Semileptonic Kaon Decays and the Precise Determination of V_{us}

Chien-Yeah Seng

University of Washington
and
FRIB, Michigan State University

seng@frib.msu.edu

The 11th International Workshop on Chiral Dynamics, Bochum, Germany

30 August, 2024

Outline

1. First-row CKM unitarity
2. Kaon Semileptonic Decays (K_{l3}) and the long-distance RC
3. Chiral Perturbation Theory
4. Sirlin's representation
5. Electroweak RC to K_{l3}
6. "Cabibbo angle anomaly"
7. Summary

CYS, Galviz and Meißner, 2020 JHEP;
CYS, Feng, Gorchtein, Jin and Meißner, 2020 JHEP;
Ma, Feng, Gorchtein, Jin and CYS, 2021 PRD;
CYS, Galviz, Gorchtein and Meißner, 2021 PLB, 2021 JHEP, 2022 JHEP;
CYS, Galviz, Marciano and Meißner, 2022 PRD

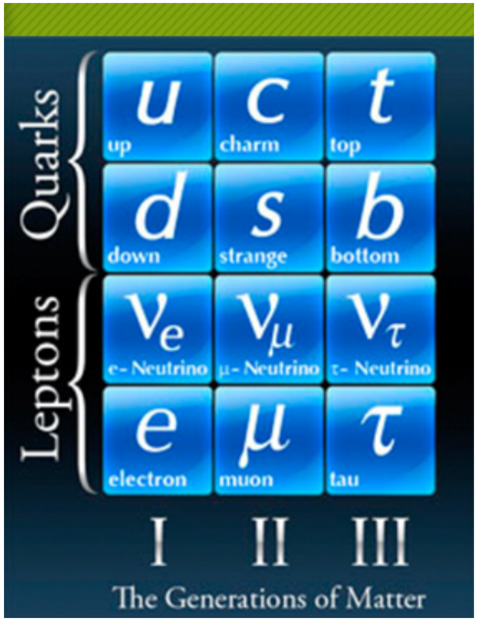
In collaboration with:
Xu Feng
Daniel Galviz
Mikhail Gorchtein
Luchang Jin
Peng-Xiang Ma
William Marciano
Ulf-G. Meißner

1. First-row CKM unitarity

Charged weak decays and CKM unitarity

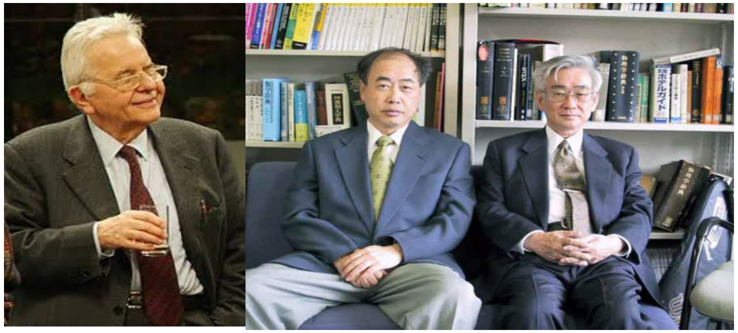
Massive quarks ==> Generation mixing

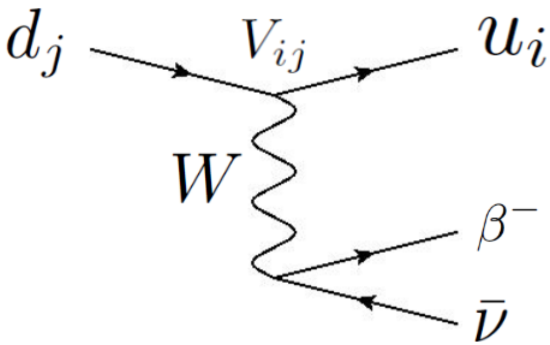
$$\psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$



Three generations of quarks and leptons

Cabibbo-Kobayashi-Maskawa (CKM) matrix





Weak interaction universality ==>
Unitarity of the measured CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

First row:

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 1}$$

$\sim 10^{-5}$

Can be tested at **0.01%** level! Probes new physics at the scale:

$$\left(\frac{v_{\mathrm{H}}}{\Lambda_{\mathrm{BSM}}}\right)^2 \sim 0.01\% \quad \Longrightarrow \quad \Lambda_{\mathrm{BSM}} \sim 20\ \mathrm{TeV}$$

Primary avenues to extract V_{ud} and V_{us}

V_{ud}: Superallowed nuclear decays

$$\phi_i(0^+) \to \phi_f(0^+)\, e^+ \nu_e$$

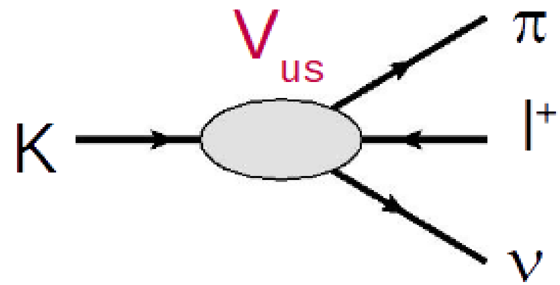
V_{us}: Semileptonic kaon decays (K_{l3})

$$K \to \pi \ell^+ \nu_\ell$$

V_{us}/V_{ud}: K/π leptonic decays ($K_{\mu 2}/\pi_{\mu 2}$)

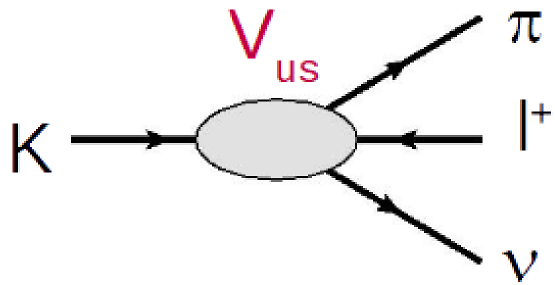
$$K/\pi \to \mu^+ \nu_\mu$$

2. Kaon semileptonic decays (K_{l3}) and the long-distance RC



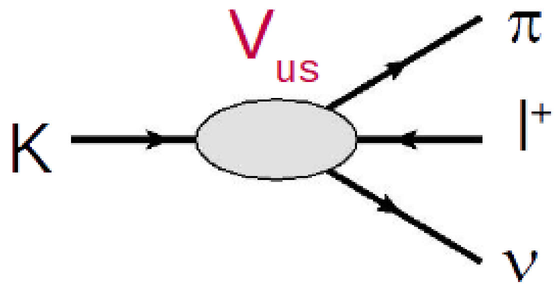
Six channels:

$$
\begin{aligned}
K^L_{e3} &: \quad K^L \to \pi^- e^+ \nu_e \\
K^L_{\mu 3} &: \quad K^L \to \pi^- \mu^+ \nu_\mu \\
K^S_{e3} &: \quad K^S \to \pi^- e^+ \nu_e \\
K^S_{\mu 3} &: \quad K^S \to \pi^- \mu^+ \nu_\mu \\
K^+_{e3} &: \quad K^+ \to \pi^0 e^+ \nu_e \\
K^+_{\mu 3} &: \quad K^+ \to \pi^0 \mu^+ \nu_\mu
\end{aligned}
$$



Master formula:

$$\boxed{\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

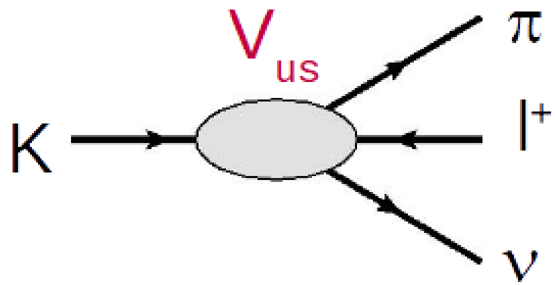


Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Theory inputs:

- Universal electroweak correction
- Kπ form factor at zero momentum transfer
- Phase space factor
- Isospin-breaking correction



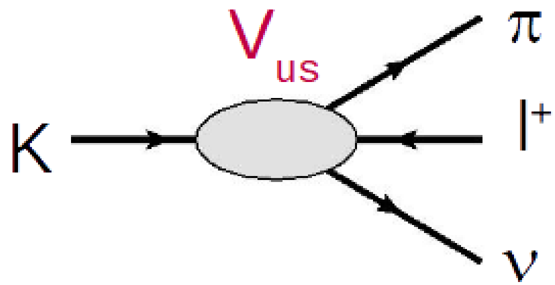
Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Universal electroweak correction:

$$\begin{aligned} S_{\mathrm{EW}} &= 1 + \frac{2\alpha}{\pi}\left(1 - \frac{\alpha_s}{4\pi}\right) \ln \frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha \alpha_S}{\pi^2}\right) \\ &= 1.0232(3) \end{aligned}$$

encodes process-independent electroweak corrections not included in G_F *Marciano and Sirlin, 1993 PRL*



