Low-energy constants from baryon masses on Lattice QCD ensembles

Matthias F.M. Lutz

GSI Helmholtzzentrum für Schwerionenforschung GmbH

- ✔ **Chiral extrapolation for QCD with light quarks**
- ✔ **The chiral SU(3) Lagrangian with baryon fields**
- ✔ **Chiral extrapolation for baryon masses on Lattice QCD ensembles**
- ✔ **Summary and outlook**

arXiv:1801.06417, arXiv:1907.00714, arXiv:2301.06387, arXiv:2404.02769, arXiv:2406.07442

Chiral extrapolation for QCD with light quarks

✔ Sustainable approach

- use Lattice QCD data where they are 'cheap'
- exploit the nonlinearity of chiral symmetry
  to predict experimental data where taking Lattice QCD data is expensive

✔ the low-energy constants (LEC) of the chiral Lagrangian
that determine the quark-mass dependence of a hadron mass
impact the scattering processes involving the Goldstone bosons and that hadron

- critical challenge: what is the convergence radius of a chiral expansion
- conventional expansion schemes appear often very slow (if at all) convergent

✔ novel expansion scheme in terms of on-shell masses

- pioneered for various hadrons on flavour SU(3) ensembles
- chiral expansion is not necessarily smooth - first order transitions are possible
- revisted for flavour SU(2) chiral expansions

The chiral SU(3) Lagrangian with baryon fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}$$

Goldstone boson octet ($J^P = 0^-$)

baryon octet ($J^P = \frac{1}{2}^+$)

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\,\Lambda \end{pmatrix}$$

✔ Leading order terms

covariant derivative $\partial_\mu = \partial_\mu + \cdots$

$$\begin{aligned} \mathcal{L} &= \mathrm{tr}\left\{\bar{B}(i\,\partial\cdot\gamma - M_{[8]})\,B\right\} + F\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,[i\,U_\mu, B]\right\} + D\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,\{i\,U_\mu,\,B\}\right\} \\ &- \mathrm{tr}\left\{\bar{B}_\mu\cdot\left((i\,\partial\cdot\gamma - M_{[10]})\,g^{\mu\nu} - i\,(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \gamma^\mu(i\,\partial\cdot\gamma + M_{[10]})\gamma^\nu\right)B_\nu\right\} \\ &+ C\left(\mathrm{tr}\left\{(\bar{B}_\mu\cdot i\,U^\mu)\,B\right\} + \text{h.c.}\right) + H\,\mathrm{tr}\left\{(\bar{B}^\mu\cdot\gamma_\nu\gamma_5\,B_\mu)\,i\,U^\nu\right\} \end{aligned}$$

- $U_\mu = \frac{1}{2}\,u^\dagger\left(\partial_\mu\,e^{i\,\frac{\Phi}{f}}\right)u^\dagger - \frac{i}{2}\,u^\dagger\,(v_\mu + a_\mu)\,u + \frac{i}{2}\,u\,(v_\mu - a_\mu)\,u^\dagger$ with $u = e^{i\,\frac{\Phi}{2\,f}}$
- from $B \to B' + e + \bar{\nu}_e$: $F \simeq 0.45$ and $D \simeq 0.80$
- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$

Chiral symmetry breaking terms

$$\mathcal{L}_\chi^{(2)} = 2\,b_0\,\mathrm{tr}\,\left(\bar{B}\,B\right)\mathrm{tr}\,(\chi_+) + 2\,b_D\,\mathrm{tr}\,\left(\bar{B}\,\{\chi_+,\,B\}\right) + 2\,b_F\,\mathrm{tr}\,\left(\bar{B}\,[\chi_+,\,B]\right)$$
$$-\;2\,d_0\,\mathrm{tr}\,\left(\bar{B}_\mu\cdot B^\mu\right)\mathrm{tr}\,(\chi_+) - 2\,d_D\,\mathrm{tr}\,\left(\left(\bar{B}_\mu\cdot B^\mu\right)\chi_+\right)$$

$$\chi_+ = \chi_0 - \frac{1}{8\,f^2}\left\{\Phi,\left\{\Phi,\,\chi_0\right\}\right\} + \mathcal{O}\left(\Phi^4\right)$$

$$\text{quark} - \text{mass matrix} \qquad \chi_0 \sim \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

✔ Relevance of low-energy parameters

- quark-mass dependence of the baryon masses $\leftrightarrow$ lattice QCD
- meson-baryon scattering $\leftrightarrow$ resonances in QCD
- nucleon sigma terms, $\langle N|\bar{u}\,u|N\rangle$, $\langle N|\bar{d}\,d|N\rangle$ and $\langle N|\bar{s}\,s|N\rangle$
  relevant in WIMP scenarios – ATLAS

  see e.g. arXiv:1805.09795

Lattice QCD for baryon octet and decuplet masses

✔ Baryon masses on CLS ensembles from Regensburg

- large set of ensembles at different β values, quark masses and volumes
- ensembles at fixed m_s or $m_u = m_d = m_s$ or $m_u + m_d + m_s$ :: crucial for chiral SU(3)
- there are about 400 data points with $m_\pi, m_K < 550$ MeV
- a significant continuum limit extrapolation appears feasible

arXiv:2211.03744

✔ The challenge of a global fit

- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratice lattice scale dependence
- global scale-setting with baryon octet and decplet masses

  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at N^3LO

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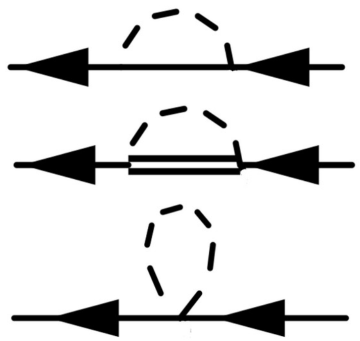
✔ Ensembles with physical pion masses: challenges

