



Role of the left-hand cut on the $T_{cc}(3875)$ & Effective range expansion with the left-hand cut

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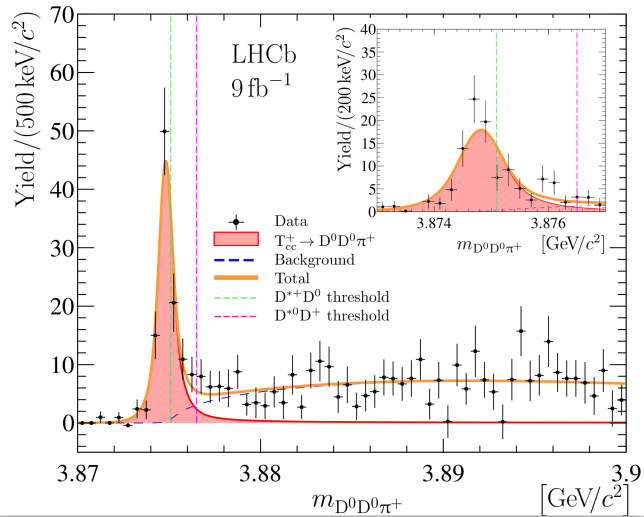
In collaboration with A. Fillin, V. Baru, X.-K. Dong, E. Epelbaum, F.-K. Guo, C. Hanhart, A. Nefedive, J. Nieves, Q. Wang, and B. Wu

Based on PRD 105, 014024(2022), PRL 131,131903 (2023), and arXiv:2408:09375[hep-ph]

Aug.26.2024 @ Ruhr University Bochum, Germany

The 11th International Workshop on Chiral Dynamics (CD2024)

Doubly charmed tetraquark (Tcc)

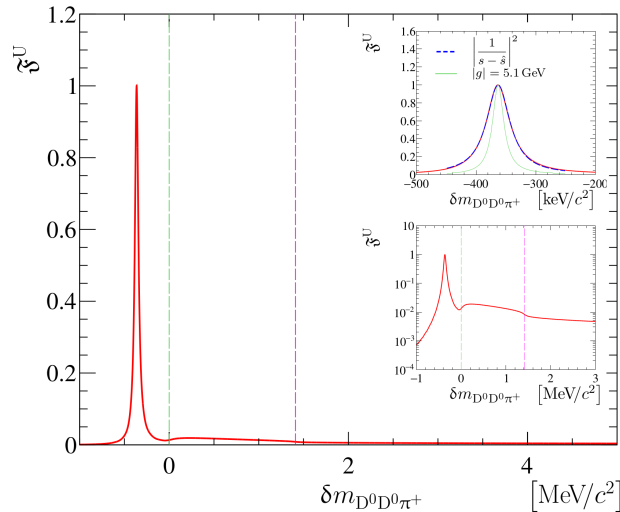


Breit-Wigner fit

LHCb, Nature Phys. 18, (2022) 751

Parameter	Value
N	117 ± 16
δm_{BW}	$-273 \pm 61 \text{ keV}$
Γ_{BW}	$410 \pm 165 \text{ keV}$

$\Rightarrow \Re \sim 400 \text{ keV}$.



Unitarized and analytical

LHCb, Nature Commun. 13 (2022), 3351

$$\delta m = m_{T_{cc}^+} - m_{D^{*+}} - m_{D^0}$$

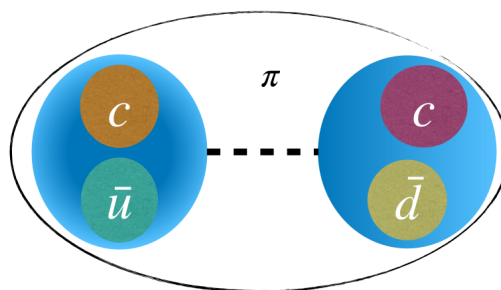
$$\delta m_{\text{pole}} = -360 \pm 40_{-0}^{+4} \text{ keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2_{-14}^{+0} \text{ keV}$$

$\Rightarrow I = 0$: isoscalar

$\hookrightarrow D^+ D^0 \pi^+, D^+ D^+ \text{ } \times$

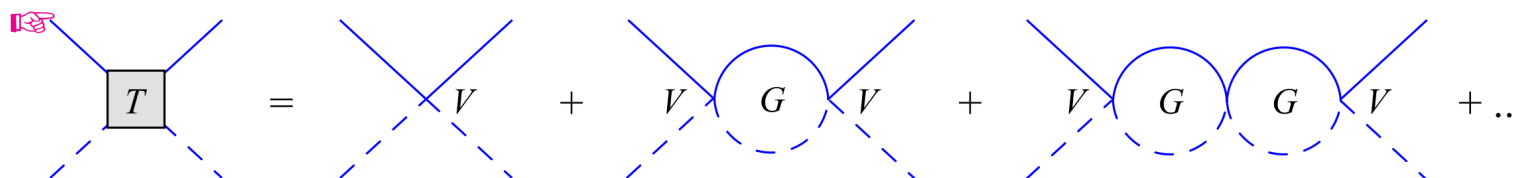
Tcc as a DD* hadronic molecule



☞ T_{cc}^+ resides near D^*D thresholds

LHCb, Nature Commun. 13 (2022)

↪ approximate 90% of $D^0D^0\pi^+$ events contain a D^{*+} .



