Gauge Theory Bootstrap:

Pion amplitudes and low energy parameters

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based on 2309.12402 and 2403.10772 with Martin Kruczenski

Strongly coupled gauge theory

confinement

energy

asymptotic freedom

$$\mathcal{L} = i\sum_{j}^{N_f} \bar{q}_j \not{D} q_j - \sum_{j}^{N_f} m_q \bar{q}_j q_j - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a$$

weakly coupled quarks/gluons (pQCD)

$$N_c \quad N_f \quad m_q \quad \Lambda_{\mathrm{QCD}}$$

Strongly coupled gauge theory

confinement

energy

chiSB

asymptotic freedom

$$\mathcal{L} = \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + m_\pi^2 \mathrm{Tr}\left(U + U^\dagger\right)\right\}$$

weakly coupled pions (EFT)

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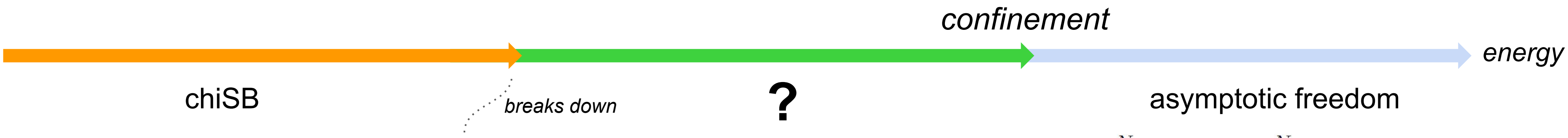
weakly coupled quarks/gluons (pQCD)

$$f_\pi \quad m_\pi$$

$$N_c \quad N_f \quad m_q \quad \Lambda_{\mathrm{QCD}}$$

Strongly coupled gauge theory



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strongly coupled regime

$$\mathcal{L} = i\sum_j^{N_f} \bar{q}_j \not{D} q_j - \sum_j^{N_f} m_q \bar{q}_j q_j - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a$$

weakly coupled pions (EFT)

weakly coupled quarks/gluons (pQCD)

$f_\pi \quad m_\pi$

$N_c \quad N_f \quad m_q \quad \Lambda_{\mathrm{QCD}}$

Strongly coupled gauge theory → Gauge Theory Bootstrap

confinement

energy

chiSB

breaks down

?

asymptotic freedom

$$\mathcal{L} = \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + m_\pi^2 \mathrm{Tr}\left(U + U^\dagger\right)\right\}$$

strongly coupled regime

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weakly coupled pions (EFT)

weakly coupled quarks/gluons (pQCD)

$f_\pi \quad m_\pi$

input

compute the strongly coupled dynamics:
amplitudes, form factors, spectrum/couplings

$N_c \quad N_f \quad m_q \quad \Lambda_{\mathrm{QCD}}$

input

Gauge Theory Bootstrap

Strongly coupled gauge theory → Gauge Theory Bootstrap

chiSB — *breaks down* — **?** — asymptotic freedom

$$\mathcal{L} = \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + m_\pi^2 \mathrm{Tr}\left(U + U^\dagger\right)\right\}$$

weakly coupled pions (EFT)

strongly coupled regime

$$\mathcal{L} = i\sum_j^{N_f} \bar{q}_j \not{D} q_j - \sum_j^{N_f} m_q \bar{q}_j q_j - \frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu}$$

weakly coupled quarks/gluons (pQCD)

$f_\pi \quad m_\pi$

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compute the strongly coupled dynamics:
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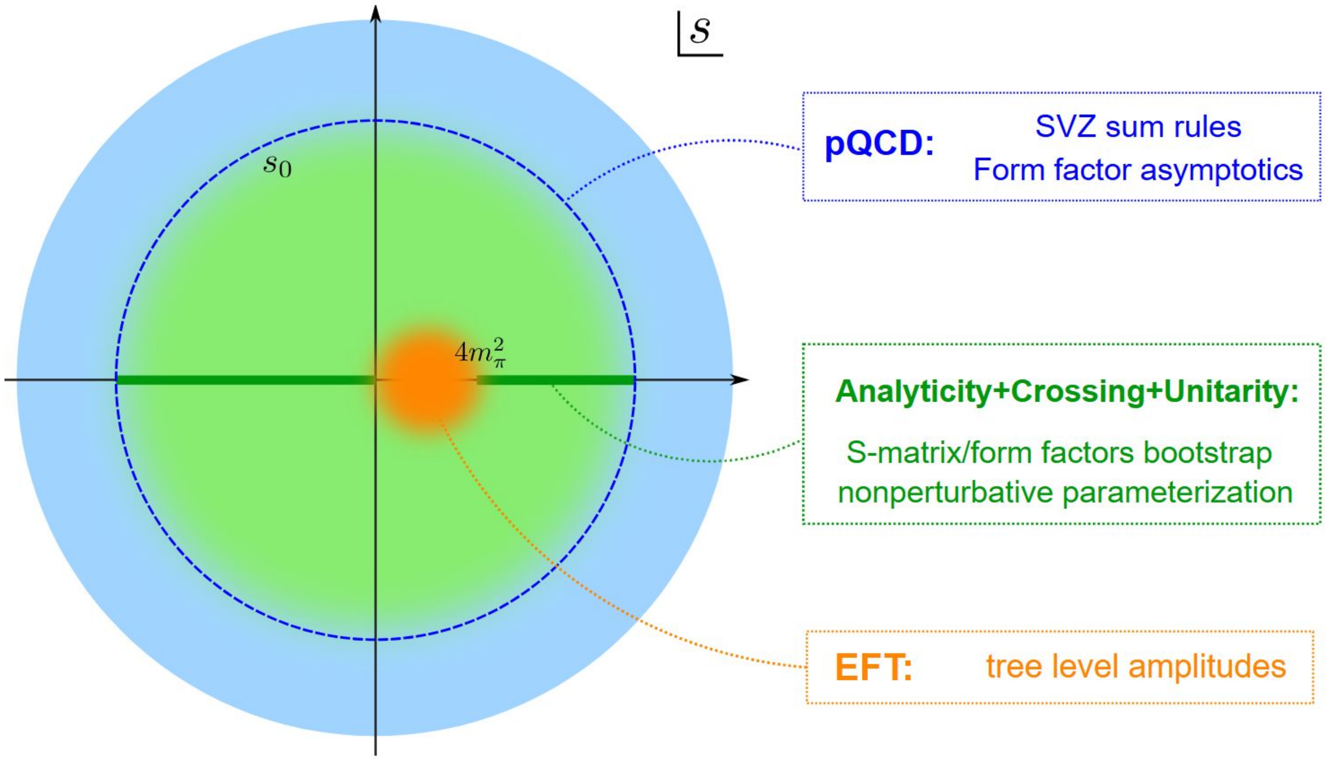
input

Gauge Theory Bootstrap

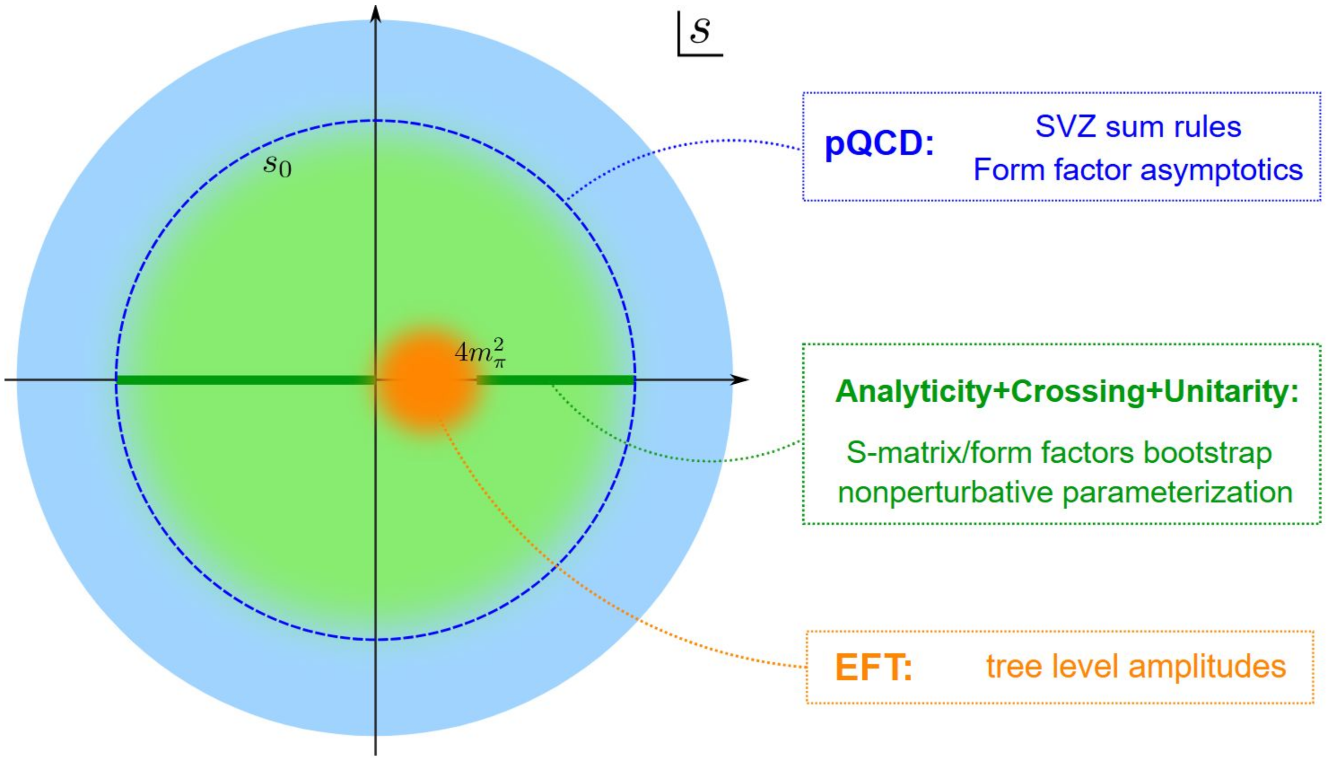
theoretical/numerical computation, not using experimental scattering data as input

Gauge Theory Bootstrap: summary



look for amplitudes/form factors that: 1, satisfy generic consistency conditions (analyticity, crossing, unitarity)
2, match low energy behavior (chiSB) and high energy (pQCD)

