The puzzles of the muon anomalous magnetic moment

Hartmut Wittig

Institute for Nuclear Physics, Helmholtz Institute Mainz, and PRISMA+ Cluster of Excellence,
Johannes Gutenberg-Universität Mainz

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Ruhr University Bochum
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WISSENSCHAFT

11. APRIL 2021 NR. 14 SEITE 57

Braut sich da was zusammen? Im Sommer 2013 wurde der Myonen-Speicherring (auf dem Lastwagen) am Fermilab nahe Chicago angeliefert. Jetzt wurden erste Ergebnisse verkündet. Foto Fermilab/Reidar Hahn

Die Macht der Myonen

Ein Teilchen schickt sich an, eine beispiellos erfolgreiche Theorie zu sprengen. Viele Physiker freuen sich wie Bolle. Andere warnen, dazu sei es noch zu früh.

Von Ulf von Rauchhaupt

Neue Zürcher Zeitung

Vor zwei Wochen bekam das Standardmodell der Teilchenphysik nasse Füsse. Jetzt steht ihm das Wasser bis zum Hals

Christian Speicher

07.04.2021, 17.00 Uhr

Frankfurter Allgemeine

HERAUSGEGEBEN VON GERALD BRAUNBERGER, JÜRGEN KAUBE, CARSTEN KNOP, BERTHOLD KOHLER

RÄTSELHAFTE MYONEN

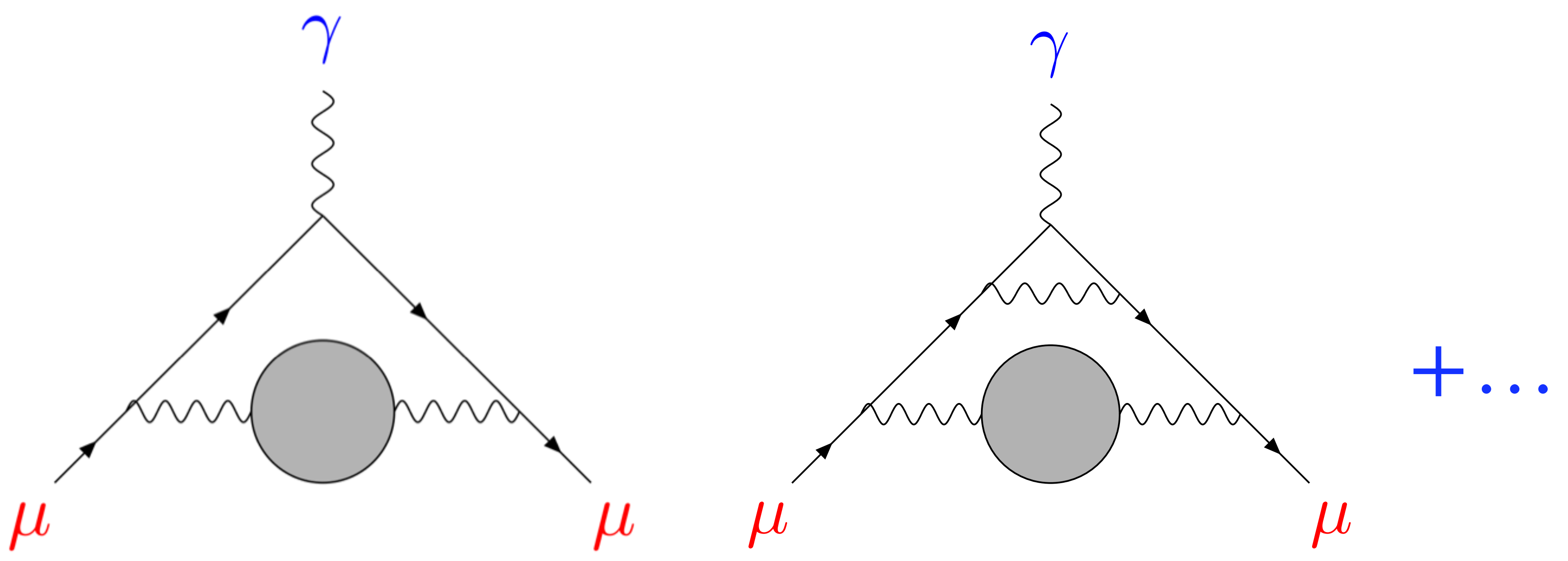
Abschied vom Standardmodell?

VON MANFRED LINDINGER - AKTUALISIERT AM 07.04.2021 - 18:55

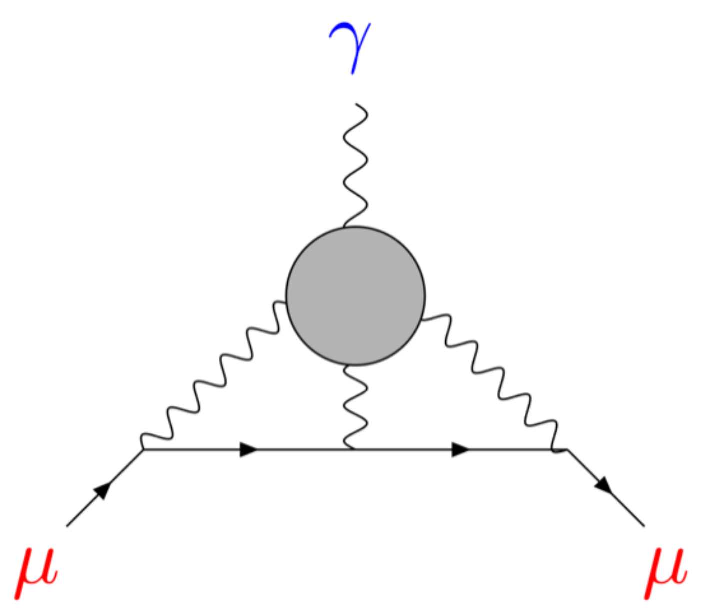
Standard Model prediction for muon $g-2$ [2020 White Paper]

QED:	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm	
Weak:	$153.6(1.0) \times 10^{-11}$	0.01 ppm	
Hadronic vacuum polarisation:	$6845(40) \times 10^{-11}$	0.34 ppm	$[0.6\%]$
Hadronic light-by-light scattering:	$92(18) \times 10^{-11}$	0.15 ppm	$[20\%]$

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} = 116\,591\,810(43) \times 10^{-11} \qquad 0.37\ \text{ppm}$$



Hadronic vacuum polarisation (HVP)



Hadronic light-by-light scattering (HLbL)

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

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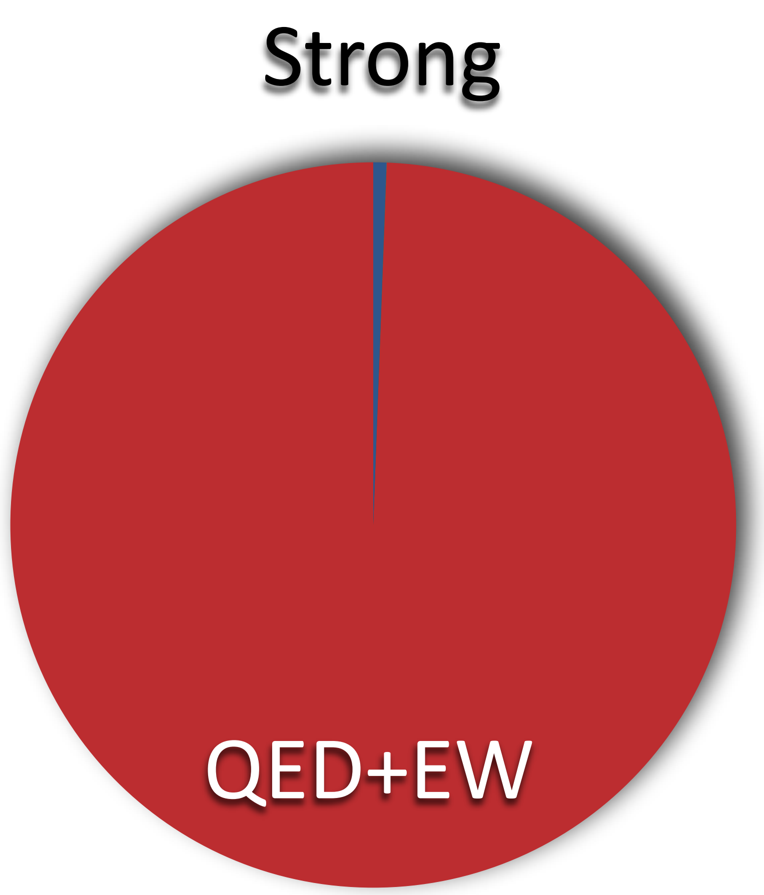
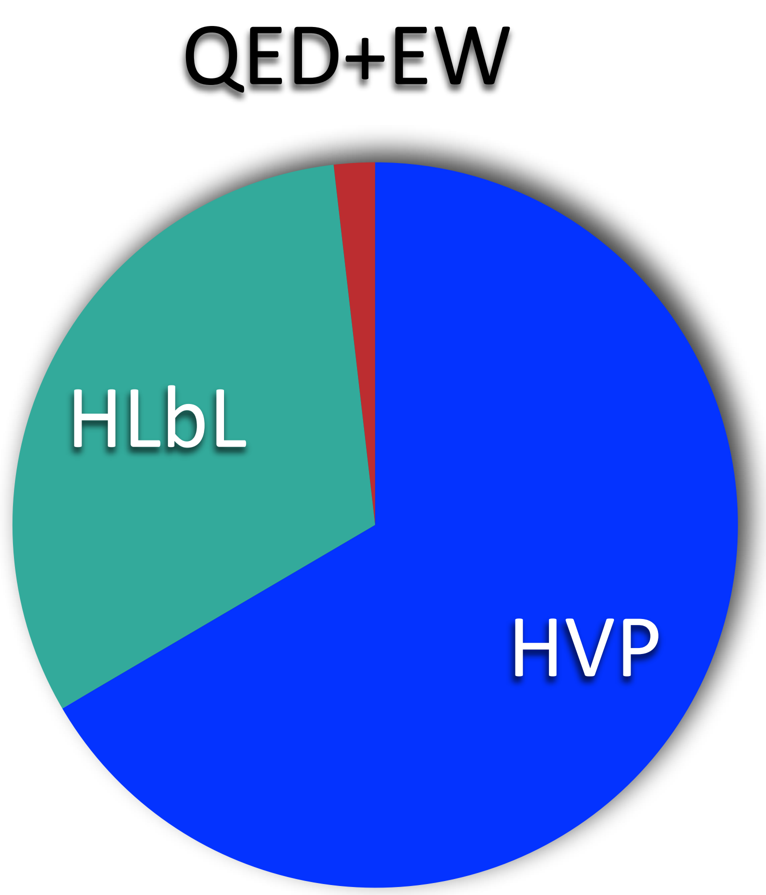
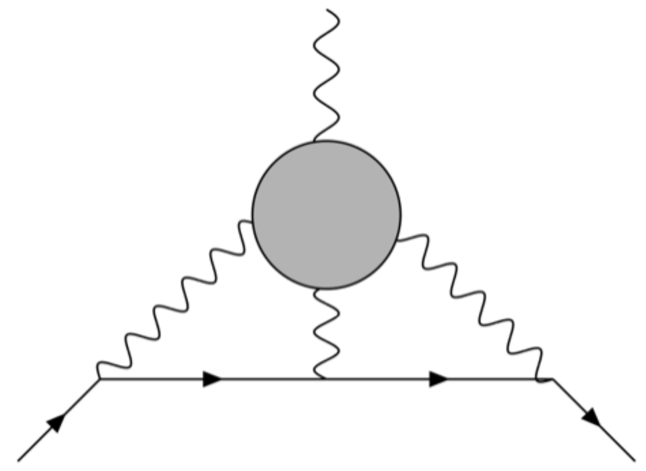
- QED and electroweak contributions account for 99.994% of the SM prediction for a_μ
- Error is dominated by strong interaction effects

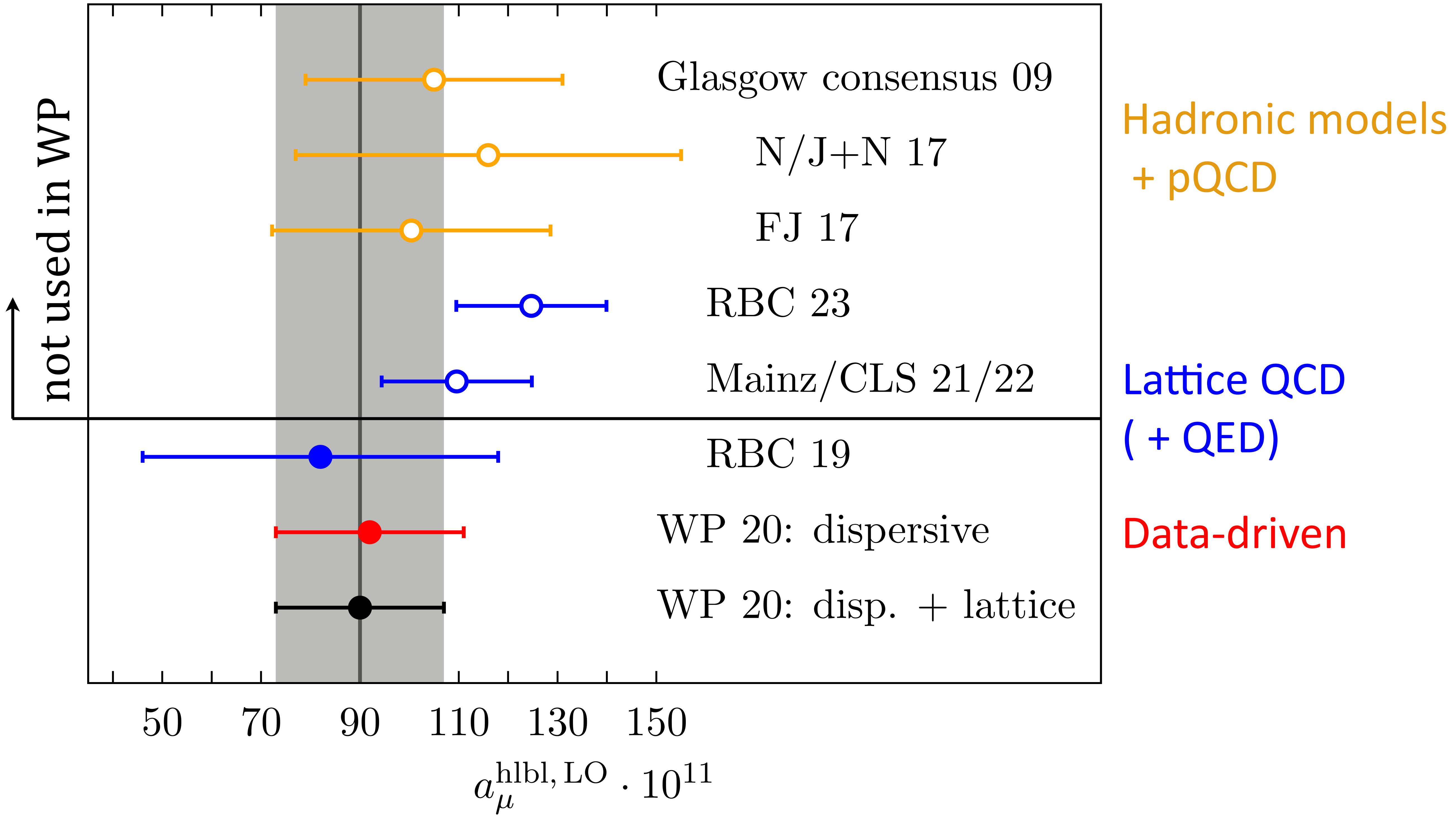




Hadronic light-by-light scattering



[Aoyama et al., Phys. Rep. 887 (2020) 1; Colangelo et al., arXiv:2203.15810]



Hadronic models, data-driven method and Lattice QCD produce compatible results

White paper recommended value:

$$a_\mu^{\rm hlbl} = (92 \pm 18)\cdot 10^{-11}$$

Recent lattice calculations:

$$a_\mu^{\rm hlbl,\,LO} = \begin{cases} (109.6 \pm 14.7)\cdot 10^{-11} & \text{Mainz/CLS} \\ (124.7 \pm 15.2)\cdot 10^{-11} & \text{RBC} \end{cases}$$

[Chao et al., EPJC 81 (2021) 651; EPJC 82 (2022) 664; Blum et al., arXiv:2304.04423]