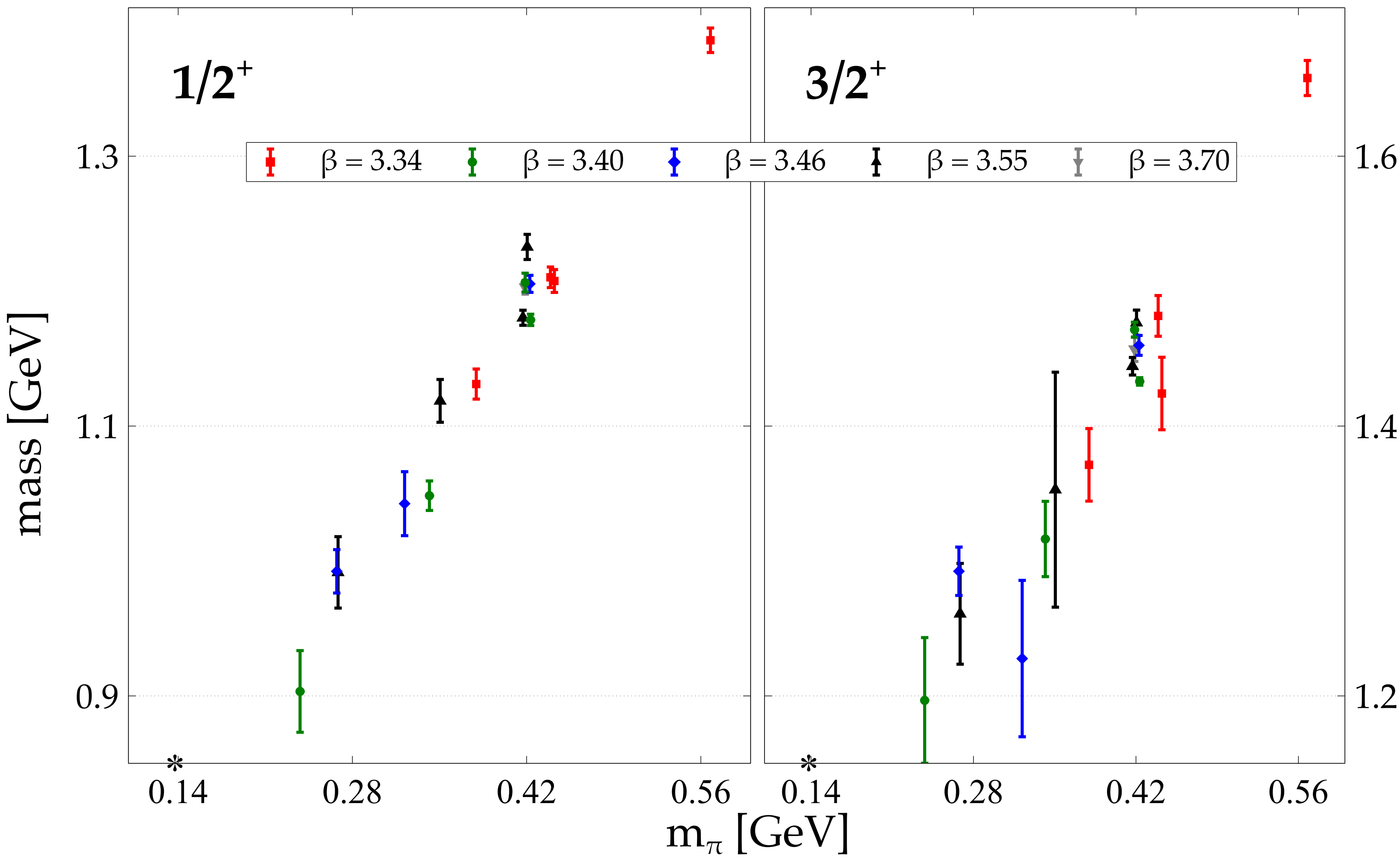
- excited state contamination of exponential signals?
- infinite volume extrapolation of isobar states?
- from an EFT point of view unphysical quark-masses are more interesting!



Chiral extrapolation for QCD with strange quarks

✔ baryon masses on CLS ensembles from Regensburg arXiv:2211.03744

- at flavor symmetric points $m_u = m_d = m_s$ (improve decuplet data !)

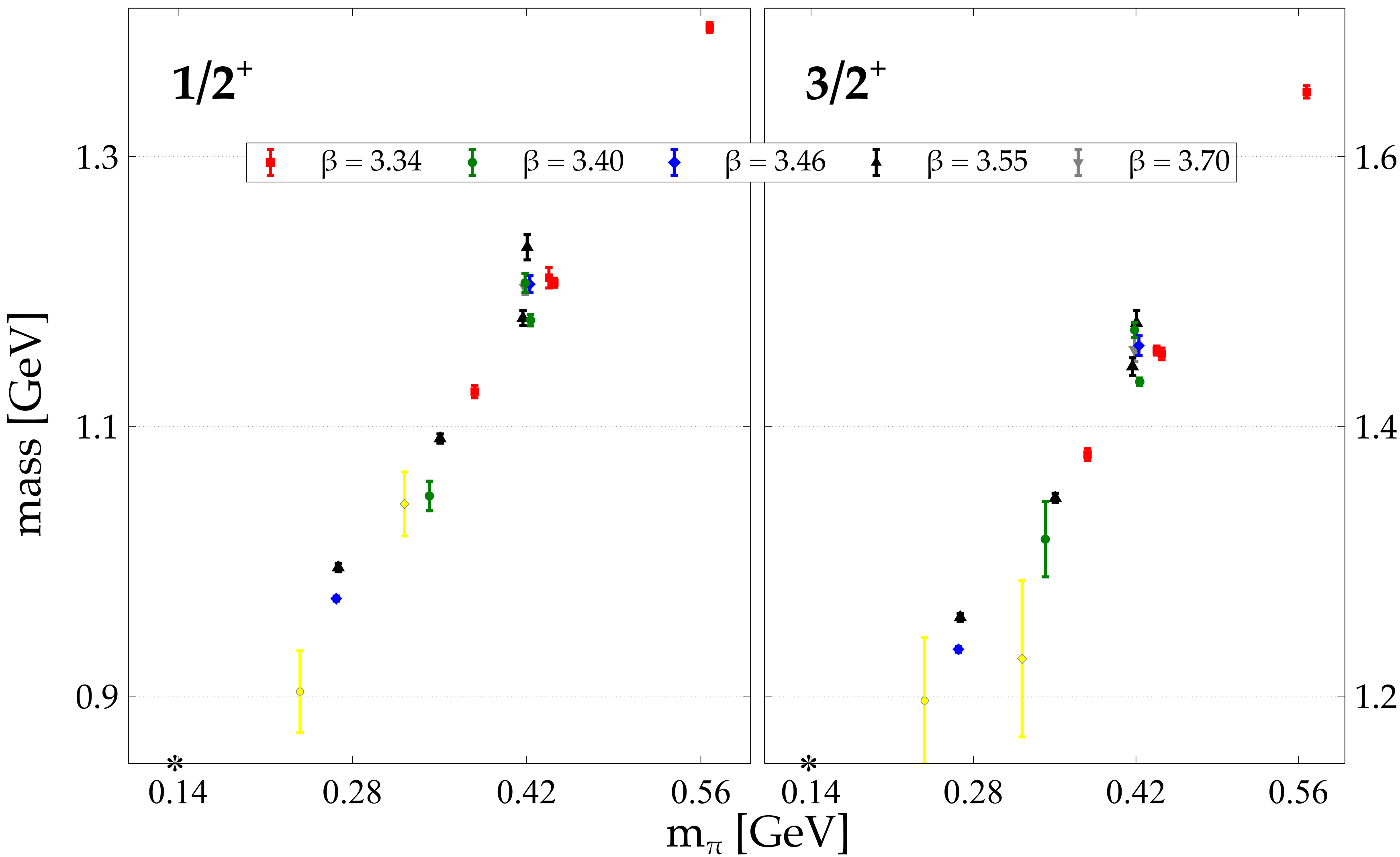


MFML, Y. Heo, X.-Y. Guo, Eur. Phys.J.C 83 (2023) 5, 440

Chiral extrapolation for QCD with strange quarks

✔ baryon masses on CLS ensembles improved at GSI by Renwick J. Hudspith

- at flavor symmetric points $m_u = m_d = m_s$ (more accurate decuplet masses !)