Gauge Theory Bootstrap: summary



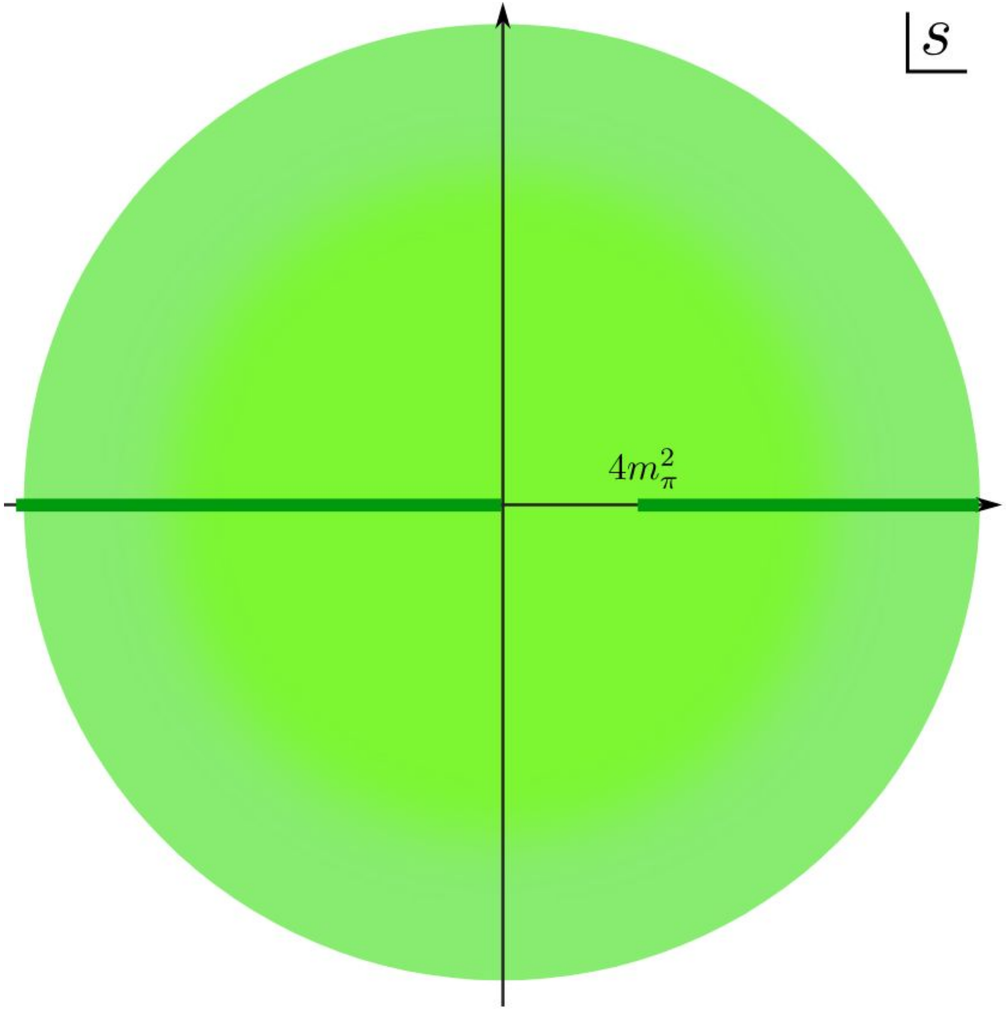
test with

$N_f = 2 \quad N_c = 3$

compare with experiments

changeable in the approach

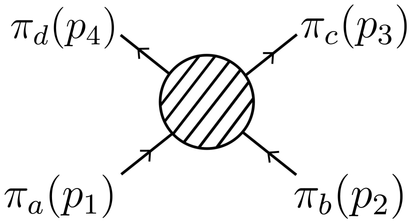
look for amplitudes/form factors that: 1, satisfy generic consistency conditions (analyticity, crossing, unitarity)
2, match low energy behavior (chiSB) and high energy (pQCD)



Analyticity+Crossing+Unitarity:

S-matrix bootstrap
nonperturbative parameterization

S-matrix bootstrap parameterization



modern S-matrix bootstrap: [Paulos, Penedones, Toledo, van Rees, Vieira, 2016&2017]

$$\langle p_1, a; p_2, b|\mathbf{T}|p_3, c; p_4, d\rangle = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

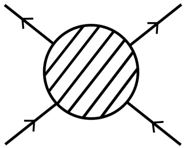
Crossing $A(s,t,u) = A(s,u,t)$

Analyticity cuts $s,t,u > 4$

$$m_\pi = 1$$

$$s + t + u = 4$$

S-matrix bootstrap parameterization



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Crossing $A(s,t,u) = A(s,u,t)$

Analyticity cuts $s,t,u > 4$

$$m_\pi = 1$$

$$s + t + u = 4$$

nonperturbative parameterization encoding **Analyticity** and **Crossing**:

$$A(s,t,u) = \frac{1}{\pi^2}\int_4^\infty dx \int_4^\infty dy \left[\frac{\rho_1(x,y)}{(x-s)(y-t)} + \frac{\rho_1(x,y)}{(x-s)(y-u)} + \frac{\rho_2(x,y)}{(x-t)(y-u)}\right]$$ +subtraction terms

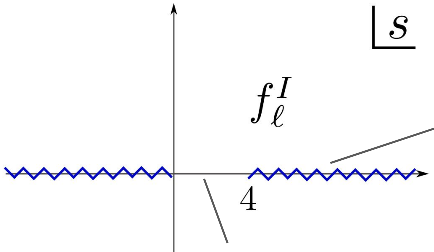
parameters: $\{\rho_{\alpha=1,2}(x,y), \ldots\}$ numerics: discretize $\{\rho_{\alpha,ij}, \ldots\}$ bootstrap variables

S-matrix bootstrap parameterization

$SU(2)_V$ isospin ***Symmetry***

$$f_\ell^I(s) = \frac{1}{4}\int_{-1}^{1} d\cos\theta P_\ell(\cos\theta) T^I(s,t)$$

analytic function of s



$f_\ell^I(0 < s < 4)$

$$S_\ell^I(s) = 1 + i\pi\sqrt{\frac{s-4}{s}} f_\ell^I(s) \quad s > 4 \quad \forall \ell, I$$

$$|S_\ell^I(s^+)| \leq 1$$ ***Unitarity***

convex constraints

real linear functionals of bootstrap variables $\{\rho_{\alpha=1,2}(x,y),\ldots\}$

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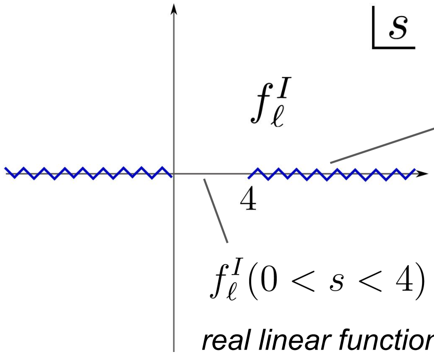

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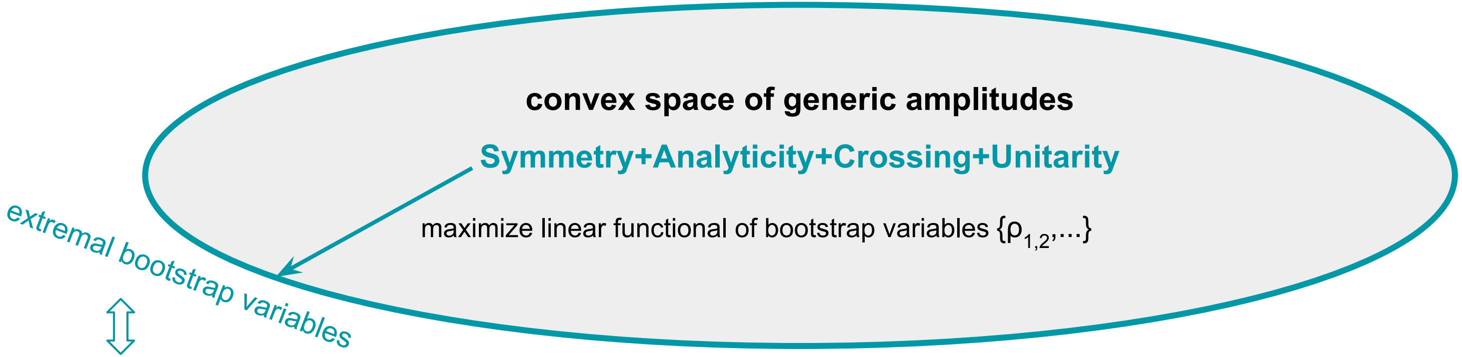
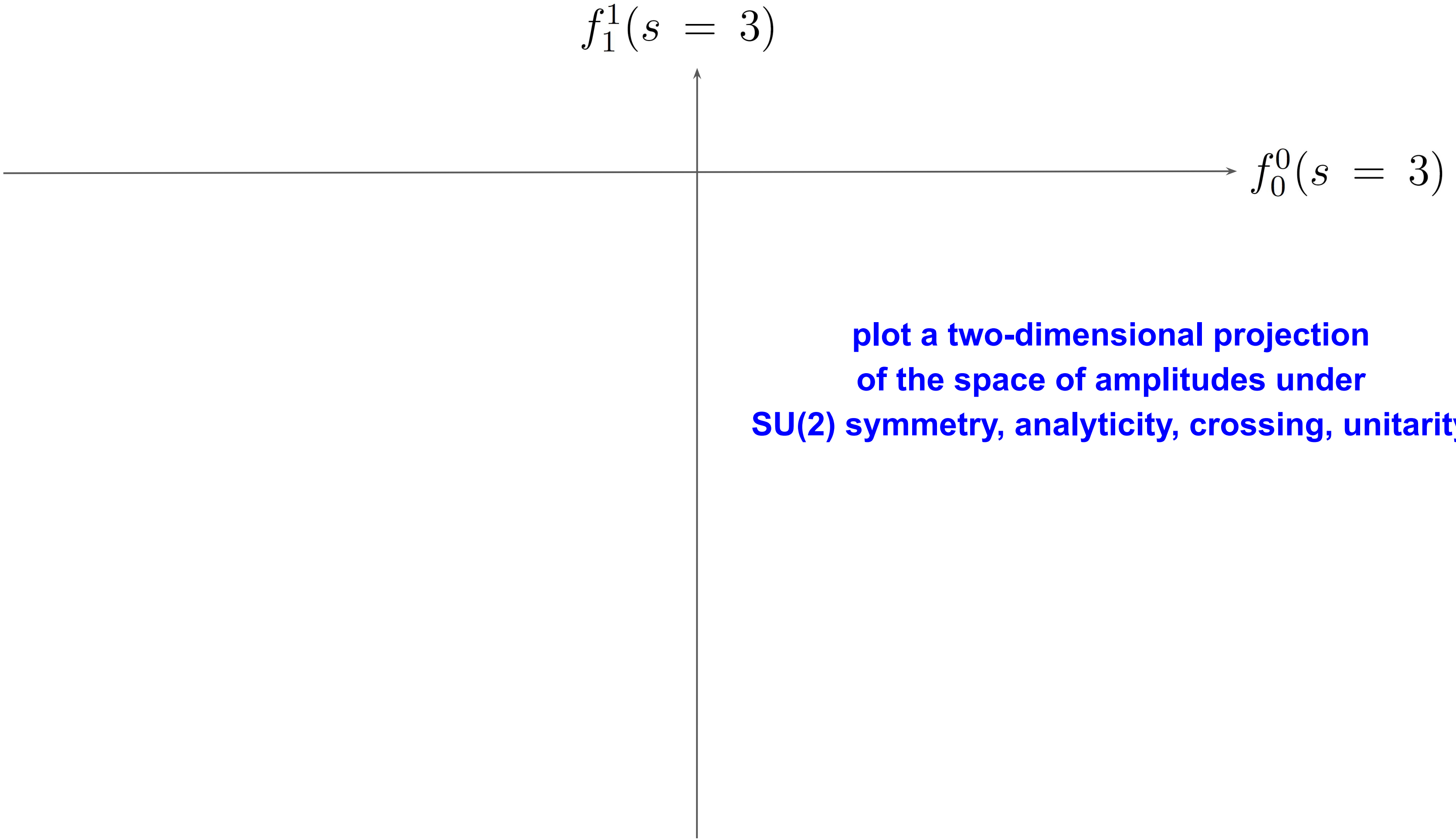
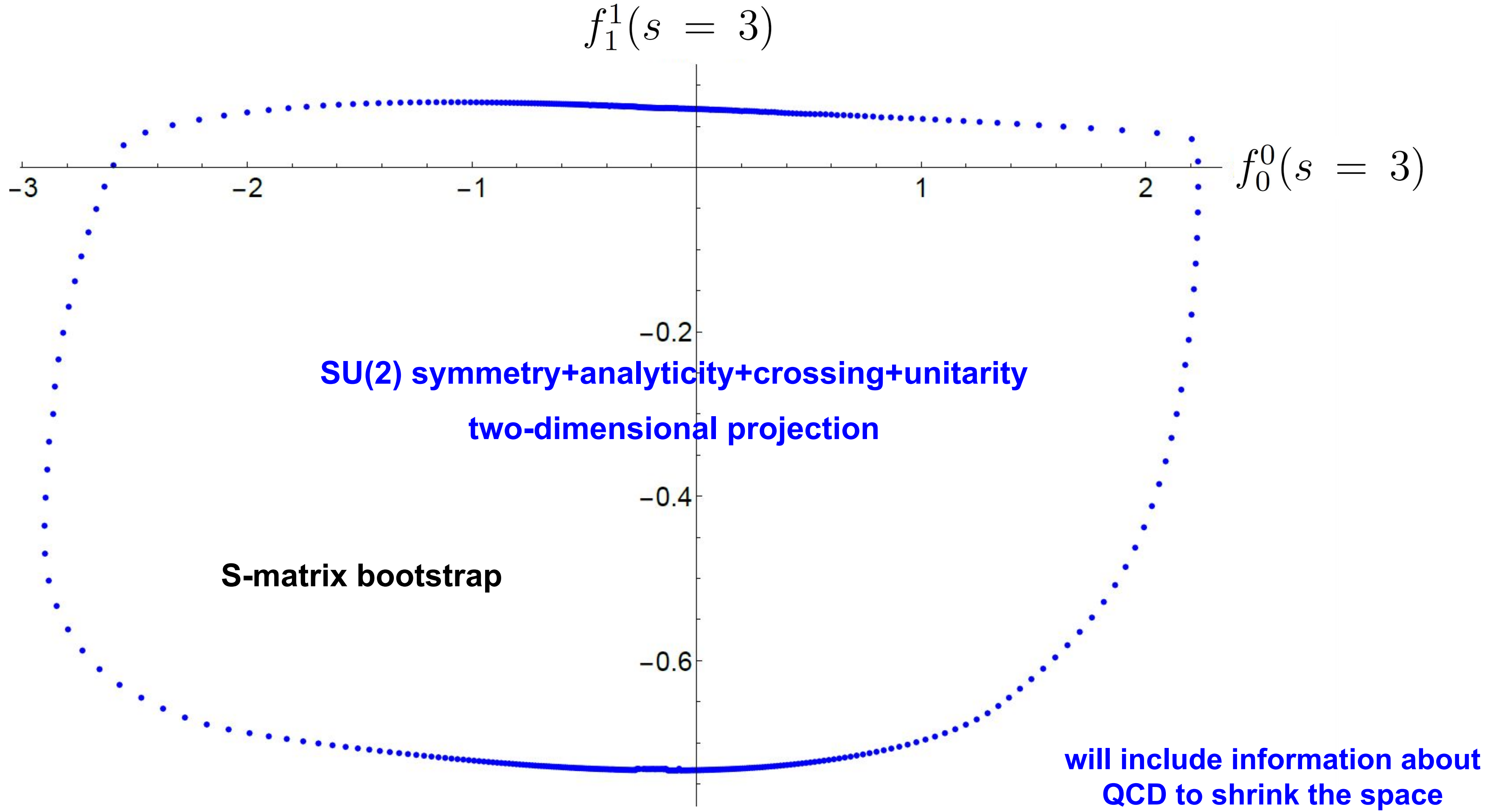


