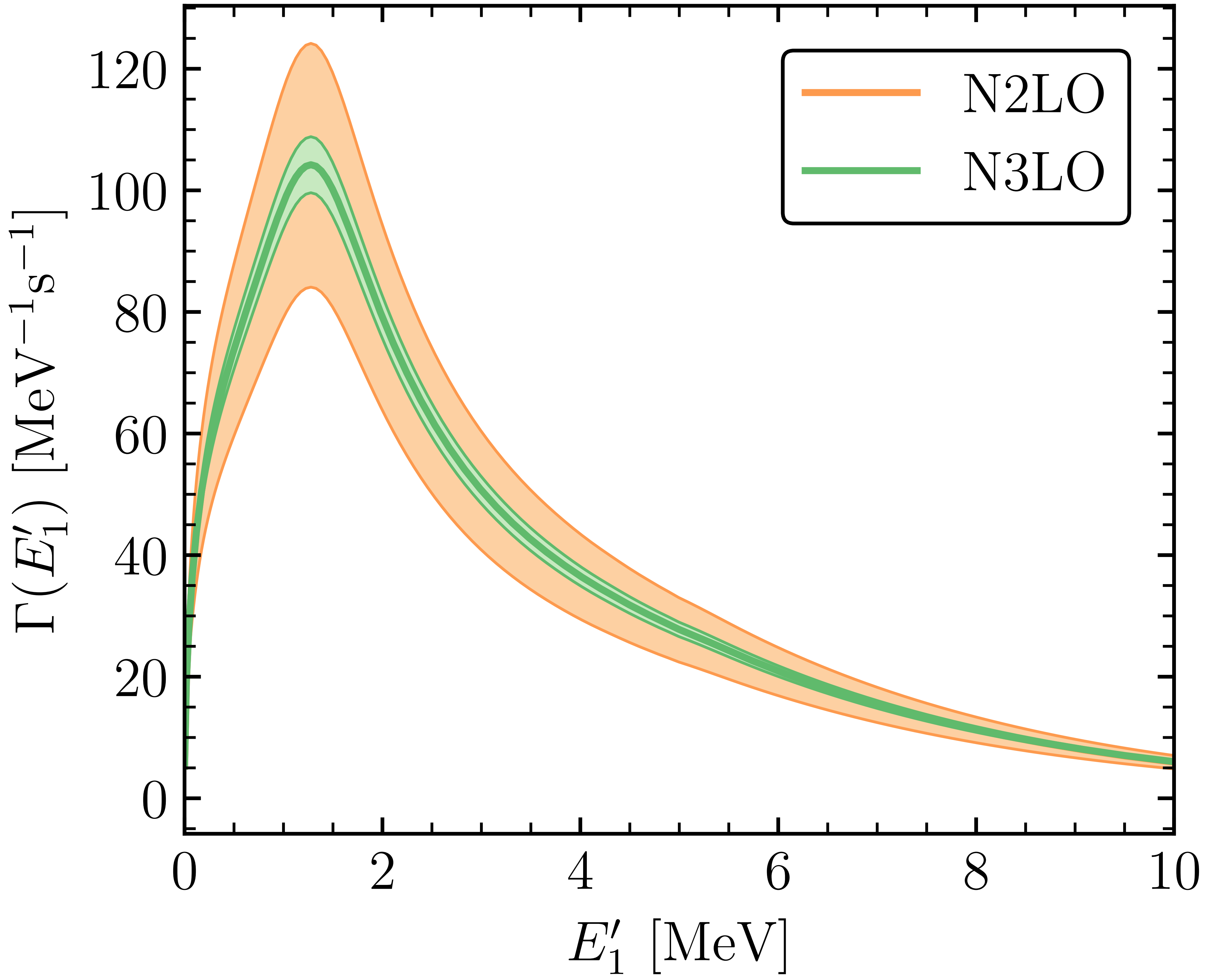
$$\mu^- + d \rightarrow n + n + \nu_\mu$$

$$\Gamma_k(p) = \Gamma_{ref}(p) \sum_{n=0}^{k} \boxed{c_n(p)} Q^n(p)$$

Gaussian process + Bayes' theorem

$$\delta\Gamma_k(p) = \Gamma_{ref}(p) \sum_{n=k+1}^{\infty} \boxed{c_n(p)} Q^n(p)$$

Analysis based on: PRC **100**, 044001 (2019) (BUQEYE Coll.)



Emitted single nucleon spectra with truncation error bands

Muon capture on deuteron

[from Phys. Rev. C 109, 035502 (2024)]

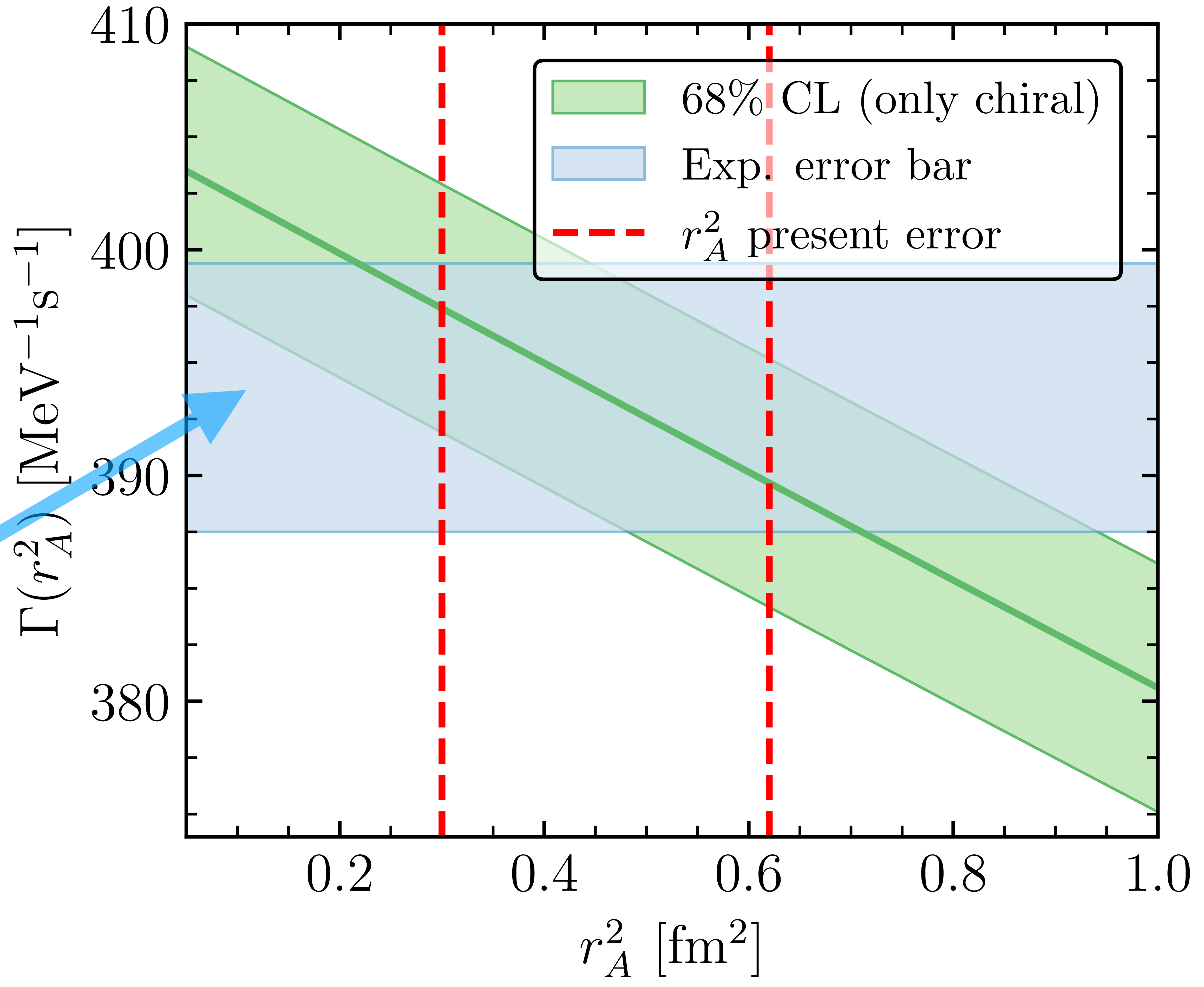
$$\mu^- + d \rightarrow n + n + \nu_\mu$$

Strong dependence on the
axial radius*

$$g_A(q^2) = g_A\left(1 - \frac{1}{6} r_A^2 q^2\right)$$

The 1.5% error bands of MuSun experiment

(hypothetical central value)



* already observed in [PRC 107, 065502 (2023)]

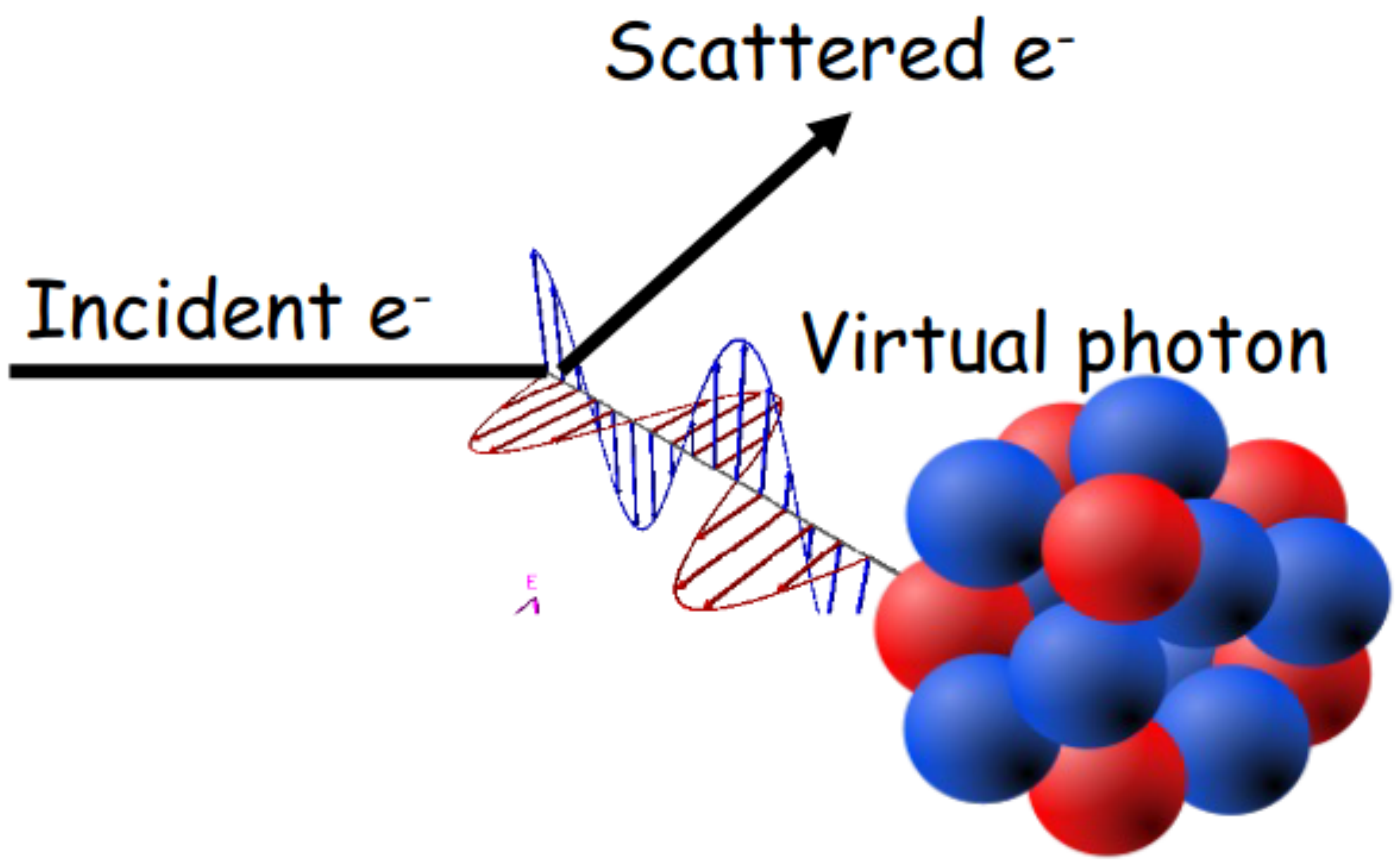
Electromagnetic currents and magnetic form factors of light nuclei

QMC Group WashU:
G. Chambers-Wall, G. B. King, S. Pastore, M. Piarulli
A.G., R. Schiavilla, R. B. Wiringa

[PRC 106, 04401 (2020),
arXiv:2407.03487,
arXiv:2407.04744]

Elastic scattering of electrons on nuclei

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\text{rec}}^{-1} \left[\frac{Q^4}{q^4} F_L^2(q) + \left(\frac{Q^2}{2q^2} + \tan^2\theta_e/2 \right) F_T^2(q) \right]$$



$$F_L(q) \propto \langle \psi | \rho(q) | \psi \rangle$$

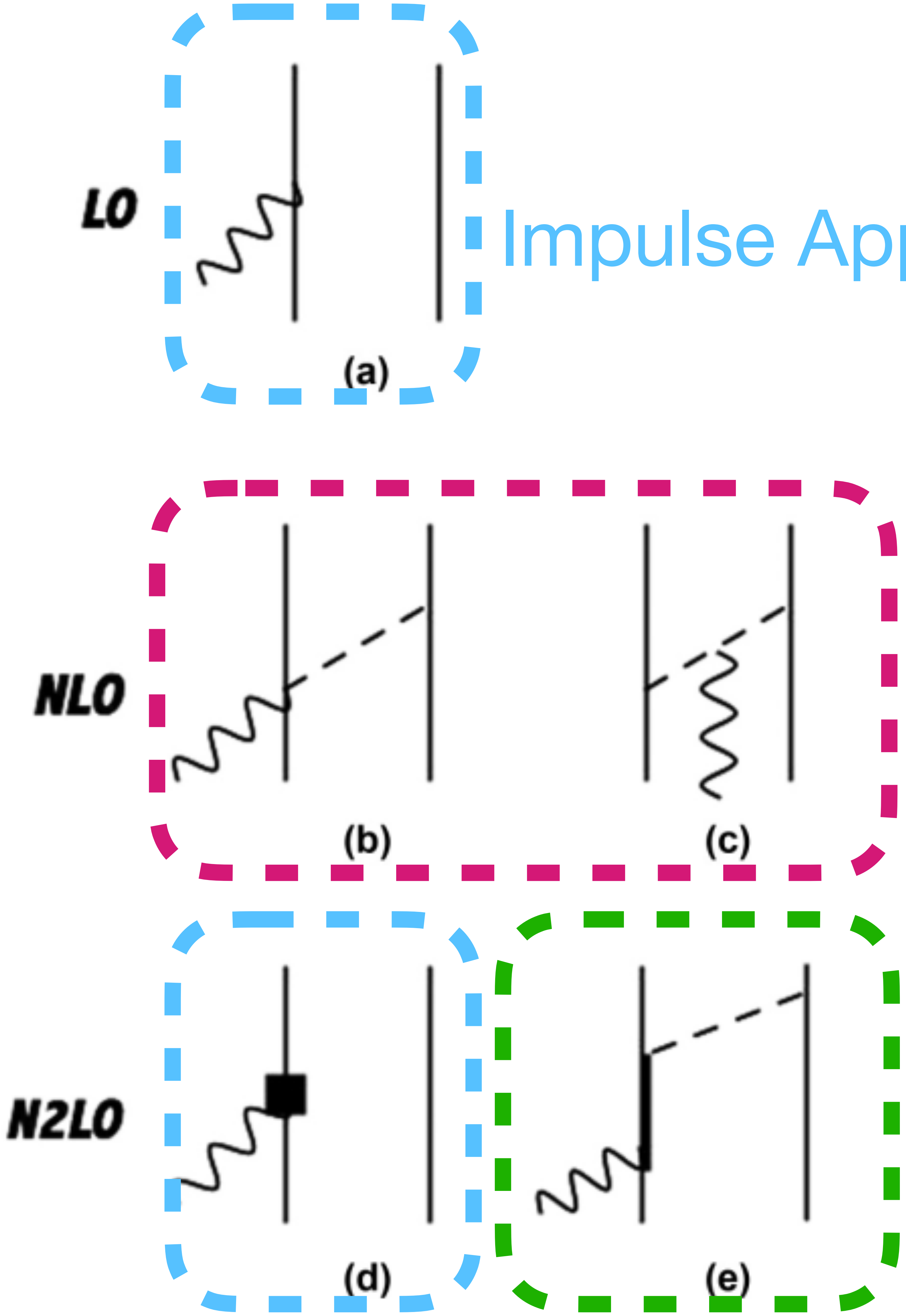
$$F_T(q) \propto \langle \psi | j_y(q) | \psi \rangle$$

$$H | \psi \rangle = E | \psi \rangle$$

Norfolk interactions [PRC 94, 054007 (2016)]

The electromagnetic currents

Currents from:
[PRC 80, 034004 (2009)]
[PRC 99, 034005 (2019)]

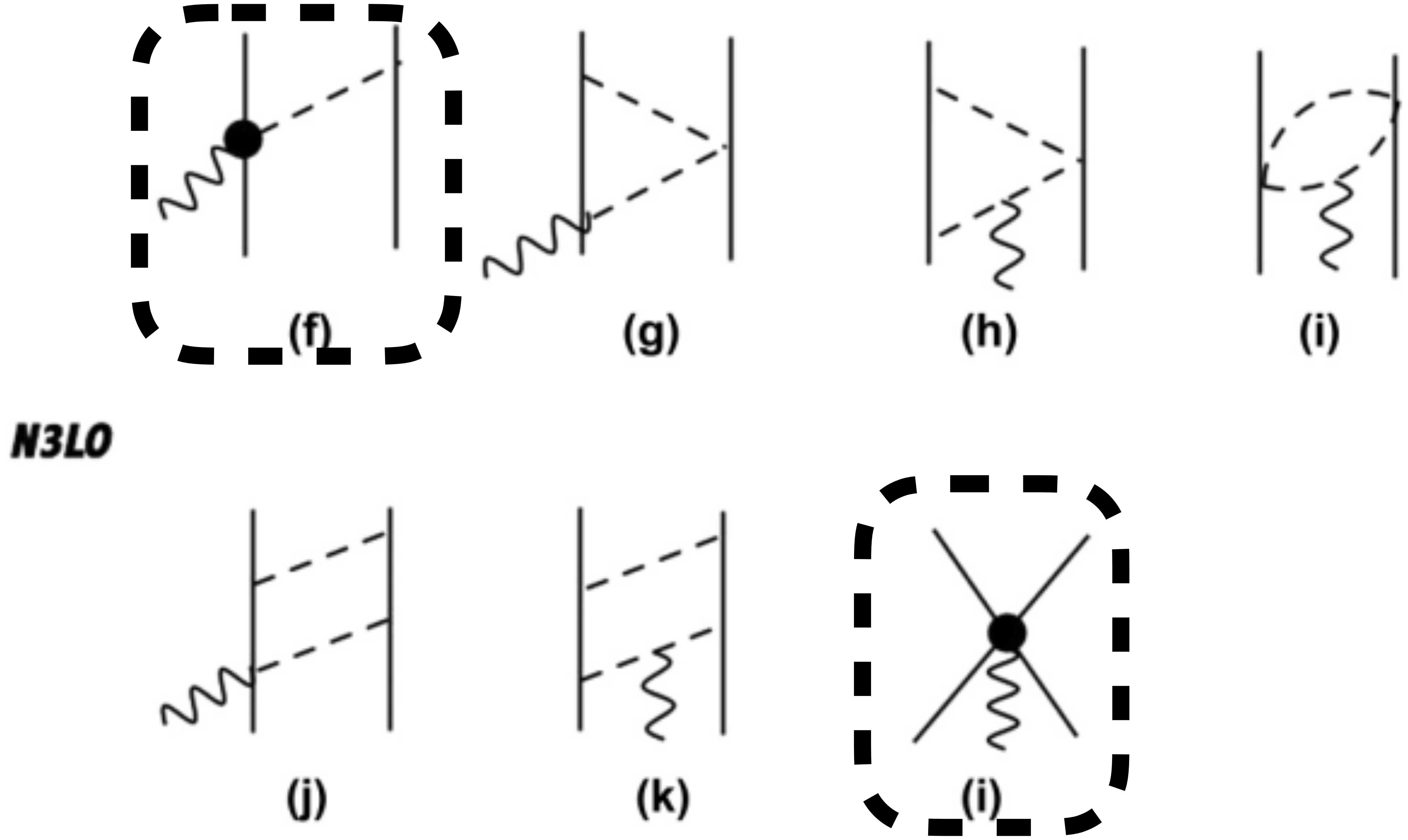


Impulse Approximation

OPE

Δ-N2LO

d_2^V d_3^V d_2^S N3LO-OPE



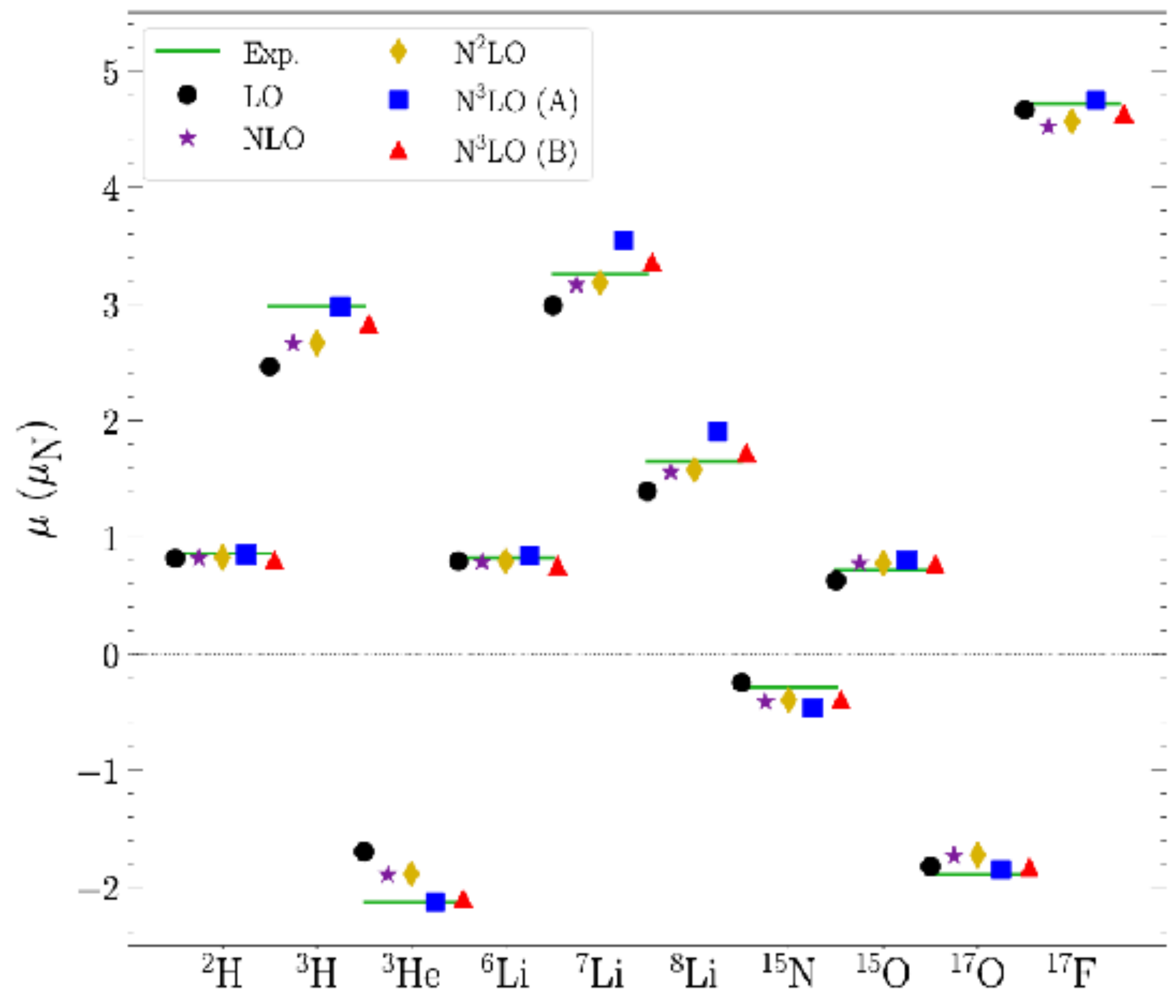
d_1^V d_1^S contact terms

Red: isoscalar Blue: isovector

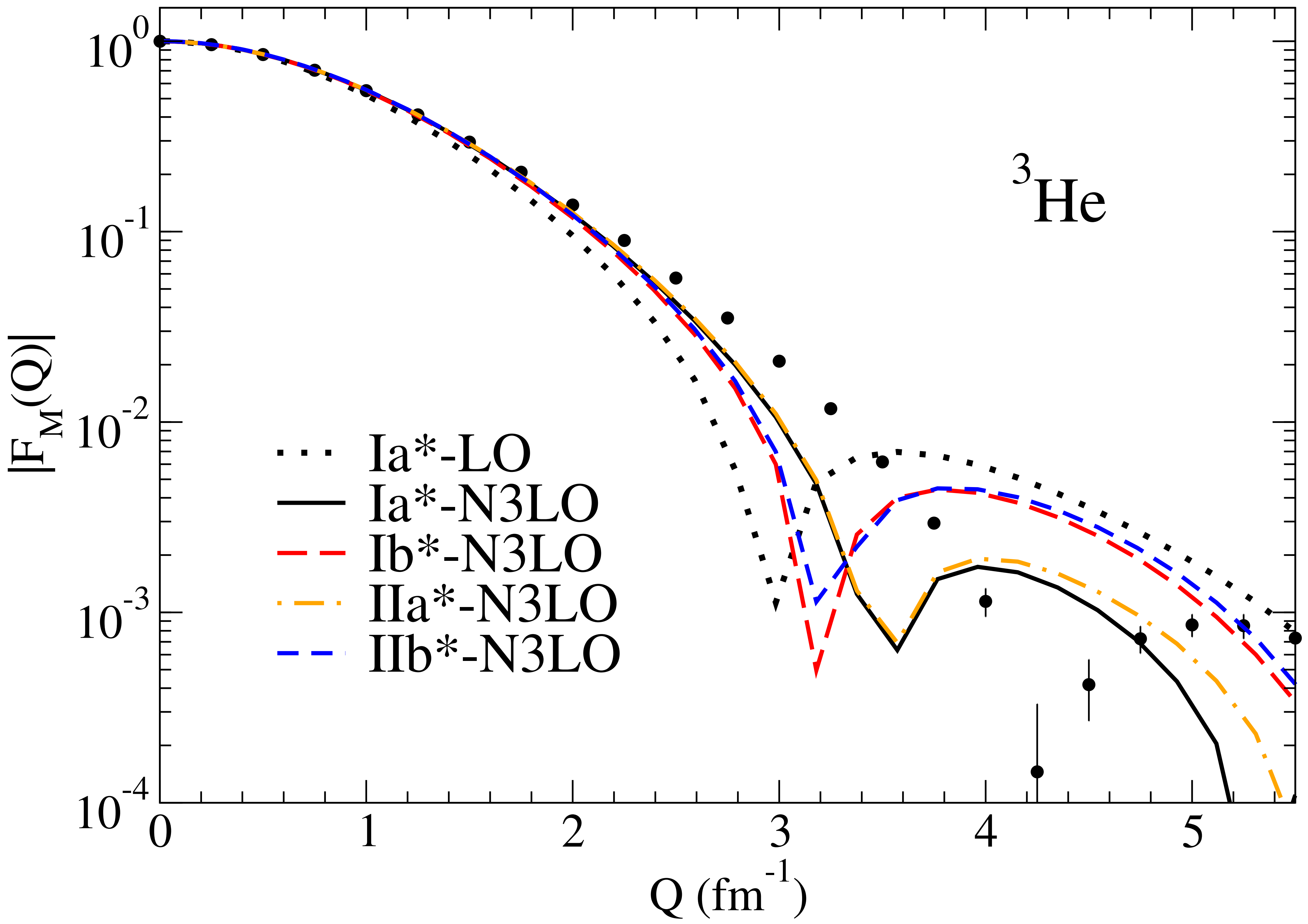
See also [H. Krebs, EPJA 56, 234 (2020)] for another derivation

LECs determination (classic approach)

Usually determined using the magnetic moments of light nuclei



[J.D. Martin et al., PRC 108, L031304 (2023)]



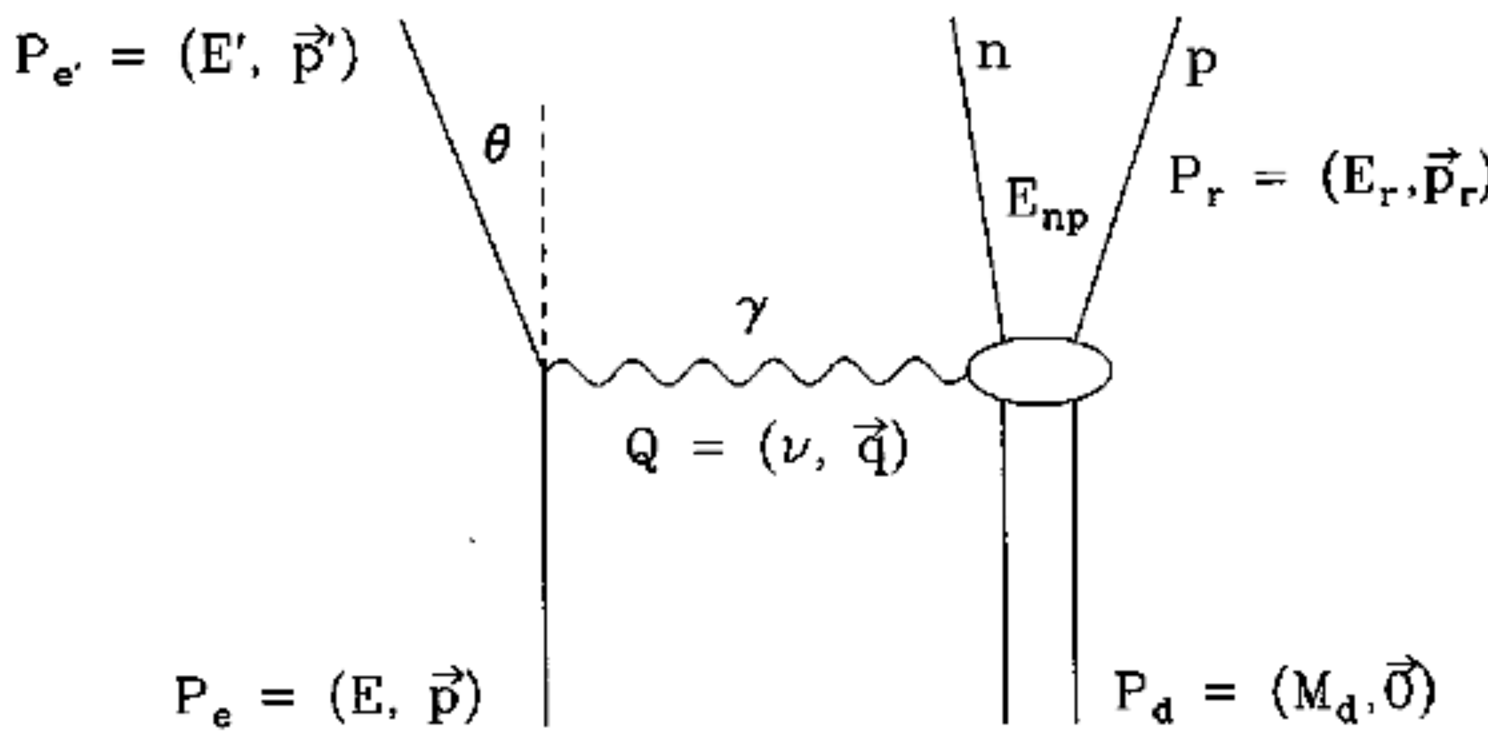
[R. Schiavilla et al., PRC 99, 034005 (2019)]

