# Proton scalar and spin polarisabilities from Compton scattering data

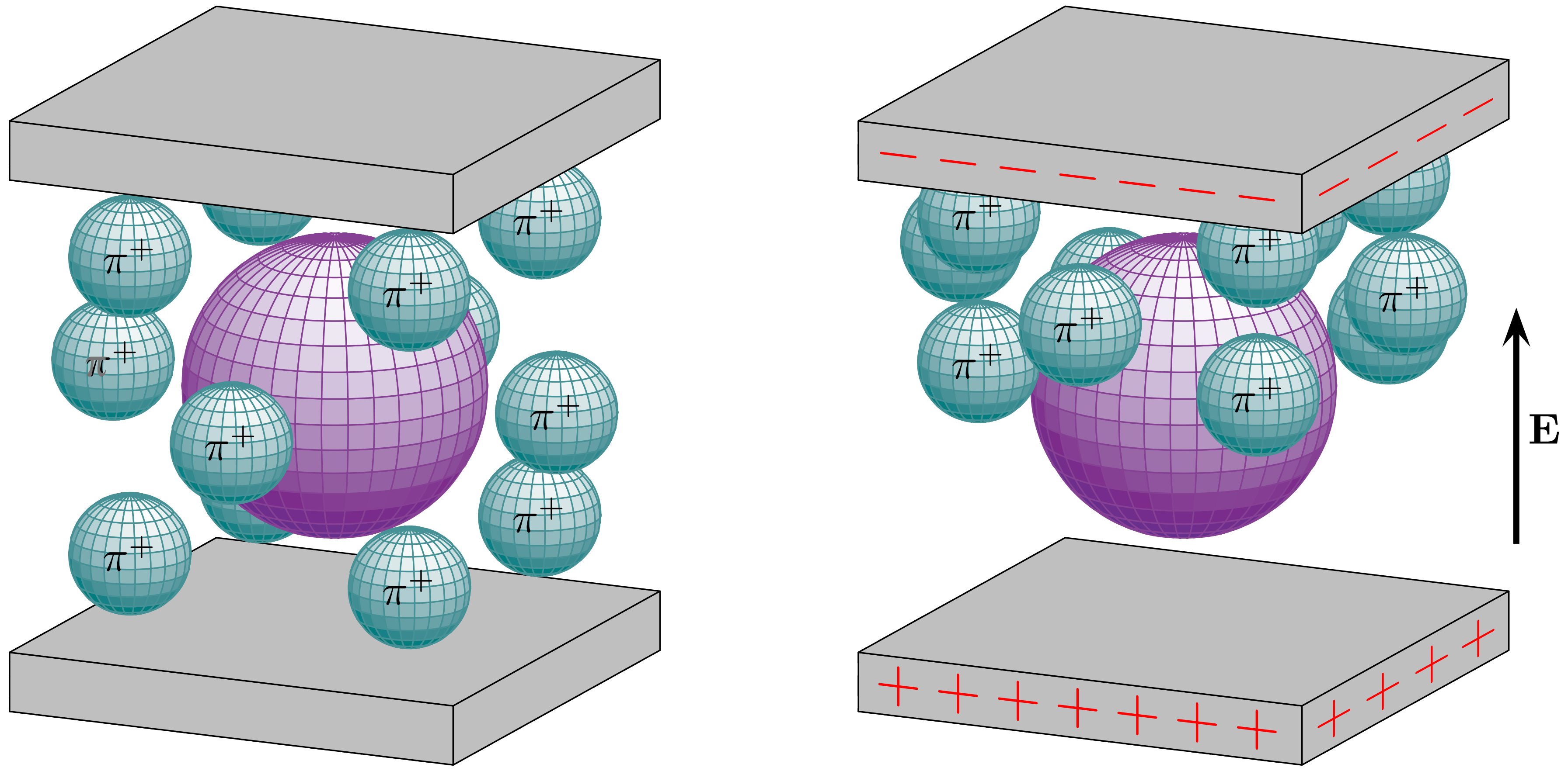
Timon Esser in collaboration with
Franziska Hagelstein, Vadim Lensky
and Vladimir Pascalutsa (JGU)

Chiral Dynamics 2024, Ruhr-Universität Bochum

26.08.2024

# Electromagnetic Polarizabilities

## Proton is 1000 times “stiffer” than naïve expectation




Electric dipole polarizability:

$$\vec{P} = \alpha_{E1}\,\vec{E}$$

induced electric dipole polarization (linear dielectric)

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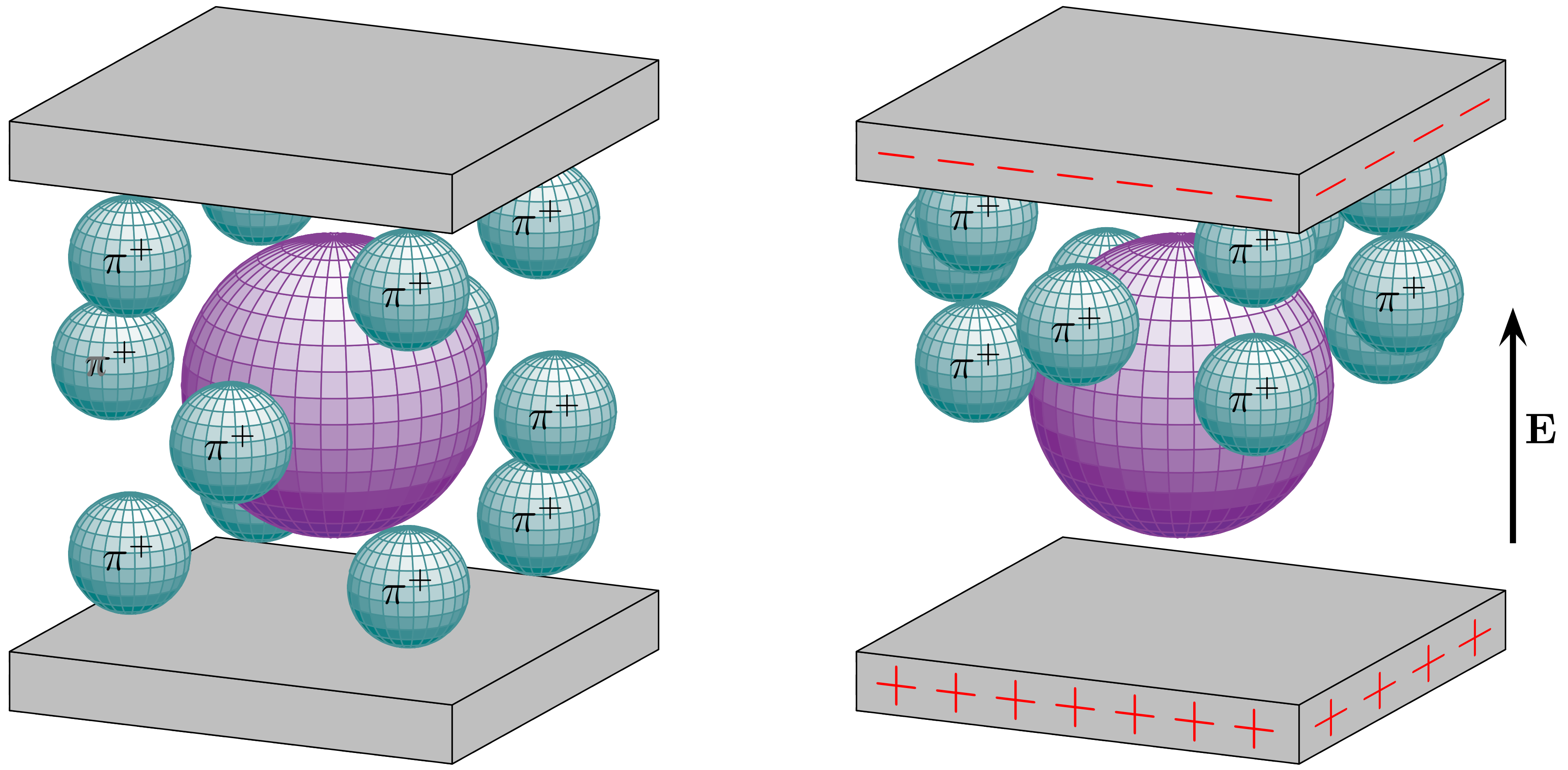


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quantum atom: $\alpha_{E1} \sim 5V$
proton: $\alpha_{E1} \sim 10^{-3}\,\mathrm{fm}^3$

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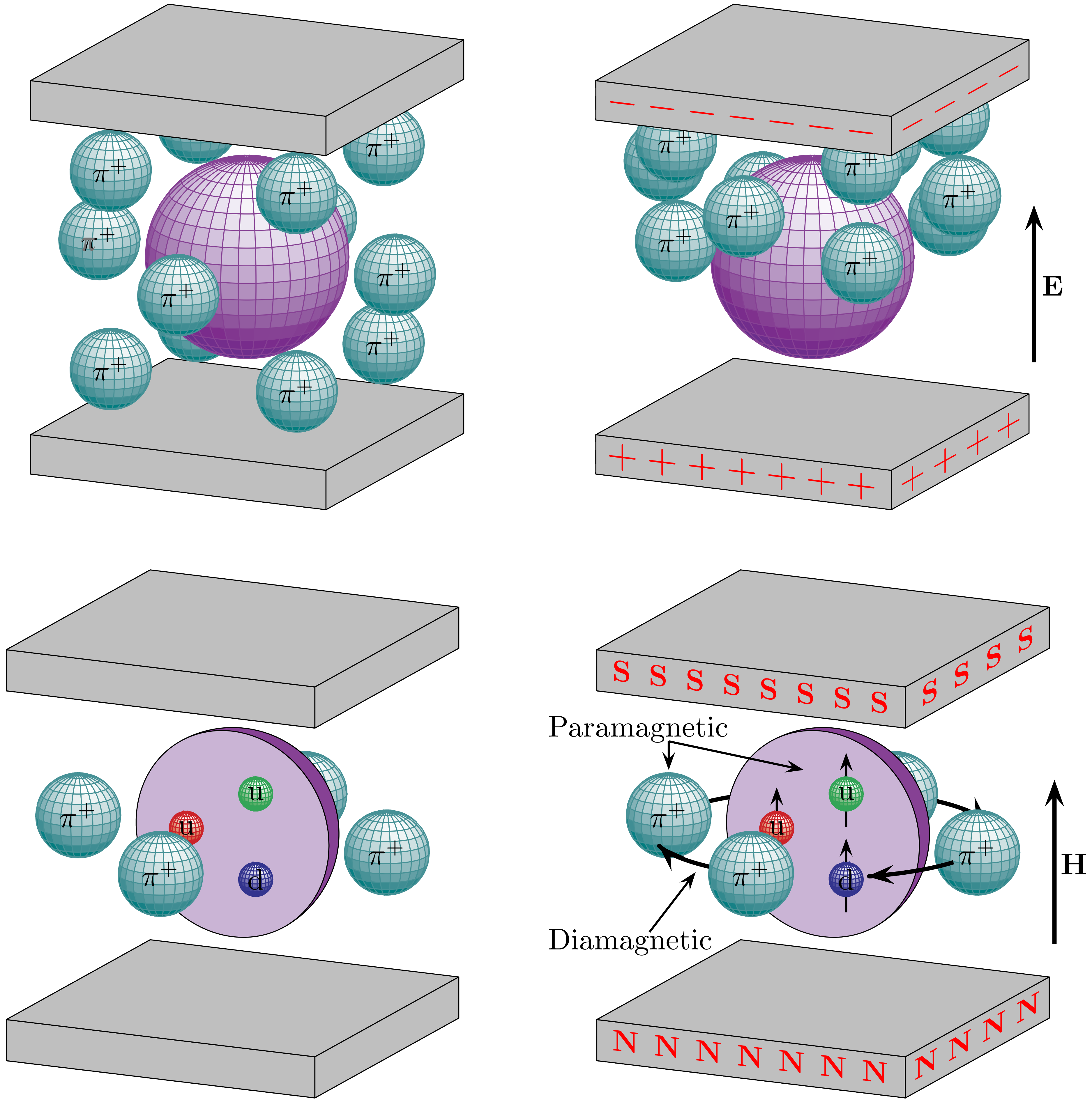