extremal amplitudes

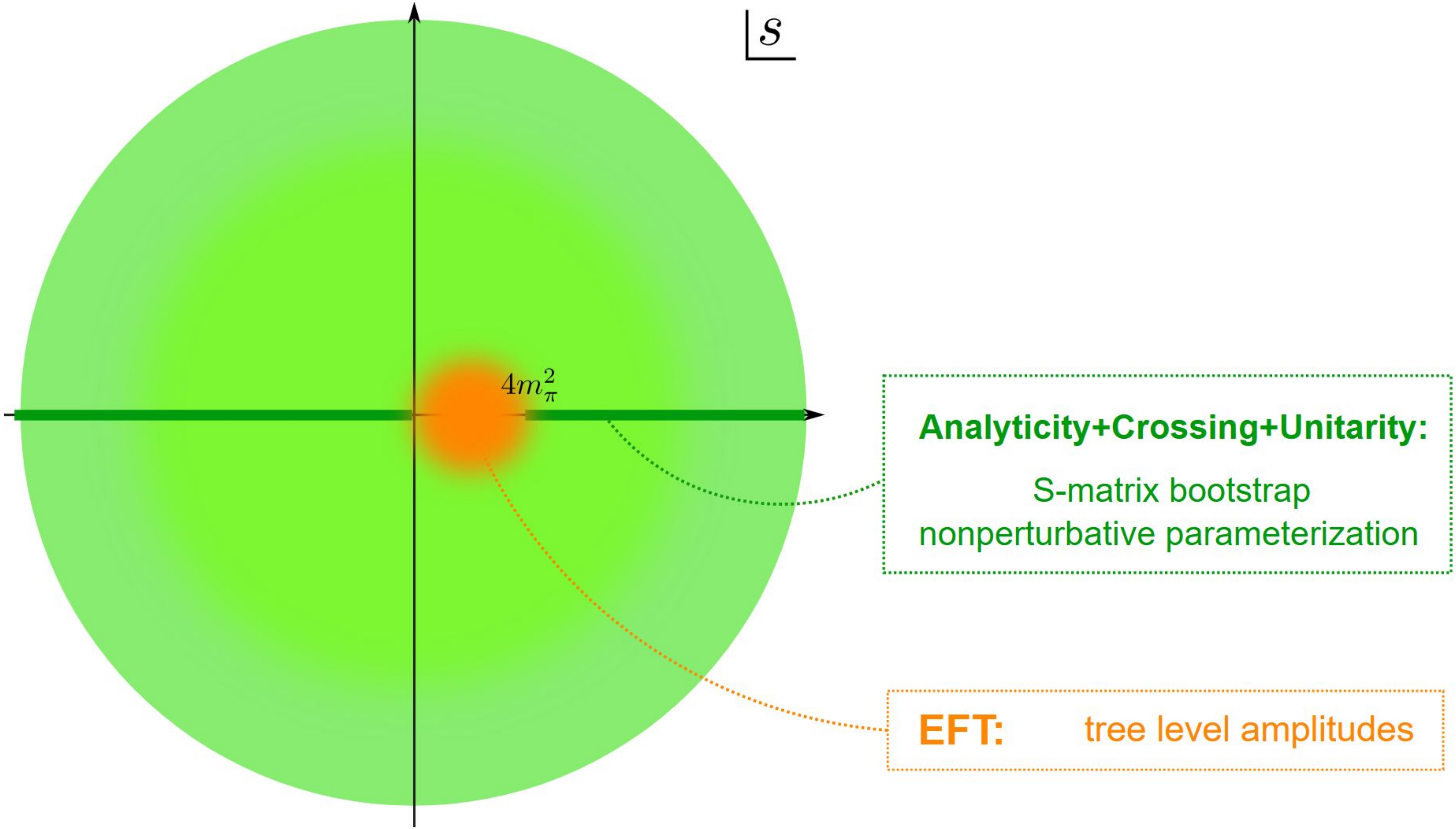
bootstrap maximization: non-perturbative numerical computation



plot a two-dimensional projection
of the space of amplitudes under
SU(2) symmetry, analyticity, crossing, unitarity



each boundary point: an extremal numerical amplitude



s
$4m_\pi^2$
Analyticity+Crossing+Unitarity:
S-matrix bootstrap
nonperturbative parameterization
EFT:
tree level amplitudes

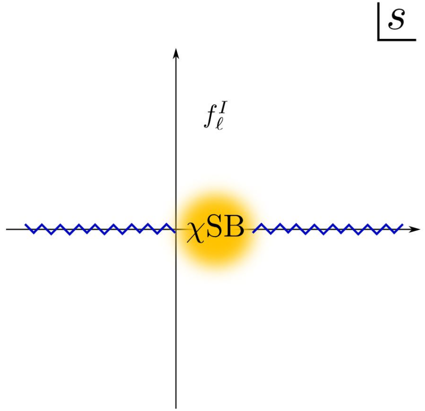
Low energy matching with EFT

interaction: $$\mathcal{L}_2^{4\pi} = \frac{1}{6f_\pi^2}\Big((\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})\Big) + \frac{m_\pi^2}{24f_\pi^2}(\vec{\pi}^2)^2$$

tree-level amplitude: $$A_{\rm tree}(s,t,u) = \frac{4}{\pi}\frac{s-m_\pi^2}{32\pi f_\pi^2}$$ [Weinberg, 1966]

S0: $f^0_{0,\rm tree}(s) = \frac{2}{\pi}\frac{2s-m_\pi^2}{32\pi f_\pi^2}$ P1: $f^1_{1,\rm tree}(s) = \frac{2}{\pi}\frac{s-4m_\pi^2}{96\pi f_\pi^2}$ S2: $f^2_{0,\rm tree}(s) = \frac{2}{\pi}\frac{2m_\pi^2-s}{32\pi f_\pi^2}$

good in the unphysical region (very low energy) $0 < s < 4m_\pi^2$



Low energy matching with EFT

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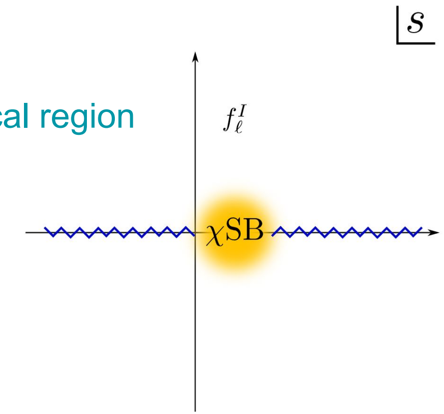
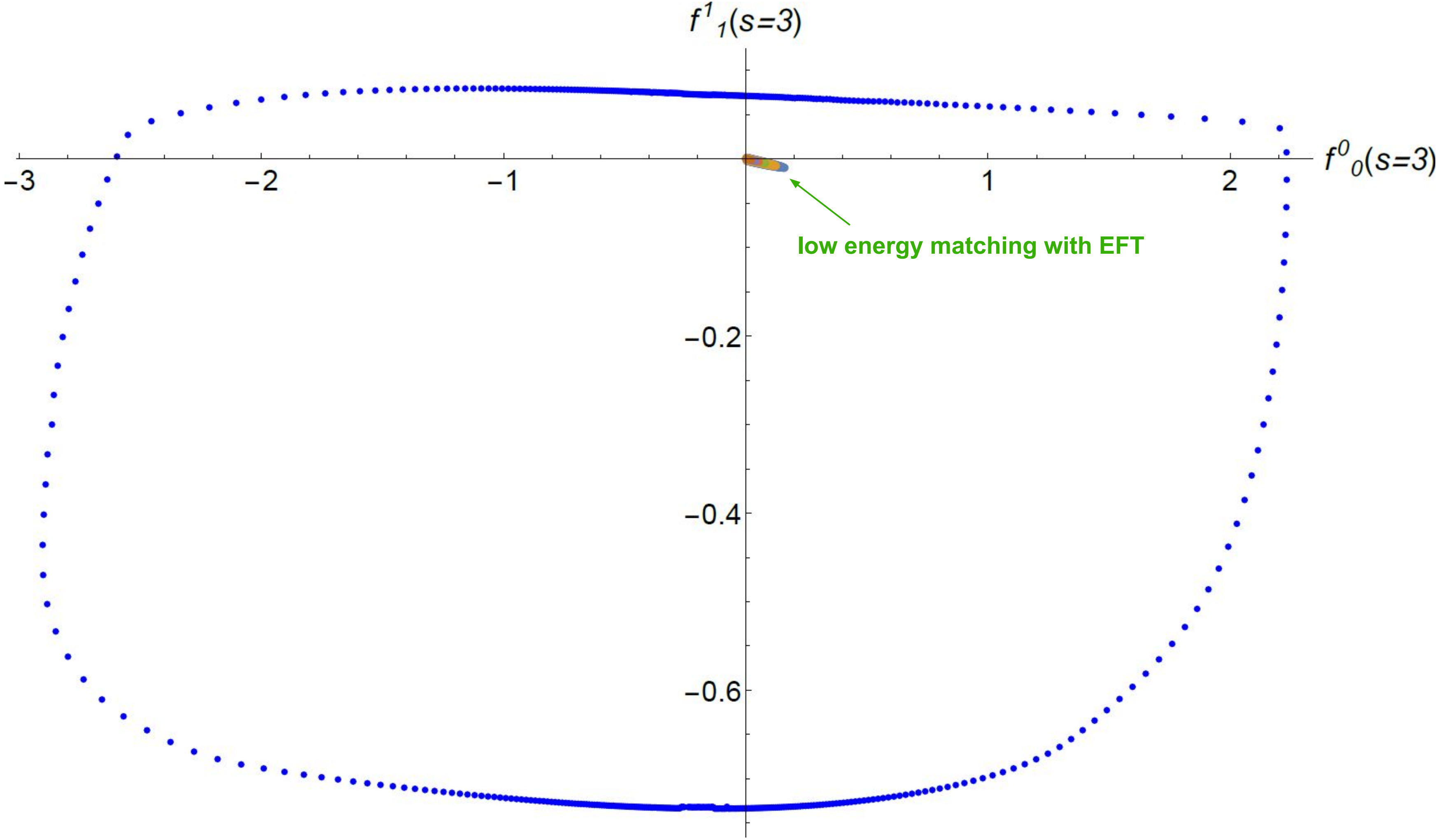
good in the unphysical region (very low energy) $0 < s < 4m_\pi^2$