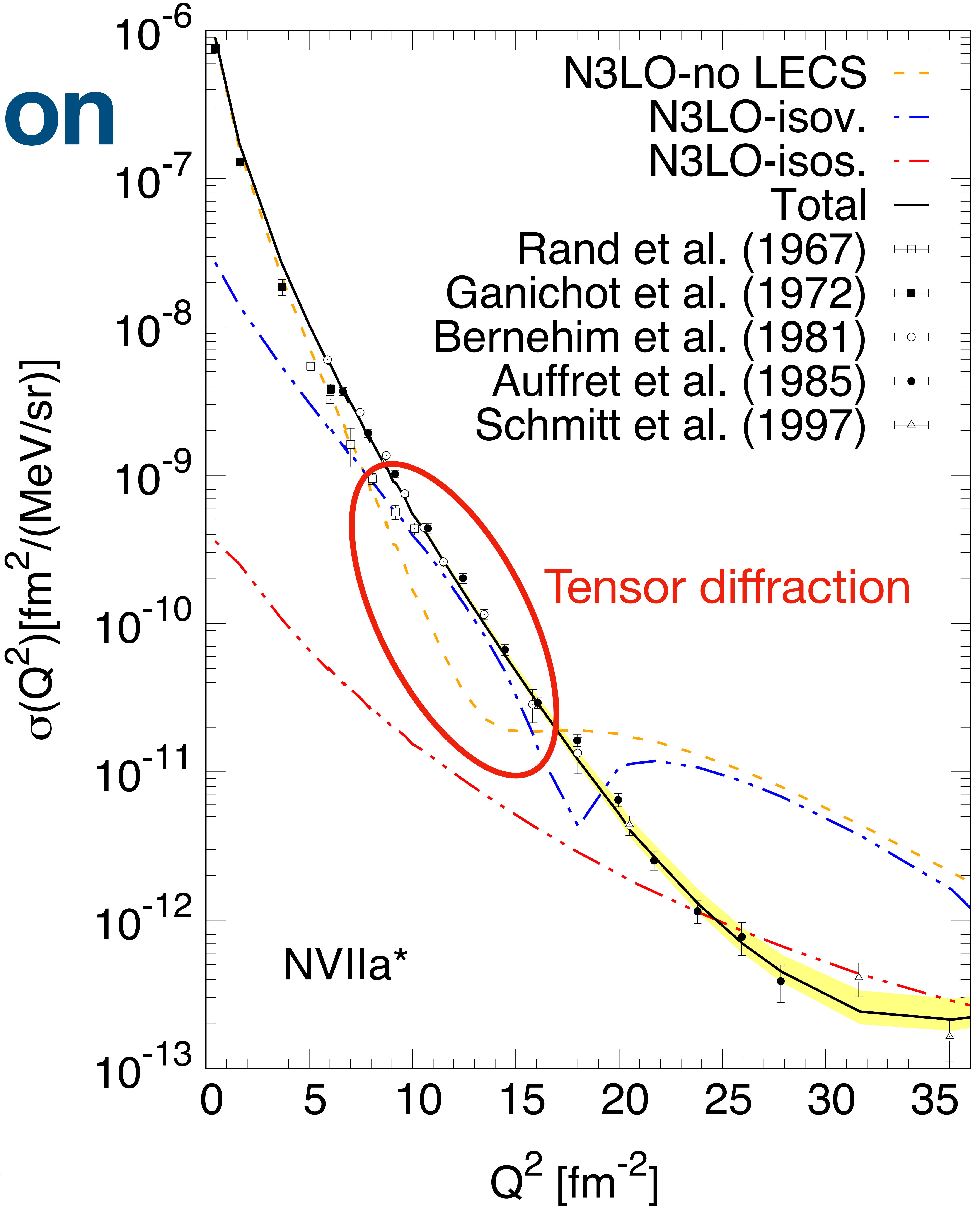
Diffraction generated by tensor forces

New LECs determination

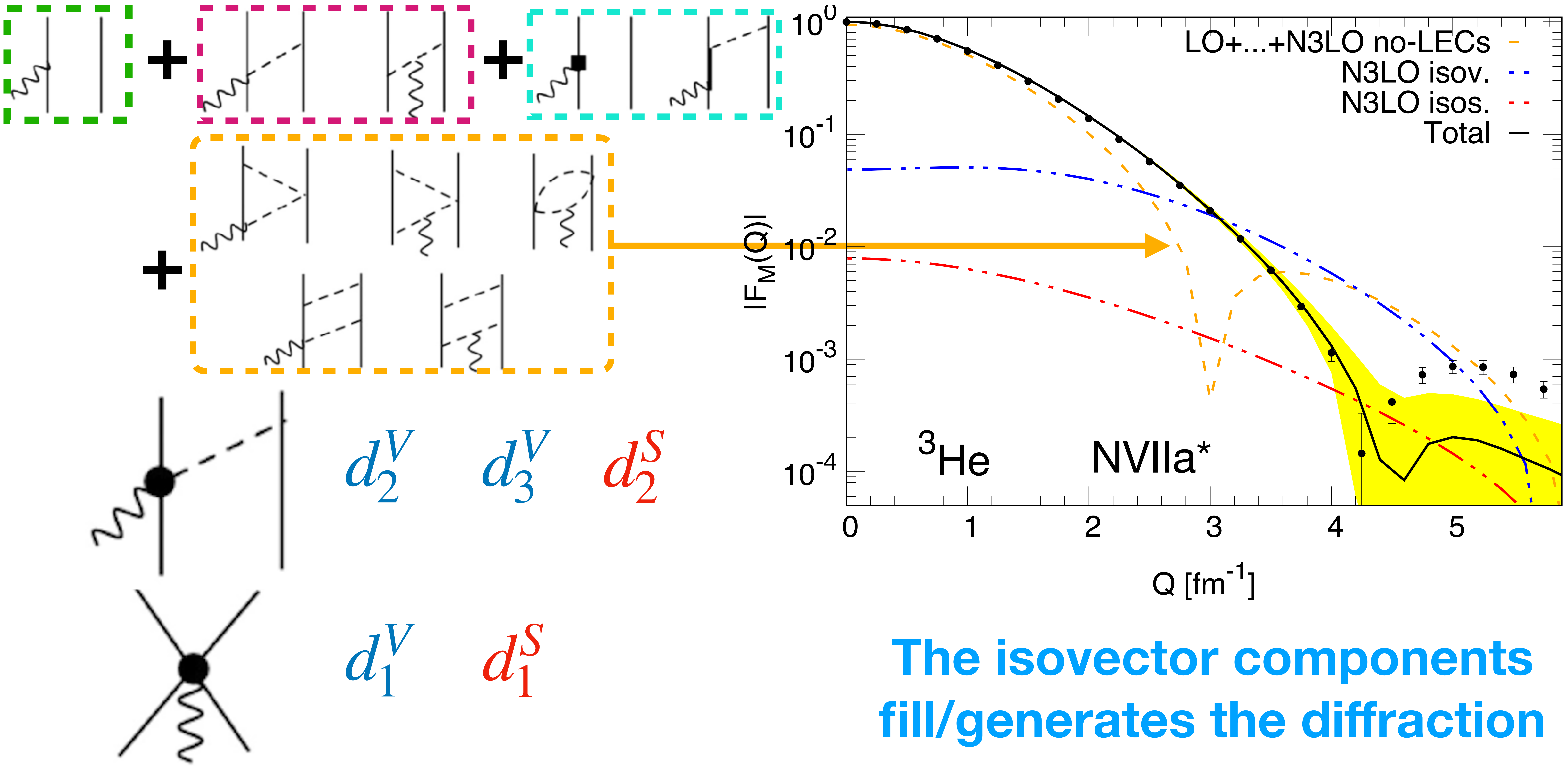
PRC 106, 04401 (2020)

- Magnetic moments of d, ^{3}He, ^{3}H (fix normalization)
- Deuteron-threshold electrodisintegration at backward angles (fix dynamics)





Prediction of A=3 Magnetic Form Factor

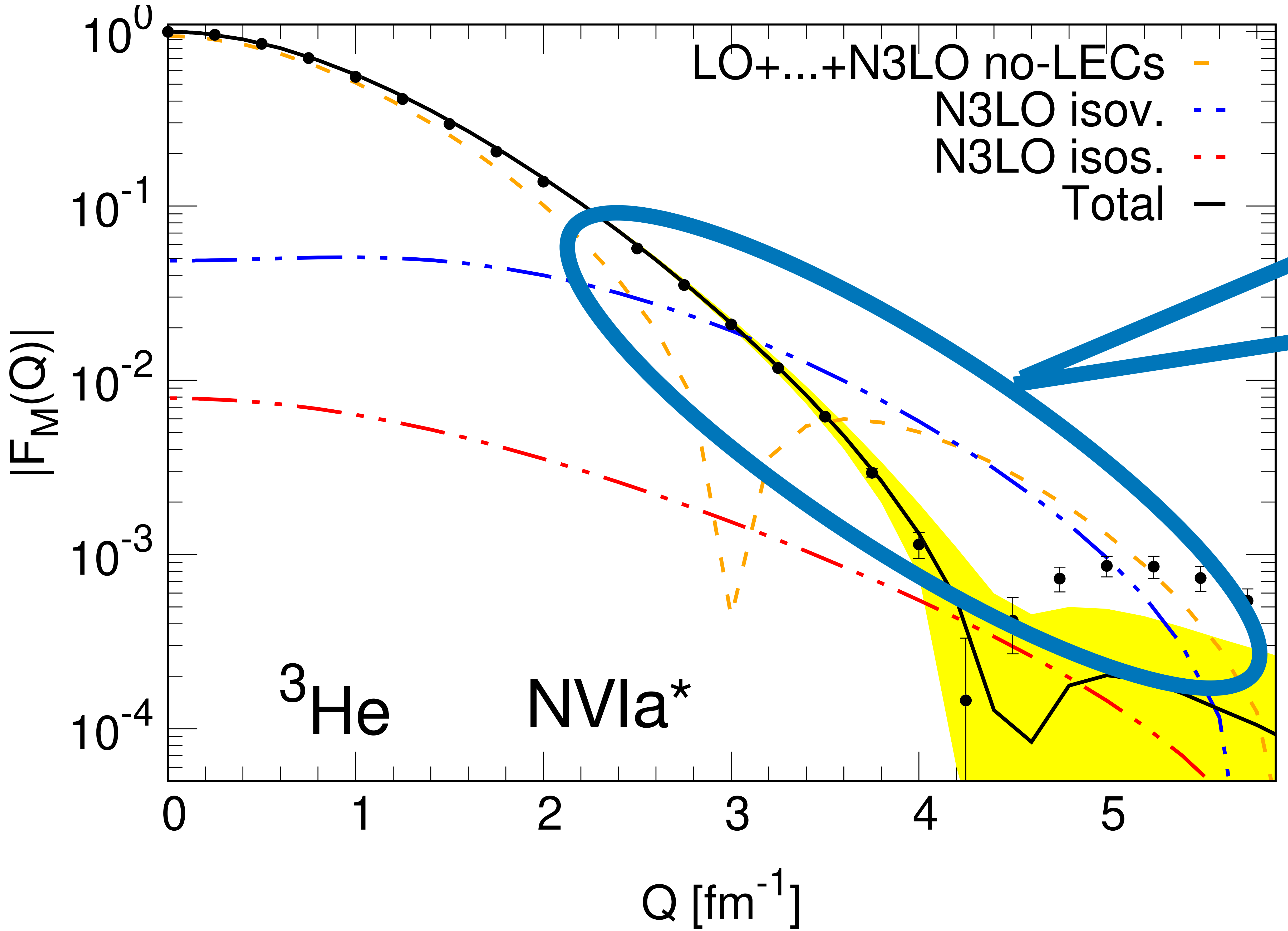


The isovector components fill/generates the diffraction

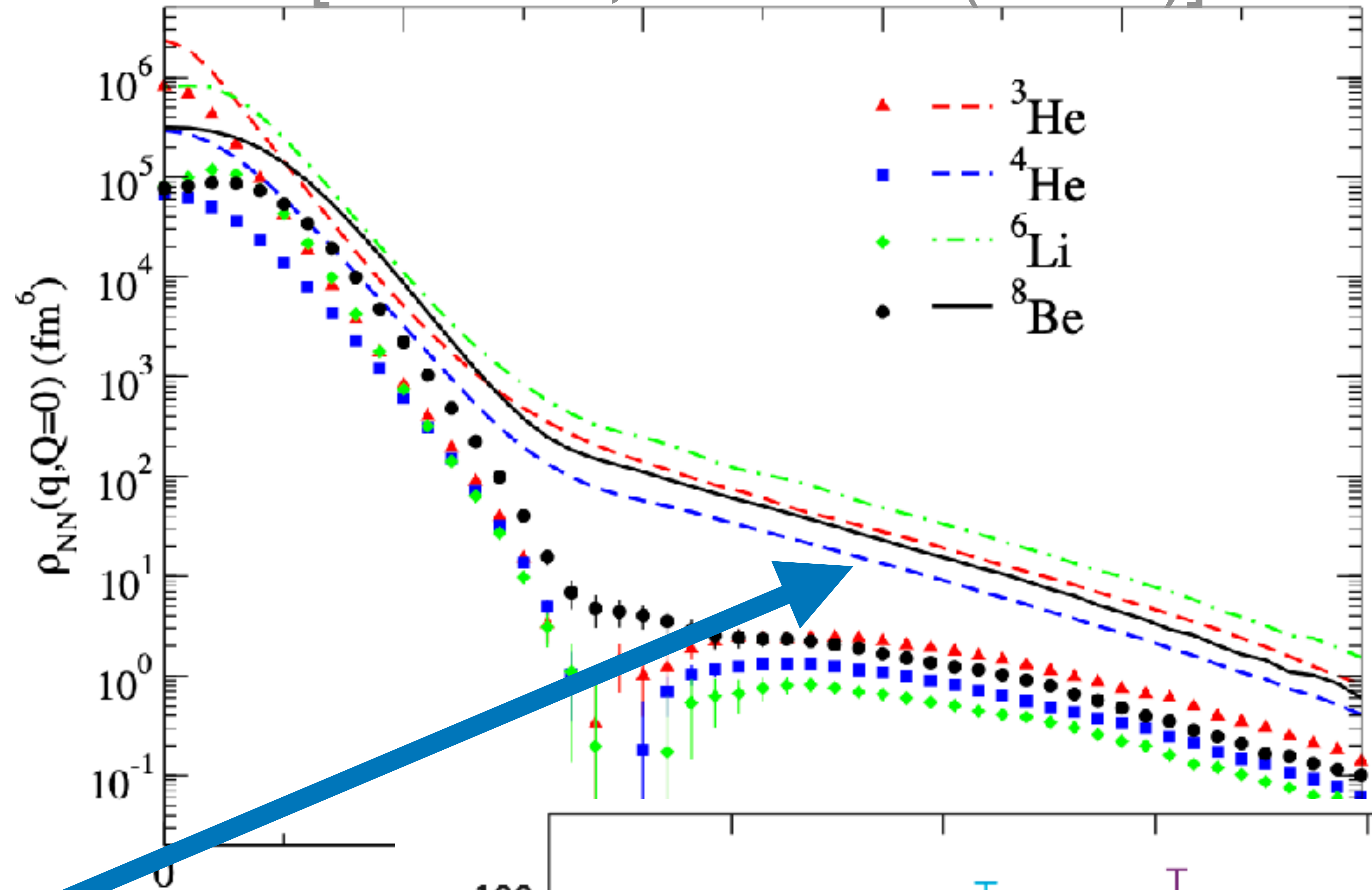
Why does it work?

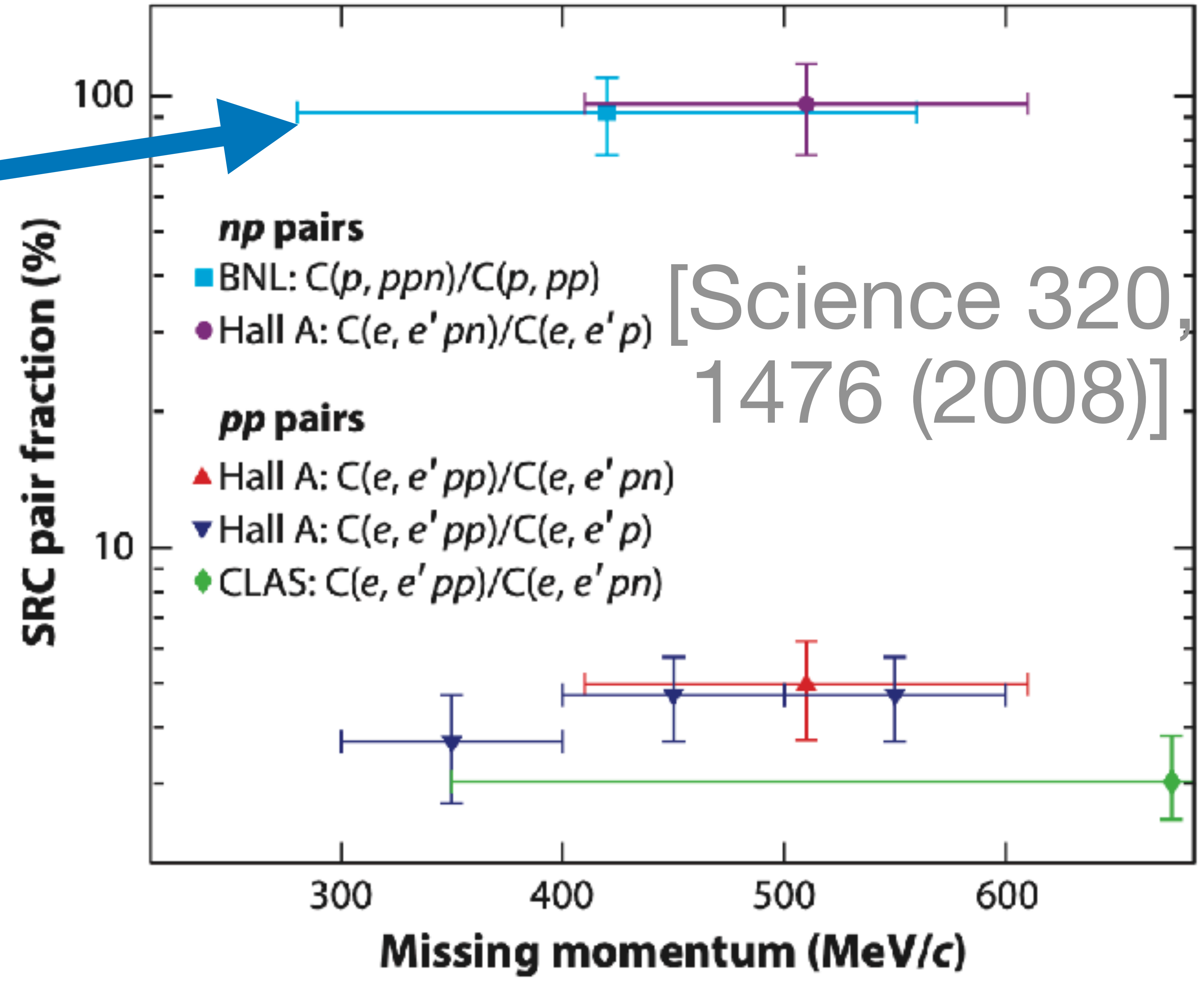
Isovector currents transform S/T=0/1 in S/T=1/0 pairs

np dominance



[PRL 98, 132501 (2007)]

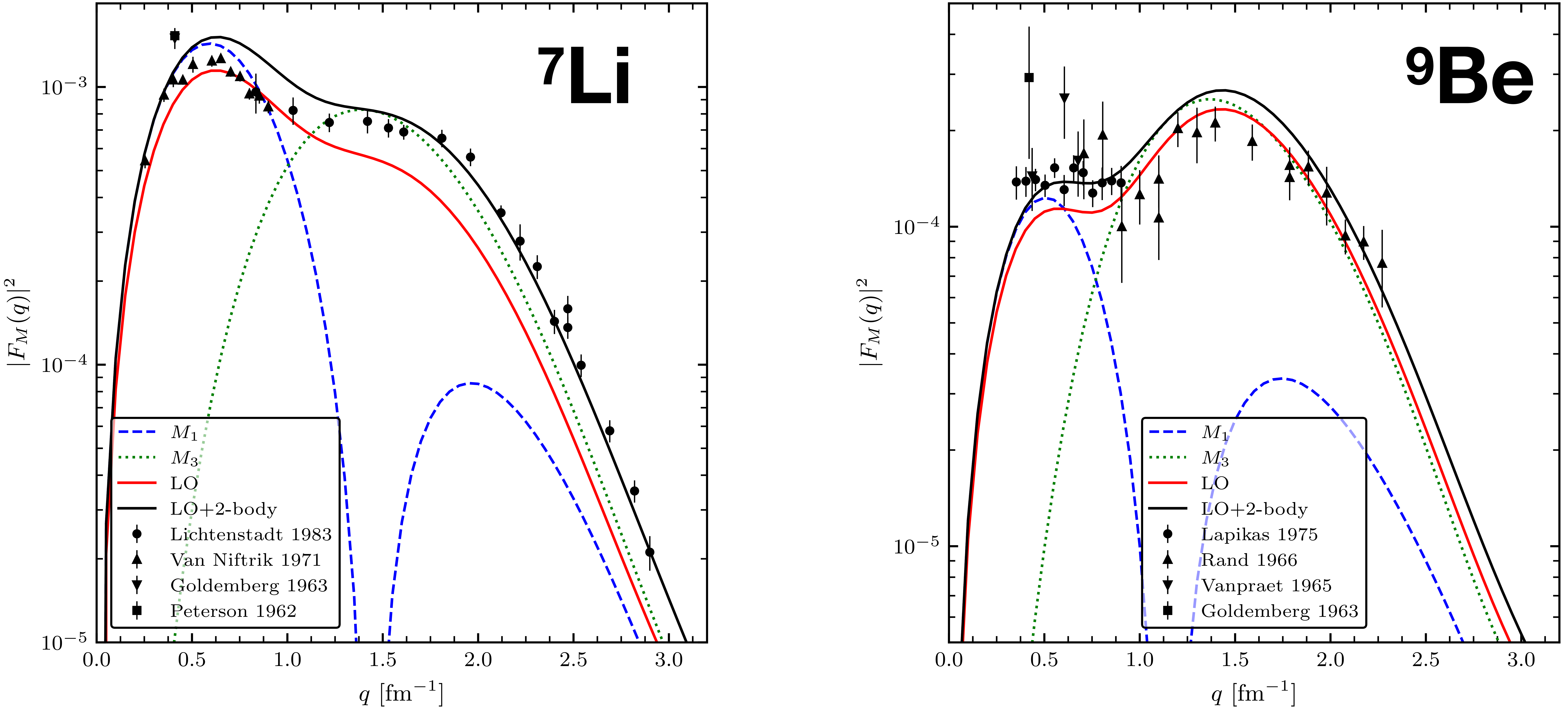




[Science 320, 1476 (2008)]

Magnetic form factor predictions

Lithium-7 and Berilium-9 (isovector dominated)



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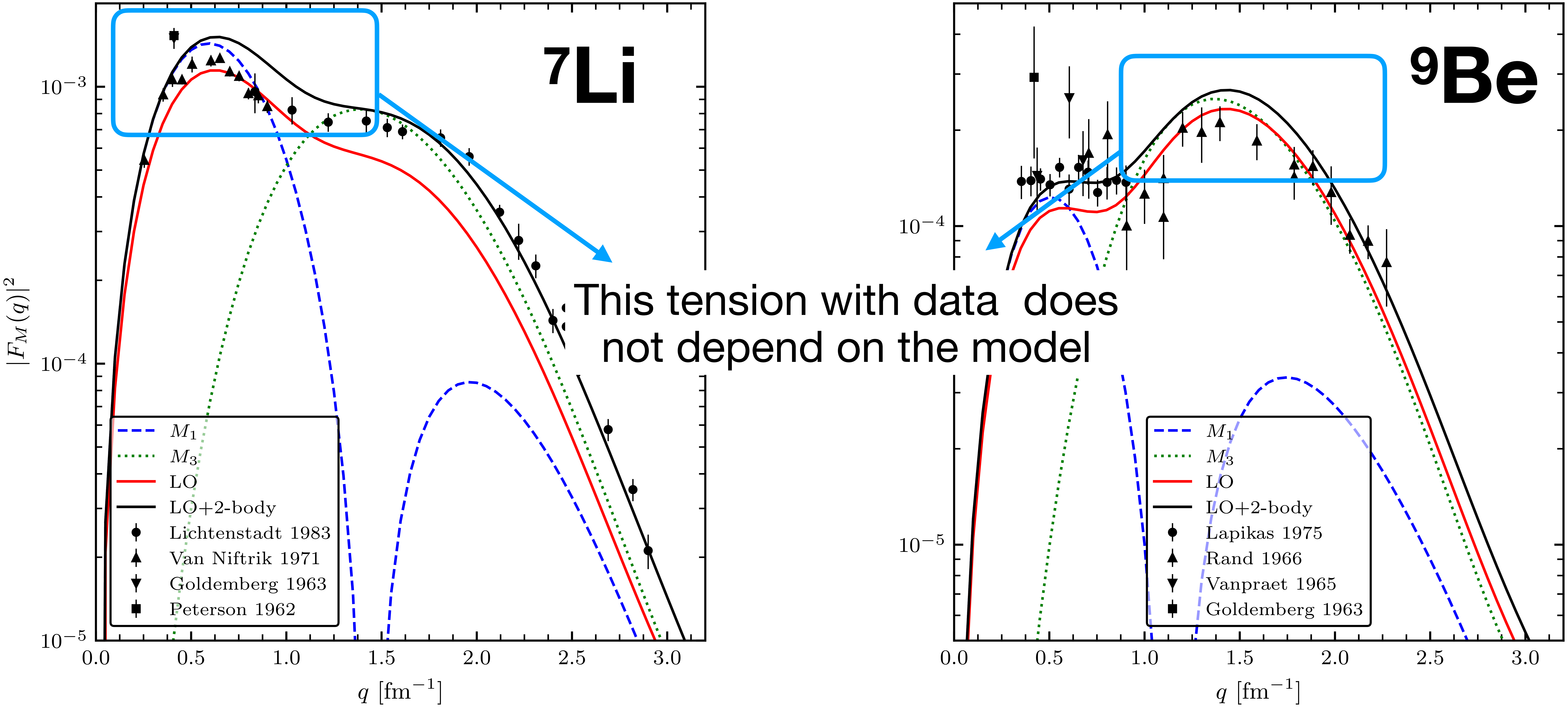
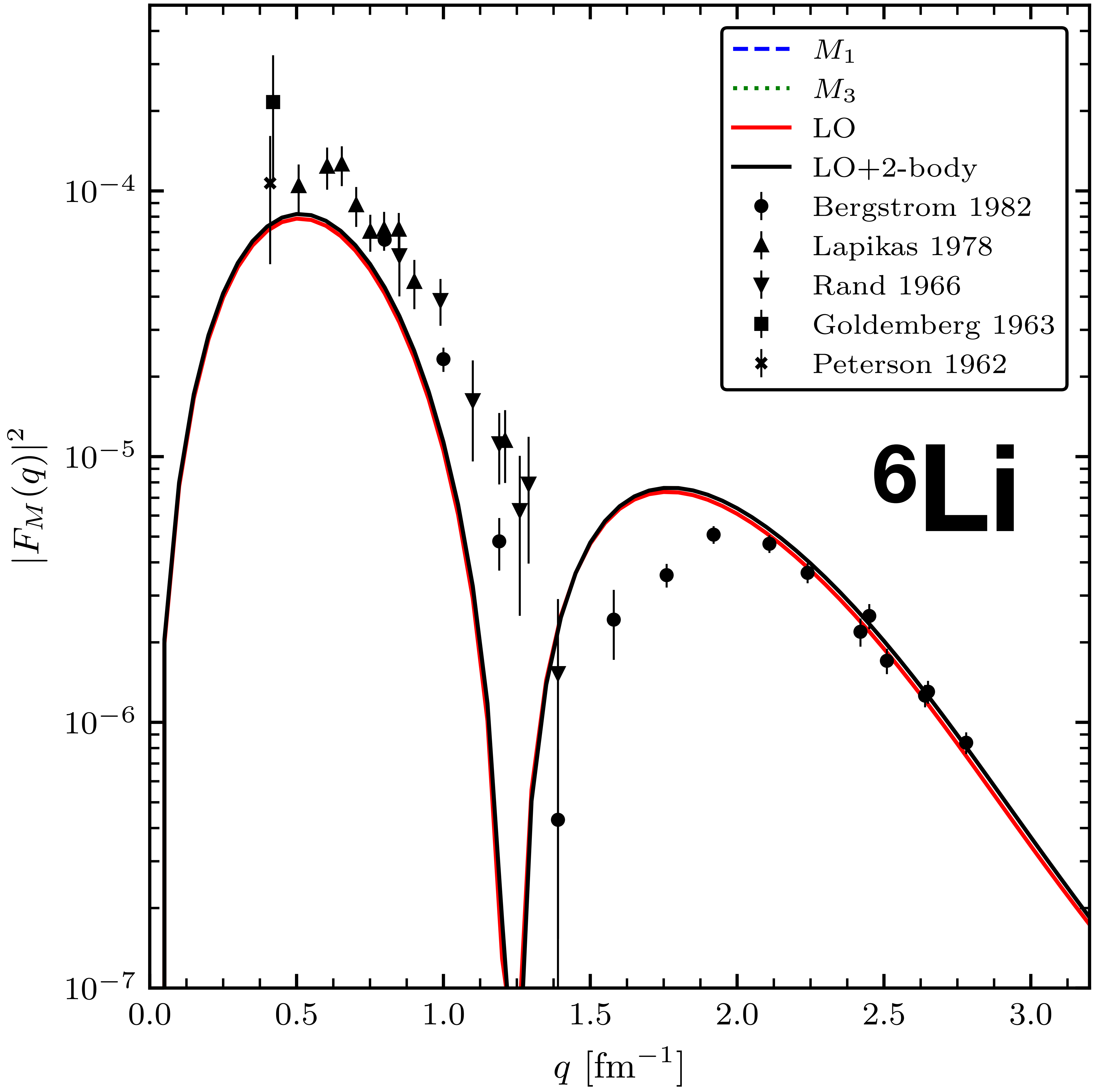
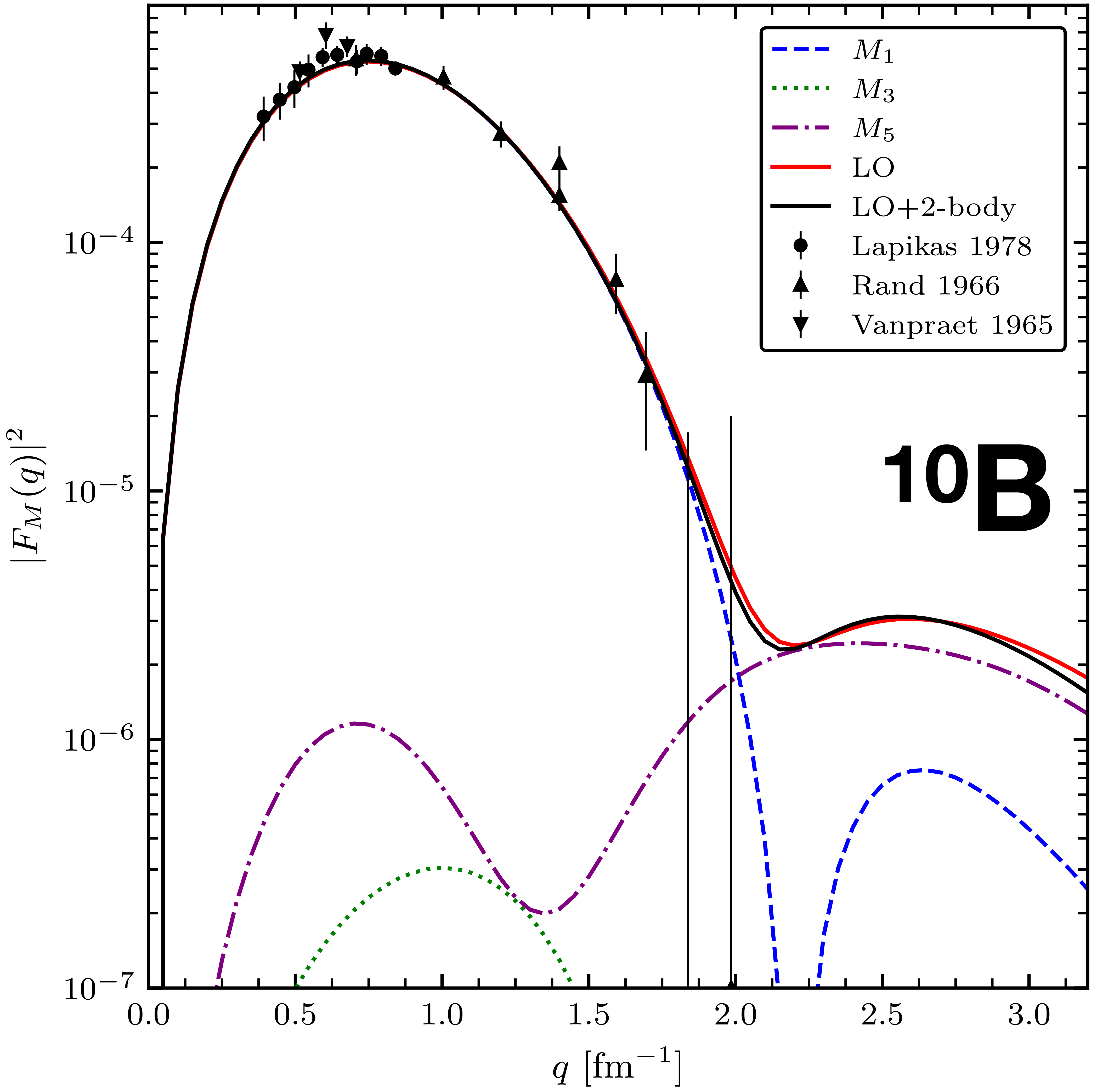