☞ D^*D isoscalar ($I = 0$) and isovector ($I = 1$)

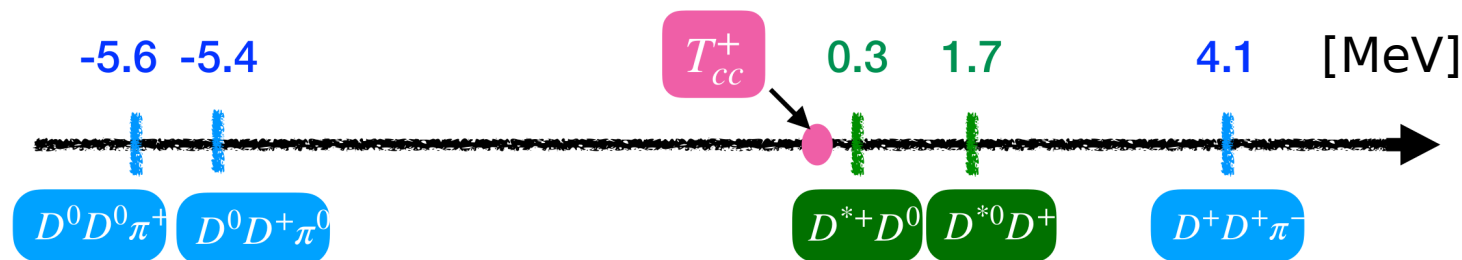
$$|D^*D, I = 0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$

$$|D^*D, I = 1\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+),$$

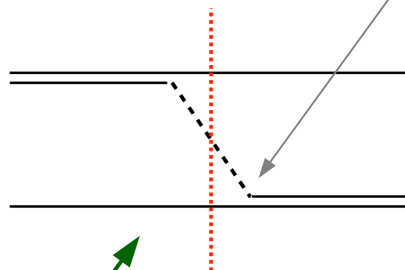
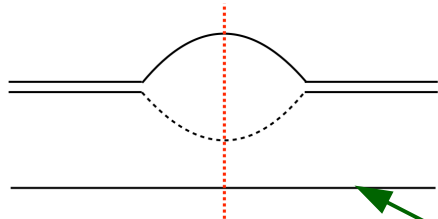
$$V_{\text{CT}}^{I=0}(D^*D \rightarrow D^*D; J^P = 1^+) = v_0,$$

$$V_{\text{CT}}^{I=1}(D^*D \rightarrow D^*D; J^P = 1^+) = v_1.$$

The three-body cut



Three-body cuts



LO Chiral Lagrangian (g determined from $D^* \rightarrow D\pi$)

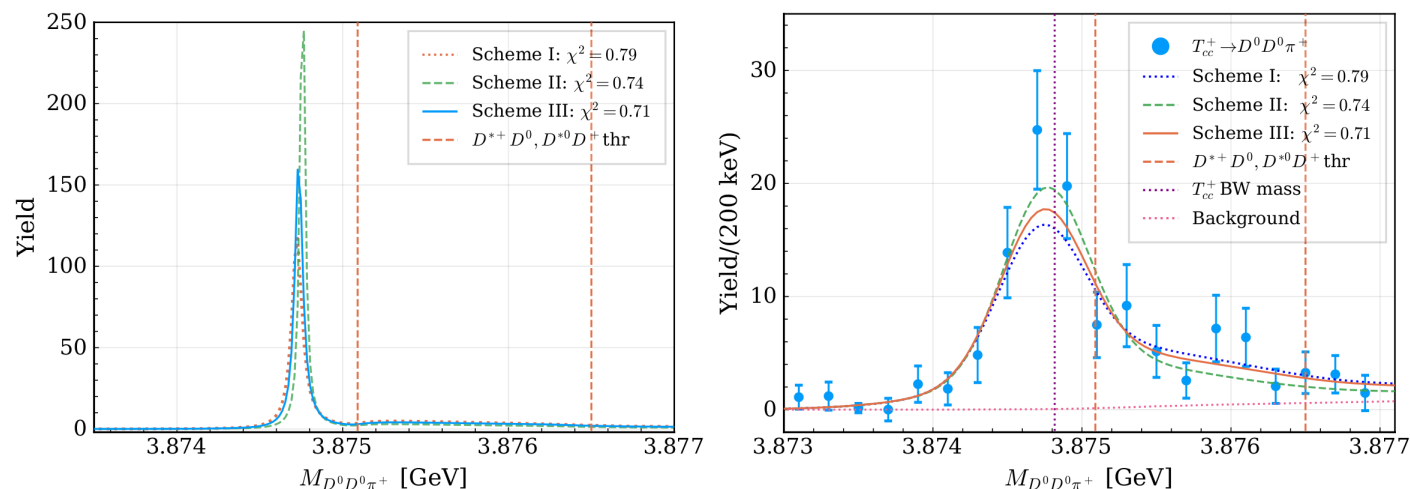
$$\mathcal{L} = \frac{1}{4} g \text{Tr} (\vec{\sigma} \cdot \vec{u}_{ab} H_b H_a^\dagger)$$

$$U_\alpha(M, p) = P_\alpha - \sum_\beta \int \frac{d^3 \vec{q}}{(2\pi)^3} V_{\alpha\beta}(M, p, q) G_\beta(M, q) U_\beta(M, q)$$

$$\hookrightarrow G_\alpha(M, p) = \frac{1}{m_\alpha^* + m_\alpha + \frac{p^2}{2\mu_\alpha} - M - \frac{i}{2} \Gamma_\alpha(M, p)}$$

Description of the experimental data

Du et al., PRD 105, 014024(2022)



Scheme	III	II	I
Pole [keV]	$-356^{+39}_{-38} - i(28 \pm 1)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-368^{+43}_{-42} - i(37 \pm 0)$