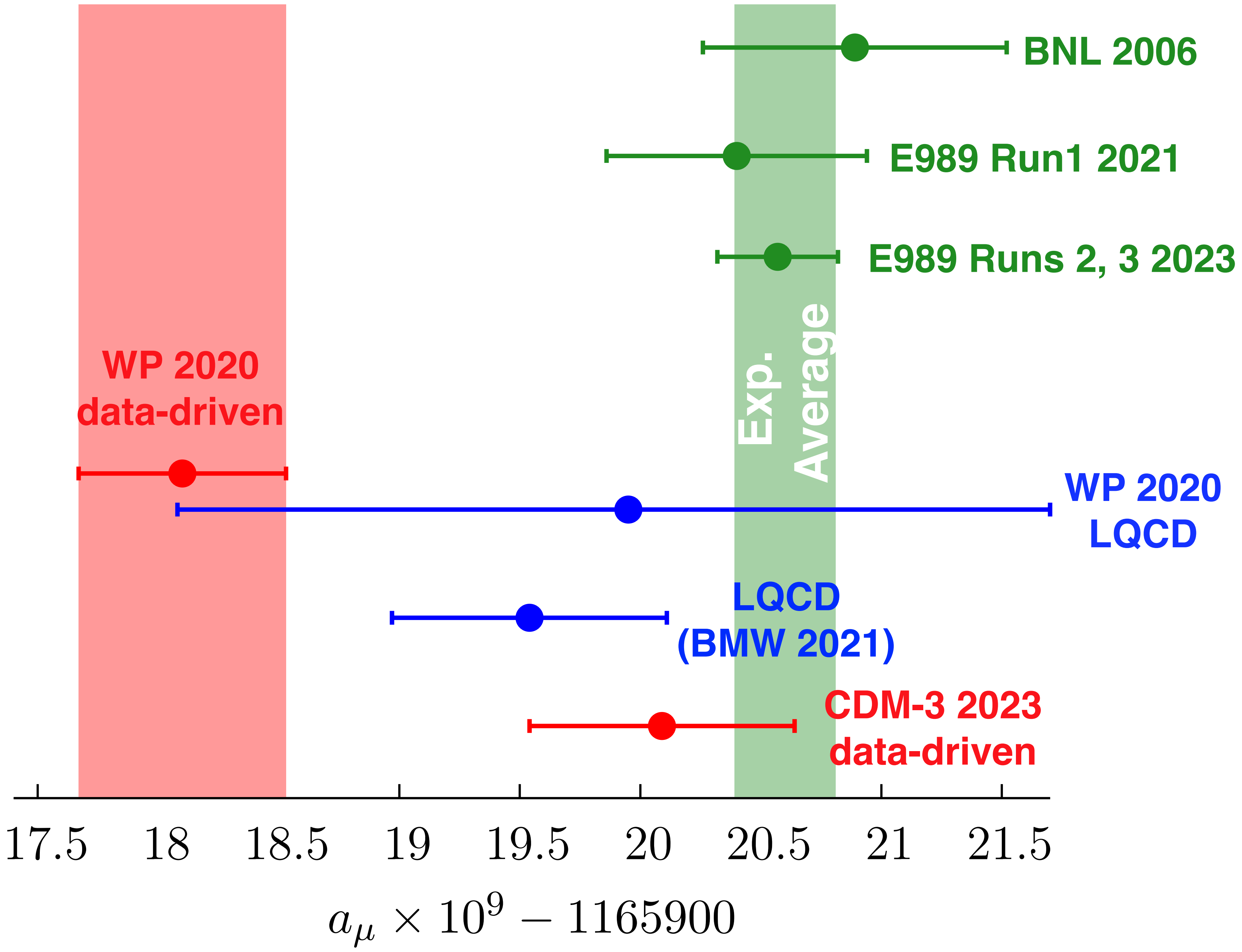
$a_\mu^{\rm hlbl}$: **Uncontroversial** — contributes $0.15\,\rm ppm$ to the total SM uncertainty of $0.37\,\rm ppm$

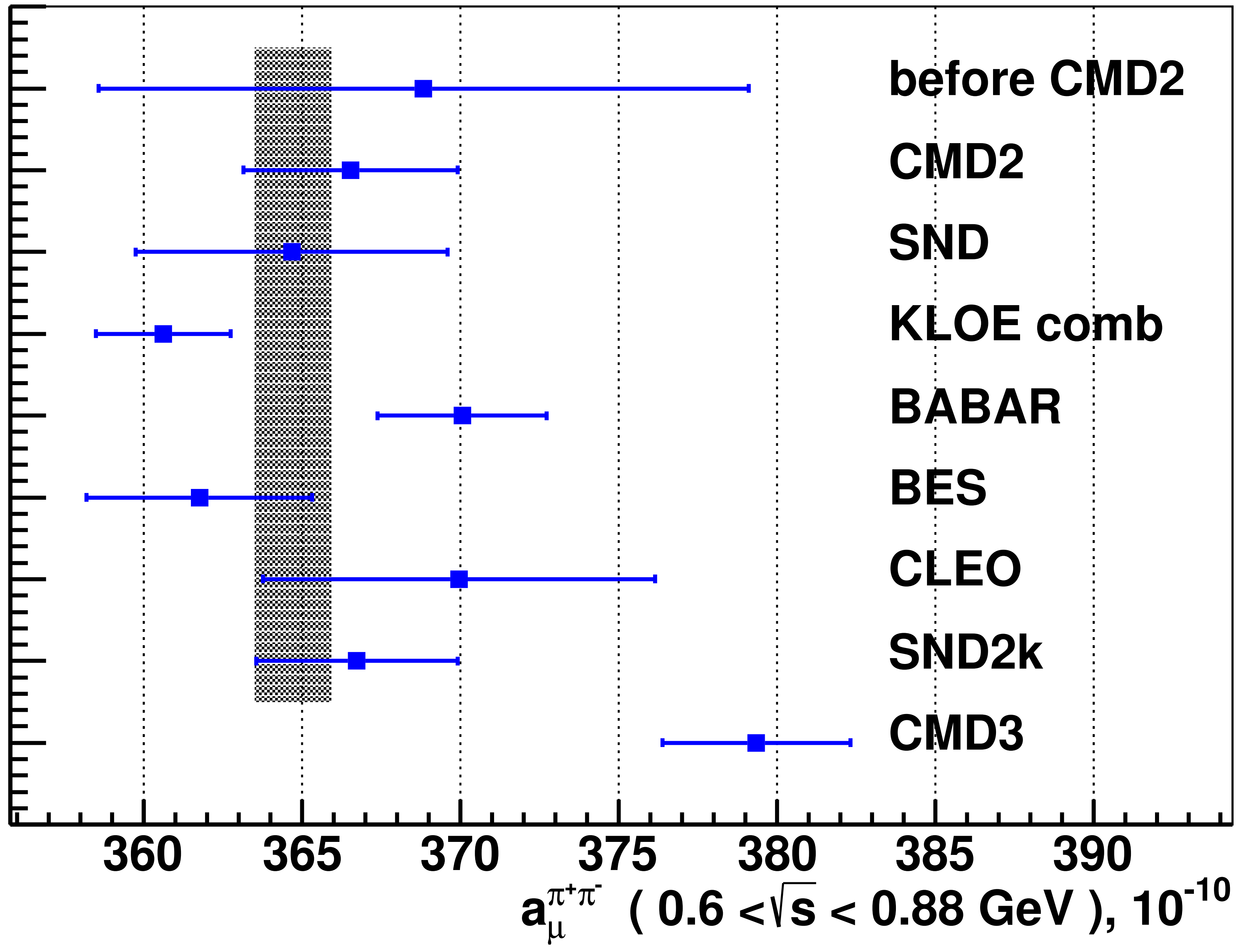
→ Focus on refinements and further reduction of uncertainty

New Physics on the horizon?

E989 @ Fermilab: $a_\mu^{\rm exp} = 116\,592\,049(22) \times 10^{-11}$ [0.19 ppm] *[Aguillard et al., Phys Rev Lett 131 (2023) 16, 161802]*

WP 2020: $a_\mu^{\rm SM} = 116\,591\,810(43) \times 10^{-11}$ [0.37 ppm] $\Rightarrow$ $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (249 \pm 48) \cdot 10^{-11}$ [5.1σ]

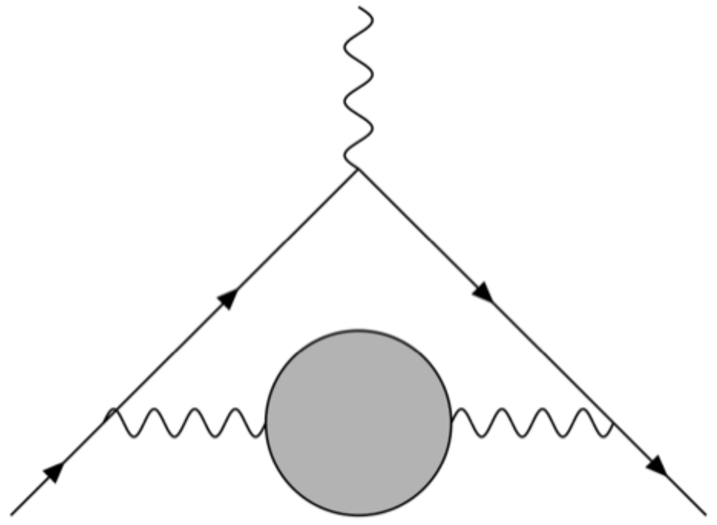




White paper estimate challenged by lattice QCD calculations *[Borsányi et al., Nature 593 (2021) 7857]*

and tensions among cross section measurements for $e^+e^- \rightarrow$ hadrons *[Ignatov et al., Phys Rev D109 (2024) 112002]*

HVP contribution: Dispersion Theory vs. Lattice QCD



Integral representations:

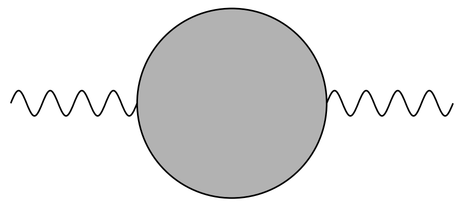
$$a_\mu^{\rm LO,\,hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

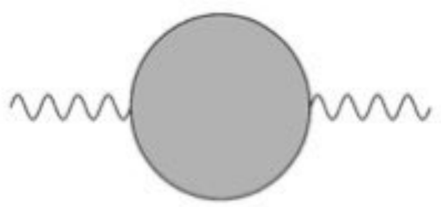
$$a_\mu^{\rm LO,\,hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt\, \tilde{K}(t)\, G(t)$$

Primary observables:

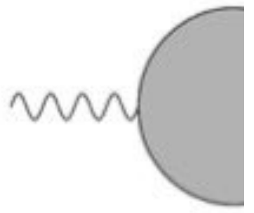
$$R_{\rm had}(s) = \frac{3s}{4\pi\,(\alpha(s))^2}\,\sigma(e^+e^- \to \text{hadrons})$$

$$G(t) = -\frac{1}{3}\sum_k \int d^3x \left\langle j_k^{\rm em}(\vec{x},t)\, j_k^{\rm em}(0)\right\rangle =$$





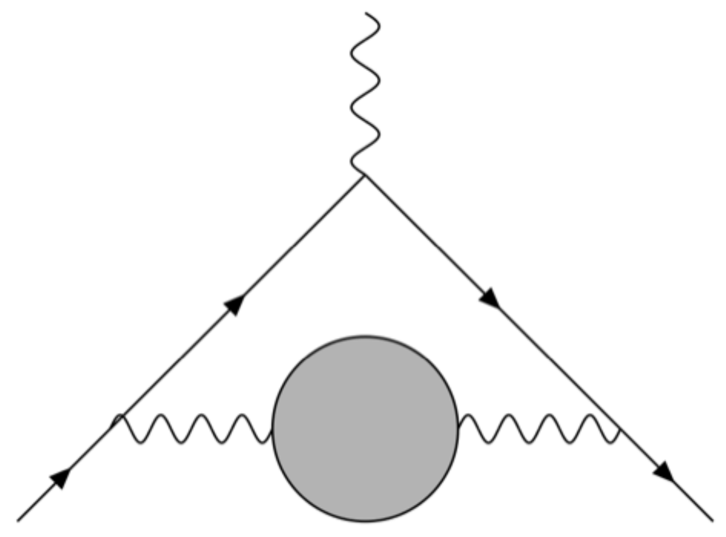
$$= \frac{1}{2}\int \frac{ds}{\pi(s-q^2)} \sum_{\rm had} \int d\Phi \left| \quad \right|^2$$



$$j_\mu^{\rm em}(x) = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c + \ldots$$

$$G(t) = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R_{\rm had}(s)\, s\, {\rm e}^{-\sqrt{s}t}$$

HVP contribution: Dispersion Theory vs. Lattice QCD



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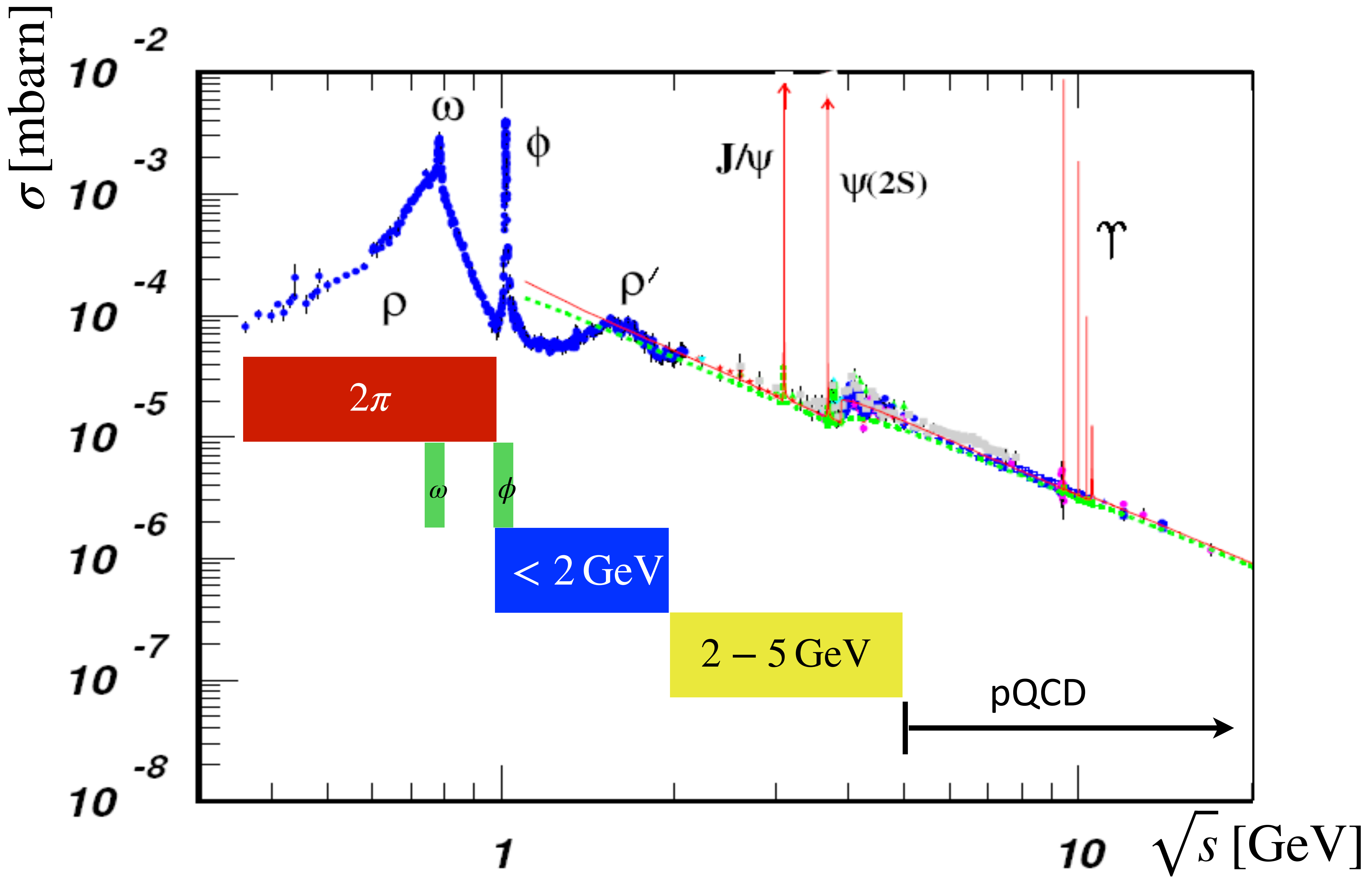

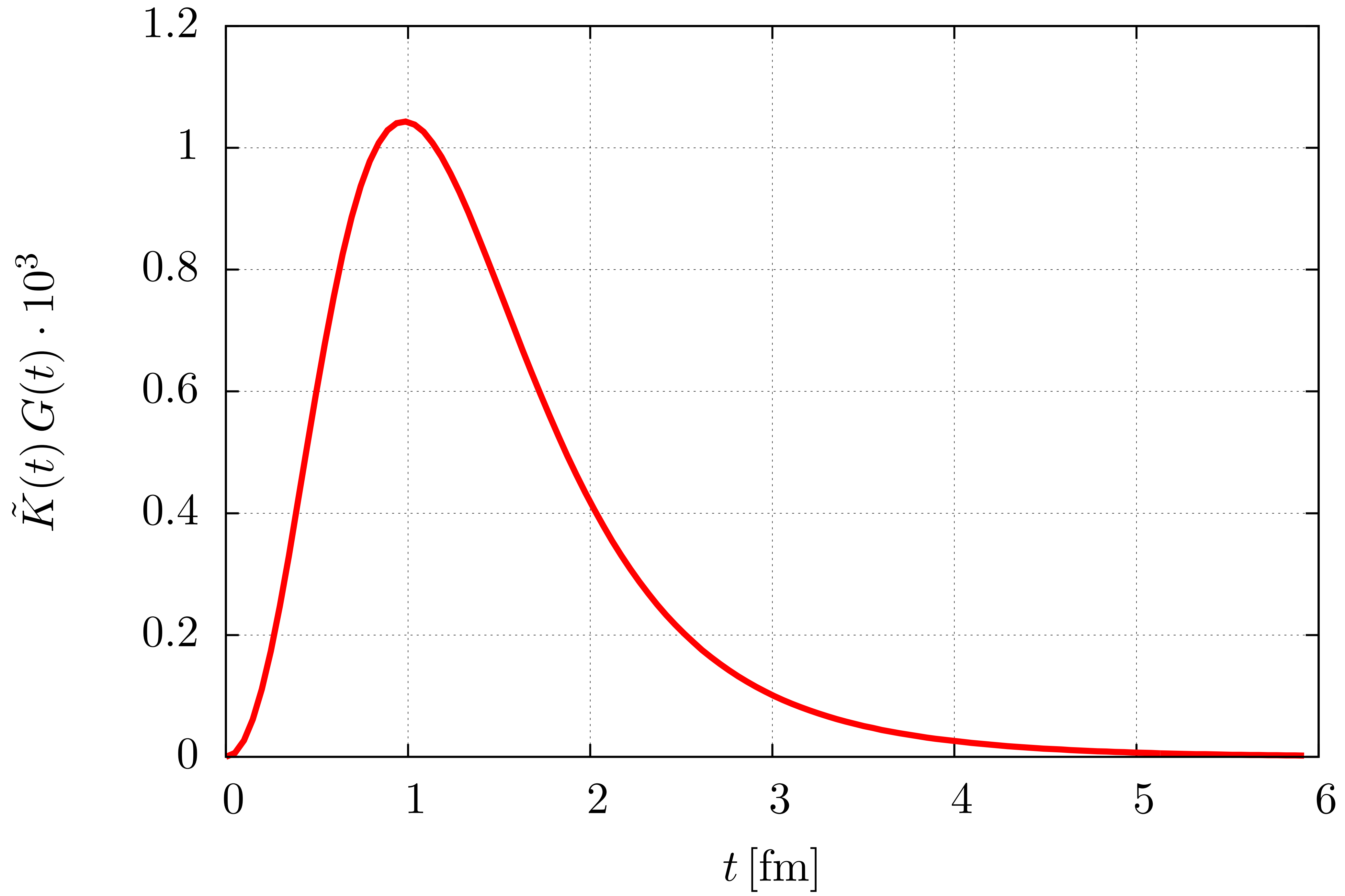

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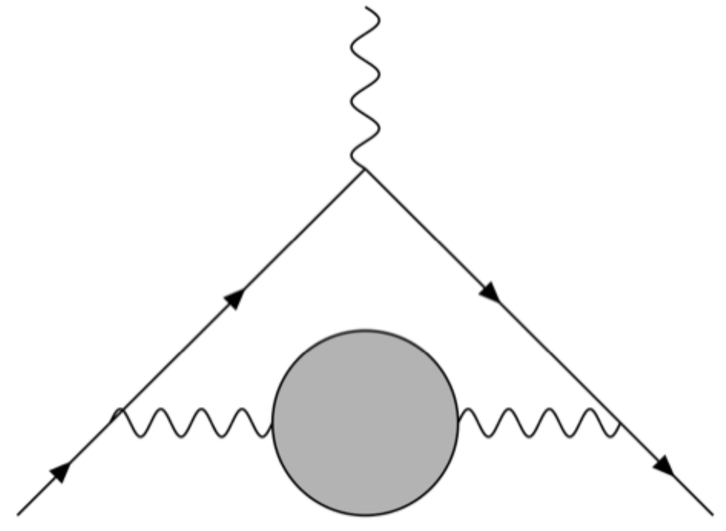