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Magnetic dipole polarizability:

$$\vec{P} = \beta_{M1}\vec{H}$$

for polarization induced by magnetic field

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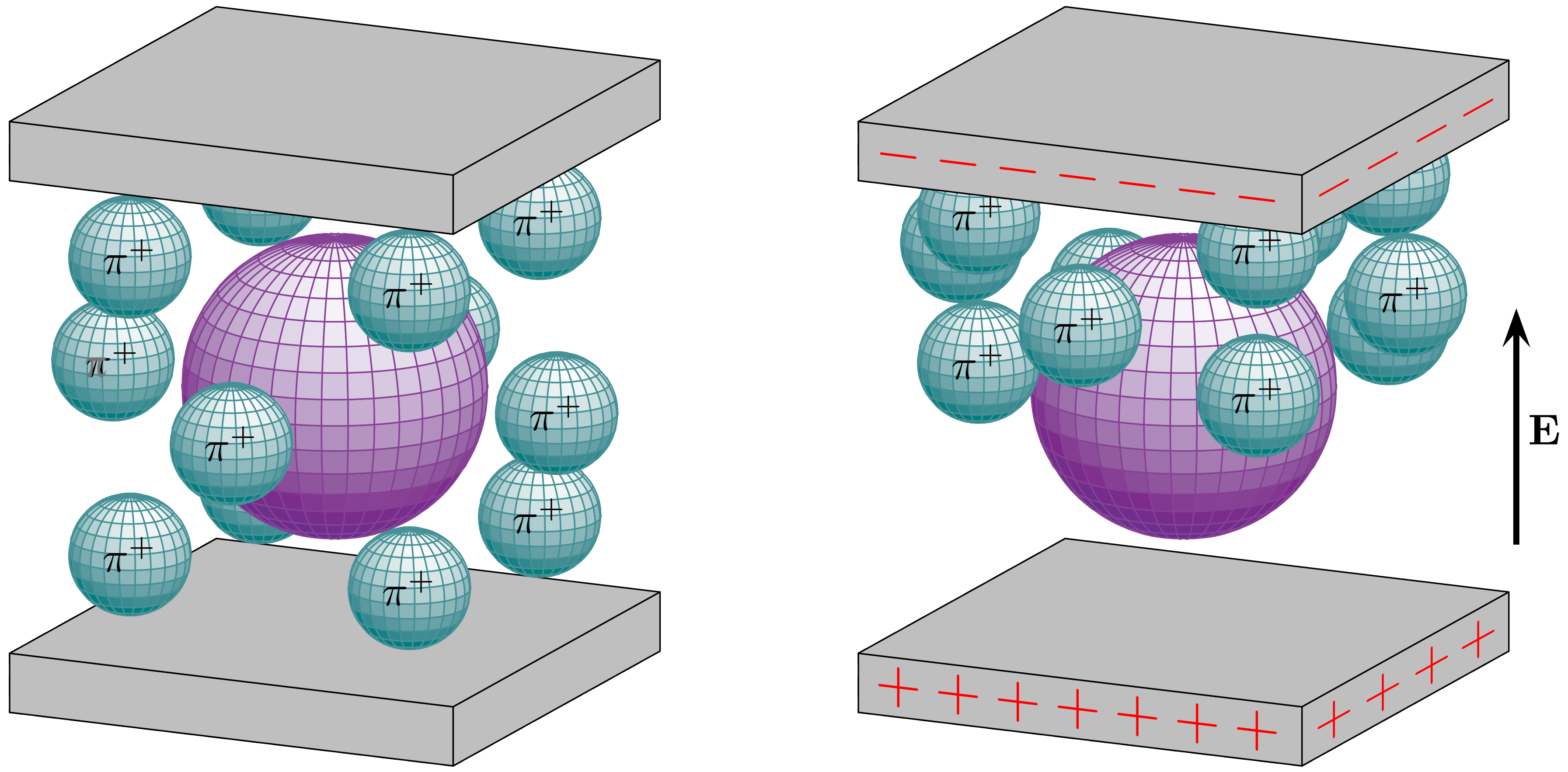


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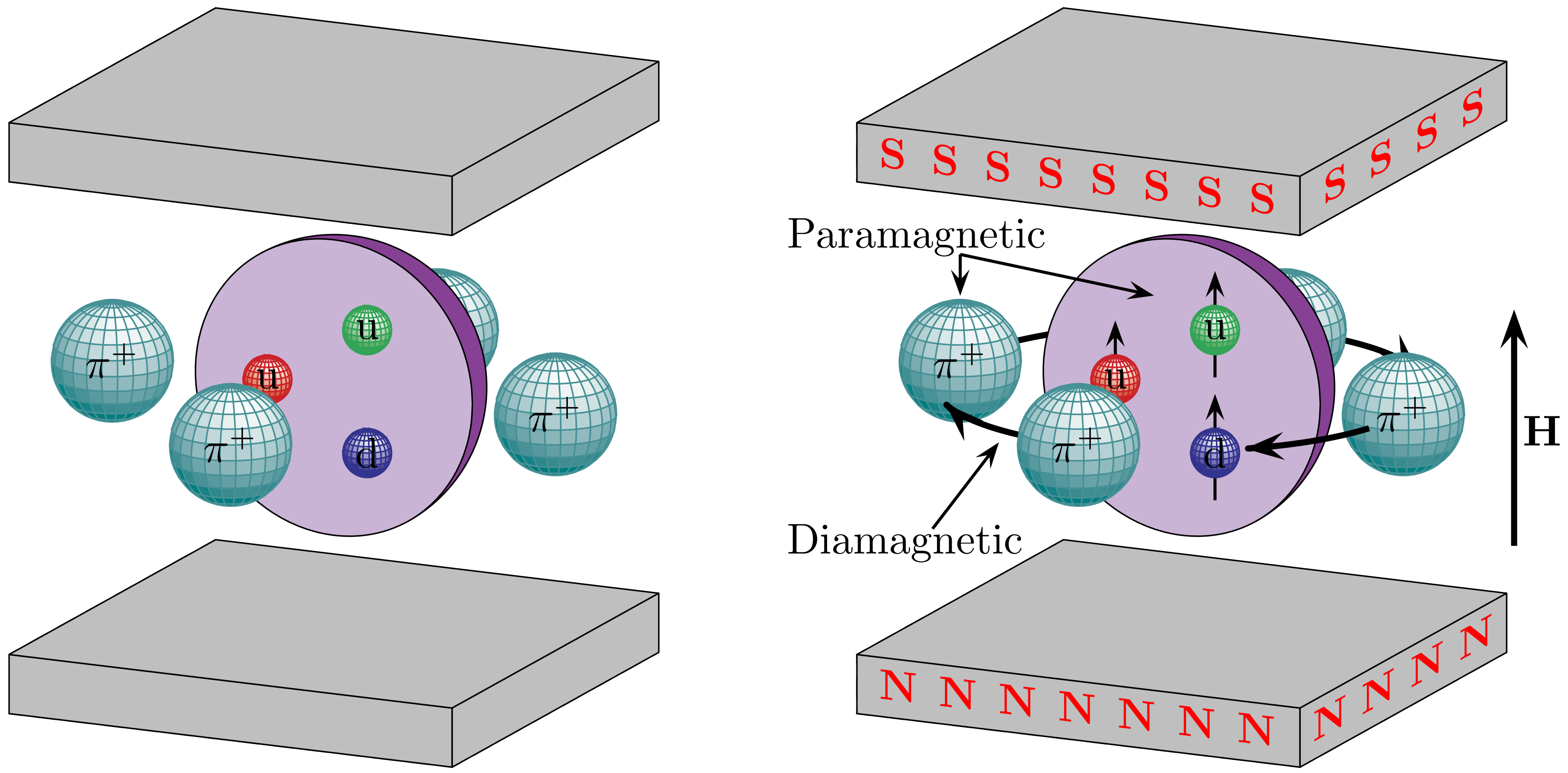



Magnetic dipole polarizability:

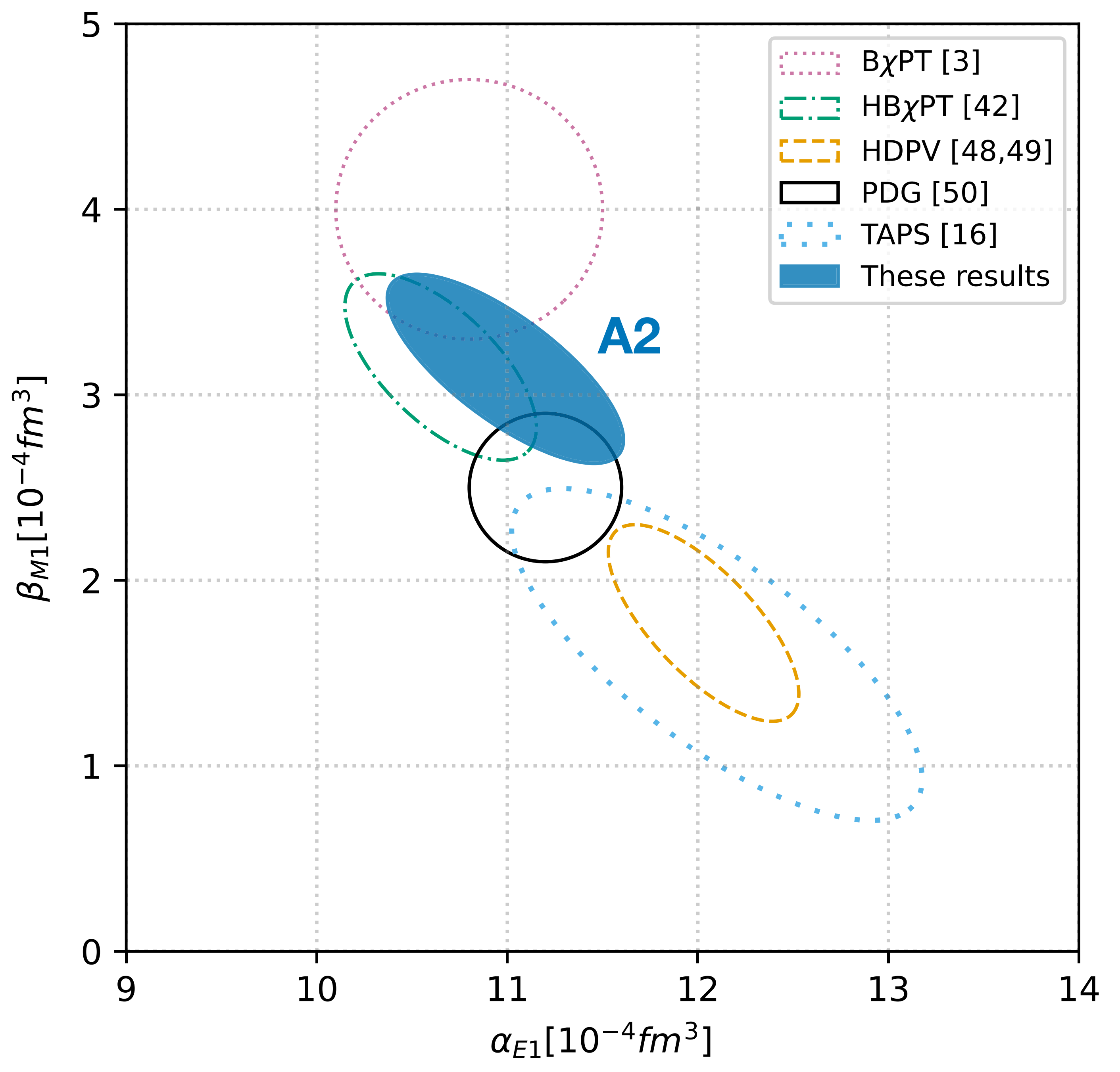
$$\vec{P} = \beta_{M1}\vec{H}$$

for polarization induced by magnetic field

| |
|---|
| diamagnetic: $\beta_{M1} < 0$ |
| paramagnetic: $\beta_{M1} > 0$ |

# Proton Magnetic Dipole Polarizability

## Dia- or paramagnetic ? $\beta_{M1} < 0$ or $\beta_{M1} > 0$



# Proton Magnetic Dipole Polarizability

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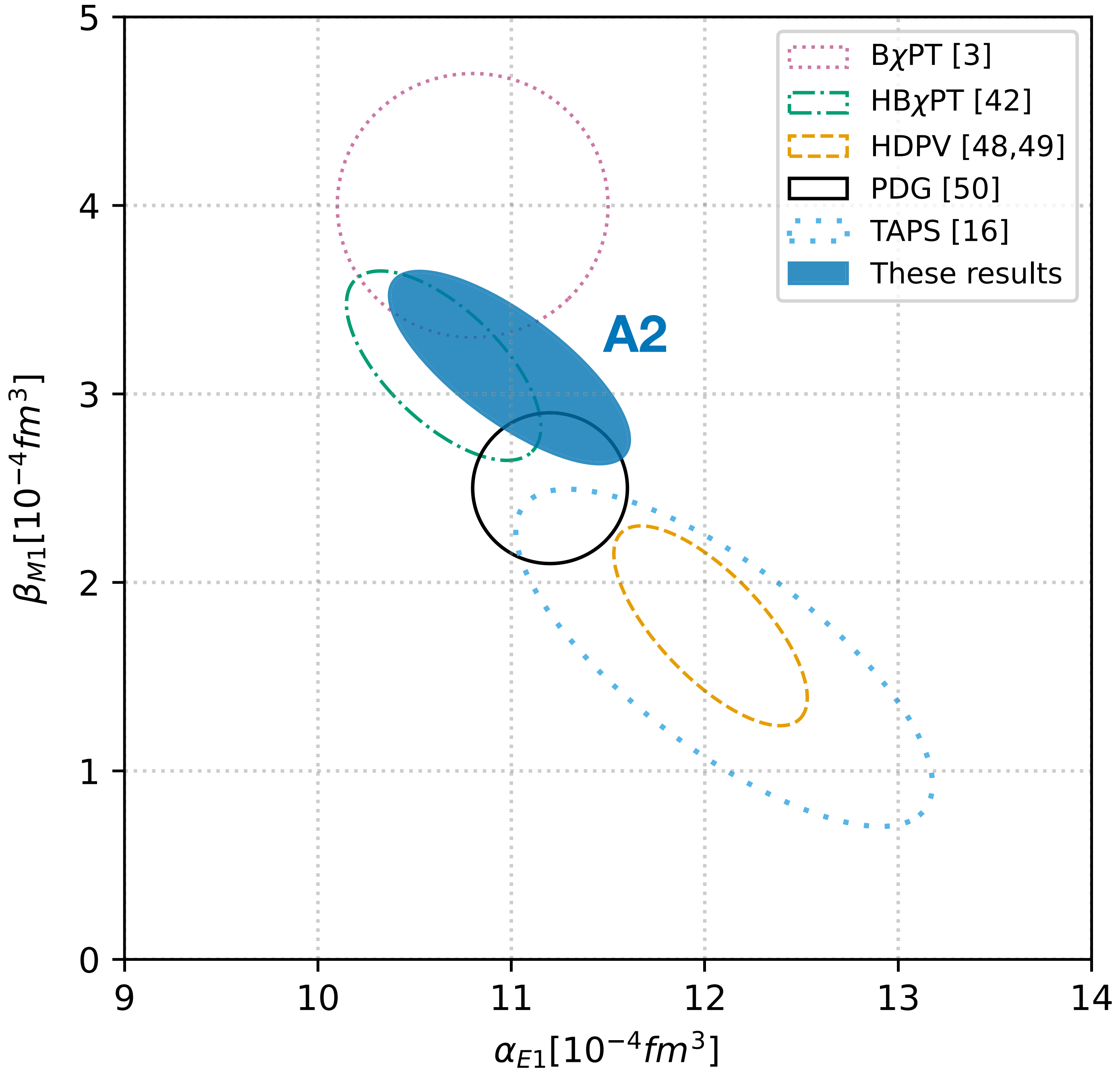
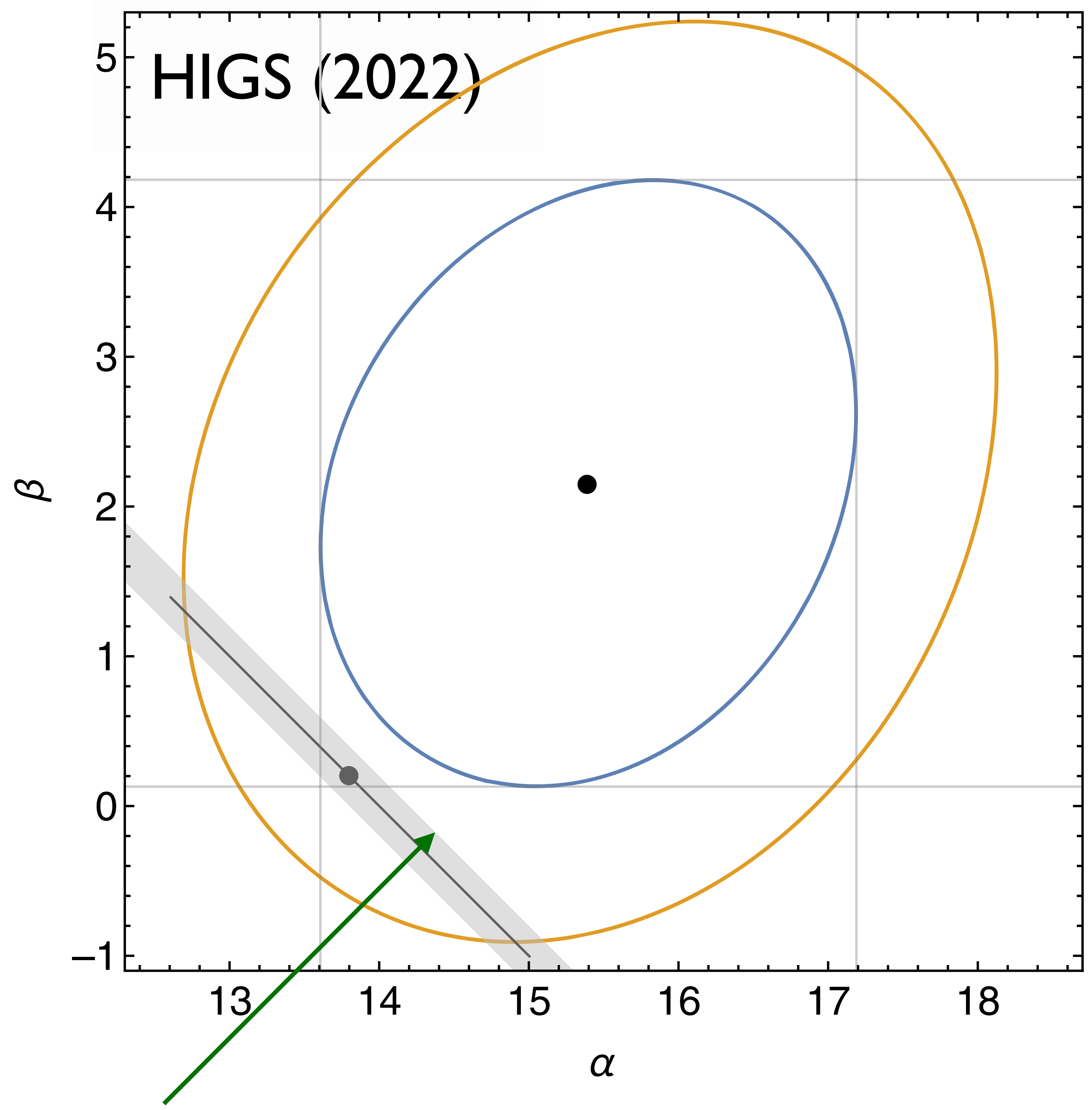