Complete 3-bdoy cut

No OPE

No 3-body cut
But constant D^* width

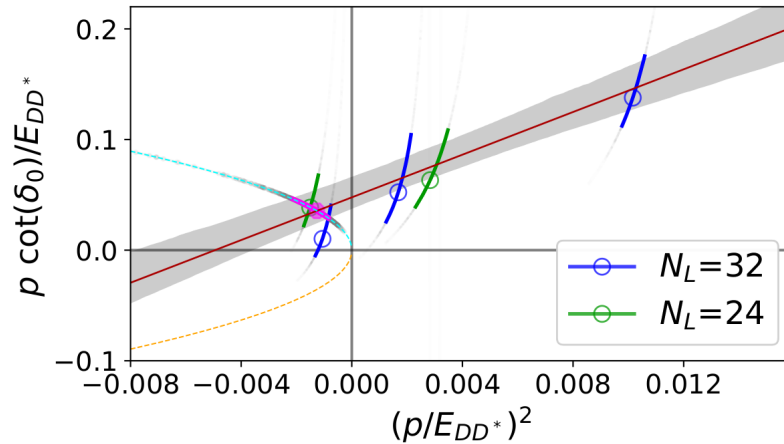
👉 The width of T_{cc}^+

$$56 \text{ keV} \xrightarrow[\text{OPE}]{\text{remove}} 36 \text{ keV} \xrightarrow[\text{M-dep. of } \Gamma^*]{\text{remove}} 74 \text{ keV}$$

Doubly Charm Tetraquark on the Lattice

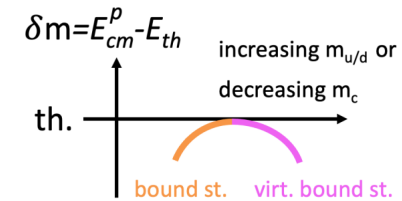
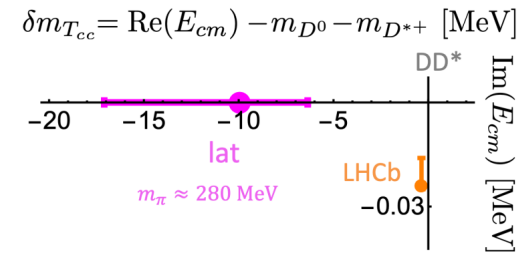
Padmanath *et al*, PRL129,032002(2022)

	m_D (MeV)	m_{D^*} (MeV)	M_{av} (MeV)	$a_{l=0}^{(J=1)}$ (fm)	$r_{l=0}^{(J=1)}$ (fm)	$\delta m_{T_{cc}}$ (MeV)	T_{cc}
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	2049(2)	3103(3)	1.04(29)	$0.96^{(+0.18)}_{(-0.20)}$	$-9.9^{+3.6}_{-7.2}$	Virtual bound st.
Lattice ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	1898(2)	2820(3)	0.86(0.22)	$0.92^{(+0.17)}_{(-0.19)}$	$-15.0^{(+4.6)}_{(-9.3)}$	Virtual bound st.
Experiment [2,41]	1864.85(5)	2010.26(5)	3068.6(1)	-7.15(51)	$[-11.9(16.9), 0]$	-0.36(4)	Bound st.