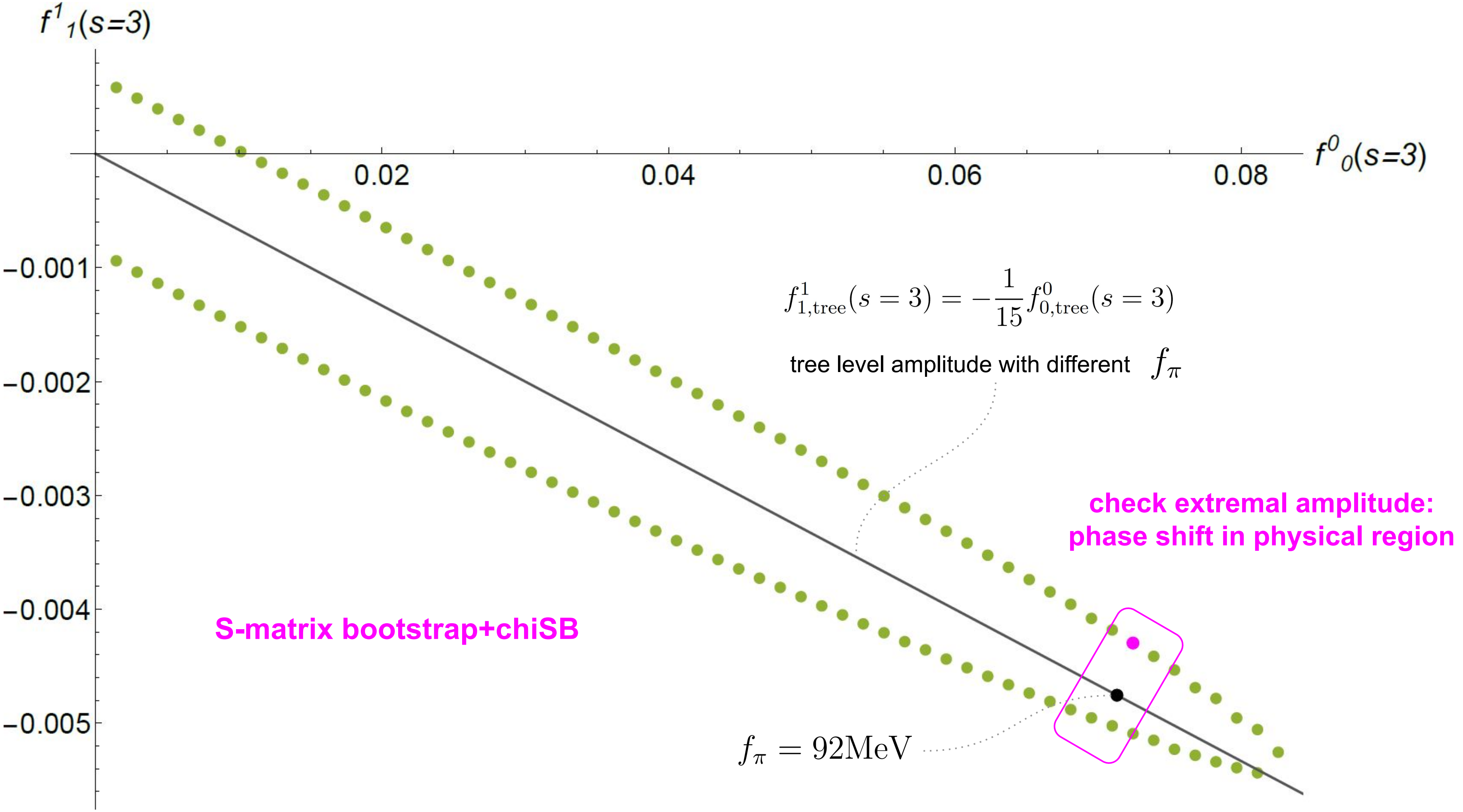
numerically: requires pw in bootstrap match tree level pw in the very low energy unphysical region

$$f_0^0(s) \simeq f^0_{0,\text{tree}}(s) \qquad f_1^1(s) \simeq f^1_{1,\text{tree}}(s) \qquad f_0^2(s) \simeq f^2_{0,\text{tree}}(s) \qquad 0 < s < 4m_\pi^2$$

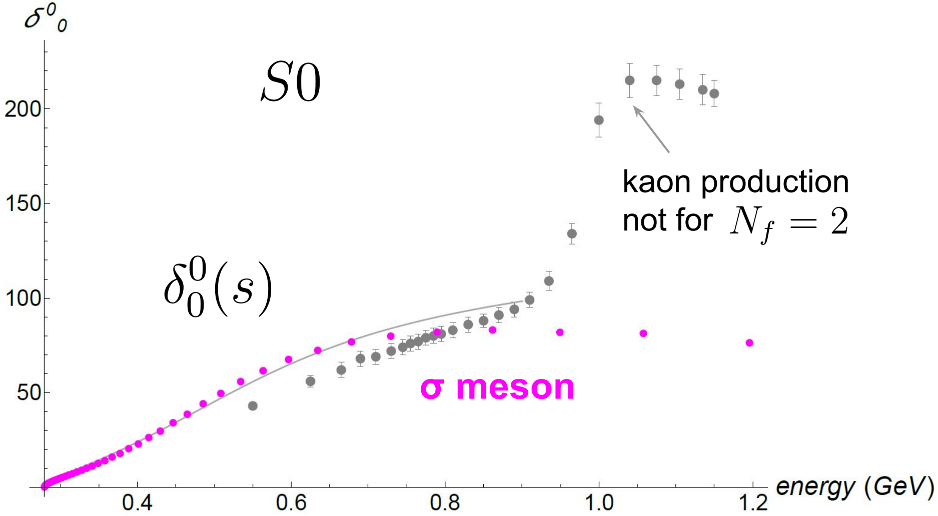


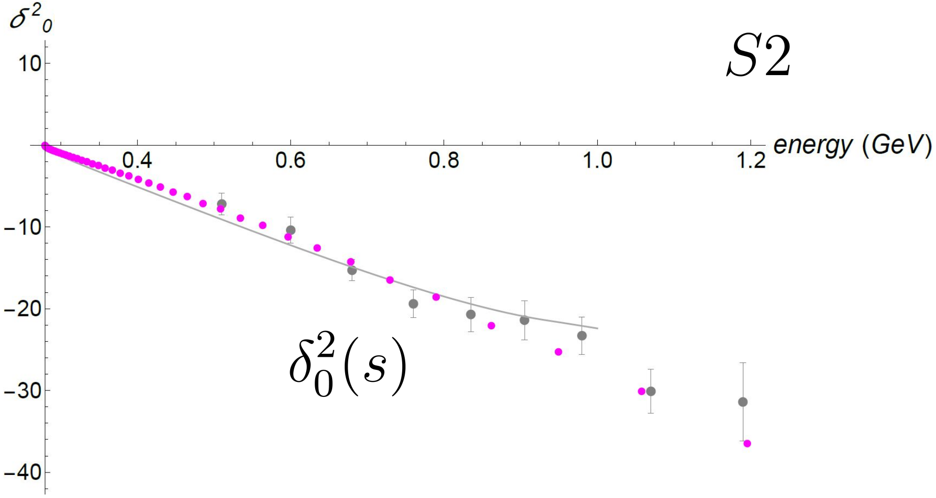


f¹₁(s=3)
f⁰₀(s=3)
−3
−2
−1
1
2
−0.2
−0.4
−0.6
low energy matching with EFT



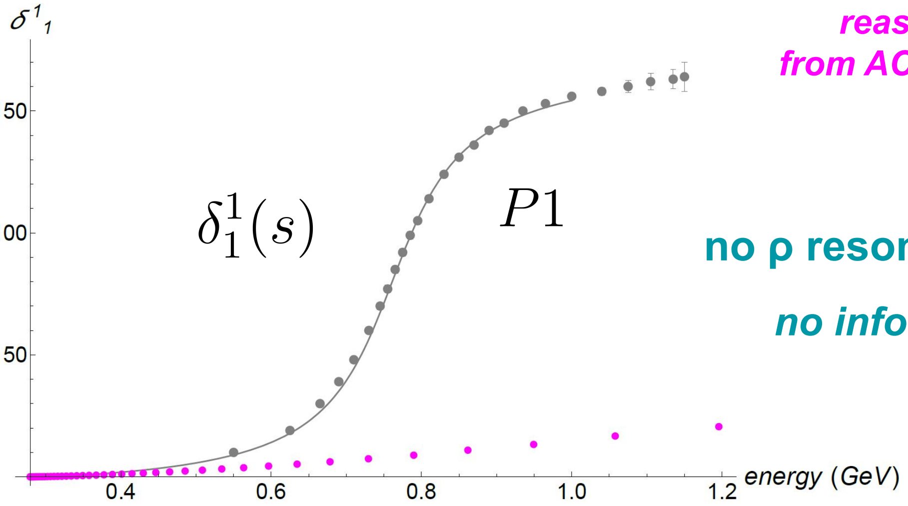
$f^1{}_1(s=3)$
$f^0{}_0(s=3)$
0.02
0.04
0.06
0.08
−0.001
−0.002
−0.003
−0.004
−0.005
$f^1_{1,\text{tree}}(s=3) = -\frac{1}{15} f^0_{0,\text{tree}}(s=3)$
tree level amplitude with different f_π
check extremal amplitude:
phase shift in physical region
S-matrix bootstrap+chiSB
$f_\pi = 92\text{MeV}$