$$\beta_{M1}\,(\mathrm{ChPT}) = 3.9(0.7) \times 10^{-4}\,\mathrm{fm}^3$$
$$\beta_{M1}\,(\mathrm{DR}) = 2.4(0.6) \times 10^{-4}\,\mathrm{fm}^3$$
$$\beta_{M1}\,(\mathrm{MAMI}) = 3.14(0.51) \times 10^{-4}\,\mathrm{fm}^3$$
$$\beta_{M1}\,(\mathrm{HIGS}) = 0.2(1.2) \times 10^{-4}\,\mathrm{fm}^3$$

Eur. Phys. J. C75 (2015) 604
Phys. Rev. Lett. 129 (2022) 10, 102501
Phys. Rev. Lett. 128 (2022) 13, 132502
Phys. Rev. Lett. 128 (2022) 13, 132503

Baldin sum rule constraint:

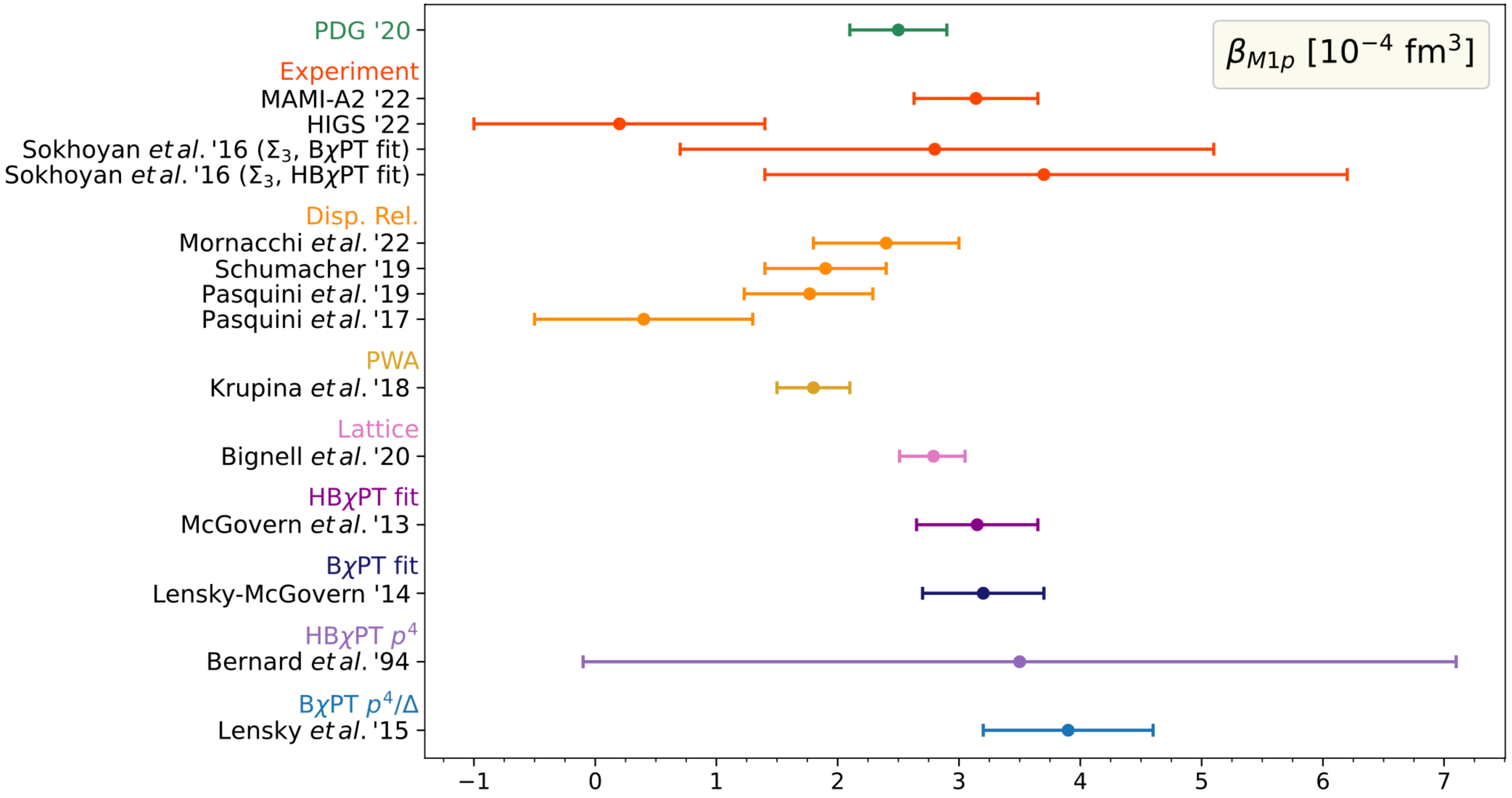
$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2}\int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4}\mathrm{fm}^3$$

# Proton Magnetic Dipole Polarizability

## Relevant input in atomic spectroscopy

- Extractions of $\beta_{M1}$ have varied in the past
- Relevant input for proton structure corrections in μH, in particular, for subtraction function contribution

  [V. Biloshytskyi on Thu.]



# Proton Magnetic Dipole Polarizability

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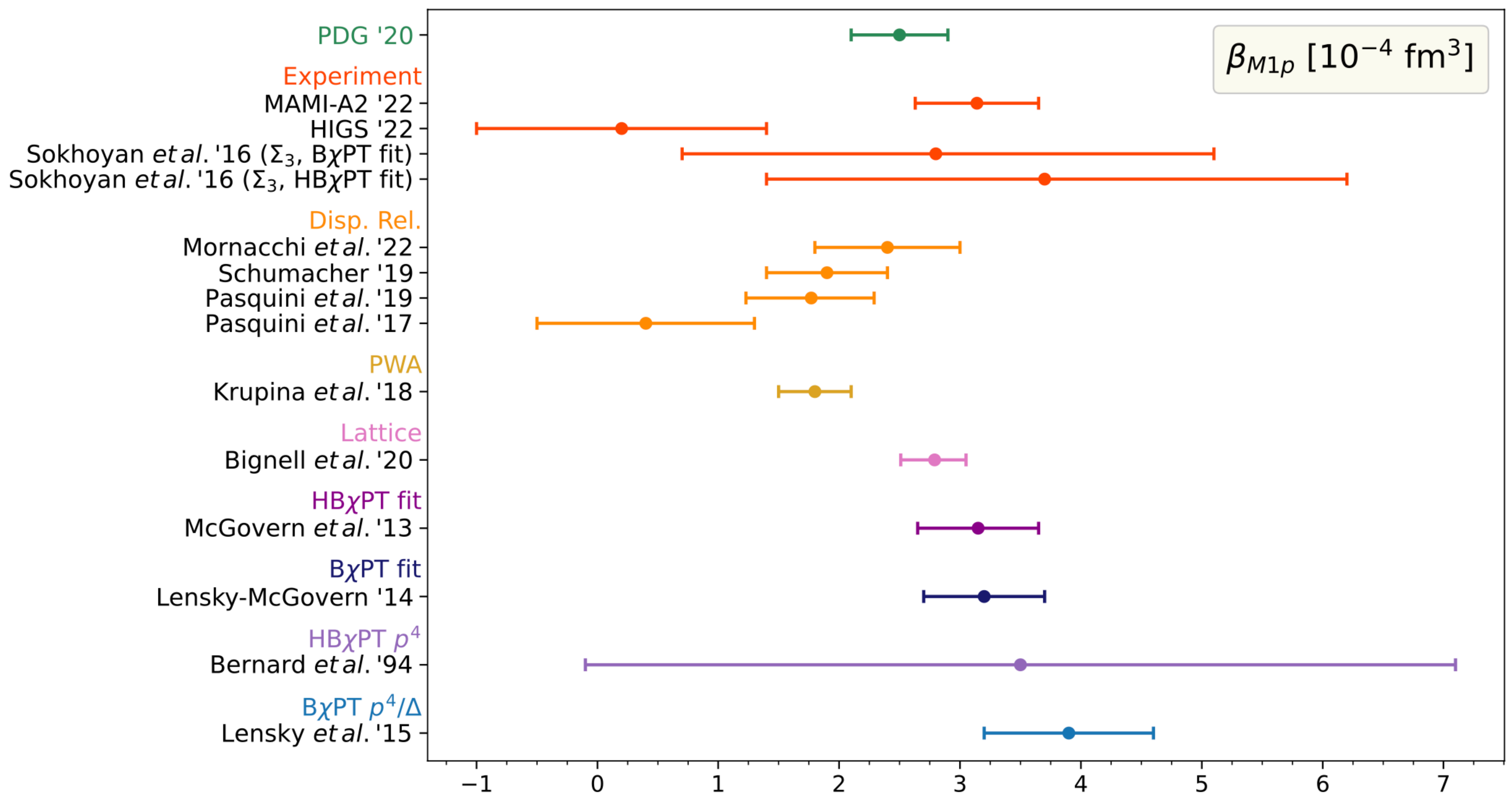
  [V. Biloshytskyi on Thu.]
- Our Aim: **Model-independent** extraction of polarizabilities through **partial wave analysis**