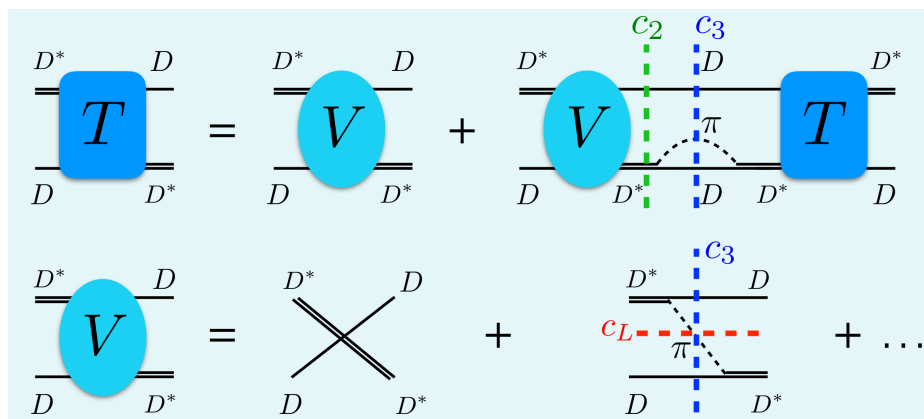


$$t = \frac{E_{cm}}{2} \frac{1}{p \cot \delta - ip},$$

$$p \cot \delta = \frac{1}{a_0} + \frac{1}{2} r_0 p^2,$$



The three-body cut vs. left-hand cut

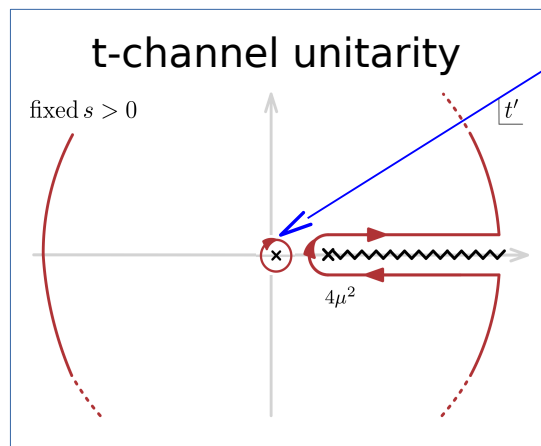


three-body cut

$$E > M_D + M_D + M_\pi$$

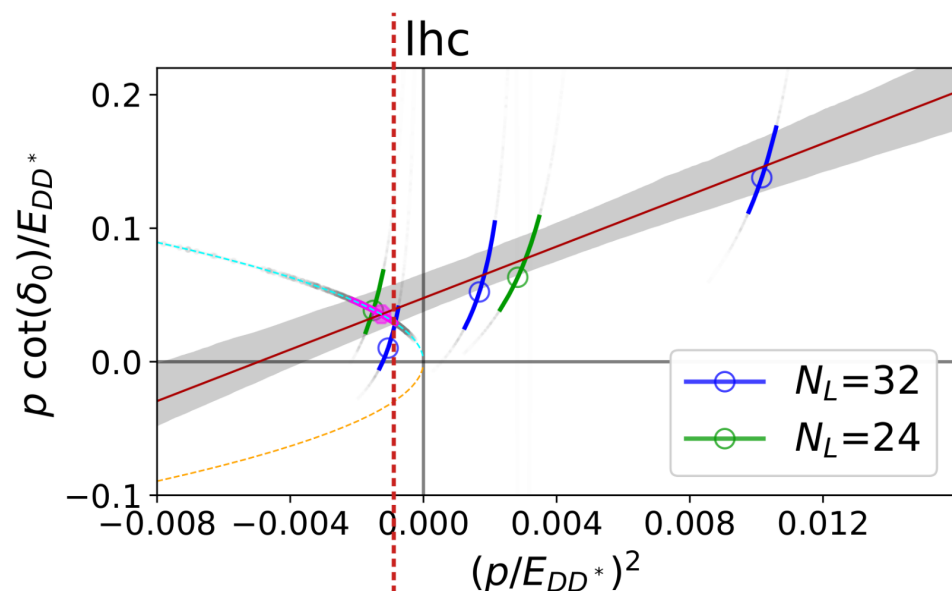
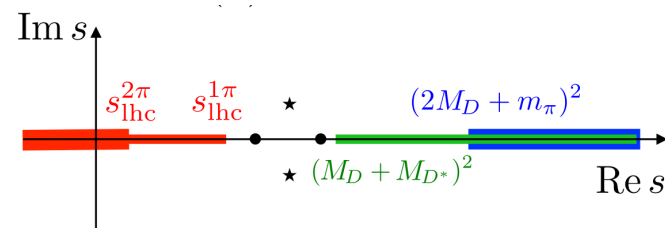
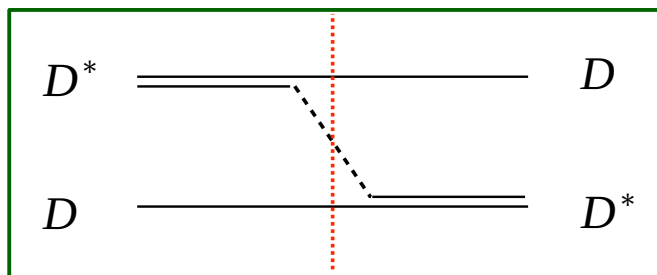
left-hand cut

$$\int_{-1}^1 d \cos \theta G_\pi(E, p, p)$$



$$G_\pi^{-1}(E, k, k') \xrightarrow[\text{on shell: } k=k'=p]{\cos \theta = \pm 1} E_{D^*}(p^2) - E_D(p^2) - \omega_\pi(4p^2/0) = 0$$

The left-hand cut



$$m_\pi = 280 \text{ MeV}$$

☞ two-body branch point:

$$E = M_D + M_{D^*}$$

$$\Rightarrow p_{\text{rhc}2}^2 = 0$$

☞ three-body branch point:

$$E = M_D + M_D + m_\pi$$

$$\Rightarrow \left(\frac{p_{\text{rhc}3}}{E_{DD^*}} \right)^2 = +0.019$$

☞ left-hand cut branch point:

$$\Rightarrow \left(\frac{p_{\text{lhc}}^{1\pi}}{E_{DD^*}} \right)^2 = -0.001$$

$$\left(\frac{\tilde{p}_{\text{lhc}}^{1\pi}}{E_{DD^*}} \right)^2 = -0.190$$

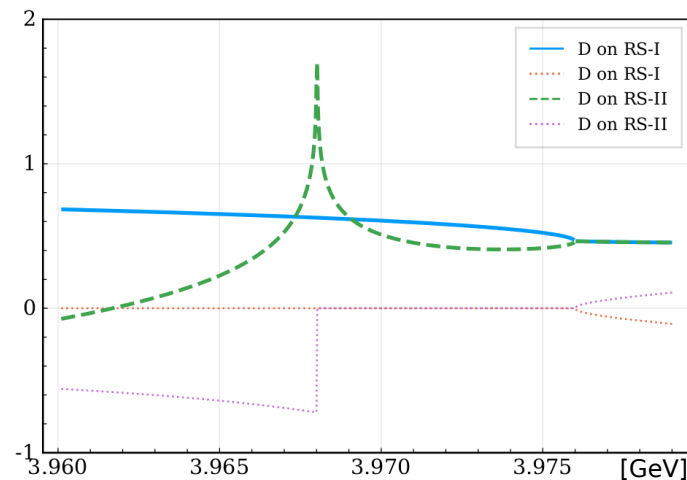
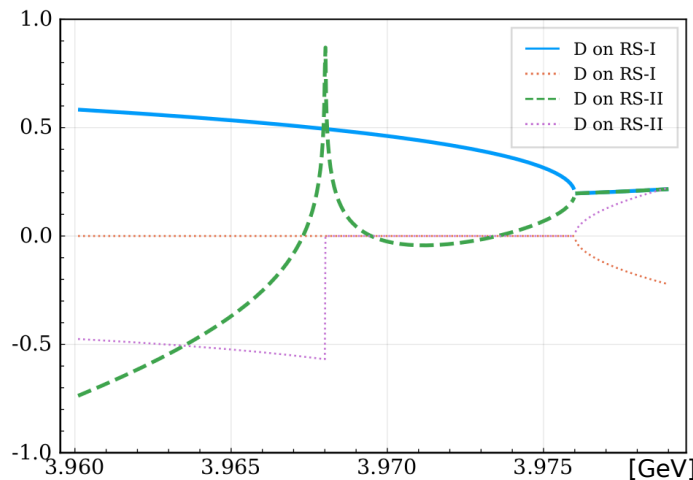
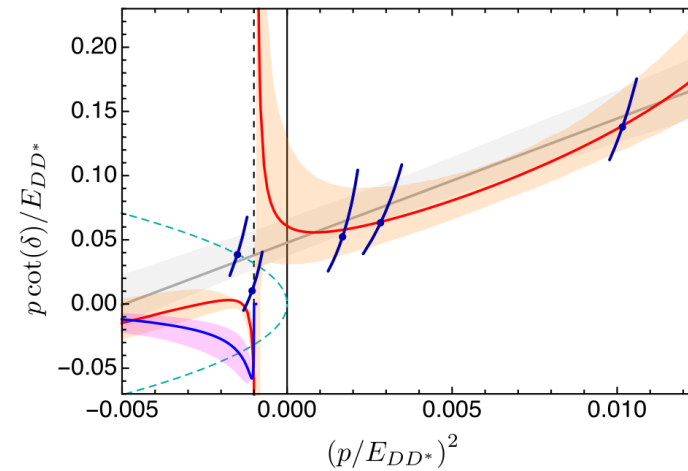
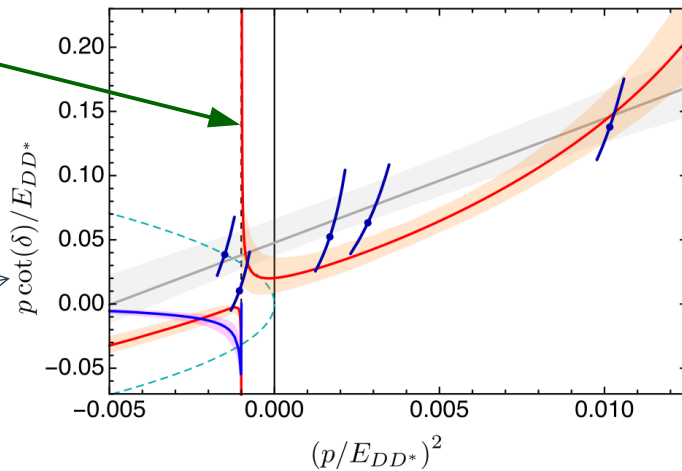
Phase shift with the left-hand cut: LSE

