RUHR-UNIVERSITÄT BOCHUM

Parity- and Time-Reversal Violating Nuclear Forces with explicit Δ-Excitations

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Beyond the Standard Model (SM)

Subtleties explaining experimental facts using the SM:

- Matter-antimatter asymmetry in observable universe.

Dine, M.; Kusenko, A. *Reviews of Modern Physics* **2003**, *76*, 1

- Dark matter.
- Non-zero neutrino mass.

One attempt in explaining these is violation of charge and parity transformations (CPV).

The Discrete Symmetries

Consider an one-particle state $\Psi(p,\sigma,n)$ where $p = (\boldsymbol{p}, \sqrt{\boldsymbol{p}^2 + M^2})$ is the four-momentum, σ is the spin and n the particle type.

$$\mathrm{P}\Psi(p,\sigma,n) \mapsto \eta_n \Psi((-\boldsymbol{p}, \sqrt{\boldsymbol{p}^2 + M^2}), \sigma, n), \quad \eta_n : \text{intrinsic parity}$$

$$\mathrm{T}\Psi(p,\sigma,n) \mapsto \phi_{n,\sigma} \Psi((-\boldsymbol{p}, \sqrt{\boldsymbol{p}^2 + M^2}), -\sigma, n), \quad \phi_{n,\sigma} : \text{arbitrary phase}$$

$$\mathrm{C}\Psi(p,\sigma,n) \mapsto \xi_n \Psi(p, \sigma, n^c), \quad \xi_n : \text{charge-conjugation parity}$$

CPV in the SM

Sources of CPV in the SM:

- Flavor-changing weak interactions associated with complex phase in Cabibbo-Kobayashi-Maskawa (CKM) matrix. Highly suppressed since it involves all three generations of quarks.

  Pospelov, M.; Ritz, A. *Annals of physics* **2005**, *318*, 119–169

- Dimension four θ-term in QCD Lagrangian. Restricted by experiments to $\bar{\theta} \lesssim 10^{-10}$, which is known as the strong CP-problem.

These effects cannot explain the observed asymmetry.

Cohen, A. G. et al. *Annual Review of Nuclear and Particle Science* **1993**, *43*, 27–70

$\rightarrow$ Consider SM as a low energy effective field theory (EFT) approximation to a more fundamental (unknown) theory.

The SM as EFT

Formulation of an EFT:
⇒ Include all possible terms into the Lagrangian, which respect the required symmetries. Hence, the CPV-Lagrangian takes the form

Engel, J. et al. *Progress in Particle and Nuclear Physics* **2013**, *71*, 21–74

CPV-Lagrangian

$$\mathcal{L}_{\mathrm{CPV}} = \mathcal{L}_{\mathrm{CKM}} + \mathcal{L}_{\bar{\theta}} + \mathcal{L}_{\mathrm{BSM}}^{\mathrm{eff}}, \quad \text{where}$$

$$\mathcal{L}_{\mathrm{CKM}} = -\frac{ig_2}{\sqrt{2}} V_{\mathrm{CKM}}^{pq} \bar{U}_L^p \not{W}^+ D_L^q + \mathrm{H.c.},$$

$$\mathcal{L}_{\bar{\theta}} = -\frac{g_3^2}{16\pi^2} \bar{\theta} \mathrm{Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu}),$$

$$\mathcal{L}_{\mathrm{BSM}}^{\mathrm{eff}} = \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} O_i^{(6)}, \quad O_i^{(6)} \text{ 15 dim.-6 CPV operators.}$$

From quark level to hadronic level

Relate interactions at quark level to P- and T-violating (PVTV) observables at hadronic, and nuclear levels.
$\rightarrow$ Heavy baryon chiral perturbation theory (HBχPT):

HBχPT

- Use the approximate chiral symmetry of QCD-Lagrangian.
- Construct EFT for hadrons: Includes baryon fields and pion fields as pseudo-Goldstone bosons of the chiral symmetry.
- Heavy baryon: Renormalization scheme in which the dynamic of the nuclei is considered non-relativistically.