bootstrap numerical computation of phase shifts

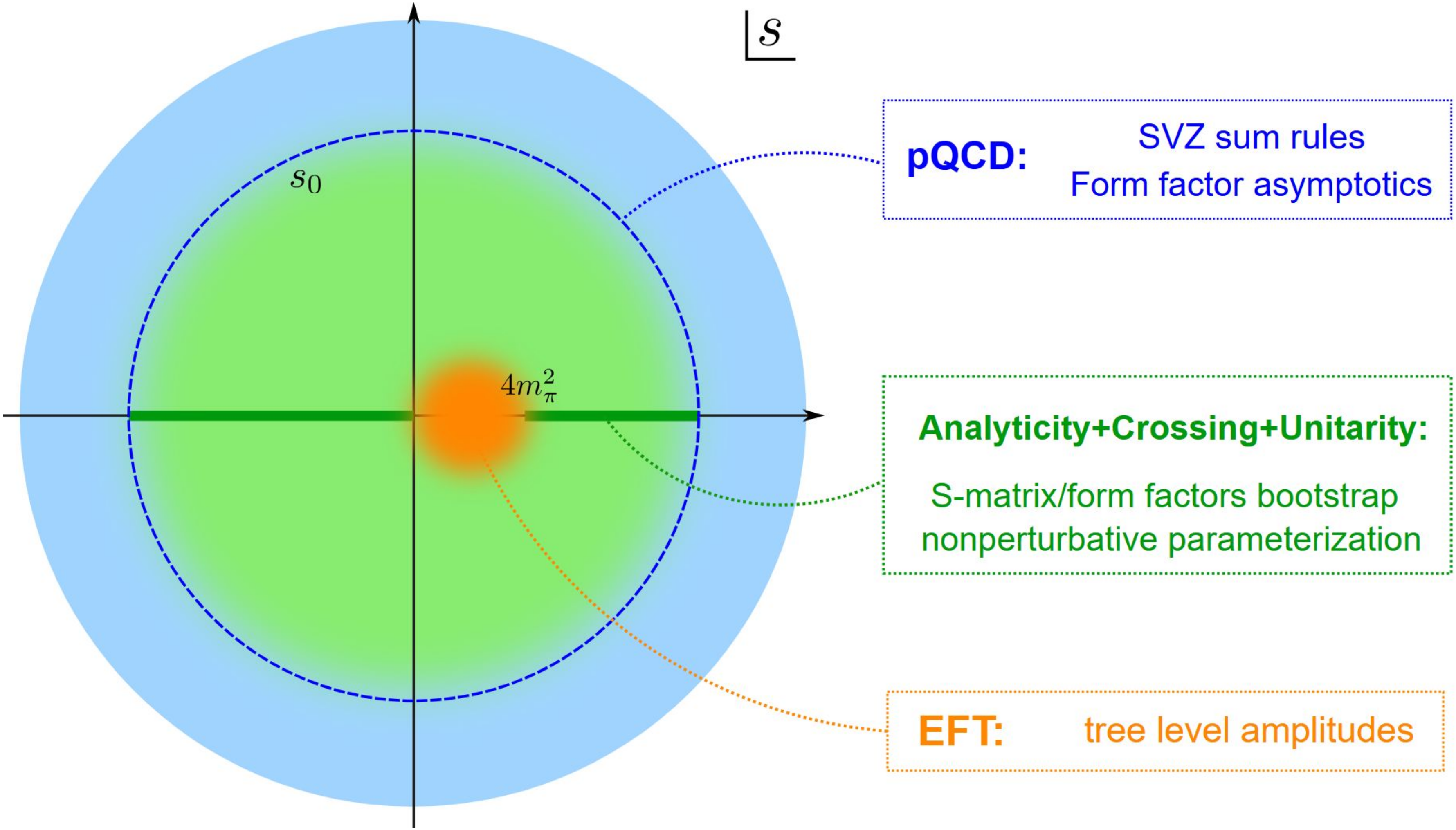
reasonable S0, S2 waves from ACU + weakly coupled EFT



experimental data (gray dots)
[Protopopescu et al, 1973]
[Losty et al, 1974]
pheno fit (gray line)
[Pelaez, Yndurain, 2005]

no ρ resonance without UV info

no information on QCD yet



s
s_0
$4m_\pi^2$
pQCD:
SVZ sum rules
Form factor asymptotics
Analyticity+Crossing+Unitarity:
S-matrix/form factors bootstrap
nonperturbative parameterization
EFT:
tree level amplitudes

S-matrix/form factor bootstrap

[Karateev, Kuhn, Penedones, 2019]

$$|\psi_1\rangle = |p_1, p_2\rangle_{in}\,, \qquad |\psi_2\rangle = |p_1, p_2\rangle_{out}\,, \qquad |\psi_3\rangle = \int dx e^{-i(p_1+p_2)\cdot x}\mathcal{O}(x)|0\rangle$$

positive semidefinite matrix $\langle\psi_a|\psi_b\rangle = \begin{pmatrix} 1 & S & \mathcal{F} \\ S^* & 1 & \mathcal{F}^* \\ \mathcal{F}^* & \mathcal{F} & \rho \end{pmatrix} \succeq 0$

S-matrix/form factor bootstrap

[Karateev, Kuhn, Penedones, 2019]

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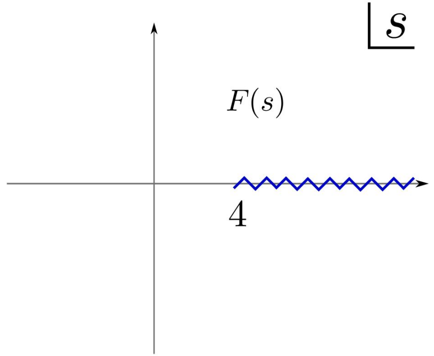
2-particle form factor: $_{\rm out}\langle p_1, p_2|\mathcal{O}(0)|0\rangle = F(s)$

analytic function of s

spectral density: $\int \frac{d^4x}{(2\pi)^4} e^{iPx}\langle 0|\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \rho(s)$

supported at $s > 4$



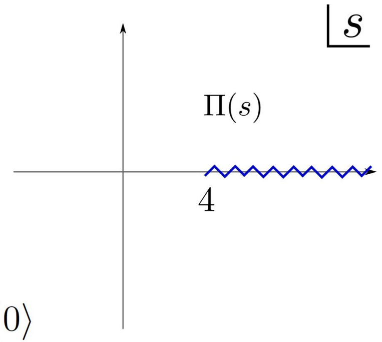
$$F(s) = \frac{1}{\pi}\int_4^\infty dx \frac{\mathrm{Im}F(x)}{x-s}$$ +subtractions

extended bootstrap variables: $\{\rho_{1,2}(x,y), \ldots, \mathrm{Im}F(x), \rho(x)\}$

Gauge theory current correlators

$$\begin{array}{c} \\ \langle \mathrm{in}|_{P',I,\ell} \\ \langle \mathrm{out}|_{P',I,\ell} \\ \langle 0|\mathcal{O}^\dagger_{P',I,\ell} \end{array} \begin{array}{c} \begin{array}{ccc} |\mathrm{in}\rangle_{P,I,\ell} & |\mathrm{out}\rangle_{P,I,\ell} & \mathcal{O}_{P,I,\ell}|0\rangle \end{array} \\ \begin{pmatrix} 1 & S^I_\ell(s) & \mathcal{F}^I_\ell \\ S^{I*}_\ell(s) & 1 & \mathcal{F}^{I*}_\ell \\ \mathcal{F}^{I*}_\ell & \mathcal{F}^I_\ell & \rho^I_\ell(s) \end{pmatrix} \end{array} \succeq 0 \quad s > 4 \quad \forall \ell, I$$

$$\rho^I_\ell(s) = 2\,\mathrm{Im}\Pi^I_\ell(x + i\epsilon)$$



construct operators from gauge theory with desired quantum numbers

e.g. vector (electromagnetic) current

$$P1 \quad : \quad j^\mu_V(x) = \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)$$

$$\Pi^1_1(s) \quad = \quad i \int \frac{d^4x}{(2\pi)^4} e^{iPx} \langle 0|\hat{T}\left\{ j^\dagger_\sigma(x) j_\sigma(0) \right\}|0\rangle$$

Gauge theory current correlators & SVZ expansion

$$\begin{array}{c} \\ \langle \text{in}|_{P',I,\ell} \\ \langle \text{out}|_{P',I,\ell} \\ \langle 0|\mathcal{O}^\dagger_{P',I,\ell} \end{array} \begin{array}{c} |\text{in}\rangle_{P,I,\ell} \quad |\text{out}\rangle_{P,I,\ell} \quad \mathcal{O}_{P,I,\ell}|0\rangle \\ \begin{pmatrix} 1 & S^I_\ell(s) & \mathcal{F}^I_\ell \\ S^{I*}_\ell(s) & 1 & \mathcal{F}^{I*}_\ell \\ \mathcal{F}^{I*}_\ell & \mathcal{F}^I_\ell & \rho^I_\ell(s) \end{pmatrix} \end{array} \succeq 0 \quad s > 4 \quad \forall \ell, I$$

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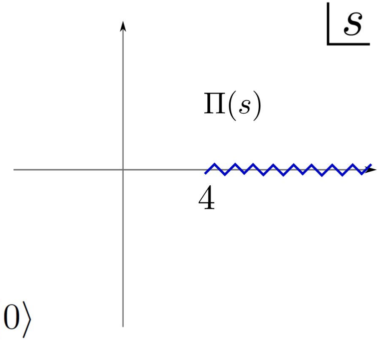

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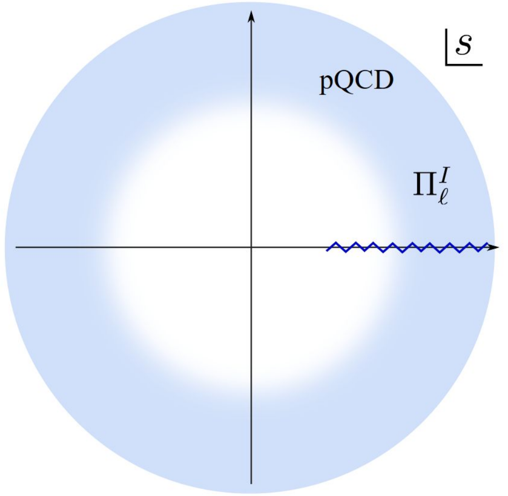
large spacelike momenta — asymptotic free region with pQCD computation

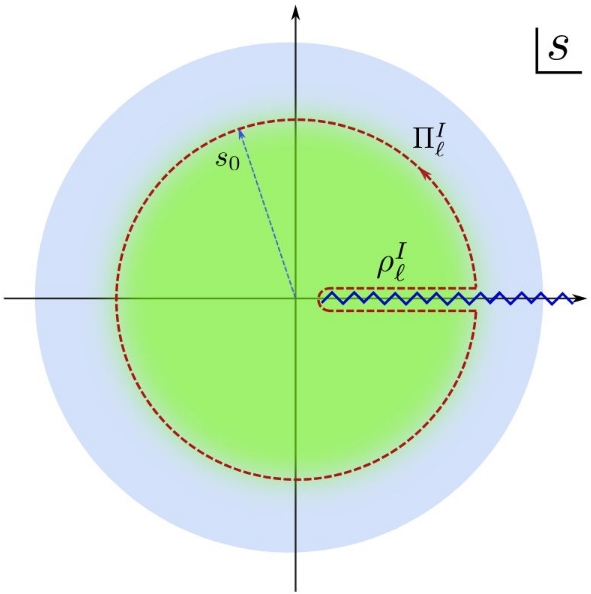
$$\Pi^1_1(s) = \frac{1}{2}\frac{1}{(2\pi)^4}\left\{ -\frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) s \ln(-\frac{s}{\mu^2}) + \dots \right\}$$

pQCD computation

[Shifman, Vainshtein, Zakharov, 1979]



