C and *CP* violation in effective field theories and applications to η-meson decays

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HISKP (Theorie) & BCTP

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Chiral Dynamics 2024

Ruhruniversität Bochum, 26/8/2024

Outline

η and η': properties, symmetries, quantum numbers

***C* and *CP* violation**

- EFT framework — Akdag, BK, Wirzba, JHEP **06** (2023) 154
- *C*-odd Dalitz plot asymmetries — Akdag, Isken, BK, JHEP **02** (2022) 137
- relating $\eta \to \pi^0\pi^+\pi^-$ to $\eta \to \pi^0\ell^+\ell^-$ — Akdag, BK, Wirzba, JHEP **03** (2024) 059
- $\eta^{(\prime)} \to \gamma\ell^+\ell^-$, $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$ — Herz BSc thesis 2023

Summary / Outlook

η and η' properties

- quantum numbers $I^G\, J^{PC} = 0^+ 0^{-+}$

  $\longrightarrow$ *C*, *P* eigenstates, all additive quantum numbers are zero

  $\longrightarrow$ flavour-conserving lab for symmetry tests

- η: (largely) (pseudo-)Goldstone boson, $\Gamma_\eta = 1.31\,$keV

  $\longrightarrow$ all decay modes forbidden at leading order by symmetries

  (*C*, *P*, angular momentum, isospin/*G*-parity. . .)

- η': no Goldstone boson due to $U(1)_A$ anomaly, $\Gamma_{\eta'} = 196\,$keV

  $\longrightarrow$ still much narrower than e.g. ω, ϕ

- theoretical methods:
  - ▷ (large-N_c) chiral perturbation theory
  - ▷ dispersion theory (final-state interactions)
  - ▷ (sometimes) vector-meson dominance

- new experiments: JLab Eta Factory (JEF), $\longrightarrow$ L. Gan this afternoon

  Rare Eta Decays with a TPC for Optical Photons (REDTOP)

Patterns of discrete symmetry breaking

- search for *CP* violation beyond the Standard Model (BSM)

Class	Violated	Conserved	Interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor-mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

- class II: *P, CP* violation
  - ▷ QCD θ-term; in general: electric dipole moments (EDMs)
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 2\pi$, $\eta^{(\prime)} \to \pi^+\pi^-\gamma^{(*)}$

    $\longrightarrow$ largely excluded indirectly via EDMs — Gan et al. 2020
- class III: *C, CP* violation ["ToPe" = *T*-odd, *P*-even]
  - ▷ far less discussed
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 3\gamma$, $\eta^{(\prime)} \to \pi^0\gamma^* \dots$

C and CP violation

- $\eta^{(\prime)}$ are $C=+1$ eigenstates: opportunity to test C violation!

Channel	Branching ratio	Note
$\eta \to 3\gamma$	$< 1.6 \times 10^{-5}$	
$\eta \to \pi^0\gamma$	$< 9 \times 10^{-5}$	Violates angular momentum conservation or gauge invariance
$\eta \to \pi^0 e^+ e^-$	$< 7.5 \times 10^{-6}$	C, CP-violating as single-γ process
$\eta \to \pi^0 \mu^+ \mu^-$	$< 5 \times 10^{-6}$	C, CP-violating as single-γ process
$\eta \to 2\pi^0\gamma$	$< 5 \times 10^{-4}$	
$\eta \to 3\pi^0\gamma$	$< 6 \times 10^{-5}$	

- example operators of "dimension 7":

$$\frac{1}{\Lambda^3}\bar{\psi}\gamma_5 D_\mu \psi\, \bar{\chi}\gamma^\mu\gamma_5\chi + \text{h.c.}\,, \qquad \frac{1}{\Lambda^3}\bar{\psi}\sigma_{\mu\nu}\lambda_a\psi G_a^{\mu\lambda} F_\lambda^\nu$$

ψ, χ: quarks, $F^{\mu\nu}$, $G_a^{\mu\nu}$: gauge fields

Khriplovich 1991; Ramsey-Musolf 1999; Kurylov et al. 2001

- electroweak radiative corrections mix class II and class III
  still weaker EDM constraints

A hierarchy of EFTs: from quarks to ToPe ChPT (1)

BSM physics	$p \geq \Lambda$	?	?
SMEFT	$\Lambda > p \geq \Lambda_{\text{EW}}$	$\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$	$L_i, Q_i, l_i, u_i, d_i,$ $H, G, A, Z, W^{\pm}$
LEFT	$\Lambda_{\text{EW}} > p \geq \Lambda_\chi$	$\text{SU}(3)_C \times \text{U}(1)_Q$	ψ, ℓ, ν, G, A
χPT	$\Lambda_\chi > p$	$\text{SU}(3)_R \times \text{SU}(3)_L \times \text{U}(1)_Q$	π, A, ℓ, ν

Standard-Model Effective Field Theory:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \ldots$$

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- ToPe operators at dimension 7?

$$\frac{1}{\Lambda^3}\bar{\psi}\gamma_5 \overleftrightarrow{D}_\mu \psi \, \bar{\chi}\gamma^\mu\gamma_5\chi \qquad \frac{1}{\Lambda^3}\bar{\psi}\sigma_{\mu\nu}\lambda_a\psi G_a^{\mu\lambda}F_\lambda^\nu$$

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- ToPe operators at SMEFT dimension 8—requires Higgs vev!

$$\frac{v}{\Lambda^4} \underbrace{\bar{\psi}\gamma_5 \overleftrightarrow{D}_\mu \psi \, \bar{\chi}\gamma^\mu \gamma_5 \chi}_{\text{chirality-breaking}} \qquad \frac{v}{\Lambda^4} \underbrace{\bar{\psi}\sigma_{\mu\nu}\lambda_a \psi G_a^{\mu\lambda} F_\lambda^\nu}_{\text{chirality-breaking}}$$

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Low-energy Effective Field Theory:

$$\mathcal{L}_{\rm LEFT} = v\tilde{\mathcal{L}}_3 + \mathcal{L}_{\rm QCD+QED} + \frac{1}{v}\tilde{\mathcal{L}}_5 + \frac{1}{v^2}\tilde{\mathcal{L}}_6 + \frac{1}{v^3}\tilde{\mathcal{L}}_7 + \frac{1}{v^4}\mathcal{L}_8 + \ldots$$

- in LEFT, below electroweak scale, retain both

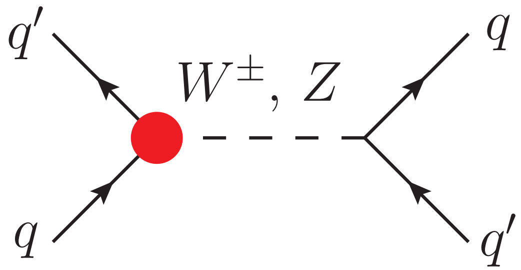
$$\frac{v}{\Lambda^4}\ \text{chirality-breaking} + \frac{1}{\Lambda^4}\ \text{chirality-conserving}$$

A loophole? *W*-exchange in SMEFT

- *claim:* the dim.-7 LEFT operator Shi, Liang, Gardner 2024

$$\bar{\psi}\gamma_5\overleftrightarrow{D}_\mu\psi\,\bar{\chi}\gamma^\mu\gamma_5\chi$$

can be generated in SMEFT using dim.-6 $W^\pm$, Z couplings:



- dimensional scaling changed according to

$$\frac{v}{\Lambda^4} \quad\longrightarrow\quad \frac{v}{\Lambda^2}\frac{1}{M_W^2} \propto \frac{1}{v\,\Lambda^2}$$

  - ▷ same order as *P*-odd, *C*-even EDM ops. beyond θ-term
  - ▷ does something similar work for other ToPe operators, too?

A hierarchy of EFTs: from quarks to ToPe ChPT (2)

How do we match to chiral perturbation theory?