Magnetic form factor predictions

Lithium-6 and Boron-10 (isoscalar transition)

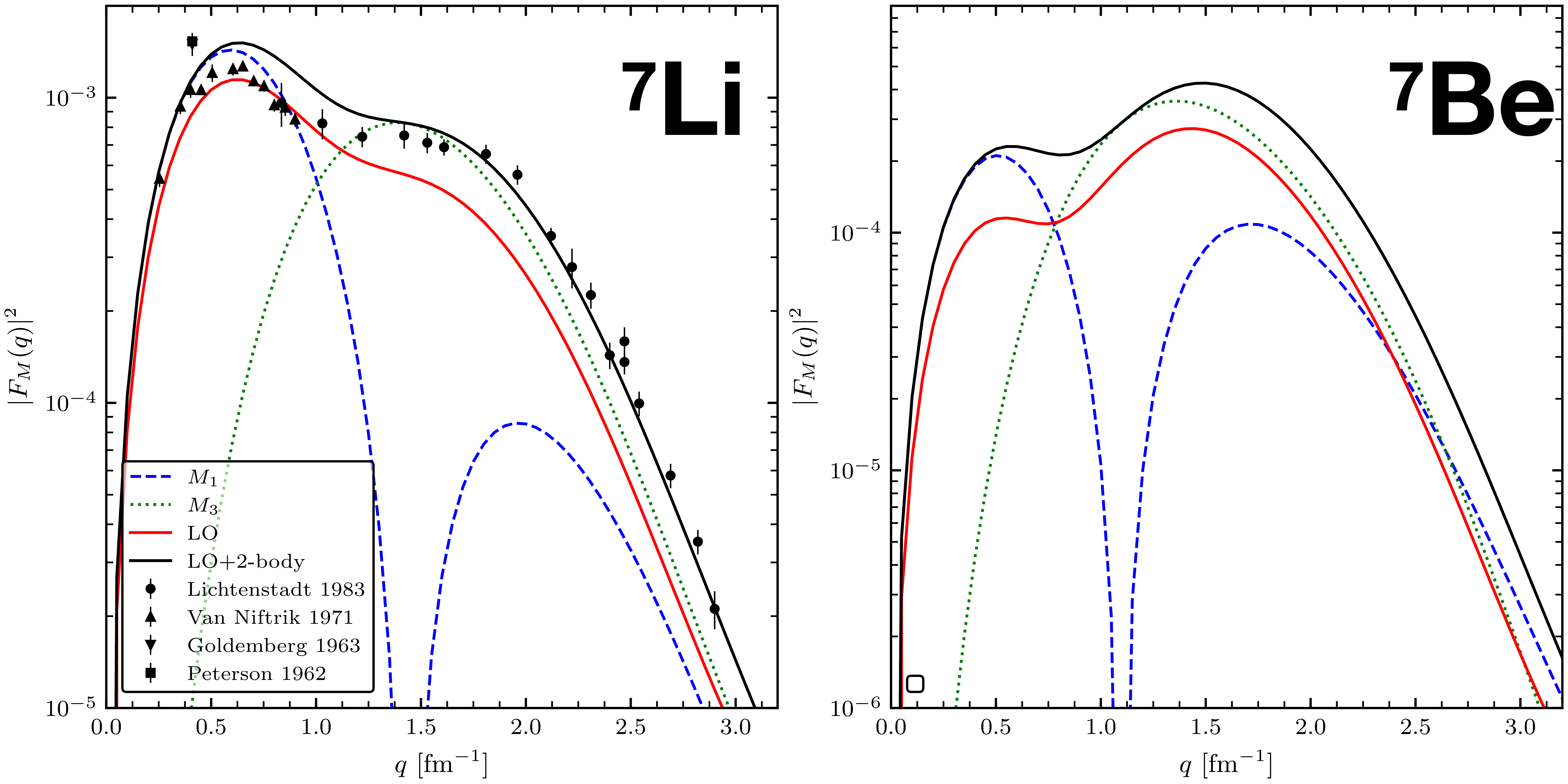




Mirror nuclei structure

- M_1 is enhanced respect to M_3 for nuclei with an unpaired neutron in the p-shell.

- We observed a similar behavior for the mirror systems ^{9}Li-^{9}C and ^{9}Be-^{9}B



Pure prediction (no previous literature) + no experimental confirmation

Mirror nuclei structure

The reason

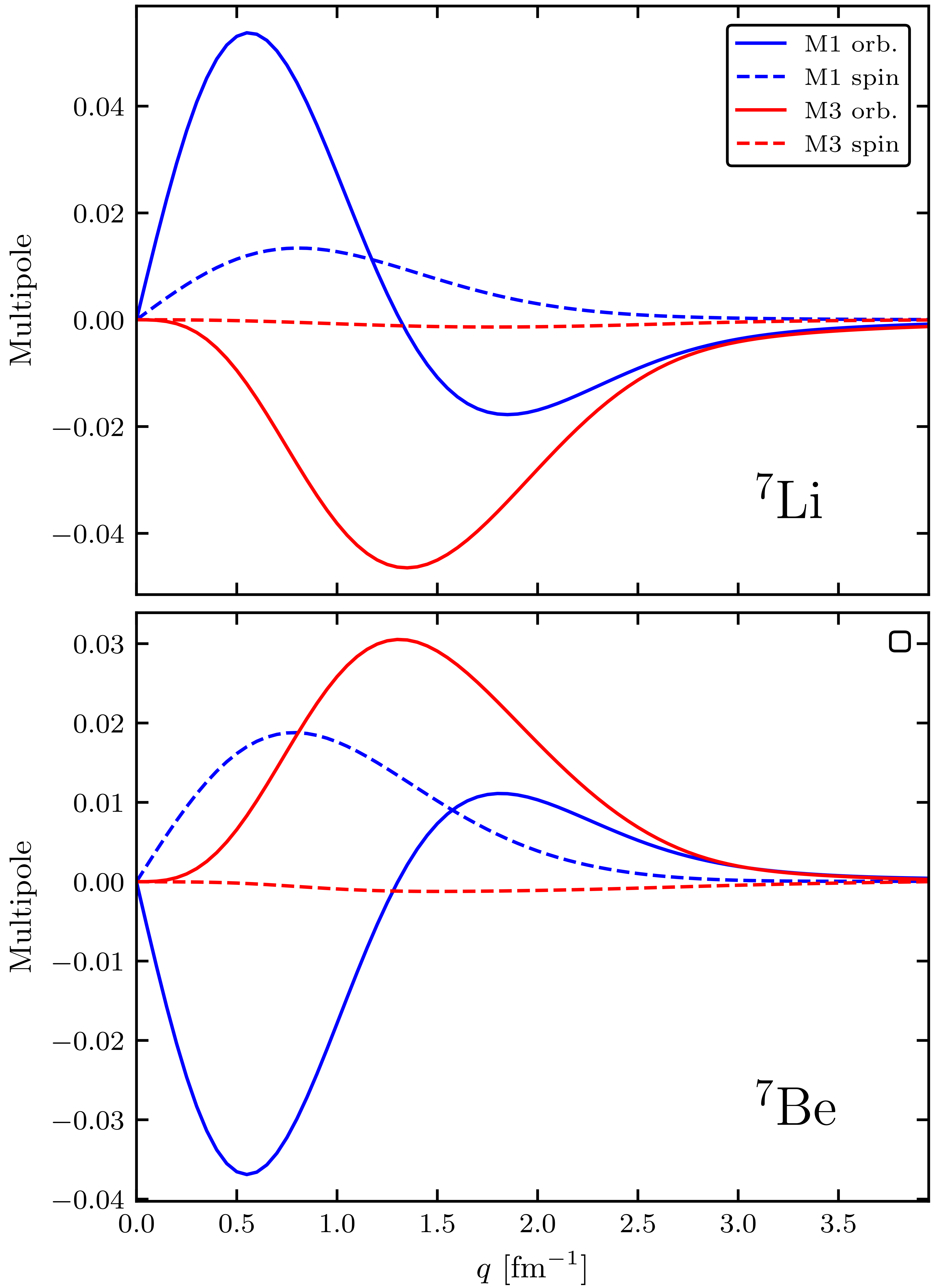
$$\mathbf{j}^{\mathrm{LO}}(\mathbf{q}) = \frac{\epsilon_i(q_\mu^2)}{2\,m}\ \left[\mathbf{p}_i\ ,\ \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_i}\right]_+ + i\,\frac{\mu_i(q_\mu^2)}{2\,m}\ \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}_i}\ \sigma_i \times \mathbf{q}\ ,$$

Convection current

Magnetic current

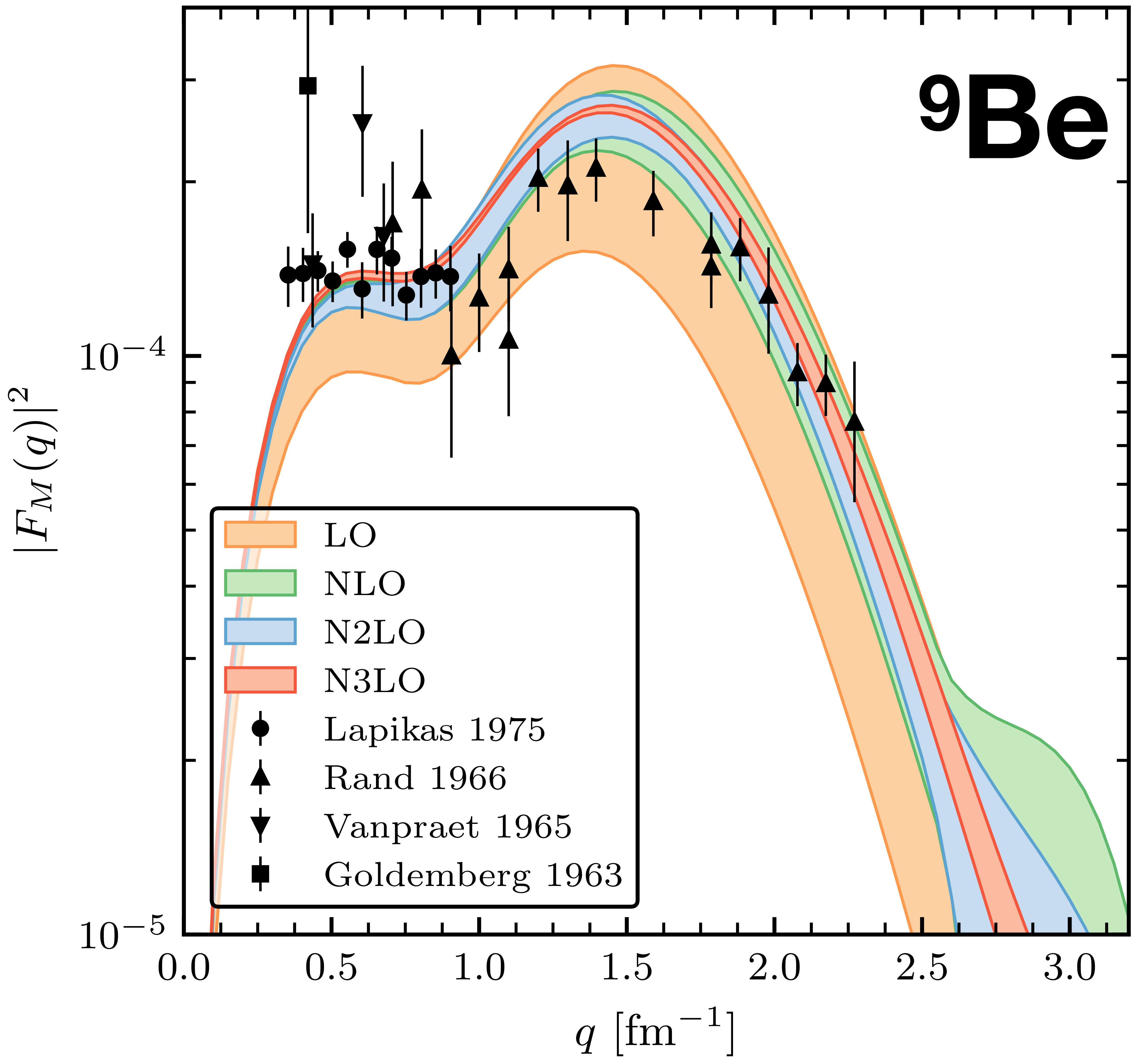
No contribution to M_3

Change sign if there is an unpaired neutron/proton



Order by order expansion

- Error analysis based on:
  [EPJA 51, 53 (2015)]
- Expansion parameter:
  $Q = (A-1)/A \times q/\Lambda_b,\ \Lambda_b = 700\,\mathrm{MeV}$
- Chiral expansion seems to have in general a good behavior.
- N3LO corrections for some nuclei are of the same size of N2LO.



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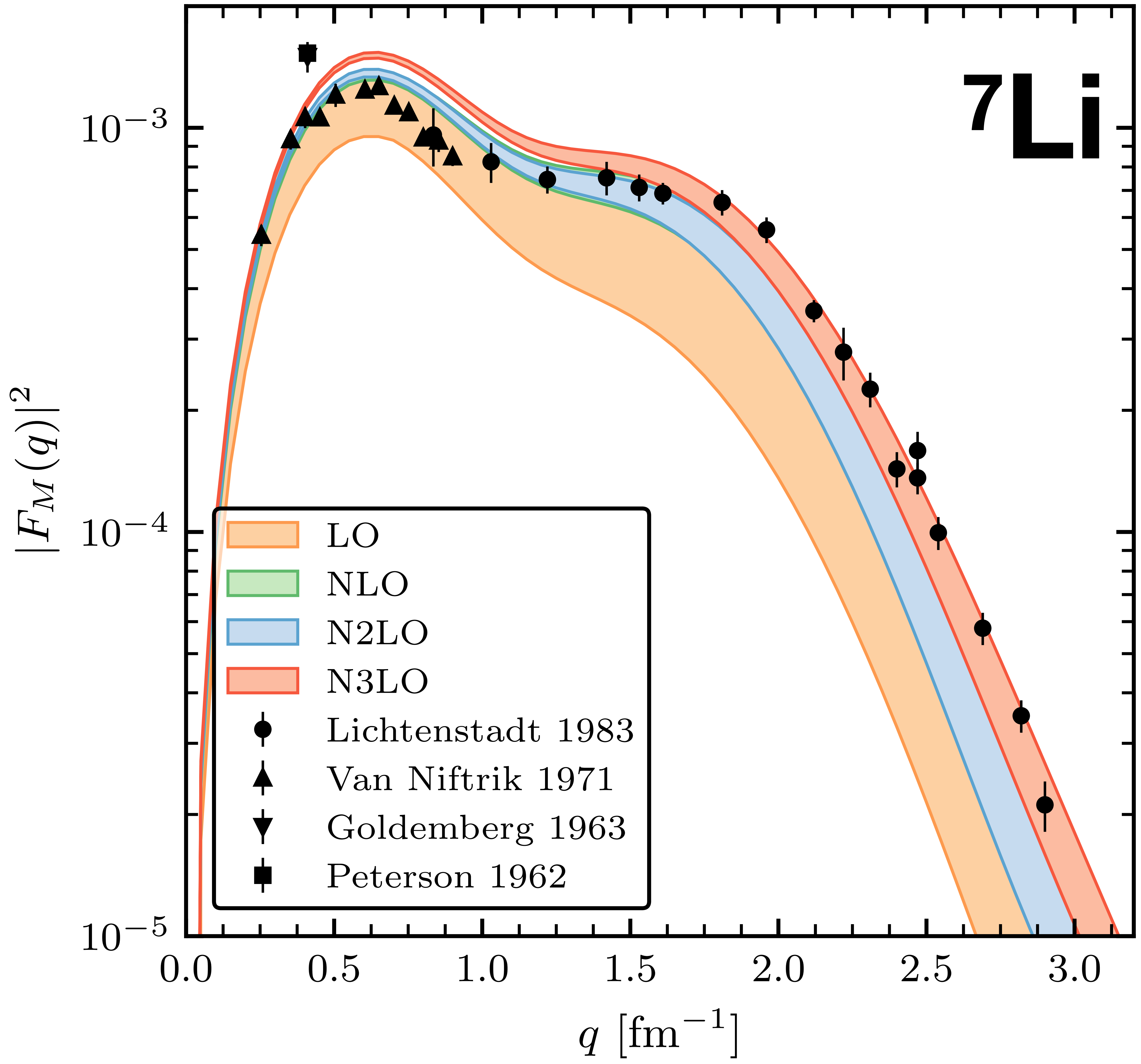
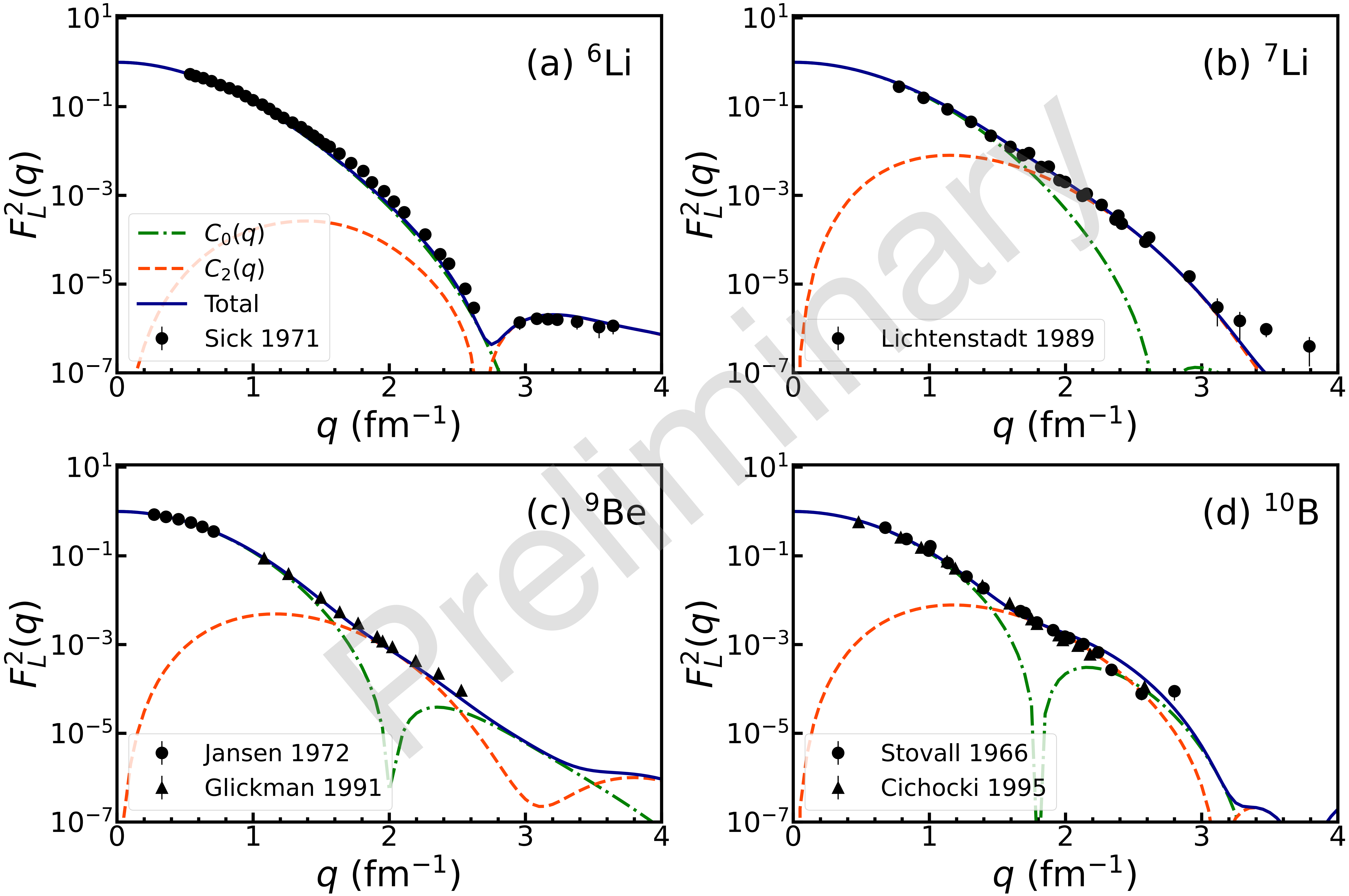


Courtesy of G.B. King

Charge form factor predictions



Summary

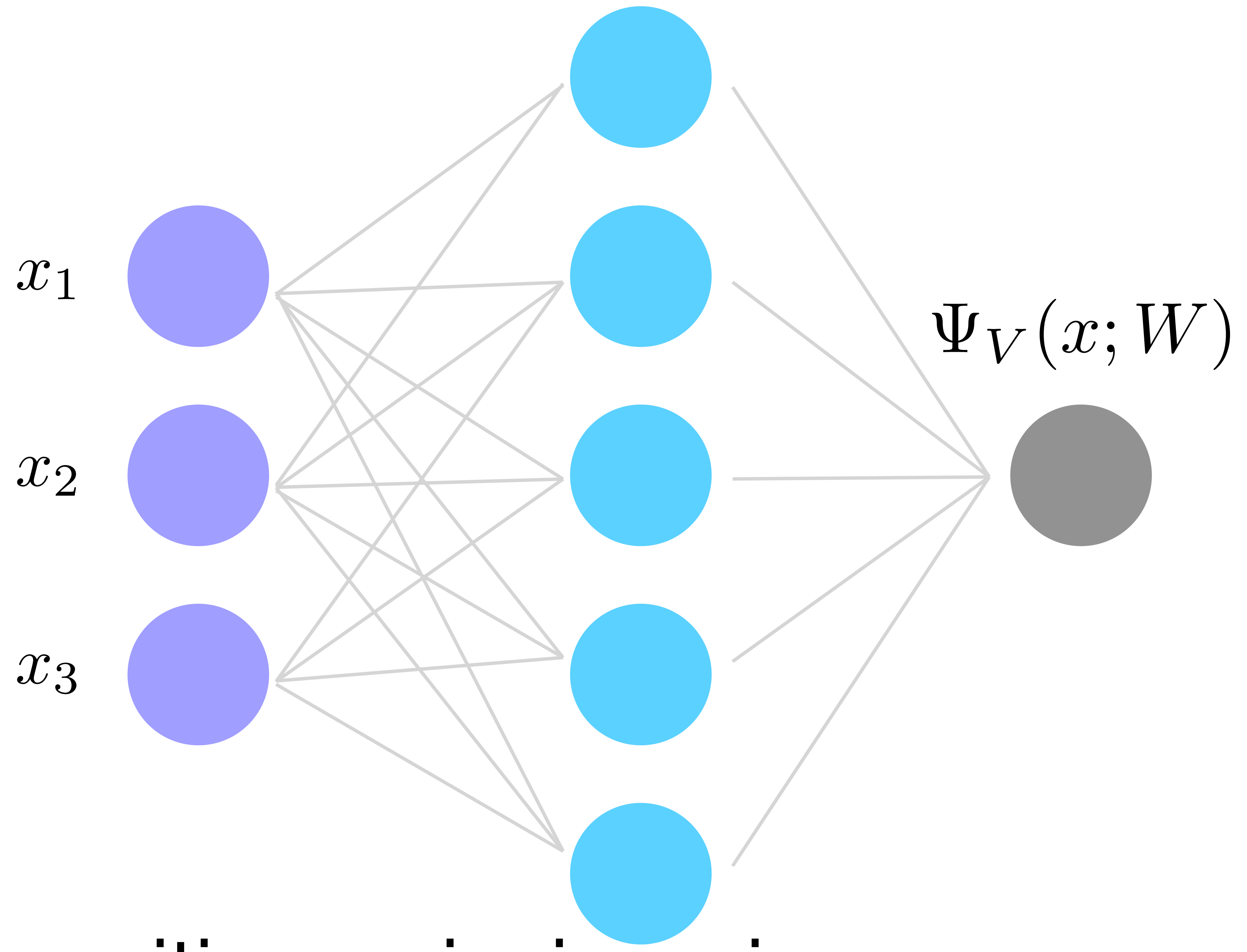
Magnetic form factors

- First ab-initio calculation of magnetic form factors of nuclei $7 \leq A \leq 10$.
- Good overall description of available magnetic form factor data.