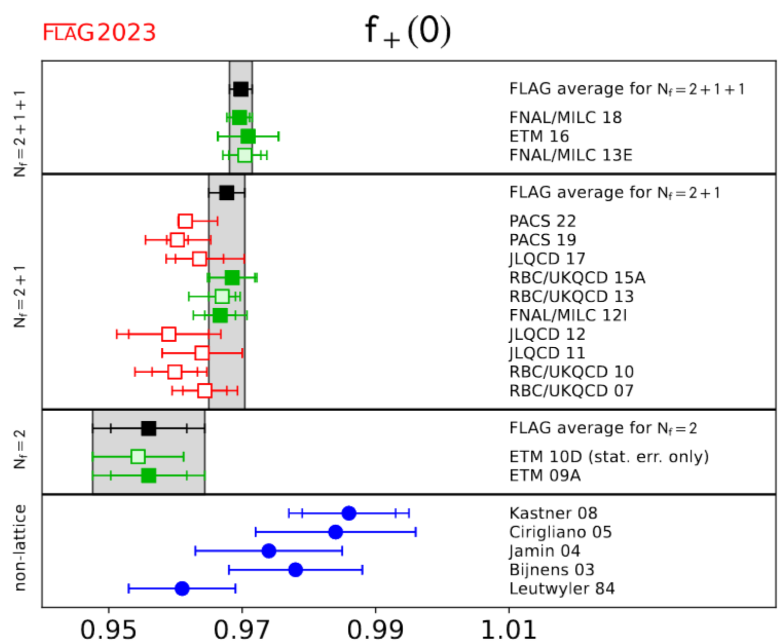
Master formula:

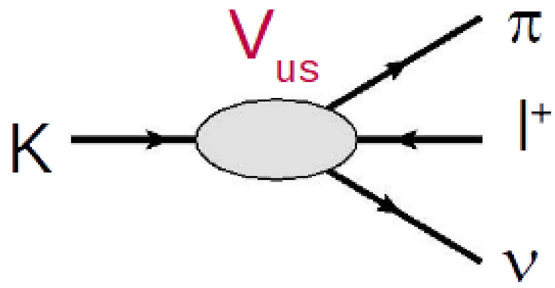
$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Kπ form factor at t=0: $\langle \pi^-(p) | J_W^\mu | K^0(p) \rangle = 2p^\mu f_+^{K^0\pi^-}(0)$

Lattice QCD inputs:

$$N_f = 2+1+1 \quad : \quad f_+(0) = 0.9698(17)$$
$$N_f = 2+1 \quad : \quad f_+(0) = 0.9677(27)$$
$$N_f = 2 \quad : \quad f_+(0) = 0.9560(57)(62)$$





Master formula:

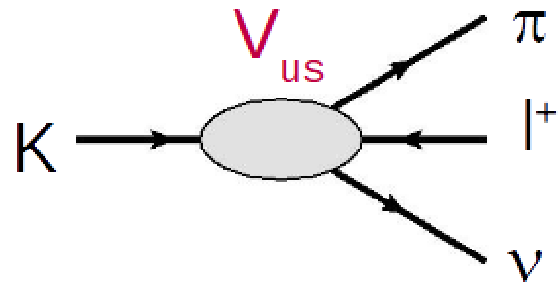
$$\Gamma_{K\ell 3} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Phase-space factor: probes the **t-dependence** of the form factors

Fit to the **K_{l3} Dalitz** plot with **dispersive parameterization**:

Mode	Update
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu 3}$	**0.10247(15)**
$K^+{}_{\mu 3}$	**0.10553(16)**

M. Moulson, in the 11th International Workshop on the CKM Unitarity Triangle, 2021



Master formula:

$$\boxed{\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

ISB correction: Difference between the **$K^+\pi^0$** and **$K^0\pi^-$** form factor

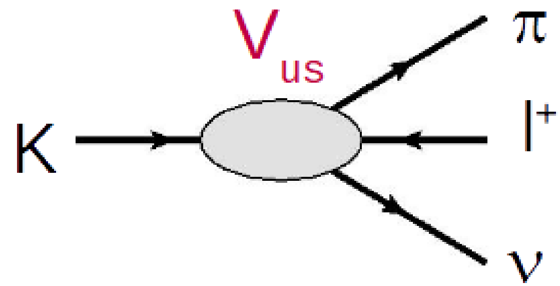
Depends on quark mass parameters:

$$m_s/\hat{m} \ , \ \ Q^2 \equiv (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2)$$

Lattice and phenomenology ($\eta \to 3\pi$) return somewhat different results:

$$\left(\delta_{\mathrm{SU(2)}}^{K^+\pi^0}\right)_{\mathrm{lattice}} = 0.0457(20)$$ *FLAG 2021 (2023 update)*

$$\left(\delta_{\mathrm{SU(2)}}^{K^+\pi^0}\right)_{\mathrm{pheno}} = 0.0522(34)$$ *Colangelo, Lanz, Leutwyler and Passemar, 2018 EPJC*

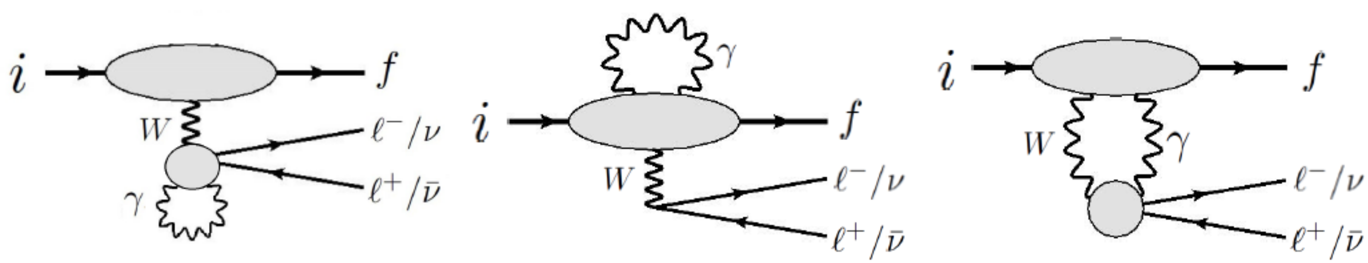


Master formula:

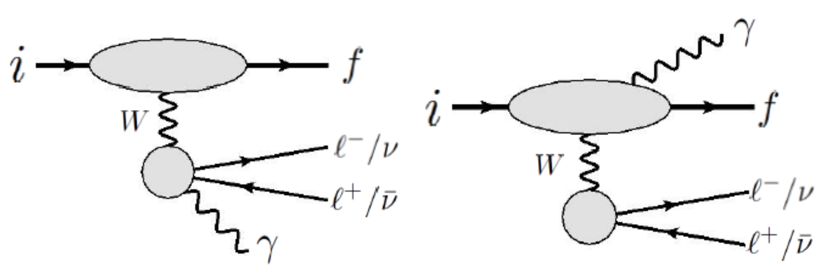
$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Long-distance electromagnetic radiative corrections (RC)

Virtual corrections:



Real corrections:



"**Long-distance**" : W-propagator shrinks to a point (Fermi's interaction)

3. Chiral Perturbation Theory

Chiral perturbation theory (ChPT)

- **Low-energy EFT** of QCD
- Constructed from **spontaneously-broken chiral symmetry**
- **DOFs:** Pseudoscalar mesons, leptons, photon
- **Chiral power-counting** scheme ensure convergence
- NP QCD contained in the **LECs**

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\gamma} + \mathcal{L}_{\text{ChPT}}}$$

$$\left.\begin{aligned} \mathcal{L}_{\text{lepton}} &= \textstyle\sum_\ell \left[\bar{\ell}(i\partial\!\!\!/ + eA\!\!\!/ - m_\ell)\ell + \bar{\nu}_{\ell L} i\partial\!\!\!/ \nu_{\ell L}\right] \\ \mathcal{L}_{\gamma} &= -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2\xi}(\partial\cdot A)^2 + \tfrac{1}{2}M_\gamma^2 A_\mu A^\mu \end{aligned}\right\}$$

Non-hadronic piece

$$\boxed{\mathcal{L}_{\text{ChPT}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots\,.}$$

$$\underbrace{\mathcal{L}^{p^2} = \frac{F_0^2}{4}\left\langle D_\mu U (D^\mu U)^\dagger + \chi U^\dagger + U\chi^\dagger\right\rangle, \quad \mathcal{L}^{e^2} = Z F_0^4 \left\langle q_L U^\dagger q_R U\right\rangle}_{\text{LO Chiral Lagrangian}}$$

NLO Chiral Lagrangian:

Pure mesonic

$$\begin{aligned}\mathcal{L}^{p^4} &= L_1\langle D_\mu U(D^\mu U)^\dagger\rangle^2 + L_2\langle D_\mu U(D_\nu U)^\dagger\rangle\langle D^\mu U(D^\nu U)^\dagger\rangle \\ &+L_3\langle D_\mu U(D^\mu U)^\dagger D_\nu U(D^\nu U)^\dagger\rangle + L_4\langle D_\mu U(D^\mu U)^\dagger\rangle\langle\chi U^\dagger + U\chi^\dagger\rangle \\ &+L_5\langle D_\mu U(D^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)\rangle + L_6\langle\chi U^\dagger + U\chi^\dagger\rangle^2 \\ &+L_7\langle\chi U^\dagger - U\chi^\dagger\rangle^2 + L_8\langle U\chi^\dagger U\chi^\dagger + \chi U^\dagger\chi U^\dagger\rangle \\ &-iL_9\left\langle f^R_{\mu\nu}D^\mu U(D^\nu U)^\dagger + f^L_{\mu\nu}(D^\mu U)^\dagger D^\nu U\right\rangle + L_{10}\left\langle U f^L_{\mu\nu}U^\dagger f^{\mu\nu}_R\right\rangle .\end{aligned}$$