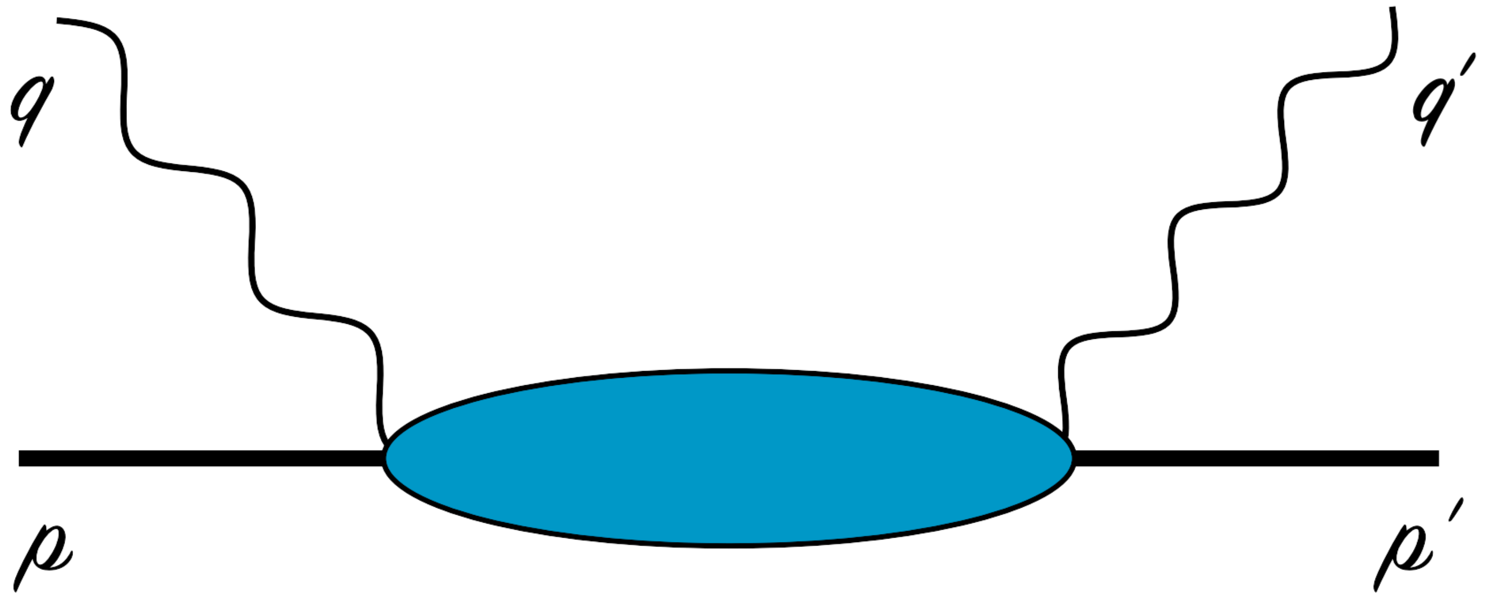


# Compton Scattering

## Low-energy expansion in terms of polarizabilities

- Low-energy (low-momentum) nucleon structure is encoded in low-energy constants (polarisabilities etc.) that parameterise the Compton scattering (CS) amplitude
- Different kinematical regimes:
  - Real CS (RCS): $q^2 = q'^2 = 0$
  - forward limit: $q = q', \quad p = p'$
  - Virtual CS (VCS) [see N. Sparveris on Wed.]
  - Forward doubly-virtual CS

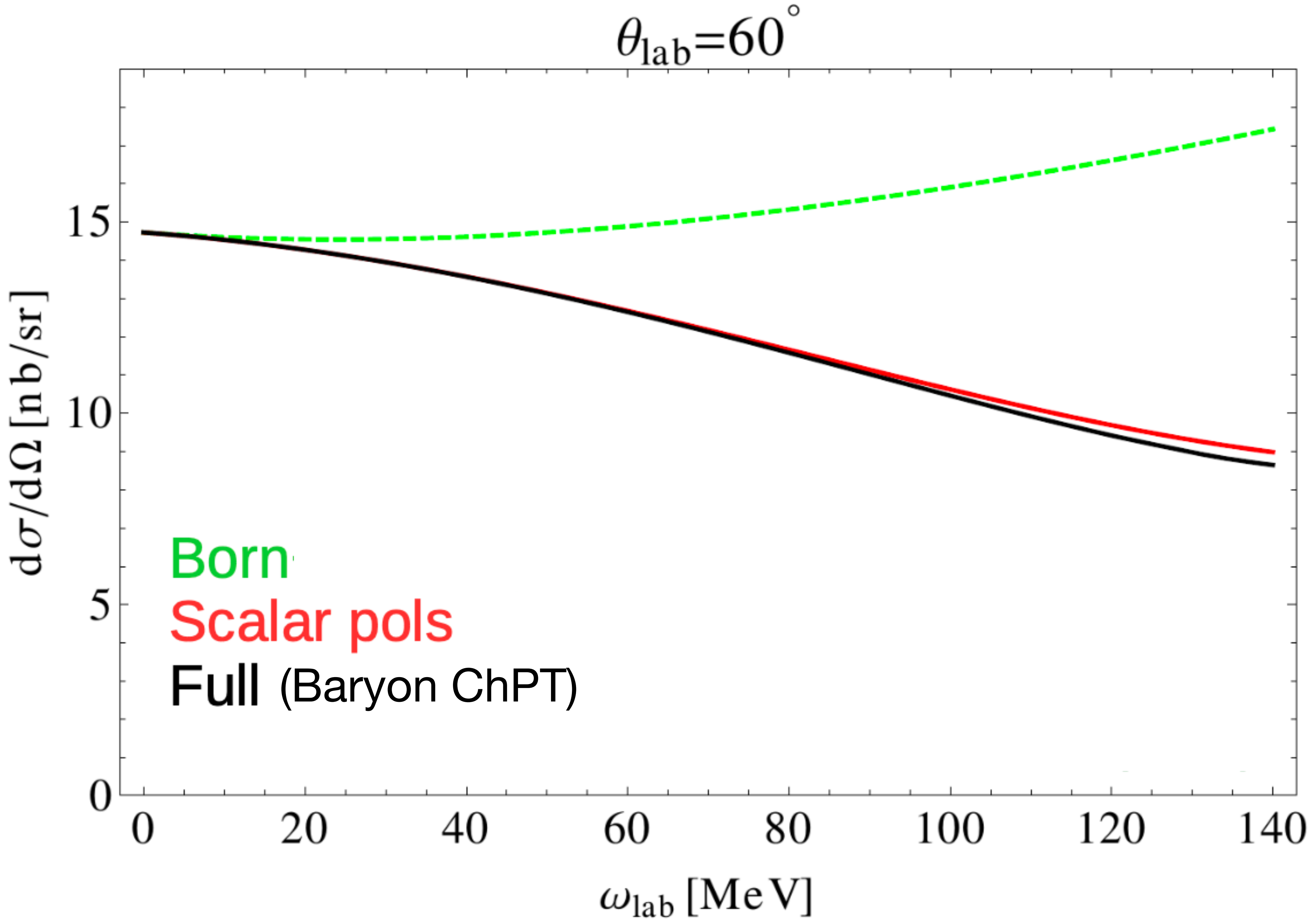
    [see F. Hagelstein, D. Ruth on Fri.]

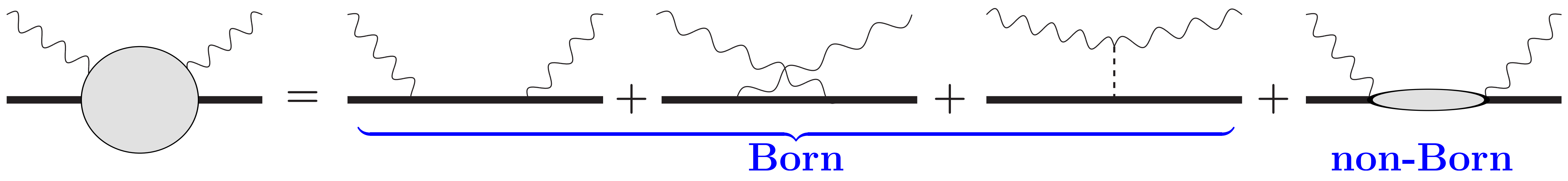


# Compton Scattering

## Low-energy expansion in terms of polarizabilities

- Born RCS is well known → mass, charge, anomalous magnetic moment and t-channel pion pole
- Non-Born RCS → polarizabilities (dipole, spin, …)

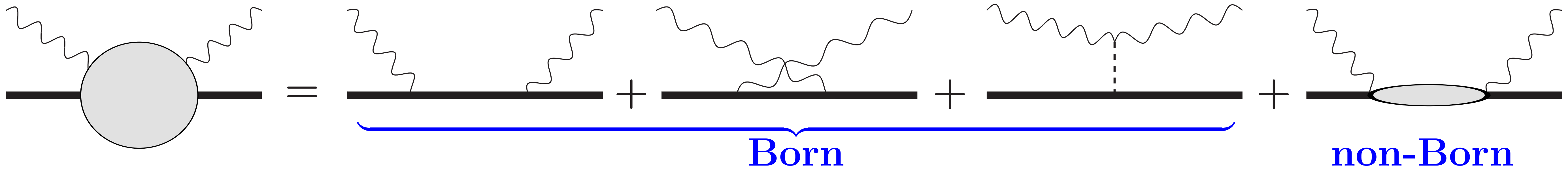



$$\boxed{\frac{d\sigma^{(NB)}}{d\Omega} = -\frac{\alpha}{M}\left(\frac{\nu'}{\nu}\right)^2 \nu\nu'\left[\alpha_{E1}(1+\cos^2\theta) + 2\beta_{M1}\cos\theta\right] + \mathcal{O}(\nu^4)}$$

W. Pfeil, H. Rollnik and S. Stankowski, “A partial-wave analysis for proton Compton scattering in the delta(1232) energy region,” Nucl. Phys. B **73**, 166 (1974).

# Multipole Expansion




$$T_{\sigma'\lambda',\sigma\lambda} = \sum_{J=1/2}^{\infty} (2J+1)\, T^J_{\sigma'\lambda',\sigma\lambda}(\omega)\, d^J_{\sigma'-\lambda',\sigma-\lambda}(\theta)$$

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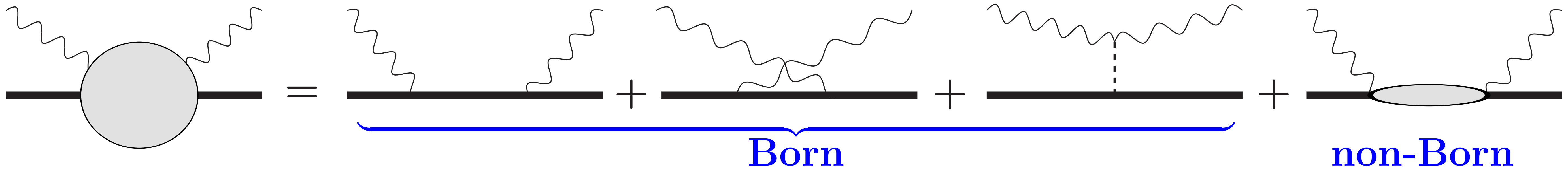


$$\boxed{\begin{aligned} \Phi_1 &= \frac{1}{8\pi\sqrt{s}}T_{\frac{1}{2},\frac{1}{2}}, \quad \Phi_2 = \frac{1}{8\pi\sqrt{s}}T_{-\frac{1}{2},\frac{1}{2}}, \quad \Phi_3 = \frac{1}{8\pi\sqrt{s}}T_{-\frac{3}{2},\frac{1}{2}}, \\ \Phi_4 &= \frac{1}{8\pi\sqrt{s}}T_{\frac{3}{2},\frac{1}{2}}, \quad \Phi_5 = \frac{1}{8\pi\sqrt{s}}T_{\frac{3}{2},\frac{3}{2}}, \quad \Phi_6 = \frac{1}{8\pi\sqrt{s}}T_{-\frac{3}{2},\frac{3}{2}}, \end{aligned}}$$

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Helicity amplitudes

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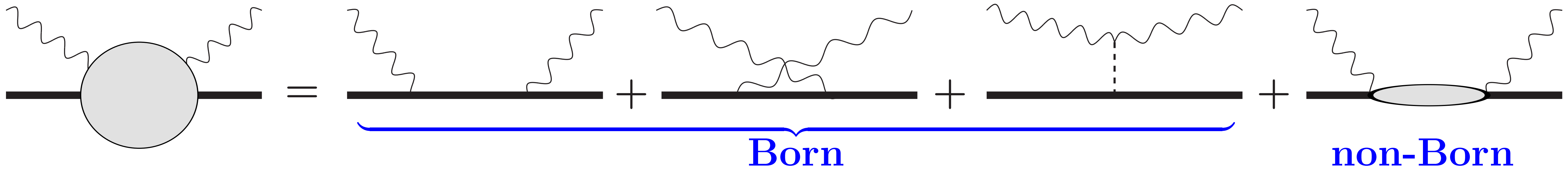


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Wigner d-functions

$$T_{\sigma'\lambda',\sigma\lambda} = \sum_{J=1/2}^{\infty} (2J+1)\, T^J_{\sigma'\lambda',\sigma\lambda}(\omega)\, d^J_{\sigma'-\lambda',\sigma-\lambda}(\theta)$$

Helicity amplitudes

PW amplitudes

$$\Phi^J_{\frac{1}{2}} = \frac{1}{4}\left\{(J+3/2)^2 f^{(J+1/2)-}_{EE\pm MM} \pm (J-1/2)^2 f^{(J-1/2)+}_{EE\pm MM} \mp 2(J+3/2)(J-1/2) f^{(J-1/2)+}_{EM\pm ME}\right\}$$
$$\Phi^J_{\frac{3}{4}} = \frac{1}{4}\sqrt{(J+3/2)(J-1/2)}\left\{(J+3/2) f^{(J+1/2)-}_{EE\mp MM} \pm (J-1/2) f^{(J-1/2)+}_{EE\mp MM} \mp 2 f^{(J-1/2)+}_{EM\mp ME}\right\}$$
$$\Phi^J_{\frac{5}{6}} = \frac{1}{4}(J+3/2)(J-1/2)\left\{f^{(J+1/2)-}_{EE\pm MM} \pm f^{(J-1/2)+}_{EE\pm MM} \pm 2 f^{(J-1/2)+}_{EM\pm ME}\right\}$$

Multipole amplitudes $f^{l\pm}_{\rho\rho'}(\omega)$

$\rho$ and $l$ define the photon multipolarity with $\rho, \rho' =$ E, or M and $l = J \pm 1/2$ the total angular momentum of the initial photon