Finite energy sum rule



connect pQCD with bootstrap at s_0

contour integral $s^n \Pi(s)$ vanishes

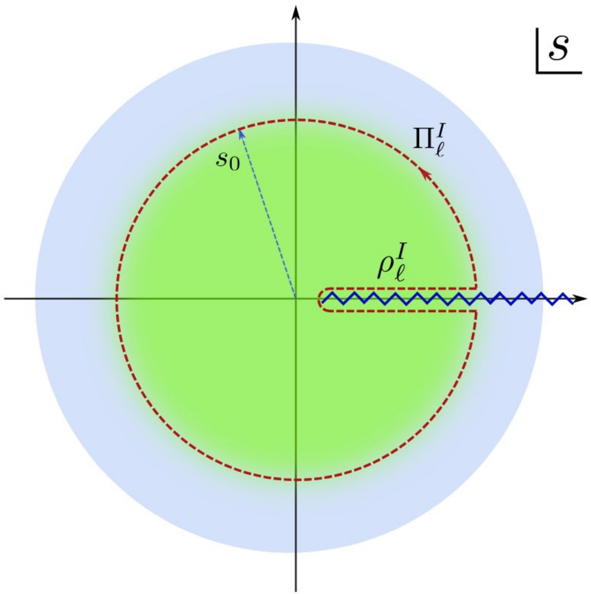
SVZ

$$\int_4^{s_0} \rho(x) x^n dx = -s_0^{n+1} \int_0^{2\pi} e^{i(n+1)\varphi} \Pi(s_0 e^{i\varphi}) d\varphi$$

bootstrap variables ← **gauge theory information**

linear constraints

Finite energy sum rule



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bootstrap variables ← **gauge theory information**

linear constraints

e.g. vector (electromagnetic) current

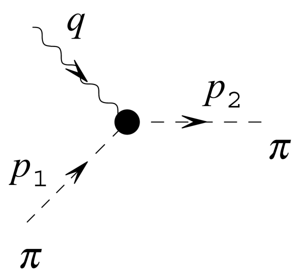
$$P1 \quad : \quad \frac{1}{s_0^{n+2}}\int_4^{s_0}\rho_1^1(x)x^n dx = \frac{1}{2(2\pi)^4}\left\{\frac{1}{2\pi(n+2)}\left(1+\frac{\alpha_s}{\pi}\right)+\dots\right\}, \quad n \geq -1$$

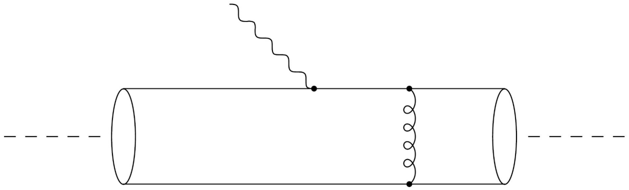
Asymptotic behavior of form factor from pQCD

perturbative QCD also controls asymptotic behavior of form factors

e.g. electromagnetic FF $\langle \pi(p_2) | J^{\mu}_{\mathrm{em}}(0) | \pi(p_1) \rangle = (p_1^{\mu} + p_2^{\mu}) F_{\pi}(q^2)$

$q = (p_2 - p_1)$





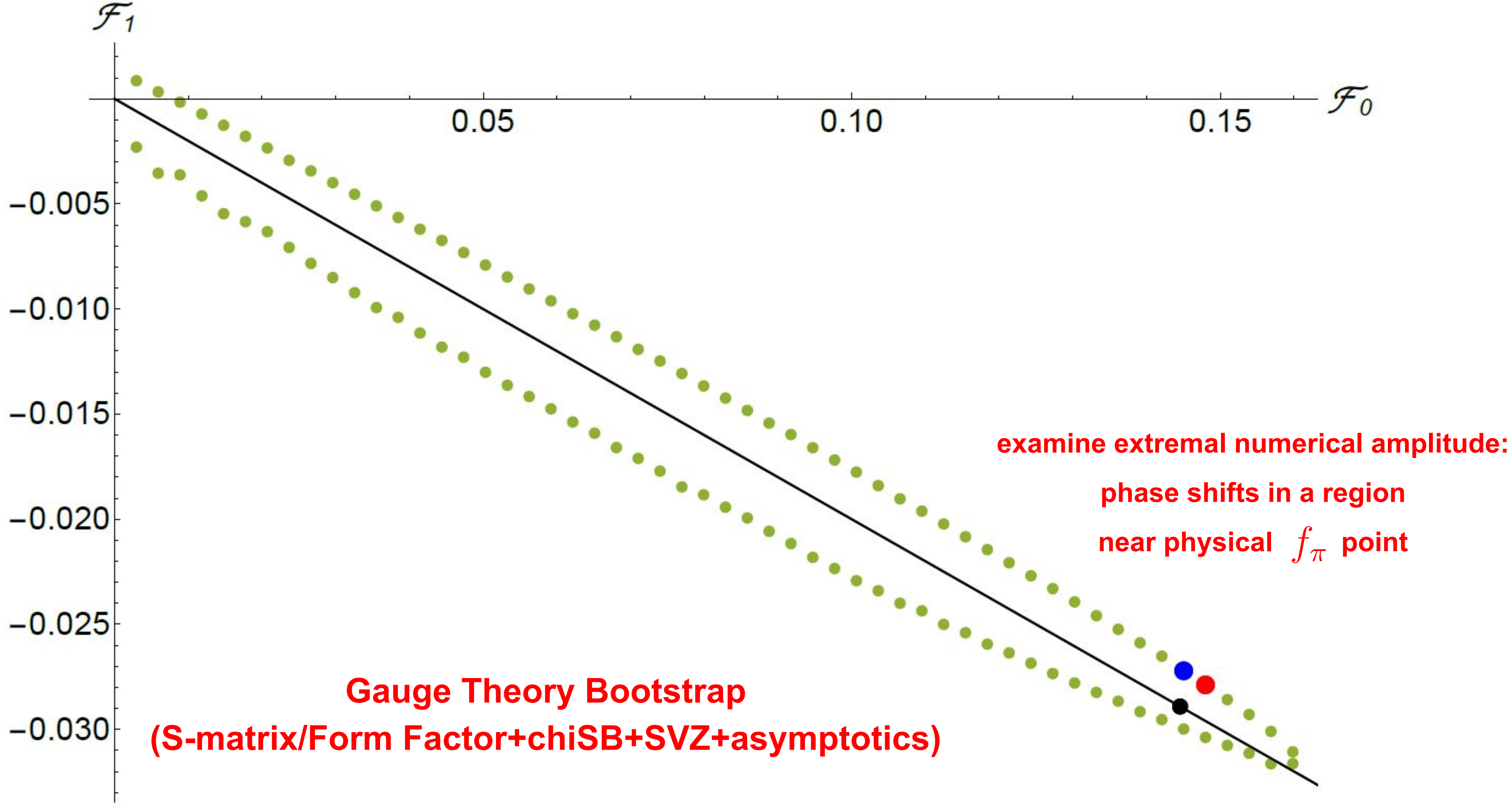
$$F_{\pi}(s) \simeq -\frac{16\pi\alpha_s(s) f_{\pi}^2}{s}$$

[Lepage, Brodsky, 1979]

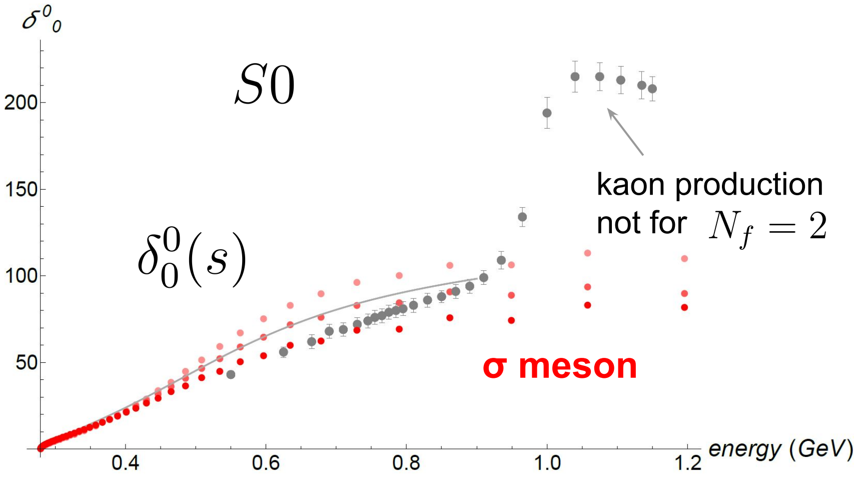
evaluate to estimate

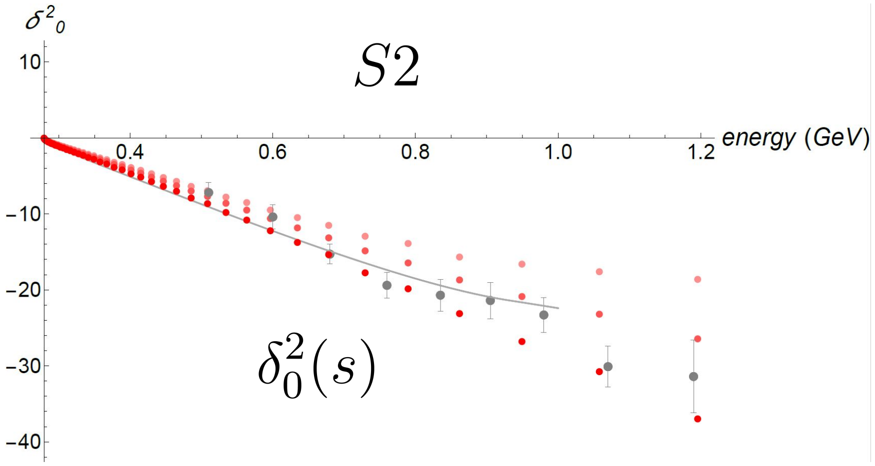
in practical numerical implementation
suffices to require smallness above $s = s_0$

$$||\mathcal{F}(s > s_0)|| \lesssim \epsilon$$



$\mathcal{F}_1$
$\mathcal{F}_0$
0.05
0.10
0.15
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030
examine extremal numerical amplitude:
phase shifts in a region
near physical f_π point
Gauge Theory Bootstrap
(S-matrix/Form Factor+chiSB+SVZ+asymptotics)