Du et al., PRL 131,131903 (2023)

$$M_D = 1927 \text{ MeV}, M_{D^*} = 2049 \text{ MeV}, m_\pi = 280 \text{ MeV}$$

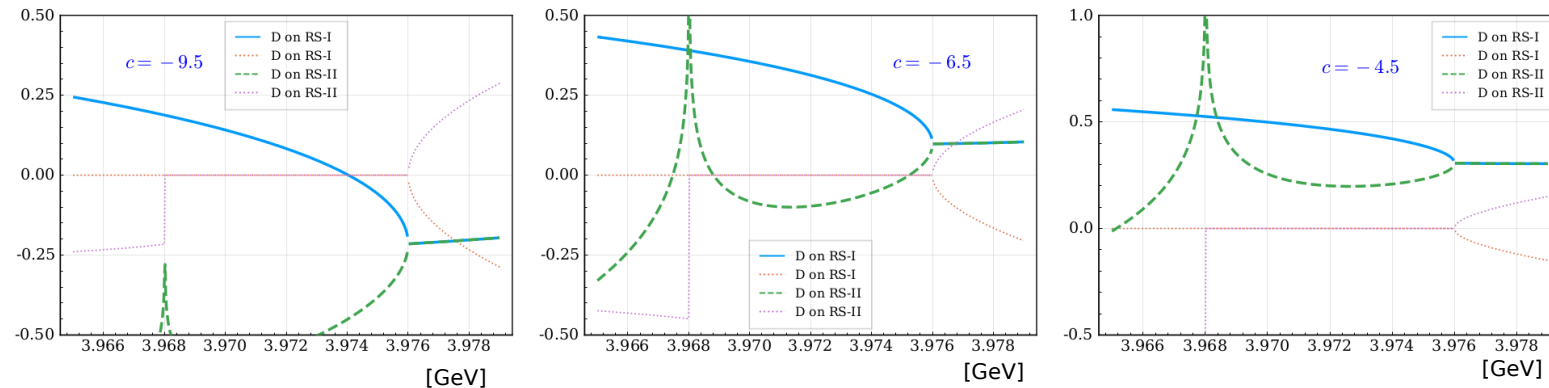
Limit ERE

$$p \cot \delta = -\frac{2\pi}{\mu} \frac{1}{T} + ik$$

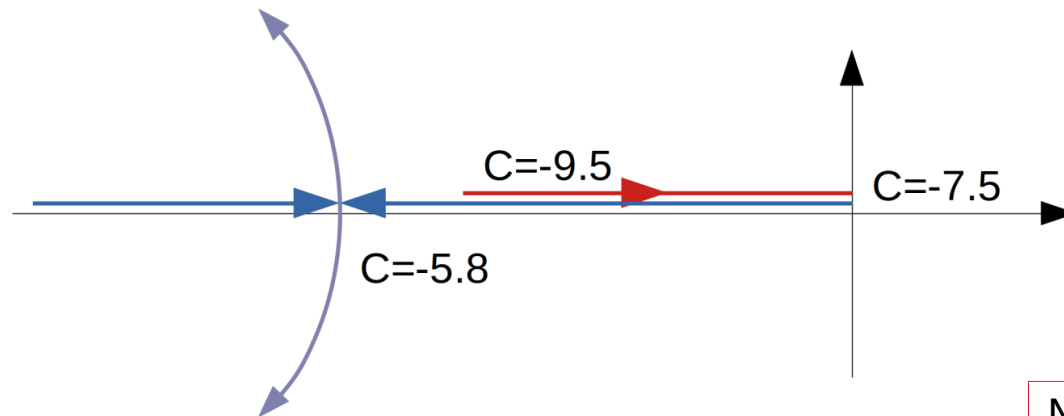


Pole trajectory

$$M_D = 1927 \text{ MeV}, M_{D^*} = 2049 \text{ MeV}, m_\pi = 280 \text{ MeV}$$



bound state $\longrightarrow$ virtual state $\longrightarrow$ resonances below threshold