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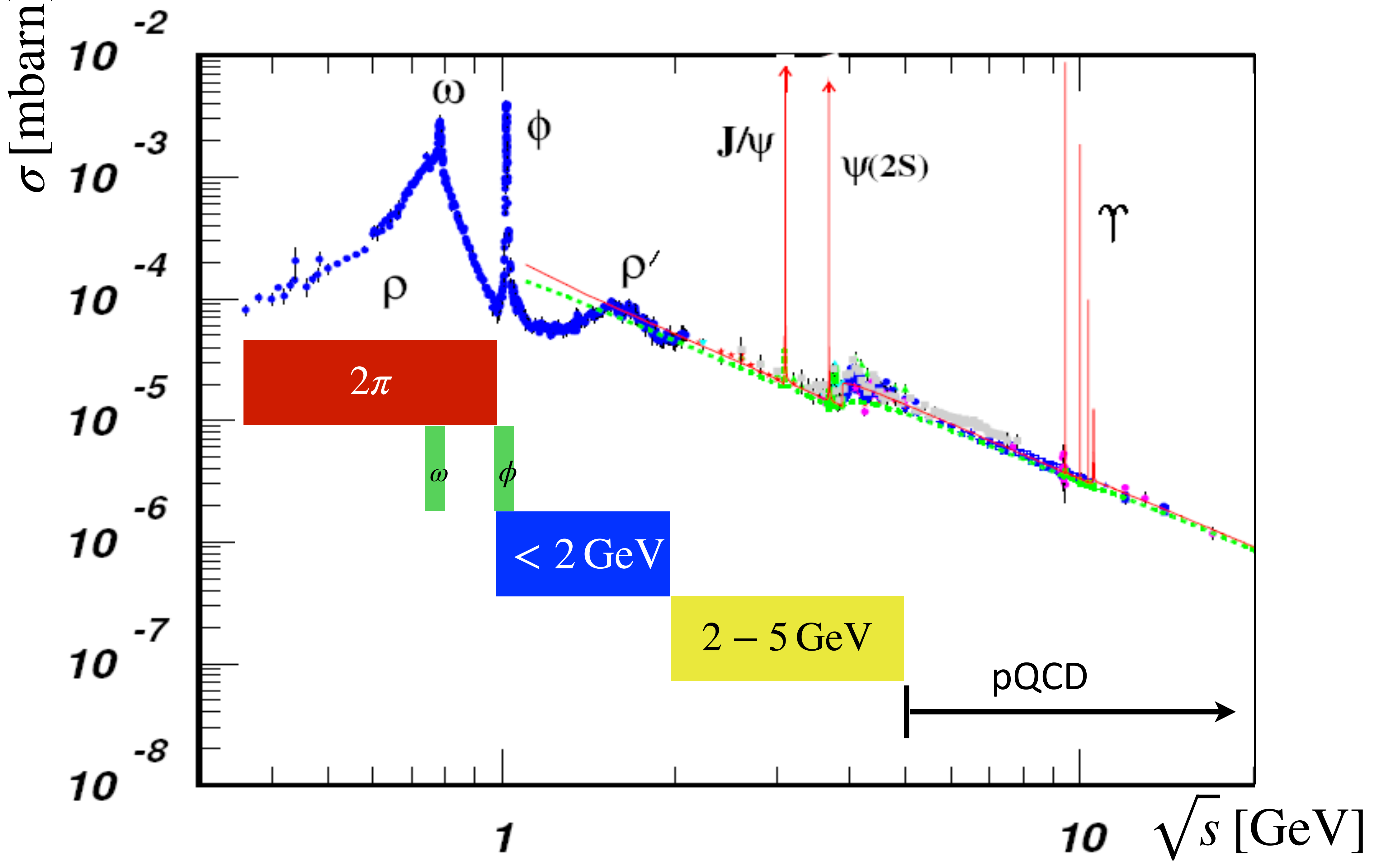
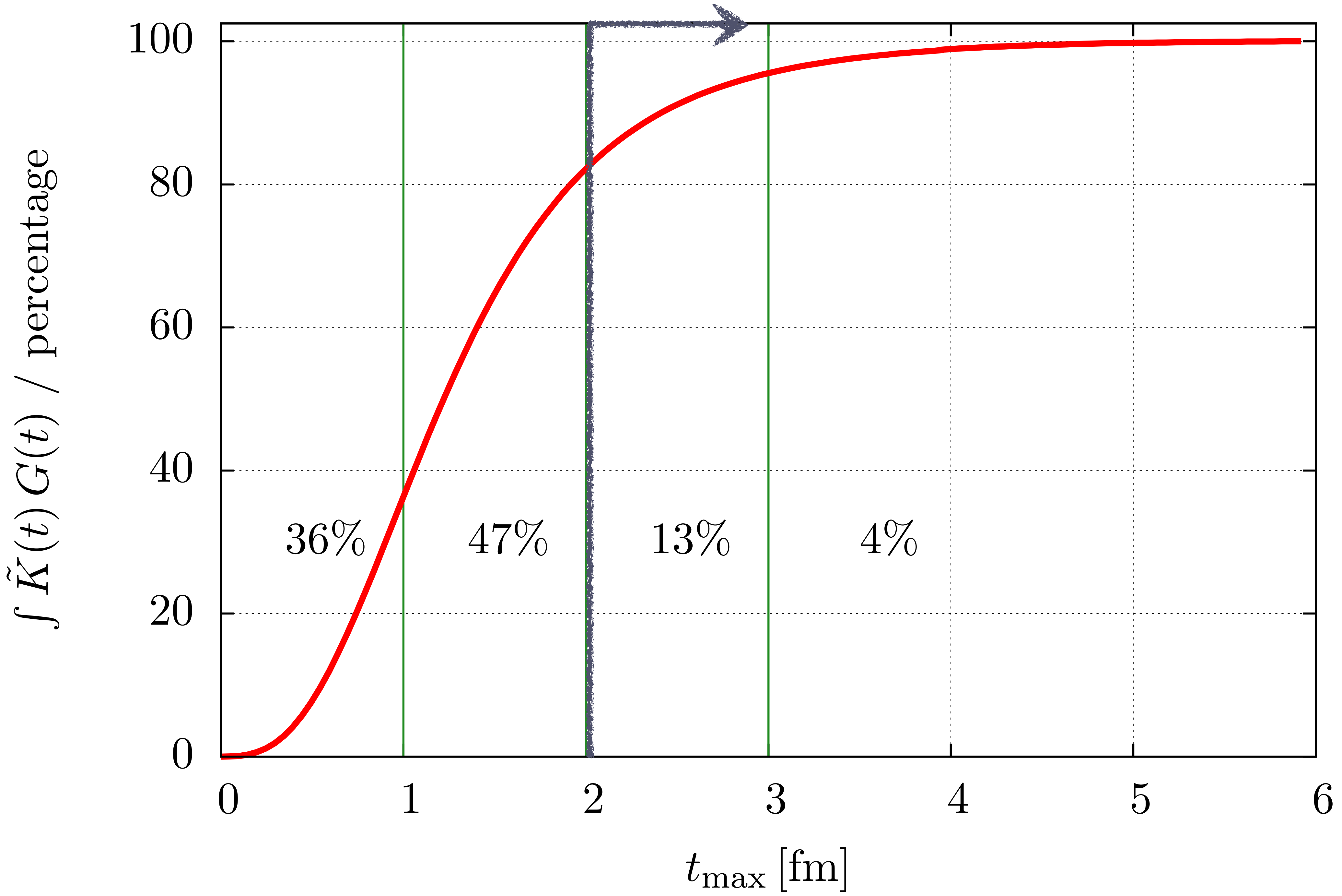


statistical noise
finite-volume effects

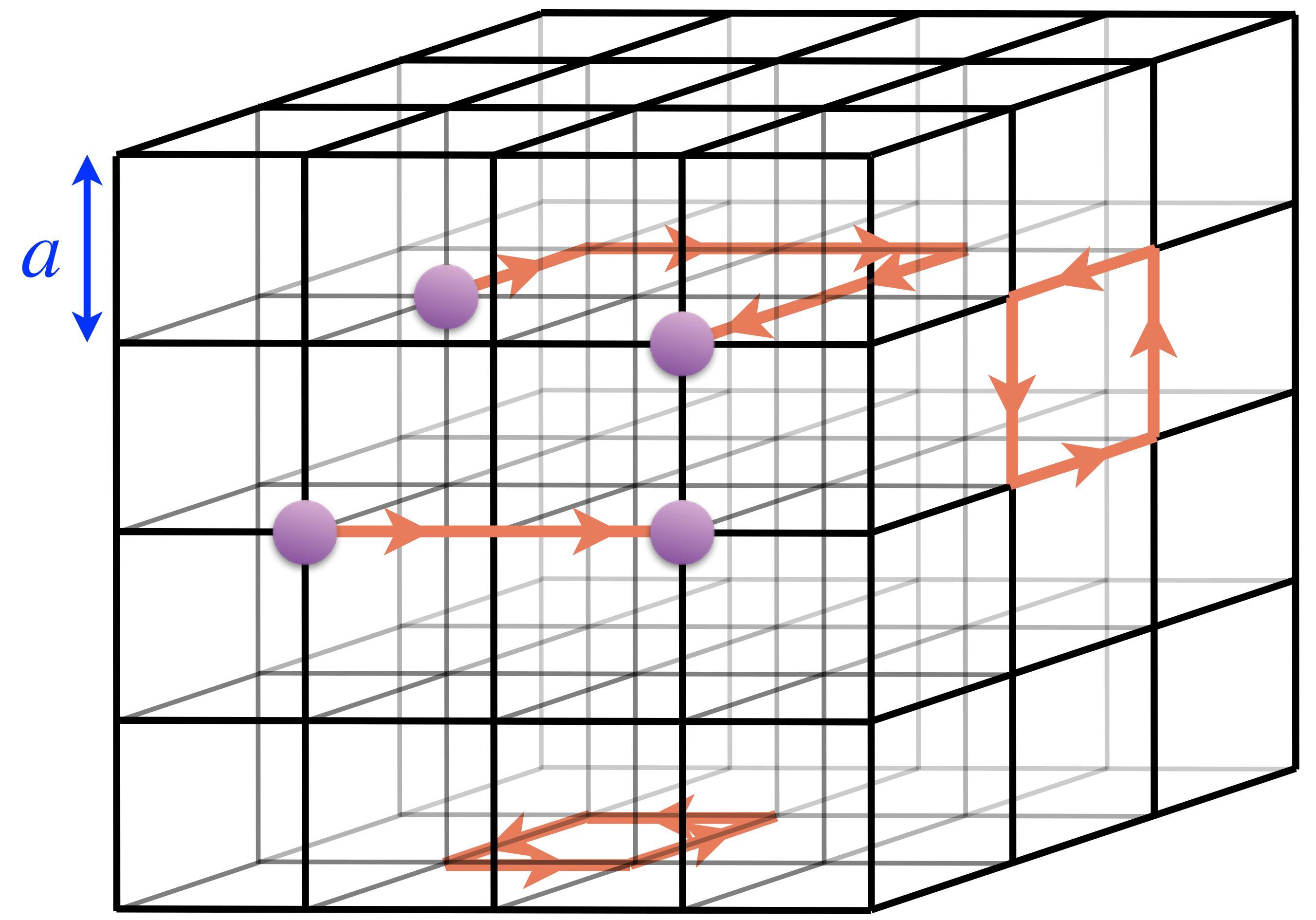


Lattice QCD Primer

Non-perturbative treatment of strong interaction via regularised Euclidean path integrals

Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\rm UV}$

Expectation value: $\langle\Omega\rangle = \frac{1}{Z}\int\prod_{x,\mu} dU_\mu(x)\;\Omega\;{\rm e}^{-S_{\rm G}^{\rm eff}[U]}$



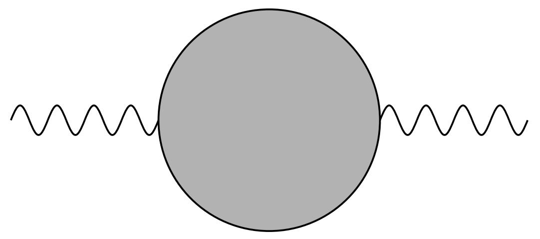
Procedure:

- Choose discretisation of QCD action
- Evaluate $\langle\Omega\rangle$ via Monte Carlo Integration:
  generate ensembles of gauge configurations via a Markov chain
- Ensemble average: $\langle\Omega\rangle \simeq \overline{\Omega}$ Statistical error: $\sqrt{\overline{\Omega^2} - \overline{\Omega}^2} \propto 1/N_{\rm cfg}^{1/2}$
- Extrapolate observables to the continuum limit: $a \to 0$ and tune quark masses to physical values

HVP from Lattice QCD: The Challenge

Lattice QCD does NOT determine the R-ratio from first principles

Time-momentum representation (TMR): *[Bernecker & Meyer EPJA 47 (2011) 148]*



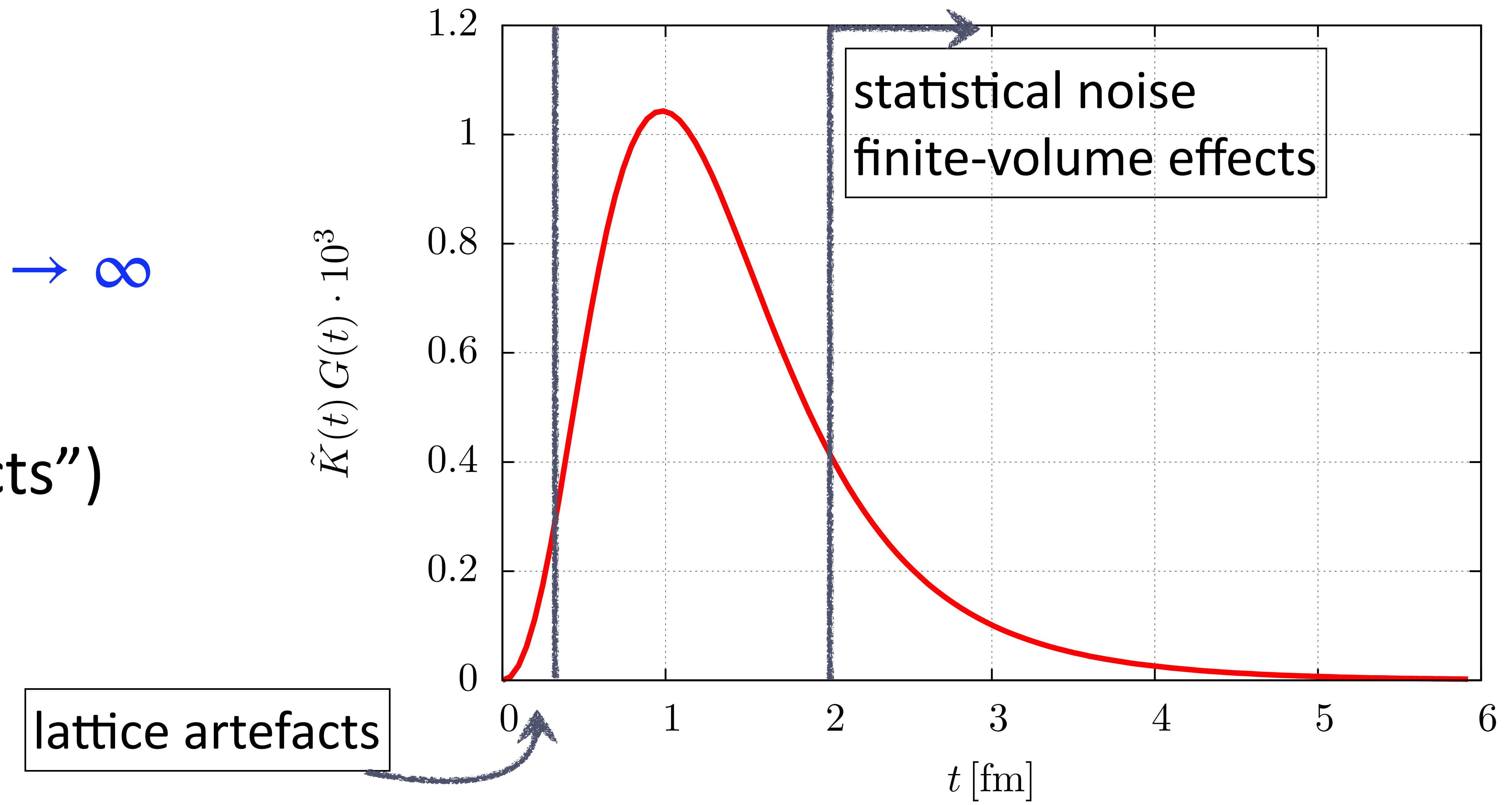
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t), \quad G(t) = -\frac{a^3}{3} \sum_k \sum_{\vec{x}} \left\langle j_k^{\rm em}(\vec{x},t)\, j_k^{\rm em}(0) \right\rangle$$