- **Two-body currents account up to 40-50% of the total contribution to the magnetic form factors.**
- **First observation of M_1/M_3 inversion in mirror p-shell nuclei (not observed experimentally yet).**

More precise data on more nuclei would permit to constrain better our models

Post-scriptum: Neural-Network Quantum States



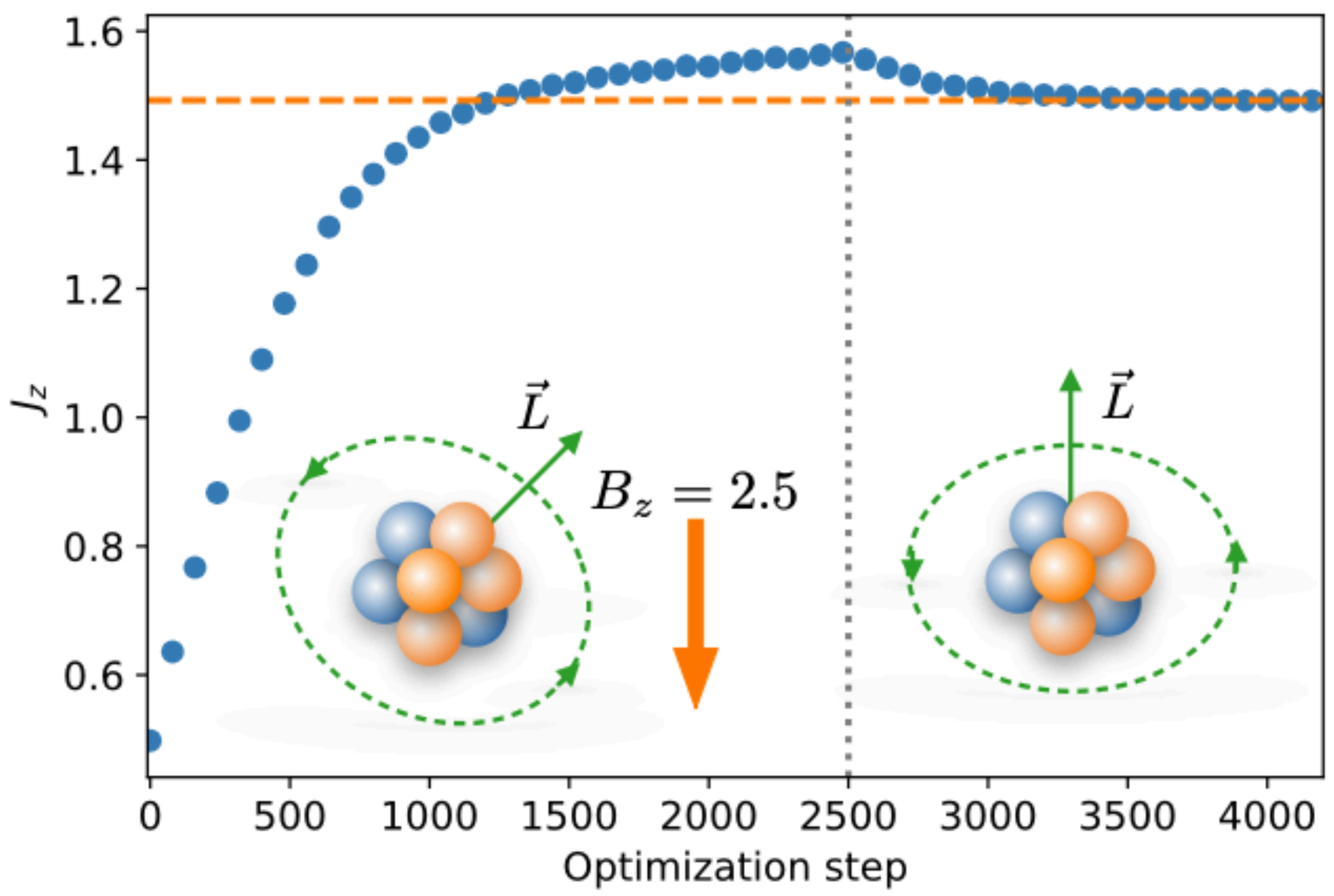
$x_i =$ position, spin, isospin

$W =$ weights of the network

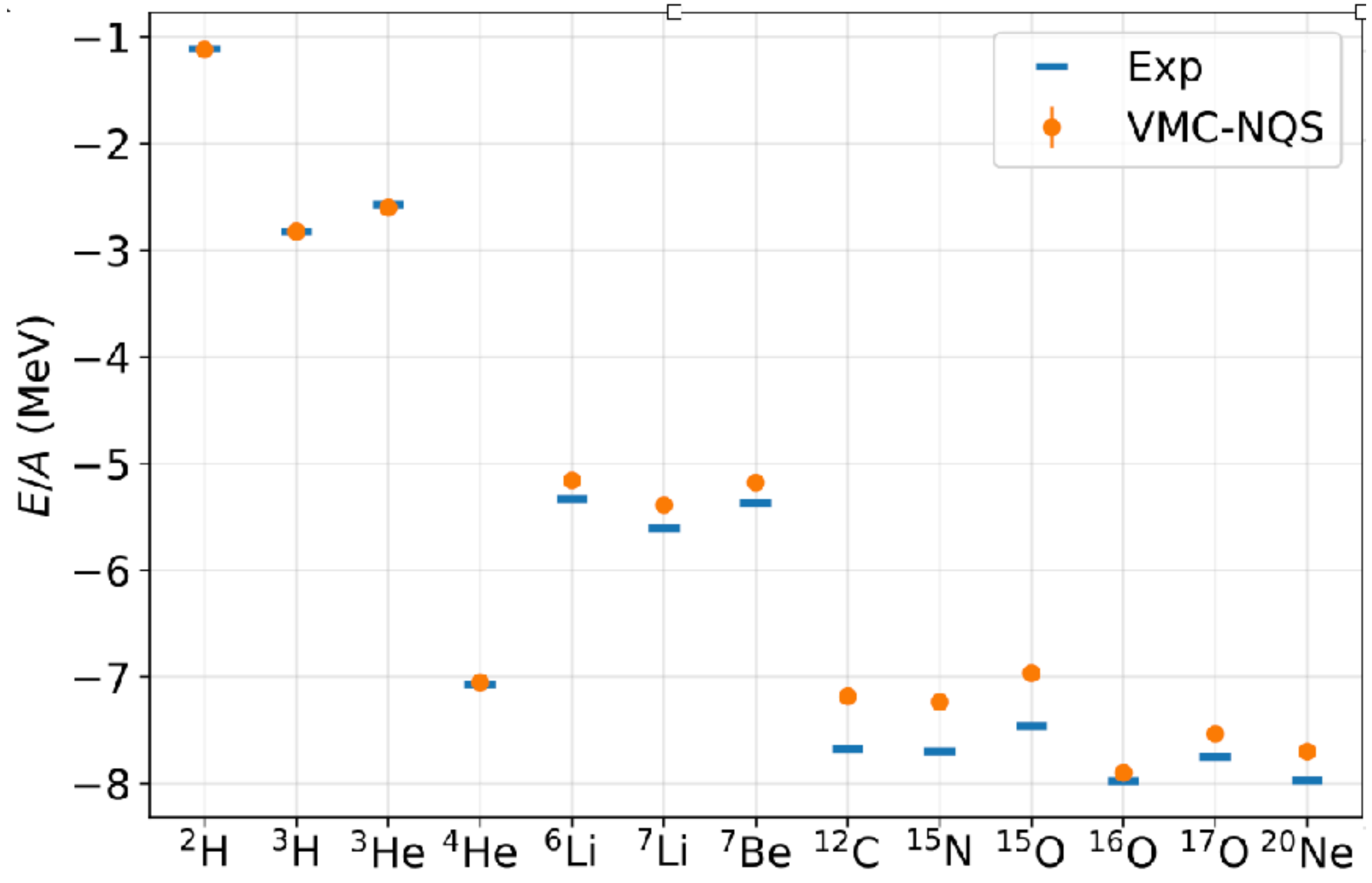
[arXiv:2308.16266 (2023)] accepted inPRL

The energy is the loss function

$$E = \min_W \frac{\langle \Psi_V(W) | H | \Psi_V(W) \rangle}{\langle \Psi_V(W) | \Psi_V(W) \rangle}$$



Post-scriptum: Neural-Network Quantum States

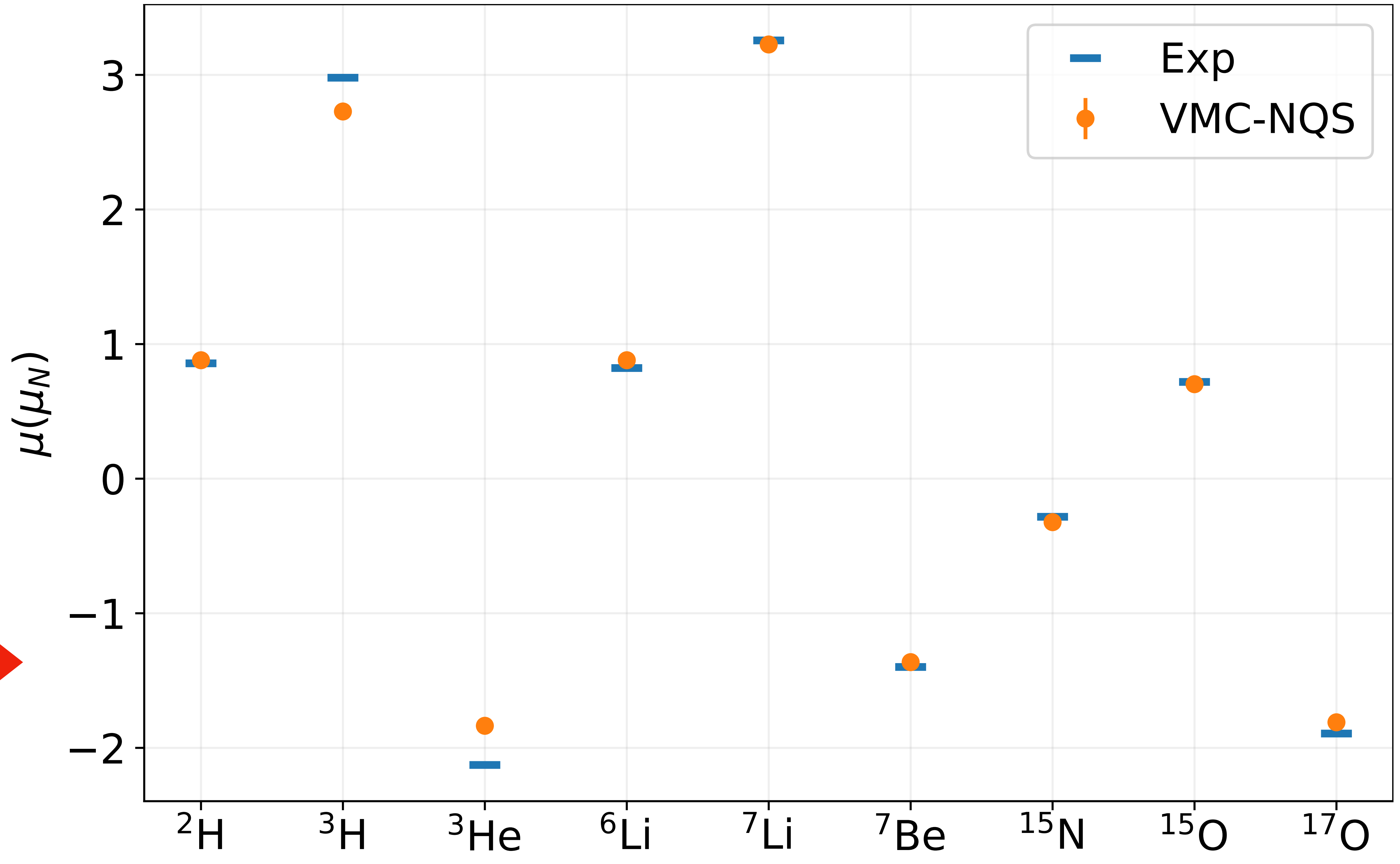


Pionless EFT + NNQS

Prediction of binding energy of light nuclei

The NNQS is learning the Hamiltonian

Prediction of static magnetic moments



Collaborators

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M. Viviani (INFN Pisa)
R. B. Wiringa (ANL)

Acknowledgments

NTNP
DOE Topical Collaboration

U.S. DEPARTMENT OF ENERGY

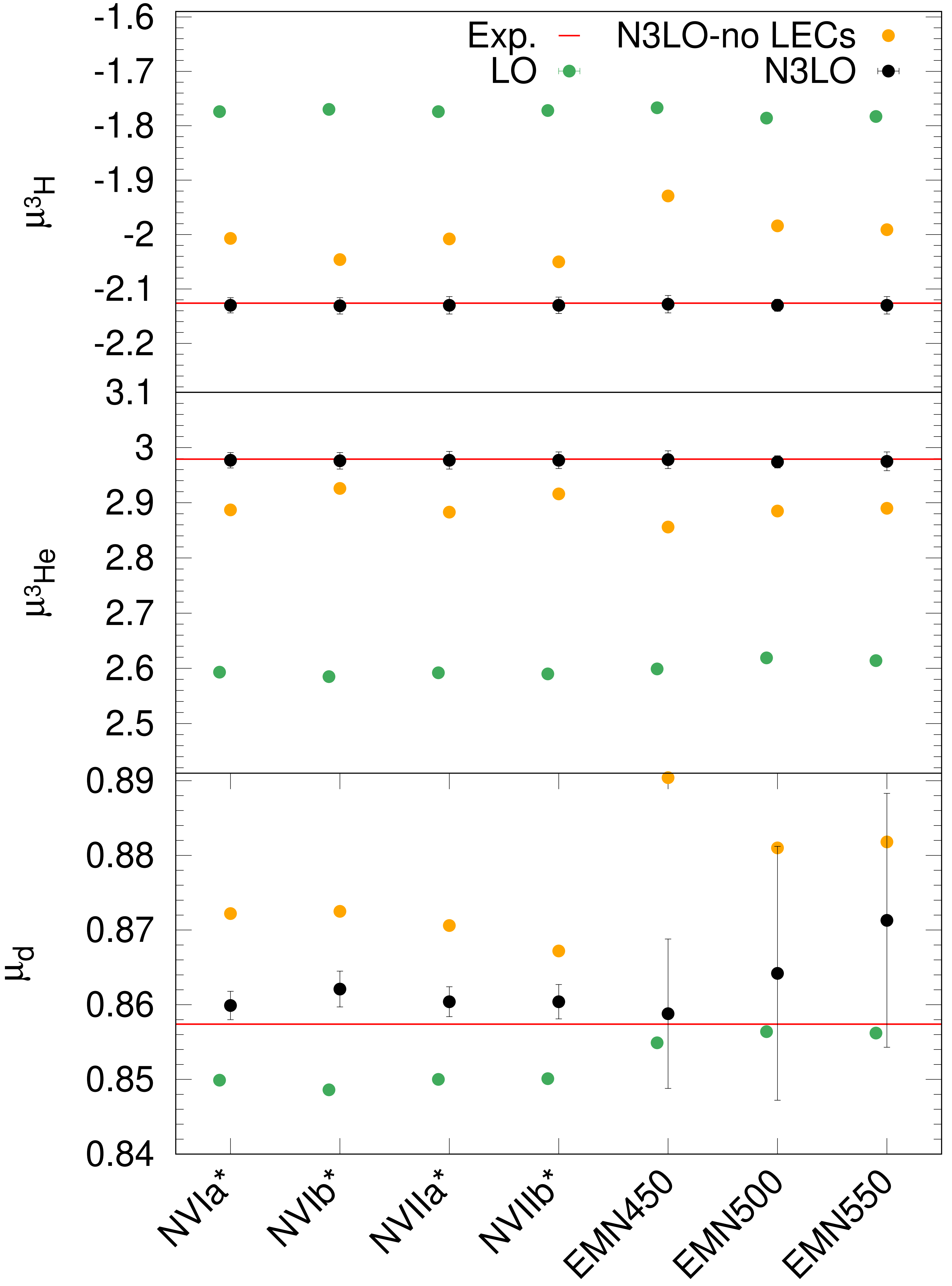
NERSC National Energy Research Scientific Computing Center

Sparse

Results of the fit

Pot.	χ^2/ndf	χ^2/ndf (no Rand)
NVIa*	9.9	2.0
NVIb*	10.2	2.3
NVIIa*	11.6	2.5
NVIIb*	11.6	2.6
EMN450	11.3	2.8
EMN500	14.7	4.7
EMN550	17.7	7.9

- ndf~40
- Removing Rand *et al.* data, χ^2improves



References

- Fit of the currents and test on few-body nuclei

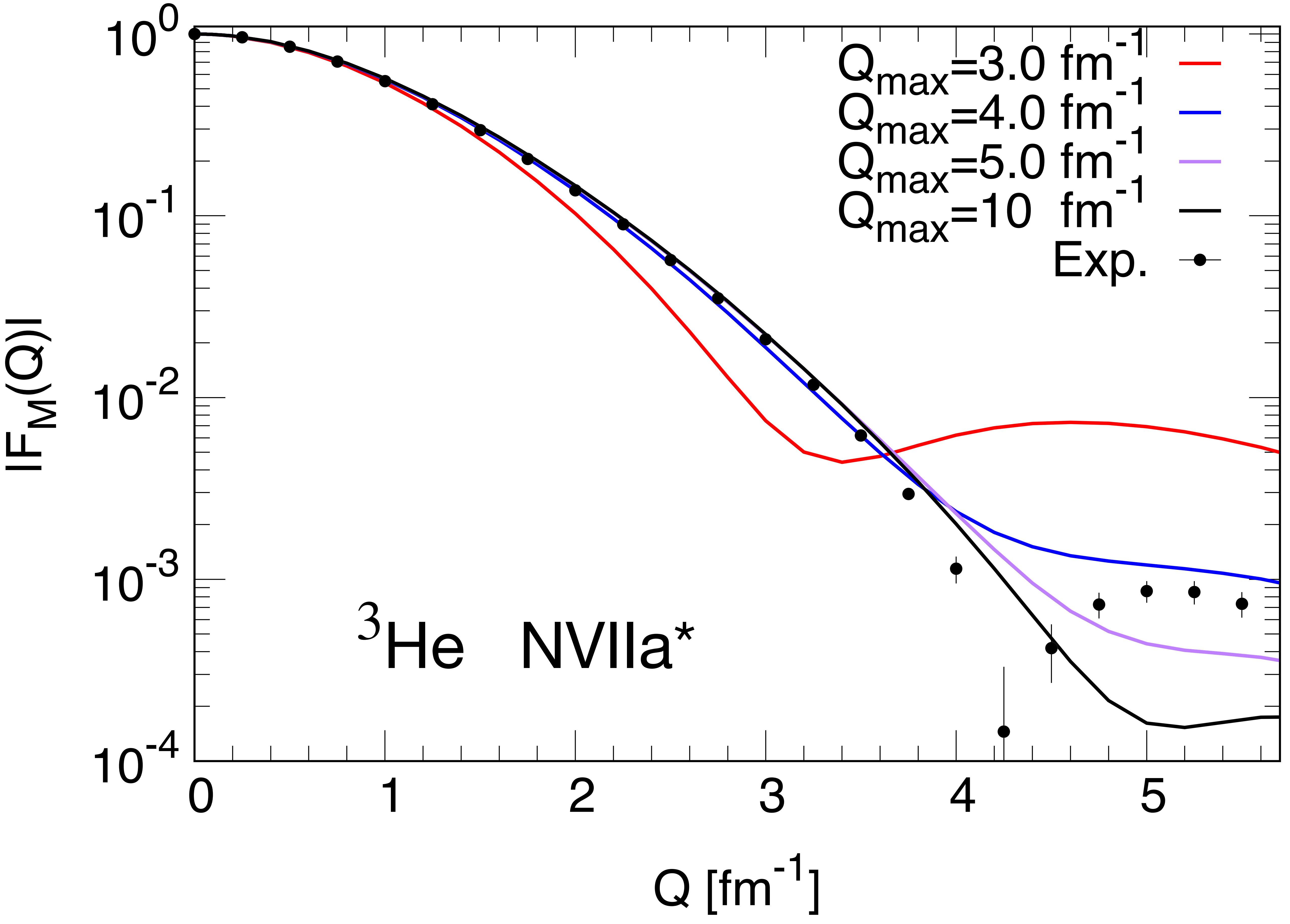
[A.G. and R. Schiavilla, Phys. Rev. C 106, 044001 (2020)]

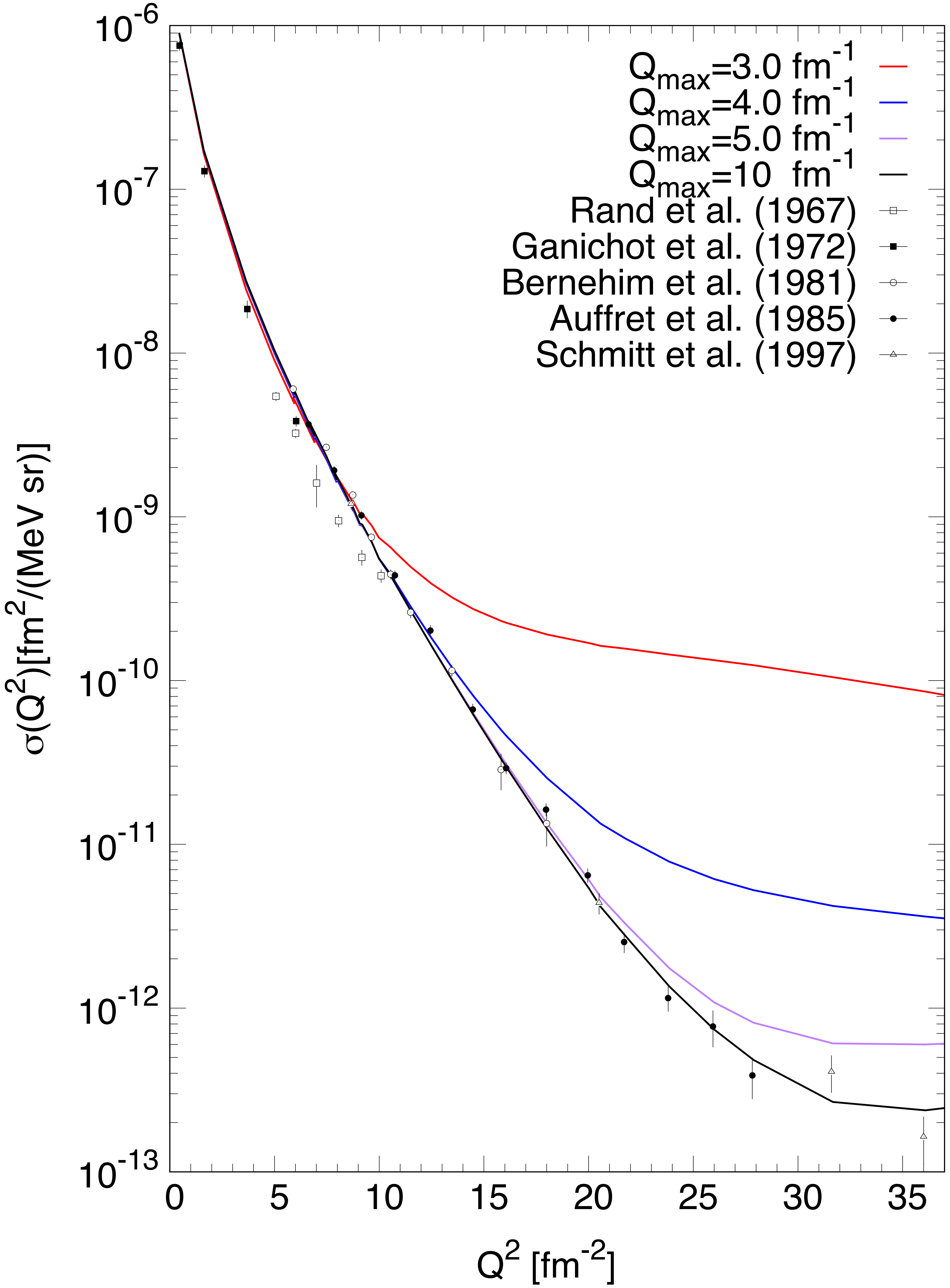
- Prediction of magnetic form factors of A=5-10

[G. Chambers-Wall, A. G., G. B. King, S. Pastore, M. Piarulli, R. Schiavilla, R. B. Wiringa, arXiv:2407.04744, arXiv:2407.03487]

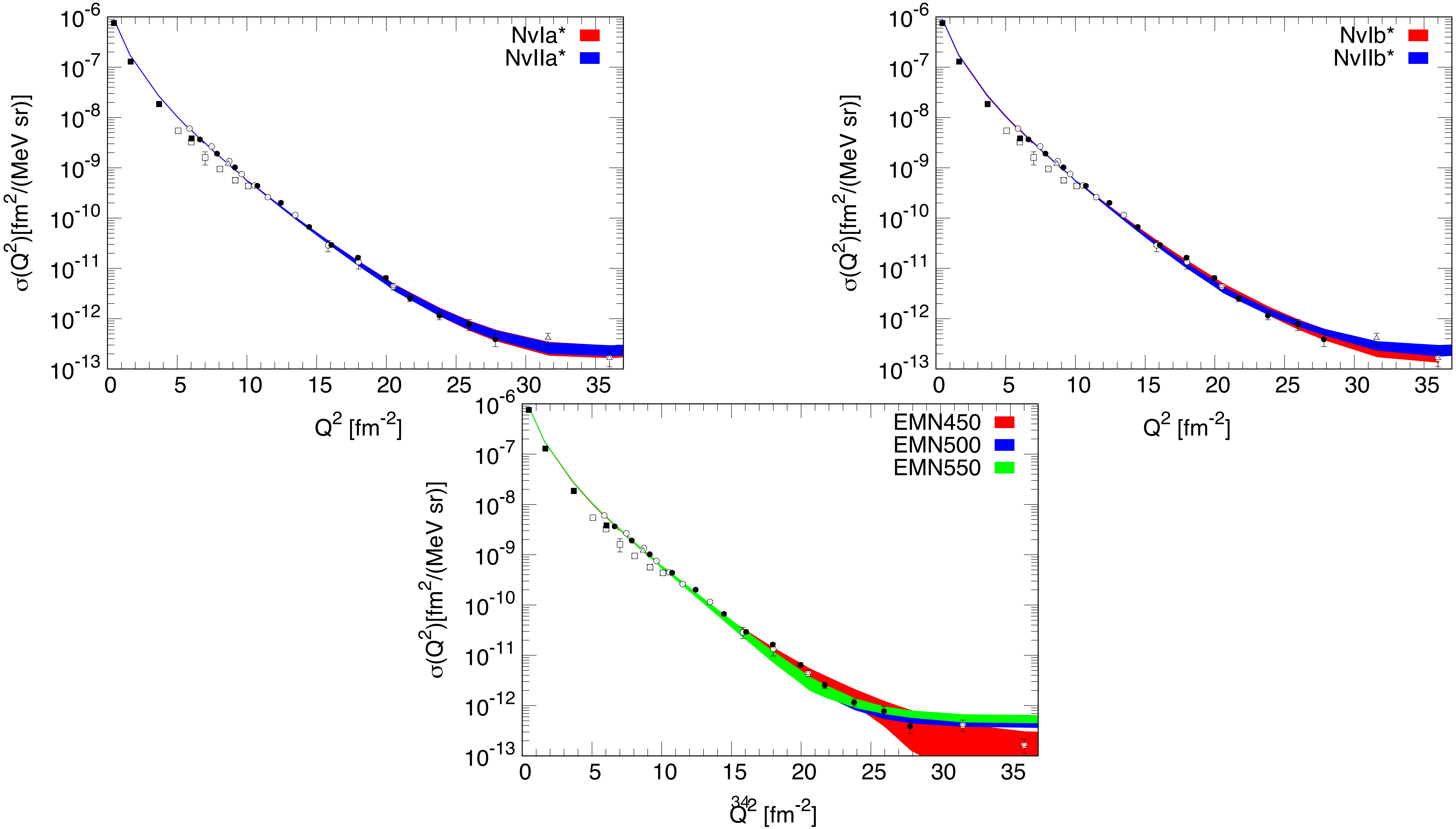
- Predictions of charge form factors (in progress)

Dependence on $Q^2_{\mathbf{max}}$

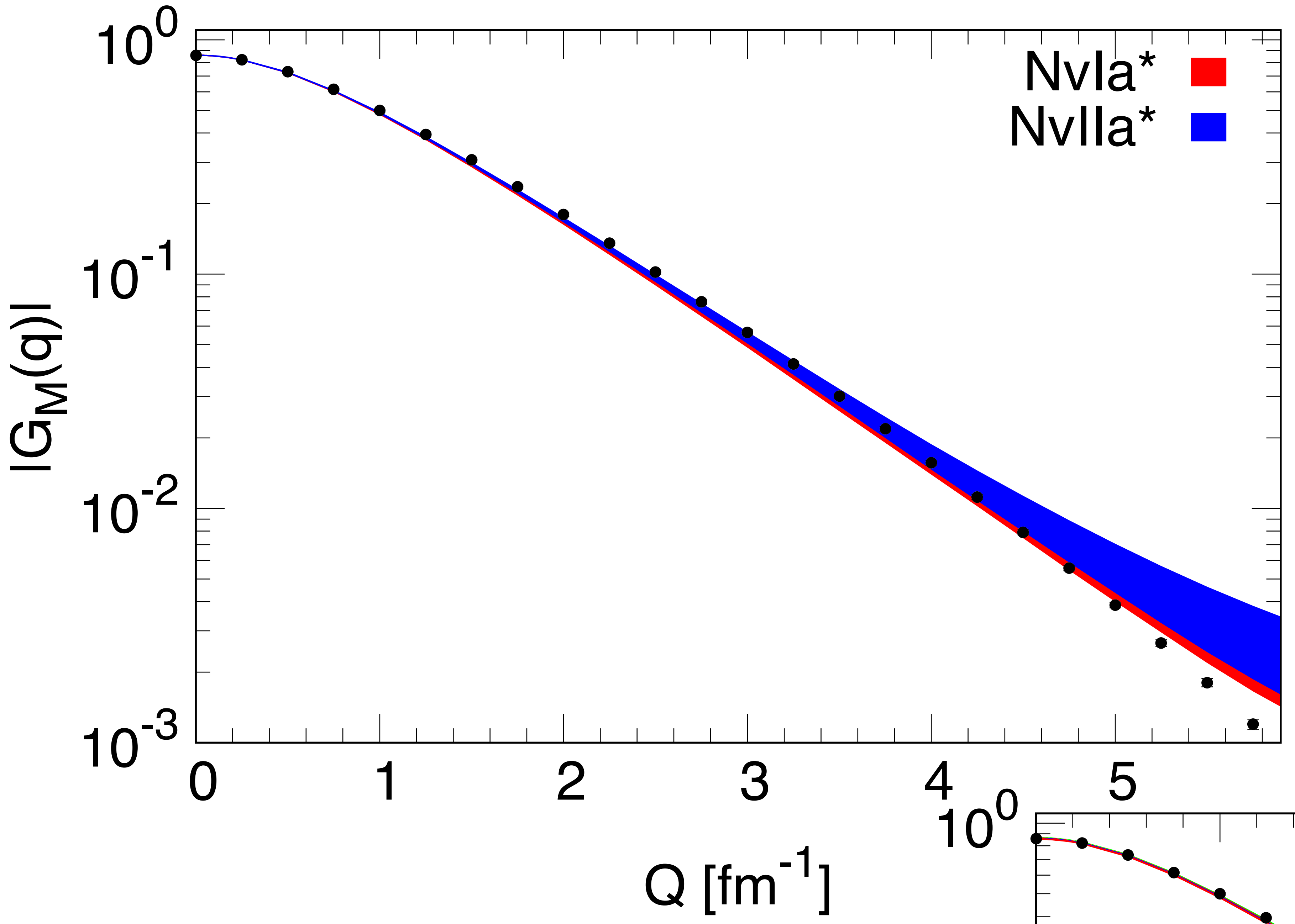


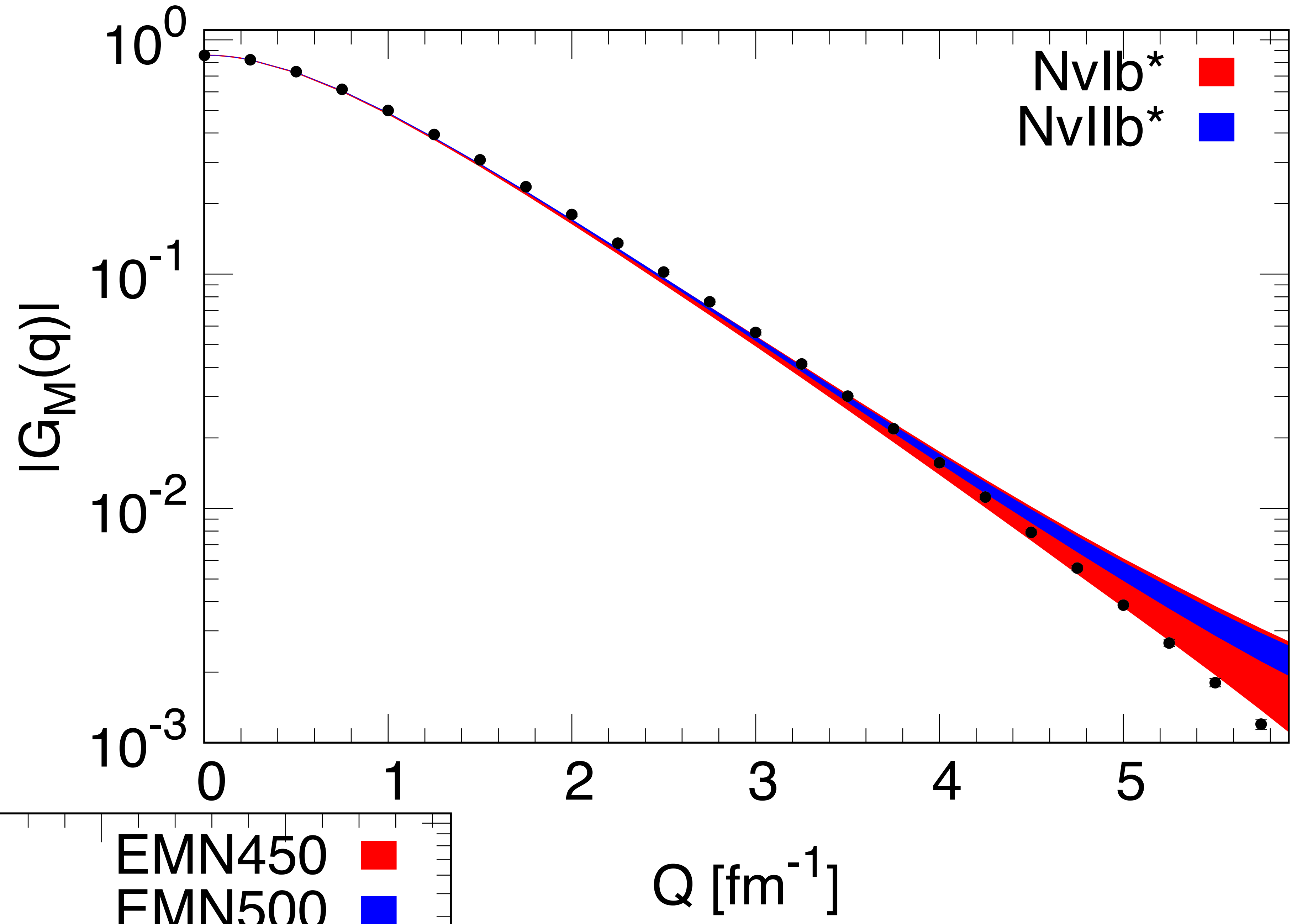


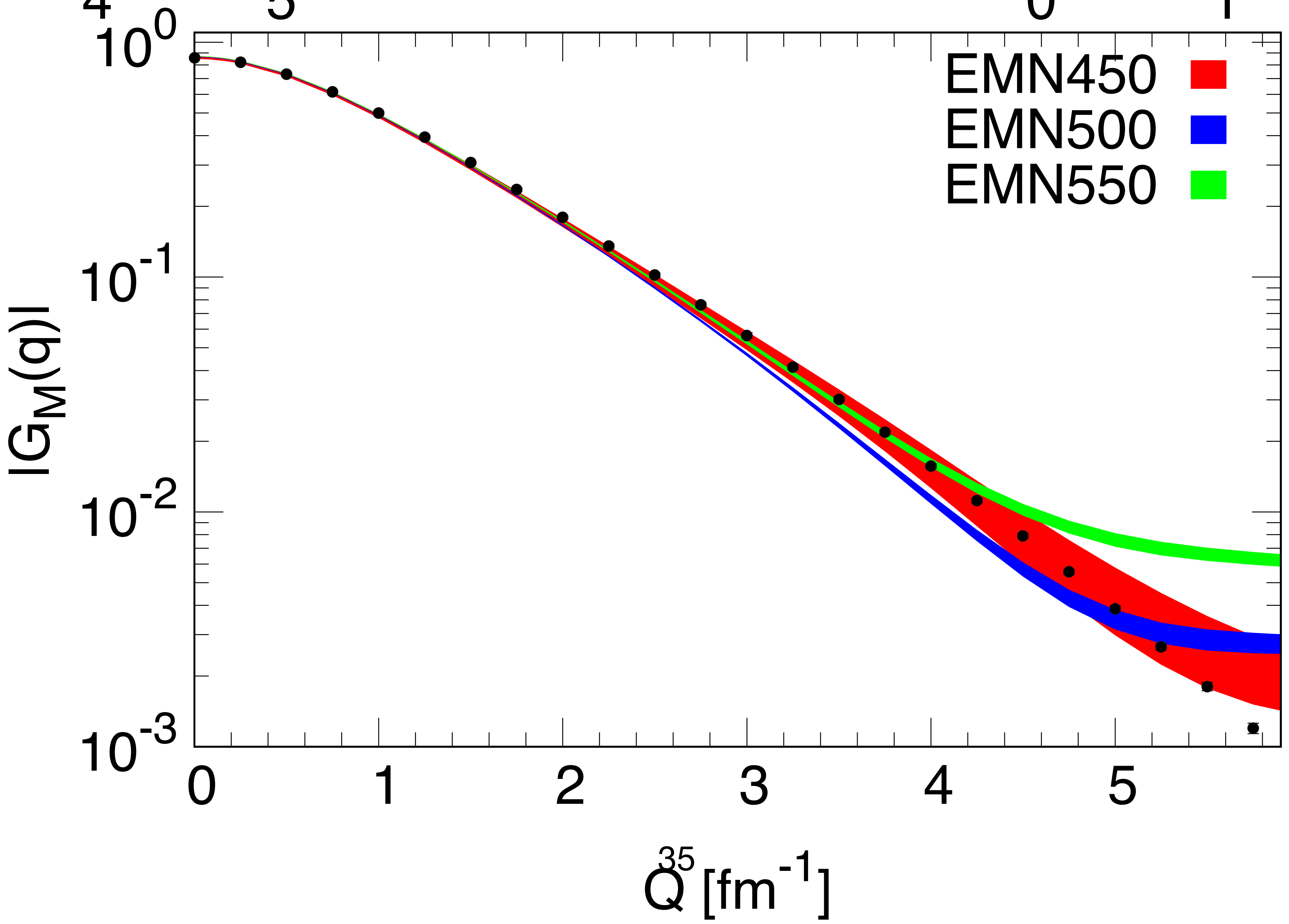
d-threshold


NvIa*
NvIIa*
NvIb*
NvIIb*
EMN450
EMN500
EMN550
$\sigma(Q^2)[\mathrm{fm}^2/(\mathrm{MeV\ sr})]$
$Q^2\ [\mathrm{fm}^{-2}]$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
0 5 10 15 20 25 30 35