MFML, Y. Heo, RJ Hudspith, arXiv:2406.07442

Quark-mass dependence of the baryon masses

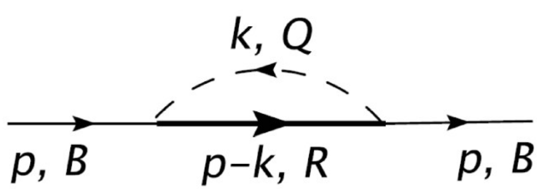
✔ A challenge

- 'poor' convergence in the heavy-baryon formulation of χPT

$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

- conventional χPT inconsistent with three-flavor QCD lattice simulations?

✔ Bubble-loops depend critically on internal masses

$$\Sigma_B(p) = \sum_{Q\in[8]} \sum_{R\in[8],[10]}$$  $$+ \cdots$$

- chiral expansion in terms of physical meson and baryon masses
- reorganize conventional χPT keeping its model independence
- renormalization scale and reparametrization invariance

Quark-mass dependence of the baryon masses

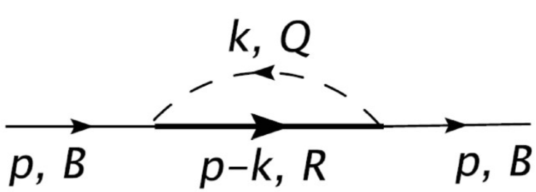
✔ Good convergence of reordered chiral expansion

- use physical meson and baryon masses
- the full one-loop contributions can be decomposed into chiral moments
- taking empirical masses the N^4LO effects are less than 8 MeV

✔ Baryon masses determined by a non-linear system

$$M_B - \Sigma_B(M_B) = \begin{cases} M_{[8]} & \text{for} \quad B \in [8] \\ M_{[10]} & \text{for} \quad B \in [10] \end{cases}$$

$$\Sigma_B(p) = \sum_{Q \in [8]} \sum_{R \in [8],[10]}$$



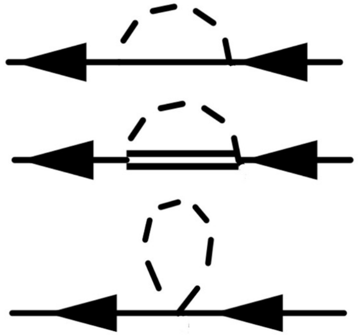
$$+ \cdots$$

- numerical challenge

Low-energy parameters from lattice QCD simulations

✔ A global fit to baryon masses on CLS ensembles

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratic lattice scale dependence
- global scale-setting with baryon octet and decplet masses
  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at $\mathrm{N}^3\mathrm{LO}$
- determine systematic error such that $\chi^2/d.o.f. \simeq 1$



✔ Sum rules from QCD in the limit of large N_c

- significant parameter reduction
- use such relations only for LEC relevant at $\mathrm{N}^3\mathrm{LO}$
- we adjust 44$= 30_{\mathrm{LEC}} + 14_{\mathrm{Lattice}}$ parameters to the lattice data set
- use GENEVA on heterogenous MPI grids with 2000 ranks ::
  Comput. Softw. Big Sci. 7 (2023) 1, 4

Predictions for LEC in the chiral limit

✔ The impact of axial couplings

- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$
- may use empirical tree-level estimates for F, D but D, H from large-N_c?
- fit all to the Lattice Data set - do not use large-N_c relations here

✔ A fit to baryon masses

	Fit arXiv:2301.06387	Fit arXiv:2406.07442
f [MeV]	92.4*	82.35(68)
F	0.51*	0.4852(73)
D	0.72*	0.4855(85)
C	1.44*	0.9740(415)
H	2.43*	1.8390(188)
M [MeV]	804.3(1)	840(16)
$M + \Delta$ [MeV]	1115.2(1)	1091(14)

- direct fit to the data set with $\chi^2/d.o.f. = 0.988$

Quark-masses from Lattice QCD ensembles

$$m_\pi^2 = 2\,B_0\,m - \frac{1}{18\,f^2}\left\{ -10\,m_\pi^2 + 4\,m_K^2 - 3\,m_\eta^2 \right\} \bar{I}_\pi - \frac{1}{6\,f^2}\,m_\pi^2\,\bar{I}_\eta$$
$$+ \frac{8}{f^2}\,m_\pi^2\,(m_\pi^2 + 2\,m_K^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\pi^4\,(2\,L_8 - L_5)\,,$$
$$\bar{I}_Q = \frac{m_Q^2}{(4\,\pi)^2}\,\log\left(\frac{m_Q^2}{\mu^2}\right) + \text{finite} - \text{box corrections}$$

✔ **Use on-shell meson masses**

- for given pion, kaon and eta masses determine $m = m_u = m_d$ and m_s
- rewrite Gasser and Leutwyler results in terms of on-shell masses
- such an analysis determines $2\,L_6 - L_4$ and $2\,L_8 - L_5$

- typically only pion and kaon masses available on Lattice ensembles - what to do?

Quark-masses from Lattice QCD ensembles

$$
\begin{aligned}
m_K^2 &= B_0\,(m+m_s) - \frac{1}{6\,f^2}\left\{m_\pi^2 - 4\,m_K^2 + 3\,m_\eta^2\right\}\bar{I}_K + \frac{1}{3\,f^2}\,m_K^2\,\bar{I}_\eta \\
&\quad + \frac{12}{f^2}\,m_K^2\,(m_\pi^2 + m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_K^4\,(2\,L_8 - L_5)\,, \\
m_\eta^2 &= \frac{2}{3}\,B_0\,(m + 2\,m_s) - \frac{1}{2\,f^2}\,m_\pi^2\,\bar{I}_\pi - \frac{1}{6\,f^2}\left\{7\,m_\eta^2 - 4\,m_K^2\right\}\bar{I}_\eta + \frac{4}{3\,f^2}\,m_K^2\,\bar{I}_K \\
&\quad + \frac{24}{f^2}\,m_\eta^2\,(2\,m_K^2 - m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\eta^4\,(2\,L_8 - L_5) \\
&\quad + \frac{16}{5\,f^2}\,(3\,m_\pi^4 - 8\,m_K^4 - 8\,m_\eta^2\,m_K^2 + 13\,m_\eta^4)\,(3\,L_7 + L_8)\,,
\end{aligned}
$$

$$
\bar{I}_Q = \frac{m_Q^2}{(4\,\pi)^2}\,\log\left(\frac{m_Q^2}{\mu^2}\right) + \text{finite} - \text{box corrections}
$$

✔ Use on-shell meson masses

- for given pion and kaon mass determine $m = m_u = m_d$ and m_s
- such an analysis determines Gasser and Leutwyler LEC

- physical m_η mass determines $3\,L_7 + L_8$

Predictions for quark-mass ratios on lattice ensembles

✔ How to fit the lattice data?