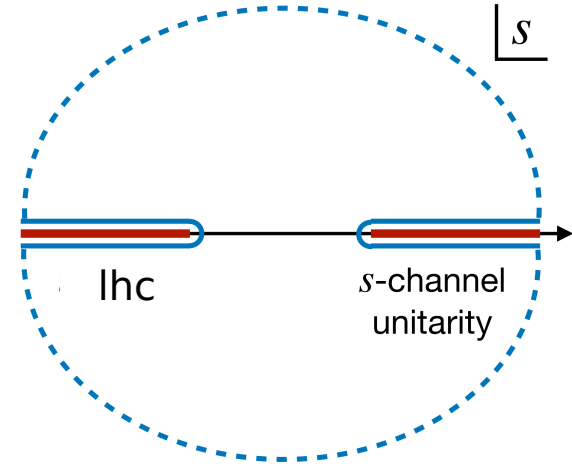
No virtual state along the $1hc$

The N/D method

$$T(s) = \frac{N(s)}{D(s)}$$

$$\text{Im}D = \text{Im}\frac{N}{T} = N\text{Im}\frac{1}{T} = \begin{cases} -N\rho, & s > s_{\text{thr}} \\ 0, & s < s_{\text{thr}} \end{cases}$$

$$\text{Im}N = \begin{cases} \text{Im}TD, & s < s_{\text{lhc}} \\ 0, & s > s_{\text{lhc}} \end{cases}$$



$$D(s) = \sum_i \frac{\gamma_i}{s - s_i} + \sum_{m=0}^{n-1} a_m s^m - \frac{(s - s_0)^n}{\pi} \int_{s_{\text{thr}}}^{\infty} ds' \frac{\rho(s') N(s')}{(s' - s)(s' - s_0)^n},$$

$$N(s) = \sum_{m=0}^{n-\ell-1} b_m s^m + \frac{(s - s_0)^{n-\ell}}{\pi} \int_{-\infty}^{s_{\text{left}}} ds' \frac{\text{Im}T(s') D(s')}{(s' - s_0)^{n-\ell}(s' - s)}.$$

$N=1$

$$D(s) = \sum_i \frac{\gamma_i}{s - s_i} + P(s) + G(s)$$

$$T(s) = \frac{1}{D(s)}$$

The N/D method

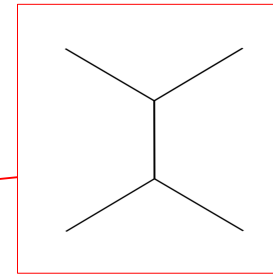
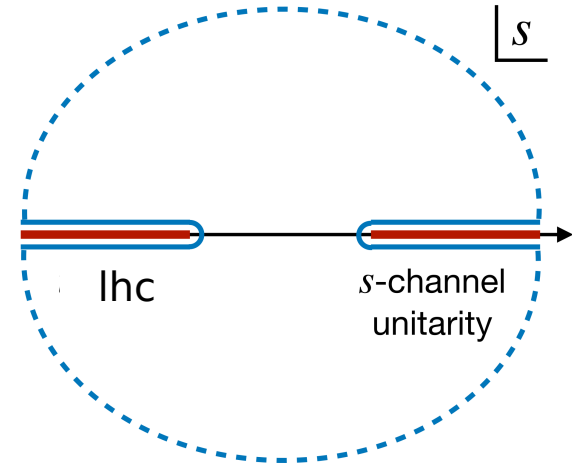
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$$N(s) = \sum_{m=0}^{n-\ell-1} b_m s^m + \frac{(s - s_0)^{n-\ell}}{\pi} \int_{-\infty}^{s_{\text{left}}} ds' \frac{\text{Im}T(s') D(s')}{(s' - s_0)^{n-\ell}(s' - s)}.$$

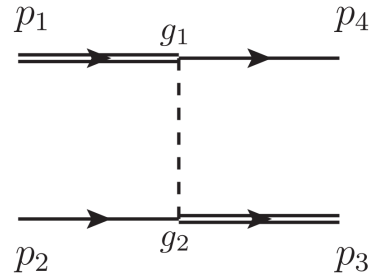
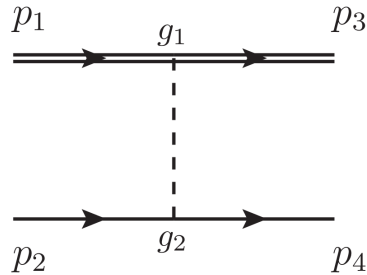


$$\frac{1}{T_\ell^\Pi} = \frac{1}{T_\ell} + 2i\rho \implies T^\Pi = \frac{1}{\frac{D}{N} + 2i\rho} = \frac{N}{D + 2i\rho N}$$

Along the lhc, $i\rho$ and D is real, N has imaginary part.

$$D + 2i\rho N \neq 0$$

The left-hand cut: nonrelativistic



$$\eta = |m_1 - m_2|/(m_1 + m_2)$$

$$\mu_{\text{ex}}^2 = m_{\text{ex}}^2 - (m_1 - m_2)^2$$

$$\mu_+^2 = 4\mu\mu_{\text{ex}}^2/(m_1 + m_2)$$

$$L_t(k^2) \equiv \frac{1}{2} \int_{-1}^{+1} \frac{d \cos \theta}{t - m_{\text{ex}}^2} = -\frac{1}{4k^2} \log \frac{m_{\text{ex}}^2/4 + k^2}{m_{\text{ex}}^2/4},$$

$$L_u(k^2) \equiv \frac{1}{2} \int_{-1}^{+1} \frac{d \cos \theta}{u - m_{\text{ex}}^2} \approx -\frac{1}{4k^2} \log \frac{\mu_+^2/4 + k^2}{\mu_+^2/4 + \eta^2 k^2},$$

$$\frac{1}{2} \int_{-1}^{+1} \frac{(\mathbf{p}_1 - \mathbf{p}_3)^2}{t - m_{\text{ex}}^2} d \cos \theta = -\frac{m_{\text{ex}}^2}{2} \int_{-1}^{+1} \frac{d \cos \theta}{t - m_{\text{ex}}^2} - 1$$

$$\frac{1}{2} \int_{-1}^{+1} \frac{(\mathbf{p}_1 - \mathbf{p}_4)^2}{u - m_{\text{ex}}^2} d \cos \theta \approx -\frac{\mu_{\text{ex}}^2}{2} \int_{-1}^{+1} \frac{d \cos \theta}{u - m_{\text{ex}}^2} - 1$$