Gasser and Leutwyler, NPB 1985

with photons

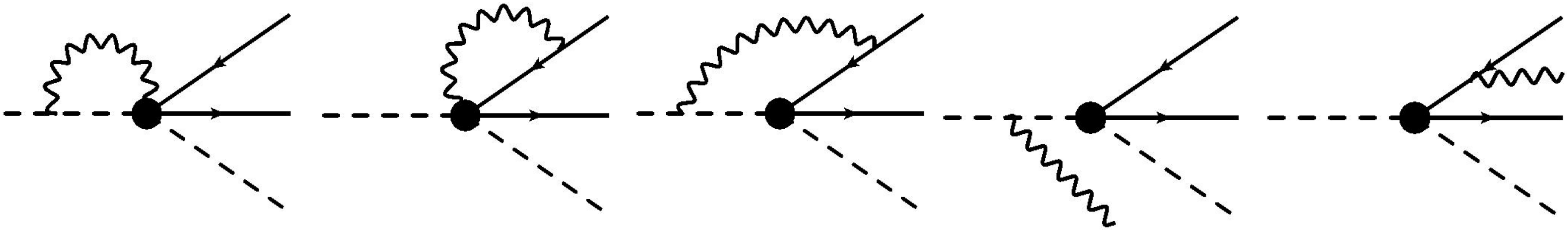
$$\begin{aligned}\mathcal{L}^{e^2p^2}_{\{K\}} &= F_0^2\Big\{\tfrac{1}{2}K_1\langle D^\mu U(D_\mu U)^\dagger\rangle\langle q_R q_R + q_L q_L\rangle + K_2\langle D^\mu U(D_\mu U)^\dagger\rangle\langle q_R U q_L U^\dagger\rangle \\ &+K_3(\langle(D^\mu U)^\dagger q_R U\rangle\langle(D_\mu U)^\dagger q_R U\rangle + \langle D^\mu U q_L U^\dagger\rangle\langle D_\mu U q_L U^\dagger\rangle) \\ &K_4\langle(D^\mu U)^\dagger q_R U\rangle\langle D_\mu U q_L U^\dagger\rangle + K_5\langle q_L q_L (D^\mu U)^\dagger D_\mu U + q_R q_R D^\mu U(D_\mu U)^\dagger\rangle \\ &+K_6\langle(D^\mu U)^\dagger D_\mu U q_L U^\dagger q_R U + D^\mu U(D_\mu U)^\dagger q_R U q_L U^\dagger\rangle \\ &+\tfrac{1}{2}K_7\langle\chi^\dagger U + U^\dagger\chi\rangle\langle q_R q_R + q_L q_L\rangle + K_8\langle\chi^\dagger U + U^\dagger\chi\rangle\langle q_R U q_L U^\dagger\rangle \\ &+K_9\langle(\chi^\dagger U + U^\dagger\chi)q_L q_L + (\chi U^\dagger + U\chi^\dagger)q_R q_R\rangle \\ &+K_{10}\langle(\chi^\dagger U + U^\dagger\chi)q_L U^\dagger q_R U + (\chi U^\dagger + U\chi^\dagger)q_R U q_L U^\dagger\rangle \\ &+K_{11}\langle(\chi^\dagger U - U^\dagger\chi)q_L U^\dagger q_R U + (\chi U^\dagger - U\chi^\dagger)q_R U q_L U^\dagger\rangle \\ &+K_{12}\langle(D^\mu U)^\dagger[\nabla_\mu q_R, q_R]U + D^\mu U[\nabla_\mu q_L, q_L]U^\dagger\rangle \\ &+K_{13}\langle\nabla^\mu q_R U\nabla_\mu q_L U^\dagger\rangle + K_{14}\langle\nabla^\mu q_R\nabla_\mu q_R + \nabla^\mu q_L\nabla_\mu q_L\rangle\Big\} ,\end{aligned}$$

Urech, NPB 1995

with leptons

$$\begin{aligned}\mathcal{L}^{e^2p^2}_{\{X\}} &= e^2F_0^2\textstyle\sum_\ell\{X_1\bar\ell\gamma_\mu\nu_{\ell L}\langle u^\mu\{\mathcal{Q}^{\rm em}_{\rm R},\mathcal{Q}^{\rm w}_{\rm L}\}\rangle + X_2\bar\ell\gamma_\mu\nu_{\ell L}\langle u^\mu[\mathcal{Q}^{\rm em}_{\rm R},\mathcal{Q}^{\rm w}_{\rm L}]\rangle \\ &+X_3 m_\ell\bar\ell\nu_{\ell L}\langle\mathcal{Q}^{\rm w}_{\rm L}\mathcal{Q}^{\rm em}_{\rm R}\rangle + h.c.\} + e^2\textstyle\sum_\ell X_6\bar\ell(i\partial\!\!\!/ + eA\!\!\!/)\ell ,\end{aligned}$$

Knecht, Neufeld, Rupertsberger and Talavera, EPJC 2000



One-loop and bremsstrahlung diagrams are calculated

Connection to the full EW theory:

Done through **a specific combination of LECs**:

$$
\begin{aligned}
X_6^{\text{phys}} &\equiv X_6^r - 4K_{12}^r \\
&= \left(X_6^{\text{phys}}\right)_{\text{SD}} + \left(X_6^{\text{phys}}\right)_{\text{LD}} \\
S_{\text{EW}} &= 1 - e^2 \left(X_6^{\text{phys}}\right)_{\text{SD}}
\end{aligned}
$$

Quantification of hadronic uncertainties:

- Uncertainties due to **neglected terms at $O(e^2p^4)$**
- Uncertainties due to **unknown LECs at $O(e^2p^2)$**

Ananthanarayan and Moussallam, JHEP 2004
Descotes-Genon and Moussallam, EPJC 2005

	$\delta_{\mathrm{EM}}^{K\ell}(\%)$
K_{e3}^0	$0.99 \pm 0.19_{e^2p^4} \pm 0.11_{\mathrm{LEC}}$
$K_{e3}^{\pm}$	$0.10 \pm 0.19_{e^2p^4} \pm 0.16_{\mathrm{LEC}}$
$K_{\mu3}^0$	$1.40 \pm 0.19_{e^2p^4} \pm 0.11_{\mathrm{LEC}}$
$K_{\mu3}^{\pm}$	$0.016 \pm 0.19_{e^2p^4} \pm 0.16_{\mathrm{LEC}}$

Uncertainty ~**10^{-3}**

"**Natural limitations**" of ChPT precision

Cirigliano, Giannotti and Neufeld, JHEP 2008

To overcome the natural limitations:

- **New theory framework** to resum the most important $O(e^2p^n)$ contributions
- **Appropriate lattice-QCD inputs** to reduce uncertainties from NP QCD

Direct lattice calculations of the **full K_{l3} RC**: ~10 years to reach 10^{-3} precision

Boyle et al., SnowMass 2021 LoI

4. Sirlin's representation

"Sirlin's representation" of the $O(G_F\alpha)$ EWRC

- First constructed by Sirlin to deal with EWRC in superallowed beta decays

Sirlin, 1978 Rev.Mod.Phys

- Re-introduced to study EWRC in general semileptonic decays

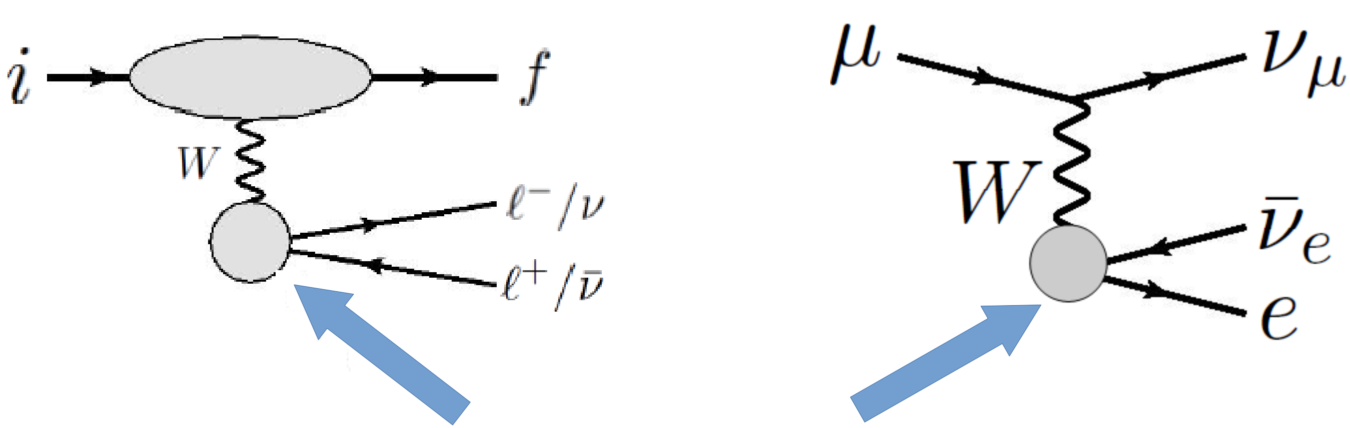
CYS, Galviz and Meißner, 2020 JHEP
CYS, 2021 Particles