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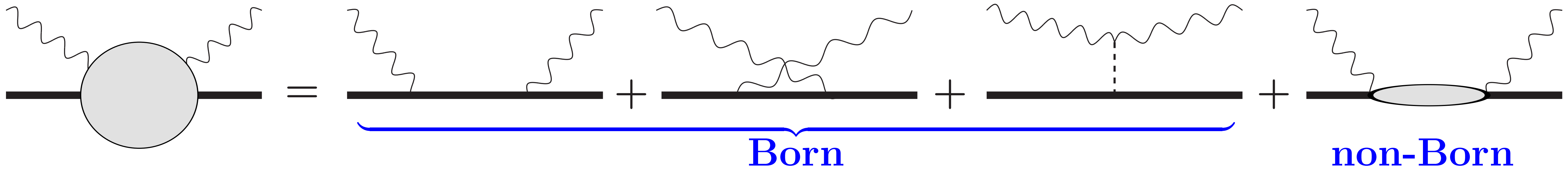


$$f = f^{\text{Born}} + \bar{f}$$

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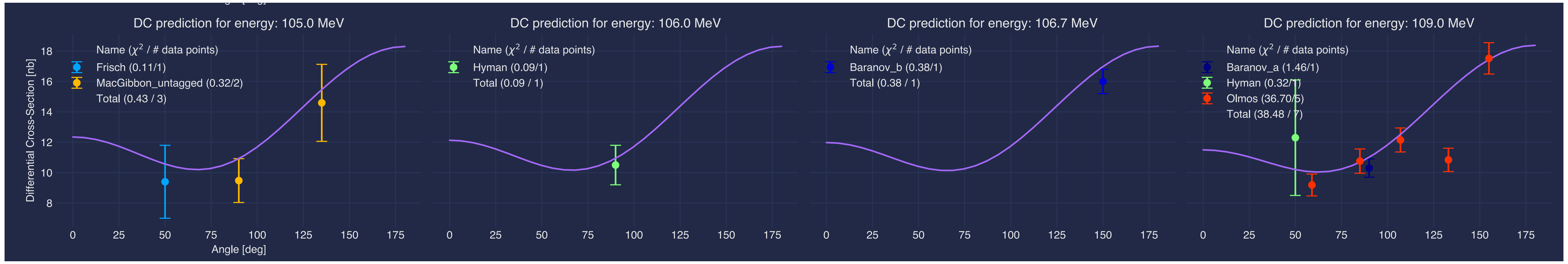
Multipole expansion of the non-Born part, truncated at J=3/2: $\bar{f} = \left(\bar{f}^{1+}_{EE}, \bar{f}^{1-}_{EE}, \bar{f}^{1+}_{MM}, \bar{f}^{1-}_{MM}, \bar{f}^{1+}_{EM}, \bar{f}^{1+}_{ME}, \bar{f}^{2+}_{EE}, \bar{f}^{2-}_{EE}, \bar{f}^{2+}_{MM}, \bar{f}^{2-}_{MM}\right)$

# Observables

## Bilinear relations

*Angular distribution*

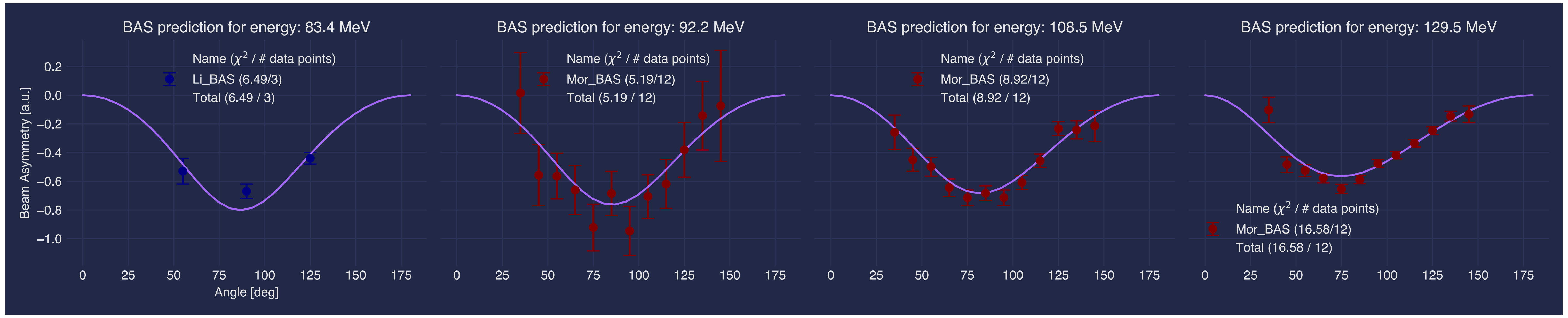
$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s} \sum_{\sigma'\lambda'\sigma\lambda} \left| T_{\sigma'\lambda',\sigma\lambda} \right|^2$$




*Beam asymmetry*

$$\Sigma_3 = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

$$\frac{d\sigma}{d\Omega}\Sigma_3 = \frac{1}{128\pi^2 s} \sum_{\sigma'\lambda'\lambda} \mathrm{Re}\left(T^*_{\sigma'\lambda',-1\lambda} T_{\sigma'\lambda',1\lambda}\right)$$



# Forward RCS Amplitudes

## Empirical evaluation

$$T_{\sigma'\lambda'\sigma\lambda} \stackrel{t=0}{=} \chi^\dagger_{\lambda'} \left\{ f(\nu)\, \vec{\varepsilon}^{\,*}_{\sigma'} \cdot \vec{\varepsilon}_\sigma + g(\nu)\, i\, (\vec{\varepsilon}^{\,*}_{\sigma'} \times \vec{\varepsilon}_\sigma) \cdot \vec{\sigma} \right\} \chi_\lambda$$

Spin-independent amplitude

Spin-dependent amplitude

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Spin-dependent amplitude

$$\boxed{f(\nu)} = \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1)^2 \left\{ (L+2)\left(f_{EE}^{(L+1)-} + f_{MM}^{(L+1)-}\right) + L\left(f_{EE}^{L+} + f_{MM}^{L+}\right) \right\}$$

$$\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f_{EE}^{1-} + 2f_{EE}^{1+} + f_{MM}^{1-} + 2f_{MM}^{1+} + 6f_{EE}^{2-} + 9f_{EE}^{2+} + 6f_{MM}^{2-} + 9f_{MM}^{2+}\right)$$

$$\boxed{g(\nu)} = \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1) \left\{ (L+2)\left(f_{EE}^{(L+1)-} + f_{MM}^{(L+1)-}\right) - L\left(f_{EE}^{L+} + f_{MM}^{L+}\right) - 2L(L+2)\left(f_{EM}^{L+} + f_{ME}^{L+}\right) \right\}$$

$$\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f_{EE}^{1-} - f_{EE}^{1+} - 6f_{EM}^{1+} - 6f_{ME}^{1+} + f_{MM}^{1-} - f_{MM}^{1+} + 3f_{EE}^{2-} - 3f_{EE}^{2+} + 3f_{MM}^{2-} - 3f_{MM}^{2+}\right)$$

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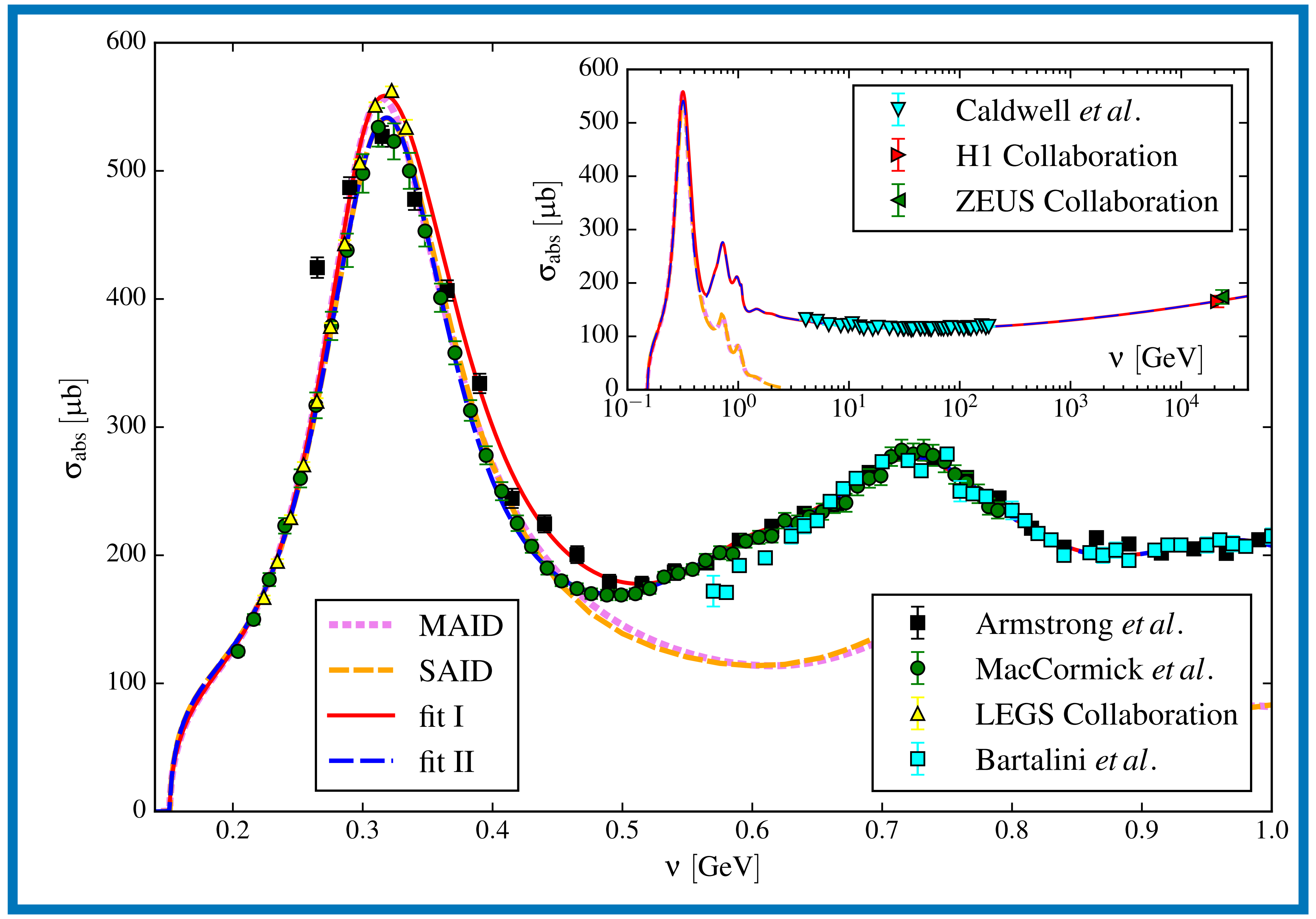


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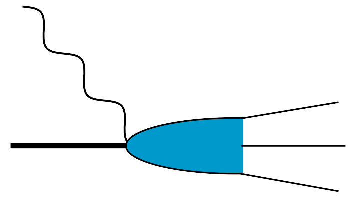
&\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left( f_{EE}^{1-} + 2 f_{EE}^{1+} + f_{MM}^{1-} + 2 f_{MM}^{1+} + 6 f_{EE}^{2-} + 9 f_{EE}^{2+} + 6 f_{MM}^{2-} + 9 f_{MM}^{2+} \right) \\
&= -\frac{\alpha}{M} + \frac{\nu^2}{4\pi^2} \int_0^\infty \frac{\mathrm{d}\nu'}{\nu'^2 - \nu^2 - i0^+} \boxed{\left[\sigma_{1/2}^{\text{abs}}(\nu') + \sigma_{3/2}^{\text{abs}}(\nu')\right]}
\end{aligned}$$



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&= -\frac{\alpha \varkappa^2 \nu}{2M^2} + \frac{\nu^3}{4\pi^2} \int_0^\infty \frac{\mathrm{d}\nu'}{\nu'} \frac{\sigma_{1/2}^{\text{abs}}(\nu') - \sigma_{3/2}^{\text{abs}}(\nu')}{\nu'^2 - \nu^2 - i0^+}
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spin-dependent amplitude

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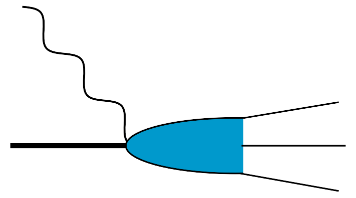