($\tilde{K}(t)$: known analytically)

- No reliance on experimental data, except for simple input quantities → scale setting, calibration
- Not sensitive to exclusive hadronic channels

Challenges

- Exponentially increasing statistical noise as $t \to \infty$
- Correct for finite-volume effects
- Control discretisation effects ("lattice artefacts")
- Include isospin-breaking corrections



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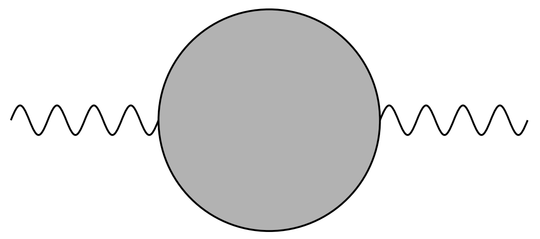

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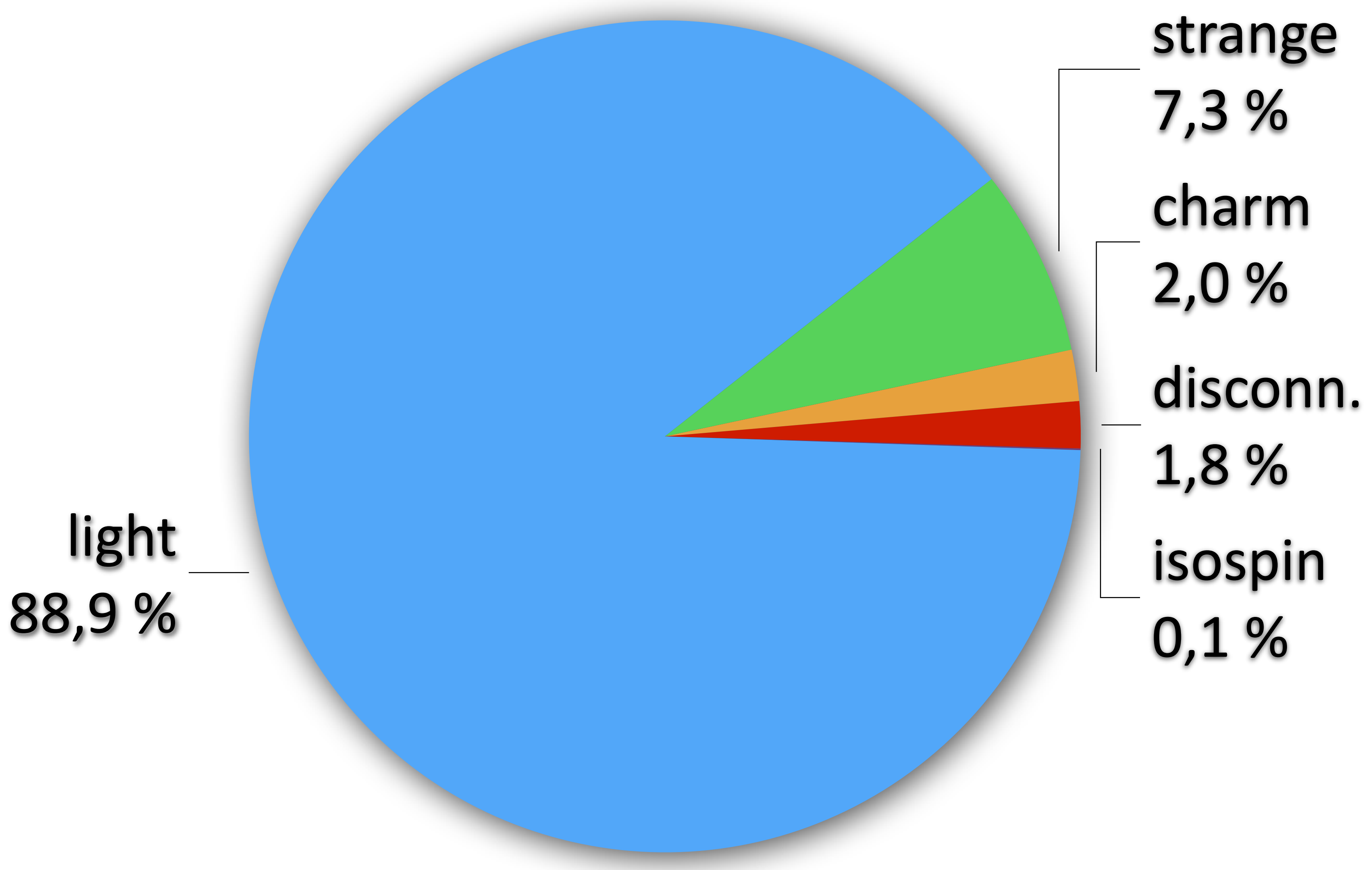
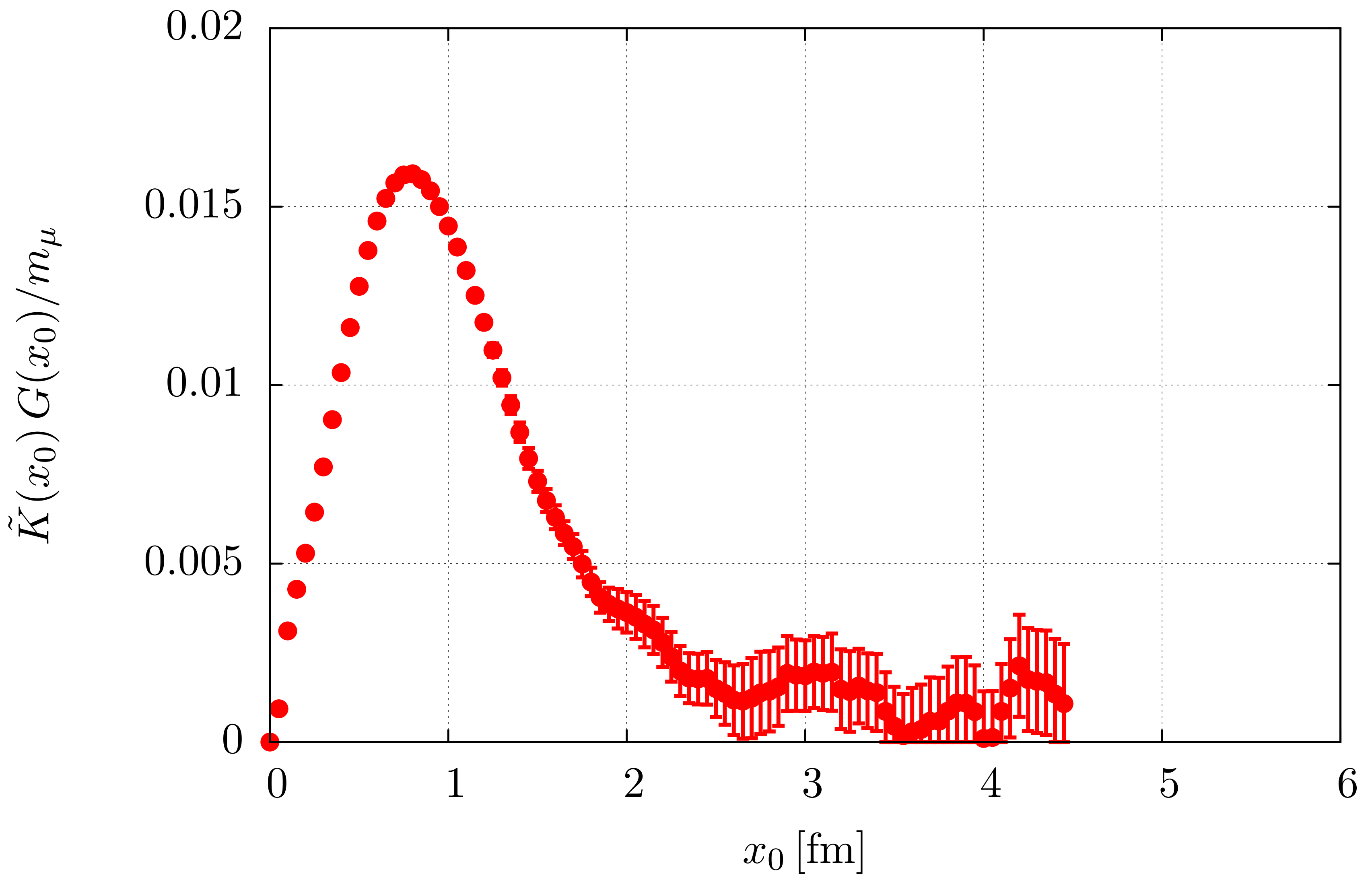
Light-quark connected contribution dominates



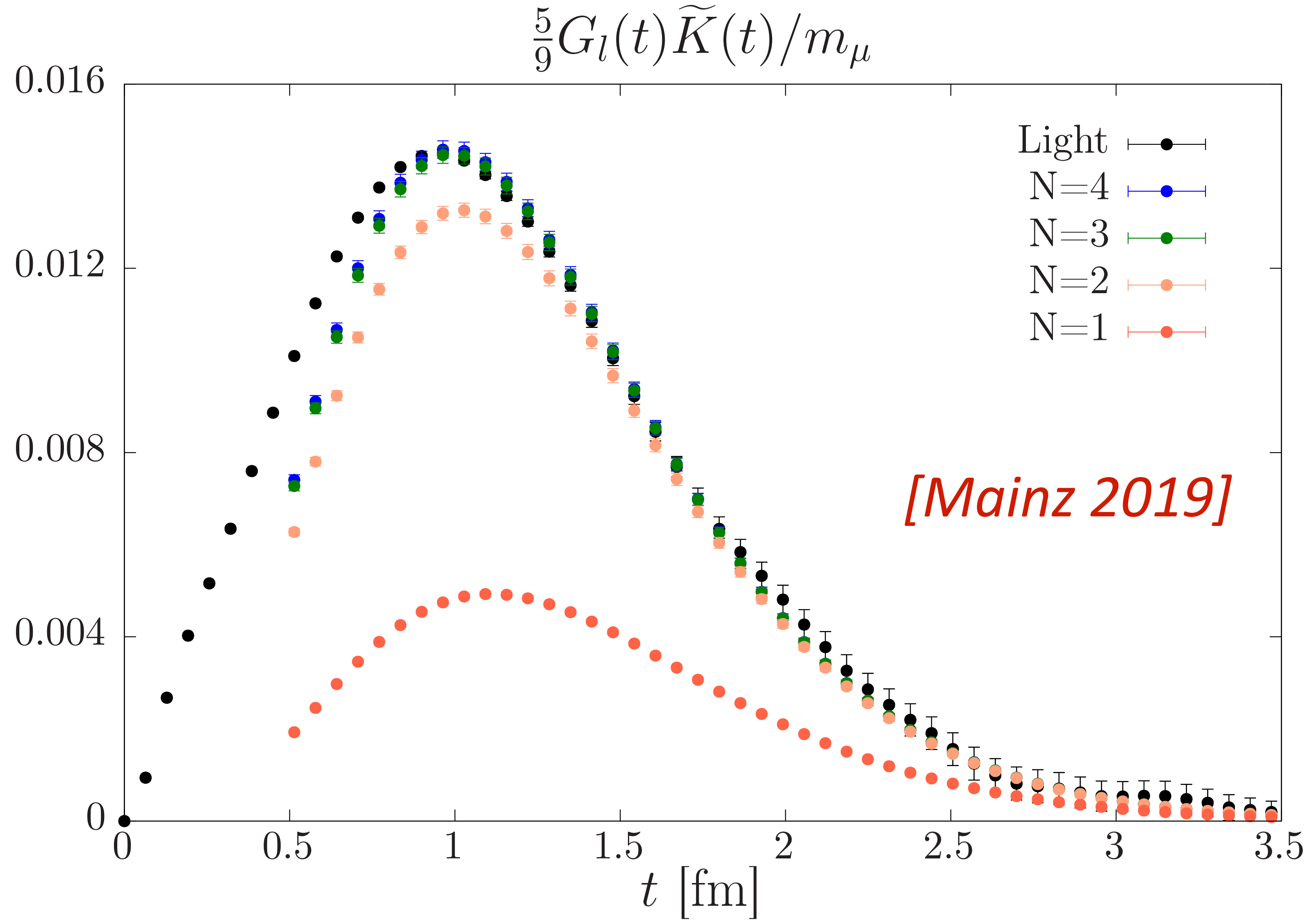
Controlling the long-distance tail of $G(t)$

- Long-distance tail of the light quark contribution to $G(t)$: limiting factor for overall statistical precision
- Correlator dominated by isovector two-pion contribution



Strategies:

- Dedicated calculations of the spectrum in isovector channel and/or pion form factor $F_\pi(\omega)$