PVTV effective Lagrangian

Calculation of PVTV nucleon-nucleon (NN) potentials requires, besides PCTC contributions, the following terms in the Lagrangian:

De Vries, J. et al. *Frontiers in Physics* **2020**, *8*, 218

Relevant PVTV terms in Lagrangian

$$\mathcal{L}^{\Delta_i=-2}_{\pi\pi,\,\mathrm{PVTV}} = \Delta_3 m_N \pi_3 \boldsymbol{\pi}^2 + \ldots\,,$$
$$\mathcal{L}^{\Delta_i=-1}_{\pi N,\,\mathrm{PVTV}} = N^\dagger \left[g_0 \boldsymbol{\tau} \cdot \boldsymbol{\pi} + g_1 \pi_3 + g_2 \pi_3 \tau_3 \right] N + \ldots\,.$$

Here, Δ_3, g_0, g_1 and g_2 are coupling constants depending on the CPV sources at quark level, and Δ_i refers to the vertex dimension.
Potentials using this Lagrangian have been calculated.

Gnech, A.; Viviani, M. *Physical Review C* **2020**, *101*, 024004

The Δ-baryons

Besides the neutron and proton, there are other baryons, which may occur in intermediate states. Of these, we consider the Δ-baryons, which are excited states carrying isospin $\frac{3}{2}$.

The Δ baryons

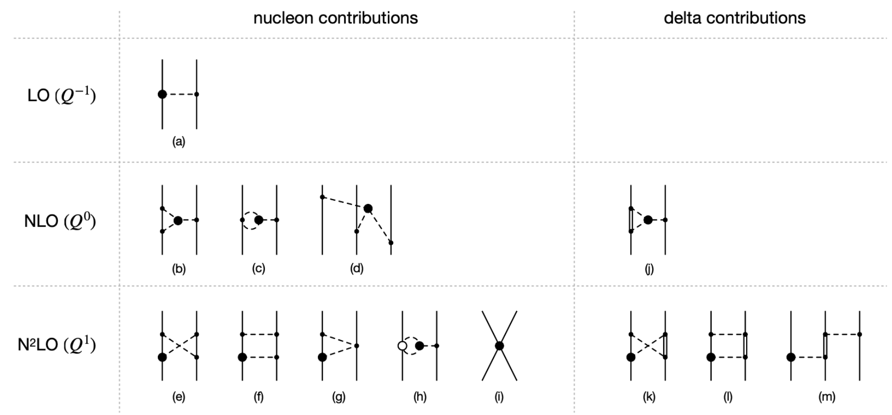
Rarita-Schwinger-formalism for Δ-field $\boldsymbol{T}_\mu$ leads to the Nucleon-Δ-pion Lagrangian:

$$\mathcal{L}_{\pi N\Delta}^{\Delta_i=0} = -\frac{\mathring{h}_A}{F} N^\dagger \boldsymbol{T}_\mu \cdot \partial^\mu \boldsymbol{\pi} + \mathrm{H.c.} + \dots .$$

Notice, that the lowest order term in the PVTV Lagrangian involving nucleon, Δ and pion fields contributes at order $\Delta_i = 2$.
$\rightarrow$ not relevant.

Feynman-Diagrams for PVTV Potential

Including the Δ allows to resum contributions without introducing new unknown parameters:



Relevant Feynman-diagrams up to N^{2}LO.

LG; Krebs, H.; Epelbaum, E. Parity and time-reversal violating nuclear forces with explicit Δ-excitations, 2024

Momentum-Space Potentials

RUB

The momentum-space potential can be decomposed into

$$V(\vec{q}\,) = \left[V_-^{\mathrm{I}} + W_-^{\mathrm{I}} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + V_-^{\mathrm{II}} \tau_1^3 \tau_2^3 + V_-^{\mathrm{III}} (\tau_1^3 + \tau_2^3)\right] i(\vec{q} \cdot \vec{\sigma}_1 - \vec{q} \cdot \vec{\sigma}_2) + V_+^{\mathrm{IV}} (\tau_1^3 - \tau_2^3)\, i(\vec{q} \cdot \vec{\sigma}_1 + \vec{q} \cdot \vec{\sigma}_2)\,,$$

where classes I, II, III, IV are isospin-invariant, charge-independence breaking, charge-symmetry breaking, isospin mixing, respectively. E.g., from the box and crossed-box diagrams we obtain