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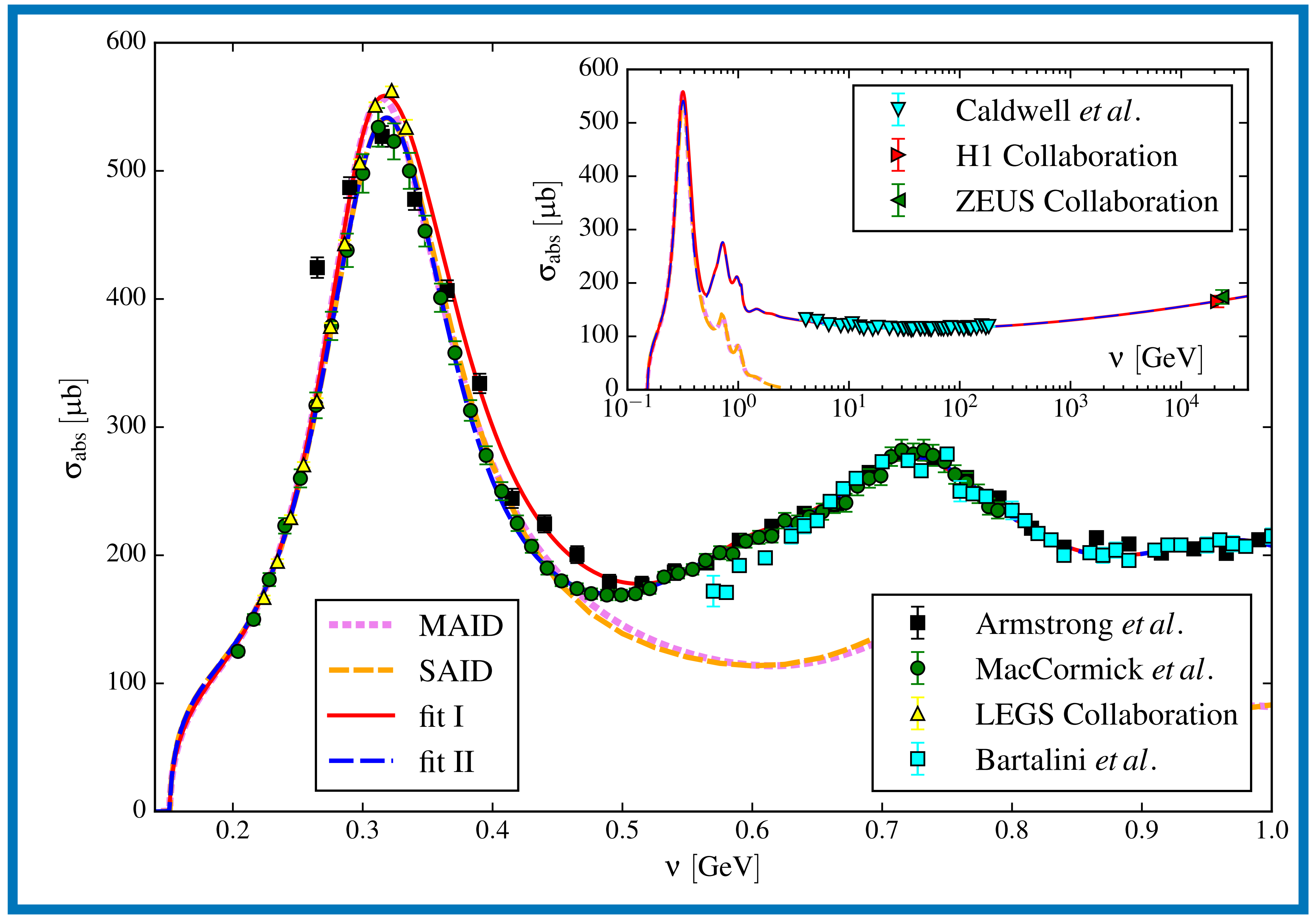
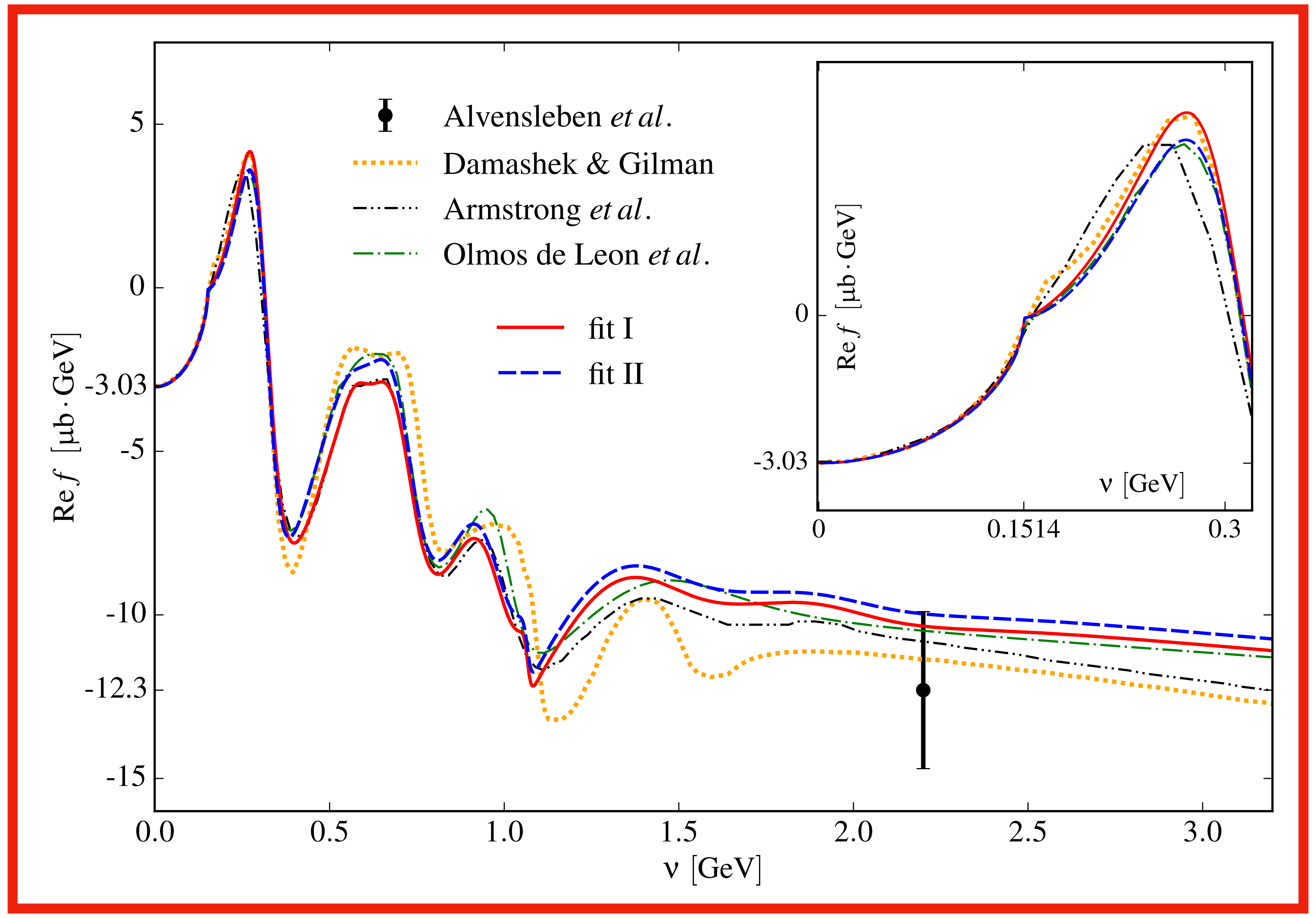
\end{aligned}$$



# RCS Sum Rules

## Empirical evaluation based on photoabsorption cross sections

Spin-independent amplitude

$f(\nu) = -\frac{Z^2\alpha}{M} + (\alpha_{E1} + \beta_{M1})\nu^2 + O(\nu^4)$

$$f(\nu) = -\frac{Z^2\alpha}{M} + \frac{\nu^2}{2\pi^2}\int_0^\infty \mathrm{d}\nu' \, \frac{\sigma_T(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

$$g(\nu) = \frac{\nu}{2\pi^2}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'\sigma_{TT}(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

Spin-dependent amplitude

$g(\nu) = -\frac{\alpha\varkappa^2}{2M^2}\nu + \gamma_0\nu^3 + O(\nu^5)$

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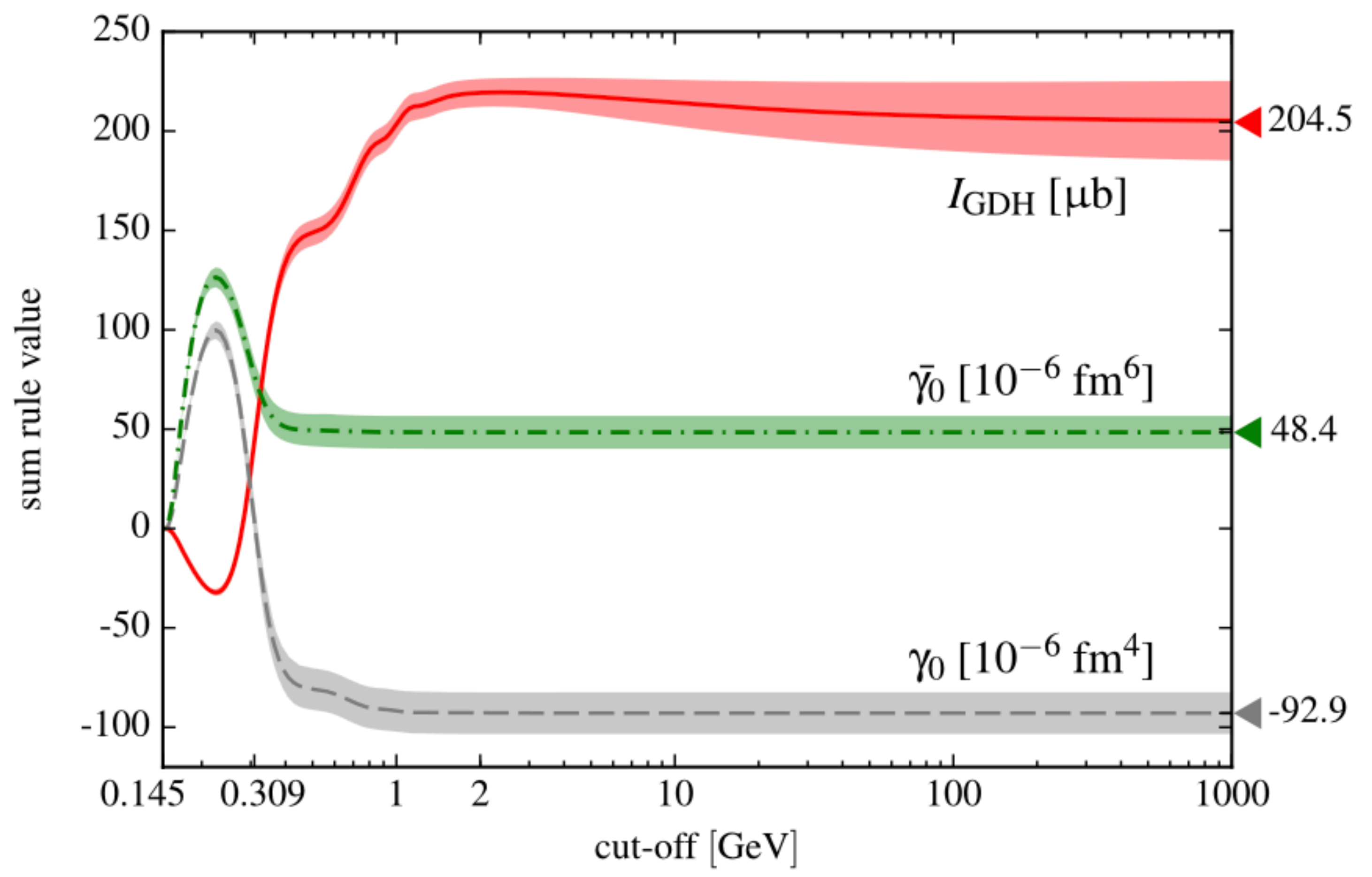
Forward spin polarizability sum rule:

$$\gamma_0 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}\nu \frac{\sigma_{TT}(\nu)}{\nu^3}, \quad = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

| | $\mathcal{O}(\nu)$ | $\mathcal{O}(\nu^3)$ | $\mathcal{O}(\nu^5)$ |
|---|---|---|---|
| | $I_{\mathrm{GDH}}$ (μb) | $\gamma_0$ ($10^{-6}$ fm$^4$) | $\bar{\gamma}_0$ ($10^{-6}$ fm$^6$) |
| GDH & A2 [9, 11] | $\approx 212$ | $\approx -86$ | |
| Helbing [21] | $212 \pm 6 \pm 12$ | | |
| Bianchi-Thomas [24] | $207 \pm 23$ | | |
| Pasquini *et al.* [12] | $210 \pm 6 \pm 14$ | $-90 \pm 8 \pm 11$ | $60 \pm 7 \pm 7$ |
| This work | $204.5 \pm 21.4$ | $-92.9 \pm 10.5$ | $48.4 \pm 8.2$ |
| GDH sum rule | 204.784481(4)[a] | | |
| B$\chi$PT [15] | | $-90 \pm 140$ | $110 \pm 50$ |
| HB$\chi$PT [17] | | $-260 \pm 190$ | |



# RCS Sum Rules

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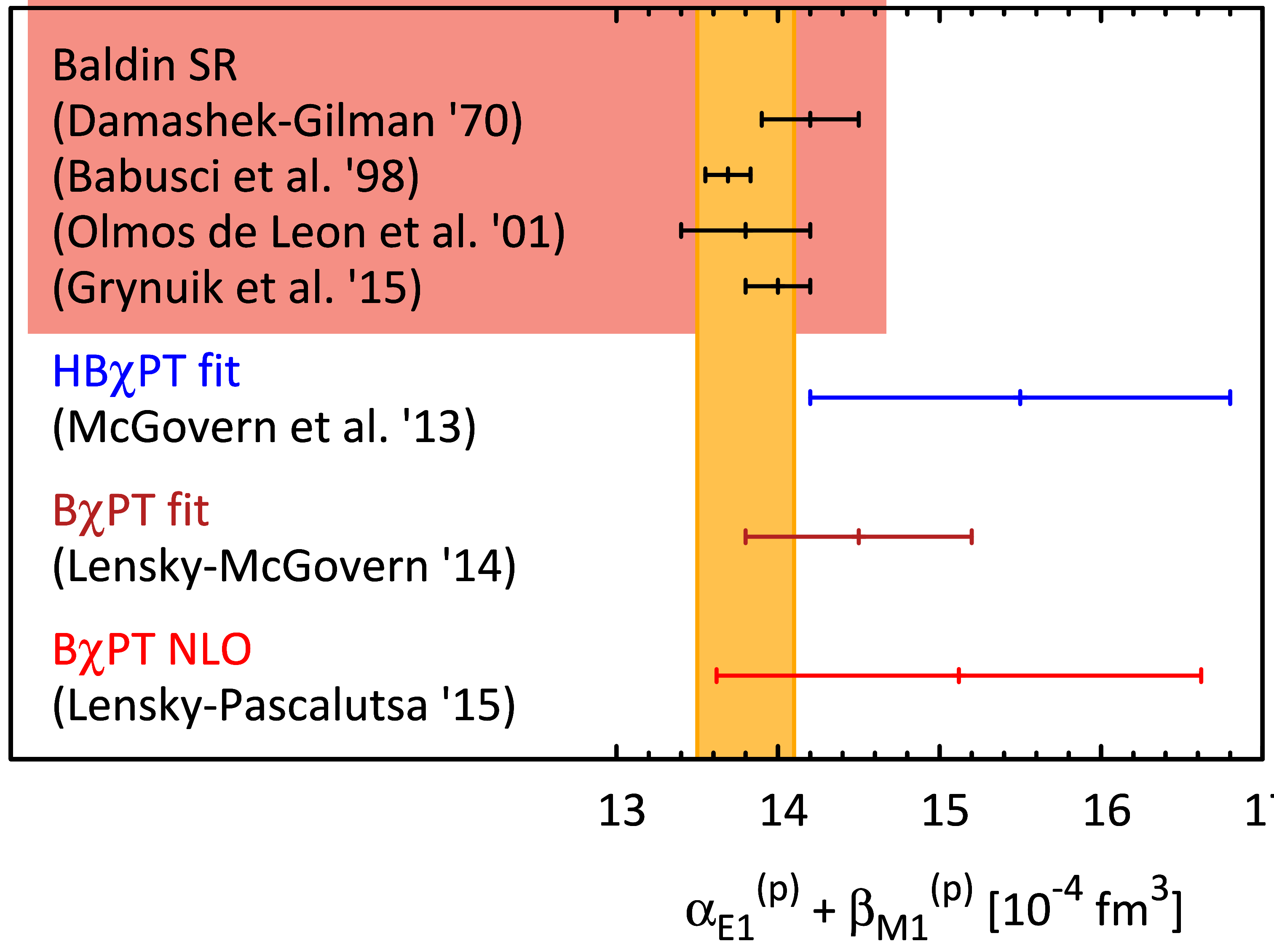
$$g(\nu) = \frac{\nu}{2\pi^2}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'\sigma_{TT}(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