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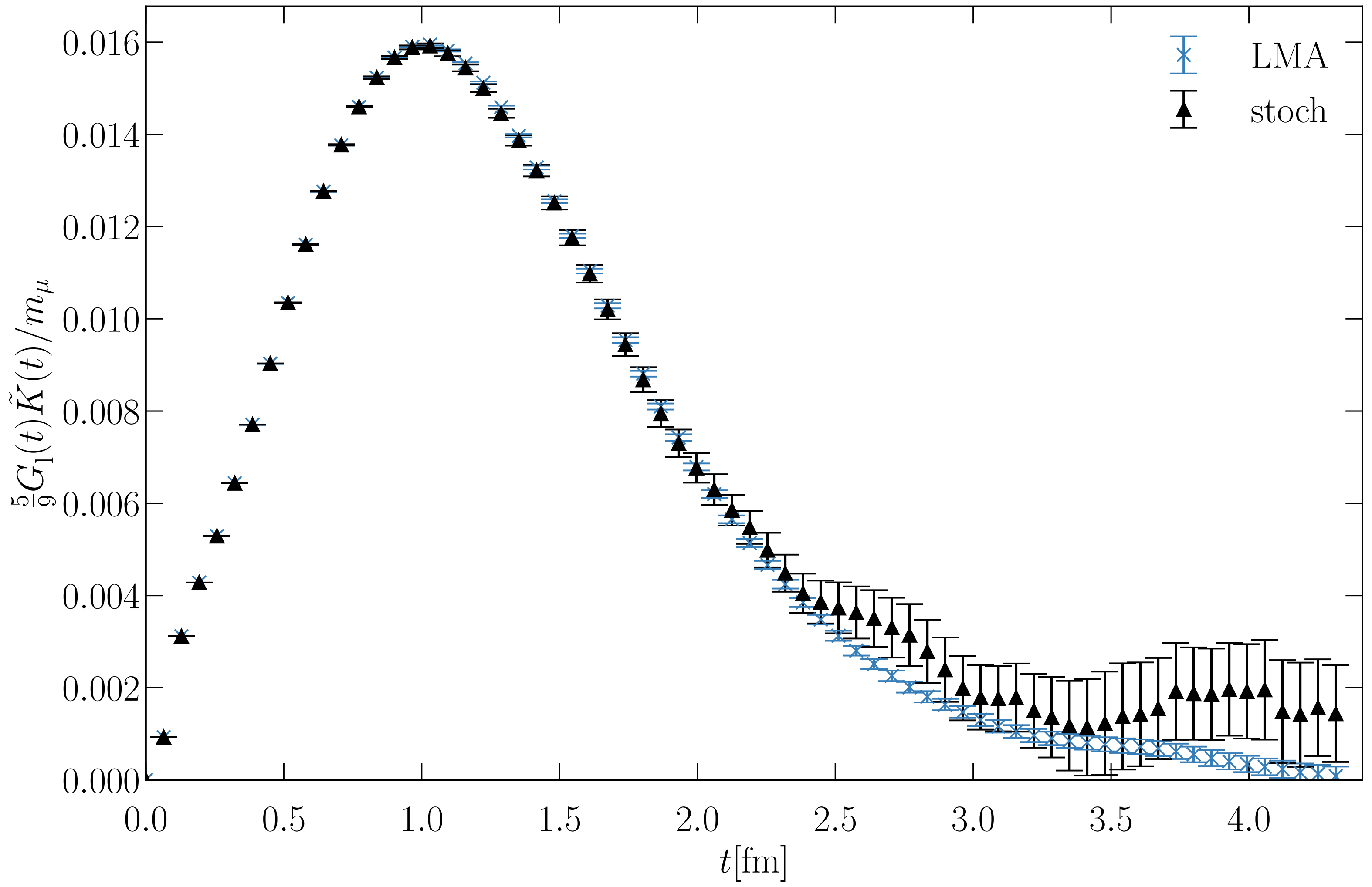
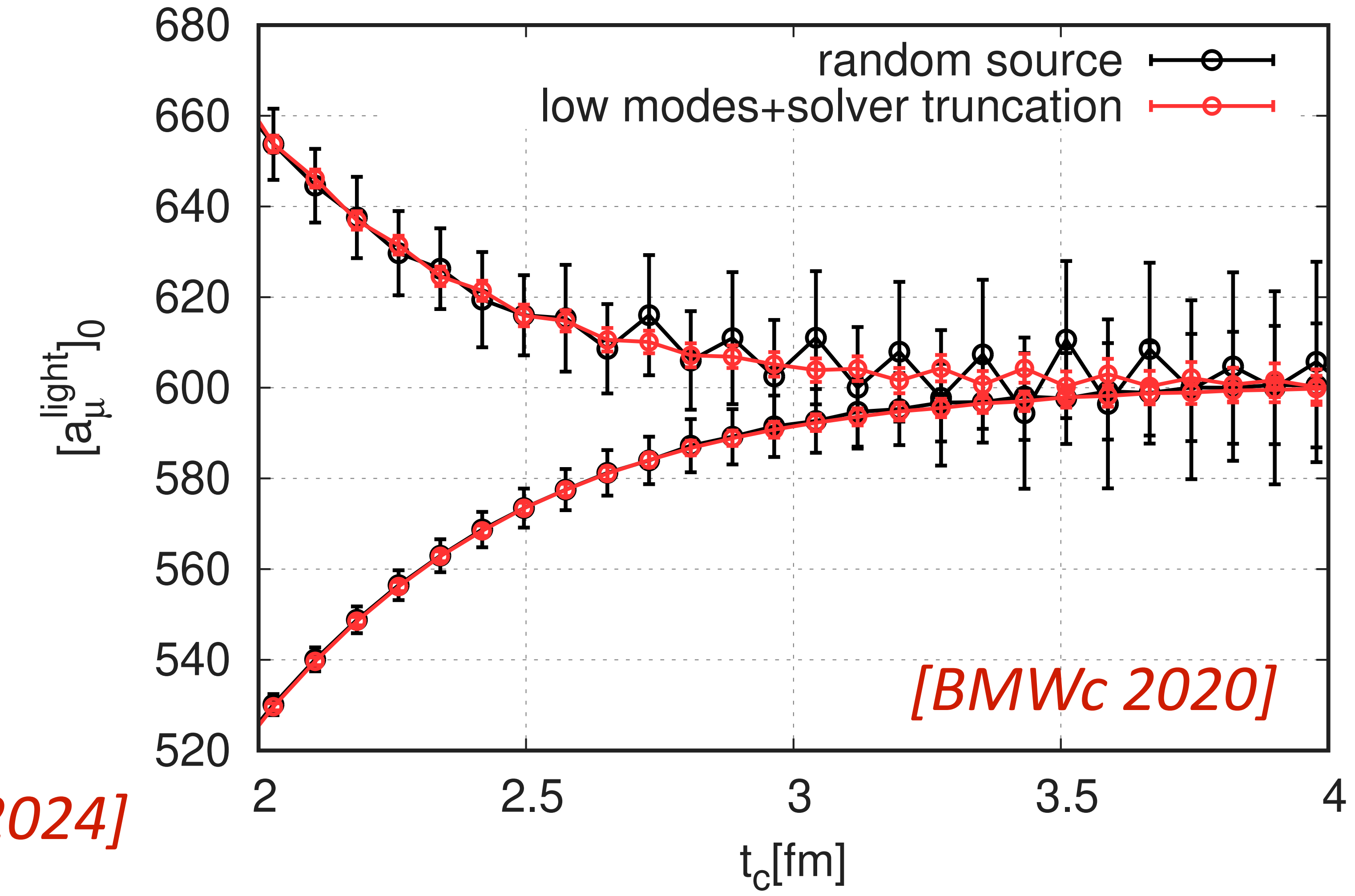
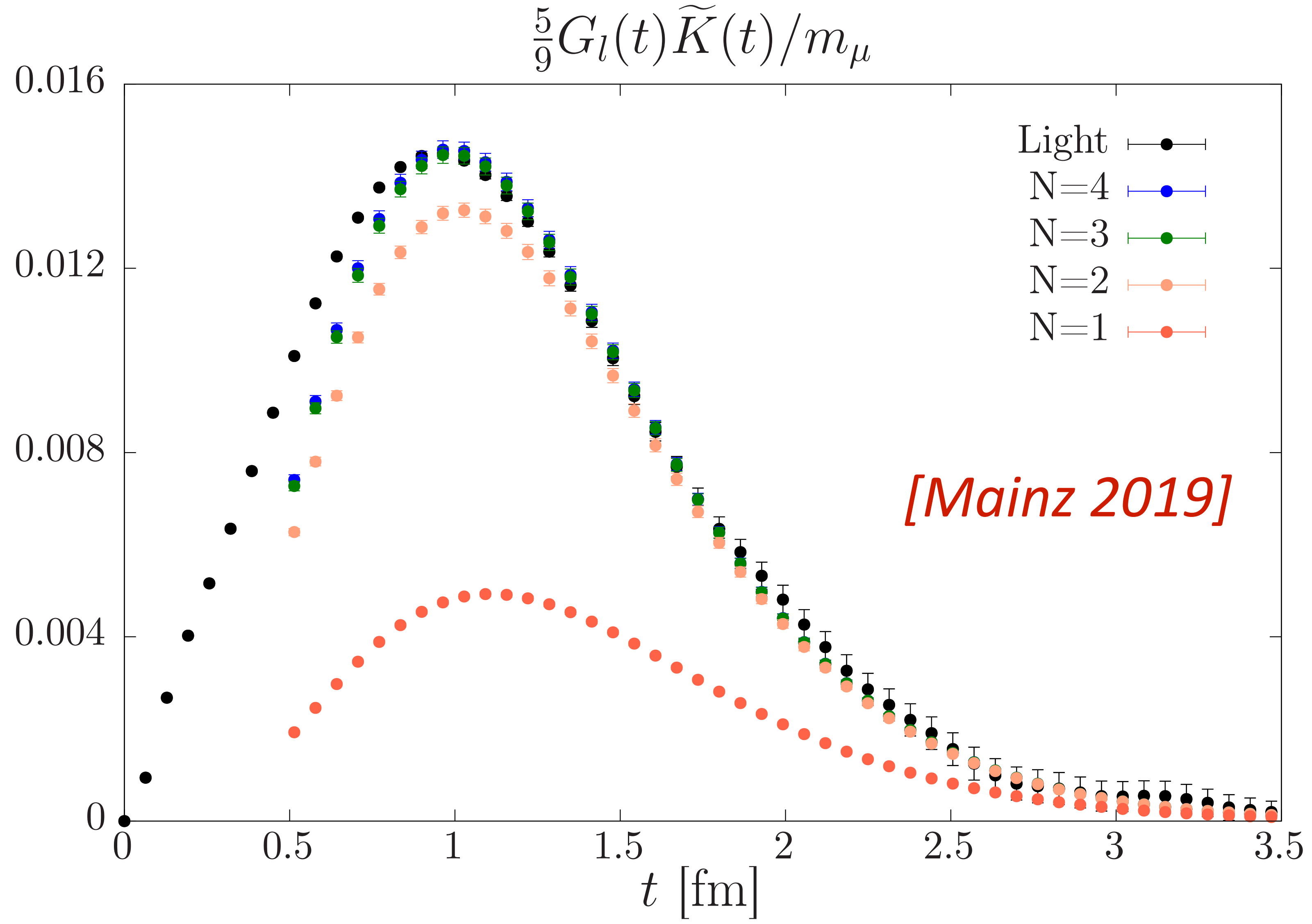


Strategies:

- Dedicated calculations of the spectrum in isovector channel and/or pion form factor $F_\pi(\omega)$
- "Bounding method":

$$0 \leq G(t) \leq G(t_c)\,\frac{G^{\pi\pi}(t)}{G^{\pi\pi}(t_c)}$$





- Noise-reduction methods: AMA, LMA, truncated solver
- Machine Learning *[H.W. @ Lattice 2024]*

Controlling finite-volume corrections

Sizeable correction to light-quark connected contribution: $\approx 3\,\%$ for $m_\pi^{\rm phys}L \approx 4$

$$G(t,L) \overset{t\to\infty}{=} \sum_n |A_n|^2\, \mathrm{e}^{-\omega_n t} \qquad G(t,\infty) = \int_0^\infty d\omega\, \omega^2 \rho(\omega^2)\, \mathrm{e}^{-\omega|t|}$$

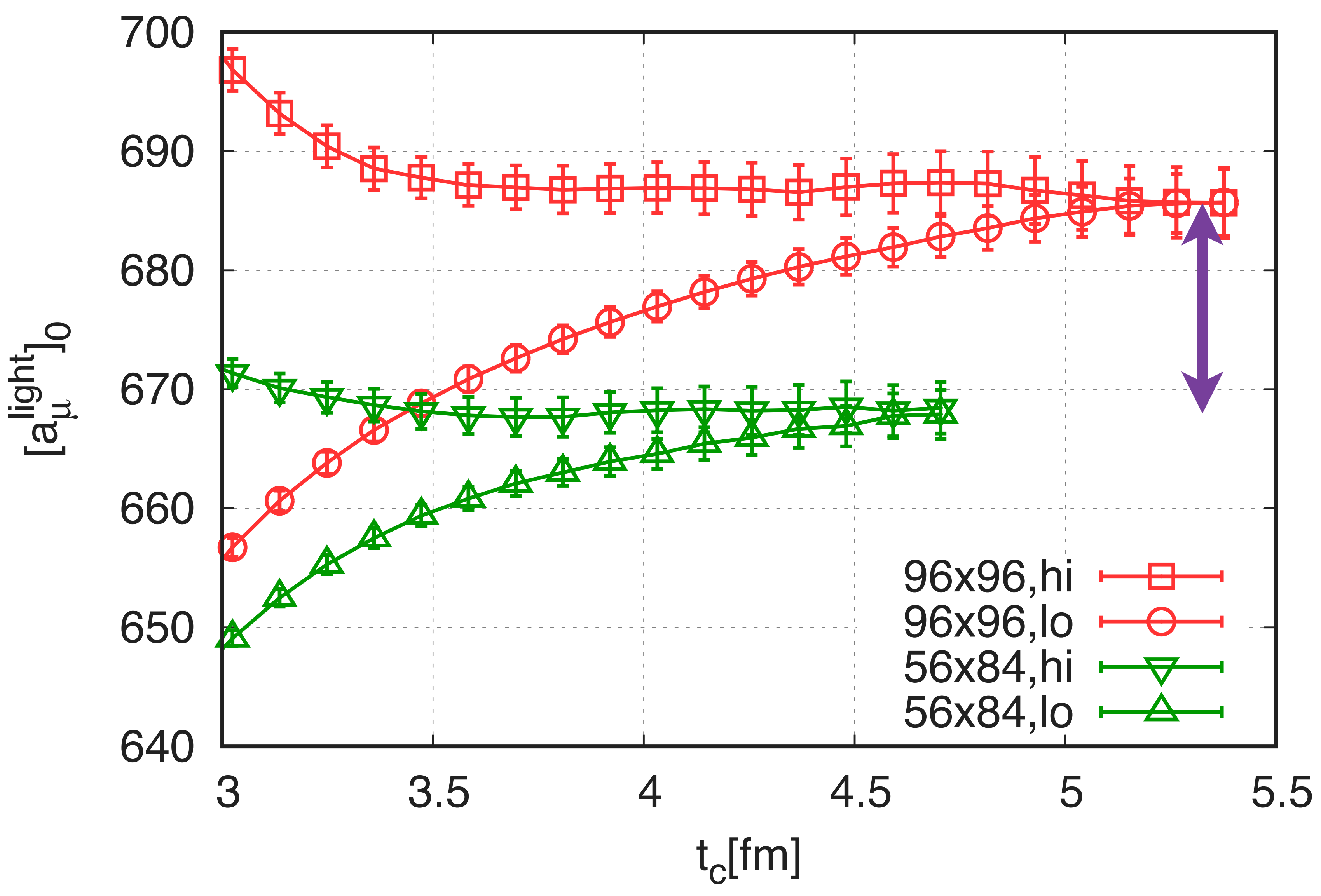
Both $|A_n|$ and $\rho(\omega^2)$ can be related to the pion form factor $F_\pi(\omega)$ $\Rightarrow$ $G(t,\infty) - G(t,L)$

[Meyer, PRL 107 (2011) 072002; Francis et al., PRD 88 (2013) 054502]

Other analytic methods:

- Chiral Perturbation Theory *[Aubin et al., 2015]*
- Expansion in pion winding number *[Hansen & Patella, 2019/20]*

Direct lattice calculation by BMWc:



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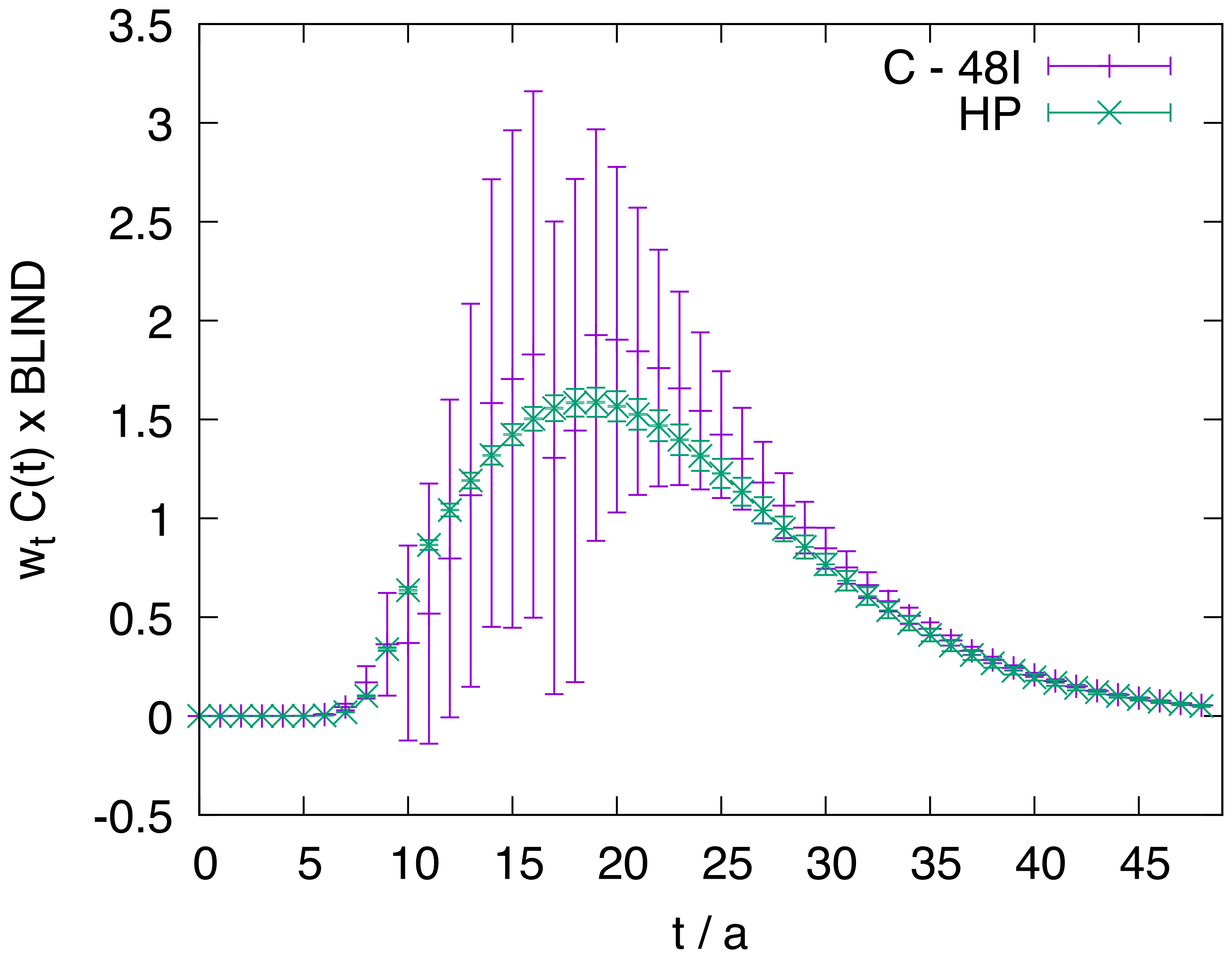
Correction/10^{-10}	Comment
17.8	Gounaris-Sakurai model for $F_\pi(\omega)$
15.7	ChPT at NNLO
16.3	Expansion in pion winding number
18.1(2.4)	Direct lattice calculation

Direct lattice calculation by RBC/UKQCD:



Common discretisations of the quark action

Computational cost depends significantly on the chosen discretisation

"Fermion doubling problem"

Domain wall /overlap quarks:

- no doublers; chiral symmetry breaking exponentially small
- live in five dimensions (dwf)
- evaluate sign function of "conventional" action (ovlp)
- used by: RBC/UKQCD, χQCD,…

Rooted staggered quarks:

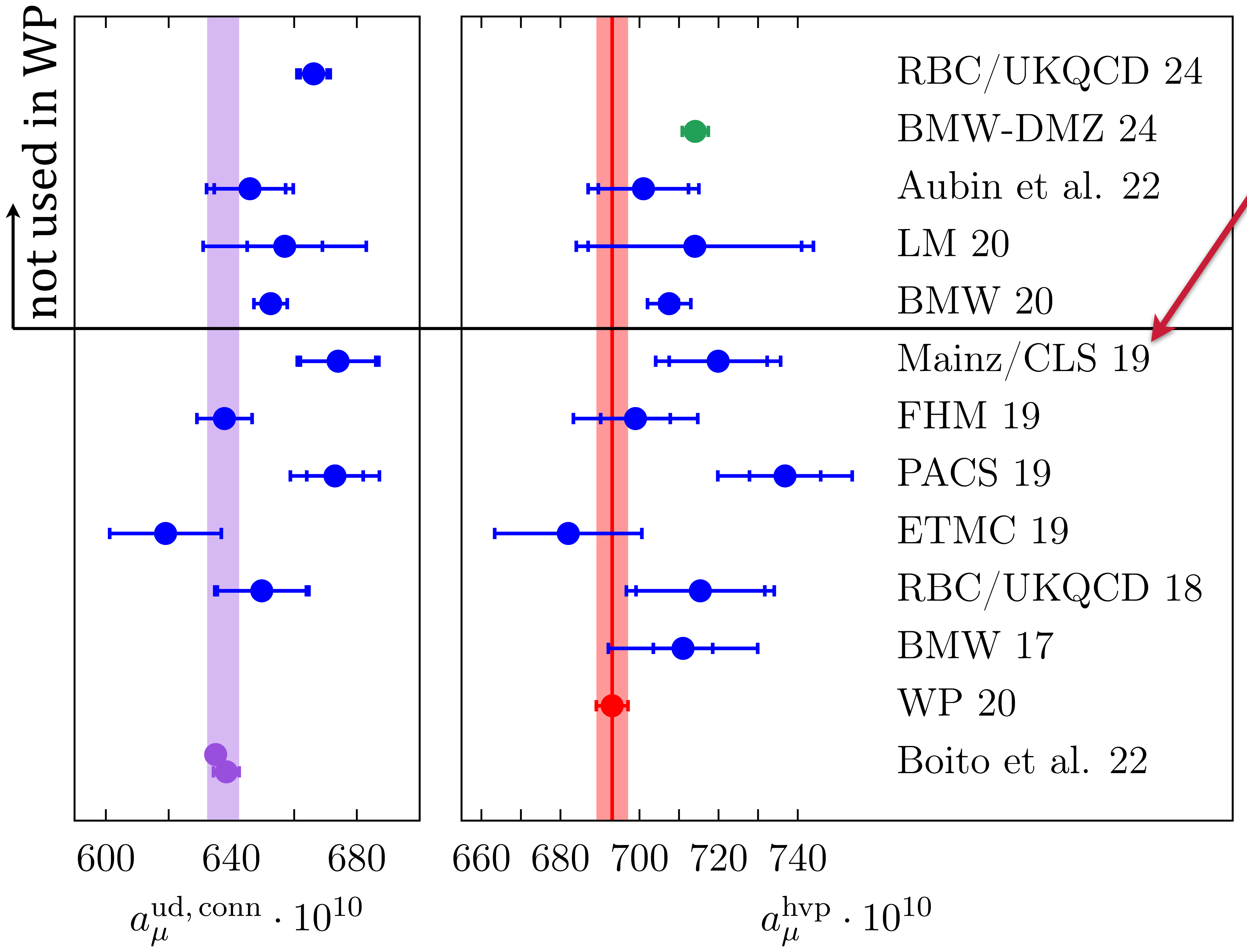
- remnant fermion doublers — "tastes"
- correct analytically for taste-induced lattice artefacts
- used by: BMW, Fermilab-HPQCD-MILC, ABGP,…

Wilson quarks:

- no doublers; chiral symmetry broken explicitly
- "exceptional configurations": negative eigenvalues of Wilson-Dirac operator
- used by: Mainz/CLS, ETM, PACS

computational cost →

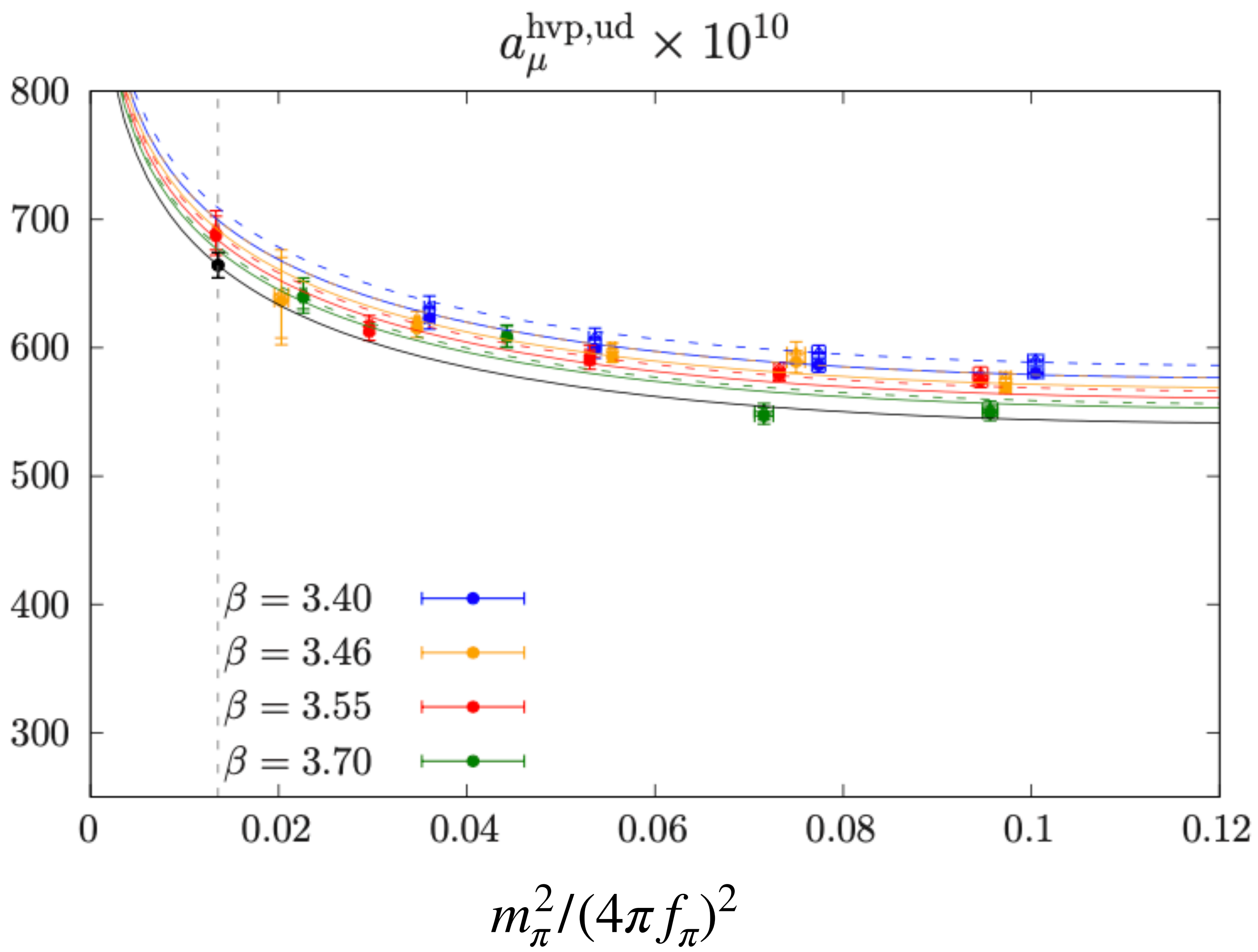
HVP in Lattice QCD



Mainz/CLS

[Gérardin et al., Phys. Rev. D 100 (2019) 014510]

- $O(a)$ improved Wilson fermions
- Four lattice spacings: $a = 0.085 - 0.050\,\mathrm{fm}$
- Pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Isospin-breaking correction by ETMC added to error
- Simultaneous chiral and continuum extrapolation



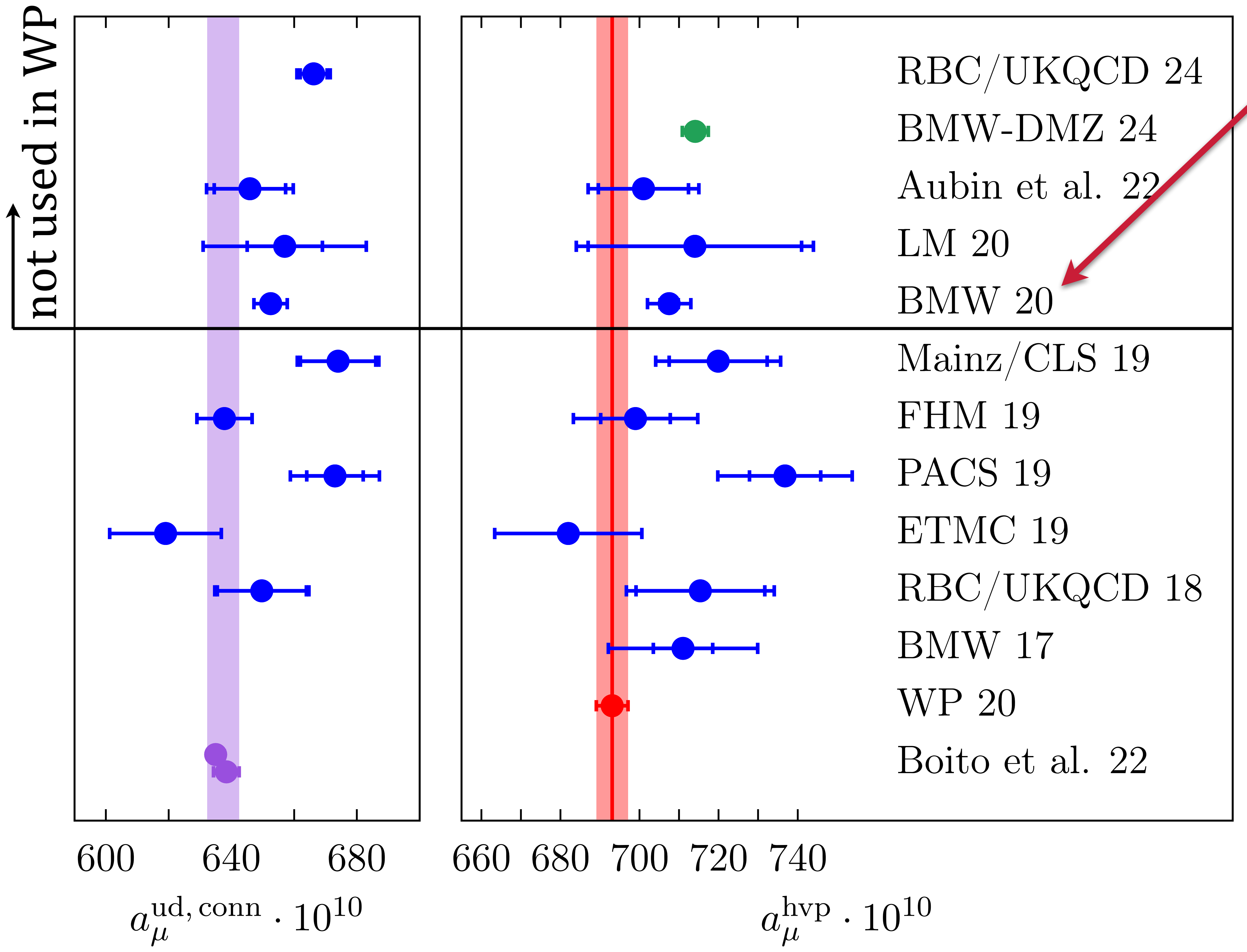
R-ratio:

WP 20: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

ud, conn: $a_\mu^{\rm ud,\,conn} = (632 - 642) \cdot 10^{-10}$

$a_\mu^{\rm hvp,\,LO} = (720.0 \pm 12.6 \pm 9.9) \cdot 10^{-10}$ [2.2%]

HVP in Lattice QCD



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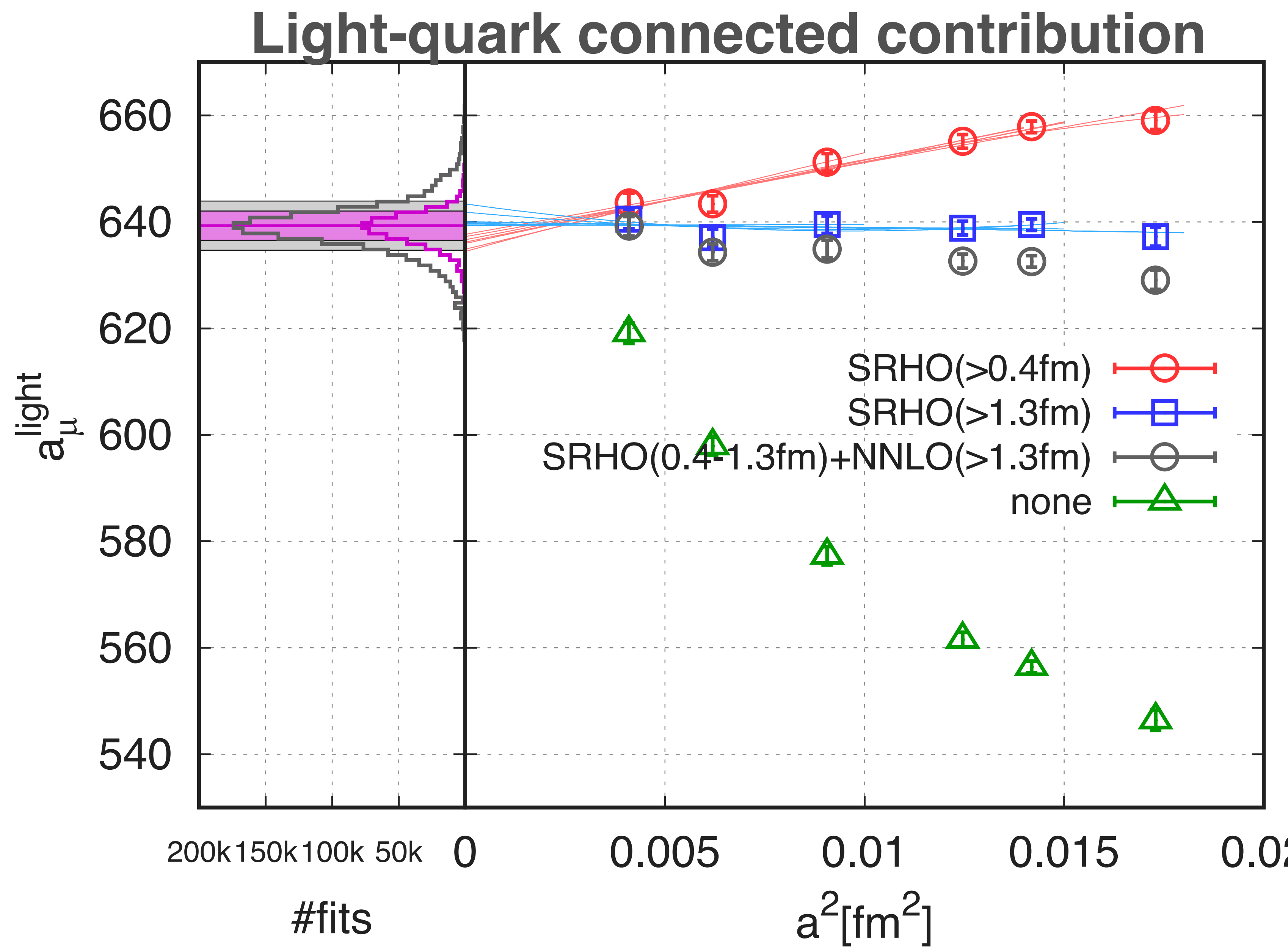
BMW

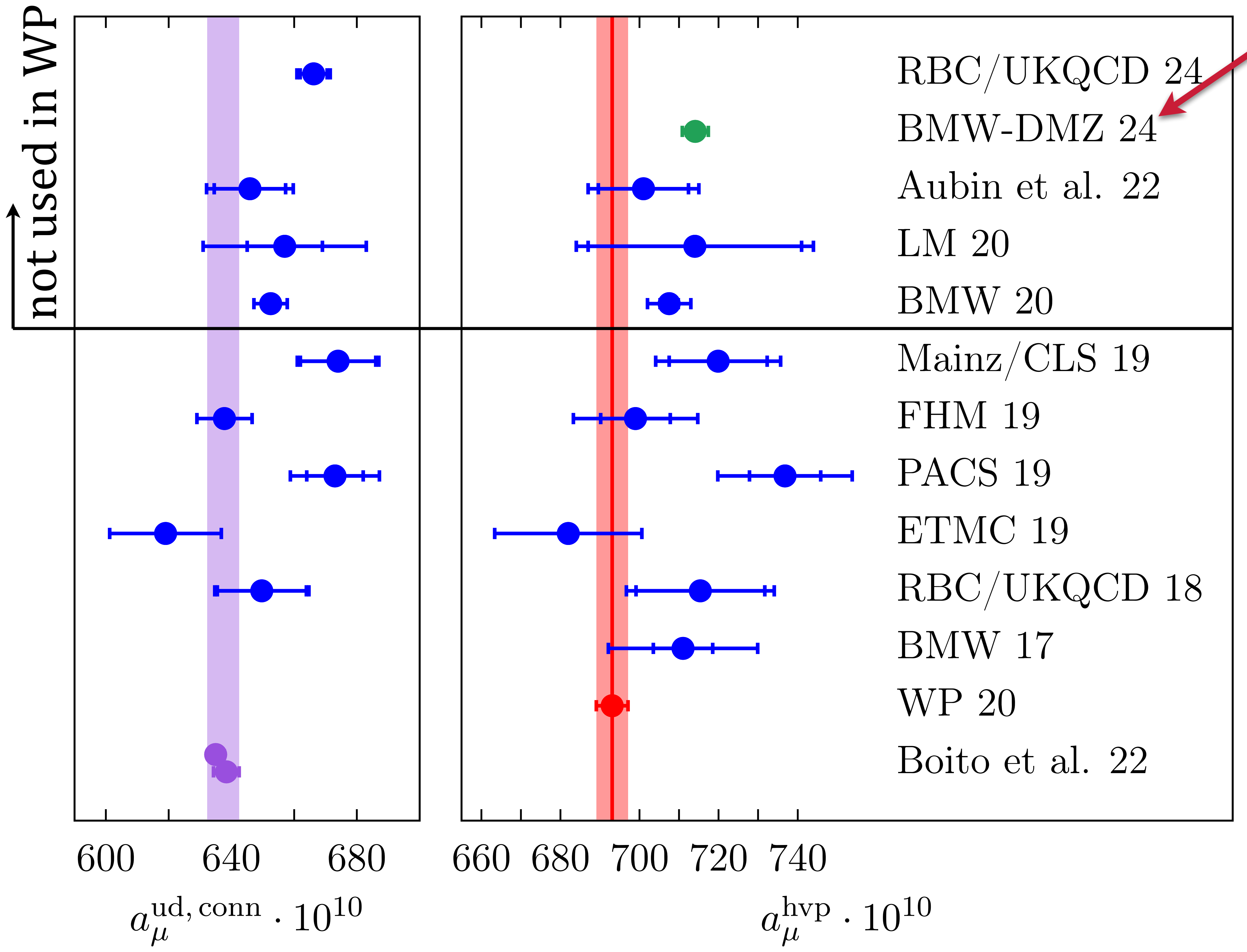
[Borsányi et al., Nature 593 (2021) 7857]