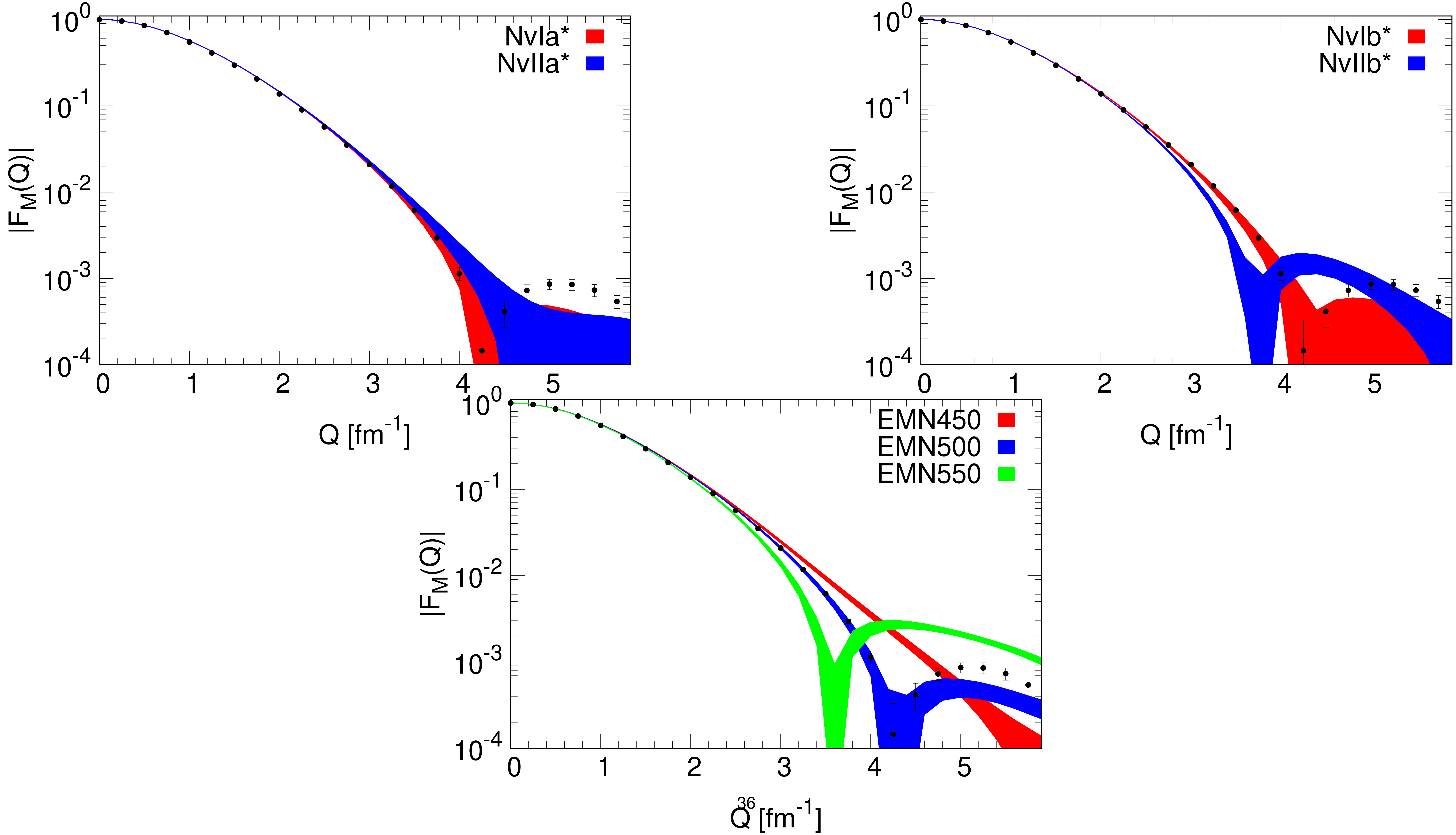
Magnetic form factors of ^{2}H







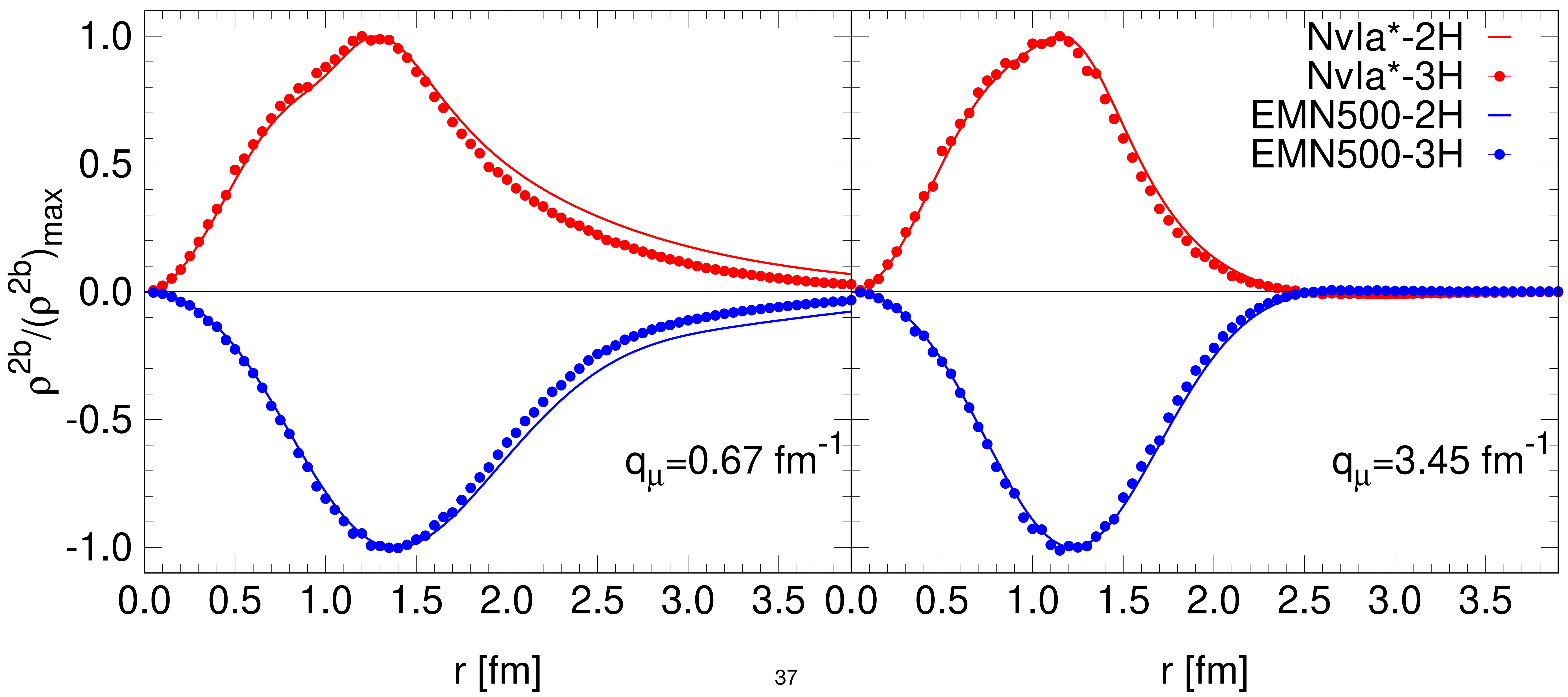
Magnetic form factors of ^{3}He



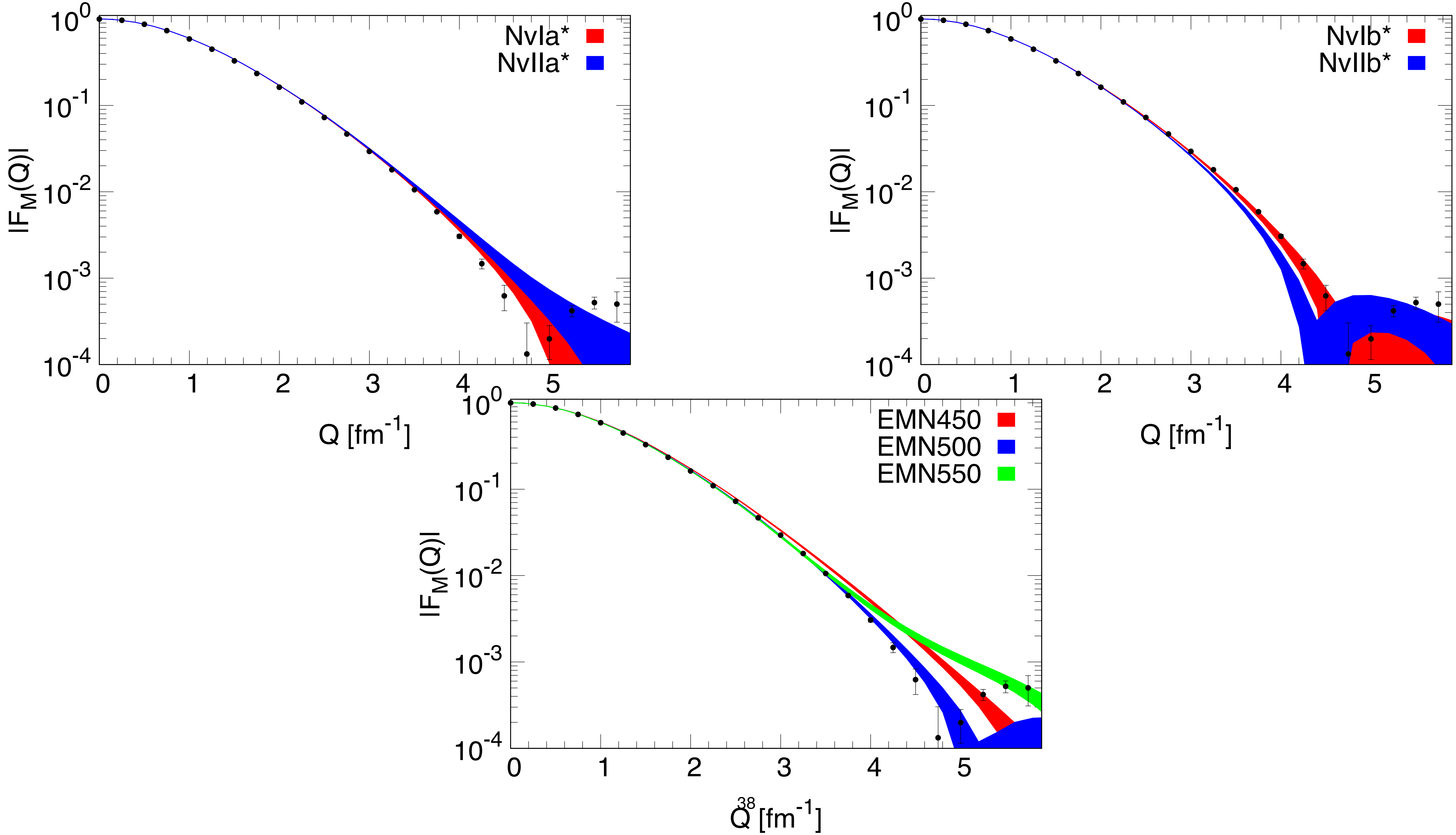
Why does it work?

Universal behavior of isovector transitions

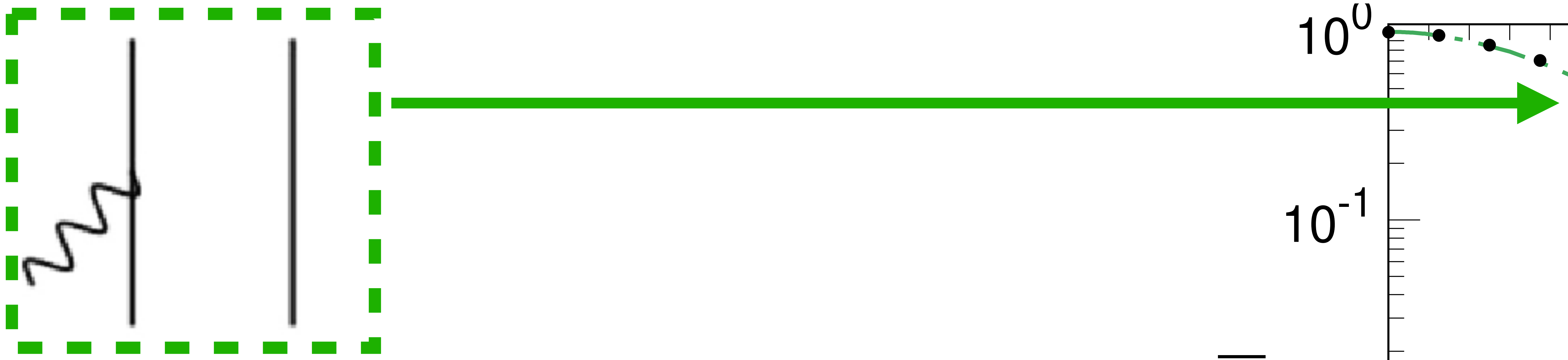
Correlated np pairs ⟷ **Universal 2-body wave functions** ⟷ **Universal 2-body transition densities**

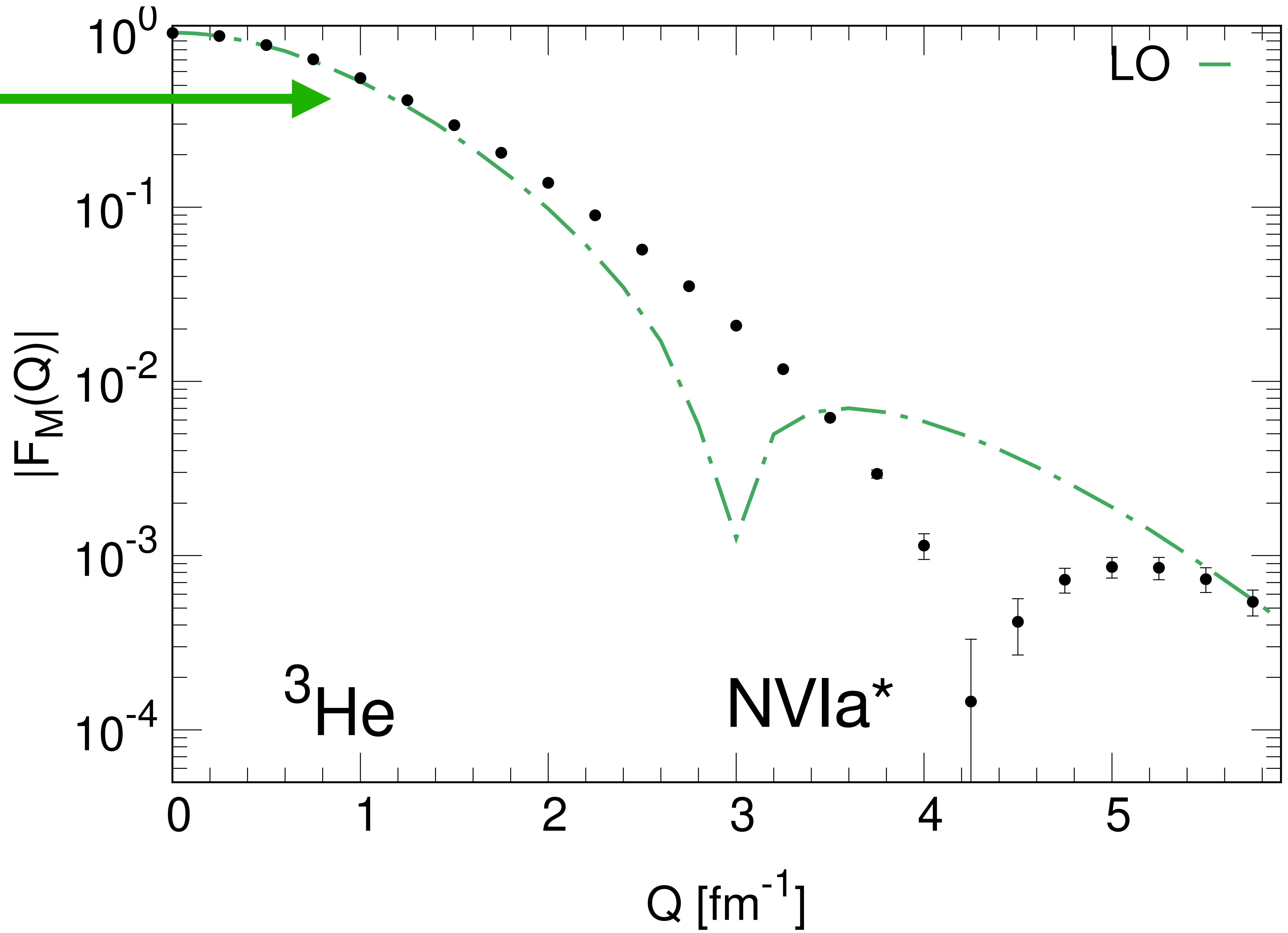


Magnetic form factors of ^{3}H

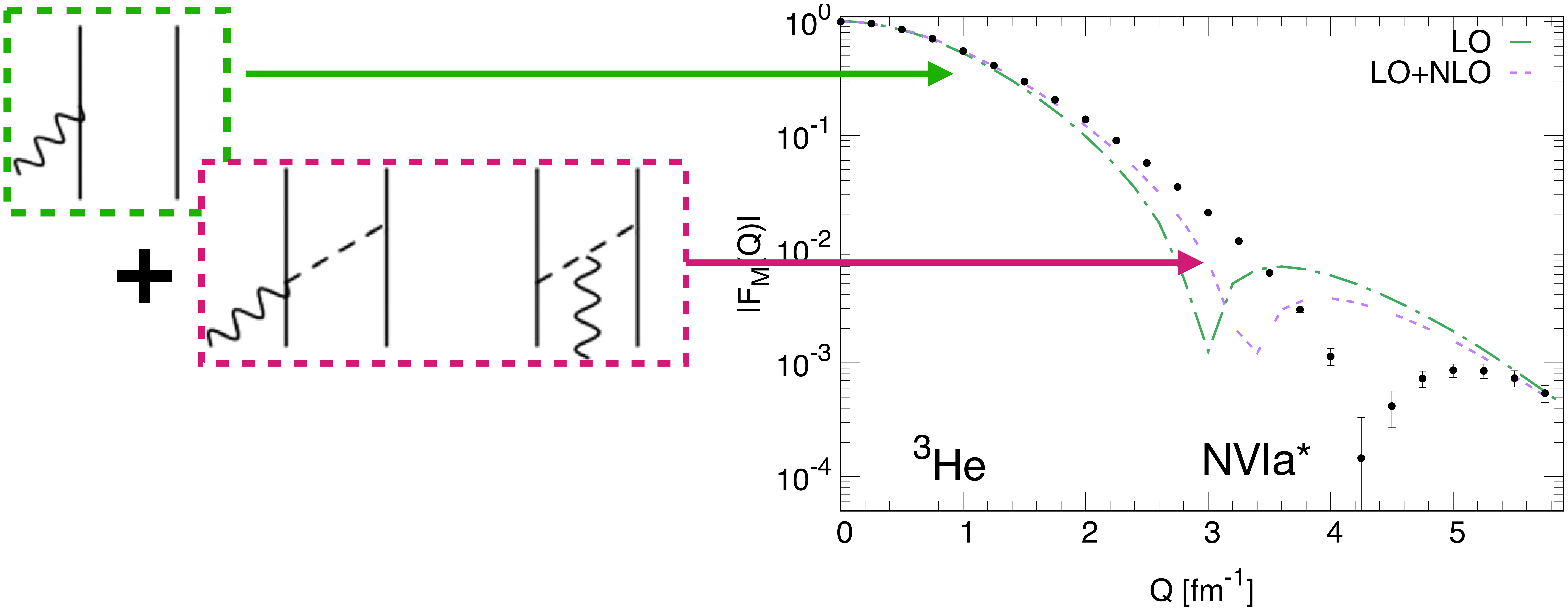


Prediction of A=3 Magnetic Form Factors





Prediction of A=3 Magnetic Form Factor



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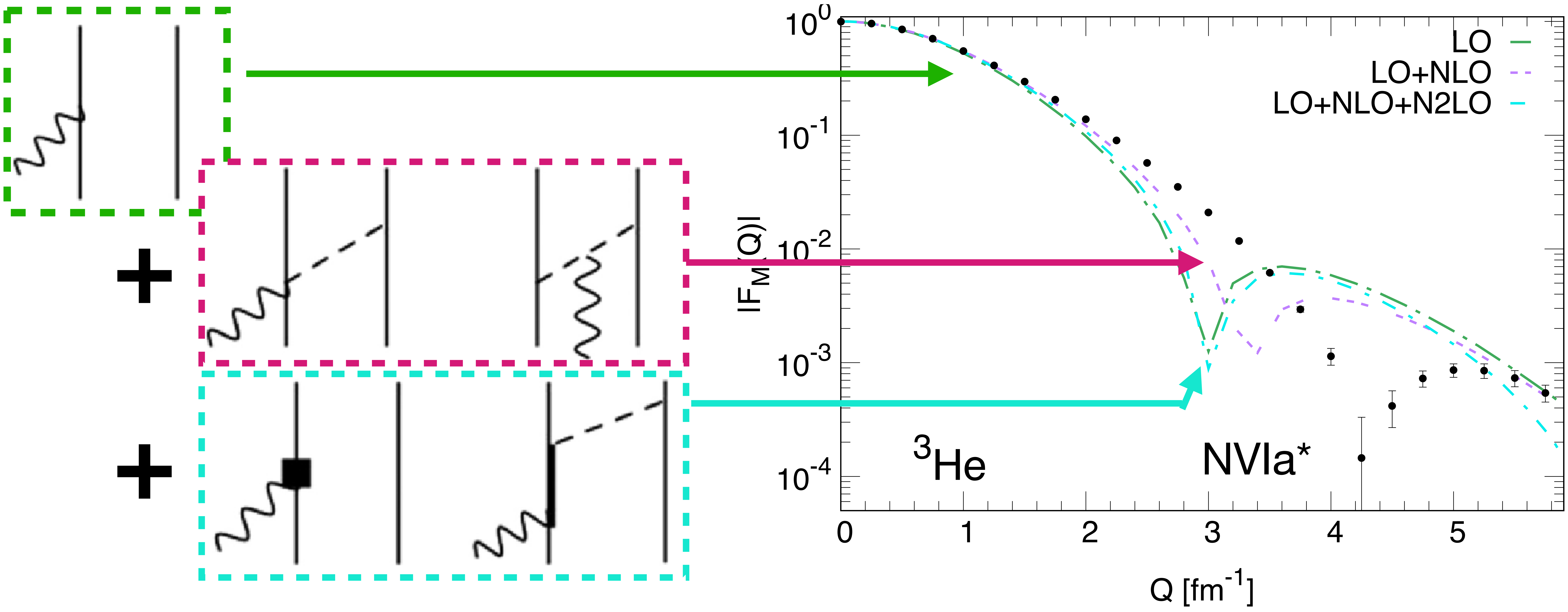

