

Precision theory for electromagnetic observables of light nuclei

Arseniy Filin

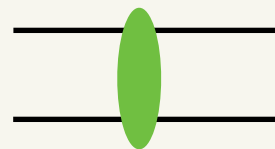
Institut für Theoretische Physik II, Ruhr-Universität Bochum, Germany

in collaboration with

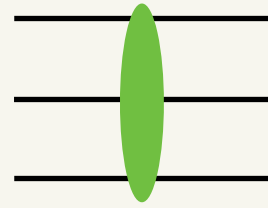
V. Baru, E. Epelbaum, C. Körber, H. Krebs, D. Möller, A. Nogga, and P. Reinert

Nuclear structure using chiral effective field theory

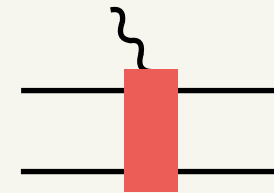
Low-energy **chiral effective field theory** of the standard model



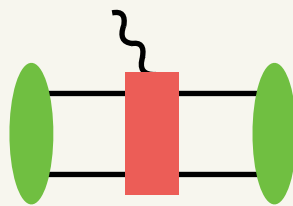
Chiral two-nucleon forces



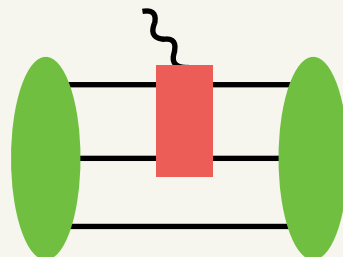
Chiral three-nucleon forces



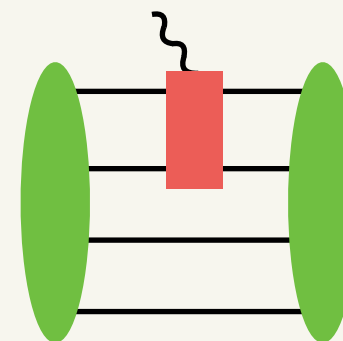
Chiral electromagnetic currents



^2H

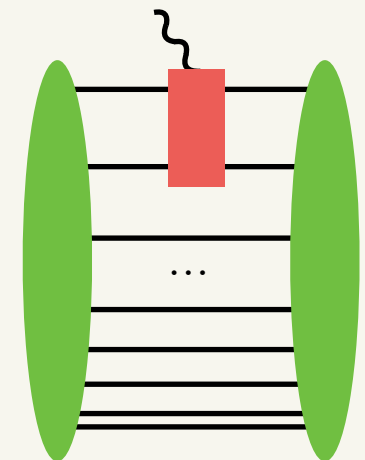


^3He ^3H



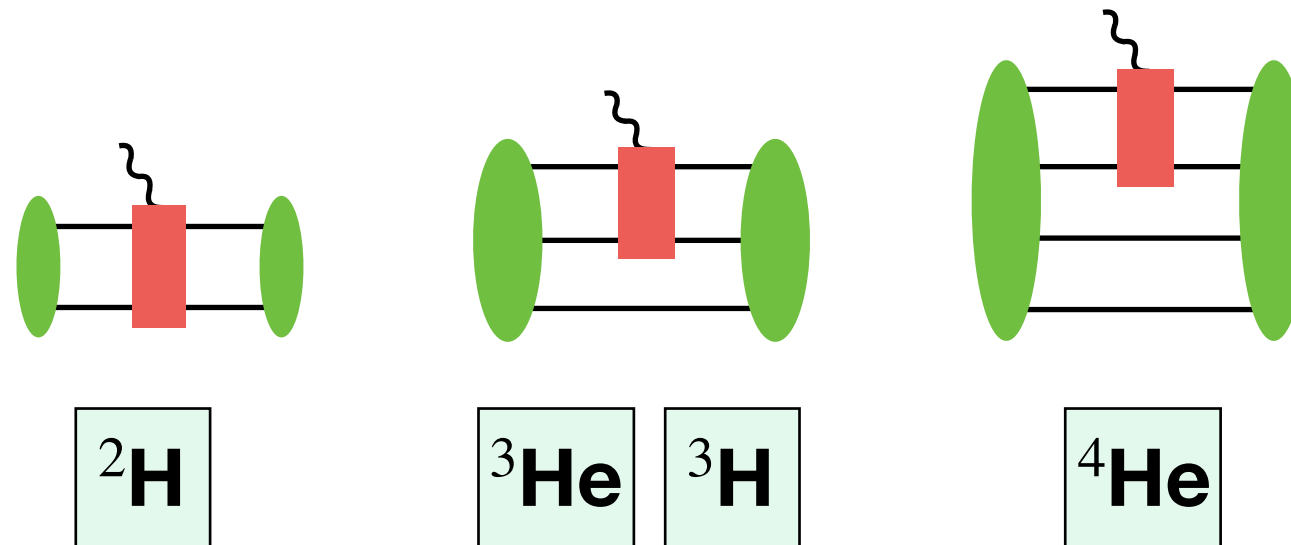
^4He

charge and magnetic form factors and **radii** of light nuclei



structure of
medium-mass nuclei

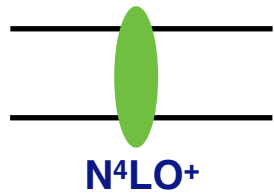
Precise calculations of charge radii of super-light nuclei



Motivation:

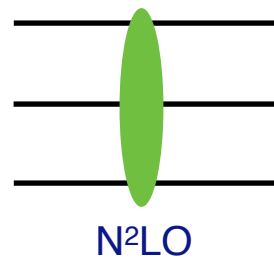
- Precision **tests of nuclear chiral effective field theory (Chiral EFT)**
- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**
- A new way to **extract the neutron and the proton charge radii** from few-nucleon data
- Search for **Beyond-Standard-Model** physics (lepton universality breaking)

Chiral effective field theory - **precise, accurate and consistent**



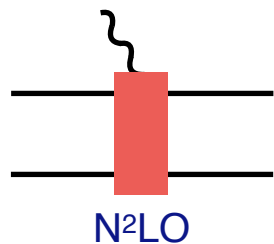
New **high-precision chiral NN forces** (N^4LO^+) Reinert et al. PRL 126, 092501 (2021)

- Nearly perfect description of pp and pn scattering data up to pion production threshold



Chiral 3N forces (general N^2LO ; selected terms at N^4LO) Epelbaum:2019kcf

- LECs cD and cE (N^2LO) are fitted to **RIKEN** Nd DCS data and 3He binding energy
- **Consistent** regularisation of N^3LO is also in progress

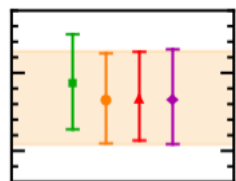


Isoscalar: N^4LO

2N Chiral electromagnetic currents (general N^2LO ; isoscalar N^4LO)

- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

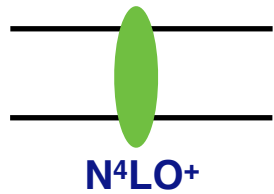
Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)



Reliable methods to quantify truncation uncertainty of the EFT expansion

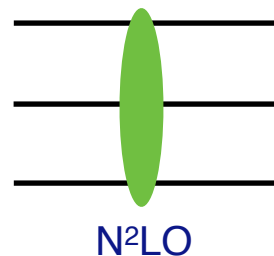
Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017),
Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

Chiral effective field theory - **precise, accurate and consistent**



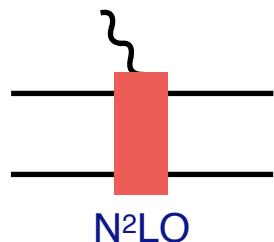
New **high-precision chiral NN forces** (N^4LO^+) Reinert et al. PRL 126, 092501 (2021)

- Nearly perfect description of pp and pn scattering data up to pion production threshold



Chiral 3N forces (general N^2LO ; selected terms at N^4LO) Epelbaum:2019kcf

- LECs cD and cE (N^2LO) are fitted to **RIKEN** Nd DCS data and 3He binding energy
- **Consistent** regularisation of N^3LO is also in progress



Isoscalar: N^4LO

2N Chiral electromagnetic currents (general N^2LO ; isoscalar N^4LO)

- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)

Goals of this study:

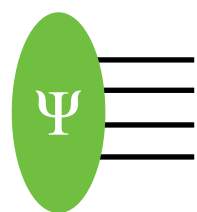
- consistent χEFT calculation of **isoscalar structure radii** of $A = 2, 3, 4$ nuclei
- aim at N^4LO level of accuracy even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, numerical and other)

Chiral EFT calculation of the nuclear charge radius

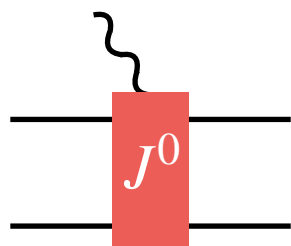
Charge radius r_C is related to the charge form factor $F_C(Q^2)$ $r_C^2 = (-6) \frac{\partial}{\partial Q^2} F_C(Q^2) \Big|_{Q=0}$

$$F_C(Q^2) = \text{Diagram 1} + \text{Diagram 2} + \dots$$

The matrix element is a convolution of nuclear wave function and charge density operator



Nuclear wave function based on high-precision chiral EFT interactions



Charge density operator - consistent with chiral nuclear forces

Structure radius

Nuclear **charge radius** can be decomposed into **structure**, **proton** and **neutron** radii

$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{N}{Z} r_n^2$$

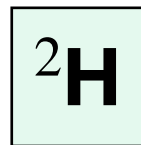
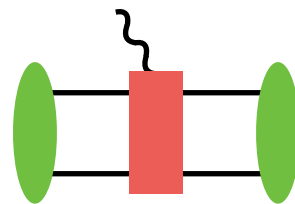
Nuclear structure radius

- depends on distribution of matter (proton and neutrons) inside the nuclei
- depends on many-body electromagnetic currents
- **can be accurately calculated using chiral nuclear forces and EM currents**

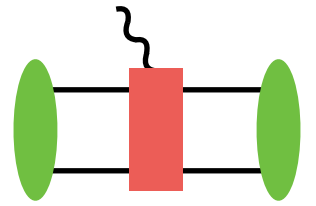
Structure radius = **charge radius** if protons and neutrons have point-like charge distributions

Deuteron structure radius

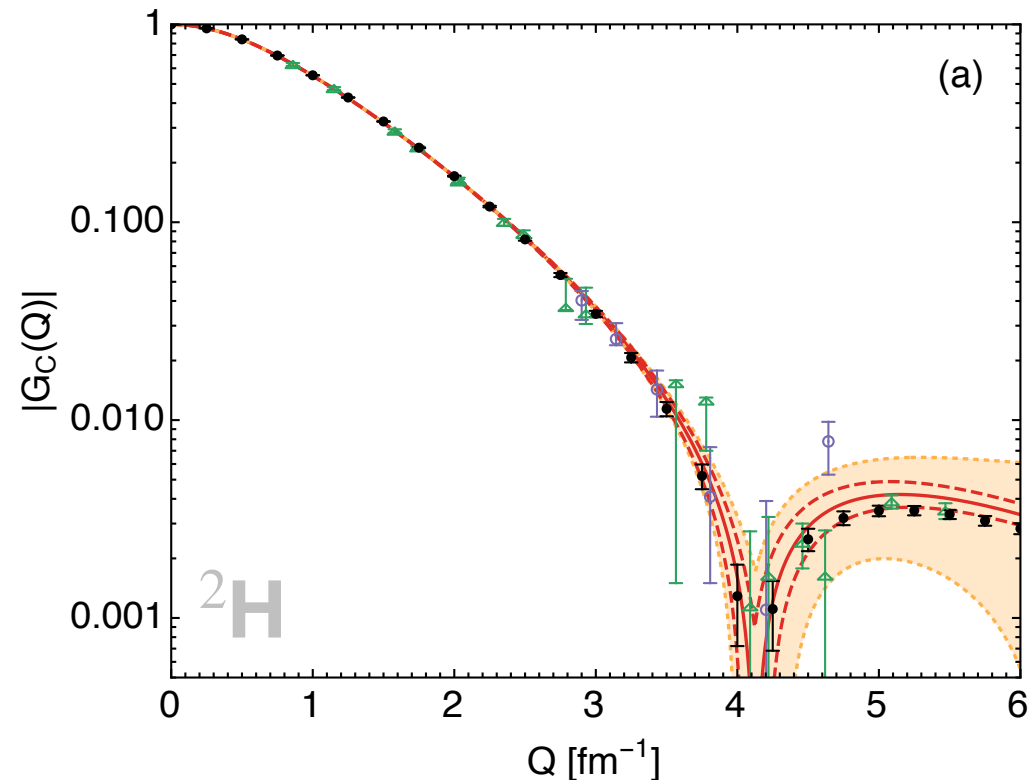
& extraction of the neutron charge radius



Deuteron charge and quadrupole form factors

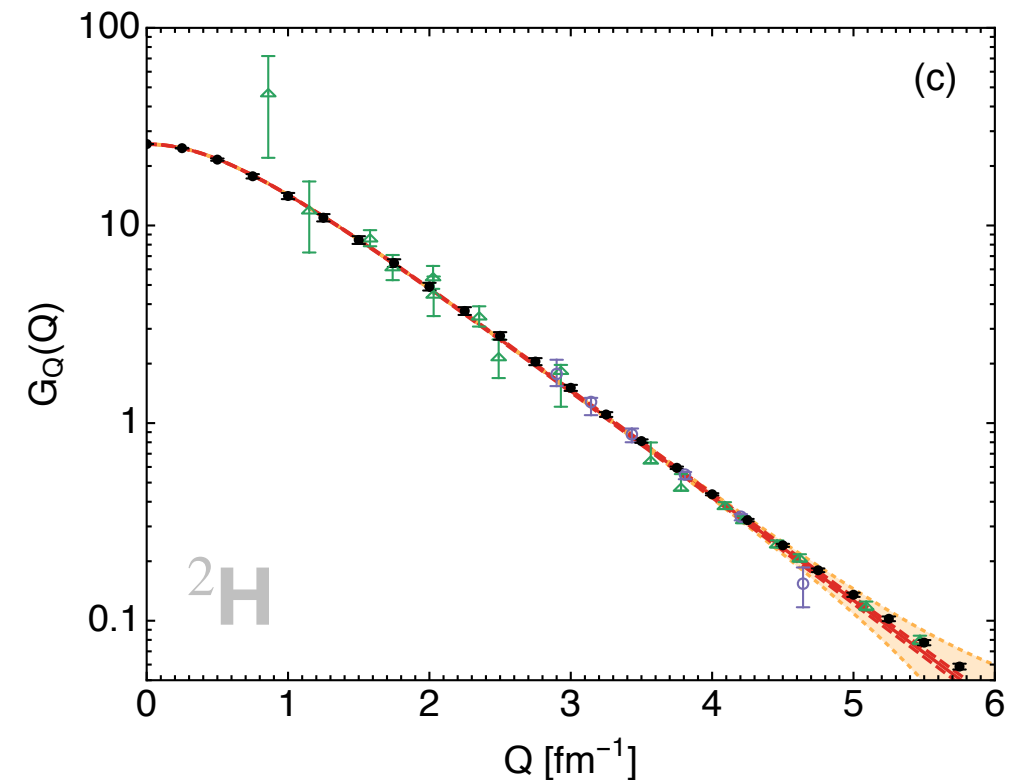


Deuteron charge form factor



 our result + N⁴LO truncation uncertainty
 statistical uncertainty

Deuteron quadrupole form factor



 Experimental data
 Parameterisation by I.Sick (not used in the fit)

Extraction of deuteron structure radius and quadrupole moment

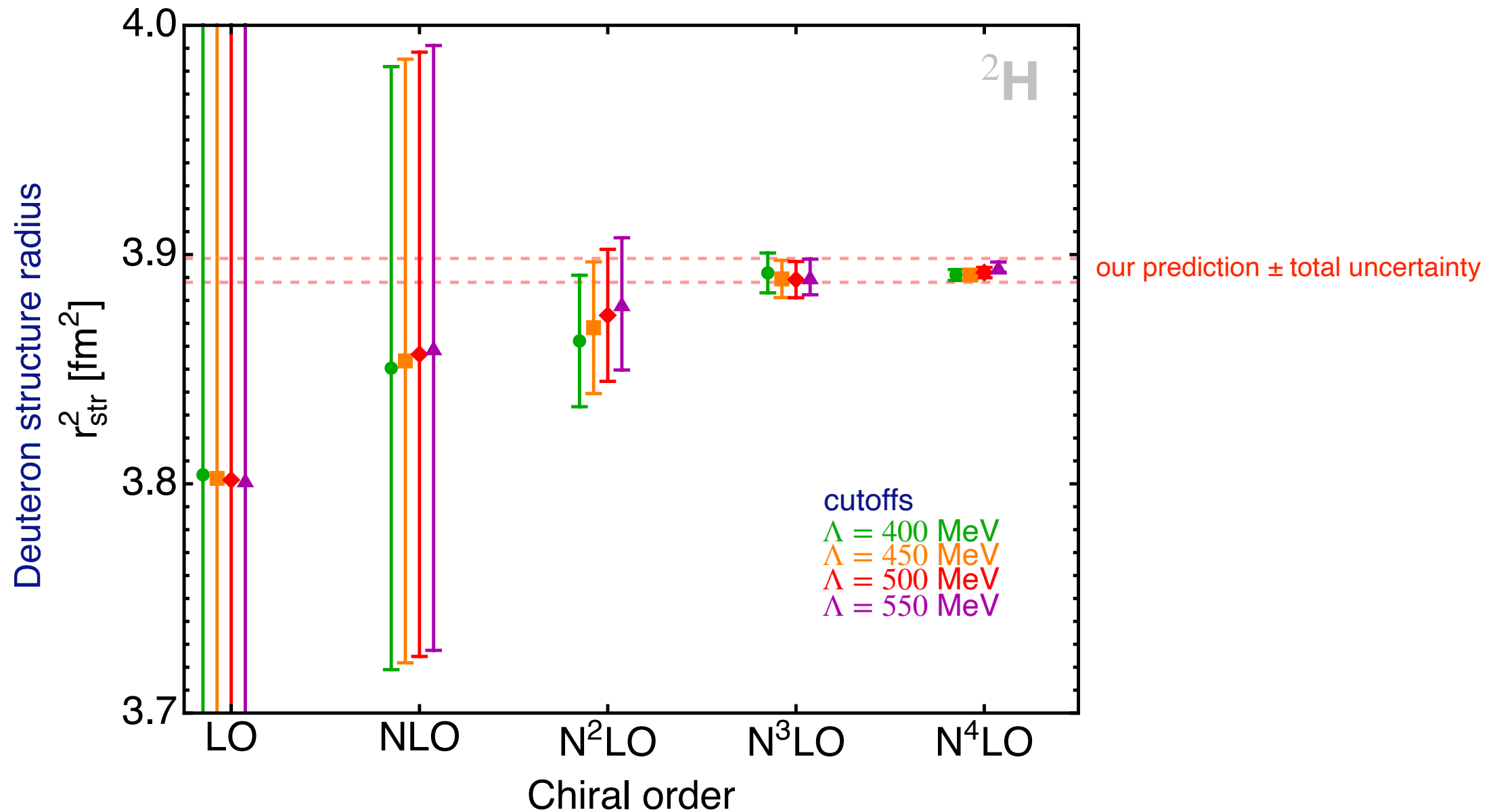
$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm$$

$$Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Truncation uncertainty of ^2H structure radius