- Rooted staggered fermions
- Six lattice spacings: $a = 0.132 - 0.064$ fm
- Physical pion mass throughout
- Correct for taste-breaking before continuum extrapol'n
- Final result selected from distribution of different fits



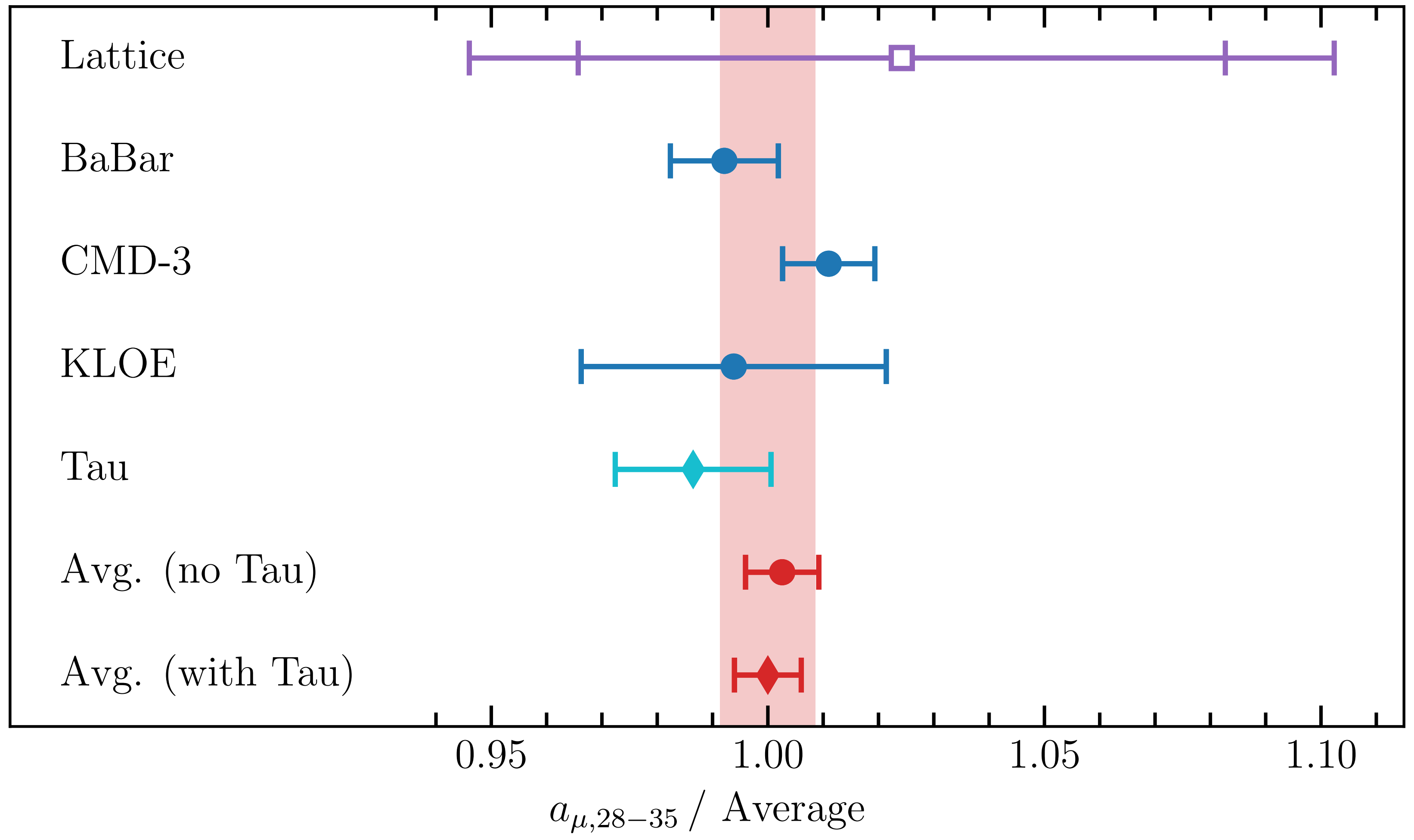
$a_\mu^{\rm hvp,\,LO} = (707.5 \pm 2.3 \pm 5.0)\cdot 10^{-10}$ [0.8%]

HVP in Lattice QCD



BMW-DMZ *[Boccaletti et al., arXiv:2407.10913]*

- Rooted staggered fermions
- Seven lattice spacings: $a = 0.132 - 0.048\,\mathrm{fm}$
- Increased statistics
- Hybrid calculation: use data-driven method to estimate long-distance tail of $G(t)$ for $t \geq 2.8\,\mathrm{fm}$ and reduce uncertainty due to finite-volume effects



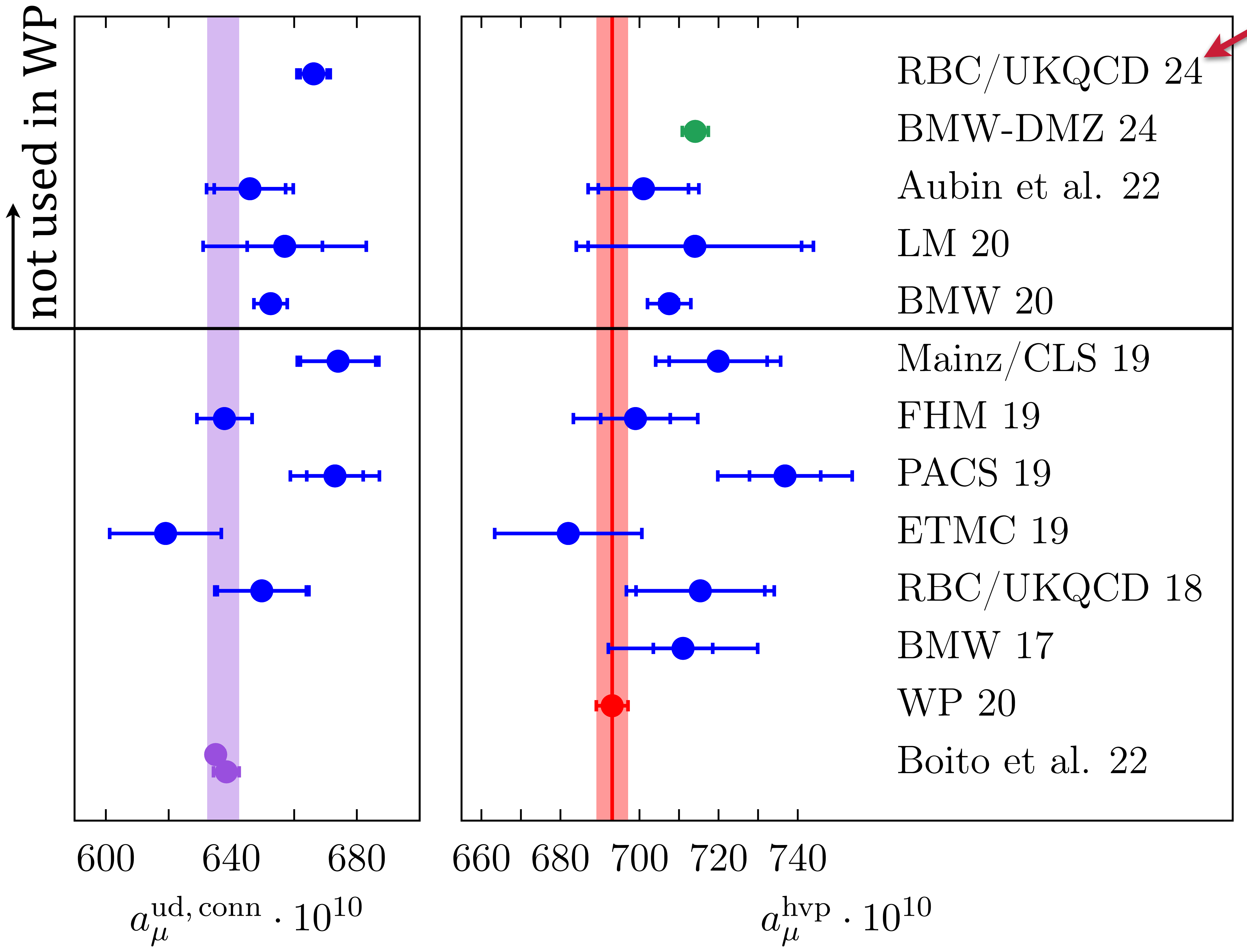
R-ratio:

WP 20: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

ud, conn: $a_\mu^{\rm ud,\,conn} = (632 - 642) \cdot 10^{-10}$

$a_\mu^{\rm hvp,\,LO} = (714.1 \pm 2.2 \pm 2.5) \cdot 10^{-10}$ [0.5%]

HVP in Lattice QCD



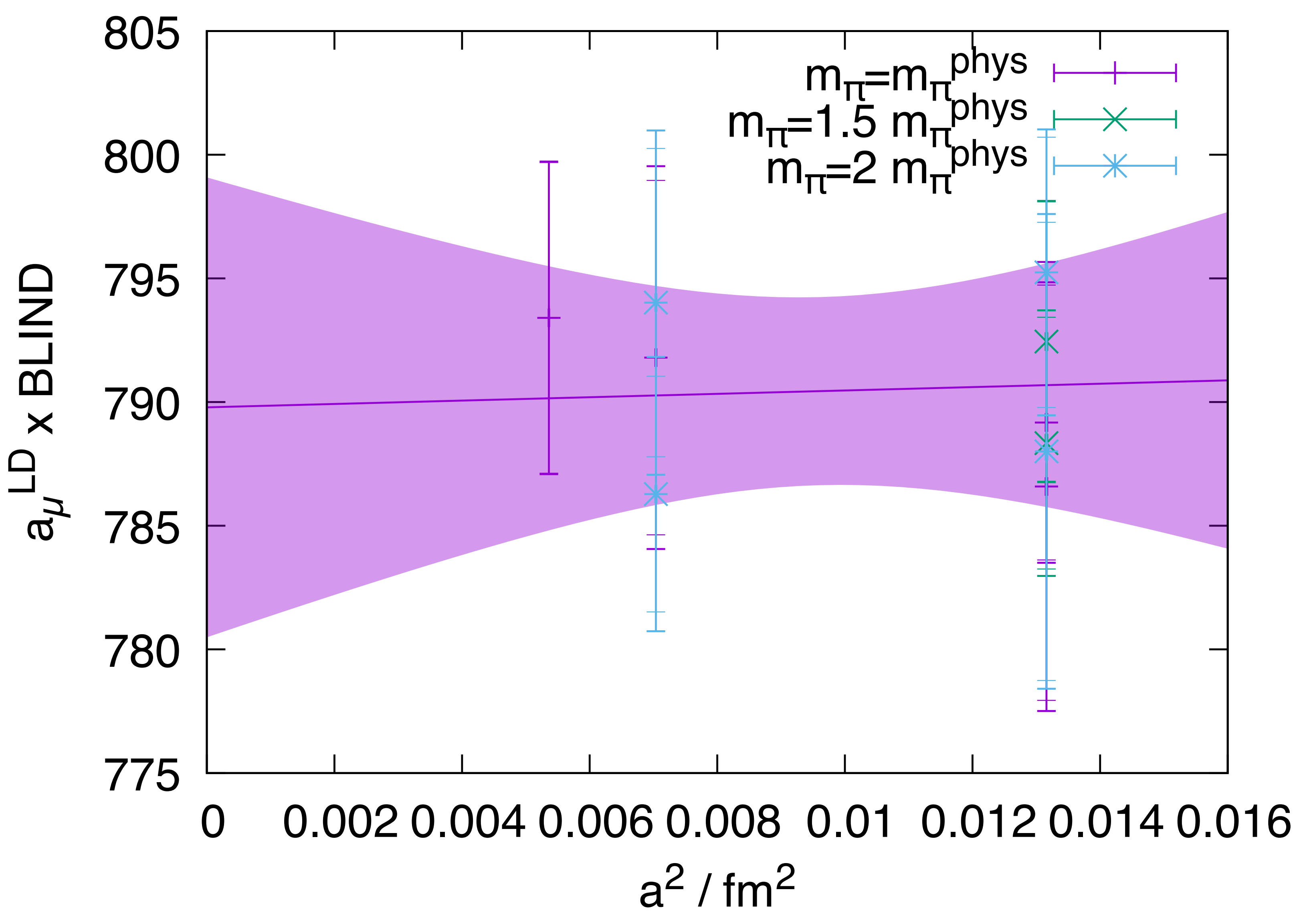
R-ratio:

WP 20: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0)\cdot 10^{-10}$ [0.6%]

ud, conn: $a_\mu^{\rm ud,\,conn} = (632 - 642)\cdot 10^{-10}$

RBC/UKQCD *[Ch. Lehner @ Lattice 2024]*

- Domain wall fermions
- Ten ensembles: $a = 0.114,\, 0.084,\, 0.073\,\mathrm{fm}$
- Pion masses: $m_\pi = 135,\, 210,\, 280\,\mathrm{MeV}$
- Spectral reconstruction of long-distance tail
- Verify analytic calculation of finite-volume corrections by comparing different volumes



$a_\mu^{\rm ud,\,conn} = (666.2 \pm 4.3 \pm 2.5)\cdot 10^{-10}$ [0.8%]

Window observables

Restrict integration over Euclidean time to sub-intervals
→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\text{hvp, win}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)\, W(t;\, t_0,\, t_1)$$

Short distance: $W^{\text{SD}}(t;\, t_0) = 1 - \Theta(t,\, t_0, \Delta)$

Intermediate distance: $W^{\text{ID}}(t;\, t_0,\, t_1) = \Theta(t,\, t_0, \Delta) - \Theta(t,\, t_1, \Delta)$

Long distance: $W^{\text{LD}}(t;\, t_1) = \Theta(t,\, t_1, \Delta)$

$$\Theta(t,\, t',\Delta) = \tfrac{1}{2}\,[1 + \tanh(t - t')/\Delta]$$

[RBC/UKQCD 2018]

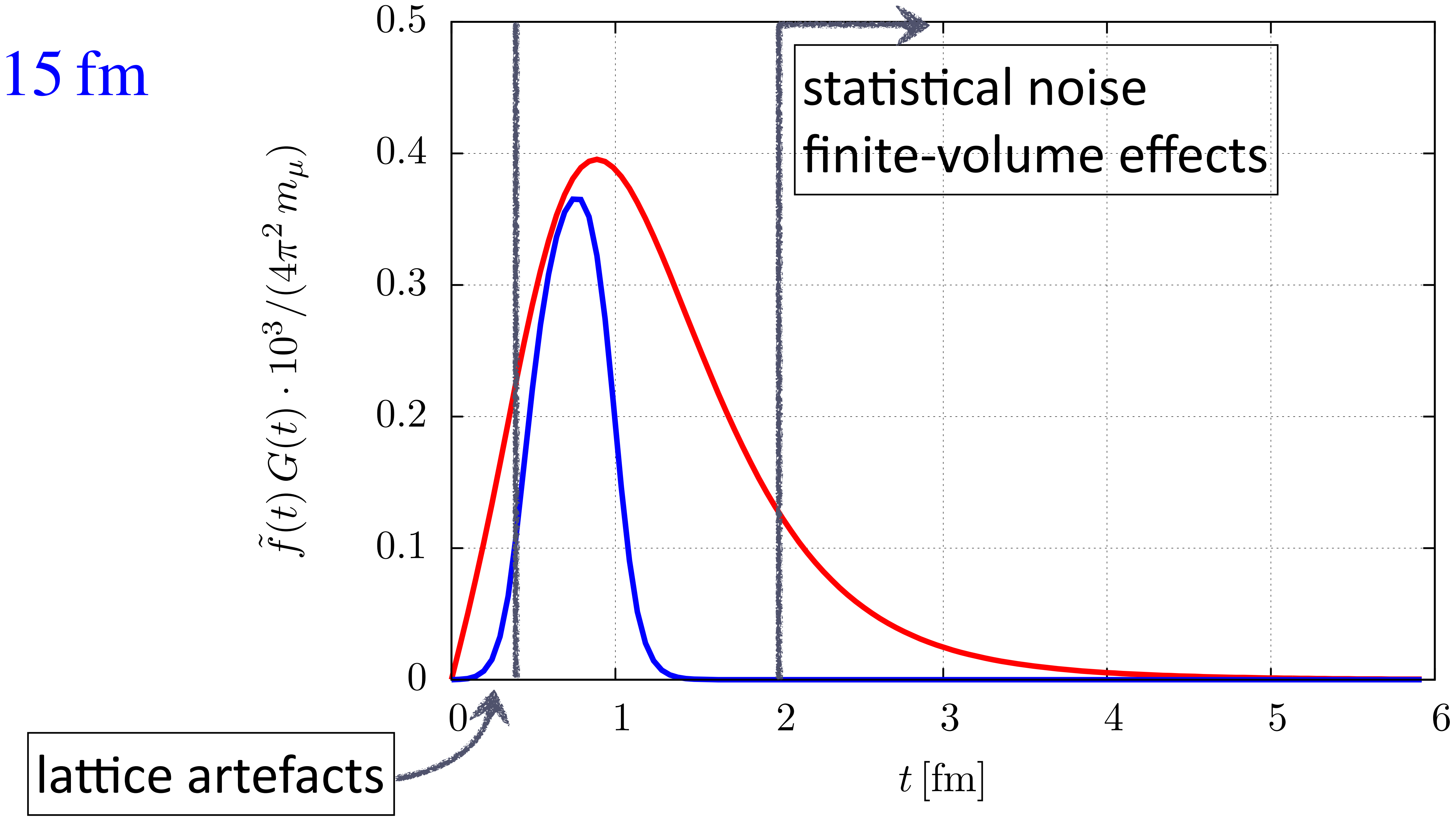
"Standard" choice: $t_0 = 0.4\,\text{fm},\ t_1 = 1.0\,\text{fm},\ \Delta = 0.15\,\text{fm}$