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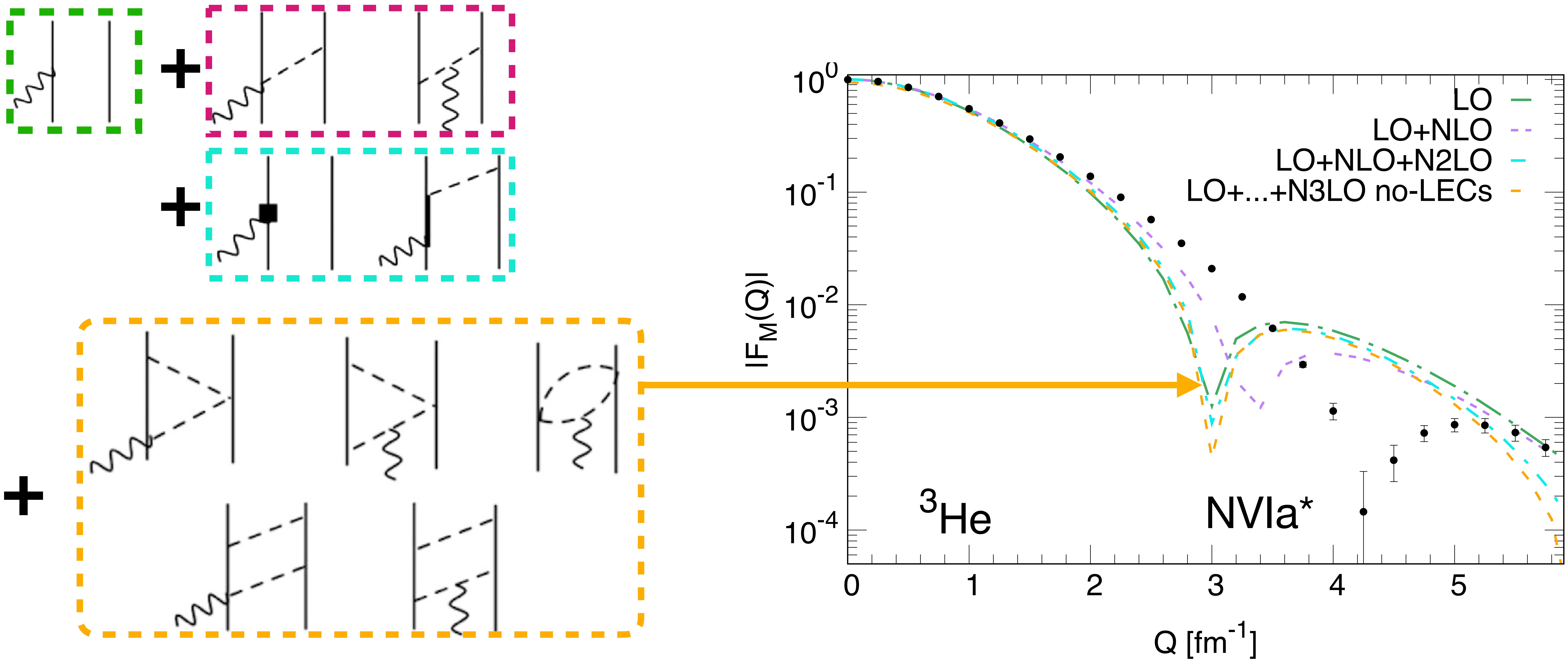
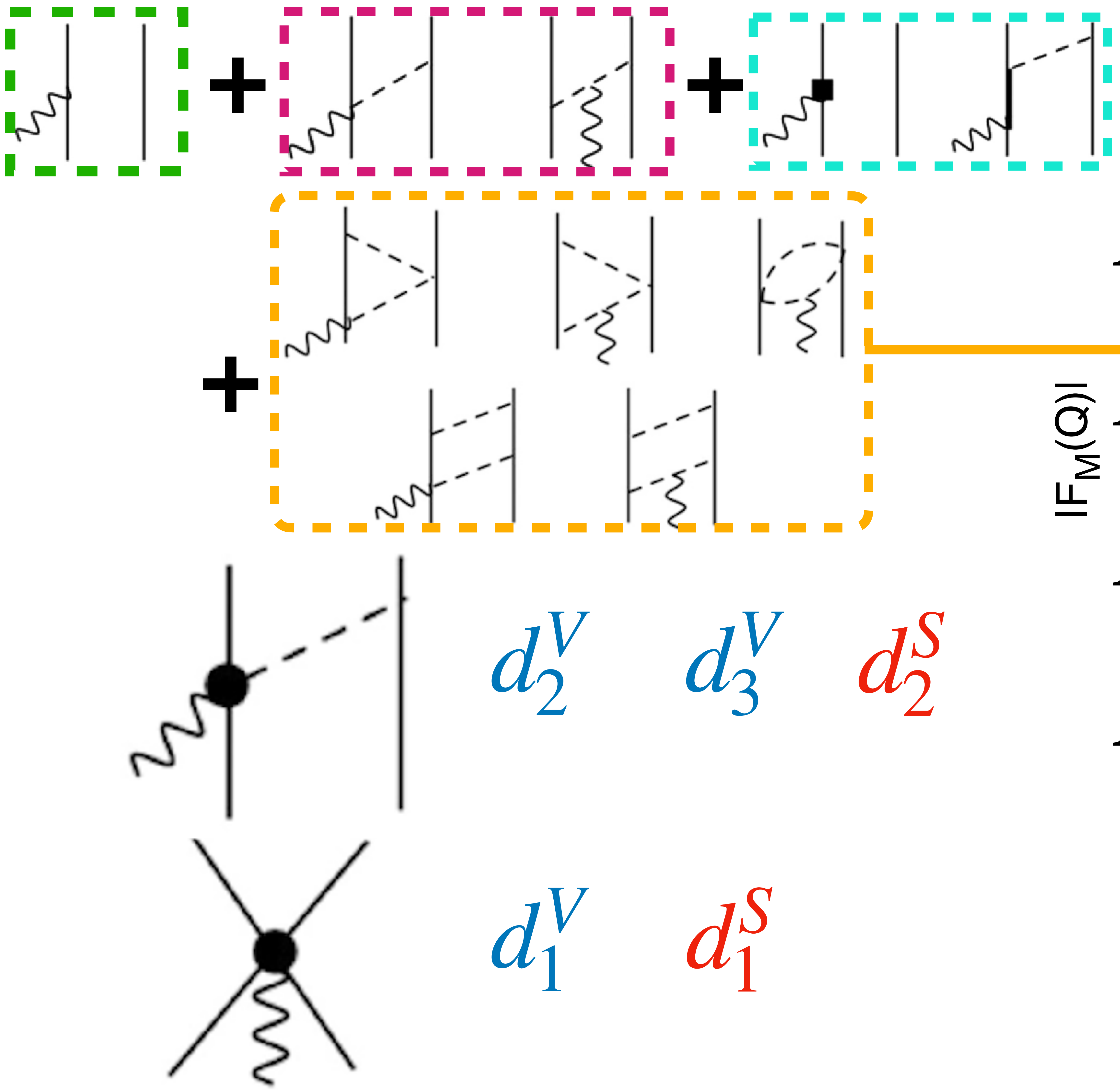
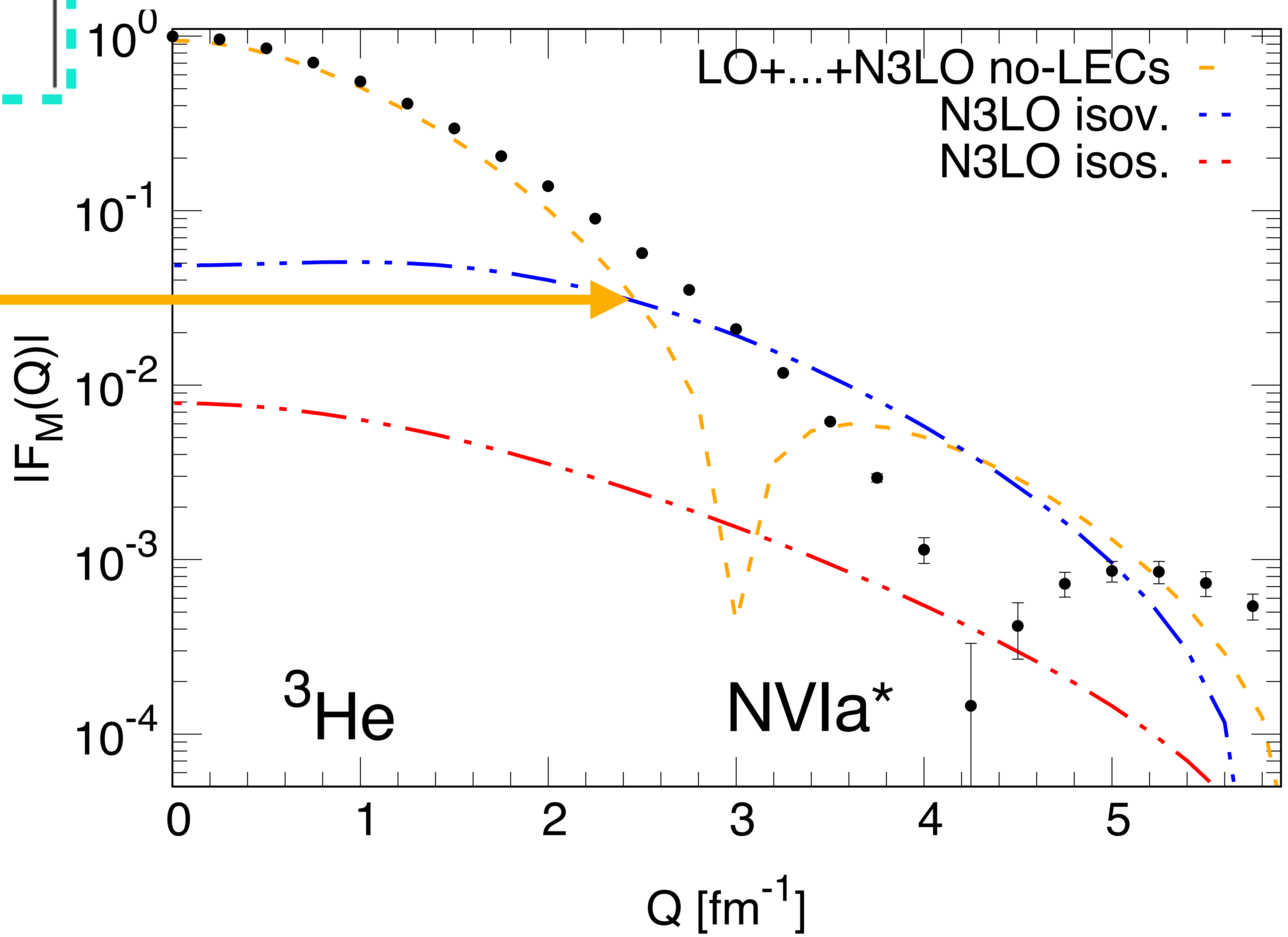


Prediction of A=3 Magnetic Form Factor





Reliability of the predictions

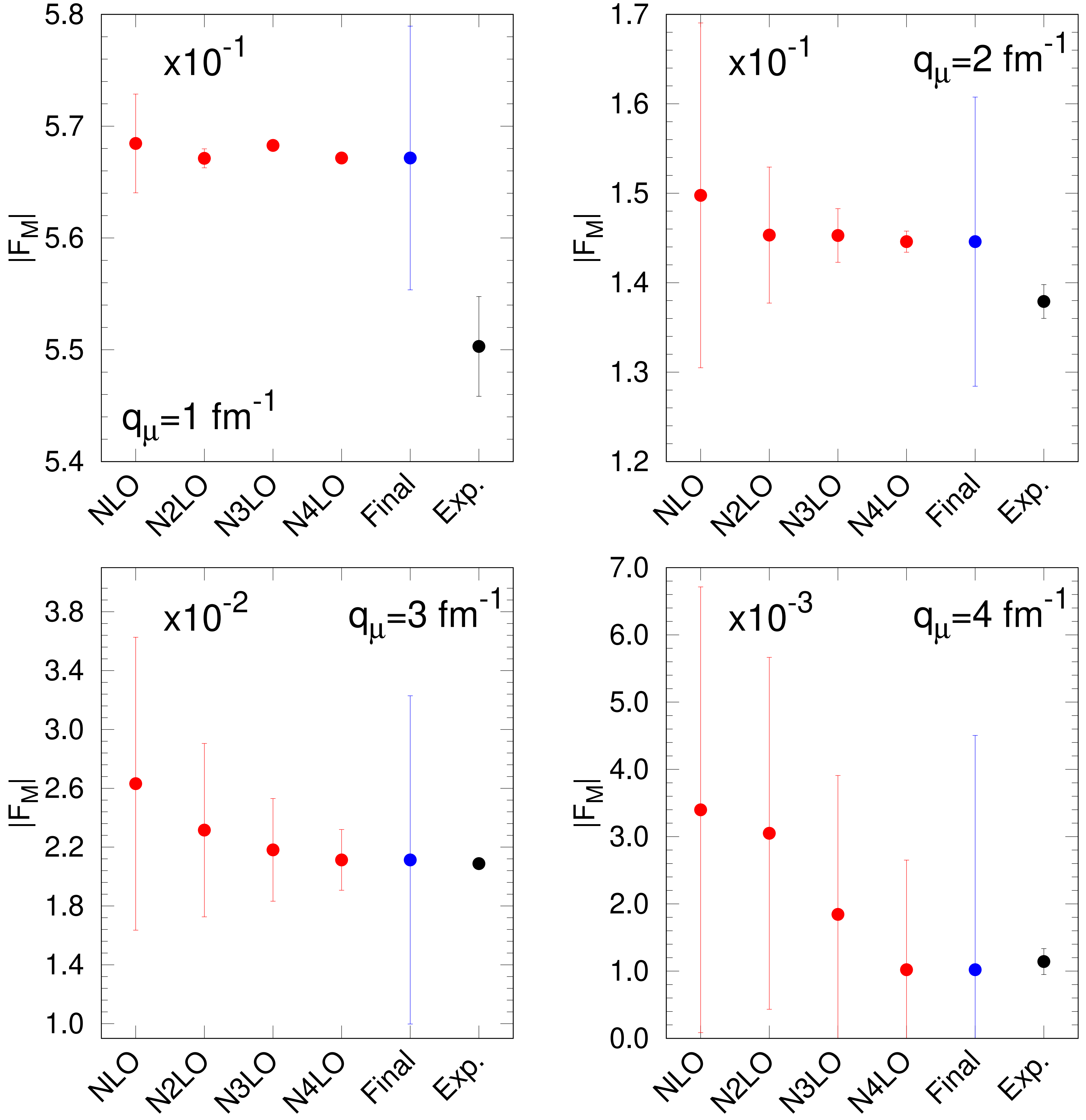
Is χEFT able to describe large Q?

- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{Q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \quad \Lambda_b = 1 \text{ GeV}$$

- Nuclear interaction + currents

- Systematic explodes after $Q^2 > 0.5\ \mathrm{GeV}^2$



^{3}He EMN500

Naive truncation error estimate

Is χEFT able to describe large Q^2?

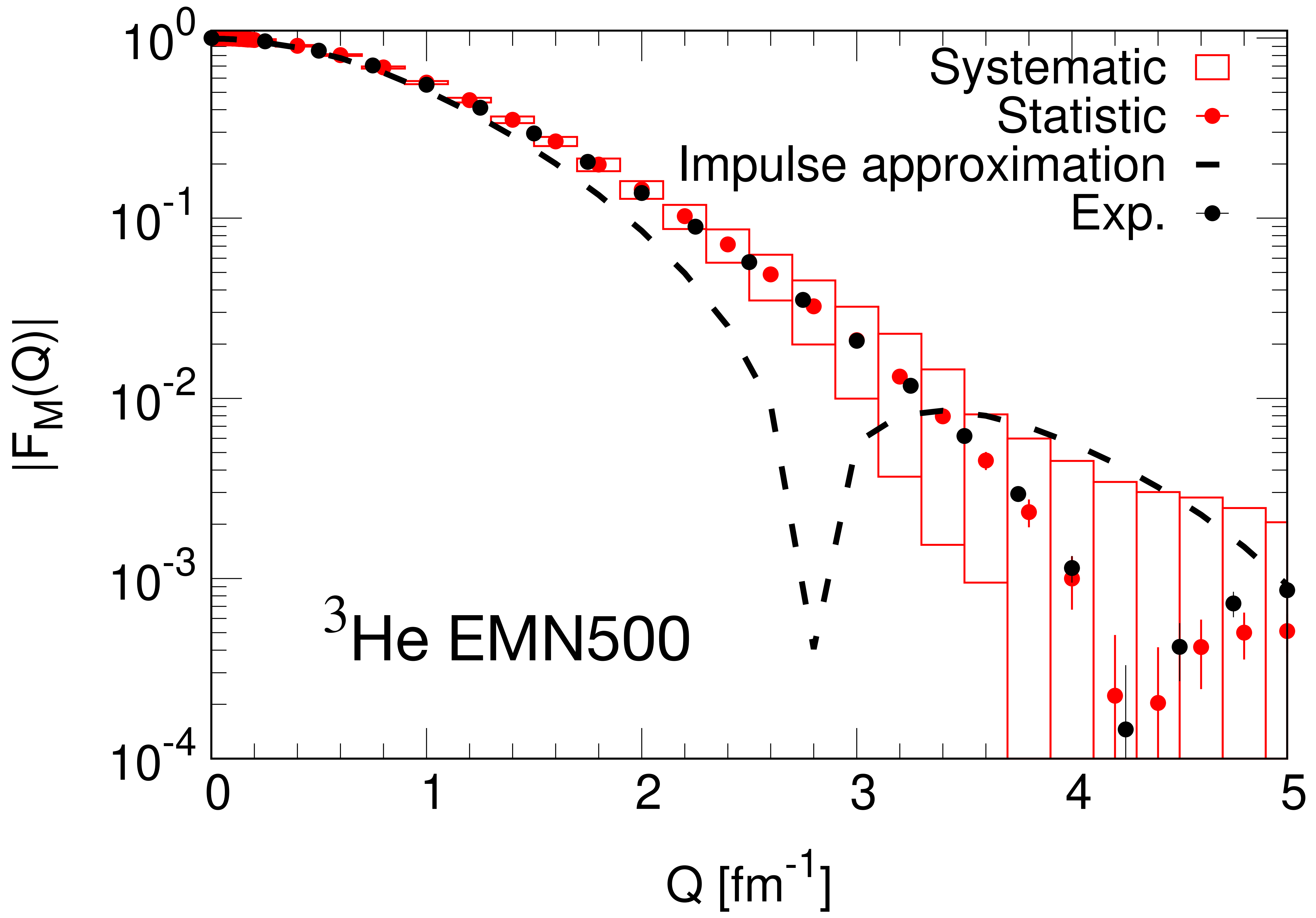
- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{Q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \quad \Lambda_b = 1 \text{ GeV}$$

- Nuclear interaction + currents

- Systematic explodes after

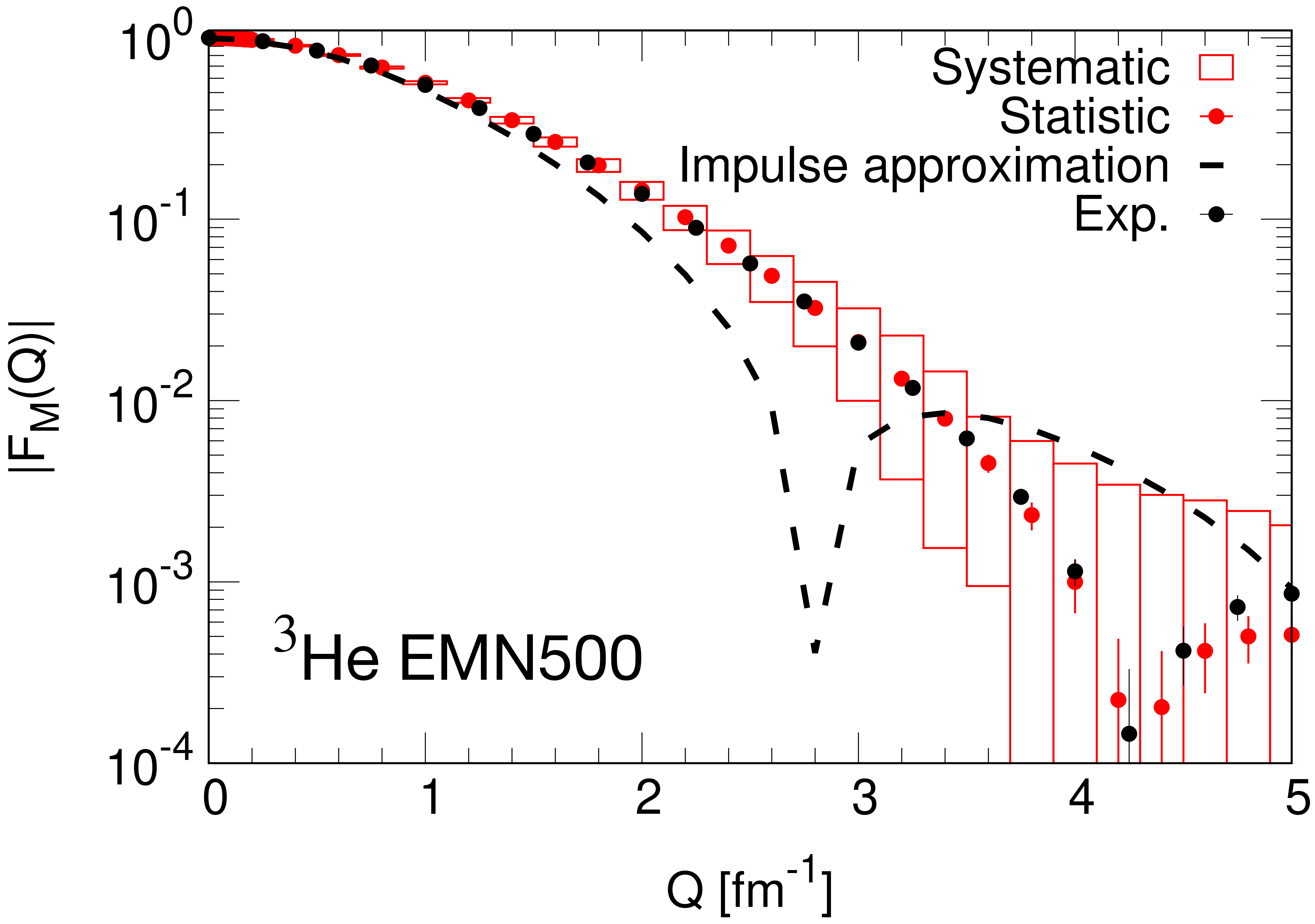
$$Q^2 > 0.5 \text{ GeV}^2$$

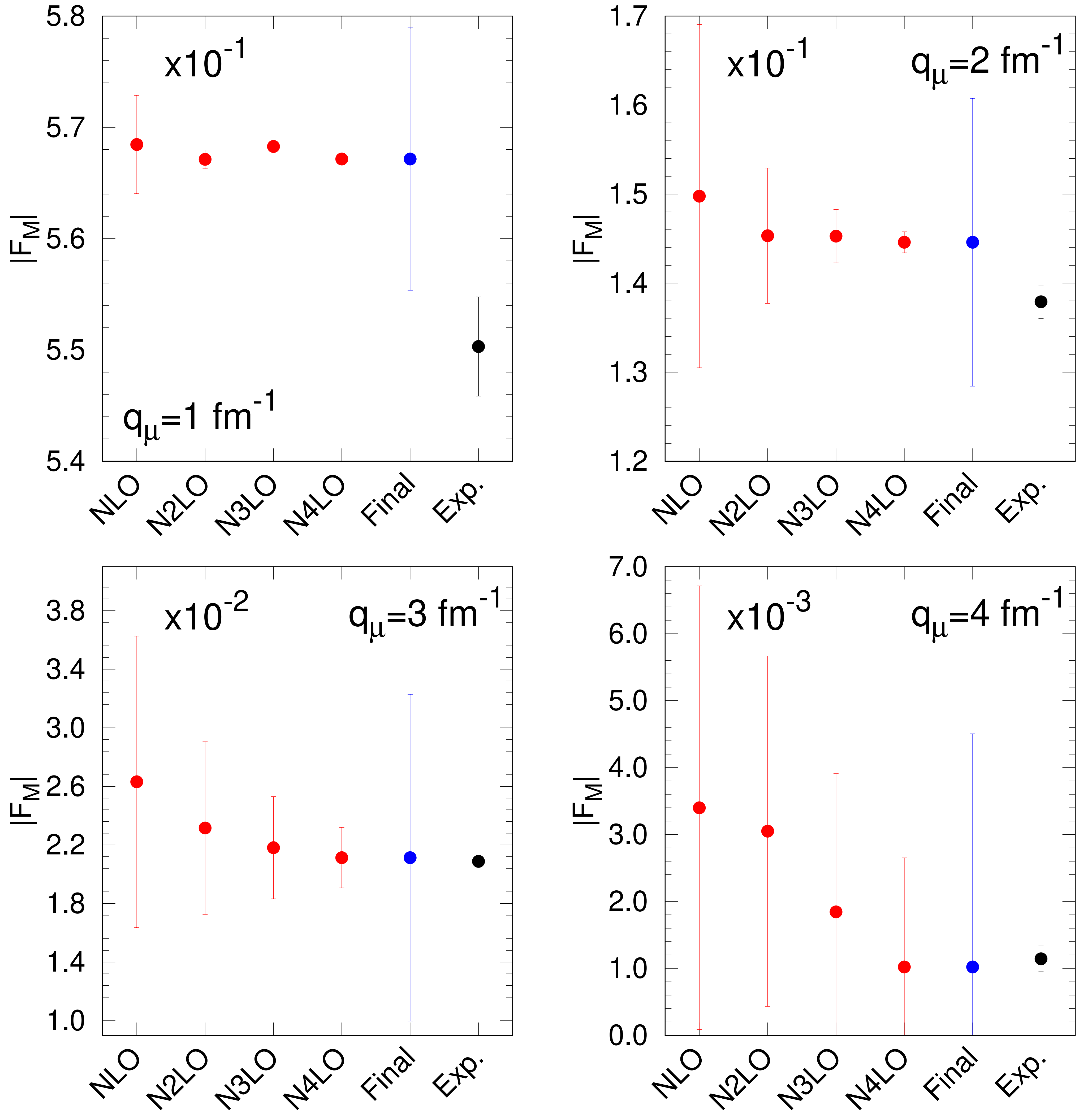


Error estimate

- Truncation errors (as [EPJA 51, 53 (2015)])

$$\alpha = \max\left\{\frac{q}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\} \qquad \Lambda_b = 1 \text{ GeV}$$





The NQS wave function

$$\Psi(x, W) \equiv \det \begin{bmatrix} \phi_v(x) & \phi_v(x_h) \\ \phi_h(x) & \phi_h(x_h) \end{bmatrix}$$

Antisymmetry = Slater-determinant

Hidden Nucleon Ansatz

Visible orbitals, visible coordinates

Visible orbitals, hidden coordinates

Hidden orbitals, visible coordinates

Hidden orbitals, hidden coordinates

[J. R. Moreno, et al., PNAS 119, 2122059119(2022)]

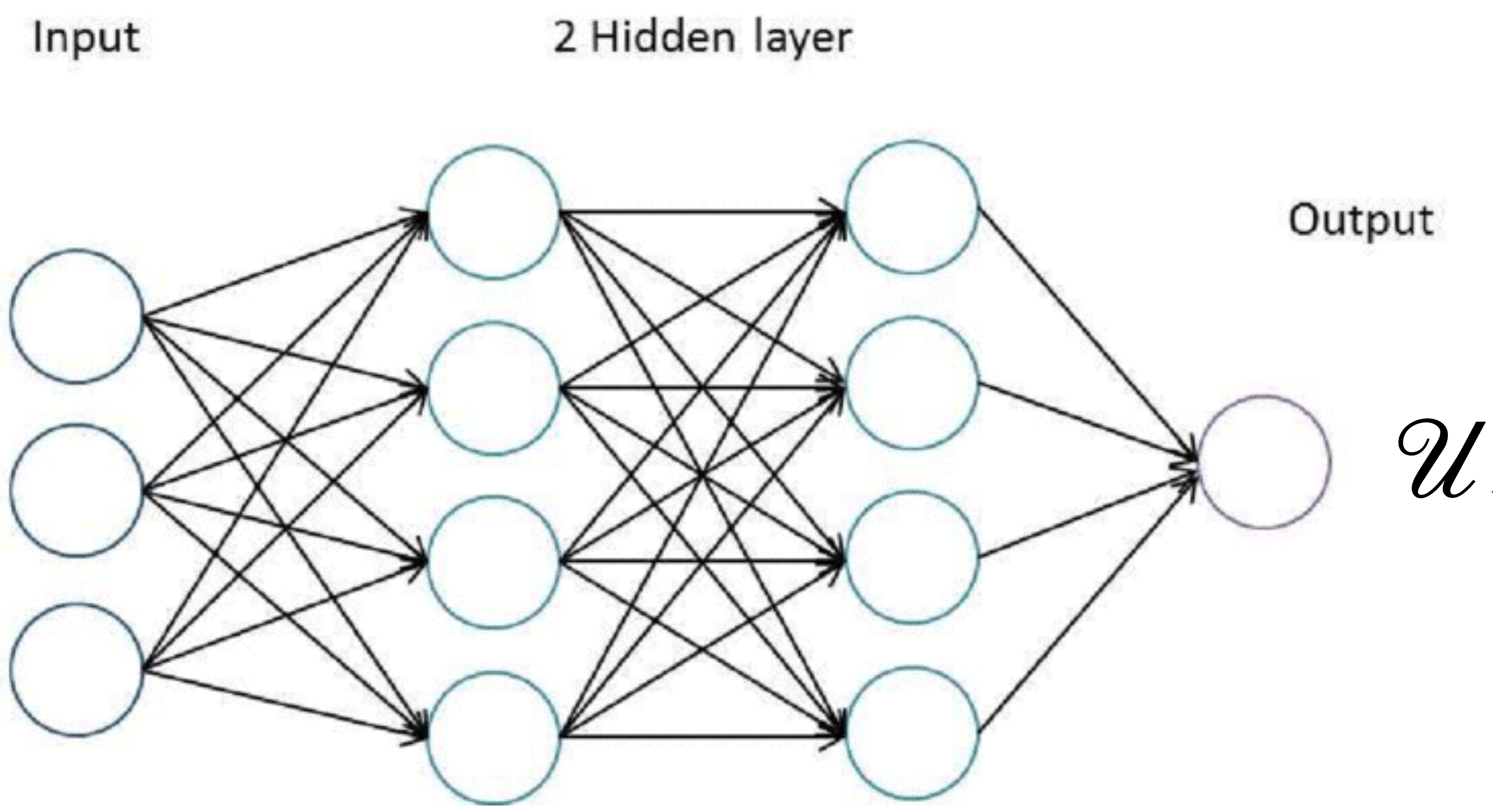
The NQS wave function

$$\Psi(x, W) \equiv \det \begin{bmatrix} \phi_v(x) & \phi_v(x_h) \\ \phi_h(x) & \phi_h(x_h) \end{bmatrix}$$

Visible orbitals, visible coordinates

$$\phi_v(x) = \exp\left[a \tanh(\mathcal{U}_v(x)/a) + i\mathcal{V}_v(x)\right]$$

Feed-forward fully connected Neural Network



$\mathcal{U}_v(x), \mathcal{V}_v(x)$

$A \times A$ matrix

$\simeq$

standard slater determinant

A simple interpretation

$$\Psi(x, W) \equiv \det \begin{bmatrix} \phi_v(x) & \phi_v(x_h) \\ \phi_h(x) & \phi_h(x_h) \end{bmatrix}$$

Let us consider $N_h = 1,\ \phi_h(x) = 0$ and, $\phi_v(x_h) = 0$

$$\Psi(x, W) \equiv \phi_h(x_h) \times \begin{vmatrix} \phi_v^1(x_1) & \cdots & \phi_v^1(x_A) \\ \vdots & \ddots & \vdots \\ \phi_v^A(x_1) & \cdots & \phi_v^A(x_A) \end{vmatrix}$$

Jastrow function

Slater determinant

Hidden nucleons generate the correlations among the single orbitals

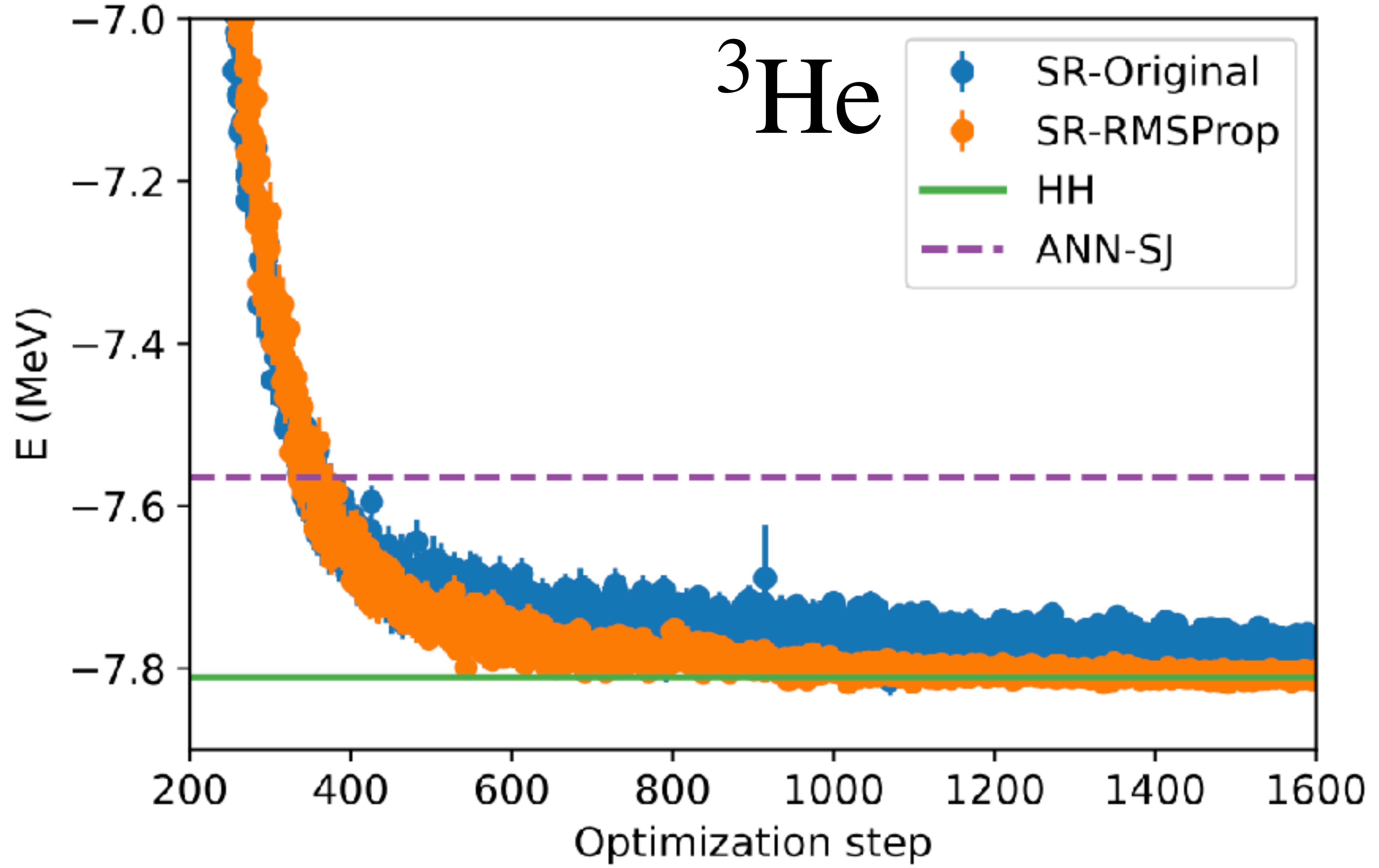
Optimization of the wave function

Monte Carlo

$$E(W) = \frac{\langle \Psi_V(W) | H | \Psi_V(W) \rangle}{\langle \Psi_V(W) | \Psi_V(W) \rangle}$$