- take pion and kaon mass of the ensemble → compute quark masses
- this requires the low-energy constants $L_4 - 2\,L_6, L_5 - 2\,L_8, L_8 + 3\,L_7$
- we do not fit to the quark-mass ratios (not given) by the lattice groups!

✔ A fit to baryon masses

- renormalization scale $\mu = 0.77$ GeV

	Fit arXiv:2301.06387	Fit arXiv:2406.07442
$10^3\,(2\,L_6 - L_4)$	0.0411(3)	0.0296(41)
$10^3\,(2\,L_8 - L_5)$	0.0826(12)	-0.0770(51)
$10^3\,(L_8 + 3\,L_7)$	-0.4768(4)	-0.3145(28)
f [MeV]	92.4*	82.4(7)
m_s/m	26.2(1)	27.6(1)

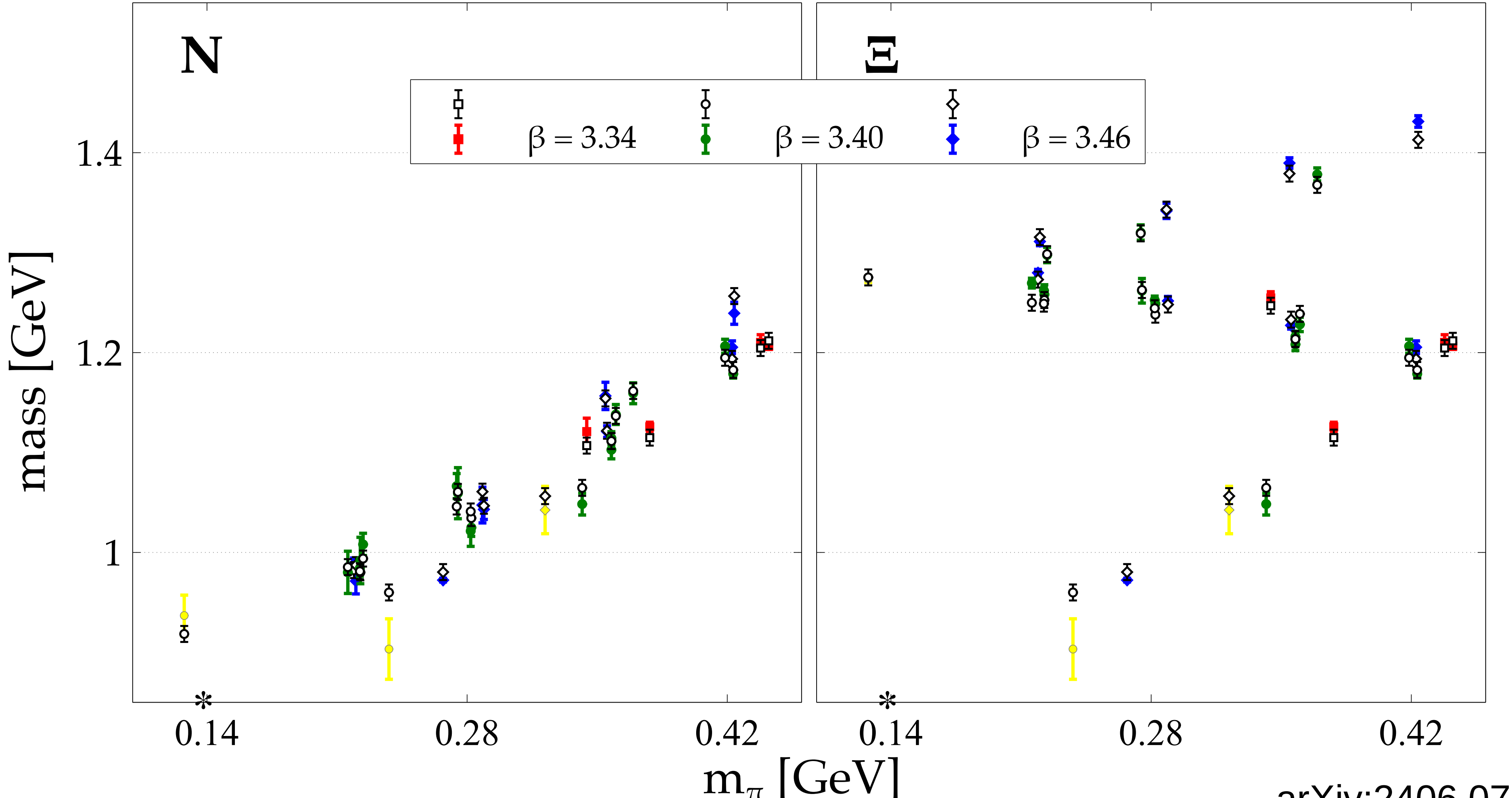
- small statistical error from global fits
- with improved data values for f and m_s/m compatible with FLAG

Pion-mass dependence of the baryon octet masses

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- leading and subleading LEC have a quadratice lattice scale dependence

$$M \to M + a^2\,\gamma_{M_8}\,,\; b_0 \to b_0 + a^2\,\gamma_{b_0}\,,\quad b_D \to b_F + a^2\,\gamma_{b_D}\,,\quad b_F \to b_F + a^2\,\gamma_{b_F}$$