- external-source method — Gasser, Leutwyler 1984
- LEFT operators as
  chirality-violating ($\lambda^{(\dagger)}$) / chirality-conserving ($\lambda_{L,R}$) spurion fields
  analogy: quark masses ($\mathcal{M}$) / quark charges ($q_{L,R}$)
- $SU(3)_L \times SU(3)_R$ invariance, hermiticity, discrete symmetries:

$$\mathcal{L}_{\text{LEFT}} = \frac{v}{\Lambda^4} c_{\psi\chi}^{(a)} \, \bar{\psi} \overleftrightarrow{D}_\mu \gamma_5 \psi \, \bar{\chi} \gamma^\mu \gamma_5 \chi + \ldots$$

$$\longrightarrow \mathcal{L}_\chi = \frac{v}{\Lambda^4} c_{\psi\chi}^{(a)} i g_1^{(a)} \langle \lambda D_\mu U^\dagger + \lambda^\dagger D_\mu U \rangle \langle \lambda_L D^\mu U^\dagger U + \lambda_R D^\mu U U^\dagger \rangle + \ldots$$

$\longrightarrow$ series of chiral operators for each LEFT operator

Applications

How can C be violated in η decays?

- C-odd decay: neutral pseudoscalars + odd no. photons $\propto 1/\Lambda^8$
- SM–C-odd interference: asymmetries $\propto 1/\Lambda^4$

Decay	Mesonic operator	$\mathcal{O}$	Current measurement	Theoretical estimate
$\eta^{(\prime)} \to \pi^0\pi^+\pi^-$	$i\,\eta^{(\prime)}\partial^\mu\pi^0(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)$	$p^2\,(\delta^0)$	$g_2 = -9(5)\cdot 10^3/\mathrm{TeV}^2$	$\lvert g_2\rvert \sim 3\cdot 10^{-4}\,\mathrm{TeV}^2/\Lambda^4$
$\eta' \to \eta\pi^+\pi^-$	$i\,\eta'\partial^\mu\eta(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)$	$p^2\,(\delta^1)$	$g_1 = 1(1)\cdot 10^6/\mathrm{TeV}^2$	$\lvert g_1\rvert \sim 3\cdot 10^{-4}\,\mathrm{TeV}^2/\Lambda^4$
$\eta \to \pi^0 e^+e^-$	$\eta\partial_\mu\pi^0\,\bar{e}\gamma^\mu e$	$p^2\,(\delta^1)$	$\mathrm{BR} < 7.5\cdot 10^{-6}$	$\mathrm{BR} \sim 7\cdot 10^{-27}\,\mathrm{TeV}^8/\Lambda^8$
$\eta' \to \eta e^+e^-$	$\eta'\partial_\mu\eta\,\bar{e}\gamma^\mu e$	$p^2\,(\delta^1)$	$\mathrm{BR} < 2.4\cdot 10^{-3}$	$\mathrm{BR} \sim 9\cdot 10^{-29}\,\mathrm{TeV}^8/\Lambda^8$
$\eta \to \pi^+\pi^-\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta\big(\partial^\nu\pi^+\partial^\rho\partial^\mu\pi^- + \partial^\nu\pi^-\partial^\rho\partial^\mu\pi^+\big)\partial_\rho F^{\alpha\beta}$	$p^6\,(\delta^2)$	$A_{LR} = 0.009(4)$	$\lvert A_{LR}\rvert \sim 5\cdot 10^{-16}\,\mathrm{TeV}^4/\Lambda^4$
$\eta \to \pi^0\pi^0\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta\big(\partial^\nu\pi^0\partial^\rho\partial^\mu\pi^0 + \partial^\nu\pi^0\partial^\rho\partial^\mu\pi^0\big)\partial_\rho F^{\alpha\beta}$	$p^6\,(\delta^3)$	$\mathrm{BR} < 5\cdot 10^{-4}$	$\mathrm{BR} \sim 1\cdot 10^{-29}\,\mathrm{TeV}^8/\Lambda^8$
$\eta' \to \eta\pi^0\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta'\partial^\mu\eta\partial^\nu\pi^0 F^{\alpha\beta}$	$p^4\,(\delta^3)$	–	$\mathrm{BR} \sim 2\cdot 10^{-28}\,\mathrm{TeV}^8/\Lambda^8$
$\eta \to 3\gamma$	$\epsilon^{\mu\nu\rho\sigma}\partial_\alpha\eta(\partial^\gamma F^{\alpha\beta})(\partial_\gamma\partial_\beta F_{\rho\sigma})F_{\mu\nu}$	$p^{10}\,(\delta^4)$	$\mathrm{BR} < 4\cdot 10^{-5}$	$\mathrm{BR} \sim 1\cdot 10^{-36}\,\mathrm{TeV}^8/\Lambda^8$

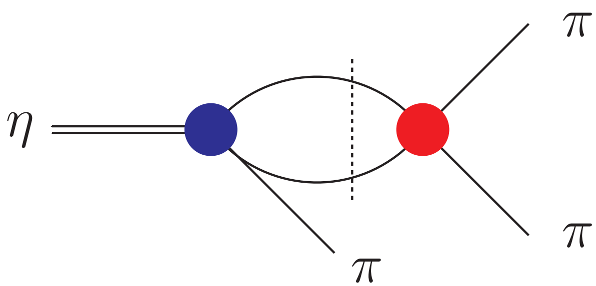
…and many more in Akdag, BK, Wirzba 2022

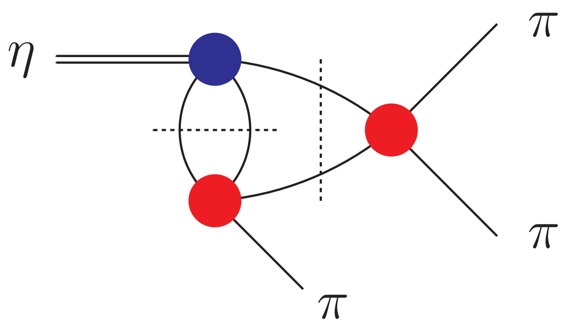
A new old proposal: Dalitz plot asymmetries

- $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$ breaks *G*-parity:
  - ▷ SM: *C* conserved, isospin broken (& el.magn. suppressed)
    $\longrightarrow$ ideal process to extract $m_u - m_d$
    see e.g. Bijnens, Ghorbani 2007; Colangelo et al. 2018 ...
  - ▷ BSM: *C* broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^C(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

- interference: $\pi^+ \leftrightarrow \pi^-$ asymmetries linear in BSM couplings
  Gardner, Shi 2019
- follow SM strategy for hadronic amplitudes: Akdag, Isken, BK 2021
  analyse $\mathcal{M}_{0,2}^{\not C}(s,t,u)$ using dispersive Khuri–Treiman framework





$\eta \to \pi^+\pi^-\pi^0$: amplitude decomposition

- Bose symm.: even (odd) $\pi\pi$ isospin $\leftrightarrow$ even (odd) partial waves
- "reconstruction theorem": symmetrised partial-wave expansion

$$\mathcal{M}_1^{C}(s,t,u) = \mathcal{F}_0(s) + (s-u)\,\mathcal{F}_1(t) + (s-t)\,\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not{C}}(s,t,u) = (t-u)\,\mathcal{G}_1(s) + (u-s)\,\mathcal{G}_1(t) + (s-t)\,\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not{C}}(s,t,u) = 2(u-t)\,\mathcal{H}_1(s) + (u-s)\,\mathcal{H}_1(t) + (s-t)\,\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

$\longrightarrow$ rescattering for *S*- and *P*-waves

Gardner, Shi 2019

cf. also Bernard et al. 2024 for $K \to 3\pi$

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- note: *C*-even/odd $\leftrightarrow$ even/odd under $t \leftrightarrow u$
- *T*-odd requires $\mathcal{M}_{0,2}^{\not{C}}$ relatively imaginary w.r.t. $\mathcal{M}_1^{C}$ at tree level

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- *T*-odd requires $\mathcal{M}_{0,2}^{\not C}$ relatively imaginary w.r.t. $\mathcal{M}_1^C$ at tree level
- Omnès solutions ($\mathcal{A}_I = \mathcal{F}_I, \mathcal{G}_I, \mathcal{H}_I$):

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^n}\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right)$$

  ▷ $P_{n-1}(s)$: subtraction polynomial, free parameters

$\eta \to \pi^+\pi^-\pi^0$: parameters, data

SM amplitude $\mathcal{M}_1^C$

- minimal subtraction scheme: 3 (real) constants
- "data" fit to
  - ▷ KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$ KLOE 2016
  - ▷ A2 Dalitz plot $\eta \to 3\pi^0$ A2 2018
  - ▷ chiral constraints [at $\mathcal{O}(p^4)$] Colangelo et al. 2018

  $\longrightarrow$ χ^2/dof ≈ 1.054, works very well!