Stochastic reconfiguration method

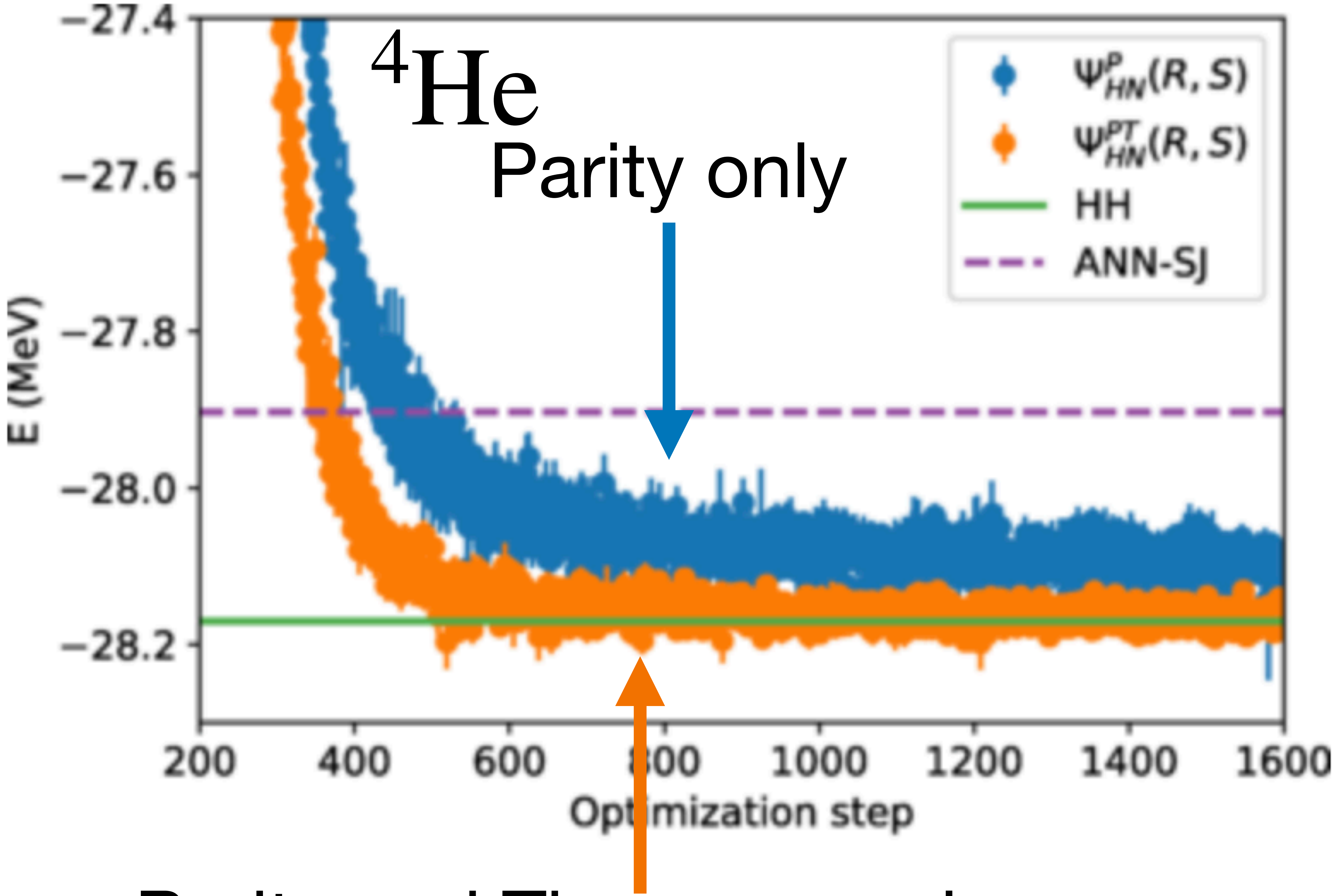
$$W_{new} = W_{old} + \delta W(\partial_W \Psi, \partial_W^2 \Psi, E(W))$$



A. Lovato, et al., Phys. Rev. Res. **4**, 043178 (2022)

Improving convergence

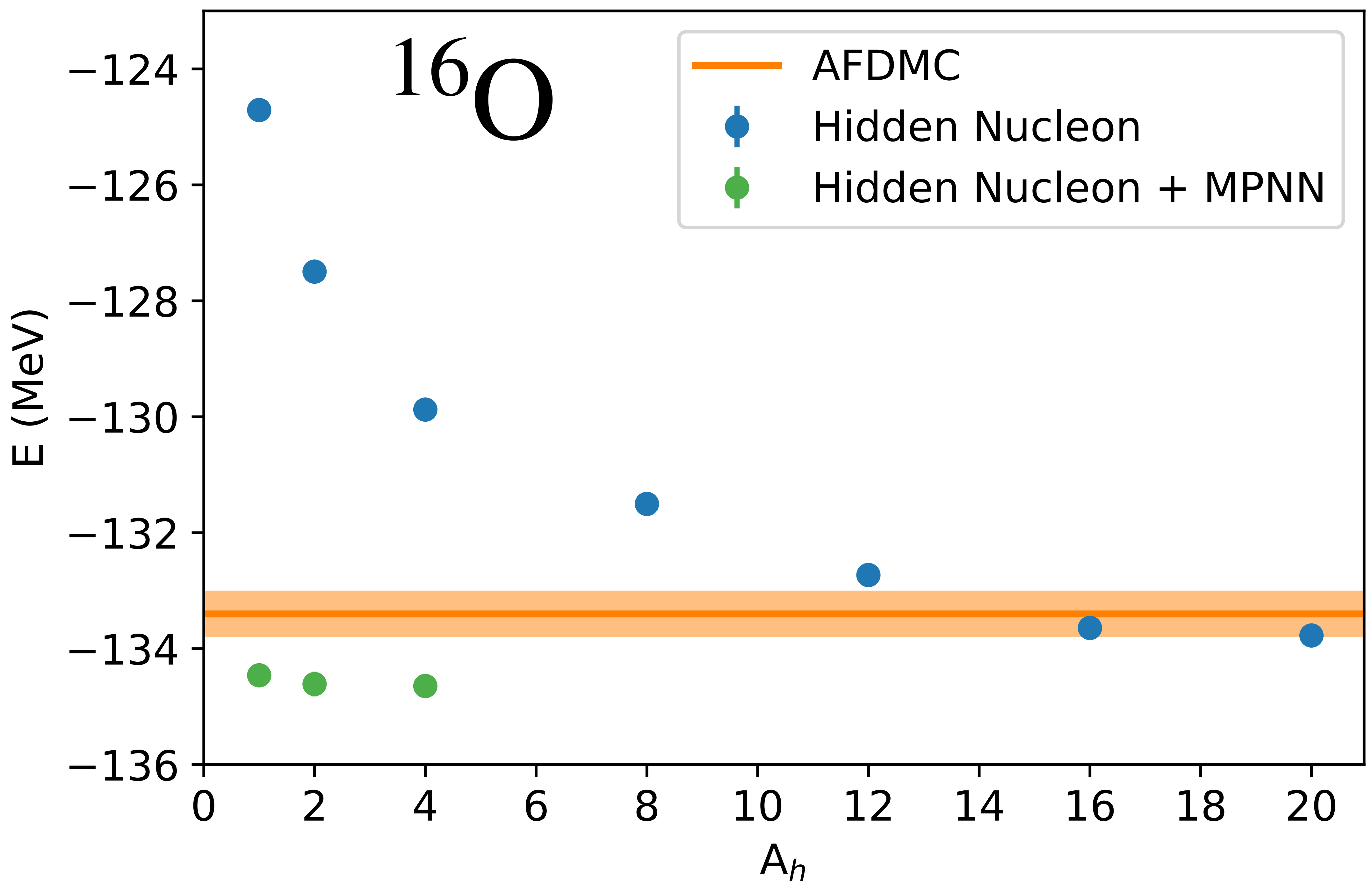
Instruct the network on discreet symmetries



Parity and Time reversal

A. Lovato, et al., Phys. Rev. Res. **4**, 043178 (2022)

Include correlations in the inputs



Message passing neural-network

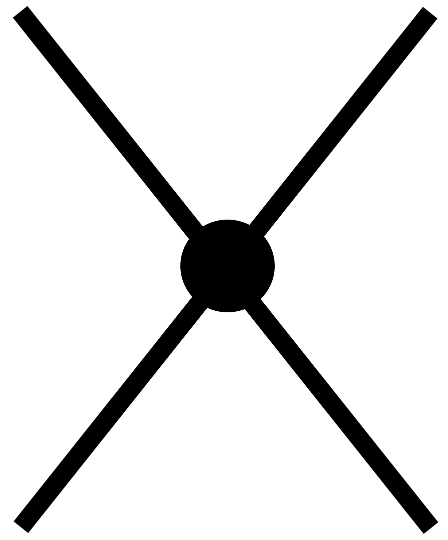
G. Pescia, et al., arXiv:2305.07240 (2023)

To the essential

Pionless EFT like model [R. Schiavilla et al. Phys. Rev. C 103, 054003 (2021)]

- Two-body interactions (with optimized cut-off — model “o”)

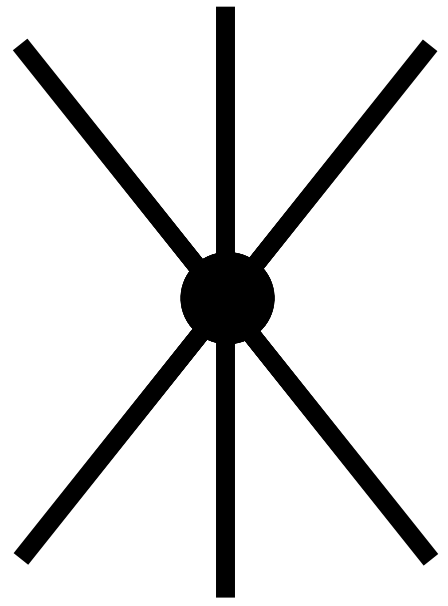
$$V = C_{01} P_0^\sigma P_1^\tau + C_{10} P_1^\sigma P_0^\tau$$



LECs fitted on np singlets scattering length and effective range + deuteron binding energy

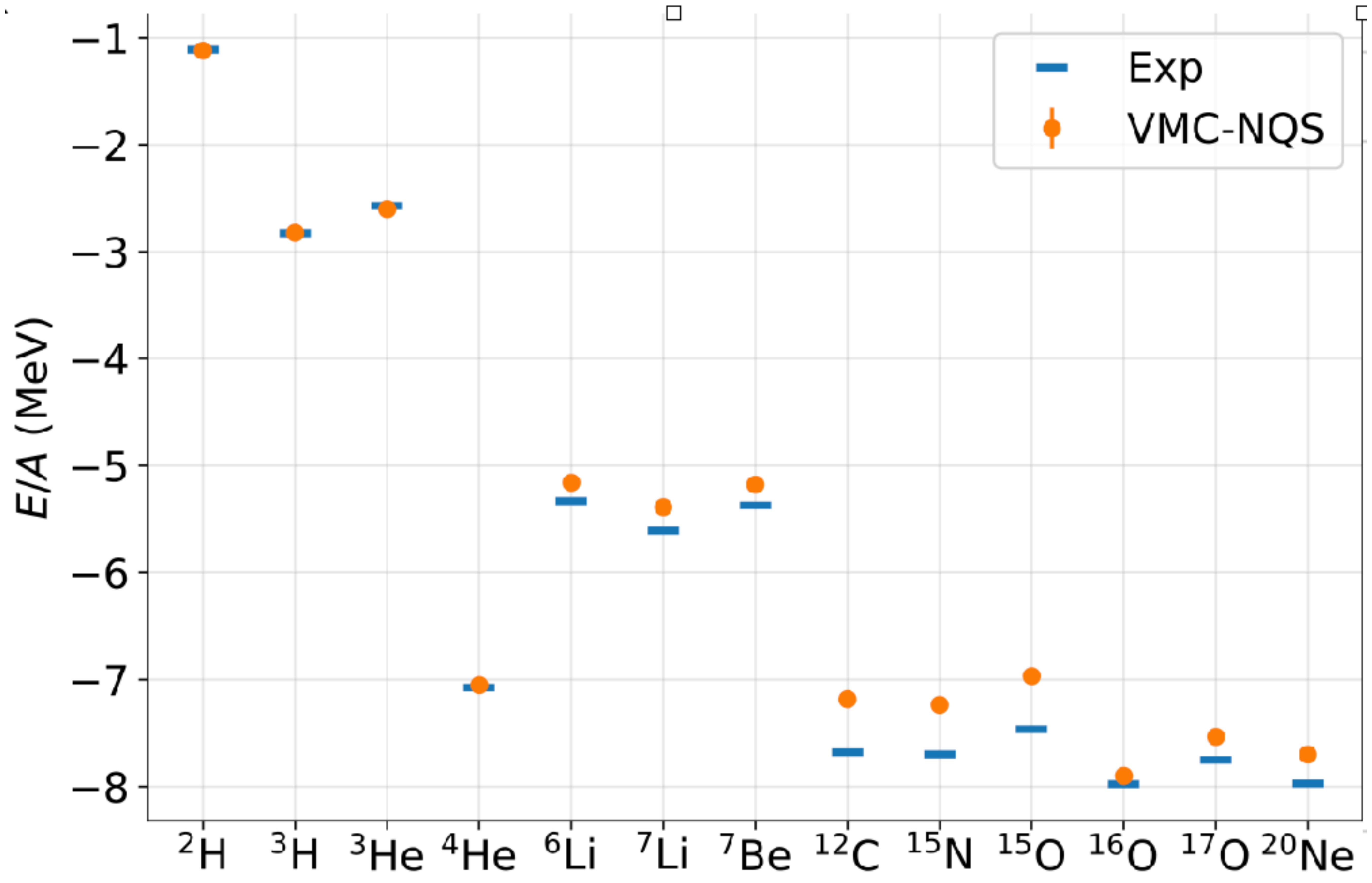
- Three-body interaction ($R_3 = 1.1$ fm)

$$V_{3b} = c_E \frac{f_\pi^4}{\Lambda_\chi} \frac{(\hbar c)^6}{\pi^3 R_3^6} \sum_{\text{cyclic } ijk} e^{-\left(r_{ij}^2 + r_{jk}^2\right)/R_3^2}$$



c_E fitted for reproducing tritium binding energy

Results: binding energies



Good agreement with the experimental values (<7%)

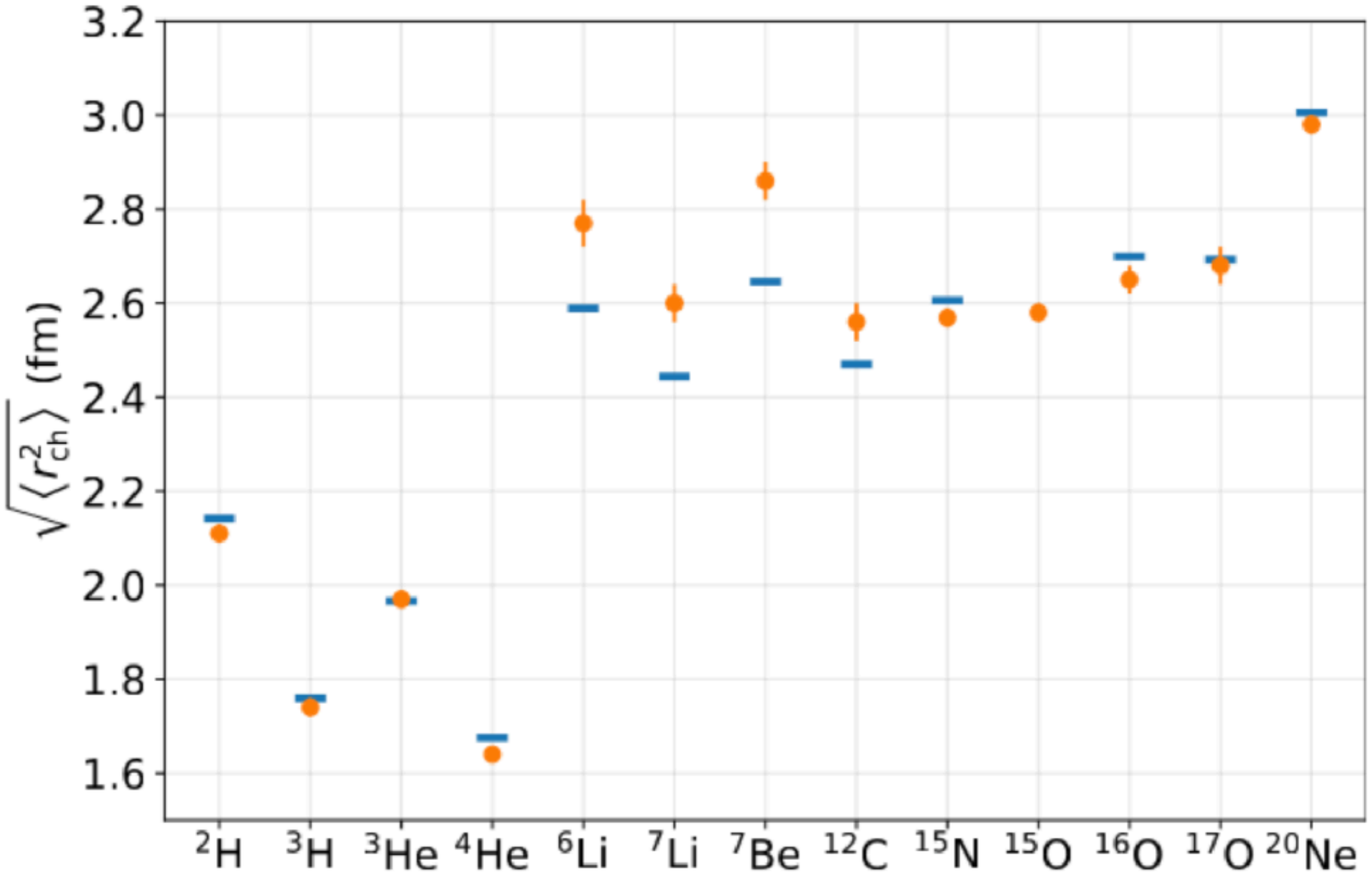
^{6}He, ^{8}Li, ^{8}B, ^{9}C, ^{17}F are unstable against cluster breakup

Computed nuclei underbind

Excessive repulsion of the Hamiltonian

No pretraining of the NQS on Hartree-Fock wave function!!

Results: charge radii



Results are stable

No need of extrapolation

No spin orbit and two-body currents corrections

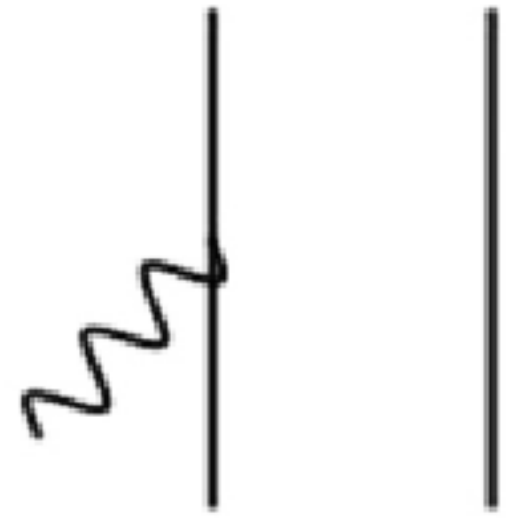
Revealing the shell structure

- **Require polarization of the wave function**

$$\mu = \langle J, J_z = J | \mu_z | J, J_z = J \rangle$$

- One body operator

$$\mu_z = \frac{1 + \tau_z}{2} L_z + \frac{\mu_s + \mu_v \tau_z}{2} \sigma_z$$



- Direct connection among shell structure and magnetic moments

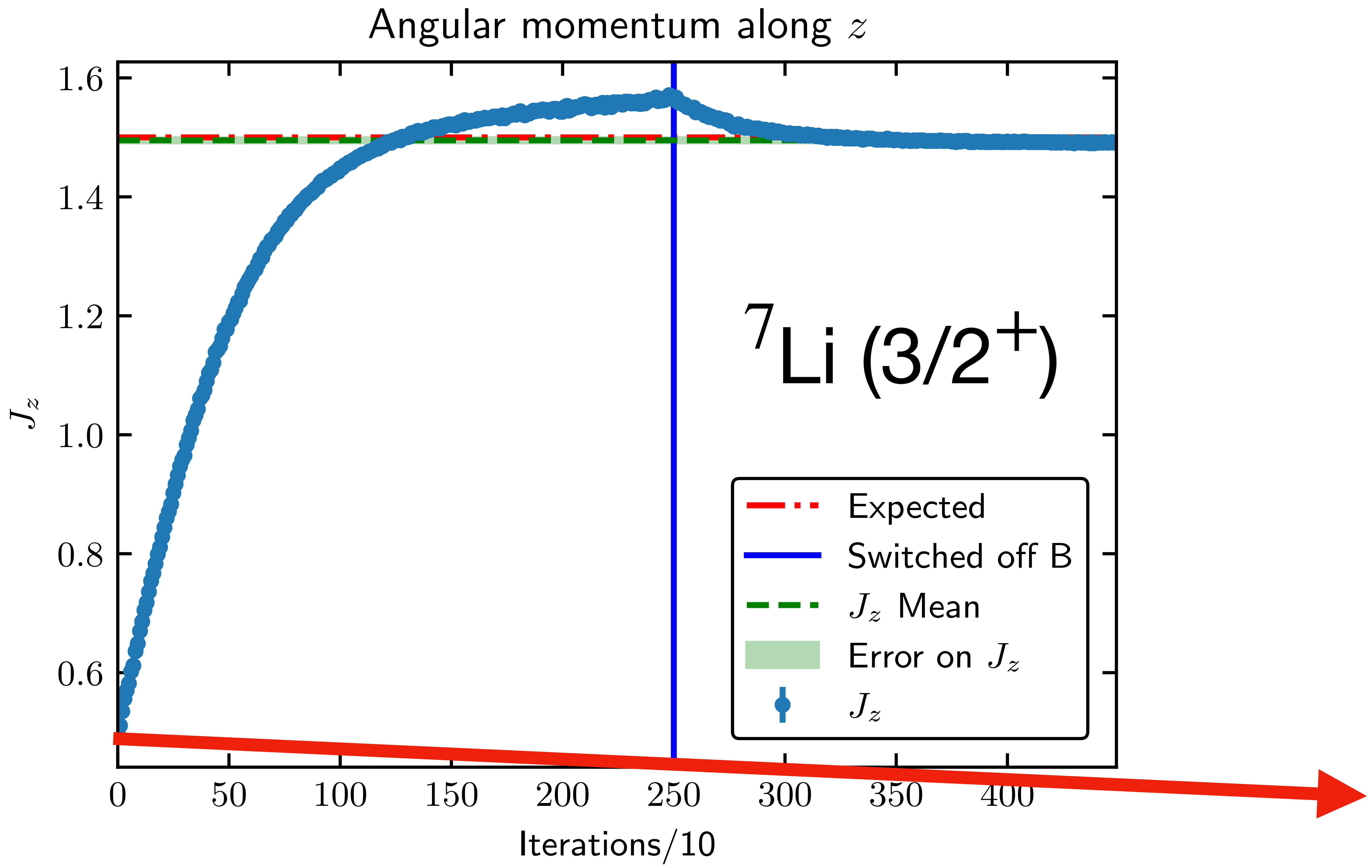
$$\mu = \mu_N \left(j - \frac{1}{2} + \lambda \right); \qquad j = l + \frac{1}{2} \qquad\qquad \mu = \mu_N \left[j + \frac{j}{j+1} \left(\frac{1}{2} - \lambda \right) \right]; \qquad j = l - \frac{1}{2}$$

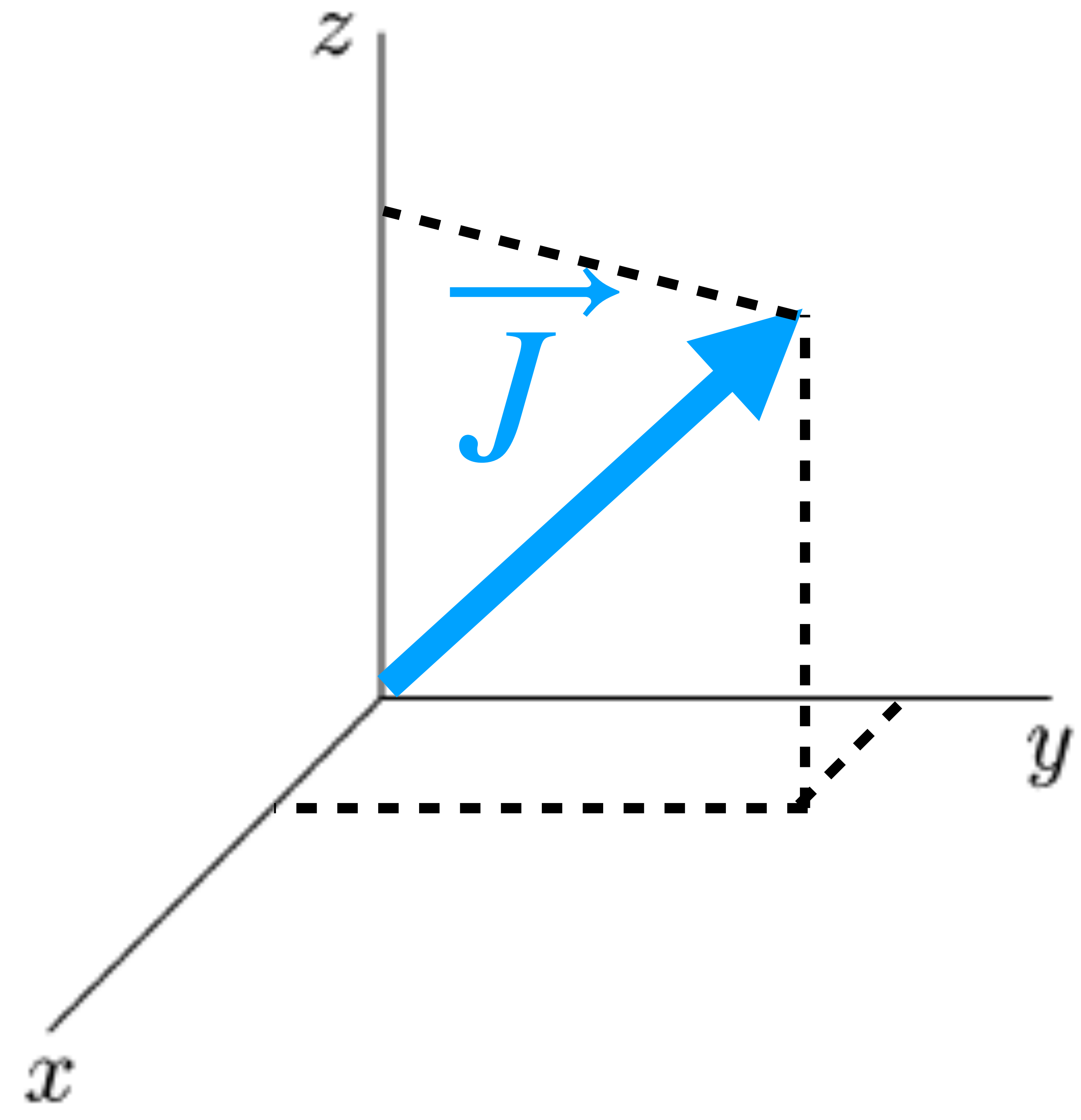
Valid for uncoupled proton (JD Walecka, Theor. Nucl. and Subnucl. Phys.)