- with systematic error of 7 MeV :: $\chi^2/d.o.f. = 0.988$



arXiv:2406.07442

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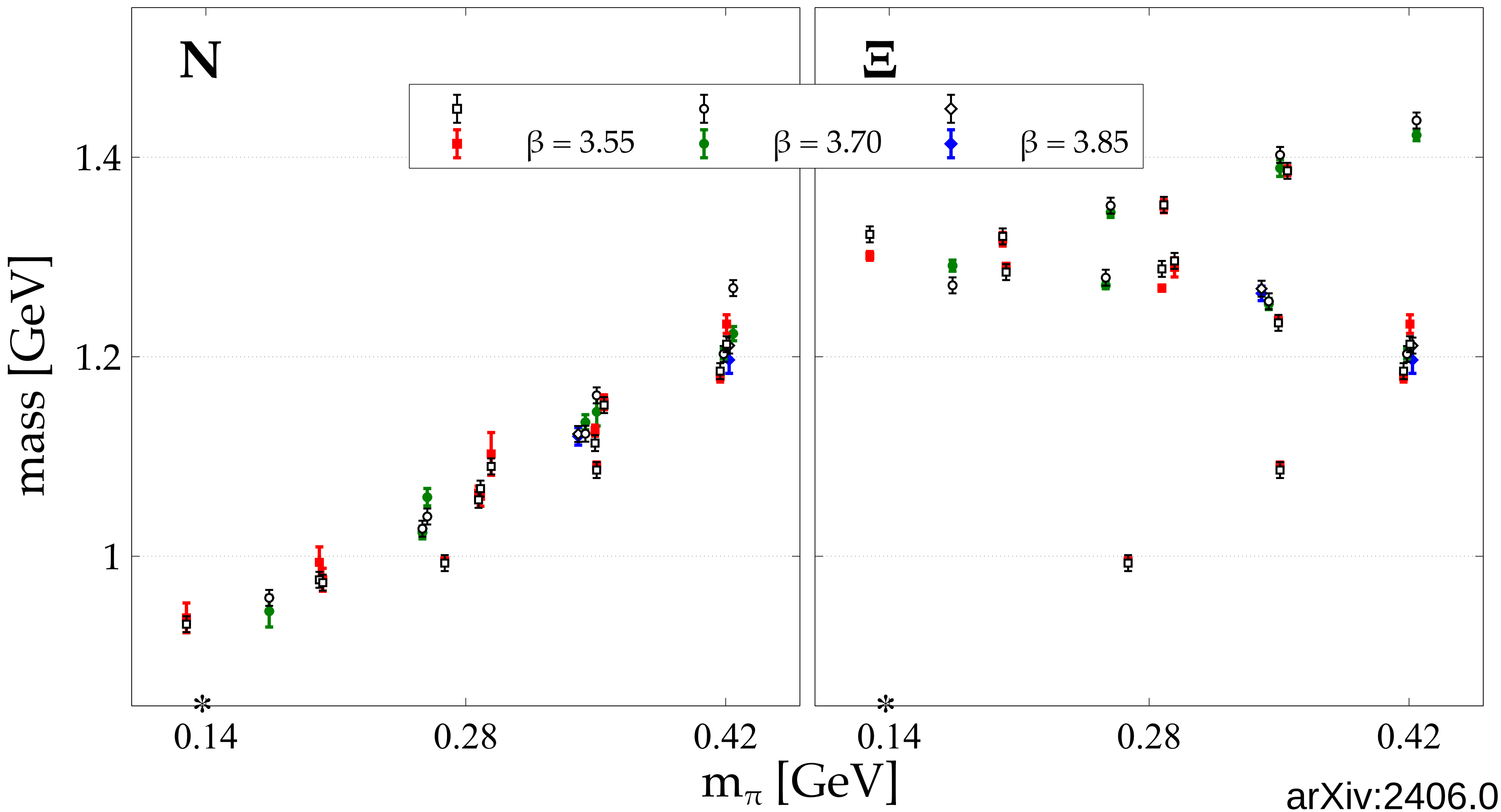

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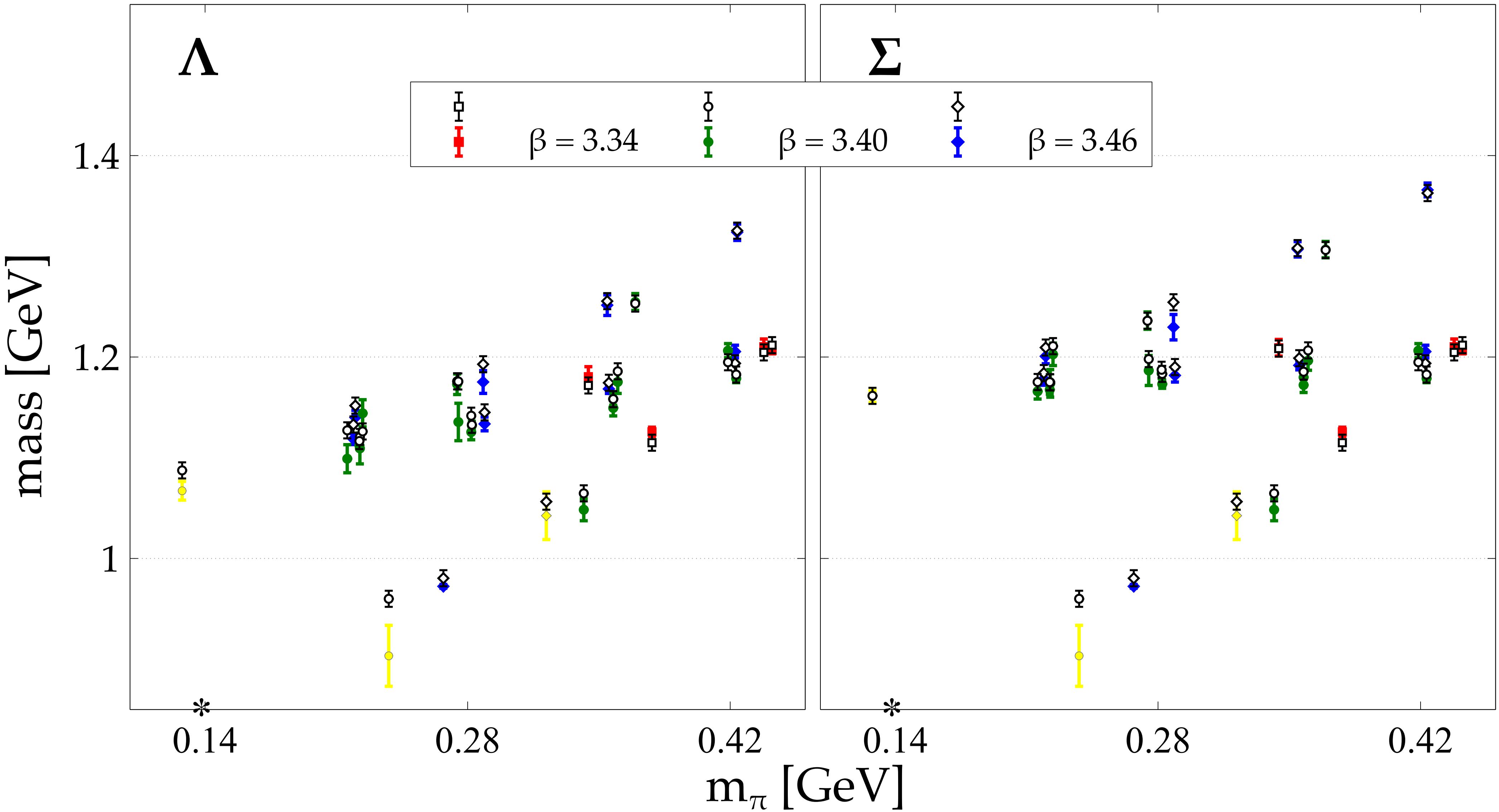