*Spin-dependent amplitude*

$$g(\nu) = -\frac{\alpha\varkappa^2}{2M^2}\nu + \gamma_0\nu^3 + O(\nu^5)$$

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}\nu \, \frac{\sigma_T(\nu)}{\nu^2}$$




Forward spin polarizability sum rule:

$$\gamma_0 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}\nu \, \frac{\sigma_{TT}(\nu)}{\nu^3}, \\ = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

| | $\mathcal{O}(\nu)$ | $\mathcal{O}(\nu^3)$ | $\mathcal{O}(\nu^5)$ |
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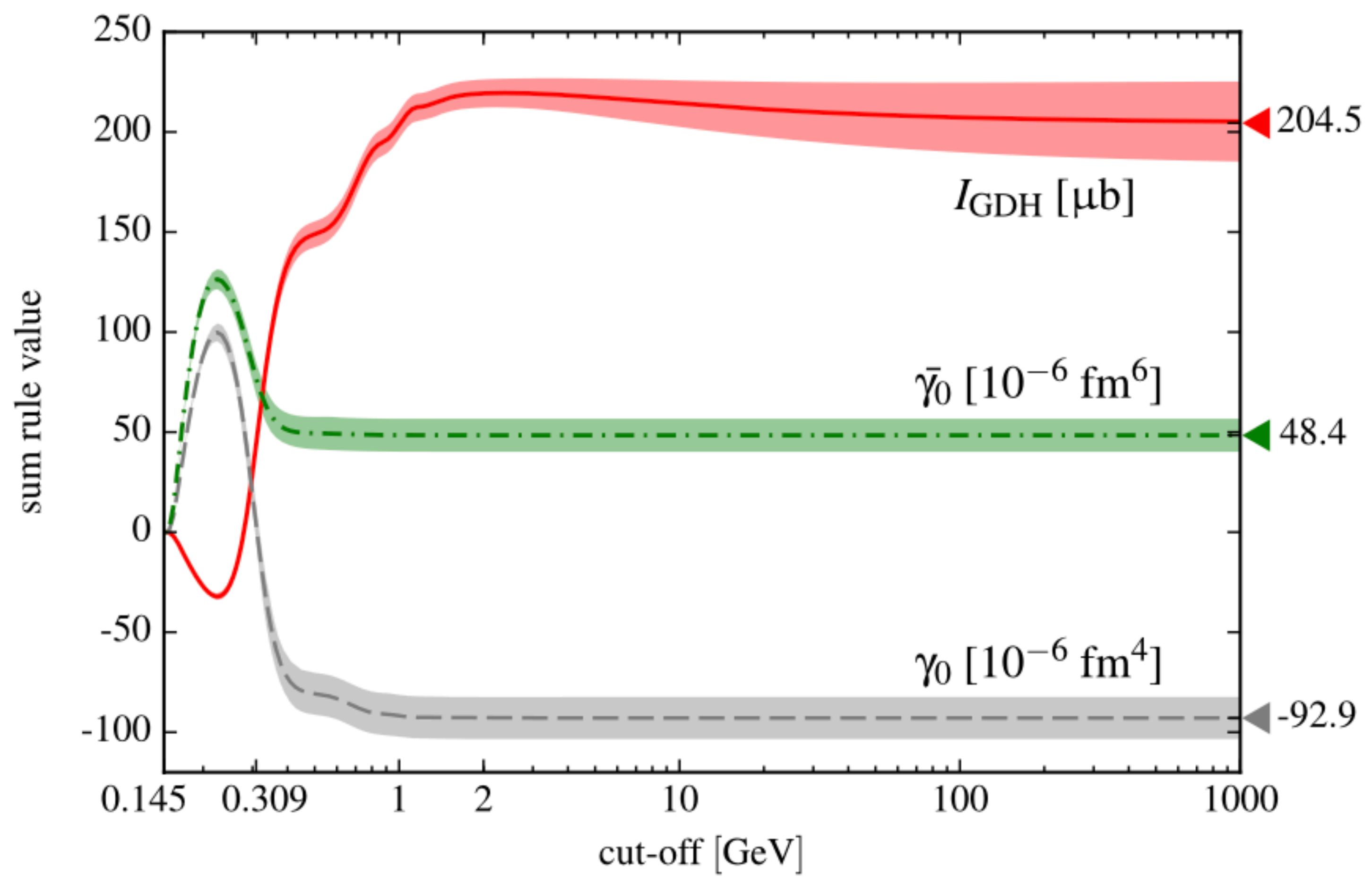


# Partial-Wave-Analysis Ansatz

- $l = 1$ multipoles determined through **low-energy expansion** in the following **model-independent** form:

$$\bar{f}^{1+}_{EE}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[ \frac{\alpha_{E1}}{3} + \frac{E_\gamma}{3} \left( \frac{-\alpha_{E1} + \beta_{M1}}{M} + \gamma_{E1E1} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_1^R(E_\gamma) \right]$$

$$\bar{f}^{1-}_{EE}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[ \frac{\alpha_{E1}}{3} + \frac{E_\gamma}{3} \left( \frac{-\alpha_{E1} + \beta_{M1}}{M} - 2\,\gamma_{E1E1} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_2^R(E_\gamma) \right]$$

$$\bar{f}^{1+}_{MM}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[ \frac{\beta_{M1}}{3} + \frac{E_\gamma}{3} \left( \frac{-\beta_{M1} + \alpha_{E1}}{M} + \gamma_{M1M1} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_3^R(E_\gamma) \right]$$

$$\bar{f}^{1-}_{MM}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[ \frac{\beta_{M1}}{3} + \frac{E_\gamma}{3} \left( \frac{-\beta_{M1} + \alpha_{E1}}{M} - 2\,\gamma_{M1M1} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_4^R(E_\gamma) \right]$$

$$\bar{f}^{1+}_{EM}(E_\gamma) = E_\gamma^3 \frac{M}{\sqrt{s}} \left[ \frac{\gamma_{E1M2}}{6} + \frac{E_\gamma}{6} \left( \frac{-6\,\gamma_{E1M2} + 3\,\gamma_{M1E2} + 3\,\gamma_{M1M1}}{4M} - \frac{\beta_{M1}}{8M^2} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_5^R(E_\gamma) \right]$$

$$\bar{f}^{1+}_{ME}(E_\gamma) = E_\gamma^3 \frac{M}{\sqrt{s}} \left[ \frac{\gamma_{M1E2}}{6} + \frac{E_\gamma}{6} \left( \frac{-6\,\gamma_{M1E2} + 3\,\gamma_{E1M2} + 3\,\gamma_{E1E1}}{4M} - \frac{\alpha_{E1}}{8M^2} \right) + \left( \frac{E_\gamma}{M} \right)^2 f_6^R(E_\gamma) \right]$$

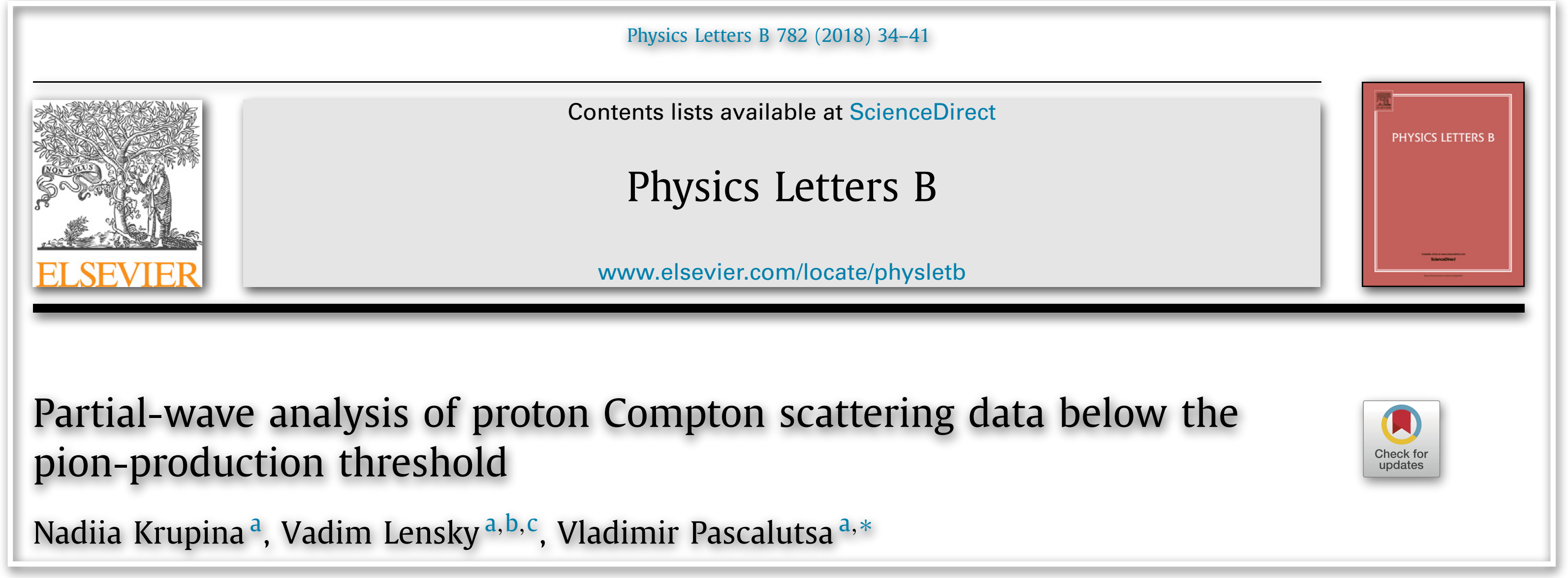
**After including the forward constraints we fit 4 polarizabilities and 4 residual functions.**

$f_i^R$ = residual functions parametrised by polynomials in photon energy $E_\gamma$

- $l = 2$ multipoles are small and will be either **neglected** or taken from **ChPT**

# PWA of RCS Below Threshold

## Updates and improvements


Physics Letters B 782 (2018) 34–41

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Physics Letters B

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Partial-wave analysis of proton Compton scattering data below the pion-production threshold

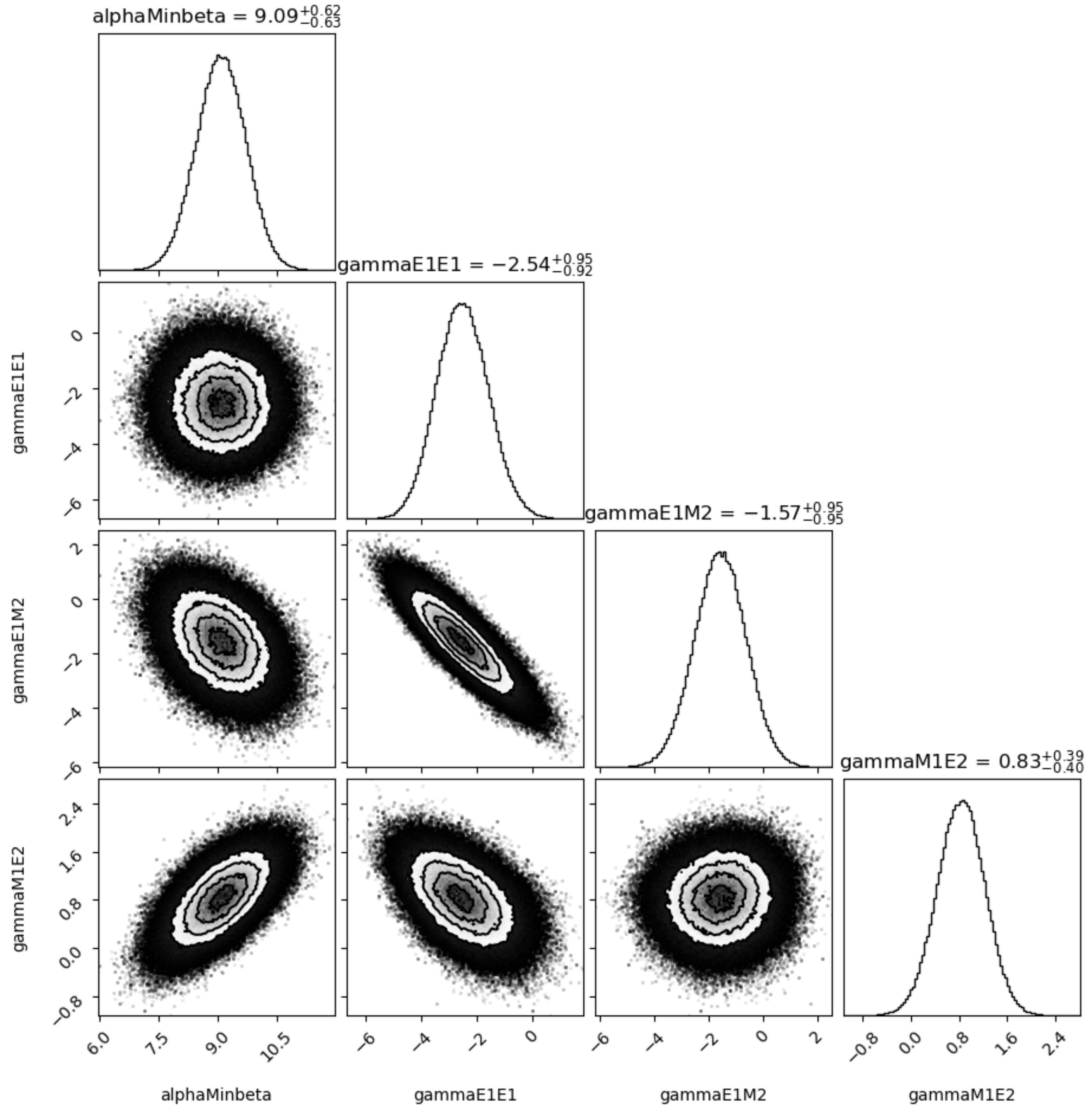
Nadiia Krupina[a], Vadim Lensky[a,b,c], Vladimir Pascalutsa[a,*]