Foundations: **Current Algebra (CA)**

$$\left[J_W^{0\dagger}(\vec{x},t), J_{\mathrm{em}}^{\mu}(\vec{y},t)\right] = J_W^{\mu\dagger}(\vec{x},t)\delta^{(3)}(\vec{x}-\vec{y})$$

Splitting the full **photon propagator**:

$$\frac{1}{q'^2} = \underbrace{\frac{1}{q'^2 - M_W^2}}_{\text{“}\gamma_>\text{”}} + \underbrace{\frac{M_W^2}{M_W^2 - q'^2}\frac{1}{q'^2}}_{\text{“}\gamma_<\text{”}}$$

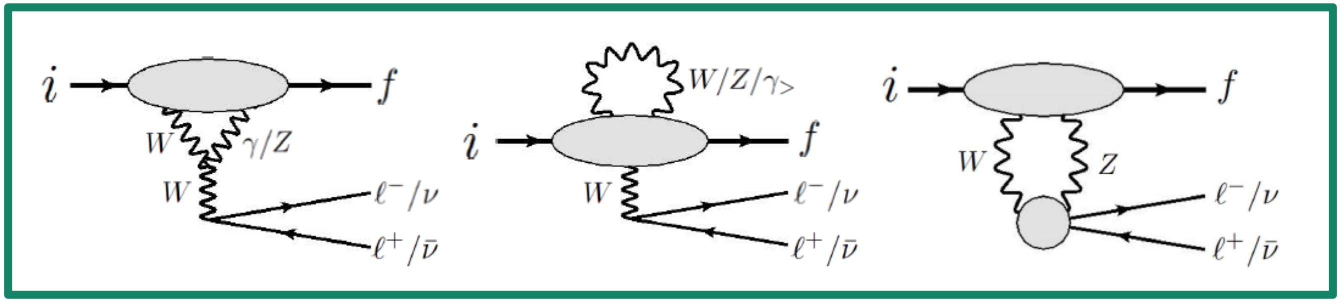


EWRCs at $q' \sim M_W$ at the **lepton side** are reabsorbed into **G_F** measured from **muon decay**

$$\frac{1}{\tau_\mu} \equiv \frac{G_F^2 m_\mu^5}{192\pi^3} F(x)(1+\delta_\mu)$$

Splitting the full **photon propagator**:

$$\frac{1}{q'^2} = \underbrace{\frac{1}{q'^2 - M_W^2}}_{``\gamma_>"} + \underbrace{\frac{M_W^2}{M_W^2 - q'^2}\frac{1}{q'^2}}_{``\gamma_<"}$$



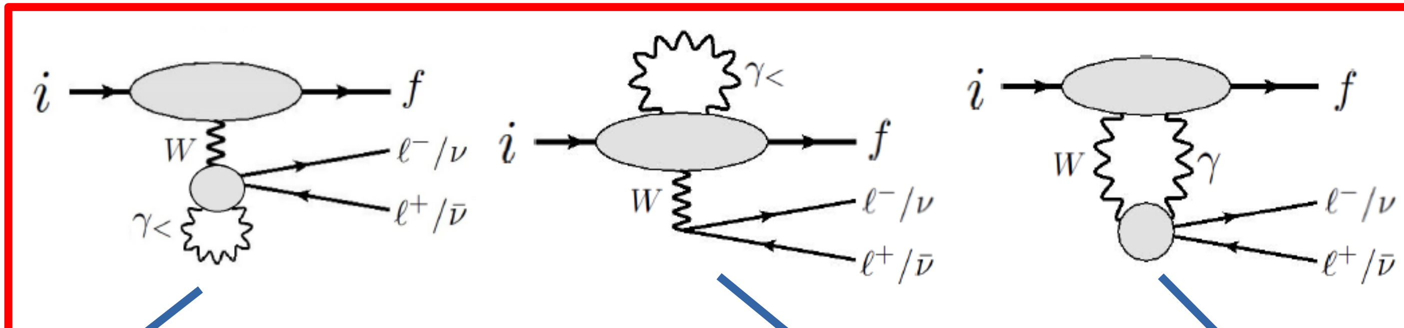
"Weak" RC:
Depends on $q' \sim M_W$

"**Modified Fermi's interaction**":

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{QED},\gamma_<} + \mathcal{L}'_{4f}$$

$$\mathcal{L}'_{4f} = \boxed{-\left\{1 - \frac{\alpha}{2\pi}\left[\ln\frac{M_W^2}{M_Z^2} + \mathcal{O}(\alpha_s)\right]\right\}}\frac{\boxed{G_F}}{\sqrt{2}}J^\mu\,\bar{l}\gamma_\mu(1-\gamma_5)\nu + h.c.$$

Virtual electromagnetic RC (EMRC)

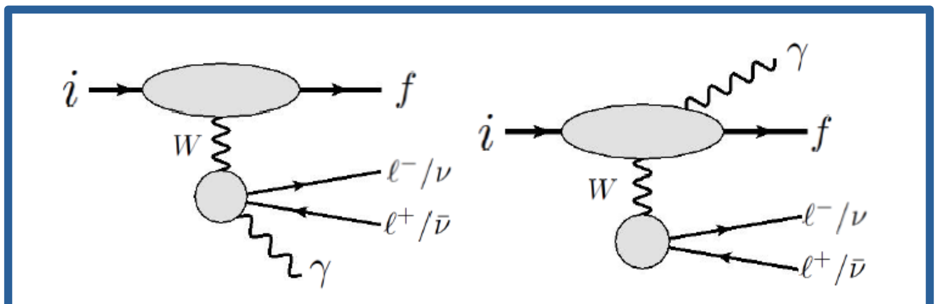


Lepton WF renormalization

EMRC to hadronic form factors

γW-box diagram

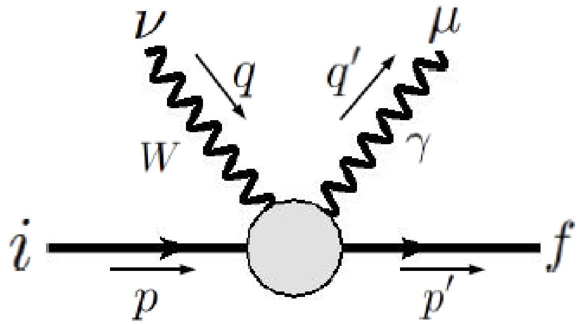
$$\mathfrak{M} = -\sqrt{Z_l}\left[1-\frac{\alpha}{2\pi}\left(\ln\frac{M_W^2}{M_Z^2}+\mathcal{O}(\alpha_s)\right)\right]\frac{G_F}{\sqrt{2}}F^\mu\,\bar{u}_v\gamma_\mu(1-\gamma_5)\nu_l-\frac{G_F}{\sqrt{2}}\delta F^\mu\,\bar{u}_\nu\gamma_\mu(1-\gamma_5)\nu_l+\Box_{\gamma W}$$



Real EMRC (Bremsstrahlung)

Key quantity: **“Generalized Compton tensor”**

$$\boxed{T^{\mu\nu}(q';p',p)=\int d^4x e^{iq'\cdot x}\langle\phi_f(p')|T\{J^\mu_{\mathrm{em}}(x)J^\nu_W(0)\}|\phi_i(p)\rangle}$$

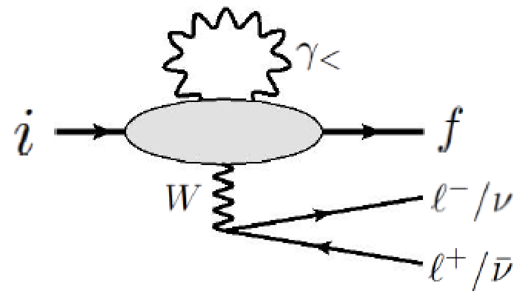


The **“convection term”:** *Meister and Yennie, PR 1963*

$$T^{\mu\nu}_{\mathrm{conv}}(q';p',p)=\frac{iZ_f(2p'+q')^\mu F^\nu(p',p)}{(p'+q')^2-M_f^2}+\frac{iZ_i(2p-q')^\mu F^\nu(p',p)}{(p-q')^2-M_i^2}$$

Simplest structure satisfying the **exact EM Ward identity;**
gives the **IR-divergent structure**

EMRC to the hadron form factors:



$$\boxed{\delta F^\lambda = \delta F_2^\lambda + \delta F_3^\lambda}$$ with current algebra

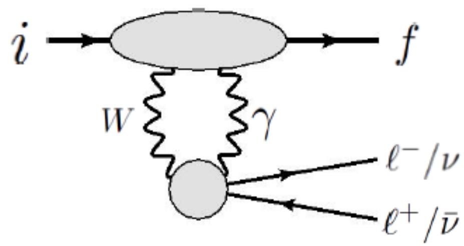
"Two-point function": contains all the UV-dependence

$$\delta F_2^\lambda(p',p) = -e^2\eta\int\frac{d^4q'}{(2\pi)^4}\left[\frac{M_W^2}{(M_W^2-q'^2)^2}\frac{q'^\lambda}{q'^2-M_\gamma^2}-\frac{M_W^2}{M_W^2-q'^2}\frac{q'^\lambda}{(q'^2-M_\gamma^2)^2}\right]T^\mu_\mu$$