Using Bayesian model to estimate truncation uncertainty at each order [Epelbaum et al. EPJA 56, 92 \(2020\)](#)

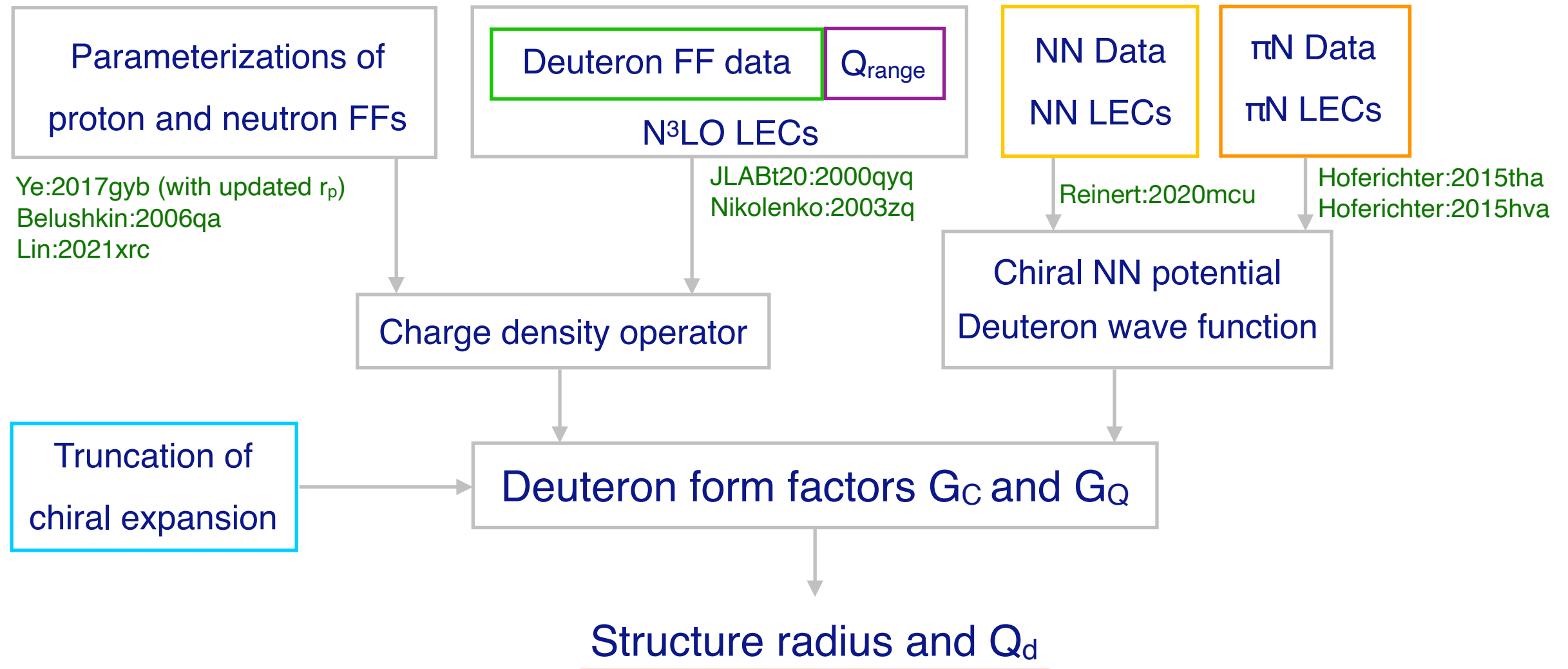


Chiral EFT expansion converges very well

Cutoff dependence is smaller than the truncation uncertainty

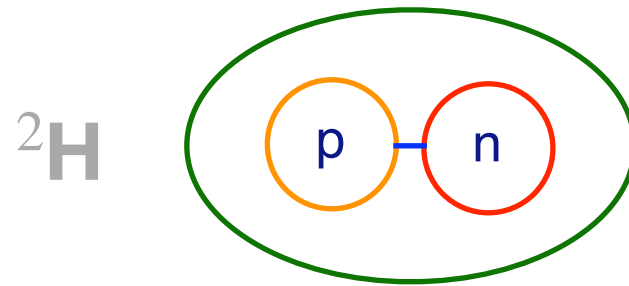
Uncertainty analysis of deuteron structure radius

We propagate uncertainties from multiple sources



	Central	Truncation	$\rho_{\text{Cont}}^{\text{reg}}$	$\pi\text{N LECs RSA}$	2N LECs and f_i^2	Q range	Total
r_{str}^2 (fm ²)	3.8925	± 0.0030	± 0.0024	± 0.0003	± 0.0025	+0.0035 −0.0005	+0.0058 −0.0046
Q_d (fm ²)	0.2854	± 0.0005	± 0.0007	± 0.0003	± 0.0016	+0.0035 −0.0005	+0.0038 −0.0017

Neutron charge radius from high-accuracy χ EFT calculation of deuteron structure radius



$$r_d^2 = r_{str}^2(^2\text{H}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Deuteron-proton radius difference

$$(r_d^2 - r_p^2) = 3.82070(31) fm^2$$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)

Jentschura et al. PRA 83 (2011)

Our accurate χ EFT calculation
of the **deuteron structure** radius

New method of determination
of the neutron charge radius

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2(^2\text{H})$$

$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

Neutron charge radius in PDG 2022

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm ²)	DOCUMENT ID	COMMENT
-0.1155 ± 0.0017 OUR AVERAGE		
$-0.115 \pm 0.002 \pm 0.003$	KOPECKY 97	ne scattering (Pb)
$-0.124 \pm 0.003 \pm 0.005$	KOPECKY 97	ne scattering (Bi)
-0.114 ± 0.003	KOESTER 95	ne scattering (Pb, Bi)
-0.115 ± 0.003	¹ KROHN 73	ne scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •		
-0.1101 ± 0.0089	² HEACOCK 21	n interferometry
$-0.106 \begin{smallmatrix} +0.007 \\ -0.005 \end{smallmatrix}$	³ FILIN 20	chiral EFT analysis
$-0.117 \begin{smallmatrix} +0.007 \\ -0.011 \end{smallmatrix}$	BELUSHKIN 07	Dispersion analysis
$-0.113 \pm 0.003 \pm 0.004$	KOPECKY 95	ne scattering (Pb)
-0.134 ± 0.009	ALEKSANDR...86	ne scattering (Bi)
-0.114 ± 0.003	KOESTER 86	ne scattering (Pb, Bi)
-0.118 ± 0.002	KOESTER 76	ne scattering (Pb)
-0.120 ± 0.002	KOESTER 76	ne scattering (Bi)
-0.116 ± 0.003	KROHN 66	ne scattering (Ne, Ar, Kr, Xe)

¹ KROHN 73 measured -0.112 ± 0.003 fm². This value is as corrected by KOESTER 76.

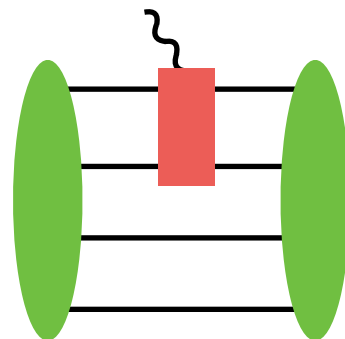
² HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

³ FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

Extracted r_n value shows 2σ deviation from the PDG (2022) weighted average $r_n^2 = -0.1155(17) \text{ fm}^2$

^4He charge radius

Precision test of the chiral EFT for ^4He



^4He

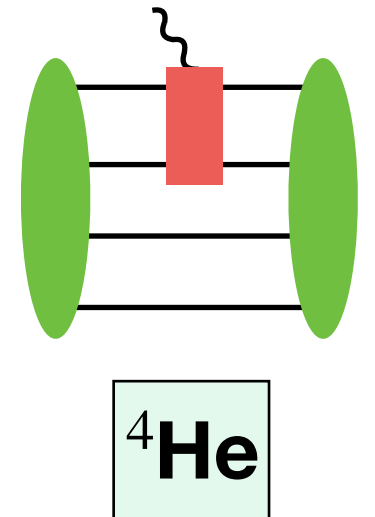
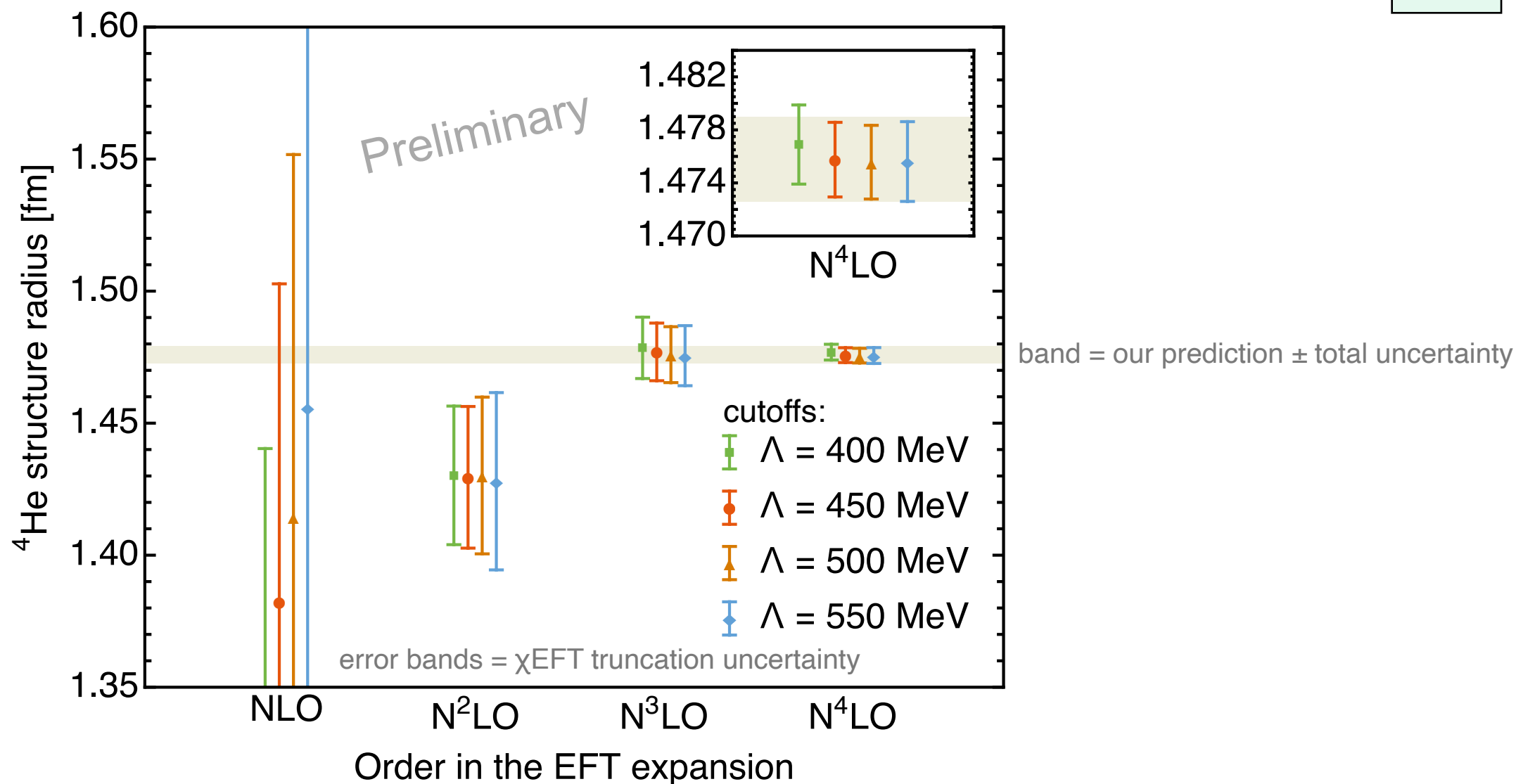
Preliminary results

Prediction of ^4He structure radius

Our preliminary prediction for ^4He structure radius:

$$r_{str}(^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nucIFF}} \text{ fm (Preliminary)}$$

Estimation of truncation error:



Cutoff dependence is smaller than the truncation uncertainty

Chiral EFT expansion converges well

Prediction for ^4He charge radius

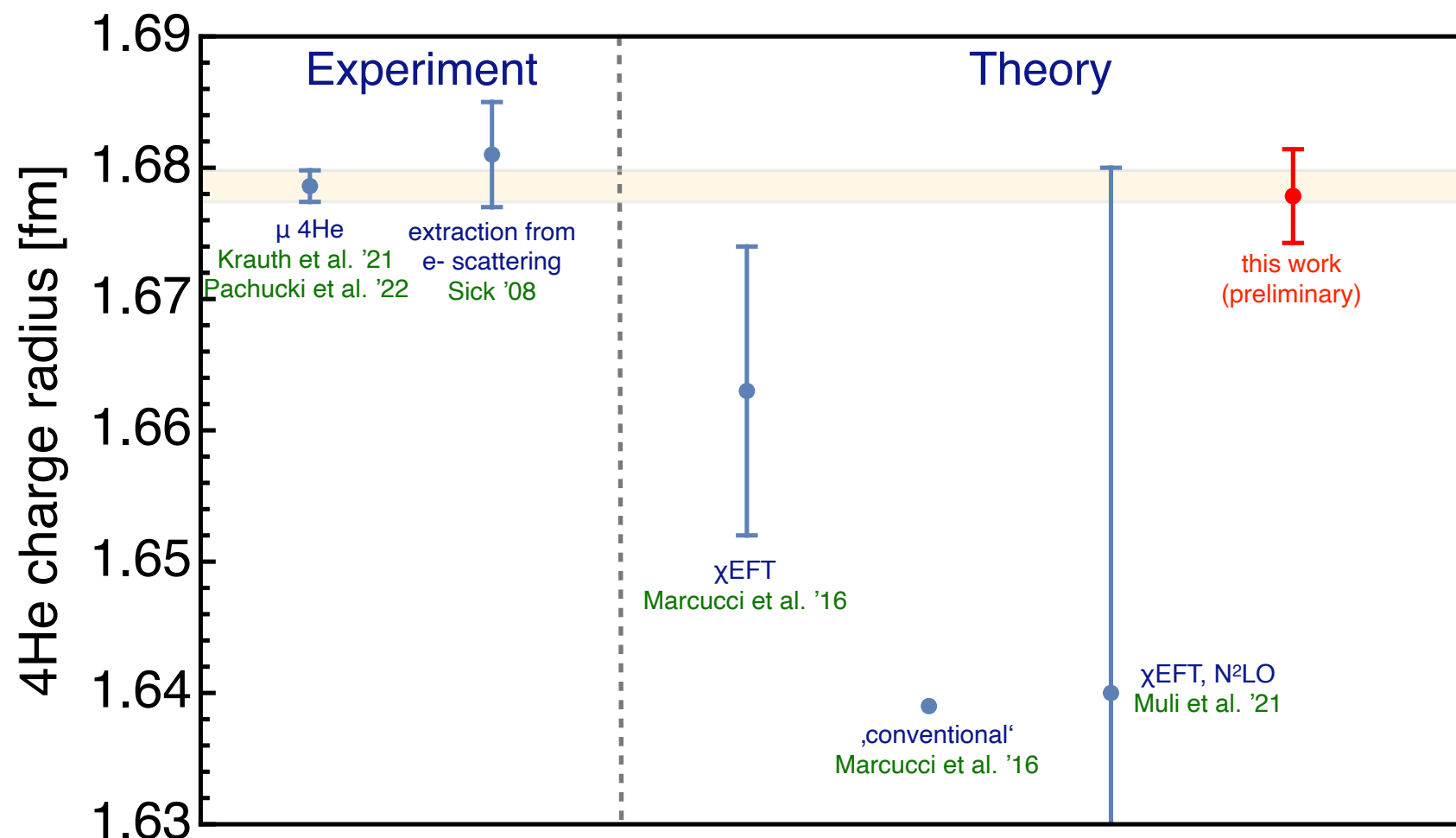
$$r_{str}(^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}} \text{ fm} \text{ (Preliminary)}$$

Our prediction for ^4He **charge** radius

$$r_C(^4\text{He}) = (1.6775 \pm 0.0035) \text{ fm}$$

$$r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

preliminary, using CODATA 2018 r_p and own determination of r_n



Our prediction for ^4He charge radius is fully consistent with the muonic-atom spectroscopy

Isoscalar nucleon charge radius from data on ^4He

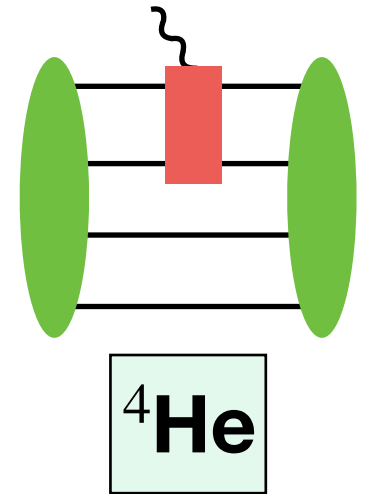
Our prediction for ^4He **structure** radius

Experimental ^4He charge radius
Krauth et al., Nature 589 (2021) 7843, 527-531
+ theory update from Pachucki et al. (2022)

$$r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \underbrace{\left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2}_{\text{Determination of the isoscalar nucleon charge radius}}$$