BSM amplitude $\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$

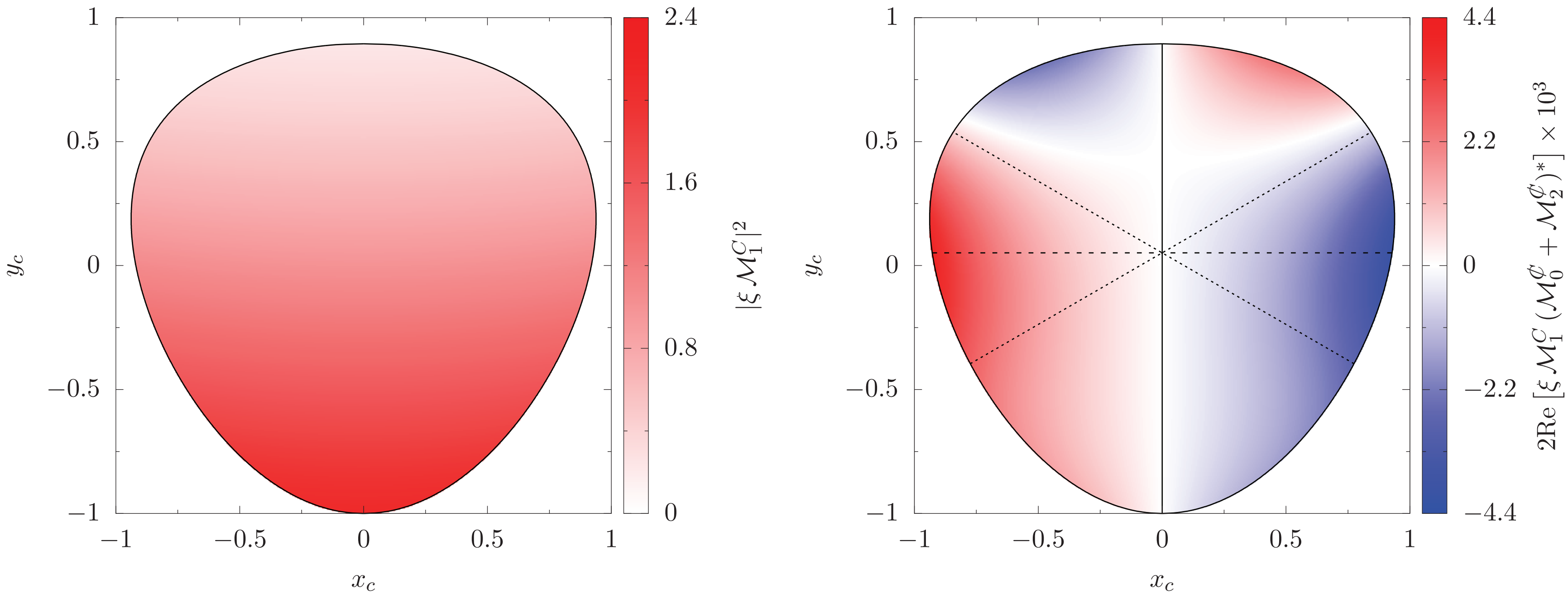
- by same assumptions: 1 imaginary subtraction each for $\mathcal{M}_{0,2}^{\not C}$

  act as overall normalisation constants $\longrightarrow$ χ^2/dof ≈ 1.048
- all *C*-/*CP*-violating signals vanish within $(1-2)\sigma$

$\eta \to \pi^+\pi^-\pi^0$: Dalitz plot asymmetries

- Dalitz plot decomposition (central fit result)

$$\left|\mathcal{M}_c\right|^2 \approx \left|\mathcal{M}_1^{C}\right|^2 + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_0^{\not C}\right)^*\right] + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_2^{\not C}\right)^*\right]$$



- asymmetries constrained to the permille level
- nonvanishing interference due to strong FSI phases!

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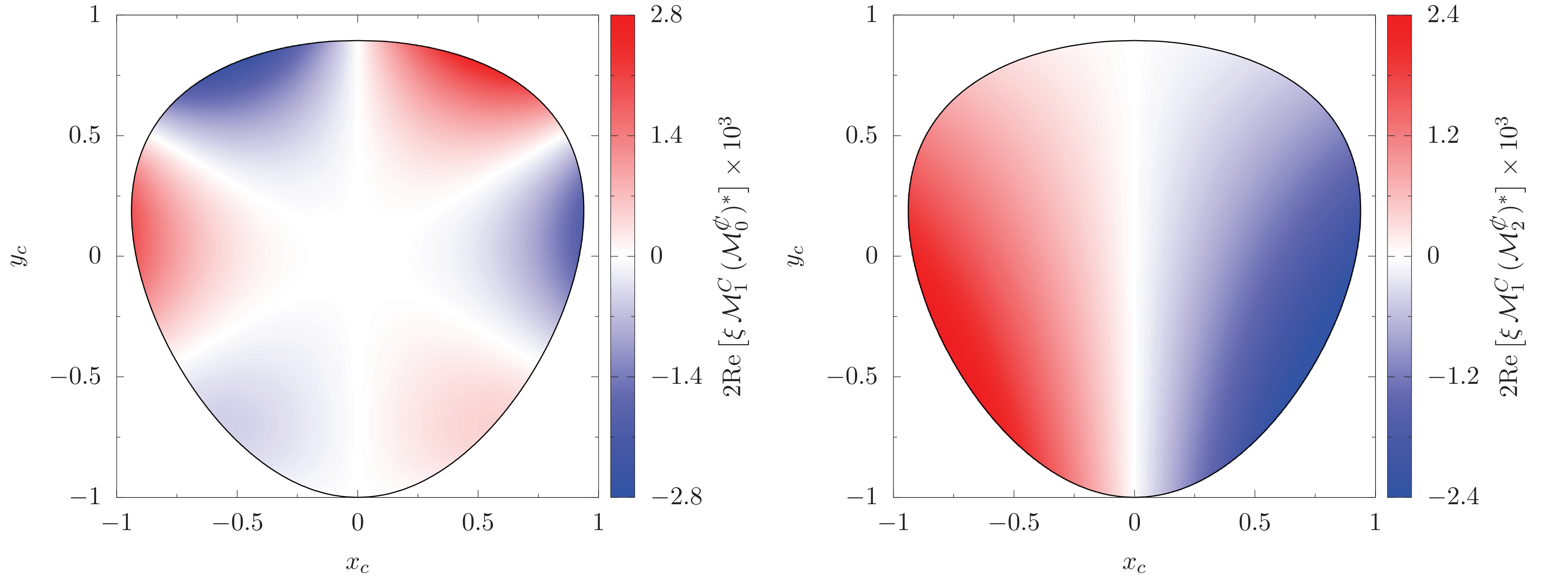


- asymmetries constrained to the permille level
- nonvanishing interference due to strong FSI phases!
- $\mathcal{M}_0^{\not C}$ and $\mathcal{M}_2^{\not C}$ lead to different interference patterns

Effective BSM couplings

Akdag, Isken, BK 2021

- polynomial ambiguities $\longrightarrow$ subtractions no good observables
- define unambigous Taylor invariants & match to these:

$$\mathcal{M}_0^{\not C}(s,t,u) = i\, g_0\,(s-t)(u-s)(t-u) + \mathcal{O}(p^8)$$
$$\mathcal{M}_2^{\not C}(s,t,u) = i\, g_2\,(t-u) + \mathcal{O}(p^4)$$