$\delta F_3^\lambda(p',p)$ **"Three-point function"**: insensitive to UV,
vanishes in the degenerate + forward limit

$$\sim\langle f|T\{J_W(x)J_{\mathrm{em}}(y)J_{\mathrm{em}}(0)\}|i\rangle$$

γW-box diagram:



$$\boxed{\delta\mathfrak{M}_{\gamma W} = \delta\mathfrak{M}^a_{\gamma W} + \delta\mathfrak{M}^b_{\gamma W}}$$

$$\delta\mathfrak{M}^a_{\gamma W} = \eta\frac{G_F e^2}{\sqrt{2}} L_\lambda \int \frac{d^4q'}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q'^2} \frac{2g^{\nu\lambda}p_\ell^\mu - g^{\mu\lambda}q'^\nu - g^{\nu\lambda}q'^\mu + g^{\mu\nu}q'^\lambda}{[(p_\ell - q')^2 - m_\ell^2][q'^2 - M_\gamma^2]} T_{\mu\nu}$$

W/O epsilon tensor

$$\delta\mathfrak{M}^b_{\gamma W} = -i\frac{G_F e^2}{\sqrt{2}} L_\lambda \int \frac{d^4q'}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q'^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q'_\alpha}{[(p_\ell - q')^2 - m_\ell^2] q'^2} T_{\mu\nu} \,.$$

With epsilon tensor

Sirlin's representation:
O(α) virtual EWRC to semileptonic decays

O(α^n) QED corrections

$$
\begin{aligned}
\mathfrak{M} \;=\; & -\frac{G_F}{\sqrt{2}} F^\lambda L_\lambda \left\{ 1 + \overbrace{\frac{\alpha}{2\pi}\left[\ln\frac{M_Z^2}{m_\ell^2} - \frac{1}{4}\ln\frac{M_W^2}{m_\ell^2} + \frac{1}{2}\ln\frac{m_\ell^2}{M_\gamma^2} - \frac{3}{8} + \frac{1}{2}\tilde{a}_g \right]}^{\text{Exactly-known O}(\alpha)\text{ terms}} + \frac{1}{2}\delta_{\text{HO}}^{\text{QED}} \right\} \\
& + \underbrace{\left(\delta\mathfrak{M}_2 + \delta\mathfrak{M}_{\gamma W}^{a} \right)_{\text{int}}}_{\text{“Residual integral”}} \underbrace{- \frac{G_F}{\sqrt{2}} \delta F_3^\lambda L_\lambda}_{\text{3-pt function}} + \underbrace{\delta\mathfrak{M}_{\gamma W}^{b}}_{\text{From }\gamma W\text{-box}} ,
\end{aligned}
$$

5. Electroweak RC to K_{l3}

Kπ charged weak form factors

suppressed in decay rate by m_ℓ^2/M_K^2

$$F_\mu^{K\pi}(p',p) \equiv \langle \pi(p') | (J_\mu^W)^\dagger | K(p) \rangle = V_{us}^* \left[\underline{f_+^{K\pi}(t)} (p+p')_\mu + \underline{f_-^{K\pi}(t)} (p-p')_\mu \right]$$

Virtual corrections expressed as corrections to form factors:

$$\boxed{f_\pm^{K\pi}(t) \to f_\pm^{K\pi}(t) + \delta f_\pm^{K\pi}(y,z)}$$

$$y = \frac{2E_\ell}{M_K} \qquad z = \frac{2E_\pi}{M_K}$$

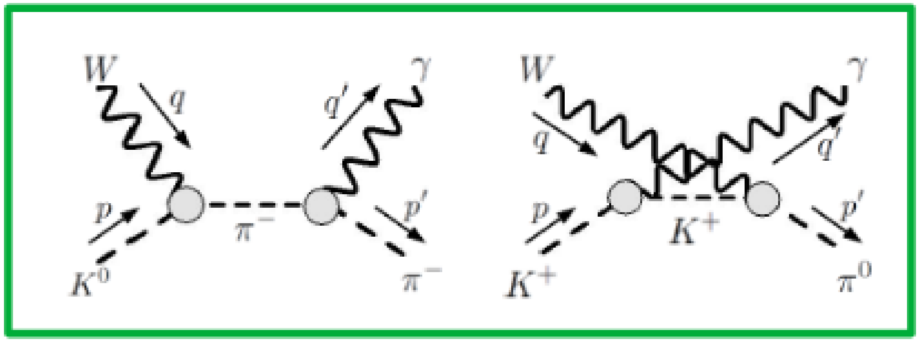
Contains IR-divergences canceled by bremsstrahlung

Calculation of δf_+ :

(A) "Residual integral" + the vector current contribution to $\delta\mathfrak{M}^b_{\gamma W}$

$$\boxed{\begin{aligned} I^\lambda_{\mathfrak{A}} \quad = \quad & -e^2 \int \frac{d^4q'}{(2\pi)^4} \frac{1}{[(p_\ell - q')^2 - m_\ell^2][q'^2 - M_\gamma^2]} \left\{ \frac{2p_\ell \cdot q' q'^\lambda}{q'^2 - M_\gamma^2} + 2p_{\ell\mu} T^{\mu\lambda} \right. \\ & \left. -(p-p')_\mu T^{\lambda\mu} + i\Gamma^\lambda - i\epsilon^{\mu\nu\alpha\lambda} q'_\alpha (T_{\mu\nu})_V \right\} \end{aligned}}$$

Contribution to f_+ is saturated by the "pole" terms" :

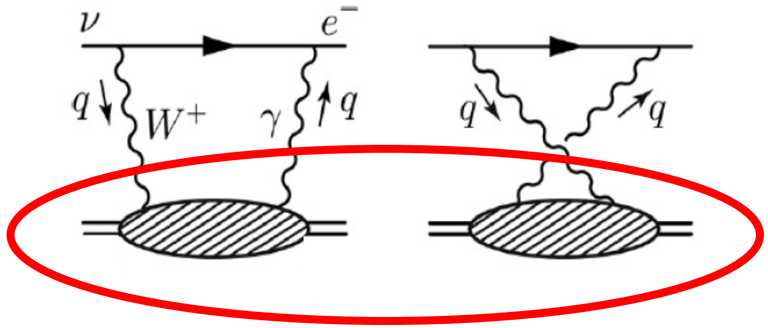


- Inputs: **K/π EM form factors** and **charged weak form factors**, well-measured in experiments
- Effectively **re-summing the most important O(e^2p^n) corrections**

Calculation of δf_+ :

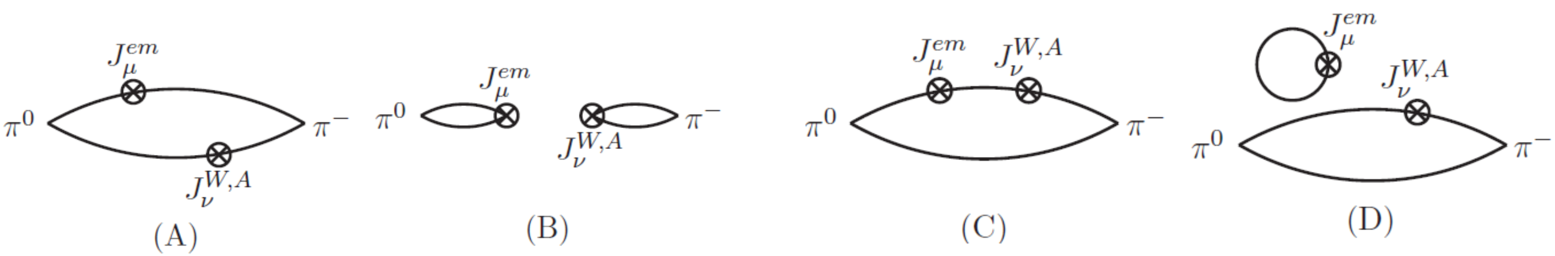
(B) Axial current contribution to $\delta \mathfrak{M}^b_{\gamma W}$

$$I^\lambda_{\mathfrak{B}} = ie^2 \int \frac{d^4q'}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q'^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q'_\alpha (T_{\mu\nu})_A}{[(p_\ell - q')^2 - m_\ell^2] q'^2}$$

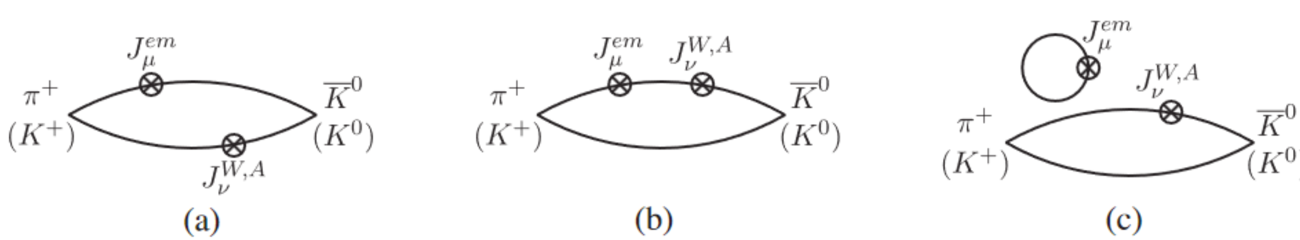


Obtained from **lattice + pQCD**

$\pi \to \pi$:



$K \to K$, $\bar{K} \to \pi$:



Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL
Ma, Feng, Gorchtein, Jin and CYS, 2021 PRD

Calculation of δf_+ :

(c) δf_+ contributed by the "3-pt function" *Seng, Galviz and Meißner, 2020 JHEP*

$$\boxed{\underbrace{\delta f_{+,3} = (\delta f_+)_{\rm III}}_{} + \underbrace{\left\{(\delta f_{+,3})^{\rm fin}_{e^2p^2} + \mathcal{O}(e^2p^4)\right\}}_{}}$$

IR-divergent terms, resummed by adding **form factors**

Regular terms calculated at fixed-order ChPT

Reabsorbed into the **tree-level form factors** and **isospin-breaking corrections,** according to standard ChPT classification

Seng, Galviz, Gorchtein and Meißner, 2021 JHEP

Calculation of δf_- :

suppressed in decay rate by m_ℓ^2/M_K^2

$$\boxed{\delta f_- = \underbrace{(\delta f_-)_{\rm I+II+III} + (\delta f_-)^{\rm fin}_{\rm conv}} + (\delta f_-)_{\rm rem}}$$

Numerically largest, exactly calculable

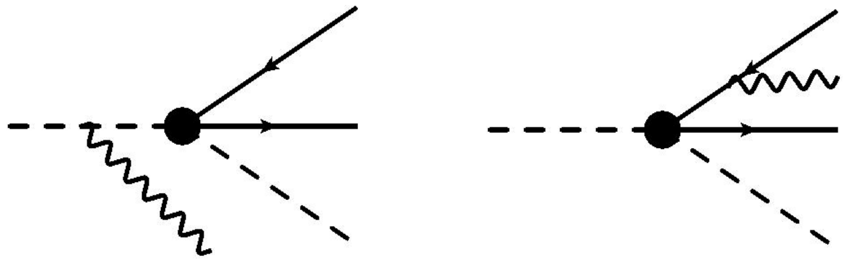
Numerically small, calculated at fixed-order ChPT

Cirigliano et al., 2002 EPJC

$$\left(\delta f_-^{K^0\pi^-}\right)^{e^2p^2}_{\rm rem} = -\frac{\alpha}{4\pi}\left[r_{0,1}\Lambda(s, M_\pi, m_\ell) + r_{0,2}\ln\frac{M_\pi^2}{m_\ell^2} + r_{0,3}\ln\frac{M_\pi^2}{\mu^2} + r_{0,4}\right]$$

$$\left(\delta f_-^{K^+\pi^0}\right)^{e^2p^2}_{\rm rem} = -\frac{\alpha}{4\sqrt{2}\pi}\left[r_{+,1}\Lambda(u, M_K, m_\ell) + r_{+,2}\ln\frac{M_K^2}{m_\ell^2} + r_{+,3}\ln\frac{M_K^2}{\mu^2} + r_{+,4}\right]$$

Bremsstrahlung:



$$\boxed{\mathfrak{M}_{\text{brems}} = \mathfrak{M}_A + \mathfrak{M}_B}$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{conv}} + \left\{ \underbrace{\left(T^{\mu\nu} - T^{\mu\nu}_{\text{conv}}\right)_{p^2} + \mathcal{O}(p^4)} \right\}$$

Convection term contribution, exactly-known and contains full IR-divergence

Regular terms calculated at fixed-order ChPT

Final Result: (in units of 10^{-3})

CYS, Galviz, Gorchtein and Meißner, 2021 PLB; 2021 JHEP

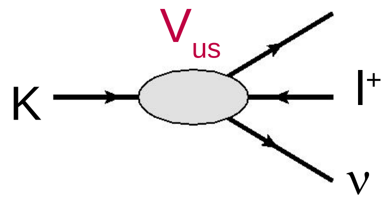
CYS, Galviz, Gorchtein and Meißner, 2022 JHEP

	$\delta_{\mathrm{EM}}^{K\ell}$	ChPT
$K^0 e$	$11.6(2)_{\mathrm{inel}}(1)_{\mathrm{lat}}(1)_{\mathrm{NF}}(2)_{e^2p^4}$	$9.9(1.9)_{e^2p^4}(1.1)_{\mathrm{LEC}}$
$K^+ e$	$2.1(2)_{\mathrm{inel}}(1)_{\mathrm{lat}}(4)_{\mathrm{NF}}(1)_{e^2p^4}$	$1.0(1.9)_{e^2p^4}(1.6)_{\mathrm{LEC}}$
$K^0 \mu$	$15.4(2)_{\mathrm{inel}}(1)_{\mathrm{lat}}(1)_{\mathrm{NF}}(2)_{\mathrm{LEC}}(2)_{e^2p^4}$	$14.0(1.9)_{e^2p^4}(1.1)_{\mathrm{LEC}}$
$K^+ \mu$	$0.5(2)_{\mathrm{inel}}(1)_{\mathrm{lat}}(4)_{\mathrm{NF}}(2)_{\mathrm{LEC}}(2)_{e^2p^4}$	$0.2(1.9)_{e^2p^4}(1.6)_{\mathrm{LEC}}$

Sources of uncertainty:

- **inel**: inelastic states contributions to the residual integral
- **lat**: Lattice uncertainty in the γW-box diagram
- **NF**: Non-forward effects in the γW-box diagram
- **e^2p^4**: Higher-order ChPT corrections
- **LEC**: Poorly-determined LECs in δf_-

- Consistent with pure ChPT result
- Significant improvement of precision: **$10^{-3} \rightarrow 10^{-4}$**



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU}(2)}^{K\pi}\right)$$

Averaging over all six channels:

	$\lvert V_{us} f_+^{K^0\pi^-}(0)\rvert$
$K_L e$	$0.21617(46)_{\mathrm{exp}}(10)_{I_K}(4)_{\delta_{\mathrm{EM}}}$
$K_S e$	$0.21530(122)_{\mathrm{exp}}(10)_{I_K}(4)_{\delta_{\mathrm{EM}}}$
$K^+ e$	$0.21714(88)_{\mathrm{exp}}(10)_{I_K}(21)_{\delta_{\mathrm{SU}(2)}}(5)_{\delta_{\mathrm{EM}}}$
$K_L \mu$	$0.21649(50)_{\mathrm{exp}}(16)_{I_K}(4)_{\delta_{\mathrm{EM}}}$
$K_S \mu$	$0.21251(466)_{\mathrm{exp}}(16)_{I_K}(4)_{\delta_{\mathrm{EM}}}$
$K^+ \mu$	$0.21699(108)_{\mathrm{exp}}(16)_{I_K}(21)_{\delta_{\mathrm{SU}(2)}}(6)_{\delta_{\mathrm{EM}}}$
Average: Ke	$0.21626(40)_K(3)_{\mathrm{HO}}$
Average: $K\mu$	$0.21654(48)_K(3)_{\mathrm{HO}}$
Average: tot	$0.21634(38)_K(3)_{\mathrm{HO}}$

N_f=2+1+1

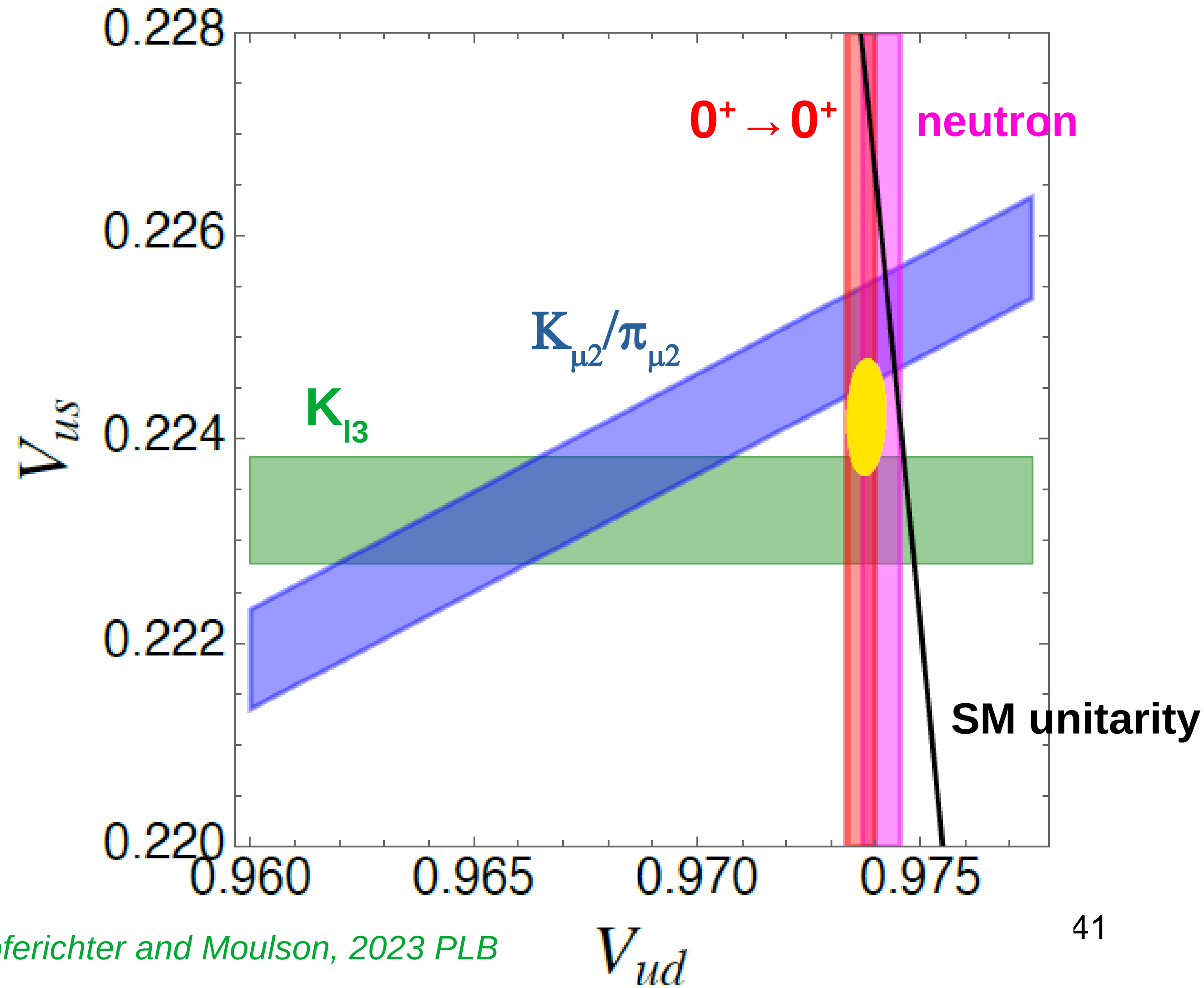
$$|V_{us}|_{K_{\ell 3}} = \begin{cases} 0.22308(39)_{\mathrm{lat}}(39)_K(3)_{\mathrm{HO}} \\ 0.22356(62)_{\mathrm{lat}}(39)_K(3)_{\mathrm{HO}} \end{cases}$$