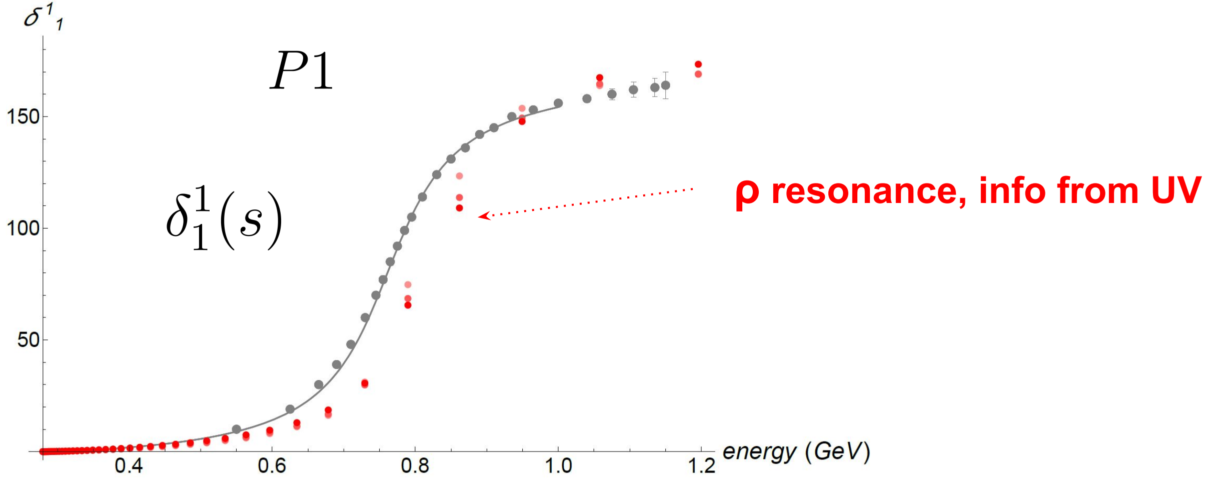




Gauge Theory Bootstrap

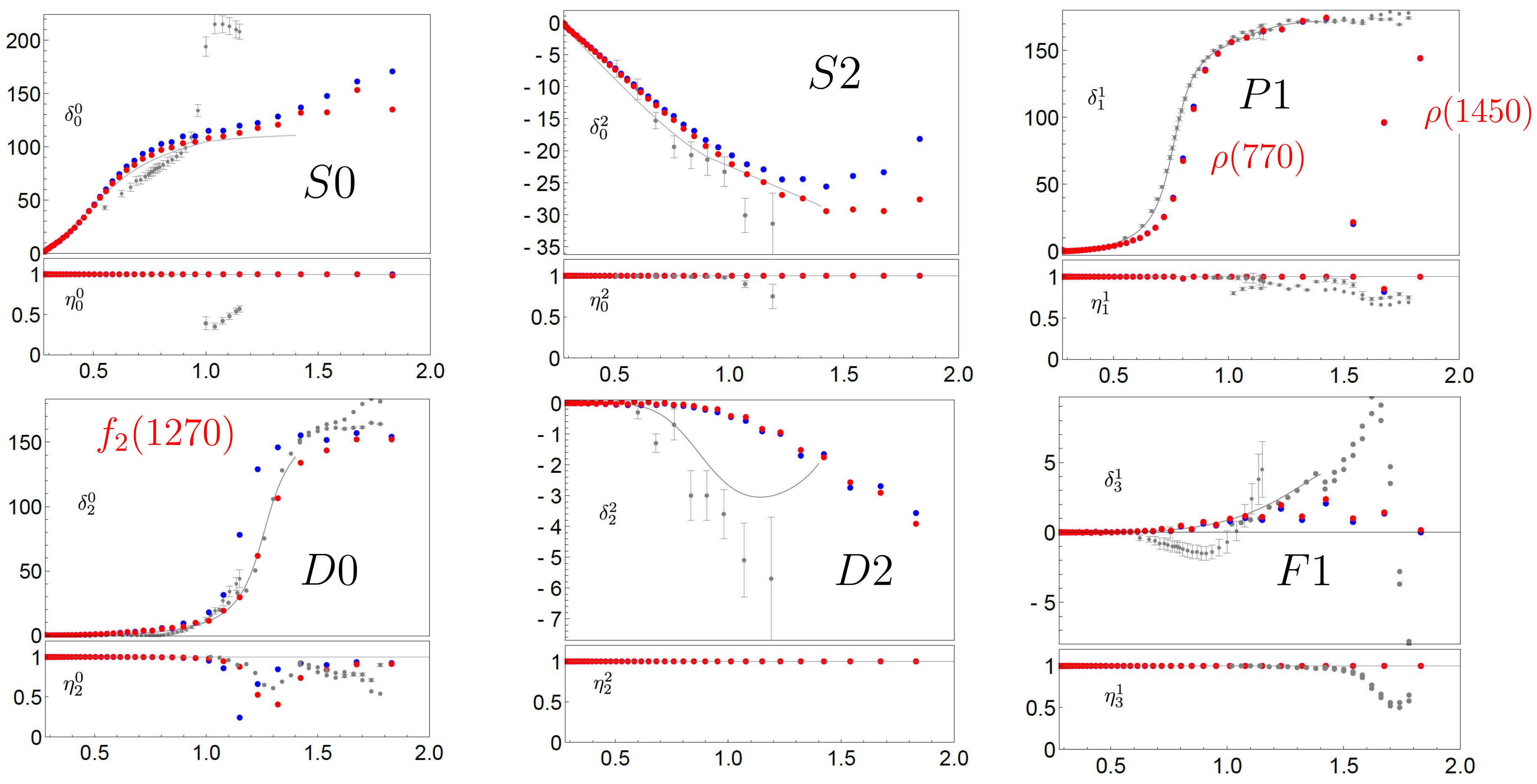
phase shifts up to 1.2 GeV

experimental data (gray dots)
[Protopopescu et al, 1973]
[Losty et al, 1974]
pheno fit (gray line)
[Pelaez, Yndurain, 2005]



Gauge Theory Bootstrap

phase shifts up to 2 GeV



Low energy parameters: threshold expansions

scattering lengths and effective range parameters

$$\mathrm{Re} f_\ell^I(s) \overset{k\to 0}{\simeq} \frac{2m_\pi}{\pi} k^{2\ell}\left(a_\ell^I + b_\ell^I k^2 + \dots\right) \qquad k = \frac{\sqrt{s-4m_\pi^2}}{2}$$

	W	**GTB**	CGL	PY
$a_0^{(0)}$	0.16	0.178, 0.182	0.220 ± 0.005	0.230 ± 0.010
$a_0^{(2)}$	-0.046	-0.0369, -0.0378	-0.0444 ± 0.0010	-0.0422 ± 0.0022
$b_0^{(0)}$	0.18	0.287, 0.290	0.280 ± 0.001	0.268 ± 0.010
$b_0^{(2)}$	-0.092	-0.064, -0.066	-0.080 ± 0.001	-0.071 ± 0.004
$a_1^{(1)}$	31	28.0, 28.4	37.0 ± 0.13	38.1 ± 1.4 $(\times 10^{-3})$
$b_1^{(1)}$	0	2.86, 3.37	5.67 ± 0.13	4.75 ± 0.16 $(\times 10^{-3})$
$a_2^{(0)}$	0	12.6, 12.3	17.5 ± 0.3	18.0 ± 0.2 $(\times 10^{-4})$
$a_2^{(2)}$	0	2.87, 2.81	1.70 ± 0.13	2.2 ± 0.2 $(\times 10^{-4})$

pion charge radii

$$F_0^0(s) = F_0^0(0)\left[1 + \frac{1}{6} s \langle r^2\rangle_S^\pi + \dots\right]$$

$$F_1^1(s) = 1 + \frac{1}{6} s \langle r^2\rangle_V^\pi + \dots$$

	GTB	Exp. fits
$\langle r^2\rangle_S^\pi$	0.64, 0.61	$0.61 \pm 0.04\,\mathrm{fm}^2$
$\langle r^2\rangle_V^\pi$	0.388, 0.381	$0.439 \pm 0.008\,\mathrm{fm}^2$

Low energy parameters: chiral Lagrangian coefficients

calculate the chiral Lagrangian coefficients

$$a_{D0} = \frac{1}{1440\pi^3 f_\pi^4}\left\{\bar{l}_1 + 4\bar{l}_2 - \frac{53}{8}\right\} + \ldots$$

$$a_{D2} = \frac{1}{1440\pi^3 f_\pi^4}\left\{\bar{l}_1 + \bar{l}_2 - \frac{103}{40}\right\} + \ldots$$

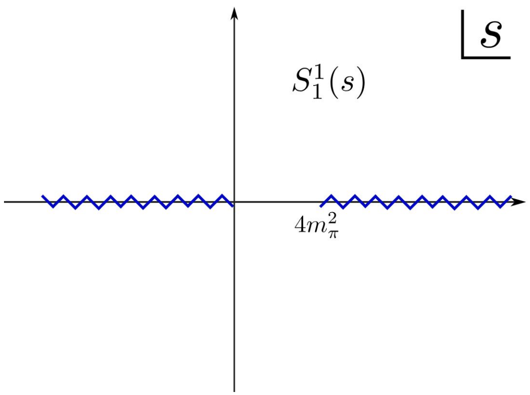
$$F_0(s) = 1 + \frac{s}{16\pi^2 f_\pi^2}\left(\bar{l}_4 - \frac{13}{12}\right) + \ldots$$

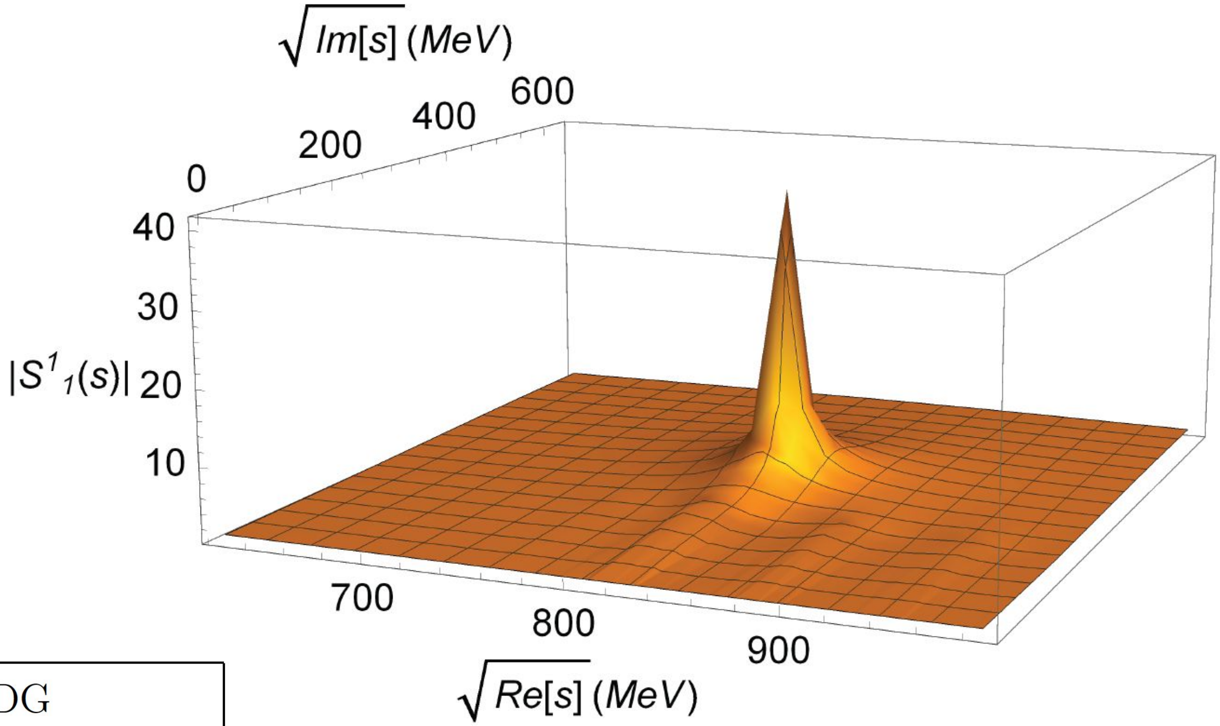
$$F_1(s) = 1 + \frac{s}{96\pi^2 f_\pi^2}(\bar{l}_6 - 1) + \ldots$$

[Gasser Leutwyler, 1984]

	GTB	GL	Bij	CGL
$\bar{l}_1$	0.92, 0.93	-2.3 ± 3.7	-1.7 ± 1.0	-0.4 ± 0.6
$\bar{l}_2$	4.1, 4.0	6.0 ± 1.3	6.1 ± 0.5	4.3 ± 0.1
$\bar{l}_4$	4.7, 4.6	4.3 ± 0.9	4.4 ± 0.3	4.4 ± 0.2
$\bar{l}_6$	14.3, 14.1	16.5 ± 1.1	$16.0 \pm 0.5 \pm 0.7$	