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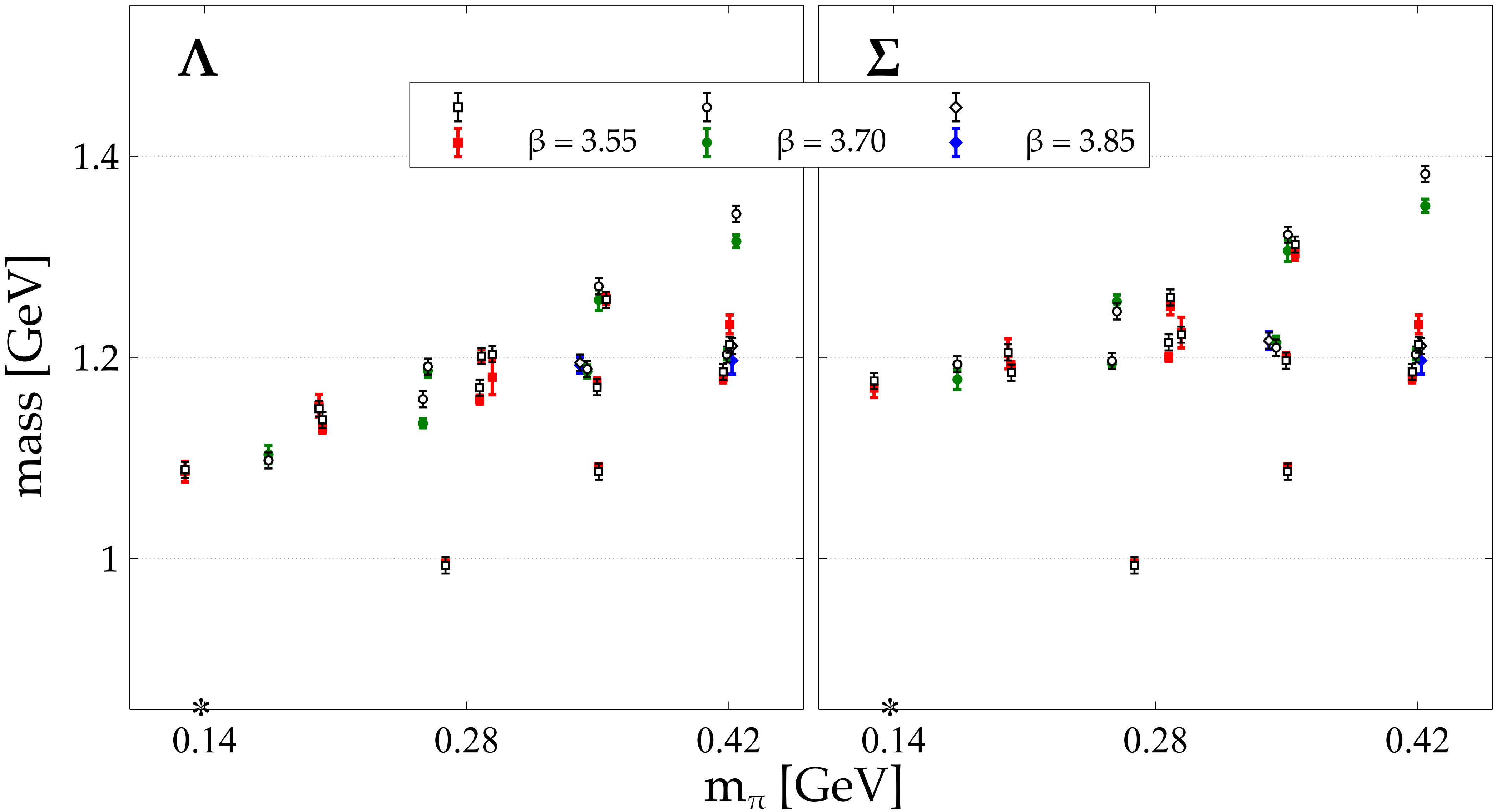
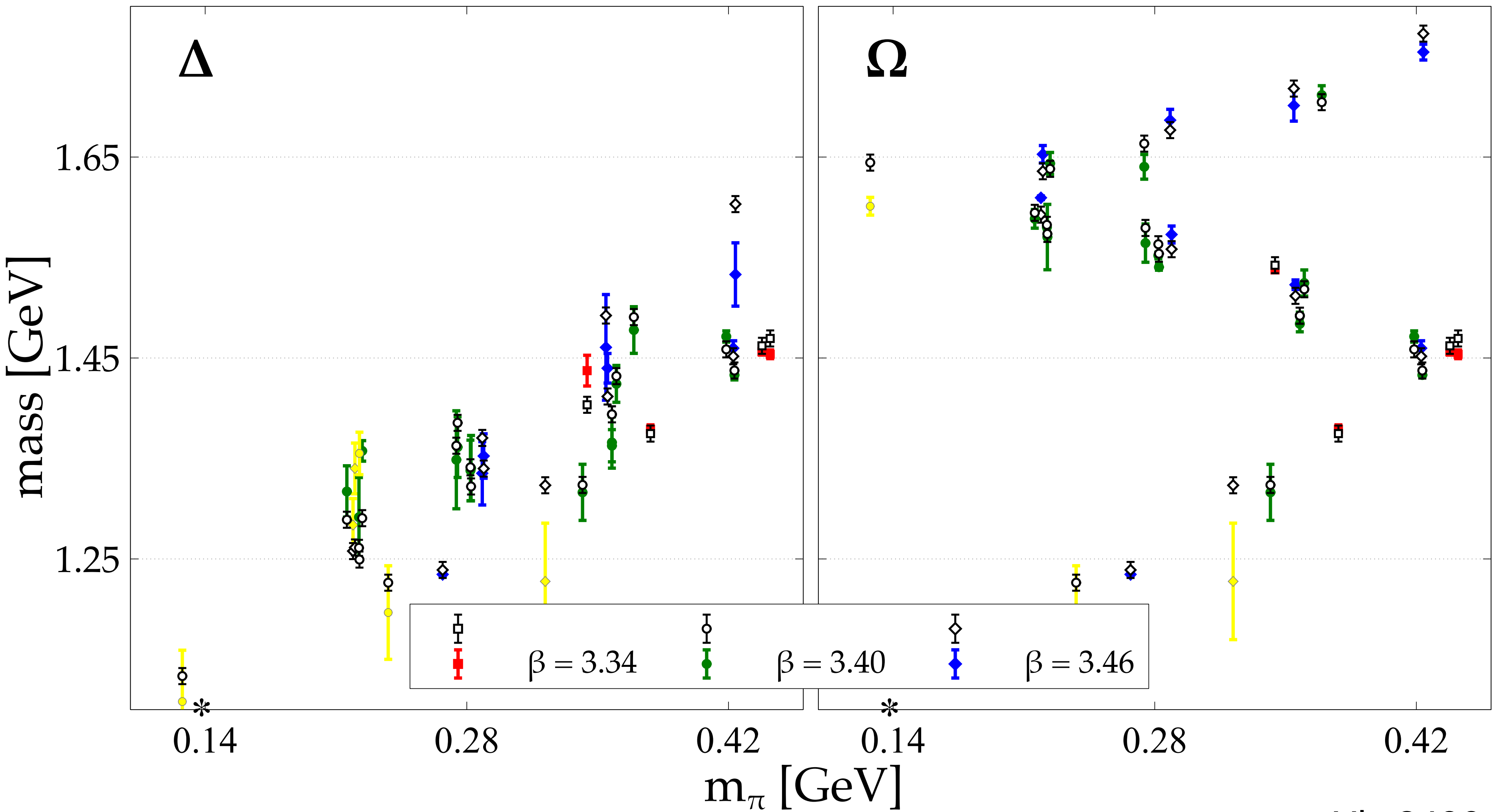


arXiv:2406.07442

Pion-mass dependence of the baryon decuplet masses

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- leading and subleading LEC have a quadratice lattice scale dependence
- with systematic error of 7 MeV :: $\chi^2/d.o.f. = 0.988$



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Pion-mass dependence of the baryon decuplet masses