Determination
of the isoscalar nucleon charge radius
 $(r_n^2 + r_p^2) = (0.607 \pm 0.010) \text{ fm}^2$

preliminary



Proton charge radius from isoscalar nucleon radius

Our determination of the
isoscalar nucleon charge radius from ^4He

$$(r_n^2 + r_p^2) = (0.607 \pm 0.010) \text{ fm}^2 \text{ preliminary}$$

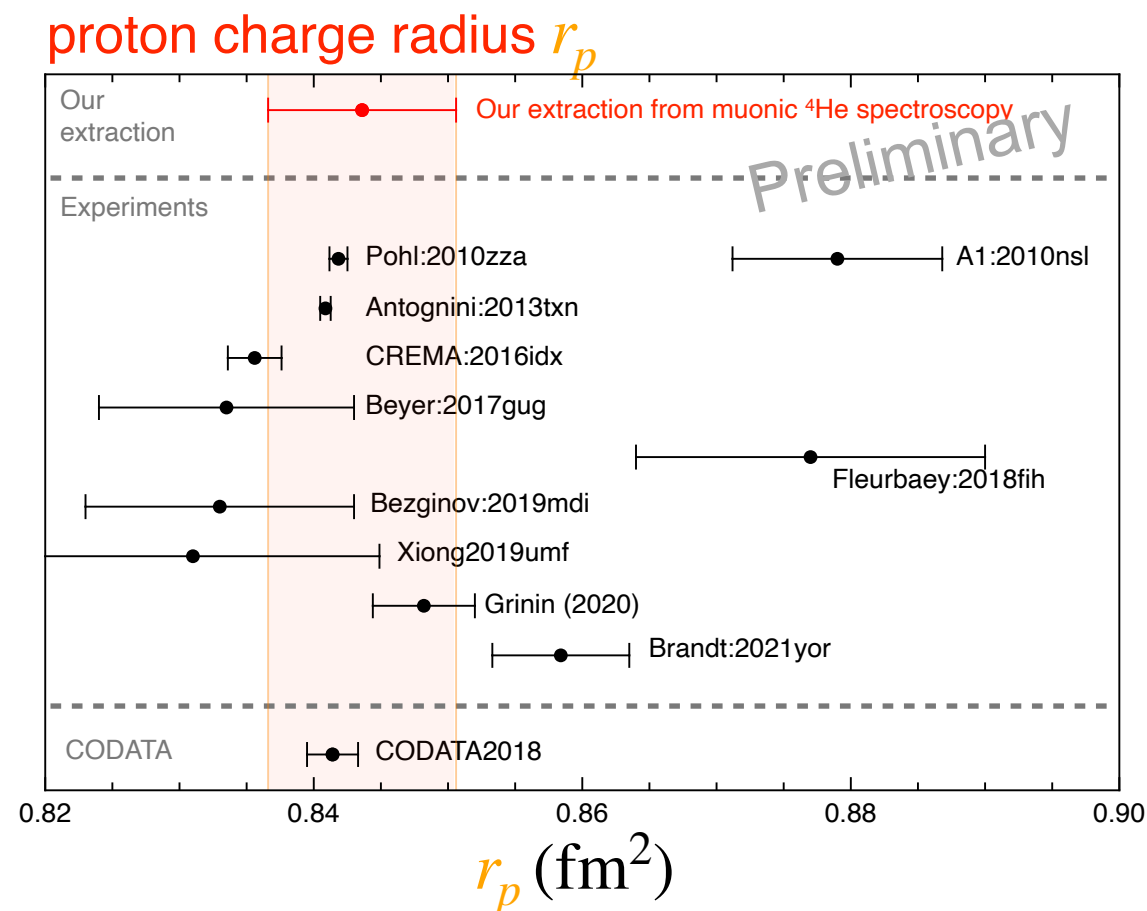
Our determination of the
neutron charge radius from ^2H

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{ fm}^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius: $r_p = (0.844 \pm 0.007) \text{ fm}$

preliminary



Our extraction supports the „small“ proton radius

Prediction for isoscalar 3N charge radius

With all LECs being fixed, we can **predict the isoscalar 3N charge radius**: $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026) fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Our result is 10x more precise than current experimental data:

the 3H charge radius from e^- scattering experiments:

$$r_C^{3H} = (1.7550 \pm 0.0860) fm \quad \text{Amroun et al. '94 (world average)} \quad 5\%$$

the 3He charge radius from muonic 3He :

$$r_C^{3He} = (1.9701 \pm 0.0009) fm \quad \text{CREMA 2023 arXiv:2305.11679} \quad 0.05\%$$

Exp. **3N isoscalar charge radius**: (using muonic 3He and old 3H)

$$r_{C,exp.}^{isoscalar3N} = (1.9010 \pm 0.0260) fm \quad 1.4\%$$

T-REX experiment in Mainz [Pohl et al.] aims at measuring r_C^{3H} within $\pm 0.0002 fm$ (400x more precise)

The isoscalar 3N radius will be then known within $\pm 0.0009 fm$

⇒ **precision tests of nuclear chiral EFT!**

Summary

Precise and accurate calculation of $A = 2, 3, 4$ isoscalar charge radii in chiral effective field theory

Extensive uncertainty analysis

Nuclear structure calculations with sub-percent accuracy!

Charge radii of neutron and proton from light nuclei:

- ^2H r_{str} combined with isotope-shift data => **extracted the neutron charge radius** (2σ tension with PDG)
- ^4He r_{str} combined with spectroscopic data => **extracted isoscalar nucleon and proton charge radii** preliminary

^4He calculation: preliminary

- calculated **^4He charge radius (0.2% accuracy)** agrees with the new $\mu^4\text{He}$ measurement
- no indications of lepton universality breaking at this accuracy level

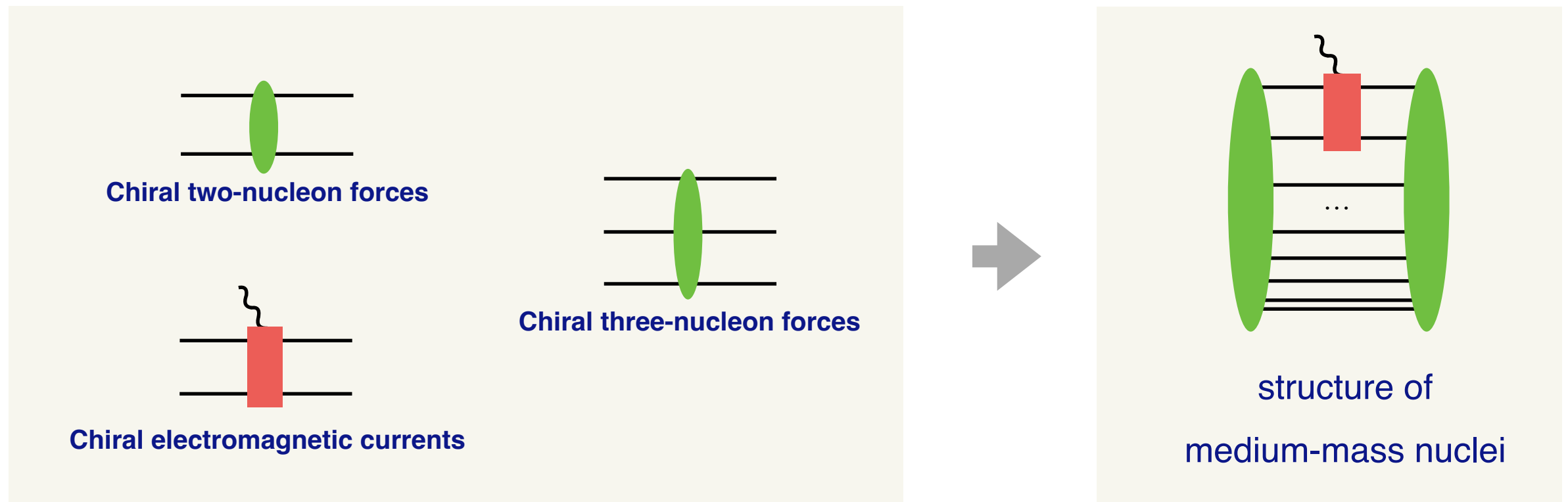
^3H - ^3He : preliminary

- predicted the **isoscalar 3N charge radius r_C (0.1% accuracy)**
- our r_C is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX (^3H) exp. in Mainz will allow for a **precision test of nuclear chiral EFT**

Outlook

In progress:

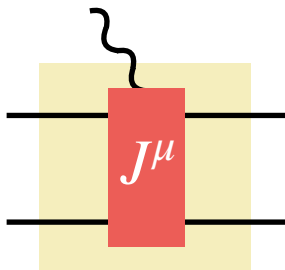
- **Consistent** inclusion of **isovector 2N currents at N³LO and N⁴LO**
- **Consistent** inclusion of **N³LO and N⁴LO three-nucleon forces**
- Analysis of **magnetic form factors** (work by Daniel Möller)
- Application to processes with two photons (**polarizabilities, ...**)
- Calculation of isoscalar N4LO **charge radii for nuclei with A>4** (work in progress by LENPIC collaboration)



Spares

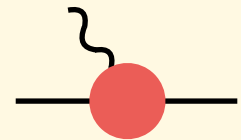
Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka
Review: H. Krebs, EPJA 56 (2020) 240

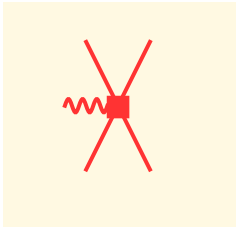


	single-nucleon		two-nucleon	
LO				
NLO				
N ² LO				
N ³ LO				

can be
parametrised
in terms of
nucleon FF

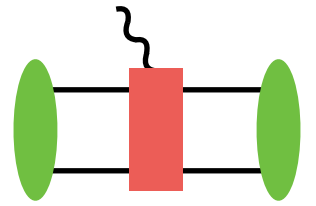


Ye:2017gyb
Belushkin:2006qa
Lin:2021umz



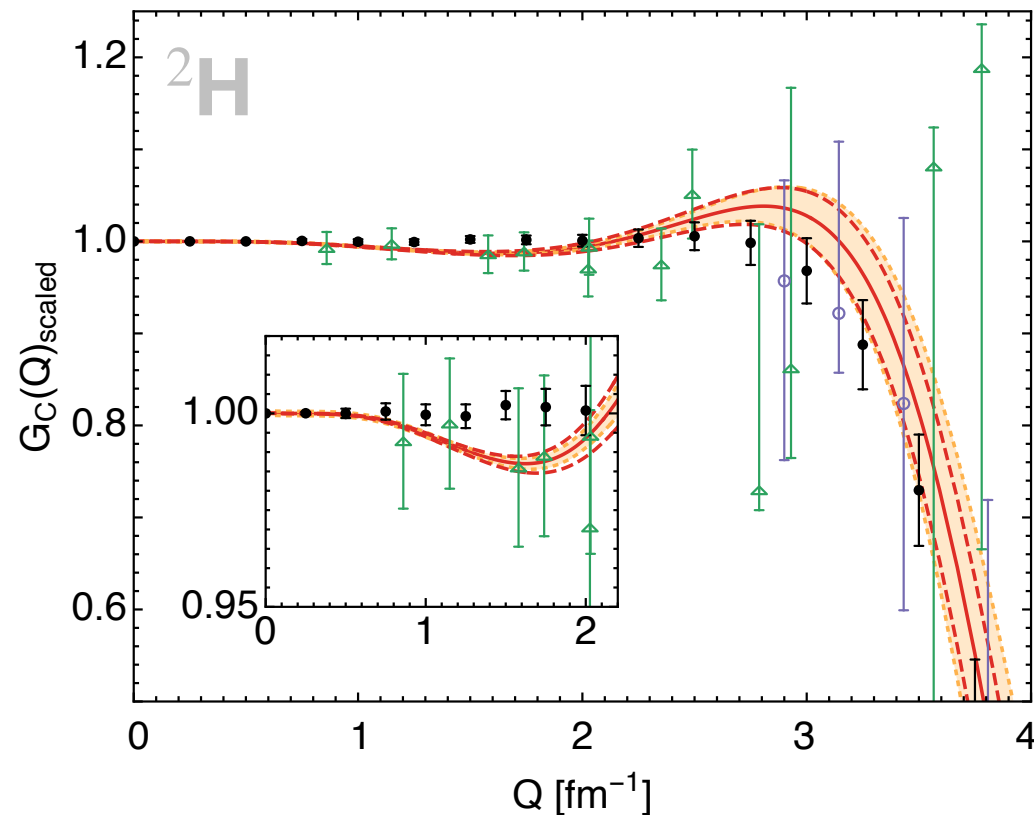
depend on **3 parameters (LECs)**
 3S_1 - 3S_1 - can be fitted to deuteron FF data
 3S_1 - 3D_1 - this one too
 1S_0 - 1S_0 - can be fitted to ^4He FF data
Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

Deuteron charge and quadrupole form factors

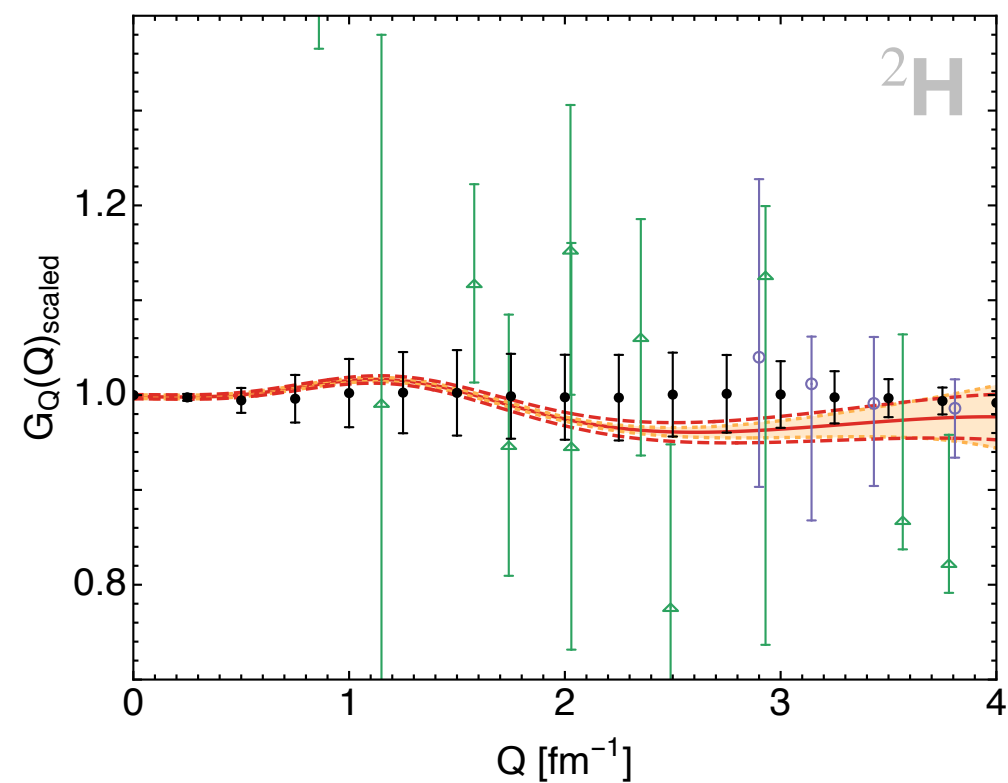


Deuteron form factors at low Q

Charge FF zoomed at low Q



Quadrupole FF zoomed at low Q



Deuteron charge FF can be used to extract neutron charge FF

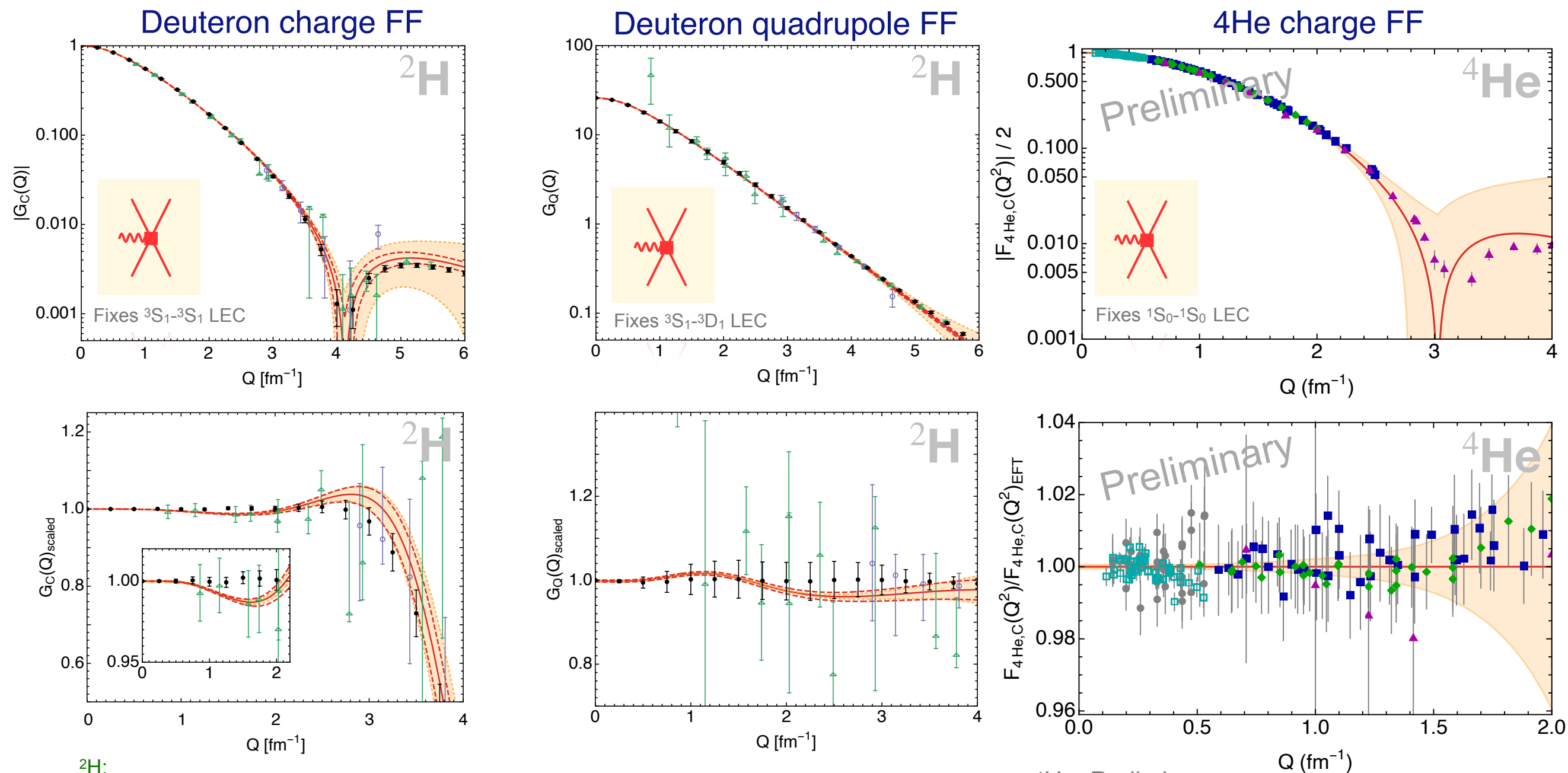
Extraction of deuteron structure radius and quadrupole moment

$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm$$

$$Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Low-energy constants from a fit to charge and quadrupole form factors



${}^2\text{H}$:
 AF, Möller, Baru, Epelbaum, Krebs, Reinert,
 PRL 124 (2020) 082501; PRC 103 (2021) 024313

— best fit + N^4LO truncation uncertainty

3 parameters (LECs) in 2N charge density J^0 are fixed from the form factor data of deuteron and ${}^4\text{He}$