Projection of NQS

What is measured

$$\mu = \langle J, J_z = J | \mu_z | J, J_z = J \rangle$$



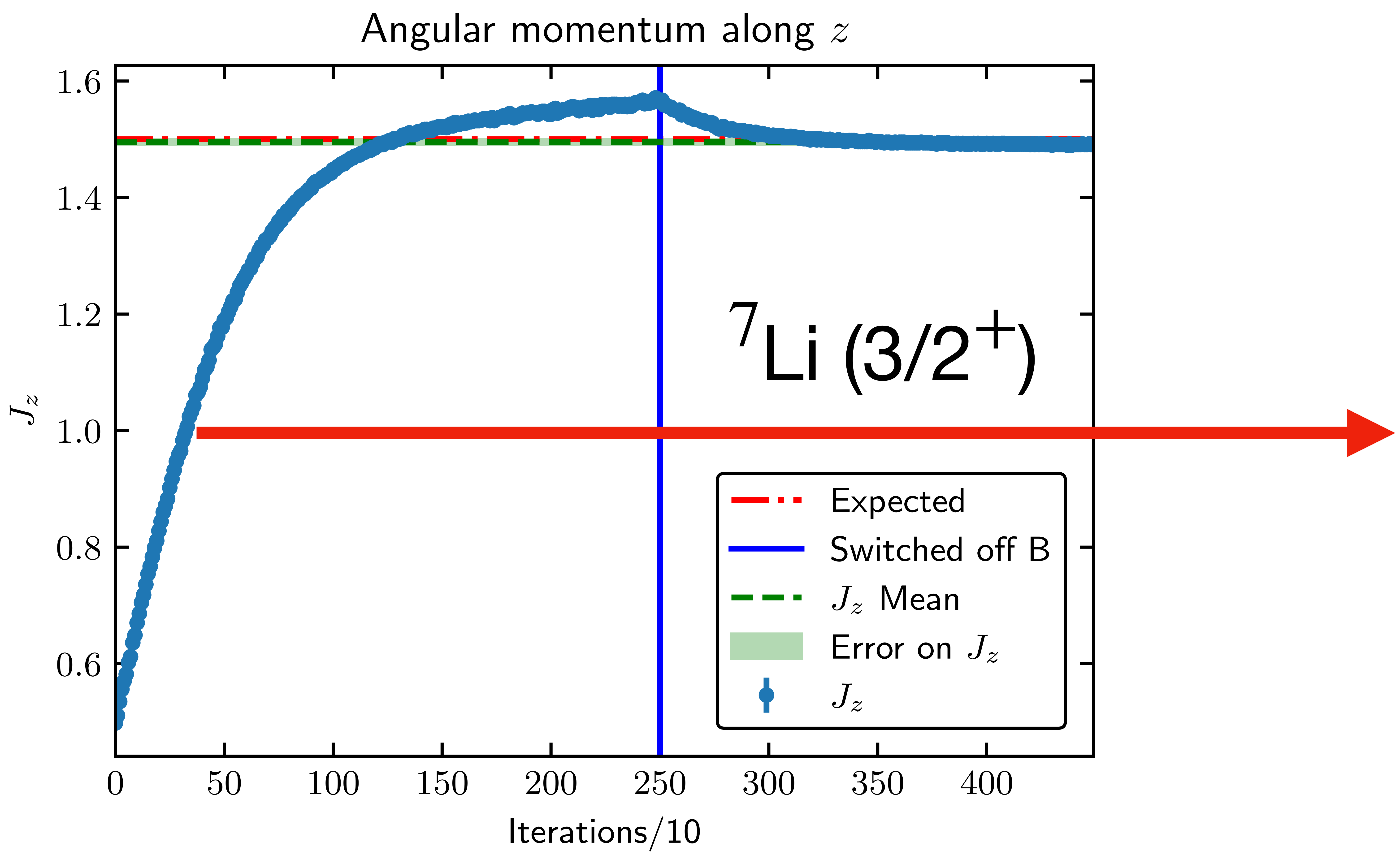


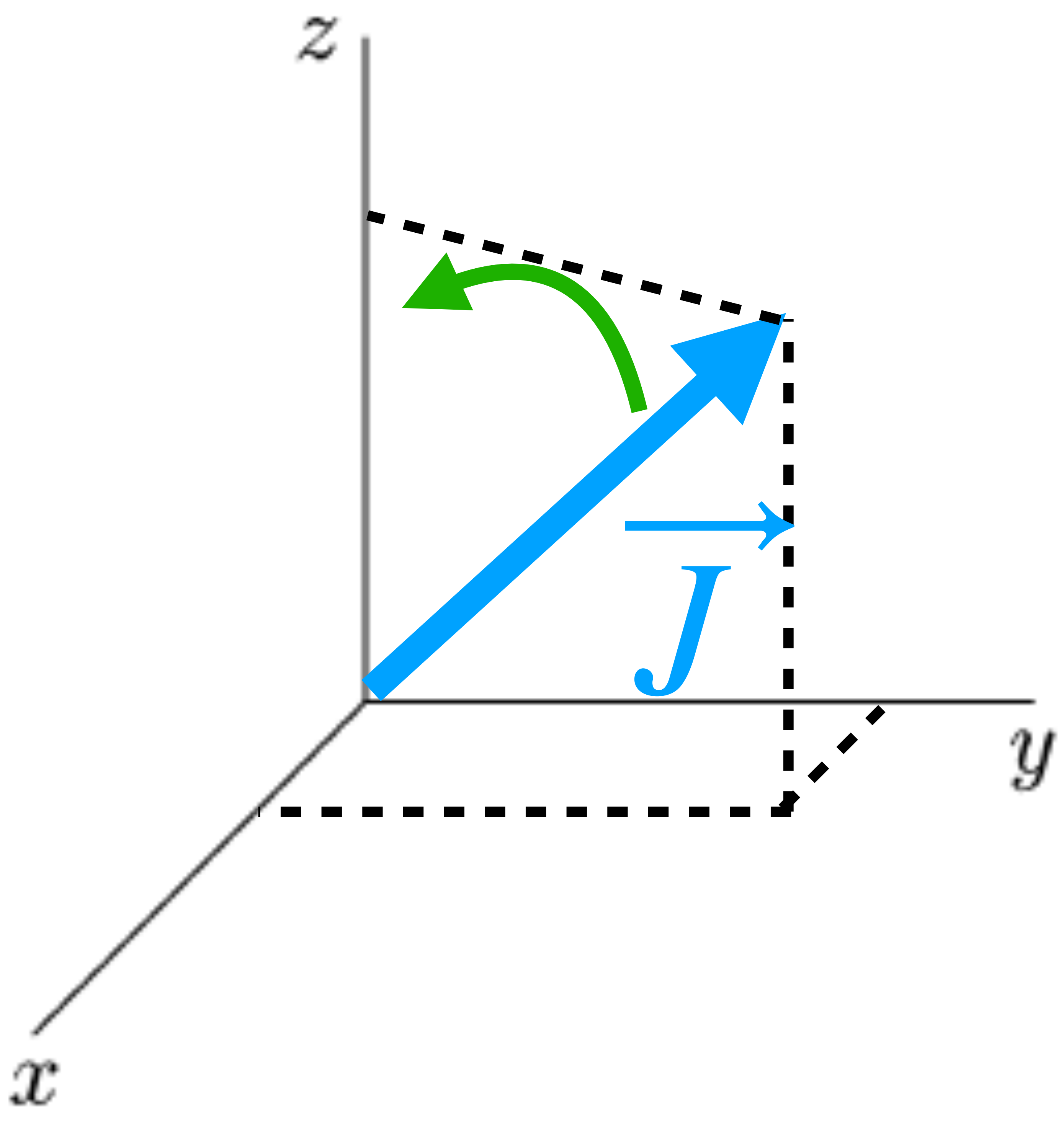
$$|\Psi_J\rangle = \sum_{J_z} c_{J_z} |\Psi_{J,J_z}\rangle$$

Projection of NQS

What is measured

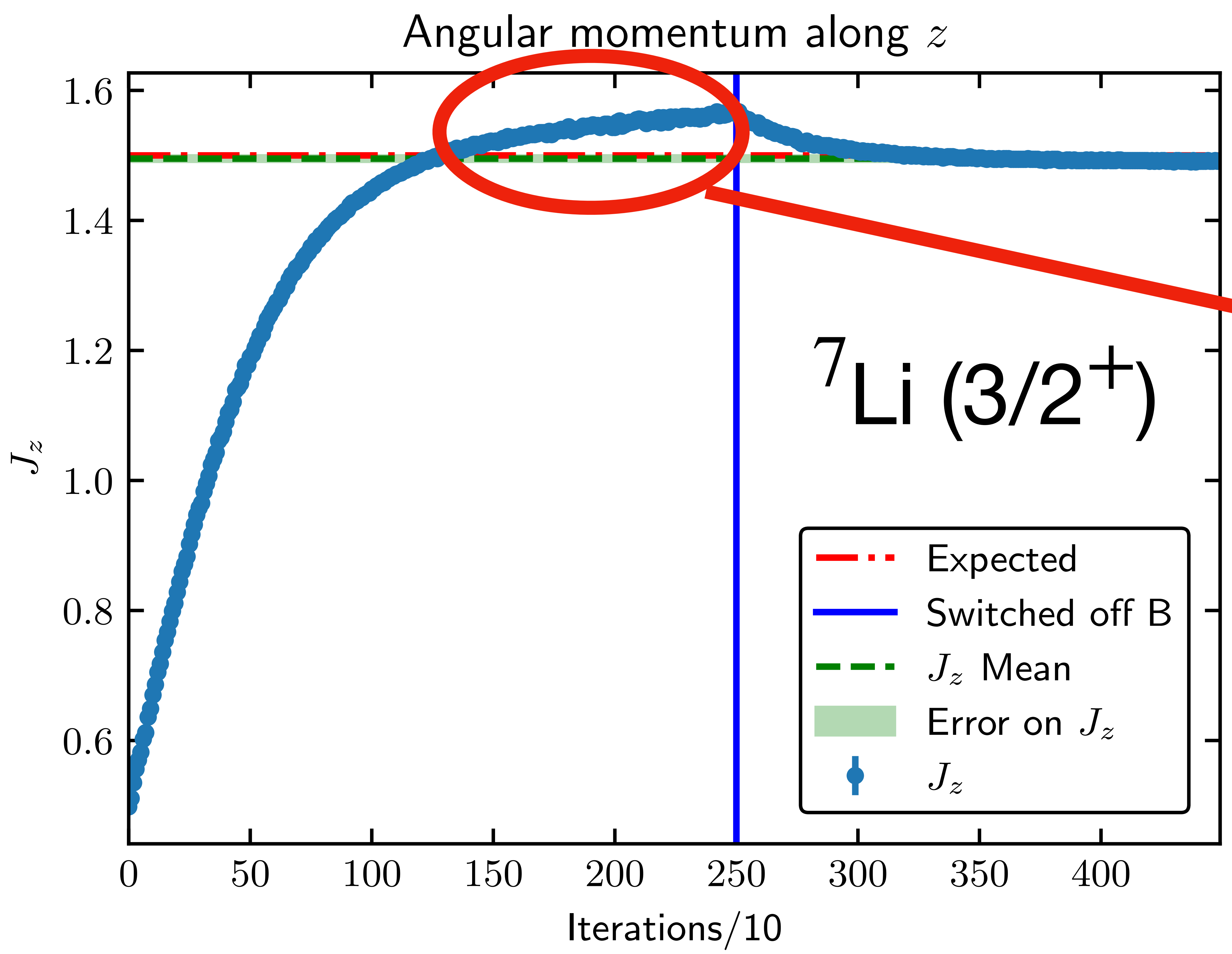
$$\mu = \langle J, J_z = J | \mu_z | J, J_z = J \rangle$$





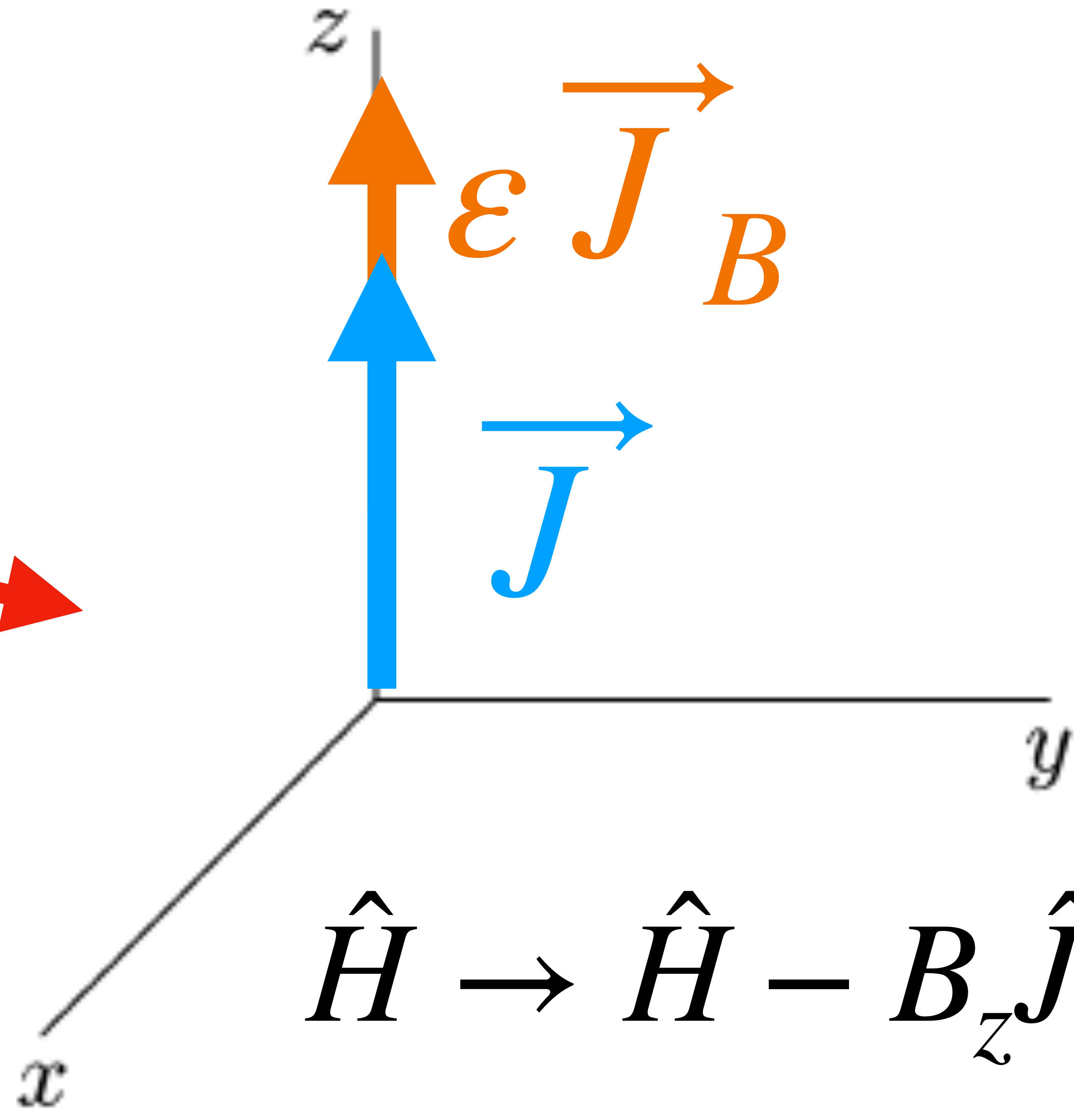
$$\hat{H} \to \hat{H} - B_z \hat{J}_z$$

Projection of NQS



What is measured

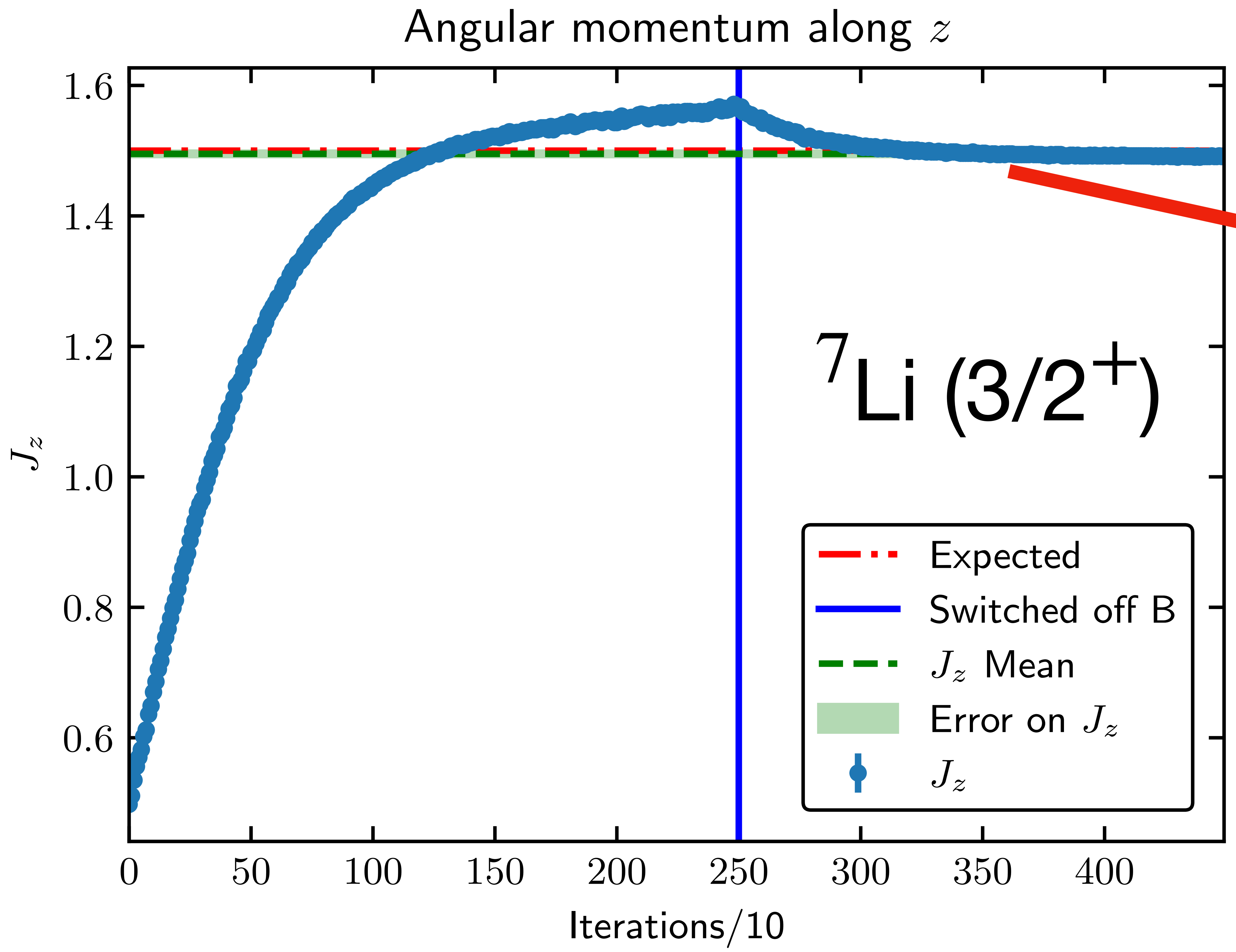
$$\mu = \langle J, J_z = J | \mu_z | J, J_z = J \rangle$$



$$\hat{H} \rightarrow \hat{H} - B_z \hat{J}_z$$

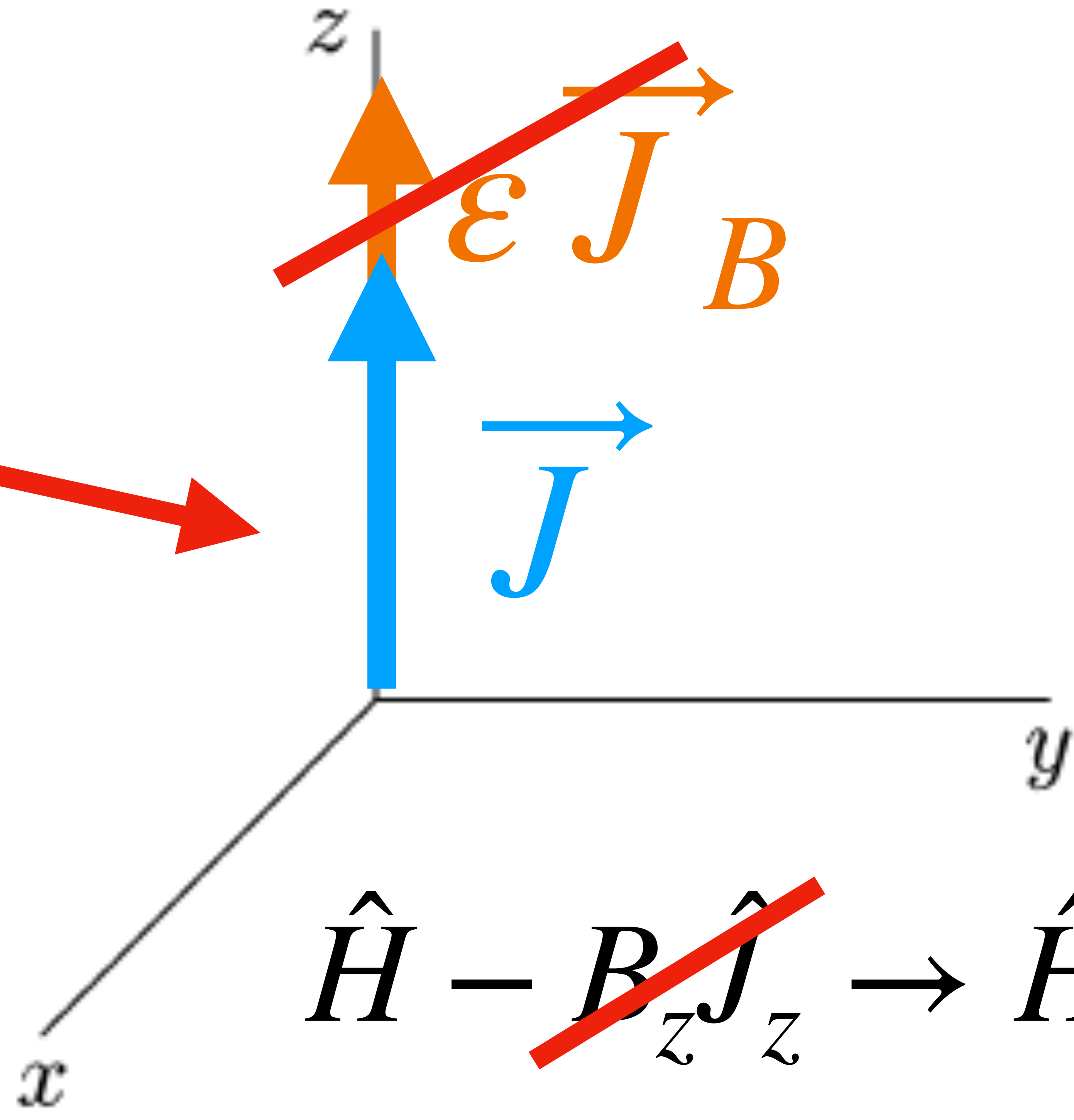
$$|\Psi_J\rangle = |\Psi_{J,J_z=J}\rangle + \varepsilon \sum_{J_B} C_{J_B} |\Psi_{J_B}\rangle$$

Projection of NQS



What is measured

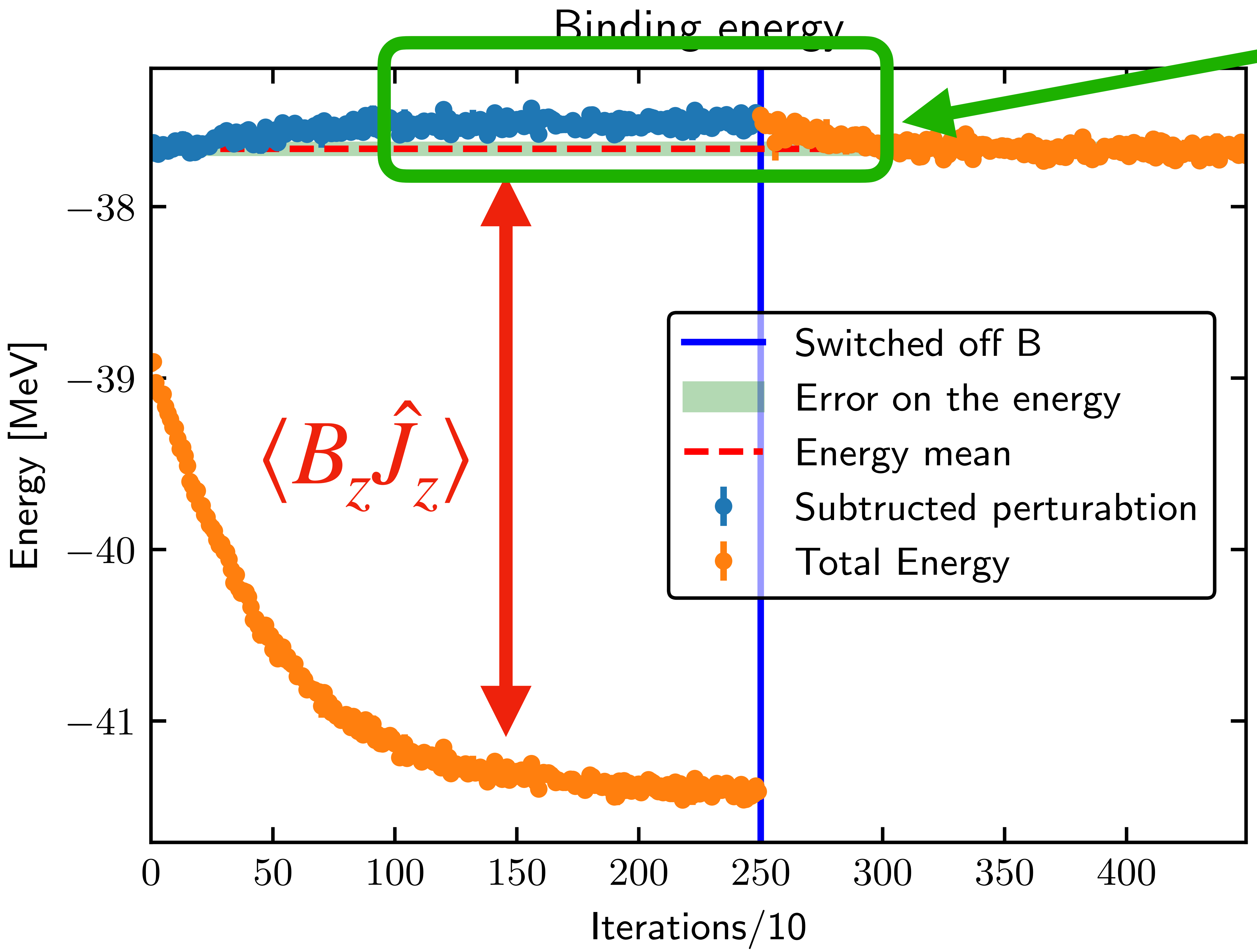
$$\mu = \langle J, J_z = J | \mu_z | J, J_z = J \rangle$$

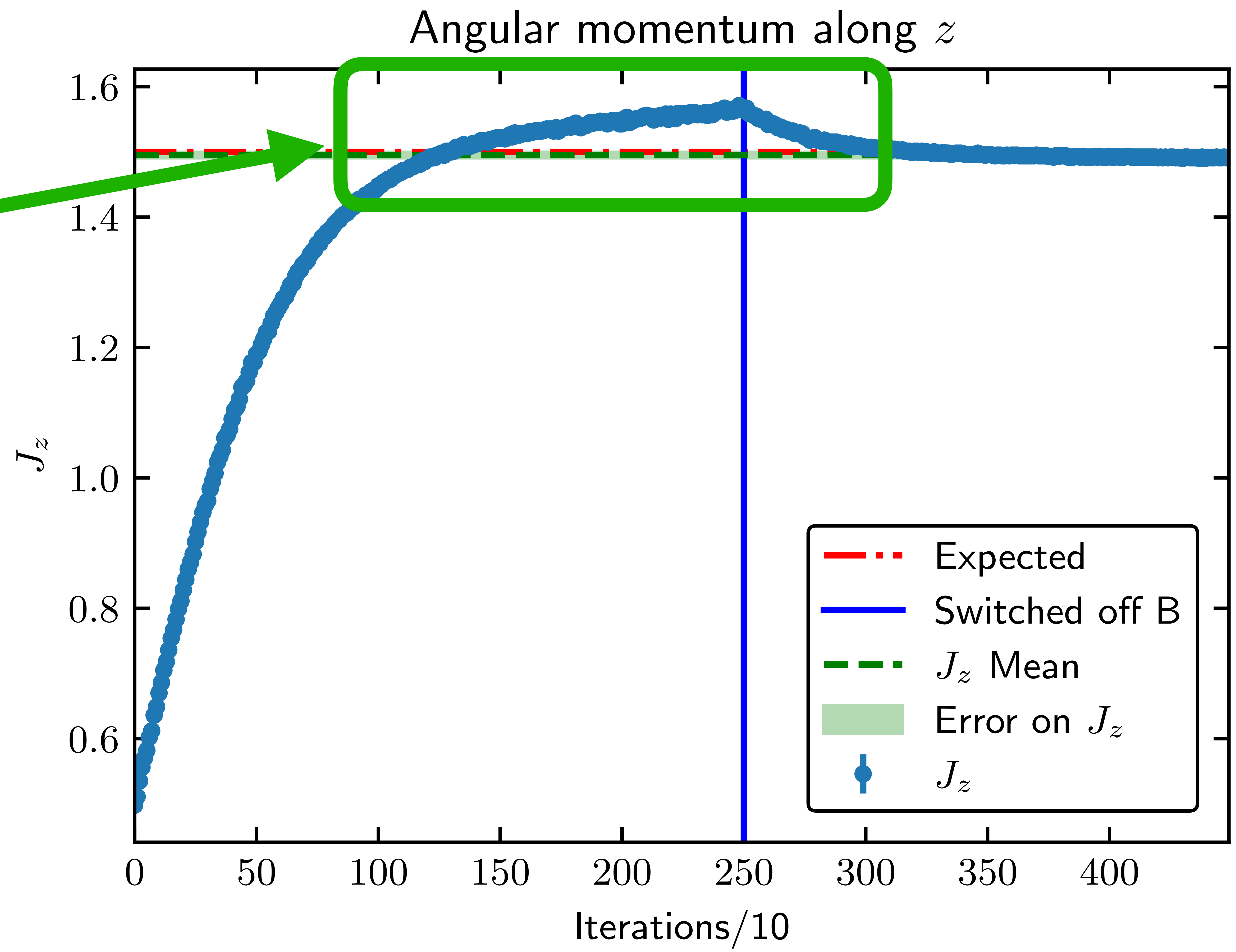


$$\hat{H} - B_z \hat{J}_z \to \hat{H}$$

$$|\Psi_J\rangle = |\Psi_{J,J_z=J}\rangle + \varepsilon \sum_{J_B} C_{J_B} |\Psi_{J_B}\rangle$$

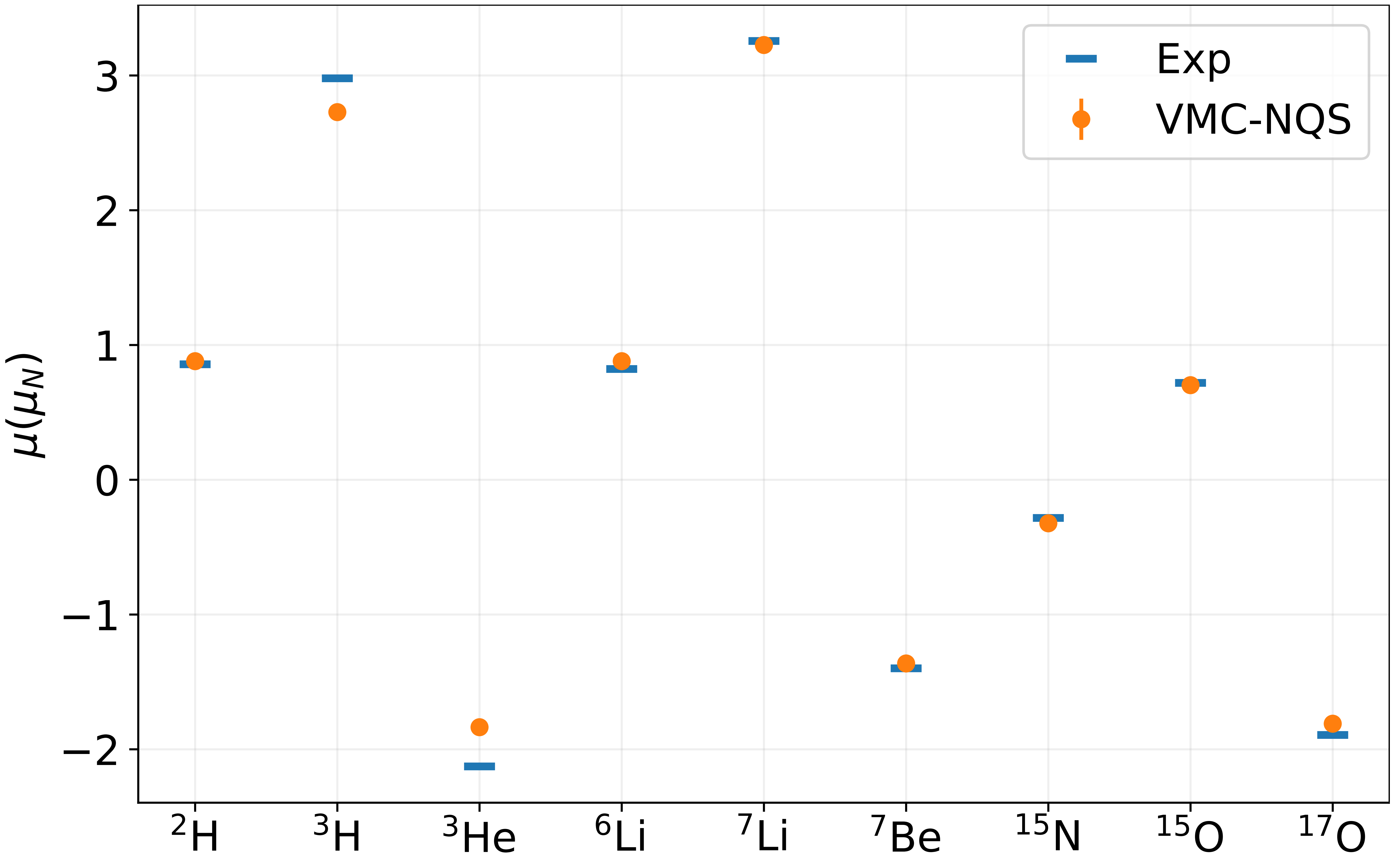
Projection of NQS





- The magnetic field B_z needs to be maintained as a small perturbation

Magnetic moments of selected light nuclei



- ^{3}H and ^{3}He require two-body currents
- Good agreement for the remaining:

  NQS reconstruct with the pionless EFT hamiltonian the shell-model structure without any ansatz

- More sophisticated interactions need two-body currents [see AFDMC and GFMC results]