LG; Krebs, H.; Epelbaum, E. Parity and time-reversal violating nuclear forces with explicit Δ-excitations, 2024

$$V_{-,\,2\pi,\,\Delta}^{\mathrm{I}\,(Q^1)} = -\frac{g_A h_A^2 (3\bar{g}_0 + \bar{g}_2)}{18 F_\pi^3 \pi \Delta} (2M_\pi^2 + q^2) A(q)\,,$$

$$W_{-,\,2\pi,\,\Delta}^{\mathrm{I}\,(Q^1)} = -\frac{2(\bar{g}_0 + \bar{g}_2) g_A h_A^2}{36 F_\pi^3 \pi^2} \left[(2M_\pi^2 + q^2 - 2\Delta^2) D(q) - L(q)\right]\,,$$

$$V_{-,\,2\pi,\,\Delta}^{\mathrm{II}\,(Q^1)} = \frac{\bar{g}_2 g_A h_A^2}{36 F_\pi^3 \pi^2} \left[(2M_\pi^2 + q^2 - 2\Delta^2) D(q) - L(q)\right]\,,$$

$$V_{-,\,2\pi,\,\Delta}^{\mathrm{III}\,(Q^1)} = V_{+,\,2\pi,\,\Delta}^{\mathrm{IV}\,(Q^1)} = -\frac{g_A h_A^2 \bar{g}_1}{36 F_\pi^3 \pi \Delta} (2M_\pi^2 + q^2) A(q)\,.$$

Δ-Resonance-Saturation

In the Δ-less approach contributions from intermediate Δ-resonances appear in the LECs one order higher.

Consistency Check

- Expand the potentials in $1/\Delta$.
- Calculate $\mathrm{N}^3\mathrm{LO}$ contributions $\propto c_i$, e. g., triangle diagram where Weiberg-Tomozawa vertex is replaced by subleading c_i-vertices:

$$V_{-,\,2\pi}^{\mathrm{I}\,(Q^1)} = \frac{g_A(3\bar{g}_0 + \bar{g}_2)}{8\pi F_\pi^3}\left(M_\pi^2(4c_1 - 2c_3) - q^2 c_3\right)A(q)$$

- Reproduce Δ-full result by setting the LECs c_i to their Δ-resonance-saturation values

Bernard, V. et al. *Nuclear Physics A* **1997**, *615*, 483–500

$$c_1 = 0, \quad c_2 = -c_3 = \frac{4h_A^2}{9\Delta}.$$

Large-Distance Behavior

For 2N potentials we can move to coordinate space easily by using their spectral representation:

$$\tilde{X}_{\pm}(r) = -\frac{1}{2\pi^2 r^2}\int_{2M_\pi}^{\infty} d\mu\mu e^{-\mu r}\,(1+\mu r)\,\mathrm{Im}X_{\pm}(-i\mu)\,,$$

Where X is V or W. This leads, e. g., to contributions arising from

- the leading 2π-exchange

$$\tilde{V}^{\mathrm{III}\,(\mathrm{Q}^0)}_{-,\,2\pi}(r) = \frac{5g_A^3\Delta_3 m_N}{1024\pi^2 F_\pi^3}\frac{1}{r^3}\left[x(x-1)e^x\mathrm{Ei}(-3\mathrm{x}) + 4(1+\mathrm{x})\mathrm{e}^{-2\mathrm{x}} + \mathrm{x}(1+\mathrm{x})\mathrm{e}^{-\mathrm{x}}\mathrm{Ei}(-\mathrm{x})\right].$$

- box and crossed-box diagrams

$$\tilde{W}^{\mathrm{I}\,(\mathrm{Q}^1)}_{-,\,2\pi}(r) = -\frac{(2\bar{g}_0+\bar{g}_2)g_A M_\pi}{64\pi^3 F_\pi^3 r^3}\left[2x(4g_A^2-1)K_0(2x) + (g_A^2(9+4x^2)-3)K_1(2x)\right].$$

Charge-Independence Breaking Potentials

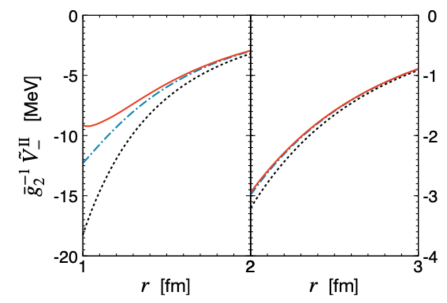


Fig. 1: Black-dotted: LO. Blue dash-dotted: N^{2}LO Δ-less. Red: N^{2}LO Δ-full.