- fit corresponds to

$$g_0 = -2.8(4.5)\,\text{GeV}^{-6}\,, \quad g_2 = -9.3(4.6)\times 10^{-3}\,\text{GeV}^{-2}$$

$\longrightarrow$ sensitivity $|g_0/g_2| \sim 10^3\,\text{GeV}^{-4} = \mathcal{O}(M_\pi^{-4})$

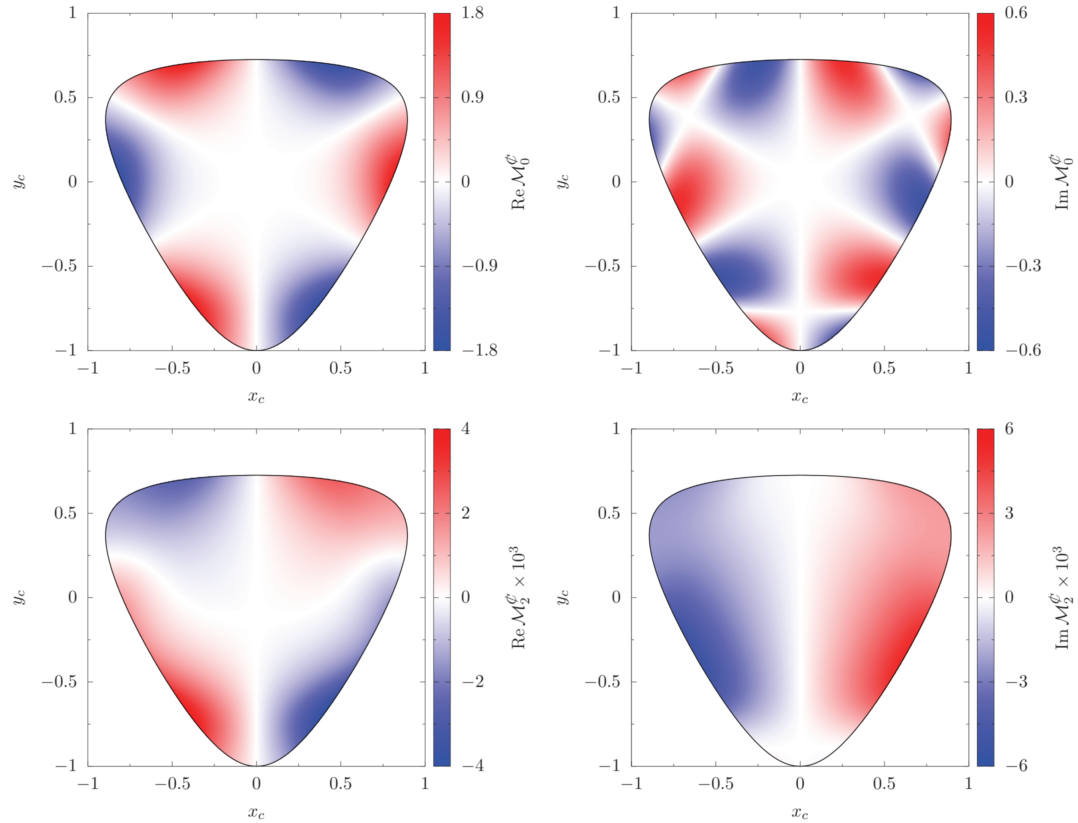
$\longrightarrow$ theoretical/chiral expectation: $|g_0/g_2| \sim \text{GeV}^{-4}$

- small phase space ($M_\eta - 3M_\pi \sim M_\pi$) reduces sensitivity to $\mathcal{M}_0^{\not C}$

Generalisation to η' decays

$\eta' \to \pi^+\pi^-\pi^0$

- rather rare, $\mathcal{B} \sim 3.6 \times 10^{-3} \longrightarrow$ data not so precise BESIII 2016
- rescale $\eta \to \pi^+\pi^-\pi^0$ with same $g_{0,2} \longrightarrow$ more sensitive to g_0 by factor ~ 100



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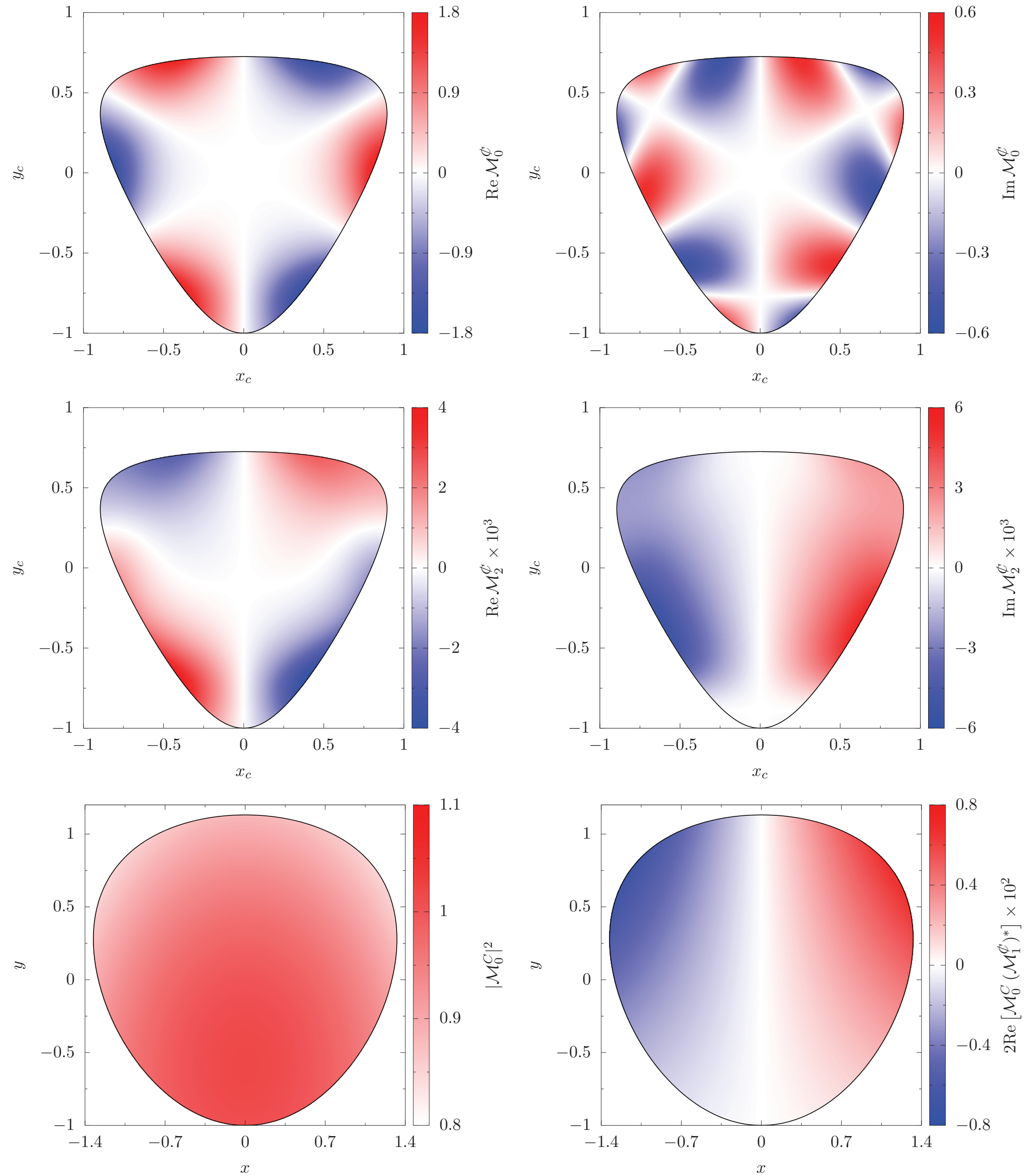
$\eta' \to \eta\pi^+\pi^-$