Prediction for ^4He charge radius

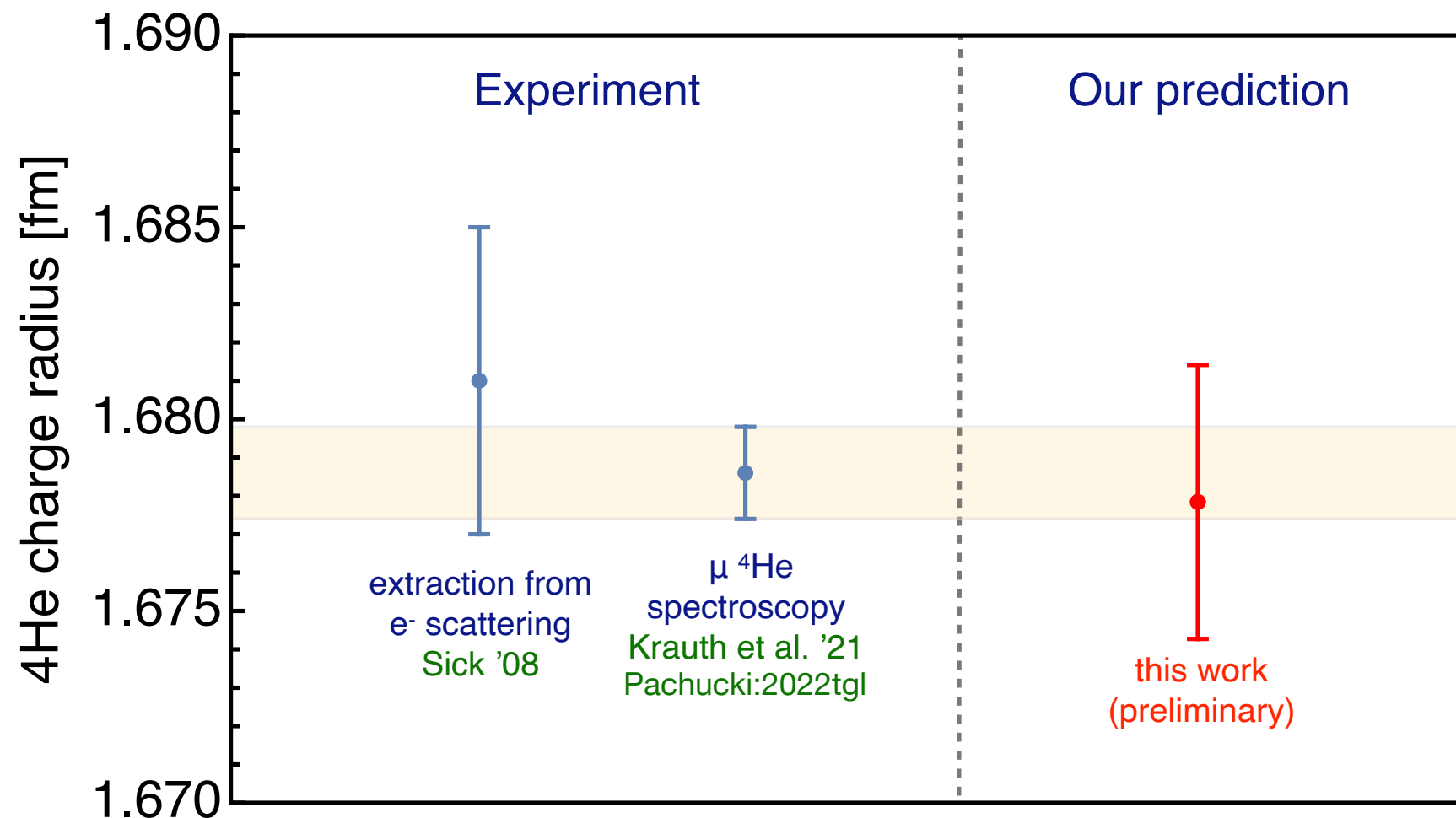
$$r_{str}(^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}} \text{ fm (Preliminary)}$$

Our prediction for ^4He **charge** radius

$$r_C(^4\text{He}) = (1.6775 \pm 0.0035) \text{ fm}$$

$$r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

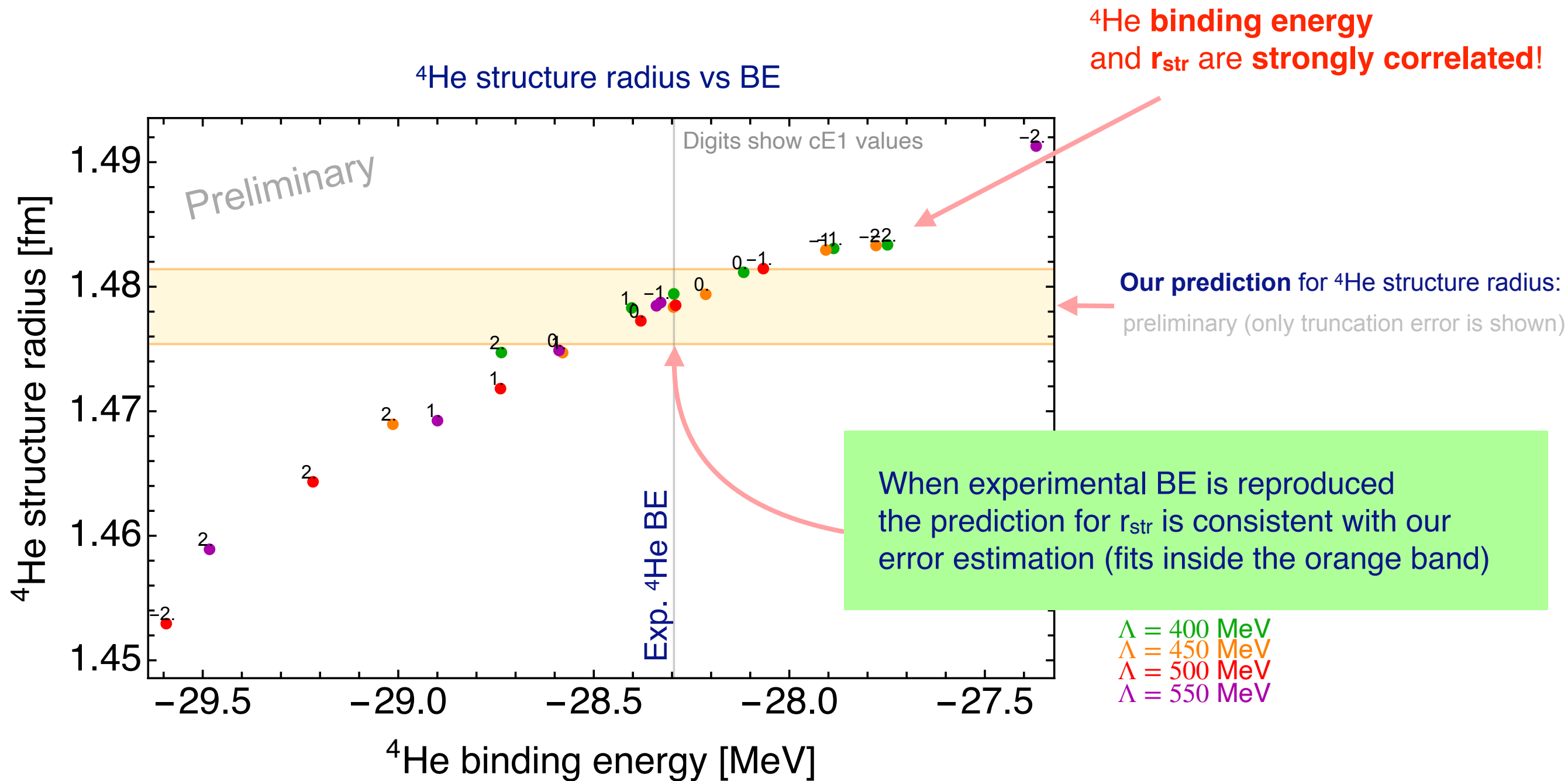
preliminary, using CODATA 2018 r_p and own determination of r_n



Our prediction for ^4He charge radius is fully consistent with the muonic-atom spectroscopy

Correlation between ^4He structure radius and binding energy

using variation of $\mathbf{cE1}$ (N^4LO contact 3NF LEC)



Many effects (higher-order 3NFs, some relativistic corrections) change BE and radius simultaneously. Once BE is fitted back to physical value the residual effect is usually ver small.

Estimation of ^3H charge radius

Our preliminary prediction for **isoscalar 3N charge** radius:

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026) fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Isoscalar 3N charge radius definition:

$$(r_C^{isoscalar3N})^2 = \frac{(r_C^{3H})^2 + 2(r_C^{3He})^2}{3}$$

Expression for ^3H radius:

$$(r_C^{3H})^2 = 3(r_C^{isoscalar3N})^2 - 2(r_C^{3He})^2$$

^3He charge radius [CREMA 2023 2305.11679]

$$r_C^{3He} = (1.9701 \pm 0.0009) fm$$

Coefficients 2 and 3 amplify both theoretical and experimental uncertainties

Our ^3H radius estimation:

$$r_C^{(3H)} = (1.7714 \pm 0.0087) fm$$

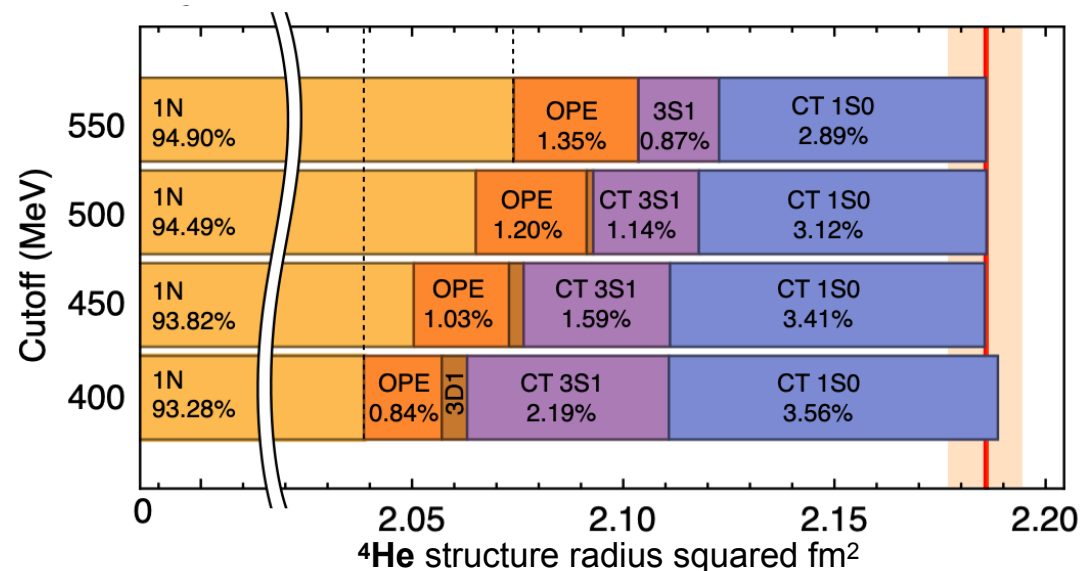
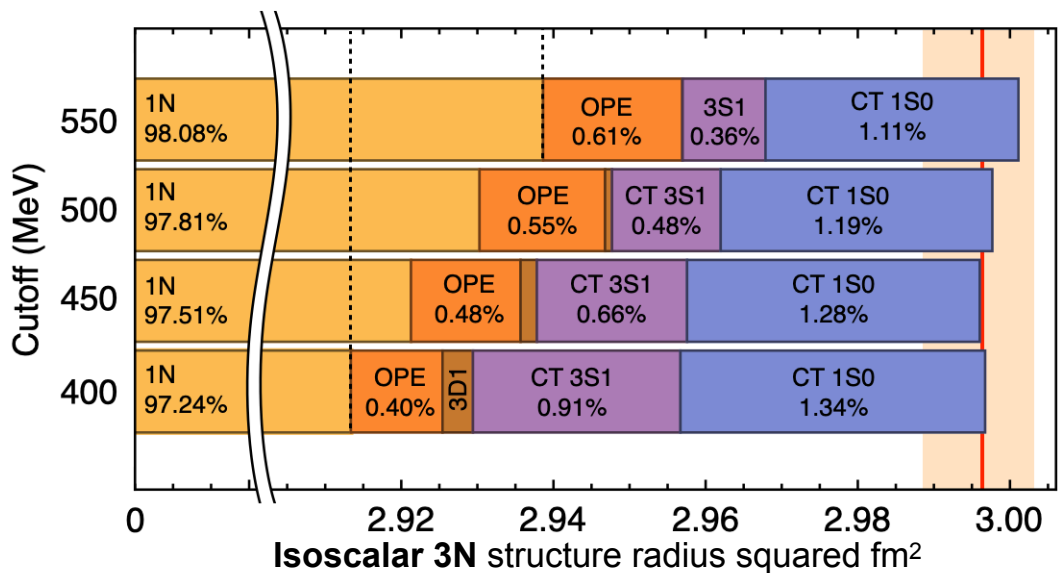
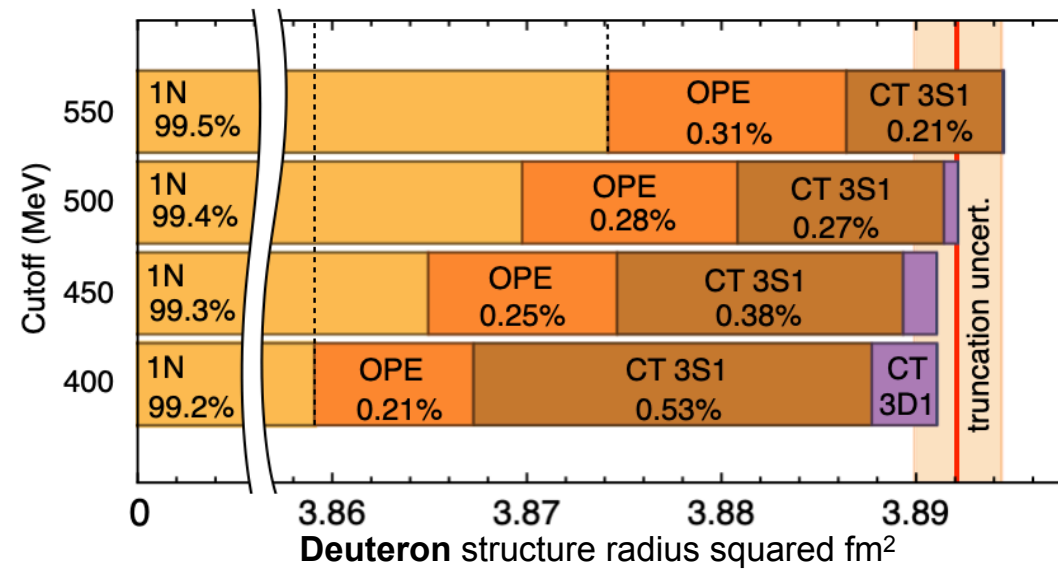
preliminary

This estimation is 10x more precise than e⁻ data $r_C^{3H} = (1.7550 \pm 0.0860) fm$ Amroun et al. '94 (world average)

But it suffers from parametric amplification of uncertainties (both from theory and from ^3He data)

=> isoscalar 3N charge radius should be used for precision tests

Importance of 2N charge density



Individual contributions to $A=2,3,4$ structure radii from

- single-nucleon charge density (1N)
- 2N one-pion exchange density (OPE)
- 2N contact densities (CT 3S1, 3D1, 1S0)

2N charge density contribution to structure radii squared:

deuteron $\sim 0.7\%$

isoscalar 3N $\sim 2.5\%$

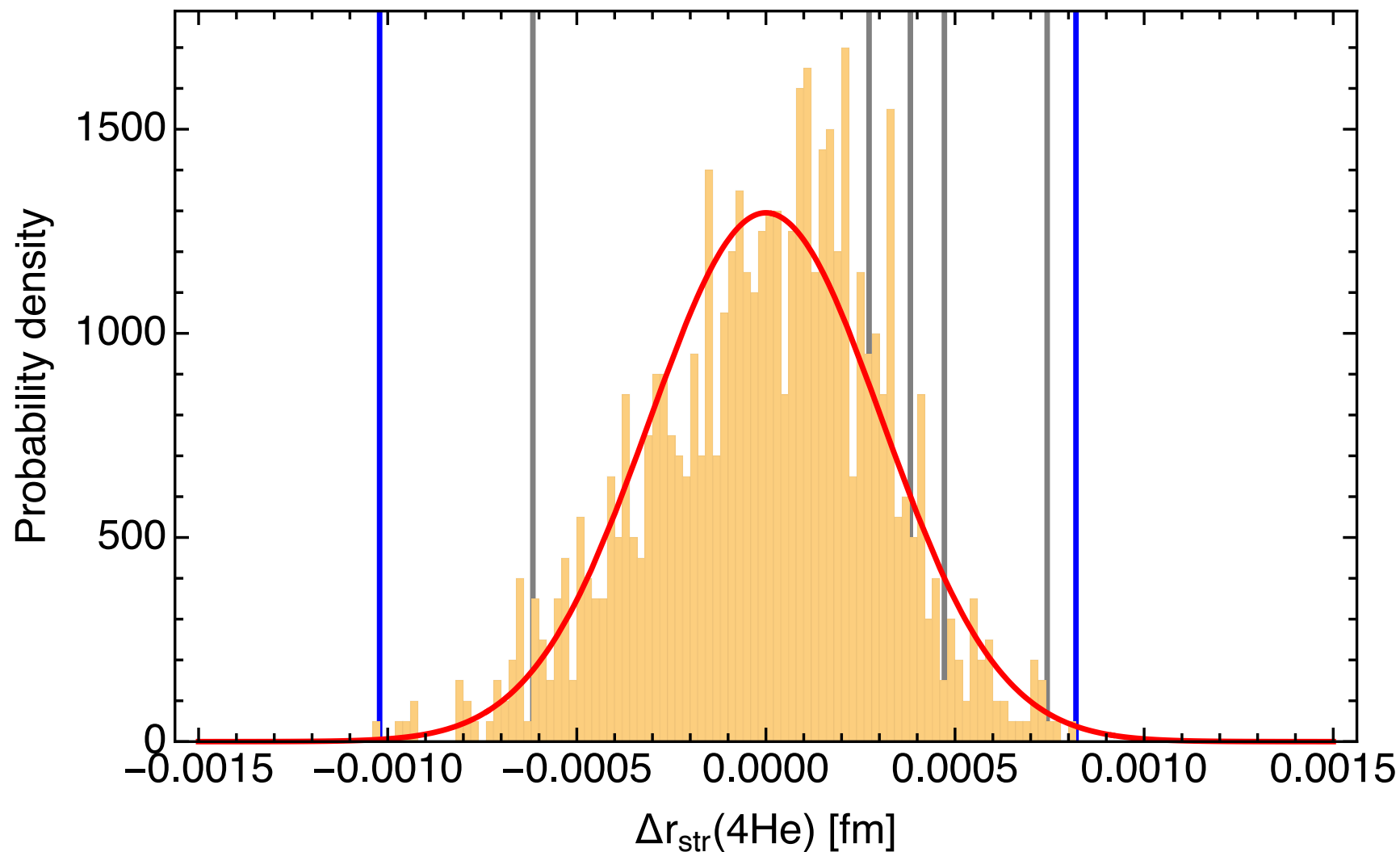
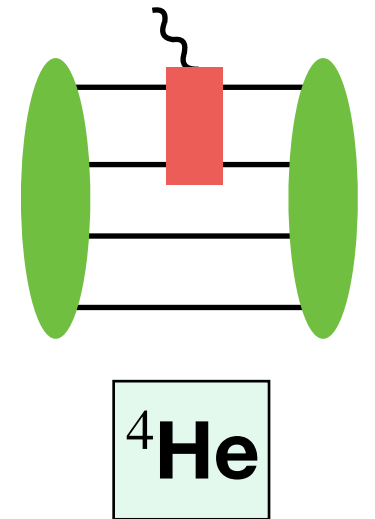
^4He $\sim 6\%$

For $A=2,3,4$ importance of 2N charge grows with A

Sensitivity to nucleon FF parameterisation

Results for ^4He structure radius fit with ~ 1000 variations of nucleon isoscalar FF

GES from dispersive analysis of Bonn group [Lin, Hammer and Meißner, PRL 128 2022](#)



Uncertainty from FF parametrisation:

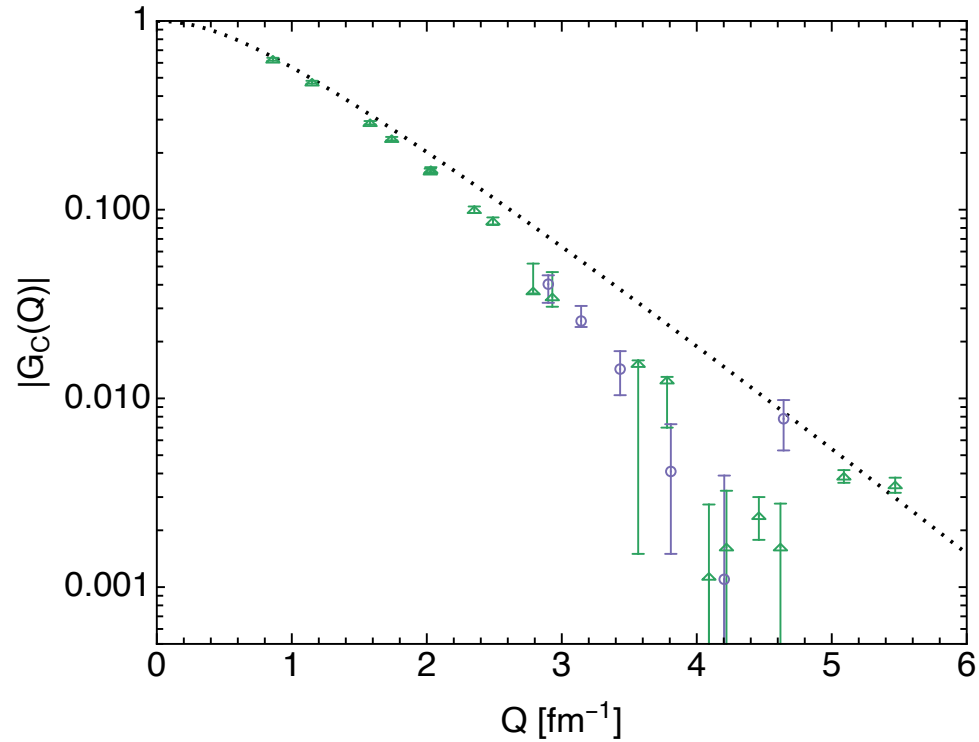
$$r_{\text{str}}(^4\text{He}) = 1.4758 \pm 0.0010_{\text{nucFF}} \text{ fm} \quad (\text{Preliminary})$$

We have also tested other modern nucleon FF parametrisations and they produce results in the same range

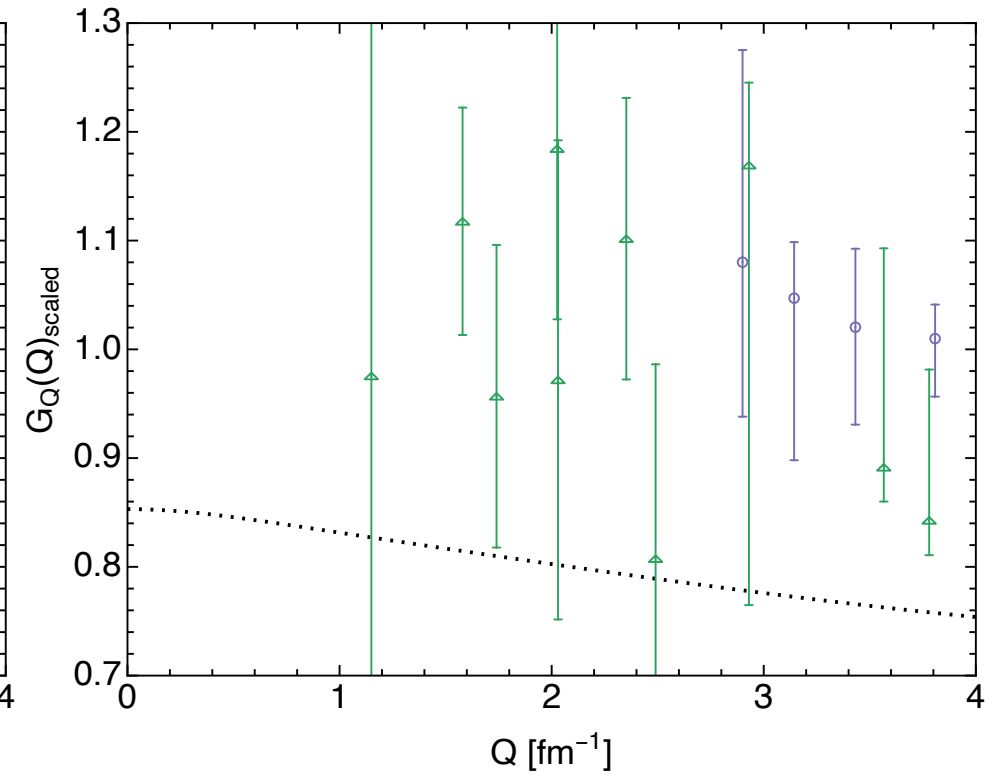
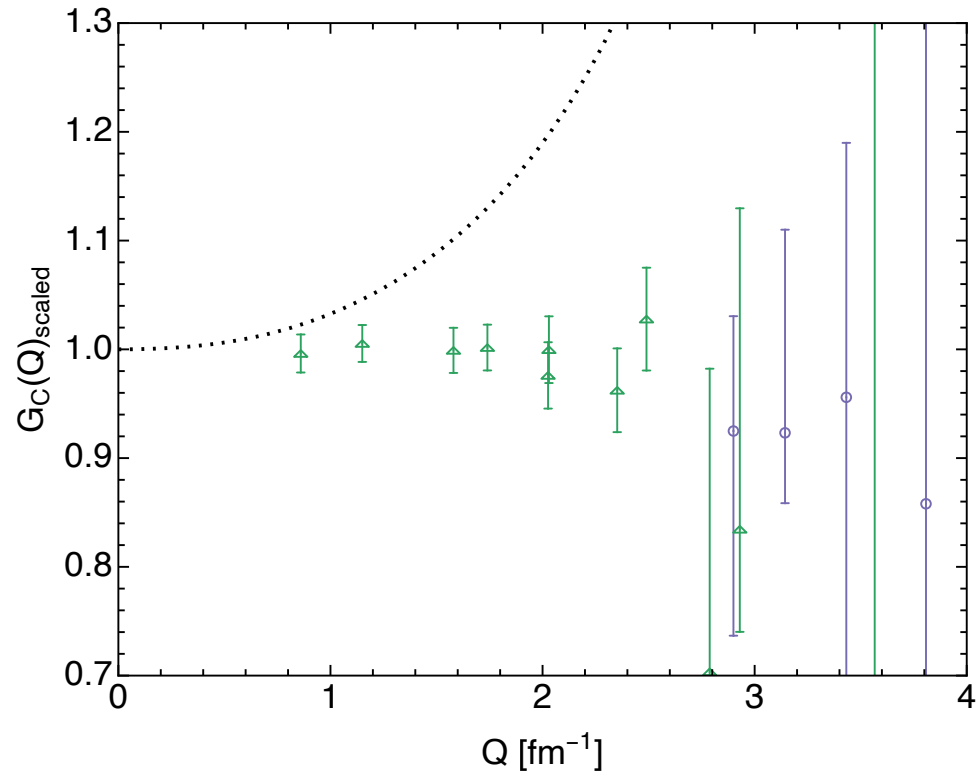
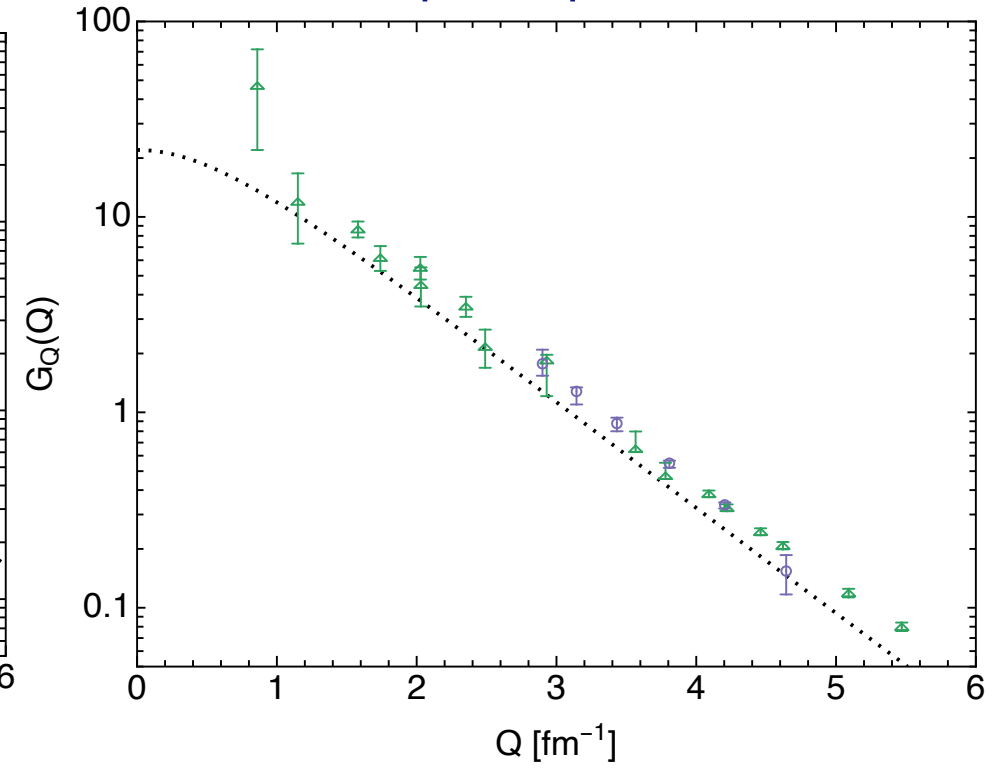
Deuteron charge and quadrupole form factors

LO

Deuteron charge form factor



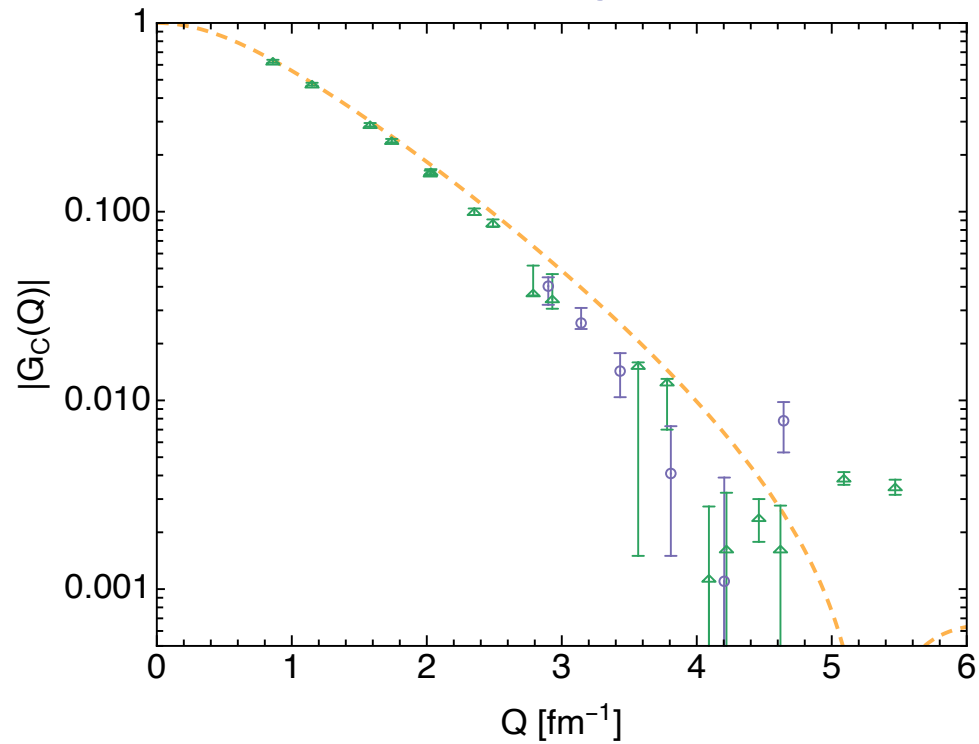
Deuteron quadrupole form factor



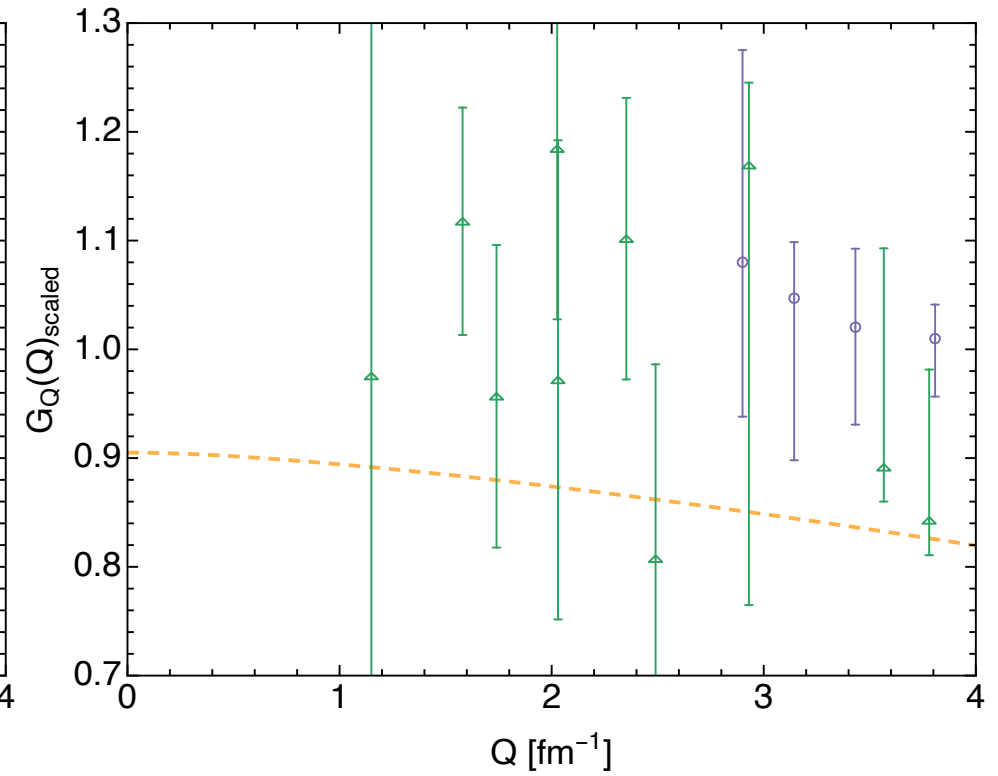
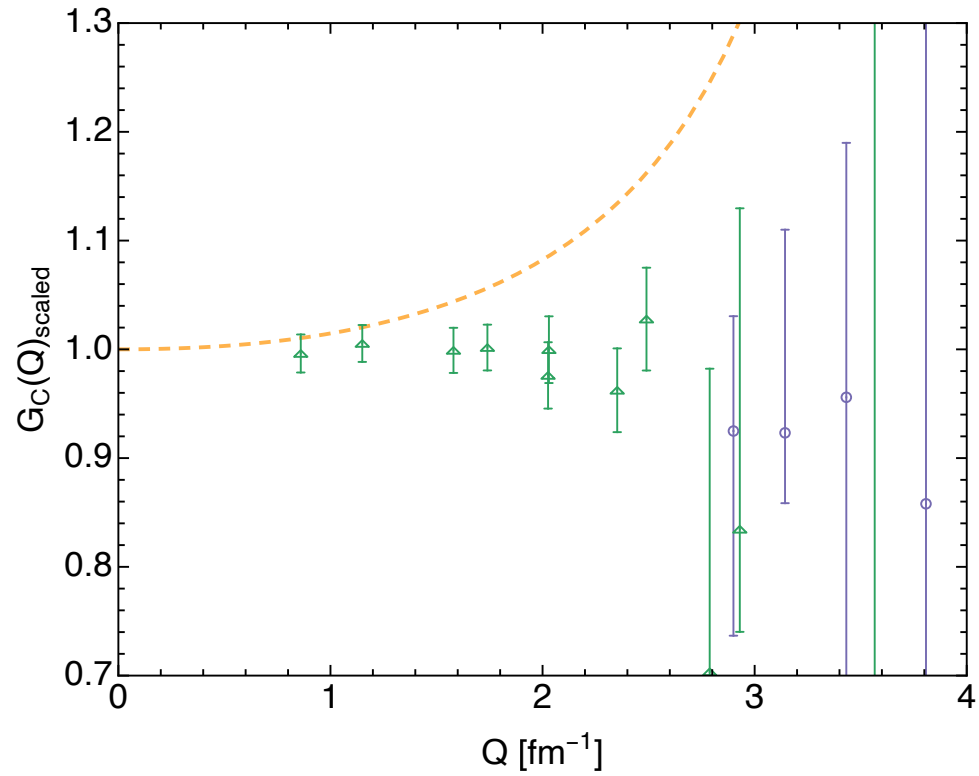
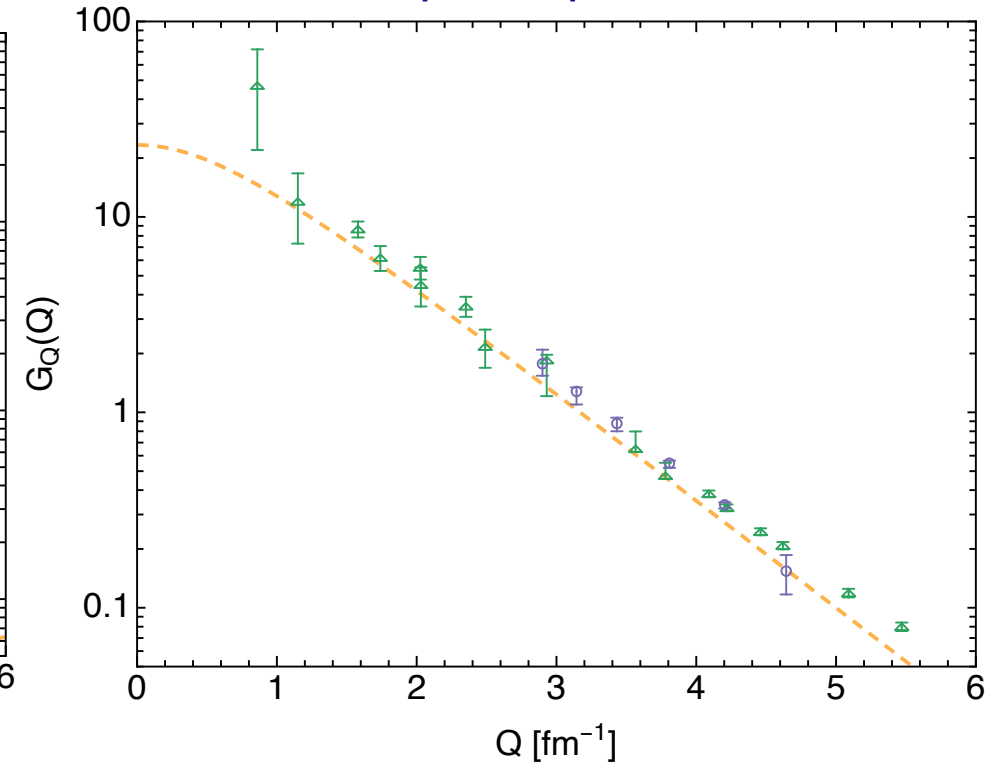
Deuteron charge and quadrupole form factors