$\mathcal{F}_\ell/2$

$$\text{Im } f(k^2) = c \text{Im } L(k^2) = -\frac{c}{4k^2} \pi, \quad \text{for } k^2 < k_{\text{lh}}^2$$

$$f(k^2) = \frac{n(k^2)}{d(k^2)} \quad \text{Im } d(k^2) = -k n(k^2), \quad \text{for } k^2 > 0,$$

$$\text{Im } n(k^2) = d(k^2) \text{Im } f(k^2), \quad \text{for } k^2 < k_{\text{lh}}^2.$$

The N/D method: nonrelativistic

$$n(k^2) = n_m(k^2) + \frac{(k^2)^m}{\pi} \int_{-\infty}^{k_{\text{lh}}^2} \frac{d(k'^2) \text{Im} f(k'^2)}{(k'^2 - k^2)(k'^2)^m} dk'^2$$

$\propto \text{Im} L$

No singularity along lh

$$n(k^2) = n'_m(k^2) + \frac{P(k^2)}{\pi} \int_{-\infty}^{k_{\text{lh}}^2} \frac{\text{Im} f(k'^2)}{k'^2 - k^2} dk'^2 = n'_m(k^2) + P(k^2) \tilde{g} L(k^2)$$

$$n(k^2) = \tilde{n}(k^2) + \tilde{g}(L(k^2) - L_0)$$

$$L_0 = L(k^2 = 0) = -1/\mu_{\text{ex}}^2$$

$$\begin{aligned} d(k^2) &= \tilde{d}(k^2) - ik(\tilde{n}(k^2) - \tilde{g}L_0) - \frac{\tilde{g}}{\pi} \int_0^\infty \frac{k' L(k'^2)}{k'^2 - k^2} dk'^2 \\ &= \tilde{d}(k^2) - ik n(k^2) - \tilde{g} d^{\text{R}}(k^2) \end{aligned}$$

$$d_u^{\text{R}}(k^2) = \frac{i}{4k} \left(\log \frac{\mu_+/2 + ik}{\mu_+/2 - ik} - \log \frac{\mu_+/2 + i\eta k}{\mu_+/2 - i\eta k} \right)$$

Effective range expansion with the left-hand cut

$$\frac{1}{f(k^2)} = \frac{\tilde{d}(k^2) - \tilde{g}d^R(k^2)}{\tilde{n}(k^2) + \tilde{g}(L(k^2) - L_0)} - ik$$

$$\frac{1}{f_{[m,n]}(k^2)} = \frac{\sum_{i=0}^n \tilde{d}_i k^{2i} - \tilde{g}d^R(k^2)}{1 + \sum_{j=1}^m \tilde{n}_j k^{2j} + \tilde{g}(L(k^2) - L_0)} - ik$$

$$f_{[0,1]}(k^2) = \left[\frac{\tilde{d}_0 + \tilde{d}_1 k^2 - \tilde{g}d^R(k^2)}{1 + \tilde{g}(L(k^2) - L_0)} - ik \right]^{-1} \xrightarrow{\tilde{g} \rightarrow 0} \frac{1}{f(k^2)} = \frac{1}{a} + \frac{1}{2} r k^2 - ik$$

Scattering length

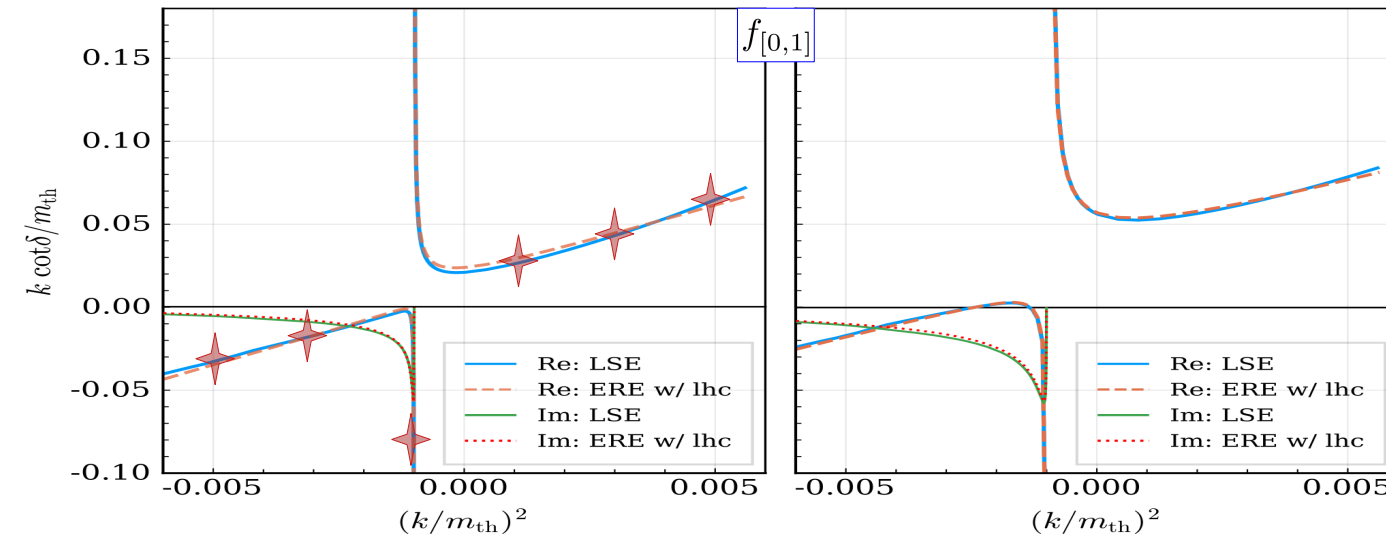
$$a = f(k^2 = 0) = \left[\tilde{d}_0 + \frac{\tilde{g}}{\mu_+} (1 - \eta) \right]^{-1}$$