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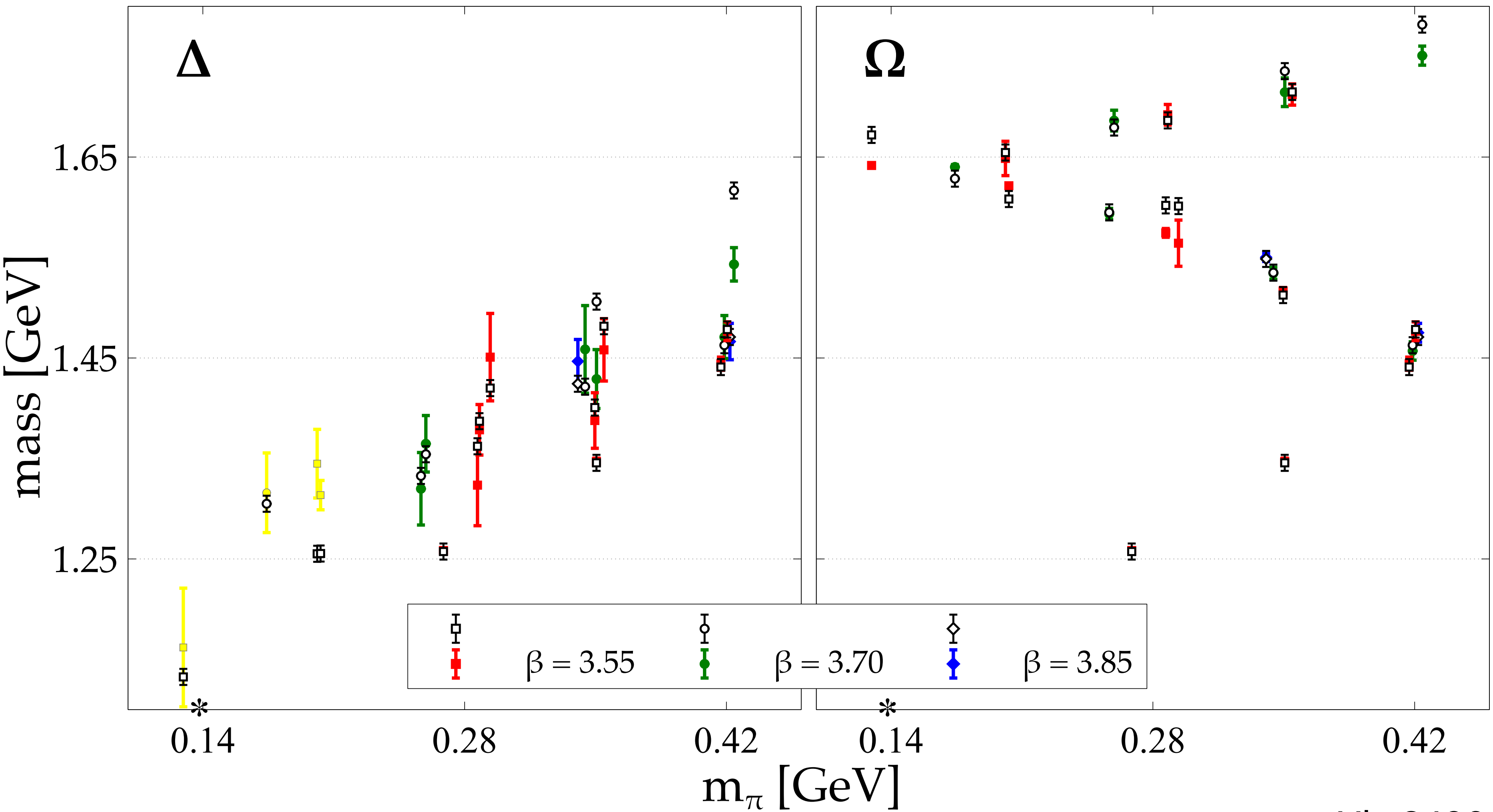

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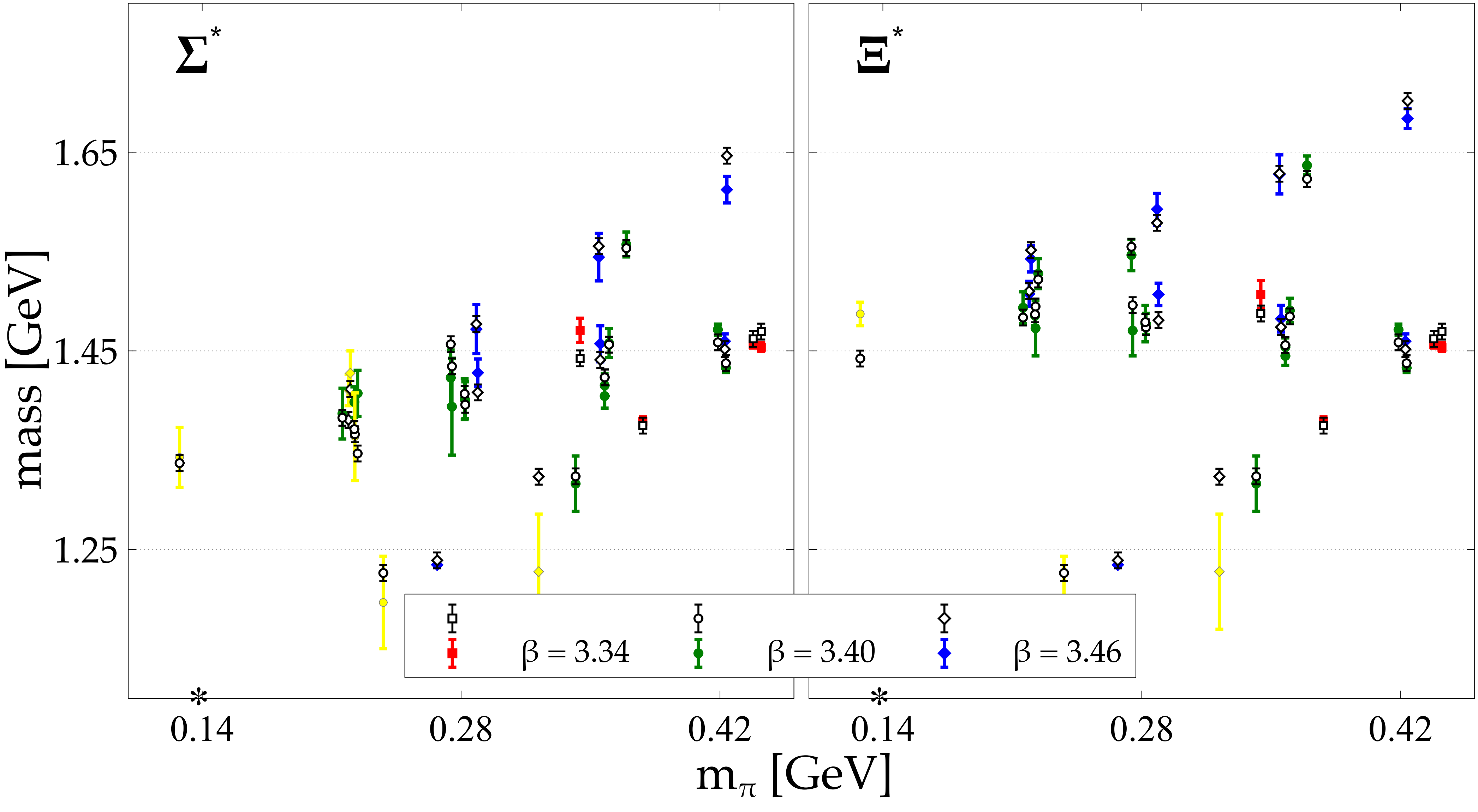
Pion-mass dependence of the baryon decuplet masses