NLO

Deuteron charge form factor



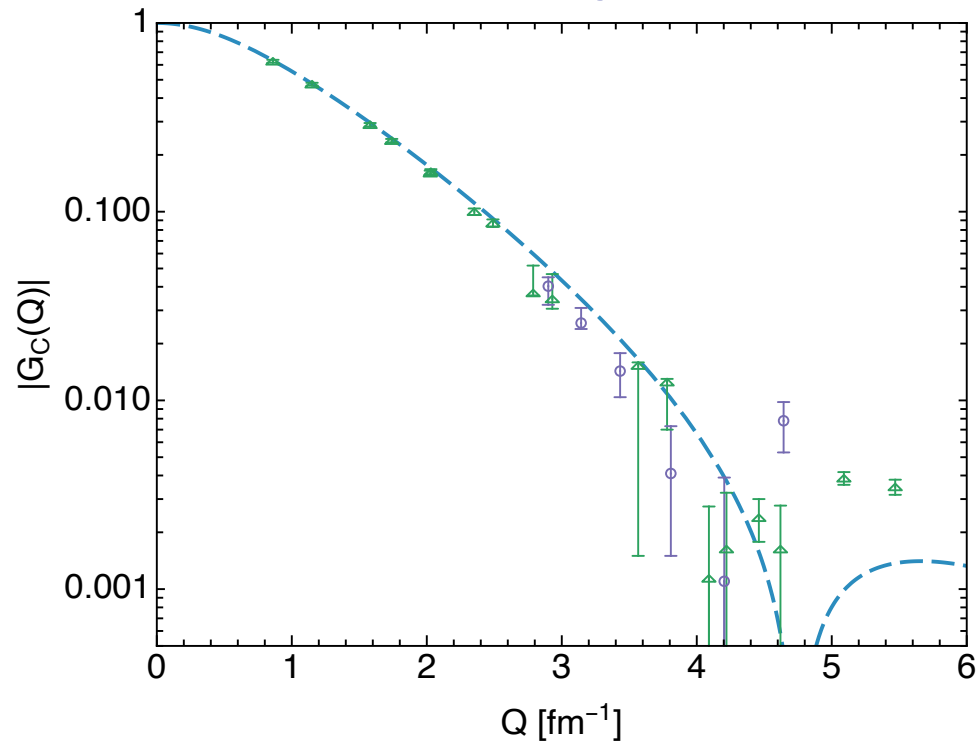
Deuteron quadrupole form factor



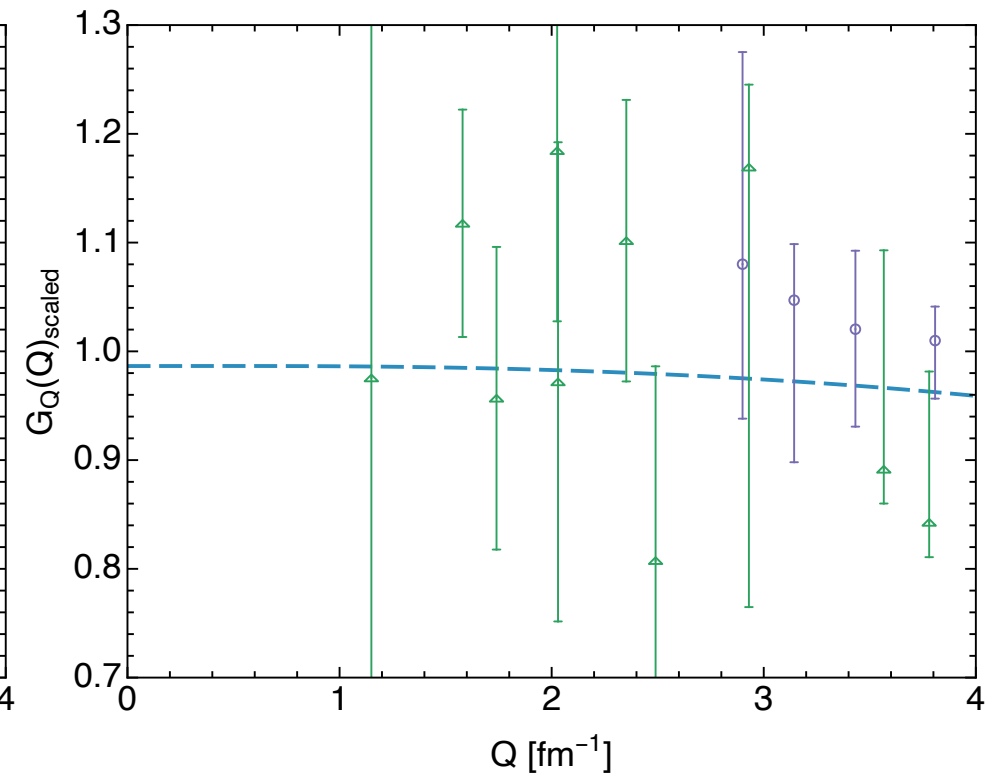
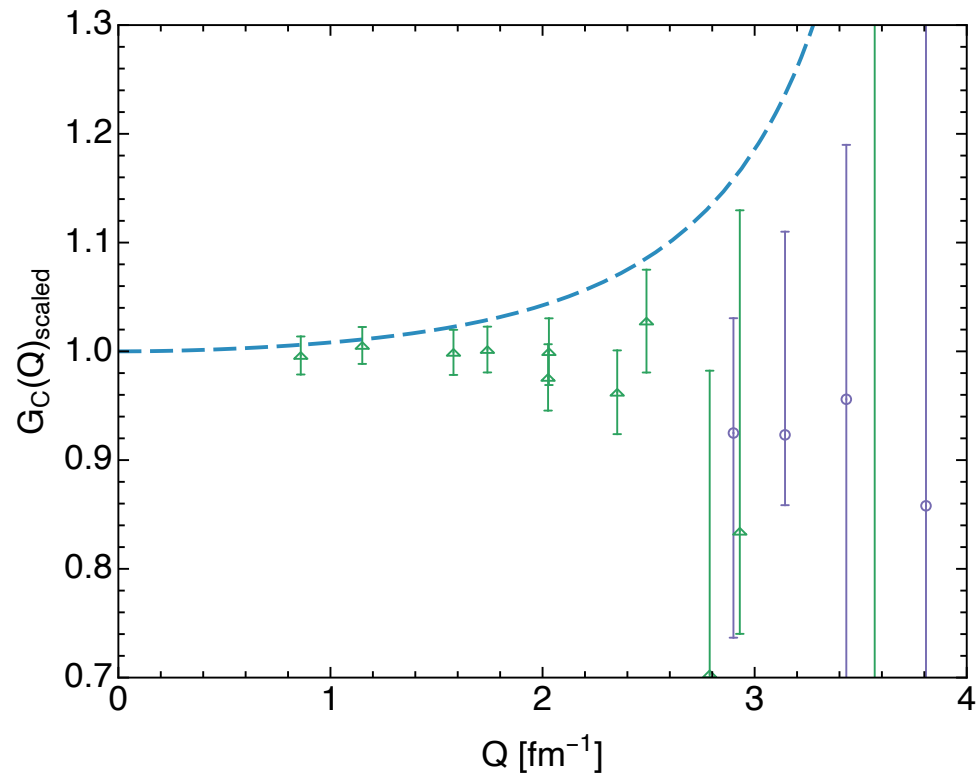
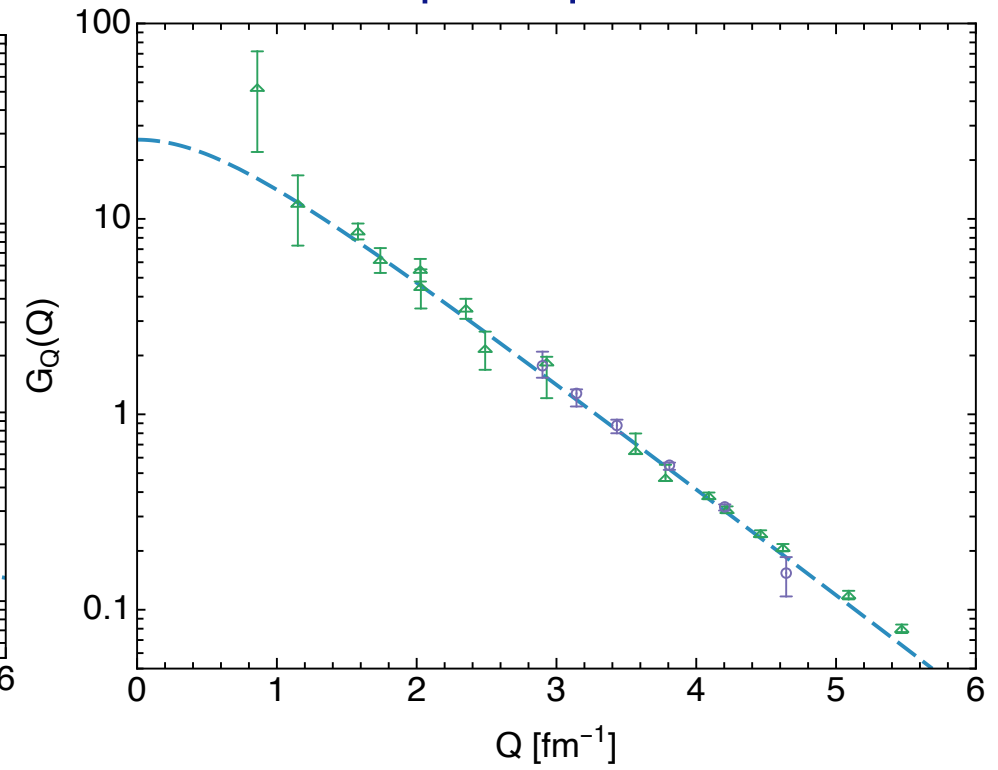
Deuteron charge and quadrupole form factors

N2LO

Deuteron charge form factor

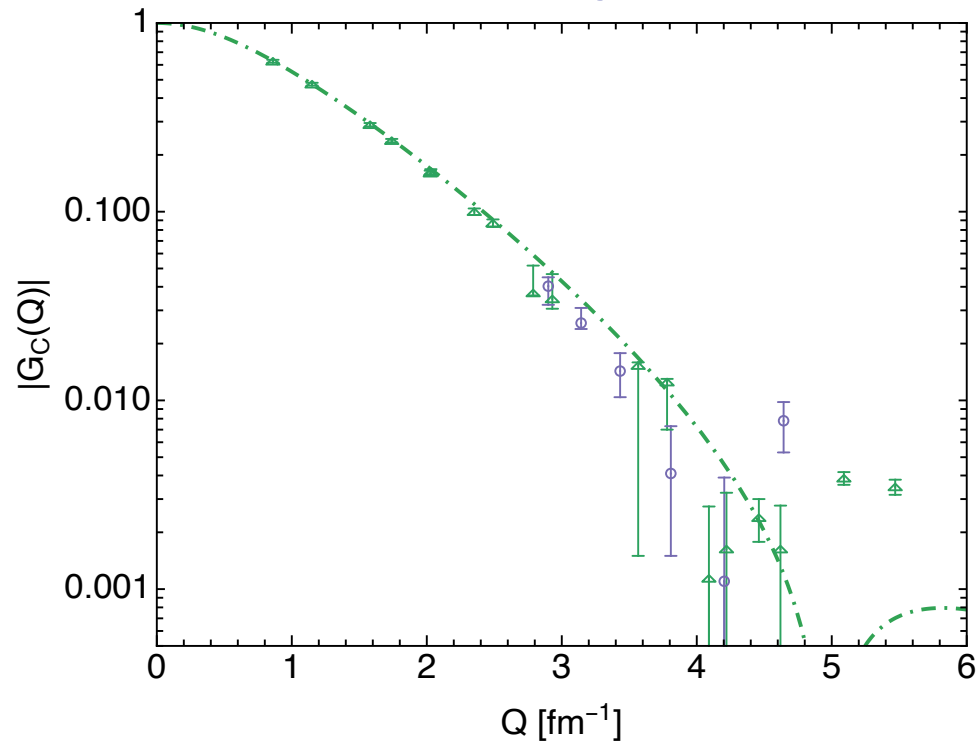


Deuteron quadrupole form factor

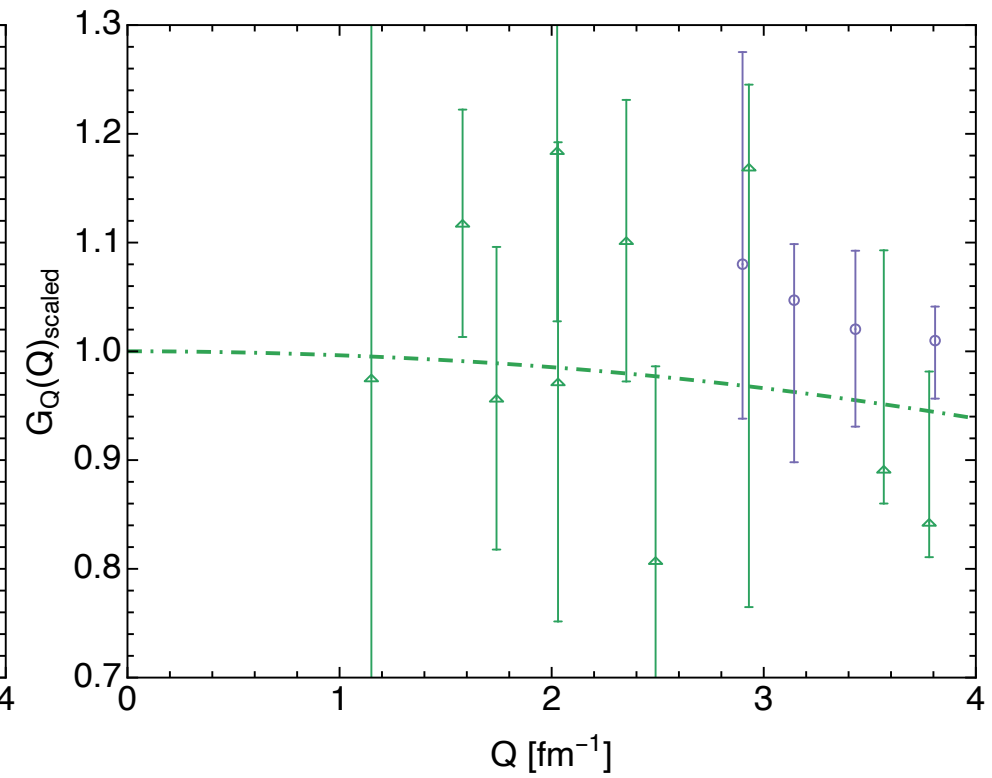
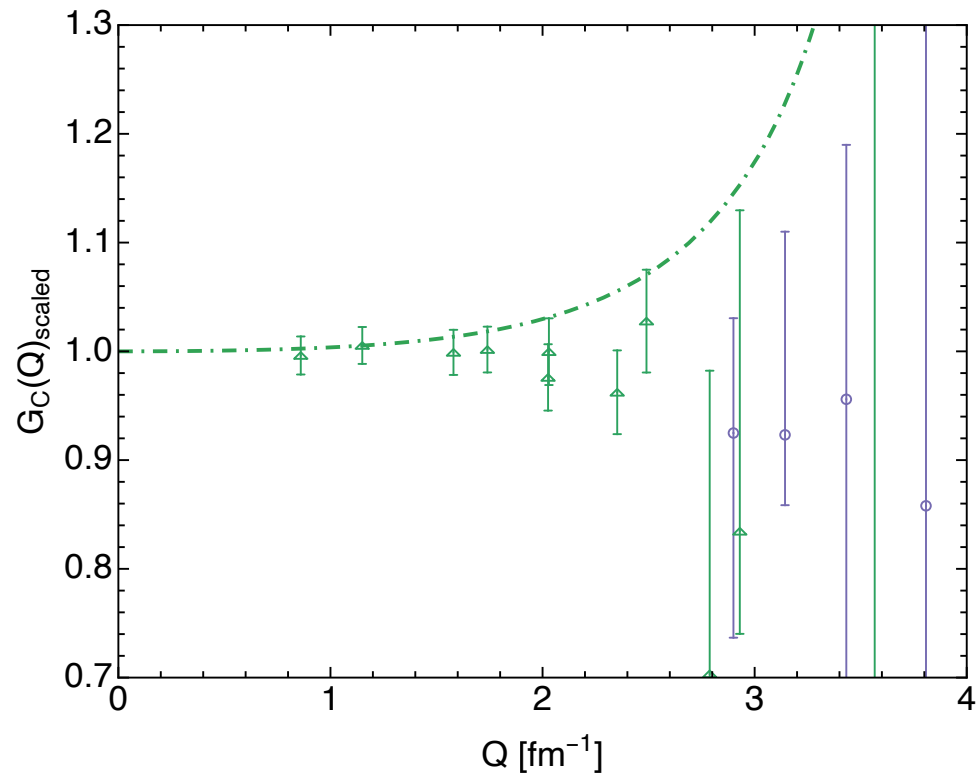
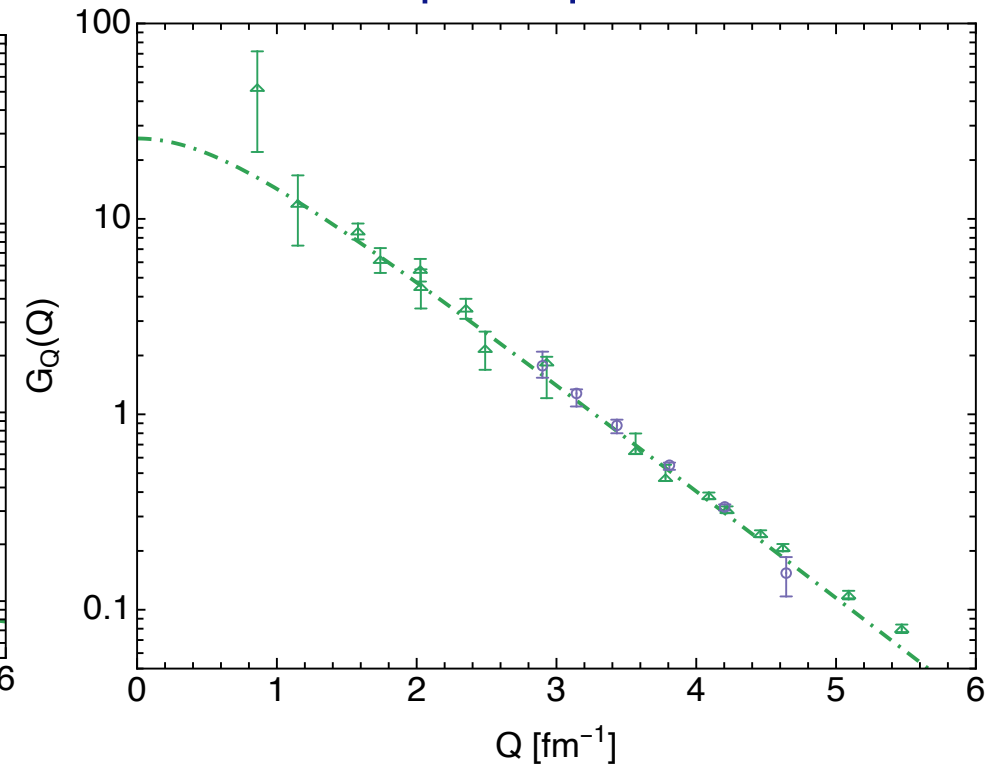