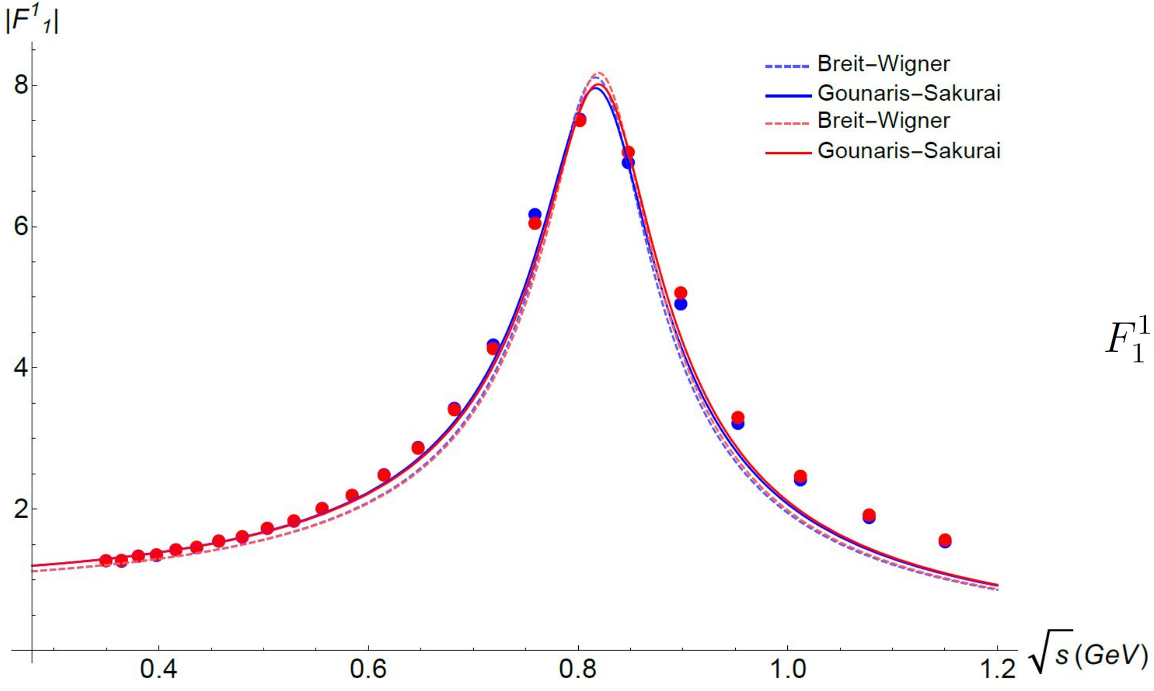
ρ(770) meson as pole on the second sheet of $S_1^1(s)$





	GTB	PDG
$\mathrm{Re}(\sqrt{s_\rho})$	829, 832	$761 - 765 \pm 0.23$ MeV
$\mathrm{Im}(\sqrt{s_\rho})$	63, 64	$71 - 74 \pm 0.8$ MeV

Vector (electromagnetic) form factor and *ρ(770)* meson



Breit-Wigner form

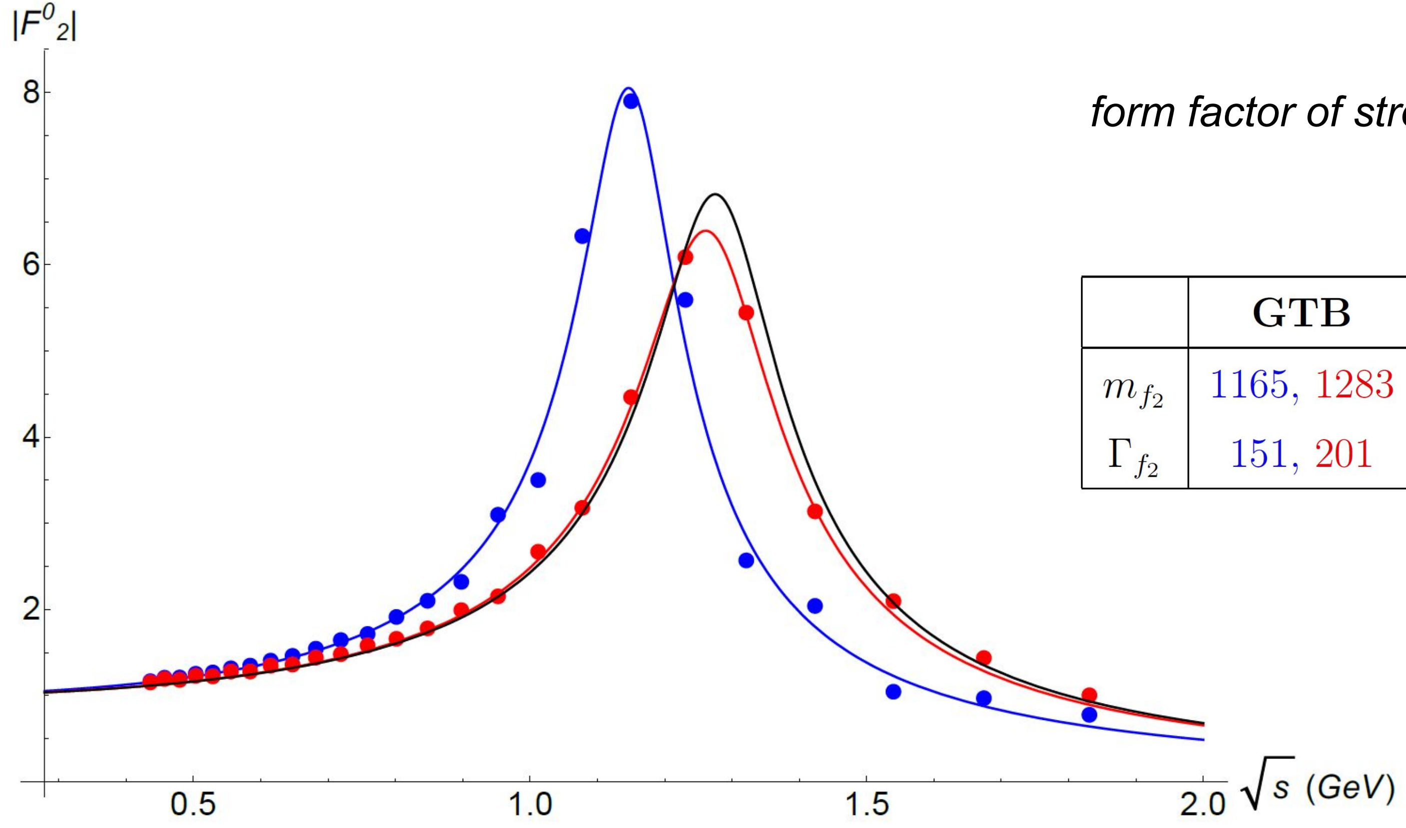
$$F_1^1(s) = -\frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho \theta(s - 4m_\pi^2)}$$

	GTB	PDG
m_ρ	836, 839	775 ± 0.23 MeV
Γ_ρ	111, 111	149.1 ± 0.8 MeV

couplings $\Gamma_\rho = g_{\rho\pi\pi}^2 \frac{m_\rho}{48\pi}\left[1 - \frac{4m_\pi^2}{m_\rho^2}\right]^{\frac{3}{2}}$

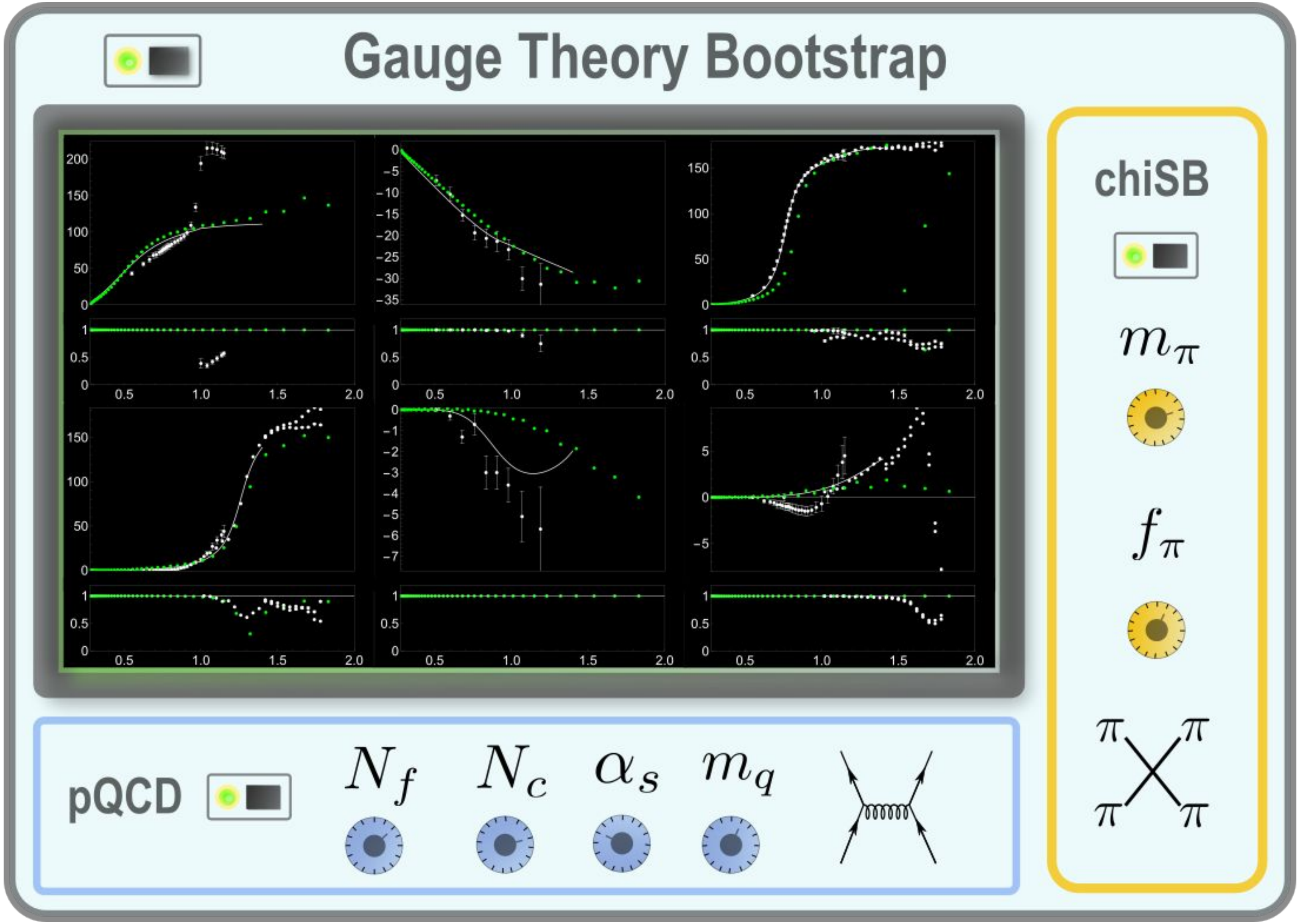
$$g_{\rho\pi\pi} = 4.9, \quad 4.9$$
$$g_{\rho\pi\pi} = 6$$

Gravitational form factor and f_2 meson



form factor of stress energy tensor

	GTB	PDG
m_{f_2}	1165, 1283	1275.4 ± 0.6 MeV
Γ_{f_2}	151, 201	186.6 ± 2.3 MeV



- theoretical/numerical computation
- good track for solving QCD (gauge theories)
- improvements to make: more precise and robust
- fast: 20min on average laptop

Ancillary files (details):

- GTB_numerics.m
- GTB_numerics.nb

Thank you !