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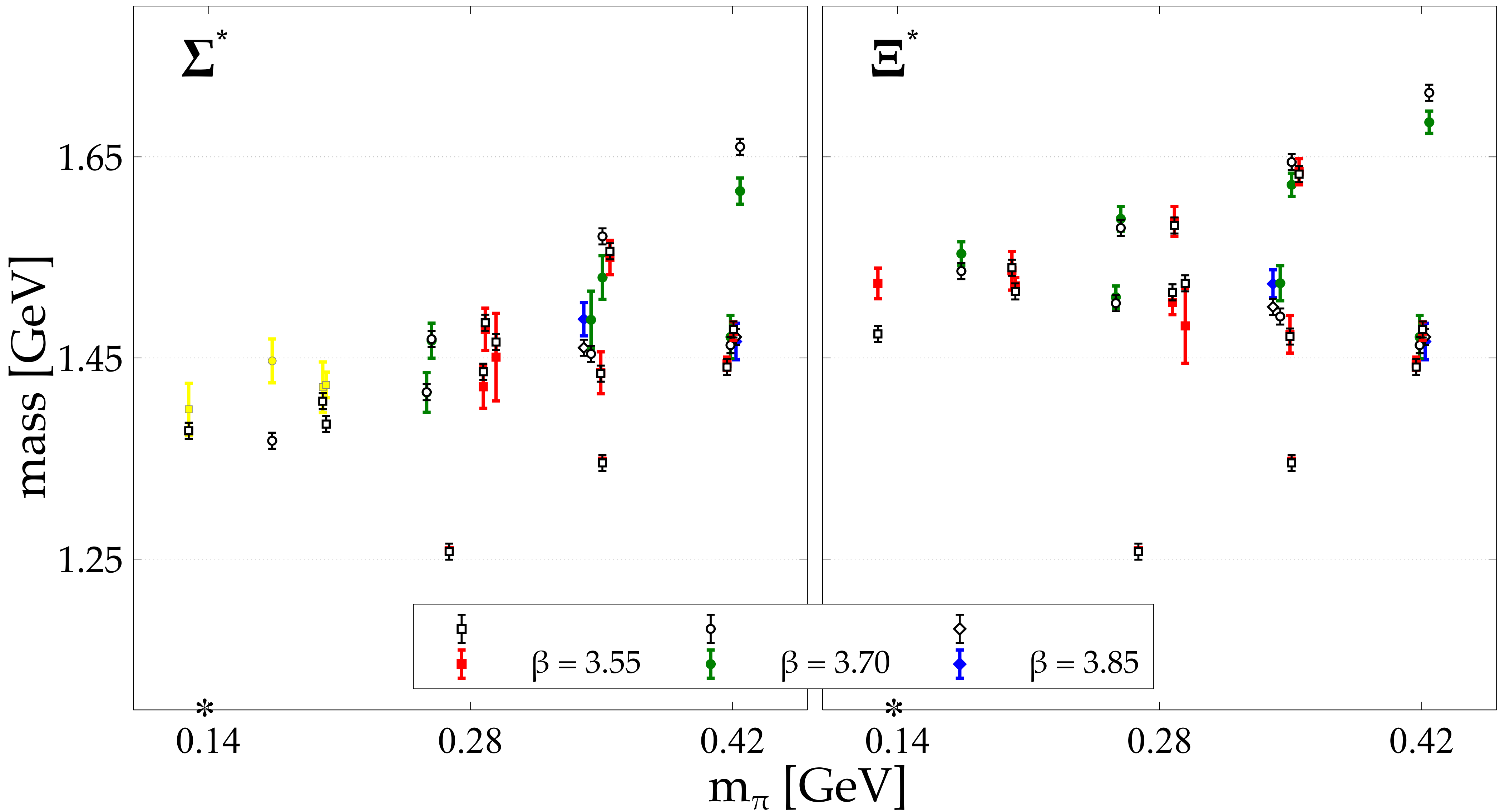
Pion-mass dependence of the baryon decuplet masses



arXiv:2406.07442

Predictions for sigma terms

$$\sigma_{\pi N} = m \frac{\partial}{\partial m} m_N$$

✔ $\sigma_{\pi N}$ from pion-nucleon scattering and pionic atom data

- empirical value $\sigma_{\pi N} = 59.0(3.5)$ MeV (M. Hoferichter et al., arXiv:2305.07045)
- significant tension with QCD lattice results (only some typical cases shown)

$\sigma_{\pi N} = 45.8(7.4)(2.8)$ MeV	Y. B. Yan et al., Phys. Rev. D 94 (2016) 054503
$\sigma_{\pi N} = 35(6)$ MeV	G. S. Bali et al., Phys. Rev. D 93 (2016) 094504
$\sigma_{\pi N} = 59.6(7.4)$ MeV	R. Gupta et al., arXiv:2105.12095

✔ From CLS ensembles

- $\sigma_{\pi N} = 43.9(4.7)$ MeV — G. S. Bali et al., arXiv:2211.03744
- $\sigma_{\pi N} = 43.6(3.8)$ MeV — A. Agadjanov et al., arXiv::2303.08741
- $\sigma_{\pi N} = 58.7(1.2)$ MeV — MFML et al., arXiv:2301.06387

  isobar masses from arXiv:2211.03744 (Regensburg)
- $\sigma_{\pi N} = 44.2(1.4)$ MeV — MFML et al., arXiv:2406.07442

  improved isobar masses (in part from Renwick J. Hudspith at GSI)
- a constrained fit with $\sigma_{\pi N} > 50$ MeV comes with $\chi^2/(d.o.f.) > 1.24$
  about 40 σ away from our best solution

Summary & Outlook

✔ Chiral extrapolation of hadron masses

- resummed χPT : use physical masses in the loops
  - chiral expansion with up, down and strange quarks is useful
- so far we considered baryon masses at N^3LO
  - fits to masses of ground states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$
  - quantitative reproduction of the available lattice data set
- predict a large number of low-energy constants for the chiral Lagrangian of QCD
  - obtain a pion-nucleon sigma term as expected form the Lattice community
  - the decuplet baryons play an instrumental role

✔ QCD spectroscopy with coupled-channel dynamcis

- curent QCD lattice data provide many LEC relevant for scattering processes
- use as input in systematic coupled-channel computations
- analyze and predict the quark-mass dependence of hadron resonances in QCD

thanks to: Yonggoo Heo and Jamie Hudspith