Deuteron charge and quadrupole form factors

Deuteron charge form factor

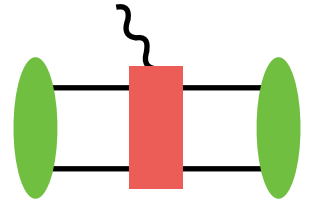


Deuteron quadrupole form factor



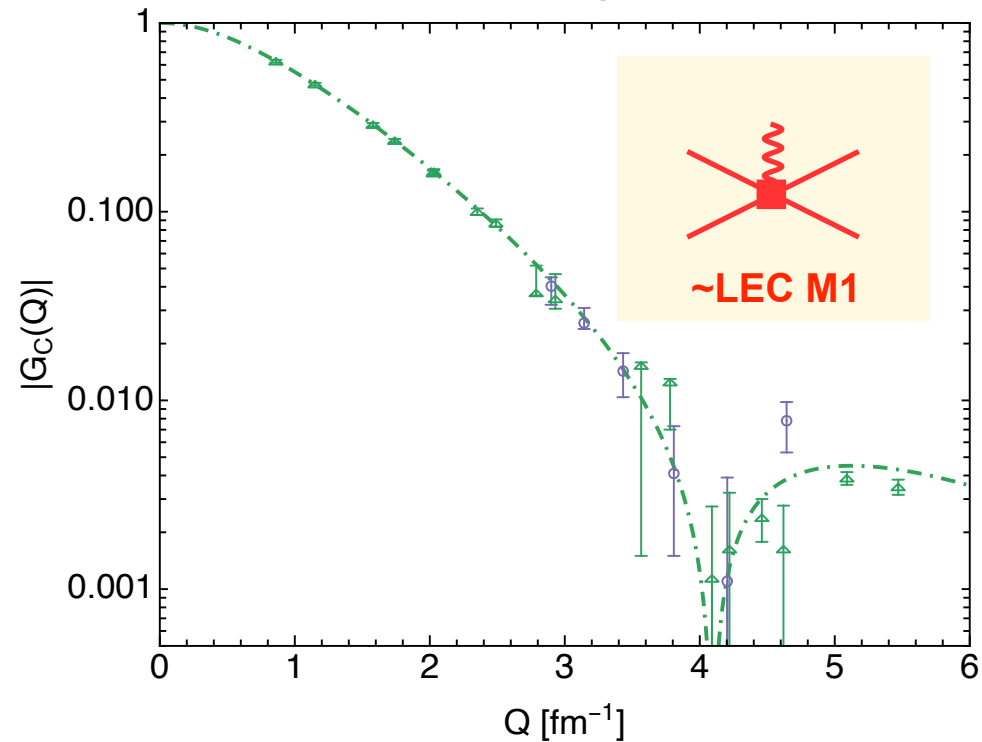
N3LO
without
2N charge
density

Deuteron charge and quadrupole form factors

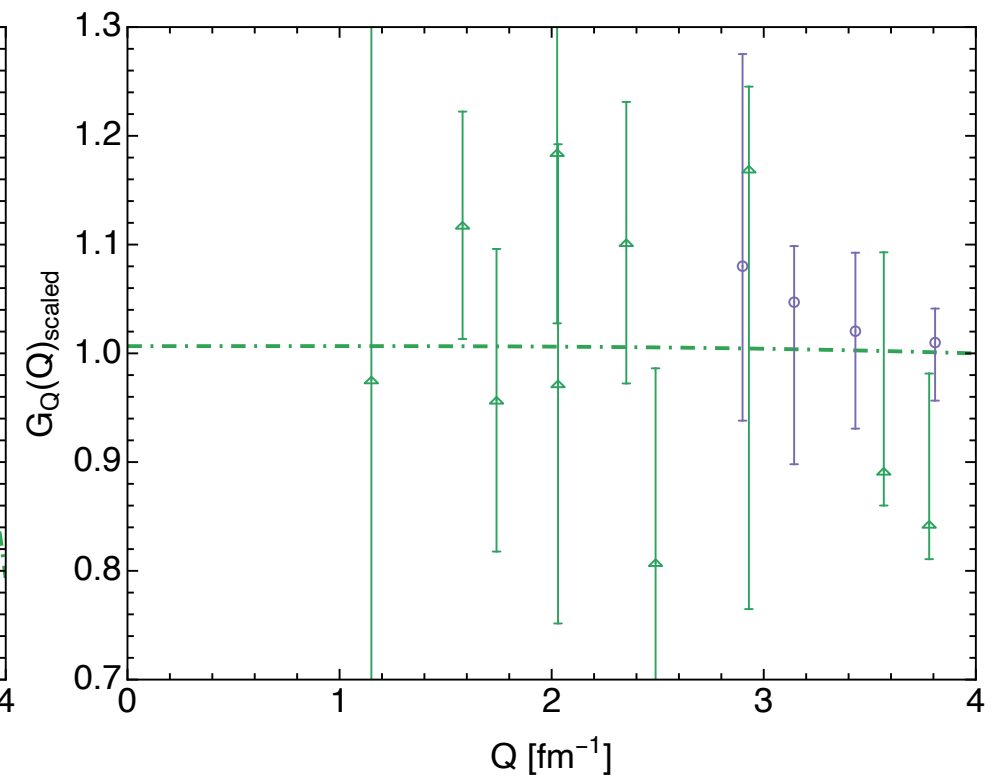
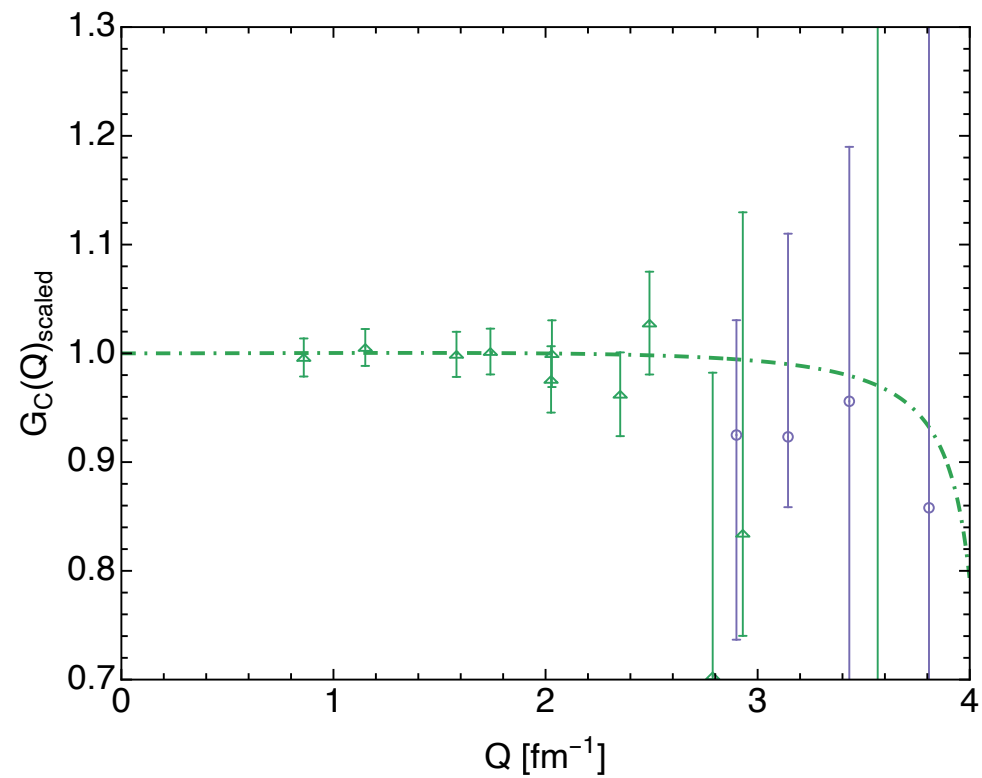
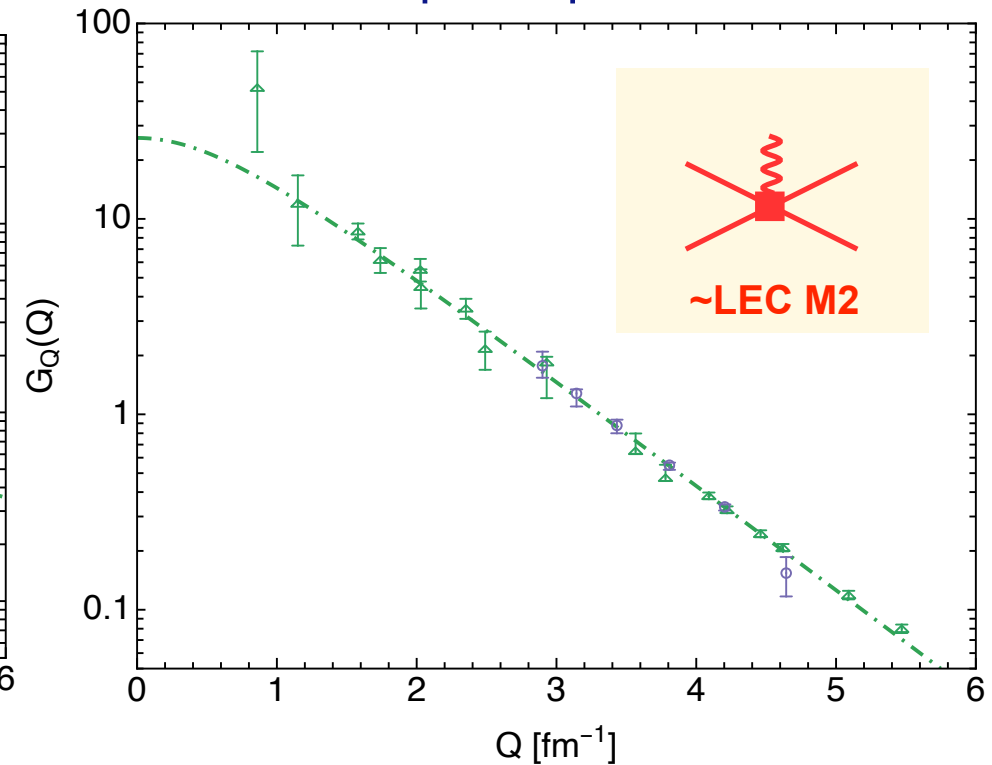


N3LO
with
2N charge
density

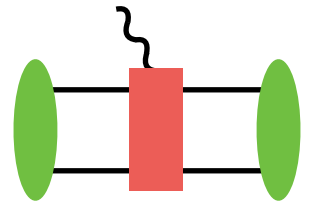
Deuteron charge form factor



Deuteron quadrupole form factor

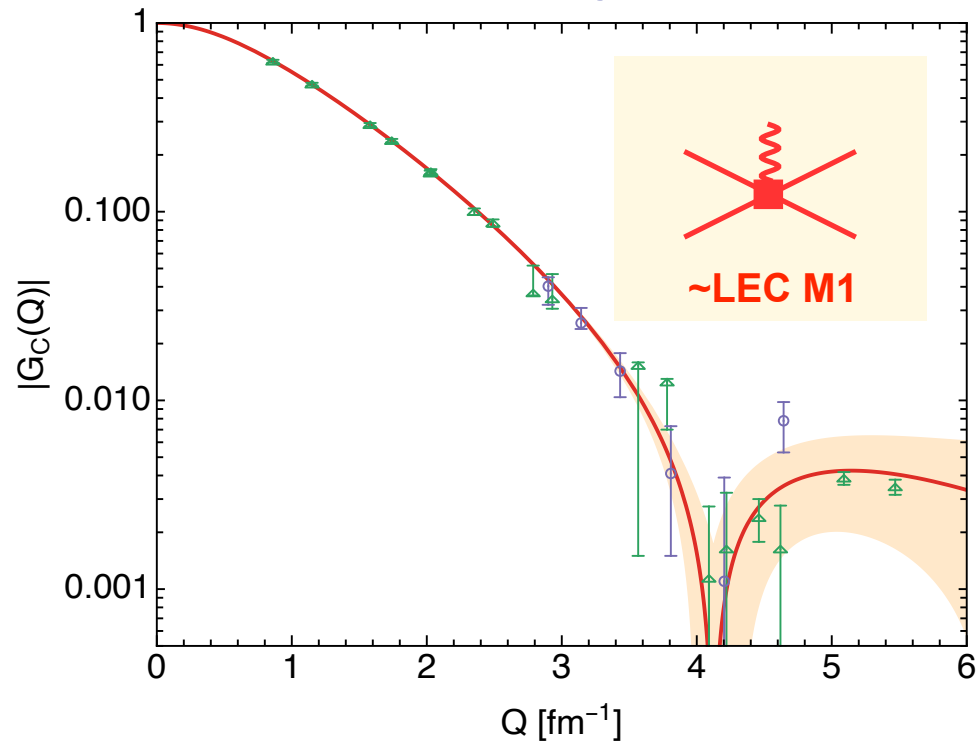


Deuteron charge and quadrupole form factors

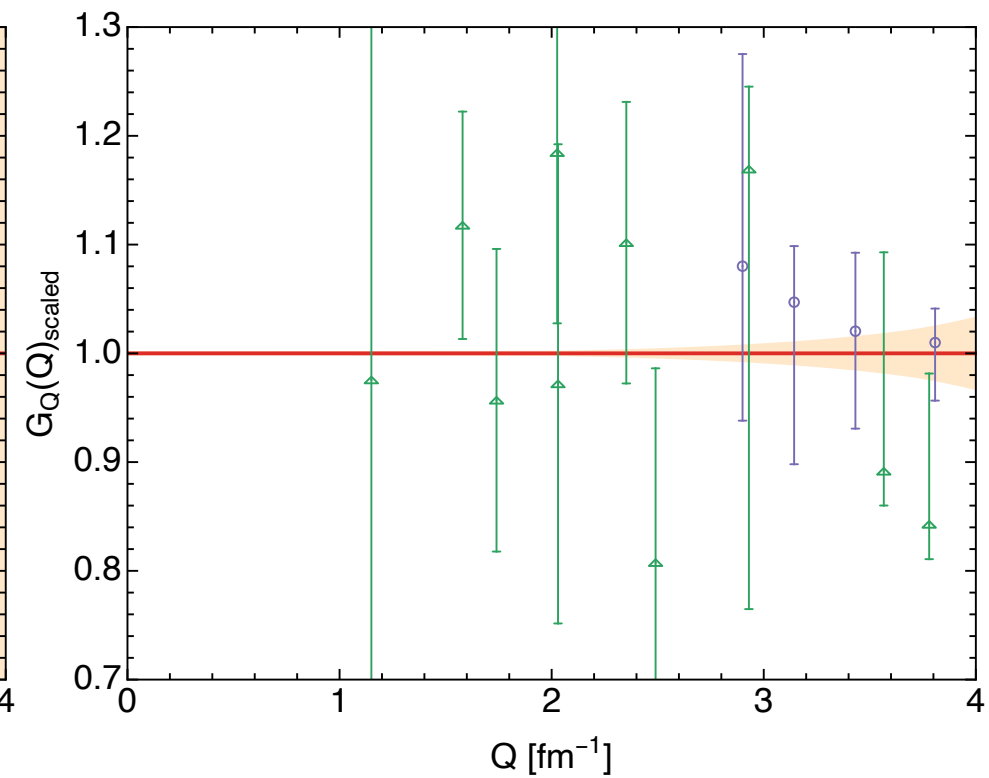
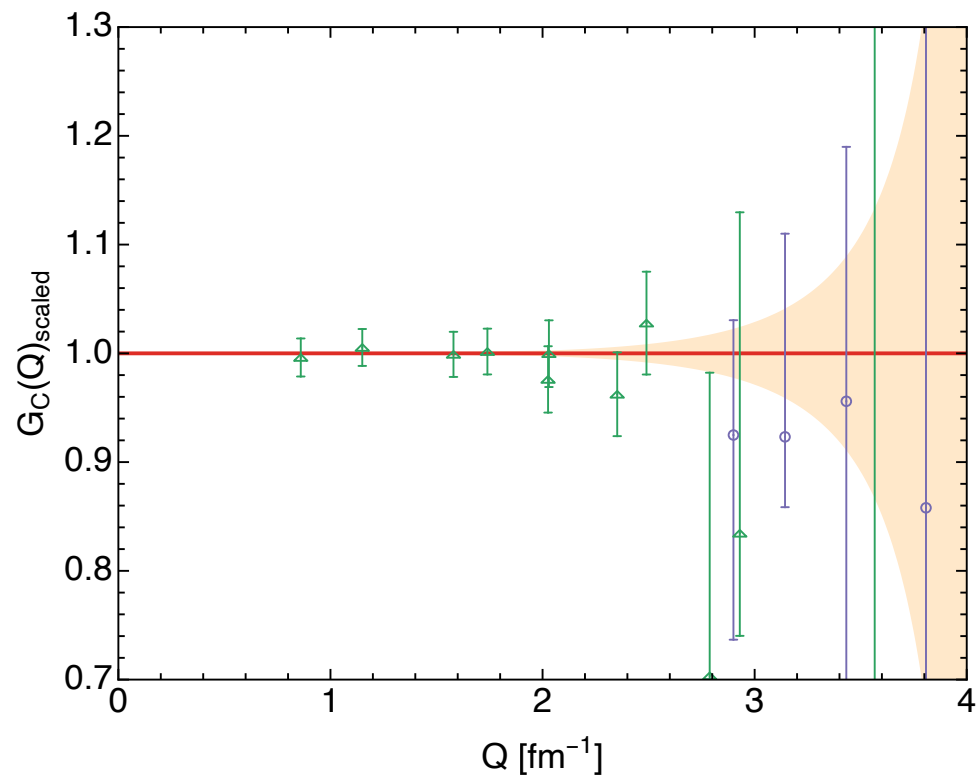
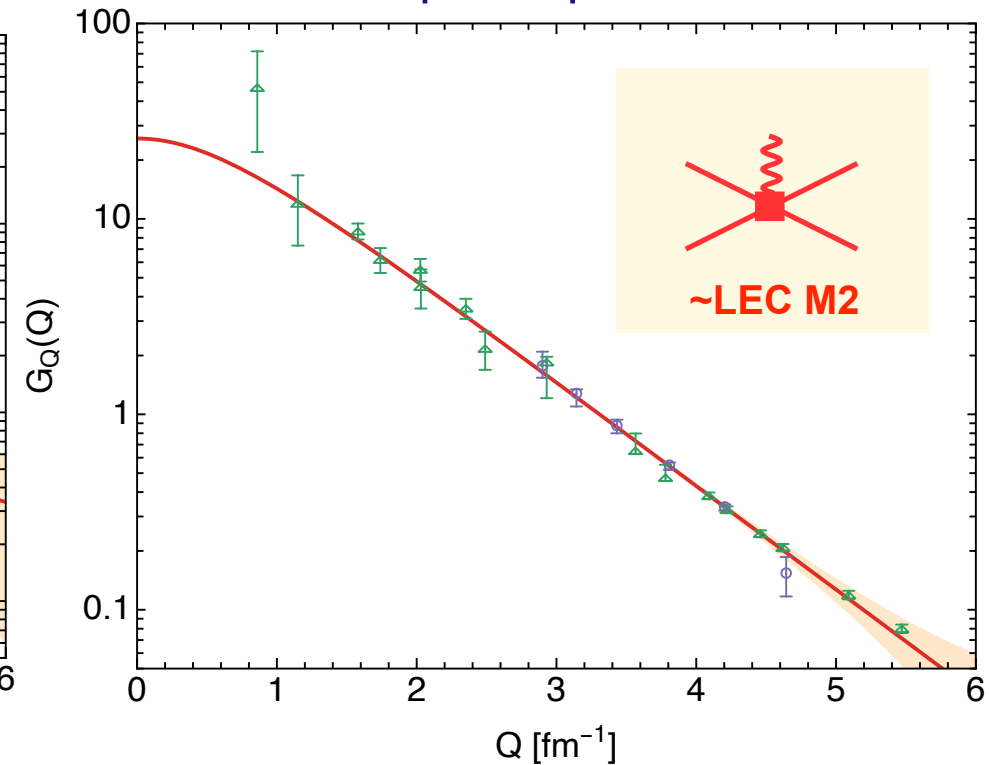