Effective range

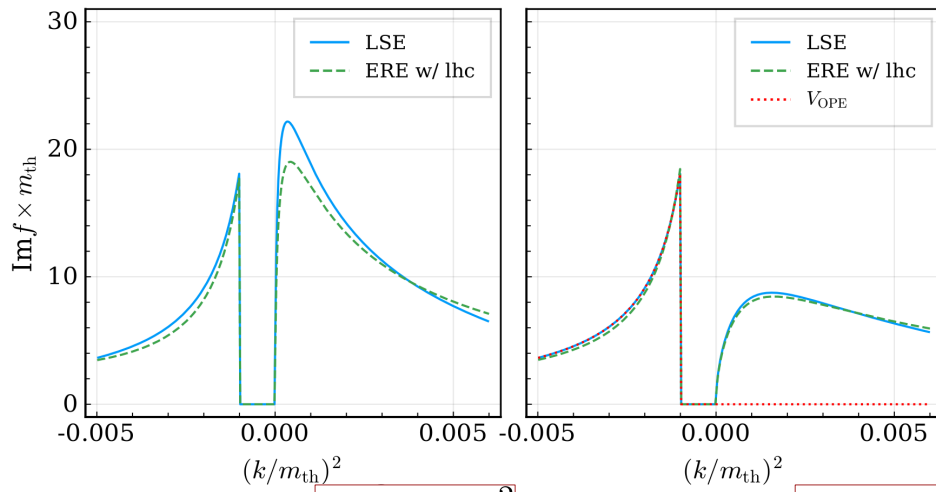
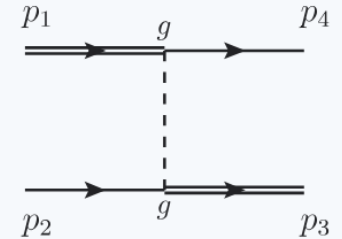
$$r = \left. \frac{d^2(1/f + ik)}{dk^2} \right|_{k=0} = 2\tilde{d}_1 - \frac{8\tilde{g}}{3\mu_+^3} (1 - \eta^3) - \frac{4\tilde{g}}{\mu_+^4 a_u} (1 - \eta^4)$$

Example: Tcc on the Lattice [3 parameters]

Du et al., 2408:09375 [hep-ph]



$$g_{D^* D \pi}^2 / (4F^2) = 9.2 \text{ GeV}^{-2}$$



$$-\frac{2\pi}{\mu} \text{Im} f = \text{Im} T = \text{Im} V_{\text{OPE}}(k^2) = g_P \frac{-\pi}{4k^2} \mathcal{F}_\ell, \quad \text{for } k^2 < k_{\text{lhc}}^2$$

$$\text{Im} n(k^2) = -\tilde{g} \frac{\pi}{4k^2}, \quad \text{for } k^2 < k_{\text{lhc}}^2$$

$$d_u^{0,\text{lhc}} = \tilde{d}_0 - \frac{\tilde{d}_1 \mu_+^2}{4} + \frac{\mu_+}{2} \left(1 + \frac{\tilde{g}}{\mu_{\text{ex}}^2} \right) + \frac{\tilde{g} \log[2/(1+\eta)]}{\mu_+}$$

$$g_P = -\frac{2\pi \tilde{g}}{\mu d^{0,\text{lhc}} \mathcal{F}_\ell}$$

Summary

- The three-body cut: one-pion exchange + self-energy of D^*

$$56 \text{ keV} \xrightarrow[\text{OPE}]{\text{remove}} 36 \text{ keV} \xrightarrow[\text{M-dep. of } \Gamma^*]{\text{remove}} 74 \text{ keV}$$

★ Unphysical pion masses on the Lattice

$$M_D = 1927 \text{ MeV}, M_{D^*} = 2049 \text{ MeV}, m_\pi = 280 \text{ MeV}$$

- ↪ the three-body cut above the two-body cut ($\sqrt{s_{\text{thc}}} = 3968 \text{ MeV}$)
- ↪ The traditional ERE valid only in a very limited range
- ↪ An accurate extraction of the pole requires the OPE implemented

★ The ERE with the left-hand cut

$$f_{[0,1]}(k^2) = \left[\frac{\tilde{d}_0 + \tilde{d}_1 k^2 - \tilde{g} d^R(k^2)}{1 + \tilde{g}(L(k^2) - L_0)} - ik \right]^{-1}$$

- ↪ correct behavior of the left-hand cut
- ↪ can be used to extract the couplings of the exchanged particle to the scattering particles
- ↪ amplitude zeros caused by the interplay between the short- and long-range interactions

$$n(k^2) = 0 \xrightarrow{m_1 - m_2 \ll m_{\text{th}}} \boxed{k_{\text{zero}}^2 = -\frac{\mu_{\text{ex}}^2}{4} \left[1 + \frac{1}{y} W(-e^{-y} y) \right]} \quad y = 1 + \frac{1 + \frac{4}{3} a \mu_{\text{ex}} (1 - \log 4) - \frac{4\pi a \mu_{\text{ex}}^2}{\mu g_F F_\ell}}{2 + a \mu_{\text{ex}} (1 - \mu_{\text{ex}} r/4)}$$

Thank you very much for your attention!