- SM conserves isospin
- *C*-odd op. $\Delta I = 1$ $\longrightarrow$ constrained at 10^{-2} BESIII 2017



Extension: $\eta \to \pi^0 \ell^+ \ell^-$

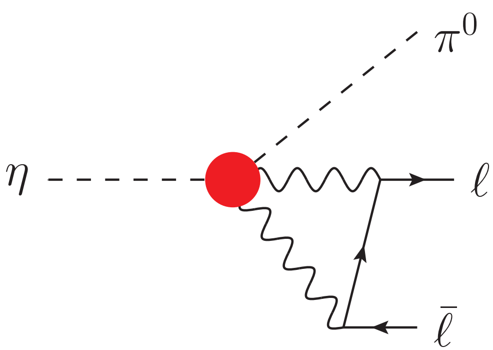
- gauge invariance: $\eta(p) \to \pi^0(k)\gamma^*(q)$ vanishes for real photon:

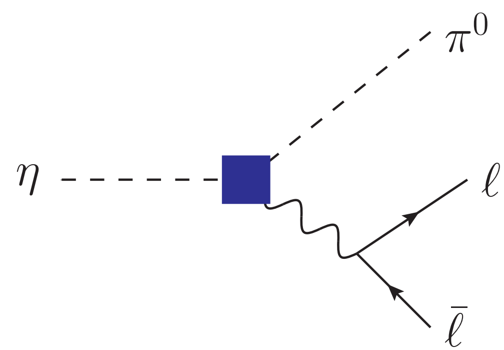
$$\langle \pi^0(k)|j_\mu(0)|\eta(p)\rangle = e\big[q^2(p+k)_\mu - (M_\eta^2 - M_\pi^2)q_\mu\big]F_{\eta\pi^0}(q^2)$$

$\longrightarrow$ can only measure dilepton decays

- *C*-even two-photon decay as Standard Model background:

$\longrightarrow$ H. Schäfer's talk this afternoon





e.g. $\mathcal{B}(\eta \to \pi^0 e^+ e^-) = 1.36(15) \times 10^{-9}$
$\mathcal{B}(\eta \to \pi^0 \mu^+ \mu^-) = 0.67(7) \times 10^{-9}$

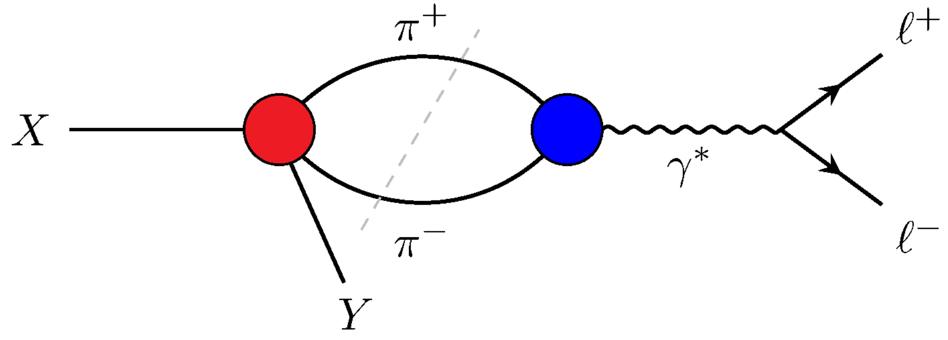
based on VMD model for $\eta \to \pi^0\gamma^*\gamma^*$ Schäfer, Zanke, Korte, BK 2023

$\longrightarrow$ 3 orders of magnitude below current experimental limits

Correlating $\eta \to \pi^0\pi^+\pi^-$ and $\eta \to \pi^0\ell^+\ell^-$ (1)

- can correlate $\eta \to \pi^0\pi^+\pi^-$ *P*-wave and (isovector) $\eta \to \pi^0\ell^+\ell^-$:

  Akdag, BK, Wirzba 2023



$$F^{(1)}_{XY}(s) = \frac{i}{48\pi^2}\int_{4M_\pi^2}^{\infty} dx\,\sigma_\pi^3(x) F_\pi^{V*}(x)\,\frac{f_{XY}(x)}{x-s}$$

pion form factor $F_\pi^V(s) = \Omega_1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} dx\,\frac{\delta_1^1(x)}{x(x-s)}\right\}$

$\longrightarrow$ analogy to $\omega \to 3\pi \leftrightarrow \omega \to \pi^0\gamma^*$ Schneider, BK, Niecknig 2012

- add isoscalar form factor (ω-exchange) using isospin + vector-meson dominance

  $\longrightarrow$ hadronic long-range contribution to *C*-odd form factor

Correlating $\eta \to \pi^0\pi^+\pi^-$ and $\eta \to \pi^0\ell^+\ell^-$ (2)

- isovector contribution scales with C-odd $\eta \to \pi^0\pi^+\pi^-$ couplings:

	isovector	isovector + isoscalar	experiment
$\mathcal{B}(\eta \to \pi^0 e^+e^-)$	$< 20 \cdot 10^{-6}$	$< 29 \cdot 10^{-6}$	$< 7.5 \cdot 10^{-6}$
$\mathcal{B}(\eta \to \pi^0 \mu^+\mu^-)$	$< 7.2 \cdot 10^{-6}$	$< 10 \cdot 10^{-6}$	$< 5.0 \cdot 10^{-6}$

- isovector and isoscalar contributions of comparable magnitude
- direct experimental limit stronger than indirect from $\eta \to \pi^0\pi^+\pi^-$
- refined regression to $\eta \to \pi^0\pi^+\pi^-$ with $\eta \to \pi^0\ell^+\ell^-$ as constraint:

$$|g_0| < 7.3\,\text{GeV}^{-6} \quad \longrightarrow \quad |g_0| < 4.4\,\text{GeV}^{-6}$$

- similar relations $\eta' \to \eta\pi^+\pi^- \leftrightarrow \eta' \to \eta\ell^+\ell^-$ worked out

  Akdag, BK, Wirzba 2023

- beware: interference with short-range effects neglected
  (C-odd photon couplings, leptonic operators)

Dalitz plot asymmetries in $\eta^{(\prime)} \to \gamma\ell^+\ell^-$

- Dalitz decays; SM: given by transition form factors $F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$

  $\longrightarrow$ S. Holz' talk this afternoon

- *C*-odd effects can be induced by quark–lepton operators

$$\bar{\ell}\gamma^\mu\gamma_5\ell\,\bar{\psi}\overleftrightarrow{D}_\mu\gamma_5\psi \quad \text{etc.}$$

- Dalitz plot asymmetries from *C*-odd/*C*-even interference

$$\mathrm{Re}\left(\mathcal{M}_{\mathsf{BSM}}\mathcal{M}_{\mathsf{SM}}^\dagger\right) \propto \mathrm{Im}\,F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$$