N4LO
with
2N charge
density

Deuteron charge form factor



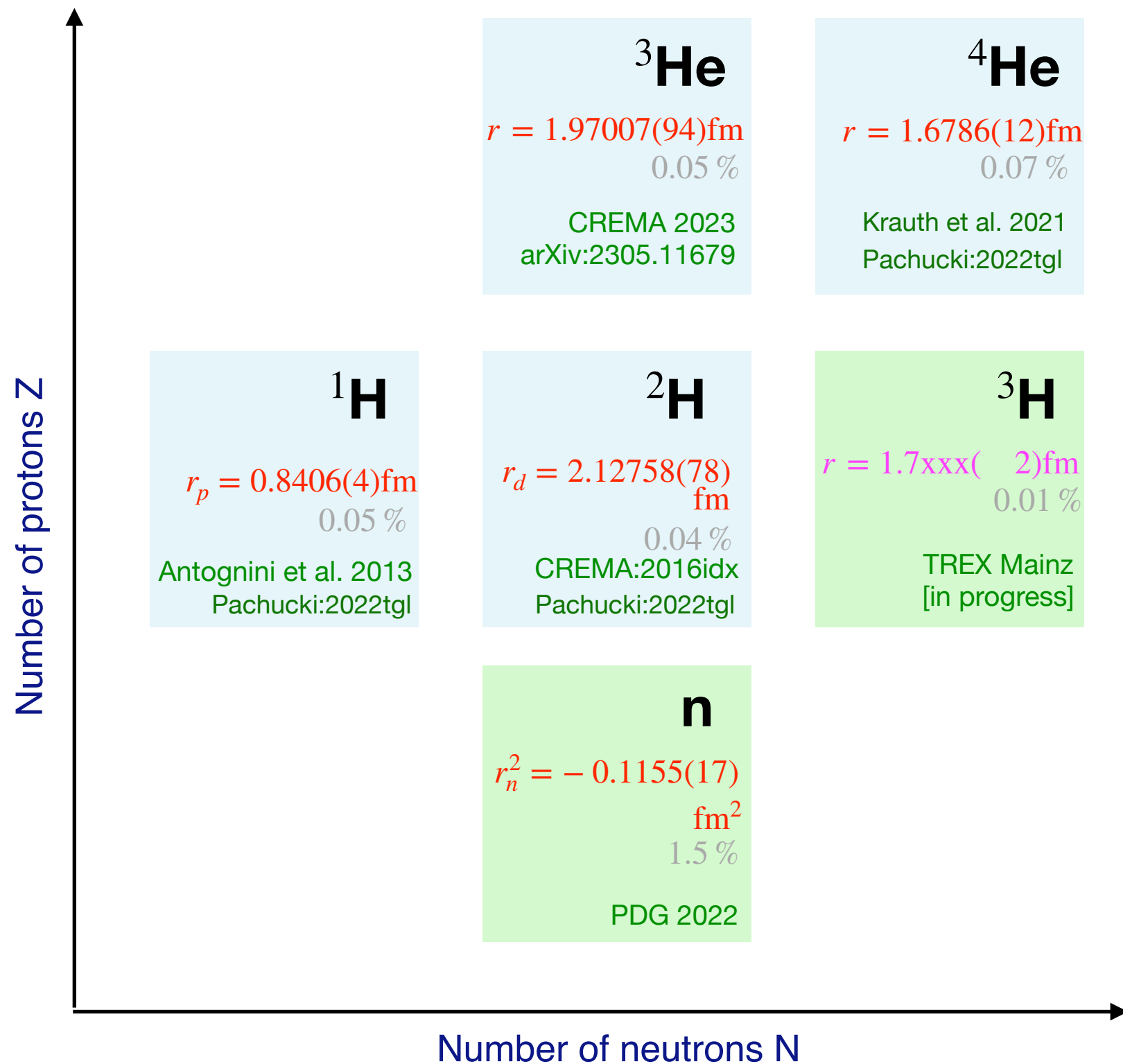
Deuteron quadrupole form factor



LECs M1 and M2 are fitted to GC and GQ data

Good description of low-Q behaviour and also node position

Charge radii of $A \leq 4$ nuclei — experimental data



Pohl:2013yb,
CREMA:2016idx,
Pohl:2016glp,
Schmidt:2018kjc,
Krauth:2021foz
...

Data from isotope shift

$$(r_d^2 - r_p^2) = 3.82070(31)\text{fm}^2$$

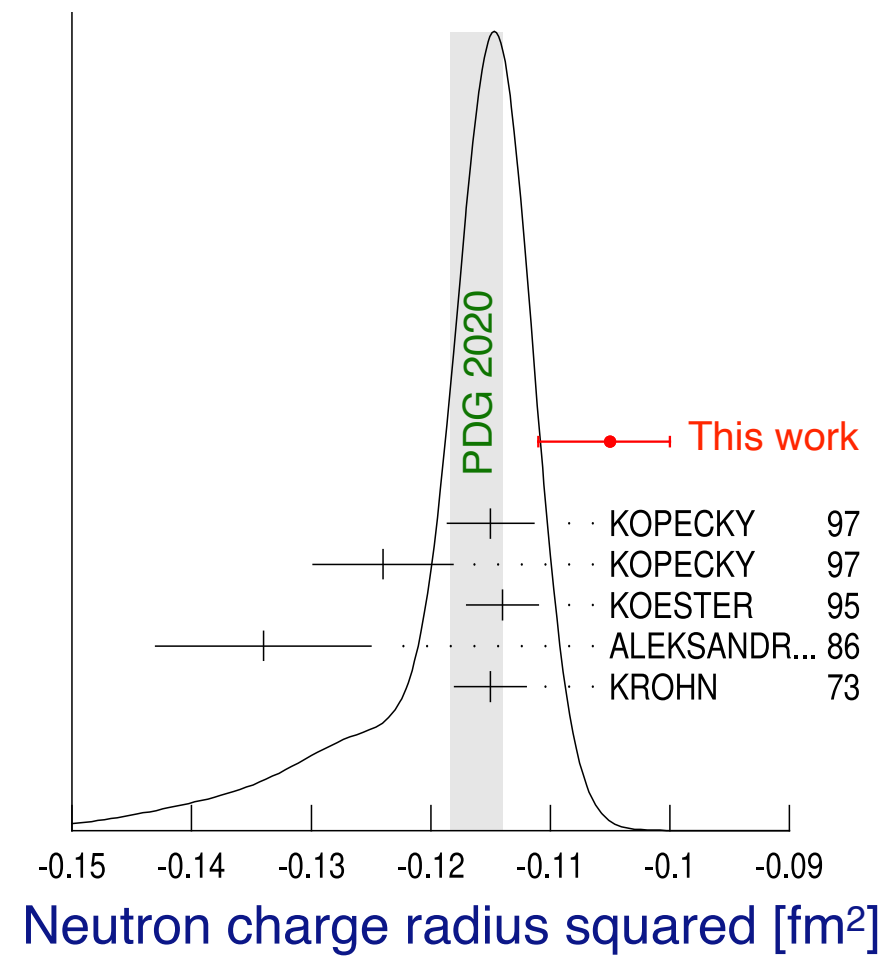
0.01 %

Pachucki et al. 2018
Jentschura et al. 2011

Our extraction of the neutron charge radius

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{fm}^2$$

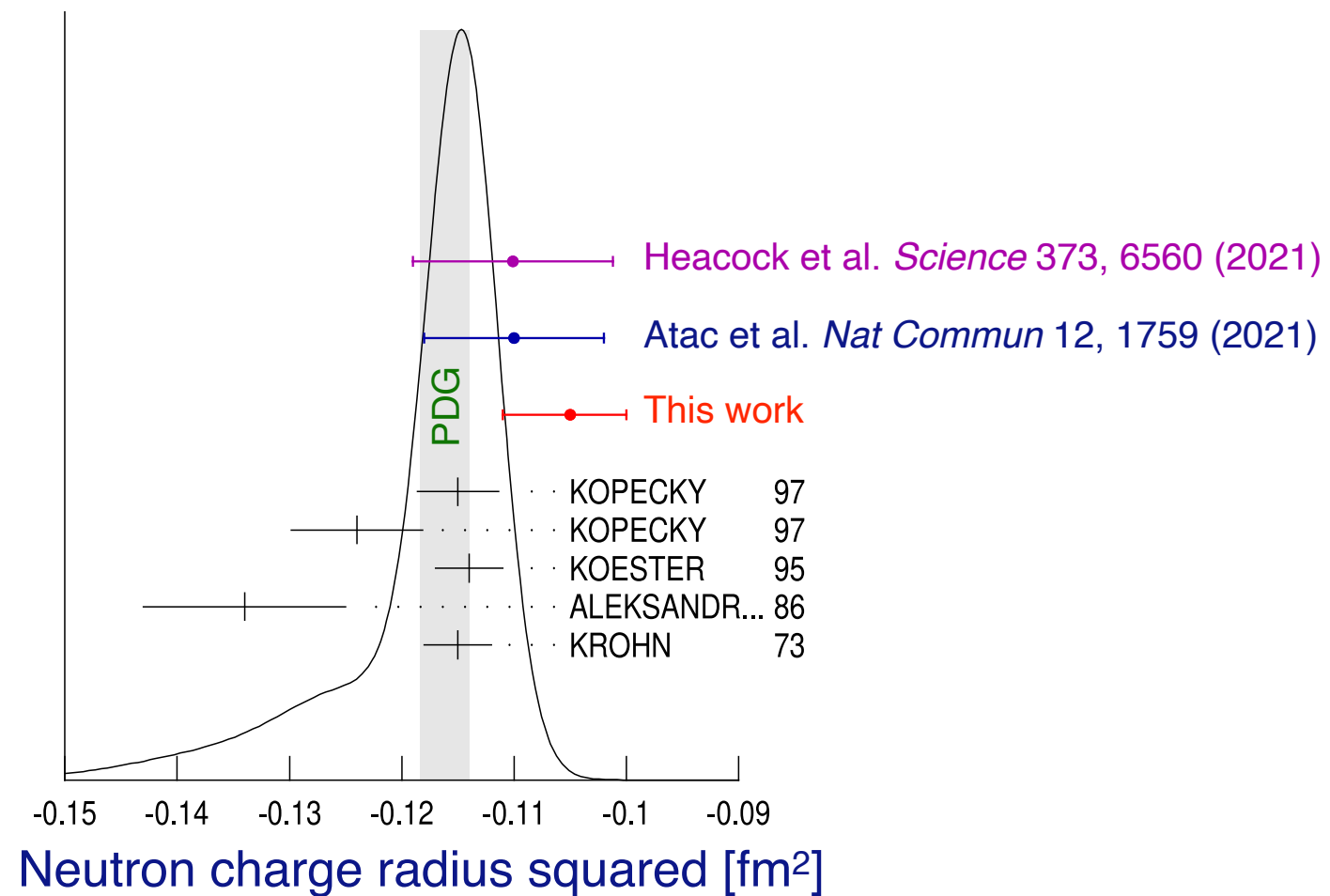
$\sim 2\sigma$ deviation from the PDG (2022) weighted average $r_n^2 = -0.1155(17)\text{fm}^2$



Our extraction of the neutron charge radius

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{fm}^2$$

$\sim 2\sigma$ deviation from the **PDG (2022)** weighted average $r_n^2 = -0.1155(17)\text{fm}^2$



Neutron charge radius in PDG 2022

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm ²)	DOCUMENT ID	COMMENT
=0.1155±0.0017 OUR AVERAGE		
−0.115 ±0.002 ±0.003	KOPECKY 97	ne scattering (Pb)
−0.124 ±0.003 ±0.005	KOPECKY 97	ne scattering (Bi)
−0.114 ±0.003	KOESTER 95	ne scattering (Pb, Bi)
−0.115 ±0.003	¹ KROHN 73	ne scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •		
−0.1101±0.0089	² HEACOCK 21	n interferometry
−0.106 $\begin{smallmatrix} +0.007 \\ -0.005 \end{smallmatrix}$	³ FILIN 20	chiral EFT analysis
−0.117 $\begin{smallmatrix} +0.007 \\ -0.011 \end{smallmatrix}$	BELUSHKIN 07	Dispersion analysis
−0.113 ±0.003 ±0.004	KOPECKY 95	ne scattering (Pb)
−0.134 ±0.009	ALEKSANDR...86	ne scattering (Bi)
−0.114 ±0.003	KOESTER 86	ne scattering (Pb, Bi)
−0.118 ±0.002	KOESTER 76	ne scattering (Pb)
−0.120 ±0.002	KOESTER 76	ne scattering (Bi)
−0.116 ±0.003	KROHN 66	ne scattering (Ne, Ar, Kr, Xe)

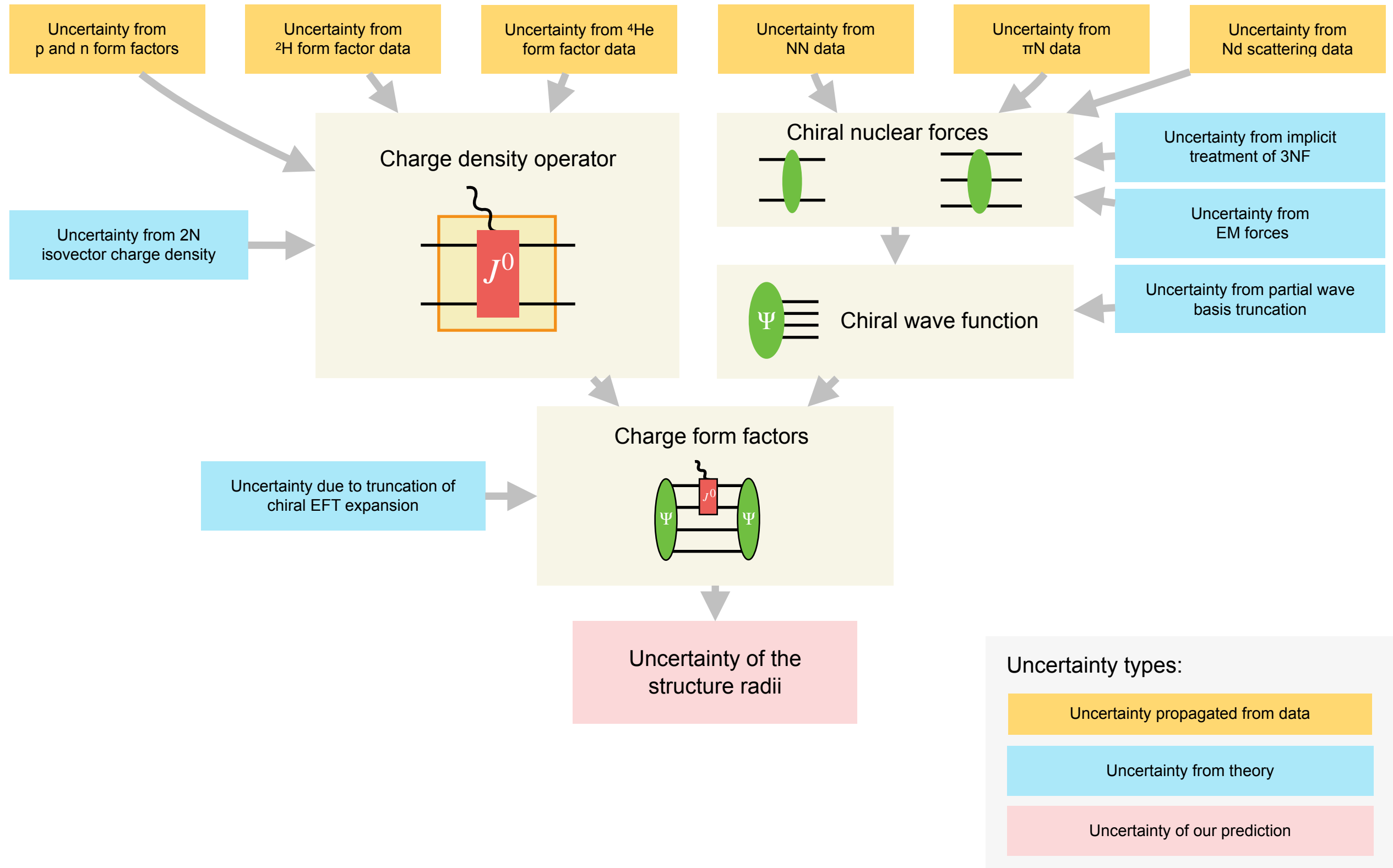
¹ KROHN 73 measured -0.112 ± 0.003 fm². This value is as corrected by KOESTER 76.

² HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

³ FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

Extensive uncertainty analysis

Propagation of uncertainties from data and theory



Indications of BSM physics?

All data used to constrain chiral EFT LECs are from strong interaction / electron-based experiments:

π N Roy-Steiner analysis [Hoferichter:2015tha](#), [Hoferichter:2015hva](#)

NN pn and pp scattering data, deuteron BE [Reinert:2020mcu](#)

Deuteron charge and quadrupole FF data [JLABt20:2000qyq](#), [Nikolenko:2003zq](#)

Deuteron-proton radii difference from atomic spectroscopy [Pachucki:2018yxe](#), [Jentschura et al. PRA 83 \(2011\)](#)

Proton charge radius [CODATA2018](#)

^4He form factor data [Erich:1971rhg](#), [Mccarthy:1977vd](#), [VonGunten:1982yna](#), [Ottermann:1985km](#), [Frosch:1967pz](#),
[Arnold:1978qs](#), [Camsonne:2013df](#)

Binding energies of ^3He and ^4He

Nd DCS minimum @ 70 MeV [RIKEN data](#)

No muonic data is used in our chiral EFT predictions

Our prediction for ^4He charge radius is consistent with the muonic experiment

No indication of lepton universality breaking at this accuracy level