- No corrections at NLO. Dominated by 1π-exchange.
- Δ-contributions negligibly small beyond $\approx 1.5\,\mathrm{fm}$.

Charge-Symmetry Breaking Potentials

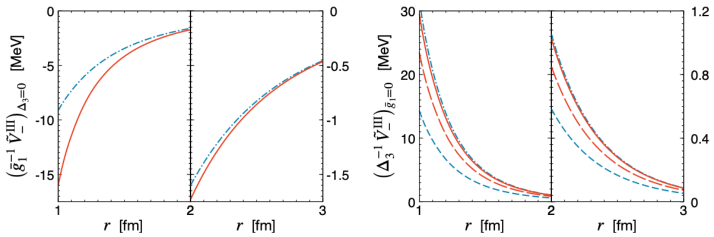


Fig. 2: Blue short-dashed: NLO Δ-less. Blue dash-dotted: N^{2}LO Δ-less. Red long-dashed: NLO Δ-full. Red: N^{2}LO Δ-full.

- No correction to 1π-exchange $\propto \bar{g}_1$ in Δ-less framework. Δ-Correction sizable at $r \lesssim 1.5\ \mathrm{fm}$.
- NLO and N^{2}LO corrections $\propto \Delta_3$. Dominant NLO-Δ-result already take into account much of the N^{2}LO-correction.
- No Δ-correction at N^{2}LO. Final results rather similar in Δ-less and Δ-full framework.

Isospin-Invariant Potentials

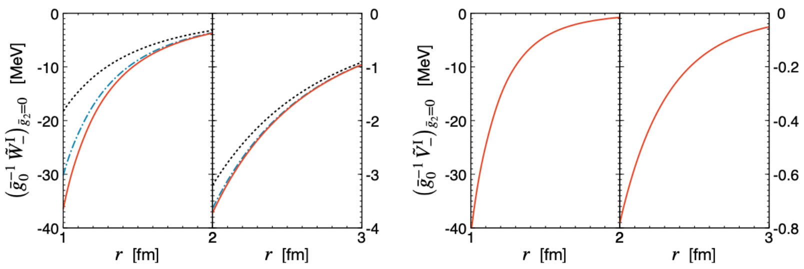


Fig. 3: Black dotted: LO. Blue dash-dotted: N^2LO Δ-less. Red: N^2LO Δ-full.

- $\bar{g}_2$-Contribution is suppressed relative to $\bar{g}_0$.
- 1π-exchange dominant for the isovector-potential at large distances.
- 2π-exchange gives large correction at $r \lesssim 2\ \mathrm{fm}$ while the Δ-correction is rather small.
- Very strong isoscalar-potential not present in the Δ-less framework up to N^2LO.

Summary and Outlook

- PVTV potentials are only very tiny component of total nucleon interactions, but are useful for analyzing fundamental symmetries of nature, e.g., non-vanishing electric dipole moments (EDMs).
- HBχPT includes LECs directly originating from effects at quark level $\rightarrow$ hints for beyond SM physics.
- Inclusion of the Δ allows to resum contributions beyond N^{2}LO without introducing additional LECs. We found improved convergence and a strong isospin-invariant PVTV two-pion exchange potential.
- The coordinate-space potentials contain at most one-dimensional integrals $\rightarrow$ well suited for applications.
- EDM of nucleon based on CKM is $\sim 10^{-18} e\,\mathrm{fm}$.

  Wirzba, A. et al. *International Journal of Modern Physics E* **2017**, *26*, 1740031

  Present experimental upper bound for proton is $\lesssim 1.2 \cdot 10^{-13} e\,\mathrm{fm}$.

  Graner, B. et al. *Physical review letters* **2016**, *116*, 161601