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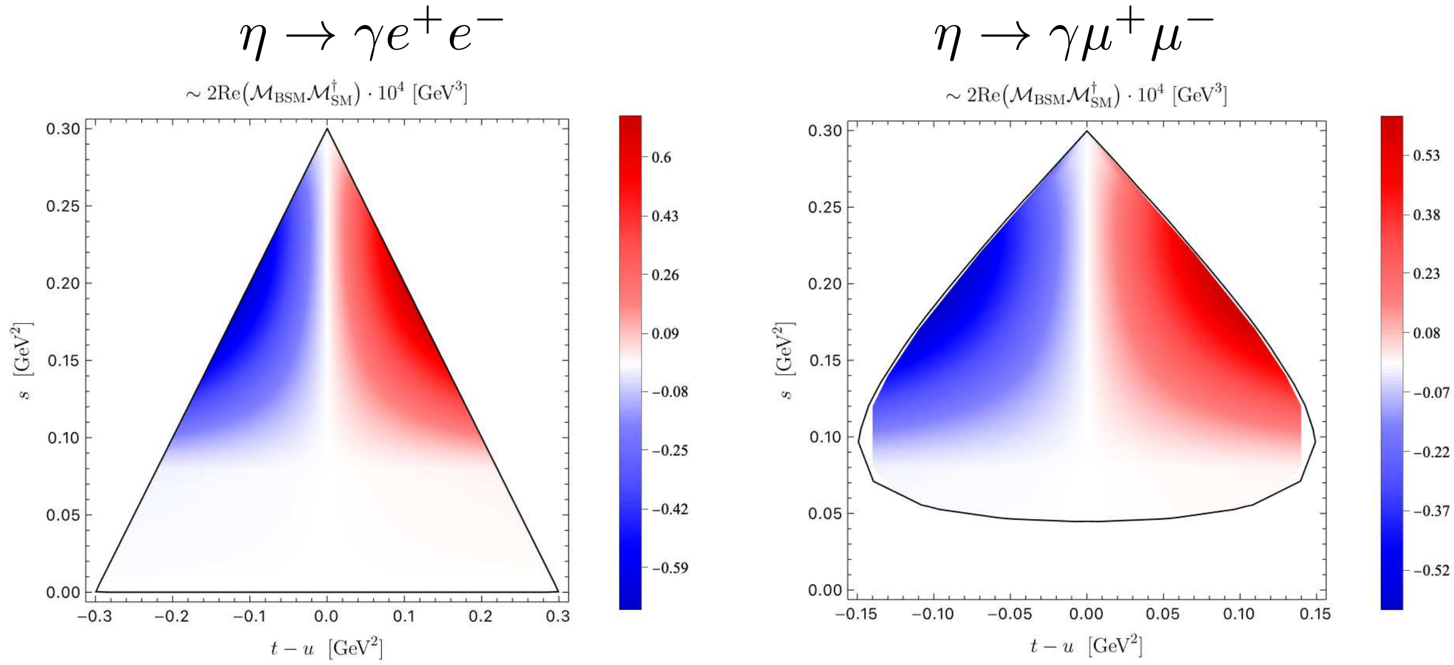

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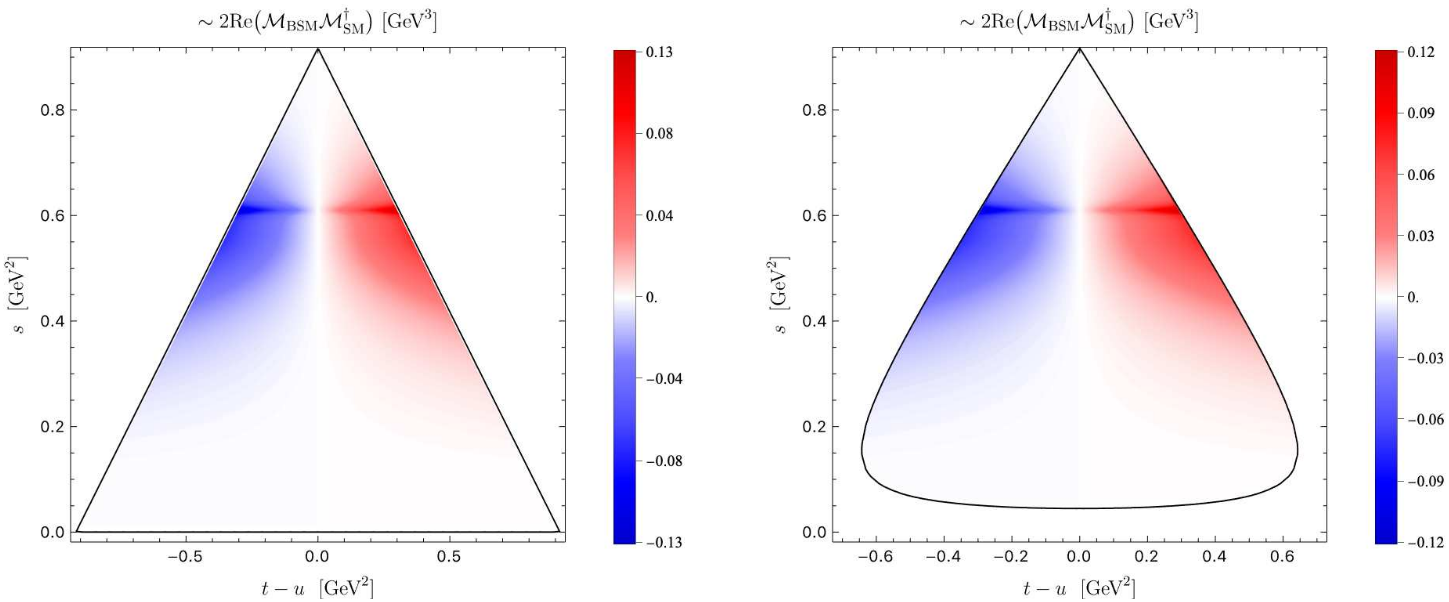
$\eta' \to \gamma e^+e^-$ $\qquad\qquad$ $\eta' \to \gamma\mu^+\mu^-$



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$$\mathrm{Re}\left(\mathcal{M}_{\text{BSM}}\mathcal{M}_{\text{SM}}^\dagger\right) \propto \mathrm{Im}\, F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$$

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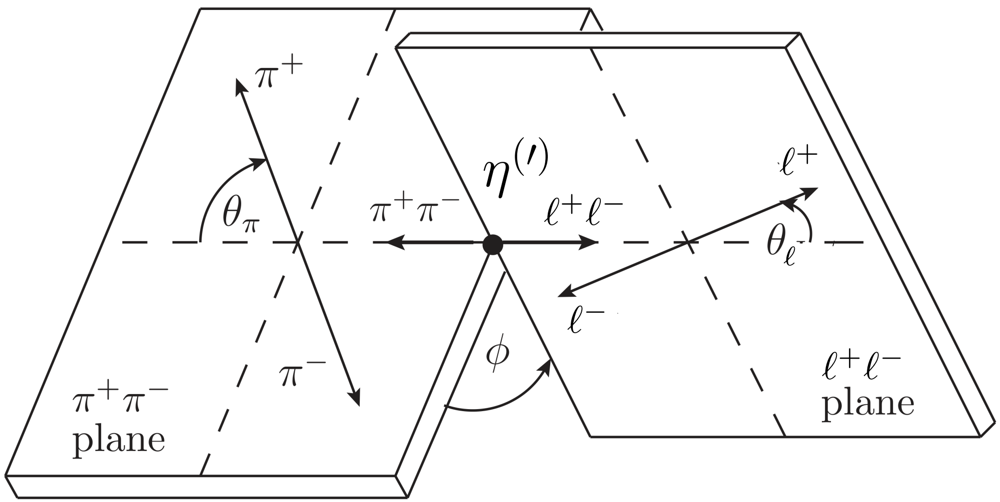
- consequences:
  - ▷ Asym$(\eta' \to \gamma\ell^+\ell^-) \gg$ Asym$(\eta \to \gamma\ell^+\ell^-)$

    $\longrightarrow \rho(770), \omega(782)$ resonances
  - ▷ Asym$(\eta^{(\prime)} \to \gamma\mu^+\mu^-) >$ Asym$(\eta^{(\prime)} \to \gamma e^+e^-)$

    $\longrightarrow$ dominance of photon pole
  - ▷ no such asymmetries generated for $\pi^0 \to \gamma\ell^+\ell^-$

    $\longrightarrow$ dilepton masses always below $\pi^+\pi^-$ cut

Asymmetries in $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$



- SM amplitude $\mathcal{F}(s, s_\ell) \propto f_1(s) \times \bar{F}(s_\ell)$ dispersively   Holz et al. 2021
- *P*-odd, *C*-even operators induce plane–plane asymmetries

  $\propto \sin 2\phi$ [Gao 2002] or $\propto \sin\phi$ [Zillinger et al. 2022]
- *P*-even, *C*-odd operators can induce   Herz BSc 2023
  - ▷ asymmetry in $\theta_\ell \to \theta_\ell + \pi$ $\propto$ $\text{Im}\,\bar{F}(s_\ell)$ ($\longrightarrow$ only η') *or*
  - ▷ asymmetry in $\theta_\pi \to \theta_\pi + \pi$ $\propto$ $\text{Im}\,f_1(s)$ ($\longrightarrow$ larger for η')

  (*C*-odd *P*-*D*-wave interference as in $\eta^{(\prime)} \to \pi^+\pi^-\gamma$)

$\longrightarrow$ exp. studies of more general asymmetries strongly advised!