N_f=2+1

CYS, Galviz, Gorchtein and Meißner, 2022 JHEP

6. “Cabibbo angle anomaly”

Grouping everything...

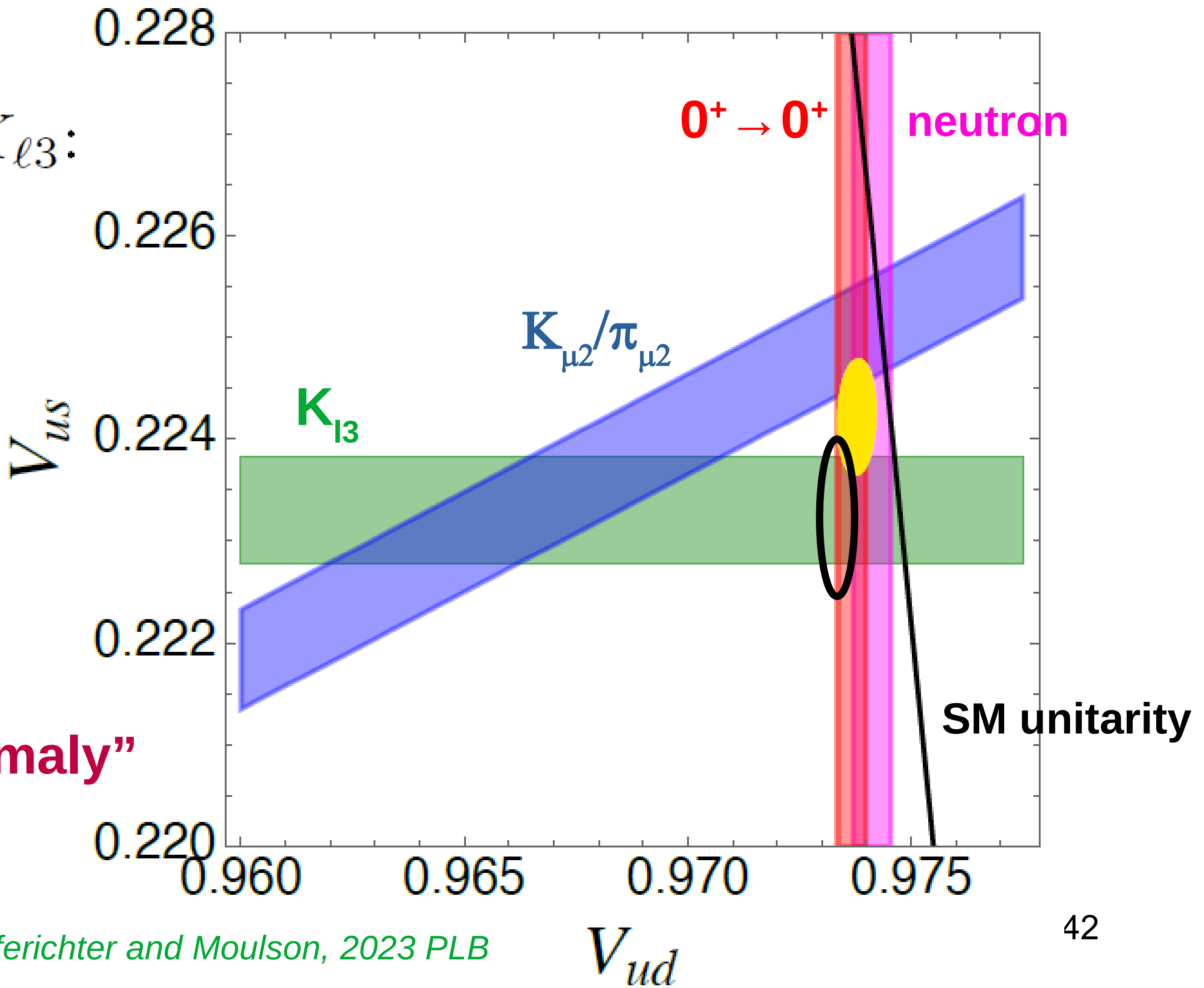


Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Grouping everything...

Superallowed + $K_{\ell 3}$:

$\boxed{3.1\sigma}$ deficit



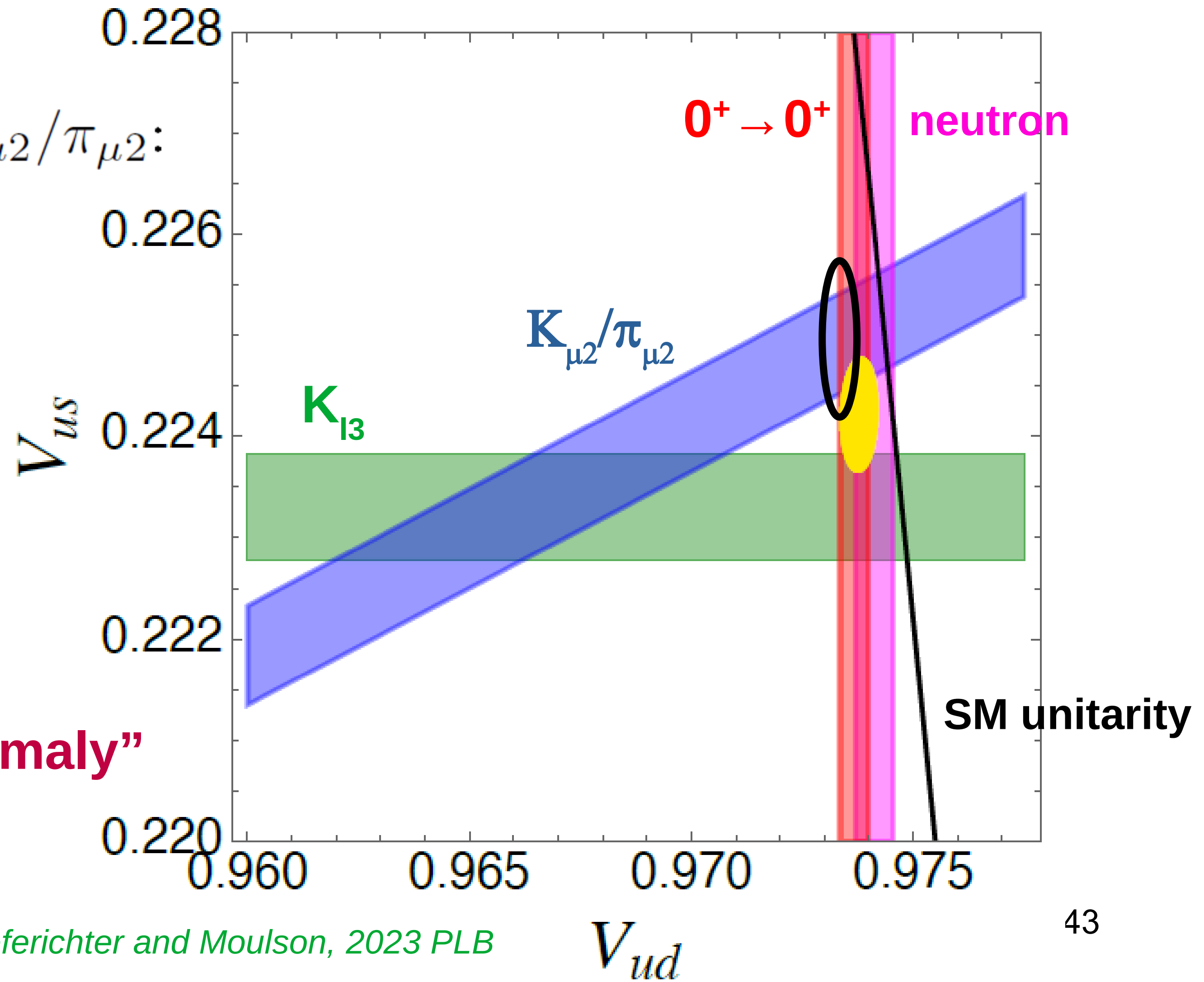
"Cabibbo angle anomaly"

Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Grouping everything...