**Mainz (2018)**

- Updated world data base including A2@MAMI and HIGS

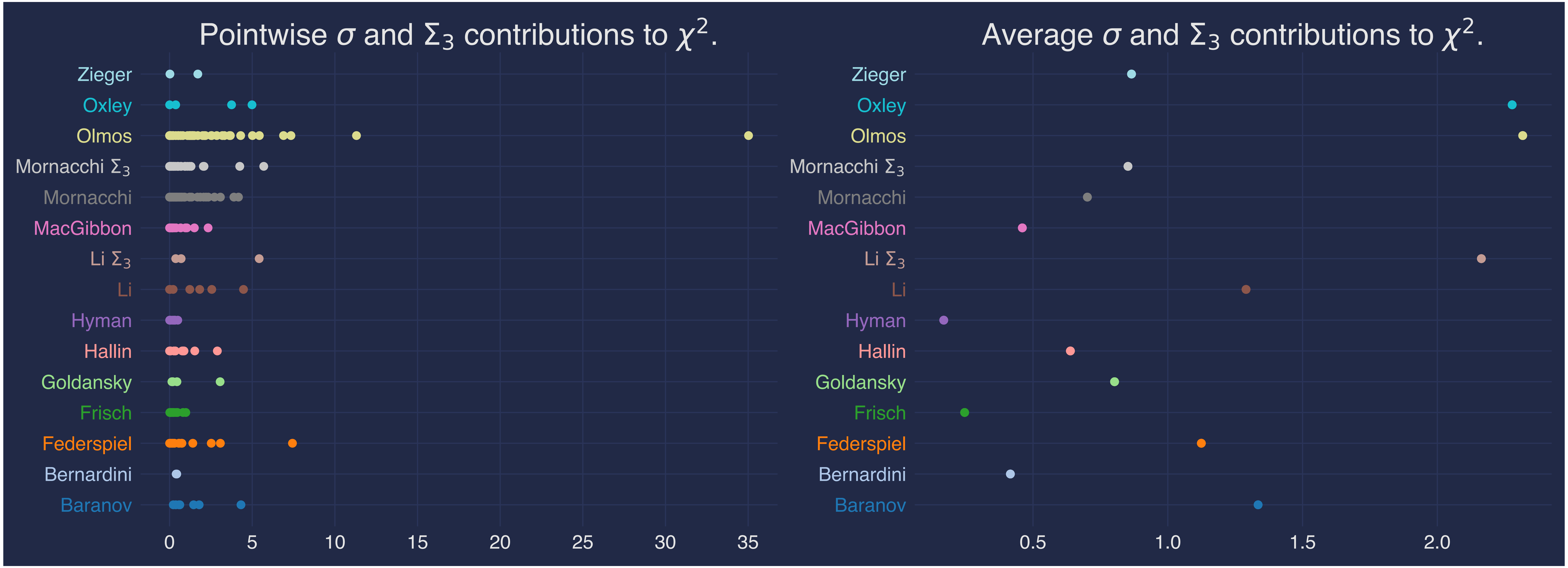
Phys. Rev. Lett. 128 (2022) 13, 132502
Phys. Rev. Lett. 128 (2022) 13, 132503

  - Old world data: 138 data points (w/o pilot $\Sigma_3$ data)
  - A2: 60 $\mathrm{d}\sigma$ and 36 $\Sigma_3$ data points
  - HIGS: 8 $\mathrm{d}\sigma$ and 3 $\Sigma_3$ data points
- Fit of 15 experiments including normalization errors
- Markov Chain Monte Carlo [emcee astro-ph.IM/1202.3665]



# Fit of World Data

## $\chi^2$ distribution



# Preliminary Results
## & comparison to literature

| Name | $\alpha_{E1}$ | $\beta_{M1}$ | $\gamma_{E1E1}$ | $\gamma_{M1M1}$ | $\gamma_{E1M2}$ | $\gamma_{M1E2}$ | $\chi^2$/dof. |
|---|---|---|---|---|---|---|---|
| Literature: | | | | | | | |
| DR, Mornacchi et al. (2022)[a] | 12.7(8)(1) | 2.4(6)(1) | $-3.0(6)(4)$ | 3.7(5)(1) | $-1.2(1.0)(3)$ | 2.0(7)(4) | p-value $= 0.24$ |
| B$\chi$PT, Lensky et al. (2015) | 11.2(7) | 3.9(7) | $-3.3(8)$ | 2.9(1.5) | 0.2(2) | 1.1(3) | |

Table 1: The proton scalar and spin pol. in units $10^{-4}$ fm$^3$ (scalar) and $10^{-4}$ fm$^4$ (spin).
$^a$ : Errors are given as (fit)(model)
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$^*$ : Values are not fitted, but taken from PHYSICAL REVIEW C 102, 035205 (2020)

# Preliminary Results

## & comparison to literature

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| Fits without $l = 2$ multipoles: | | | | | | | |
| New world data | 11.5(3) | 2.5(3) | −2.5(9) | 4.2(4) | −1.6(1.0) | 0.8(4) | 1.21 |

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| Fits without $l = 2$ multipoles: | | | | | | | |
| A2, $\sigma$ | 9.5(5) | 4.5(5) | −1.1(1.1) | 3.5(4) | −0.0(1.1) | −1.4(6) | 0.93 |

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# Preliminary Results
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| Old world data | 11.5(4) | 2.5(4) | −3.0(1.0) | 4.1(3) | −0.7(1.0) | 0.6(5) | 1.47 |
| Fits with $l = 2$ multipoles: | | | | | | | |
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| A2, $\Sigma_3$ | 7.8(6) | 6.2(6) | 1.2(1.6) | −2.1(1.7) | 0.1(1.2) | 1.8(1.1) | 1.04 |

Table 1: The proton scalar and spin pol. in units $10^{-4}$ fm$^3$ (scalar) and $10^{-4}$ fm$^4$ (spin).
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| New world data | 11.5(3) | 2.5(3) | $-2.5(9)$ | 4.2(4) | $-1.6(1.0)$ | 0.8(4) | 1.21 |
| New world data without HIGS | 11.3(3) | 2.7(3) | $-2.5(9)$ | 4.1(3) | $-1.4(1.0)$ | 0.7(4) | 1.19 |
| Old world data | 11.5(4) | 2.5(4) | $-3.0(1.0)$ | 4.1(3) | $-0.7(1.0)$ | 0.6(5) | 1.47 |
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| Fits without $l = 2$ multipoles: | | | | | | | |
| A2, $\sigma$ | 9.5(5) | 4.5(5) | $-1.1(1.1)$ | 3.5(4) | $-0.0(1.1)$ | $-1.4(6)$ | 0.93 |
| A2, $\Sigma_3$ | 7.8(6) | 6.2(6) | 1.2(1.6) | $-2.1(1.7)$ | 0.1(1.2) | 1.8(1.1) | 1.04 |
| A2, $\sigma$ and $\Sigma_3$ | 9.5(4) | 4.5(4) | $-1.0(1.1)$ | 3.5(3) | $-0.5(1.1)$ | $-1.1(6)$ | 0.90 |

Table 1: The proton scalar and spin pol. in units $10^{-4}$ fm$^3$ (scalar) and $10^{-4}$ fm$^4$ (spin).
[a] : Errors are given as (fit)(model)
[b] : Errors are given as (statistical)(systematic)(spin polarizability)(model)
[*] : Values are not fitted, but taken from PHYSICAL REVIEW C 102, 035205 (2020)

# Preliminary Results
## & comparison to literature

| Name | $\alpha_{E1}$ | $\beta_{M1}$ | $\gamma_{E1E1}$ | $\gamma_{M1M1}$ | $\gamma_{E1M2}$ | $\gamma_{M1E2}$ | $\chi^2$/dof. |
|---|---|---|---|---|---|---|---|
| Literature: | | | | | | | |
| DR, Mornacchi et al. (2022)[a] | 12.7(8)(1) | 2.4(6)(1) | −3.0(6)(4) | 3.7(5)(1) | −1.2(1.0)(3) | 2.0(7)(4) | p-value = 0.24 |
| B$\chi$PT, Lensky et al. (2015) | 11.2(7) | 3.9(7) | −3.3(8) | 2.9(1.5) | 0.2(2) | 1.1(3) | |
| Fits without $l=2$ multipoles: | | | | | | | |
| New world data | 11.5(3) | 2.5(3) | −2.5(9) | 4.2(4) | −1.6(1.0) | 0.8(4) | 1.21 |
| New world data without HIGS | 11.3(3) | 2.7(3) | −2.5(9) | 4.1(3) | −1.4(1.0) | 0.7(4) | 1.19 |
| Old world data | 11.5(4) | 2.5(4) | −3.0(1.0) | 4.1(3) | −0.7(1.0) | 0.6(5) | 1.47 |
| Fits with $l=2$ multipoles: | | | | | | | |
| New world data | 11.5(3) | 2.5(3) | −2.0(1.0) | 4.2(3) | −1.8(1.0) | 0.5(4) | 1.23 |
| Fits without $l=2$ multipoles: | | | | | | | |
| A2, $\sigma$ | 9.5(5) | 4.5(5) | −1.1(1.1) | 3.5(4) | −0.0(1.1) | −1.4(6) | 0.93 |
| A2, $\Sigma_3$ | 7.8(6) | 6.2(6) | 1.2(1.6) | −2.1(1.7) | 0.1(1.2) | 1.8(1.1) | 1.04 |
| A2, $\sigma$ and $\Sigma_3$ | 9.5(4) | 4.5(4) | −1.0(1.1) | 3.5(3) | −0.5(1.1) | −1.1(6) | 0.90 |
| Literature: | | | | | | | |
| DR, A2-collaboration (2022)[b] | 10.99(16)(47)(17)(34) | 3.14(21)(24)(20)(35) | −2.87(52)* | 2.70(43)* | −0.85(72)* | 2.04(43)* | 0.89 |

Table 1: The proton scalar and spin pol. in units $10^{-4}$ fm$^3$ (scalar) and $10^{-4}$ fm$^4$ (spin).

[a] : Errors are given as (fit)(model)

[b] : Errors are given as (statistical)(systematic)(spin polarizability)(model)

* : Values are not fitted, but taken from PHYSICAL REVIEW C 102, 035205 (2020)

# Summary and Conclusions

## PWA of RCS

- Proton polarizabilities related to 2ɣ-exchange in scattering / structure corrections in μH
- In the past: tensions between ChPT and fixed-t DR extractions of polarizabilities
  - **Model-independent ansatz** needed
- **Mainz Partial-Wave-Analysis of RCS**:
  - No resonances below the pion threshold
  - Multipoles are real
  - Forward-scattering is determined via the sum rules (photo absorption cross sections)
- **New precise data from A2 and HIGS**:
  - Preliminary PWA of new world data leads to increased value of $\gamma_{M1M1}$ as compared to Mainz PWA ’18
  - Preliminary PWA of A2 data shows **trend towards a larger magnetic polarizability**, similar to $\beta_{M1}^{\mathrm{BChPT}} = 3.9(7) \times 10^{-4}\,\mathrm{fm}^3$
- **Analysis in progress**

# Back-up

$\chi^2 = 294.9 = 244.8$ (DC) + 37.18 (BAS) + 12.9 (Scaling), Total # data points: 217
DC prediction for energy: 33.5 MeV
DC prediction for energy: 37.5 MeV
DC prediction for energy: 44.2 MeV
DC prediction for energy: 48.2 MeV
DC prediction for energy: 55.0 MeV
DC prediction for energy: 59.0 MeV
DC prediction for energy: 60.0 MeV
DC prediction for energy: 65.0 MeV
DC prediction for energy: 65.8 MeV
DC prediction for energy: 69.0 MeV
DC prediction for energy: 70.0 MeV
DC prediction for energy: 73.2 MeV
DC prediction for energy: 75.0 MeV
DC prediction for energy: 78.0 MeV
DC prediction for energy: 79.0 MeV
DC prediction for energy: 80.0 MeV
DC prediction for energy: 80.9 MeV
DC prediction for energy: 81.3 MeV
DC prediction for energy: 81.8 MeV
DC prediction for energy: 81.9 MeV
DC prediction for energy: 85.4 MeV
DC prediction for energy: 86.3 MeV
DC prediction for energy: 87.0 MeV
DC prediction for energy: 88.0 MeV
DC prediction for energy: 89.0 MeV
DC prediction for energy: 89.8 MeV
DC prediction for energy: 92.2 MeV
DC prediction for energy: 95.0 MeV
DC prediction for energy: 96.8 MeV
DC prediction for energy: 98.0 MeV
DC prediction for energy: 99.0 MeV
DC prediction for energy: 103.2 MeV
DC prediction for energy: 105.0 MeV
DC prediction for energy: 106.0 MeV
DC prediction for energy: 106.7 MeV
DC prediction for energy: 109.0 MeV
DC prediction for energy: 111.1 MeV
DC prediction for energy: 113.6 MeV
DC prediction for energy: 115.0 MeV
DC prediction for energy: 117.0 MeV
DC prediction for energy: 118.0 MeV
DC prediction for energy: 120.0 MeV
DC prediction for energy: 124.1 MeV
DC prediction for energy: 125.0 MeV
DC prediction for energy: 127.0 MeV
DC prediction for energy: 128.0 MeV
DC prediction for energy: 130.0 MeV
DC prediction for energy: 132.0 MeV
DC prediction for energy: 135.0 MeV
DC prediction for energy: 135.3 MeV
DC prediction for energy: 138.0 MeV
DC prediction for energy: 139.0 MeV
DC prediction for energy: 140.0 MeV
DC prediction for energy: 142.0 MeV
DC prediction for energy: 144.0 MeV
DC prediction for energy: 145.0 MeV
Differential Cross-Section [nb]
Angle [deg]