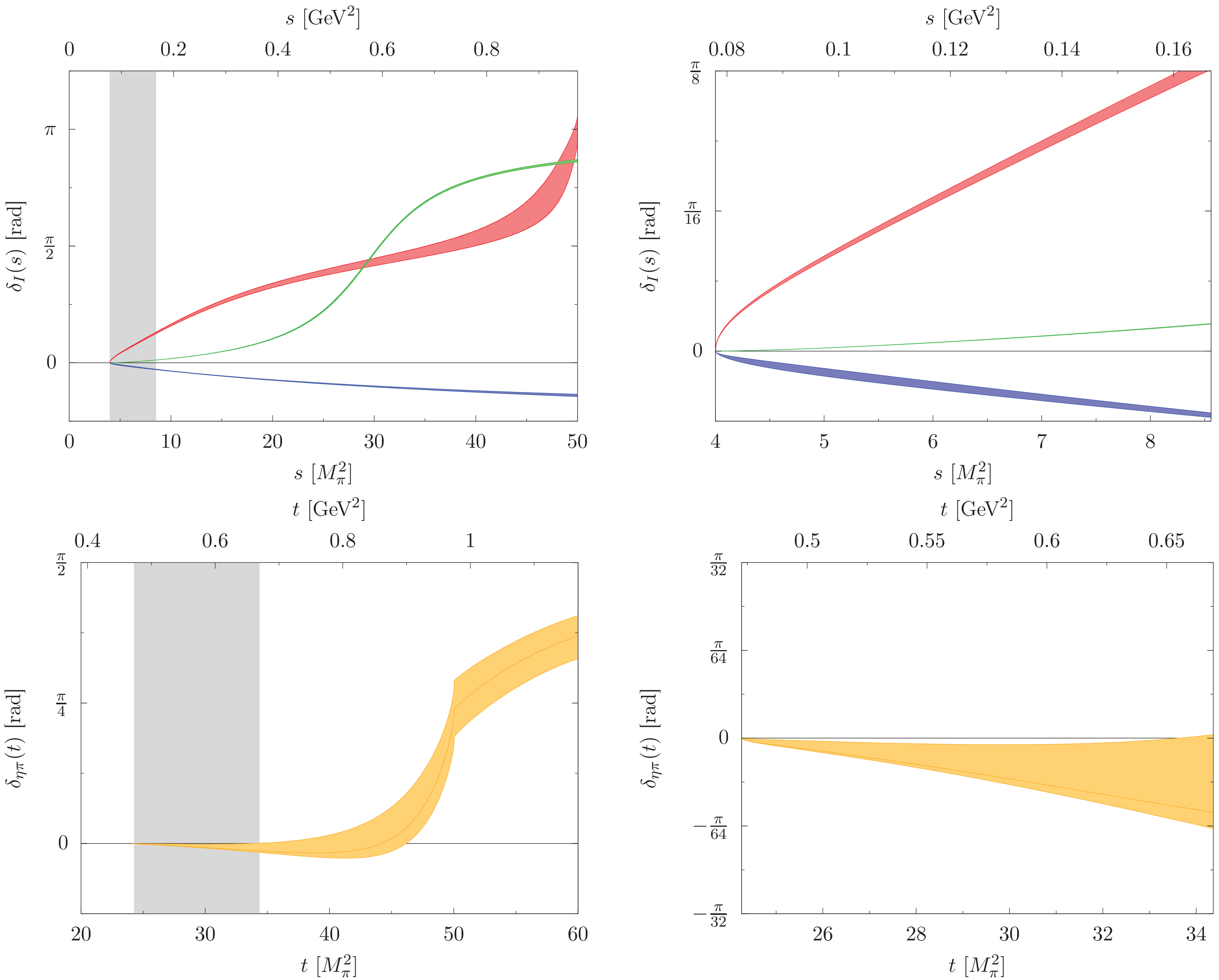
Summary & Outlook

C and *CP* violation

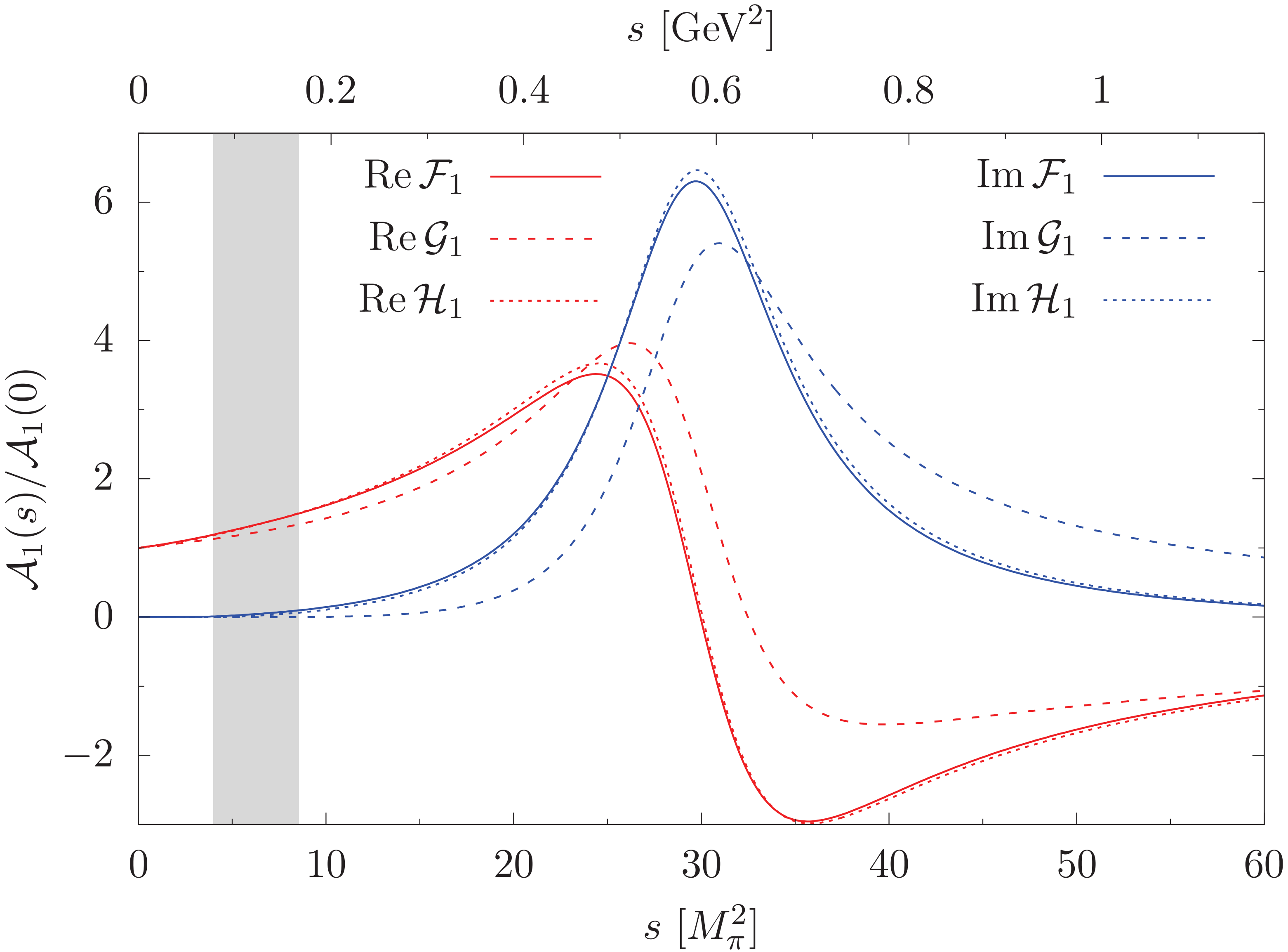
- fundamental (LEFT) operator basis established
  $\longrightarrow$ correct scaling in SMEFT to be checked
- matching to ChPT performed
- dispersive formalism for amplitude analysis of Dalitz plots
  $\eta^{(\prime)} \to \pi^+\pi^-\pi^0$, $\eta' \to \eta\pi^+\pi^-$ worked out
  $\longrightarrow$ asymmetries due to SM–BSM interferences
  $\longrightarrow$ strong rescattering central
- dispersion-theoretical correlations between Dalitz plot asymmetries and semileptonic decays $\eta \to \pi^0\ell^+\ell^-$ etc.
- $\eta^{(\prime)} \to \gamma\ell^+\ell^-$, $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$:
  further opportunities to test SM–BSM interferences
  $\longrightarrow$ sensitivity $\propto \mathrm{Im}\,(\text{SM})$ favours η' decays