Superallowed + $K_{\mu2}/\pi_{\mu2}$:

$\boxed{1.7\sigma}$ deficit



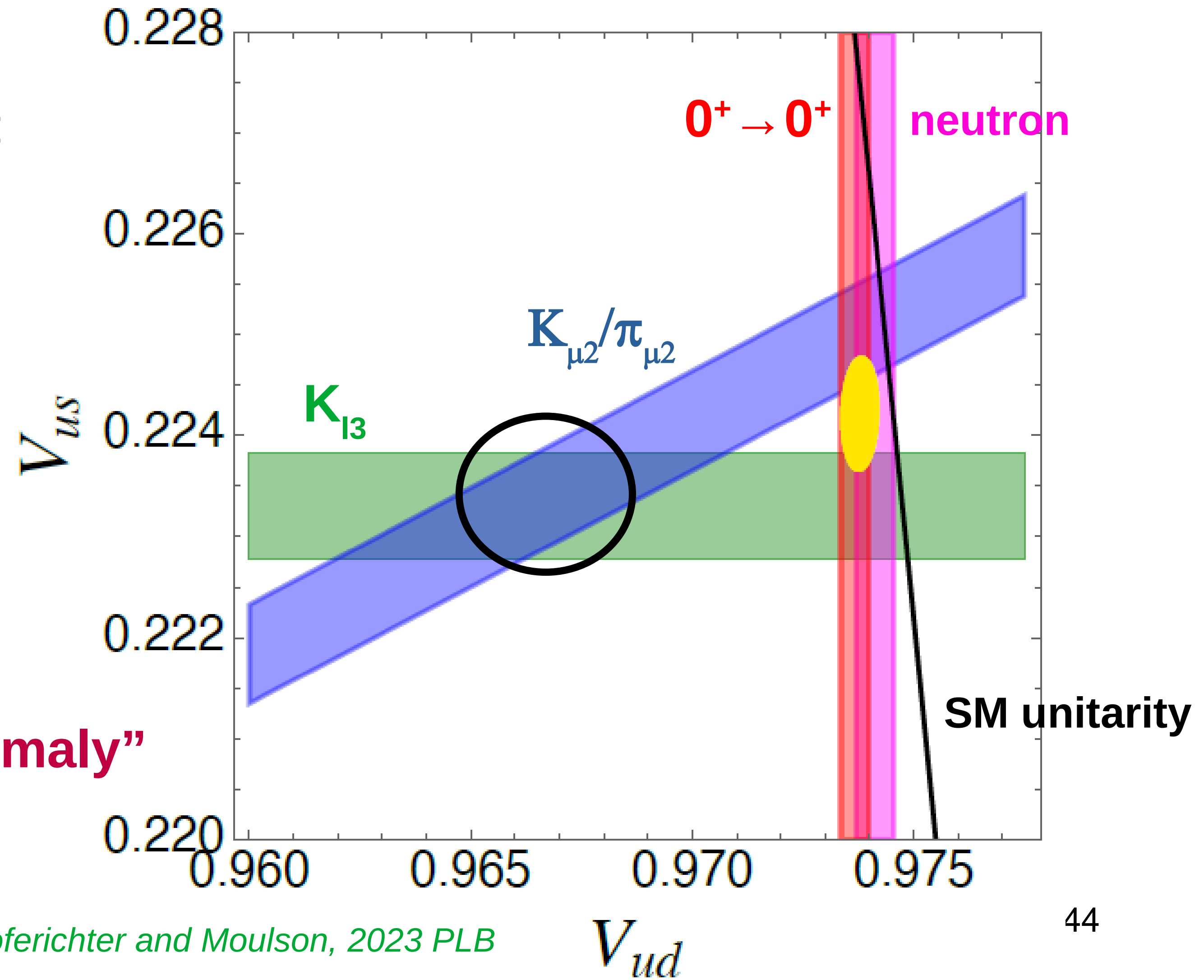
"Cabibbo angle anomaly"

Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Grouping everything...

$K_{\ell 3} + K_{\mu 2}/\pi_{\mu 2}$:

$\boxed{2.6\sigma}$ deficit



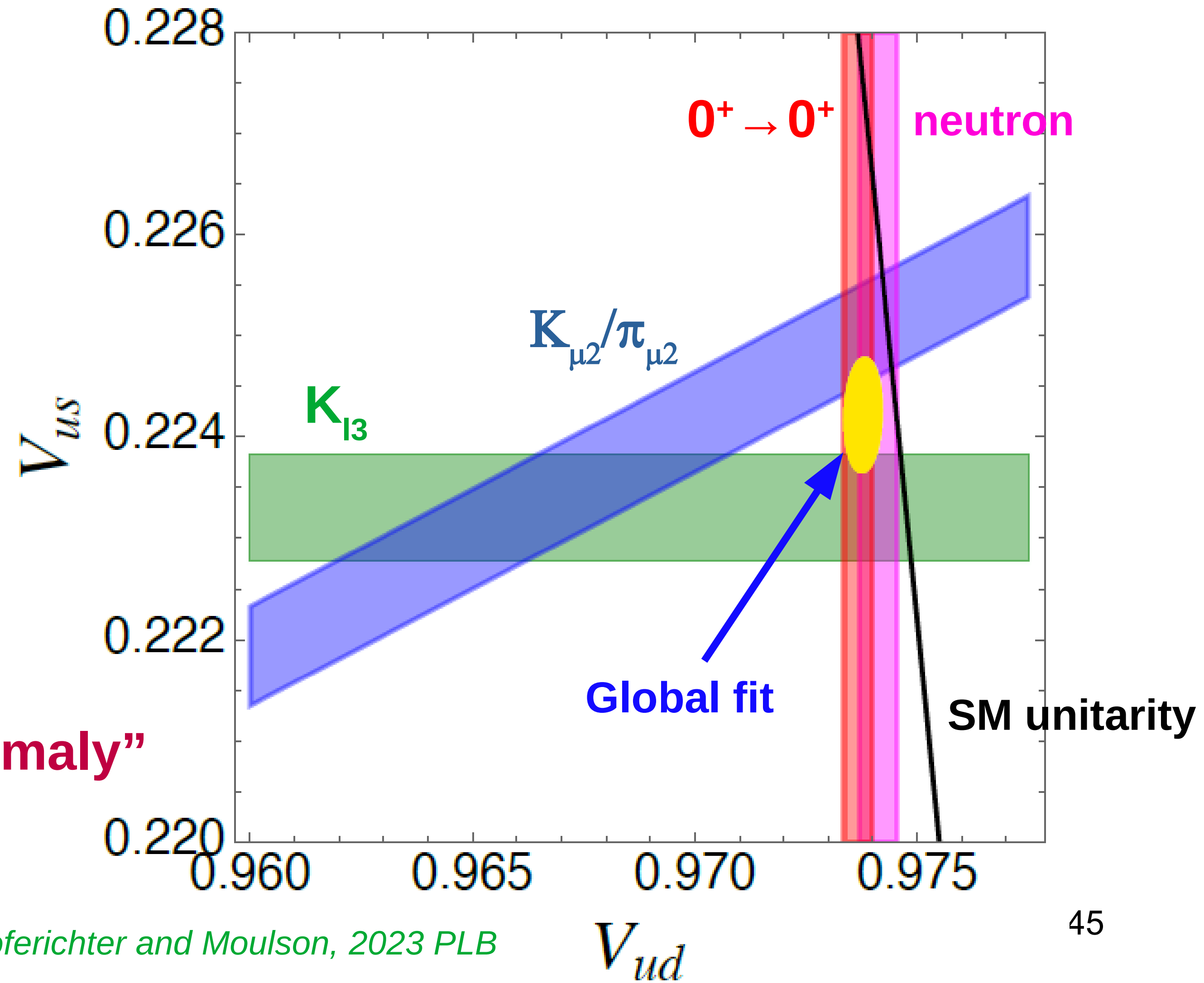
"Cabibbo angle anomaly"

Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Grouping everything...

Global fit:

$\boxed{2.8\sigma}$ deficit



"Cabibbo angle anomaly"

Cirigliano, Crivellin, Hoferichter and Moulson, 2023 PLB

Summary

- First-row CKM unitarity offers precision tests of SM; K_{l3} is the primary avenue to extract V_{us}.
- EMRC to the K_{l3} decay rate requires non-perturbative QCD. ChPT calculation is limited by unknown higher-order contributions and LECs.
- A reformulation using ChPT + current algebra allows inputs from meson form factors + lattice QCD, results in a much-improved determination of the K_{l3} RC, with central values consistent to pure ChPT result
- These new calculations helped to sharpen the "Cabibbo angle anomaly", which points towards new physics.

Thanks for your attention!