Spares

Phase shift input

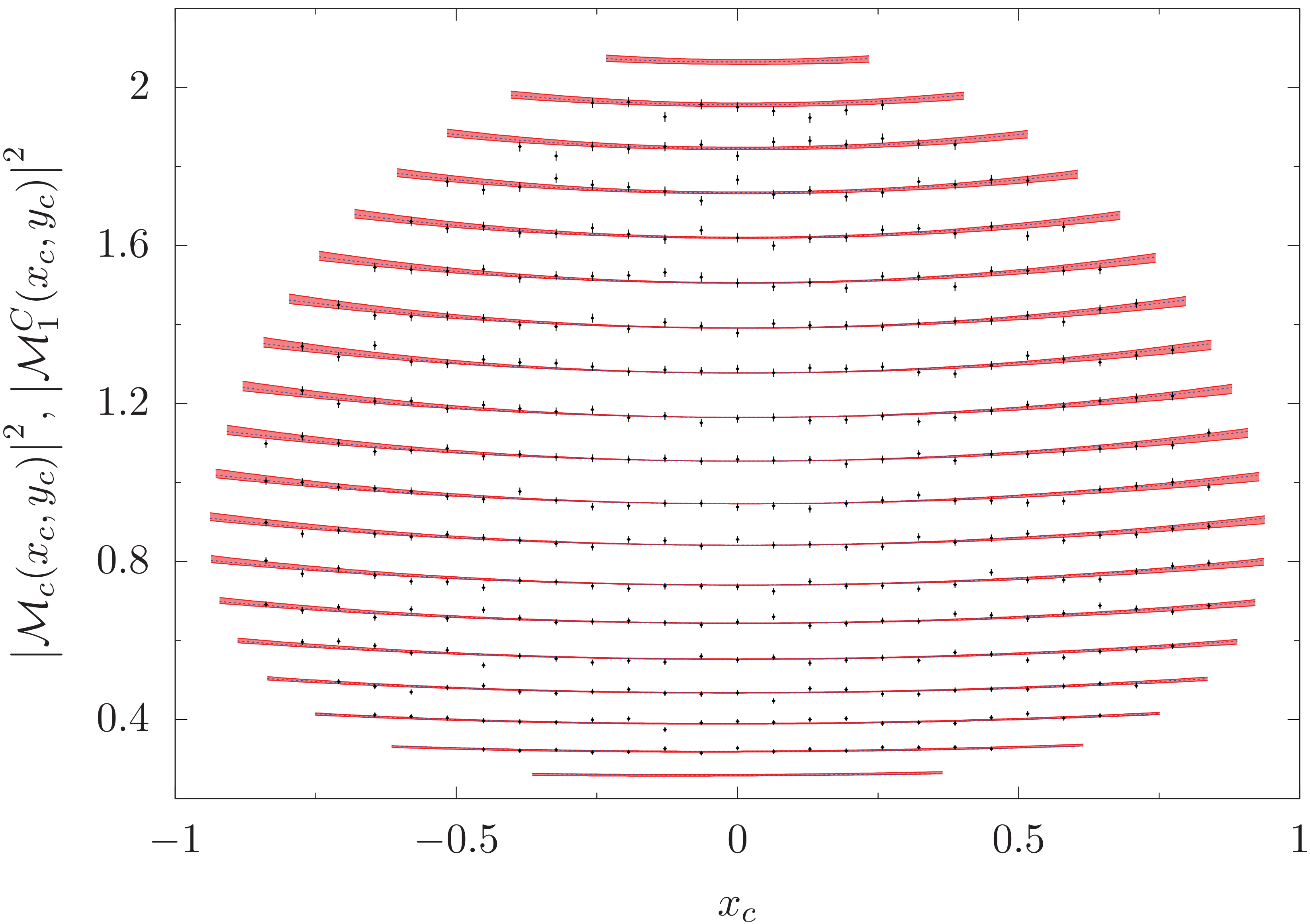


Not all *P*-waves are equal!



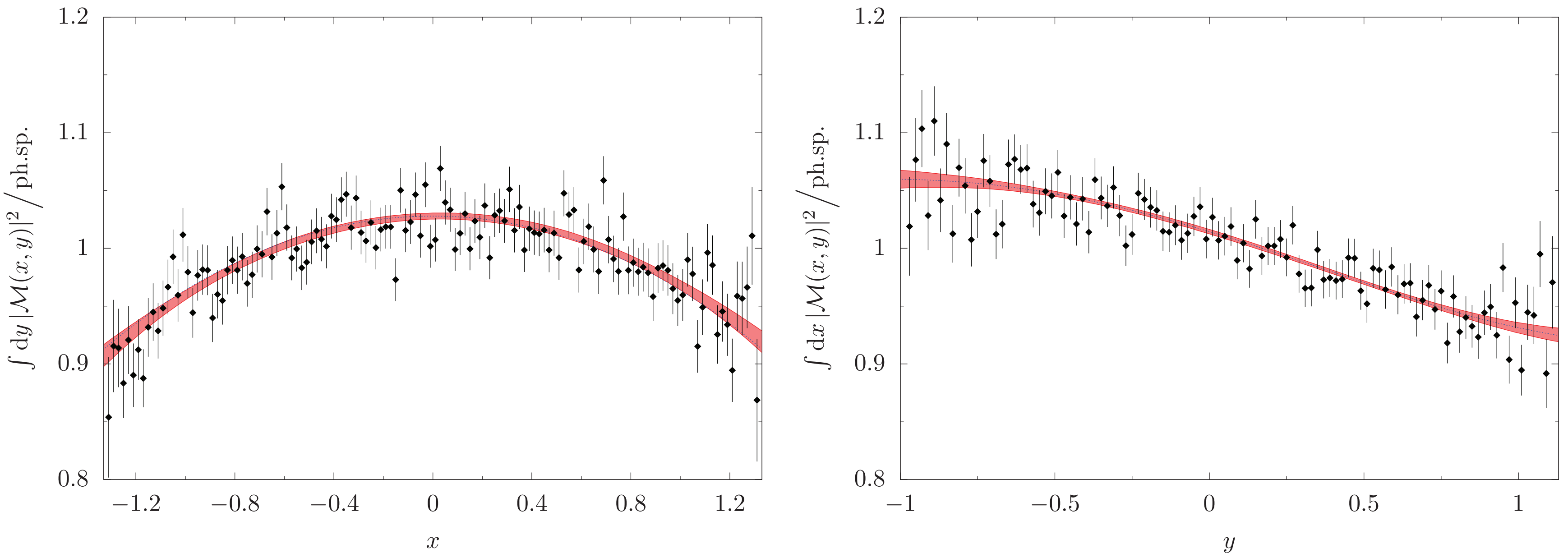
Akdag, Isken, BK 2021

Fit results KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$



KLOE2016 vs. Akdag, Isken, BK 2021

Fit results BESIII (projected) Dalitz plot $\eta' \to \eta\pi^+\pi^-$



BESIII 2017 vs. Akdag, Isken, BK 2021