Intermediate window:

- Finite-volume correction reduced to 0.25%
- Uncertainty dominated by statistics
- Data-driven approach (excluding CMD-3):

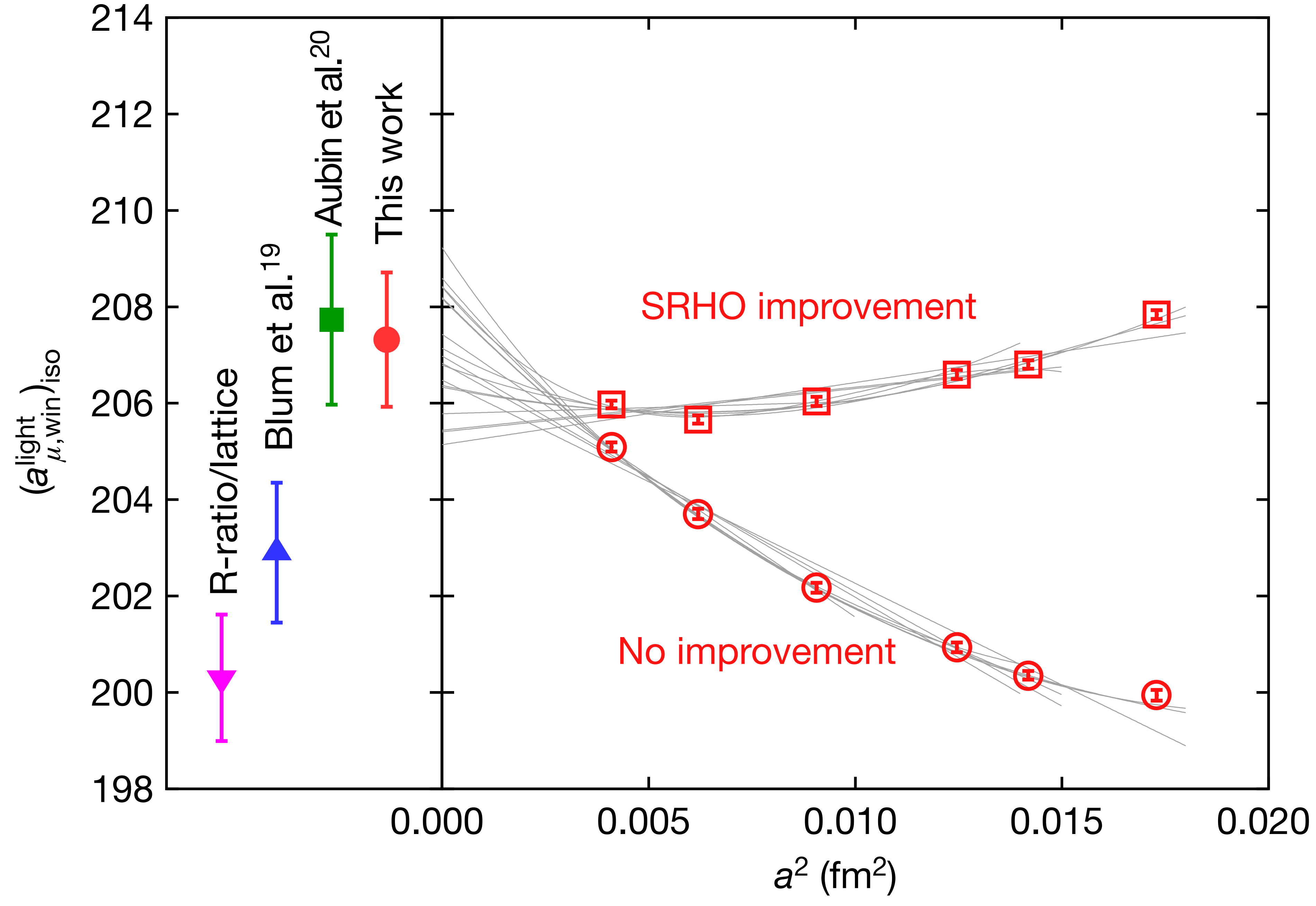
$$a_\mu^{\text{hvp, ID}} \equiv a_\mu^{\text{win}} = (229.4 \pm 1.4) \cdot 10^{-10}$$

[Colangelo et al., Phys Lett B833 (2022) 137313]



Intermediate window observable in Lattice QCD

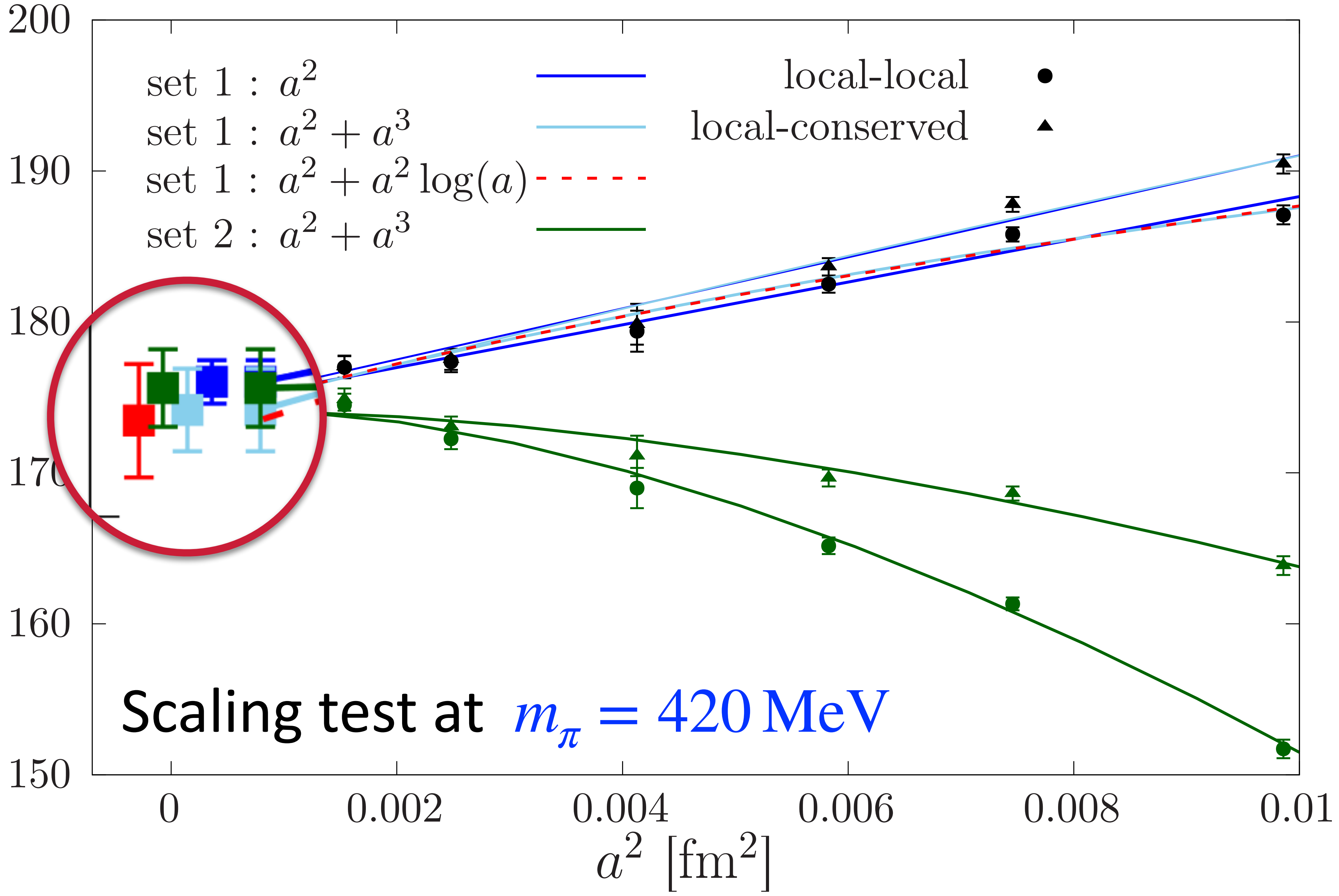
BMWc: Rooted staggered quarks



$$a_\mu^{\mathrm{win,ud}} = (207.3 \pm 0.4 \pm 1.3) \cdot 10^{-10}$$

[Borsányi et al., Nature 593 (2021) 7857]

Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks

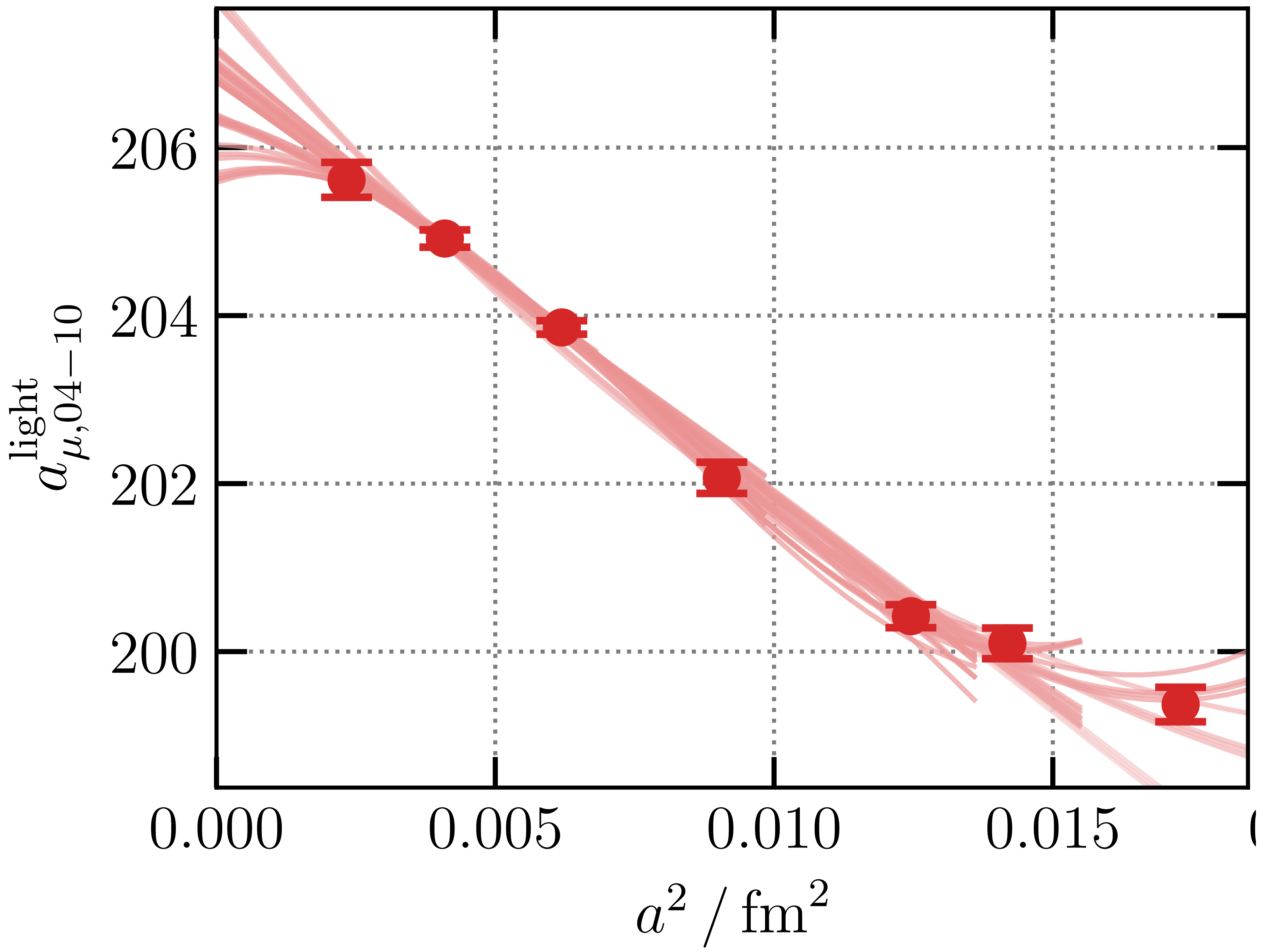


$$a_\mu^{\mathrm{win,ud}} = (207.0 \pm 0.8 \pm 1.2) \cdot 10^{-10}$$

[Cè et al., Phys Rev D106 (2022) 114502]

Intermediate window observable in Lattice QCD

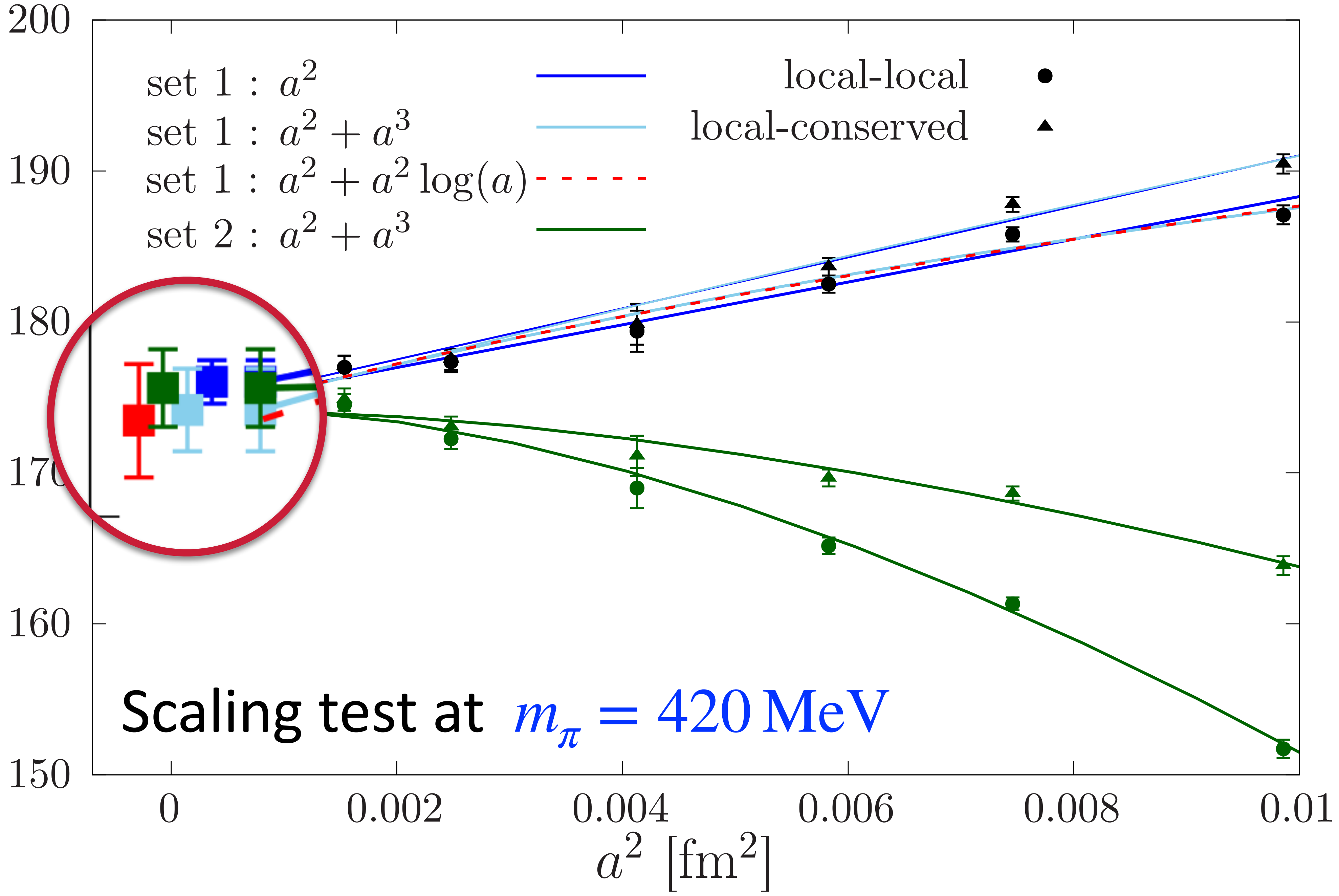
BMWc: 2024 update



$$a_\mu^{\text{win,ud}} = (206.57 \pm 0.25 \pm 0.65) \cdot 10^{-10}$$

[Boccaletti et al., arXiv:2407.10913]

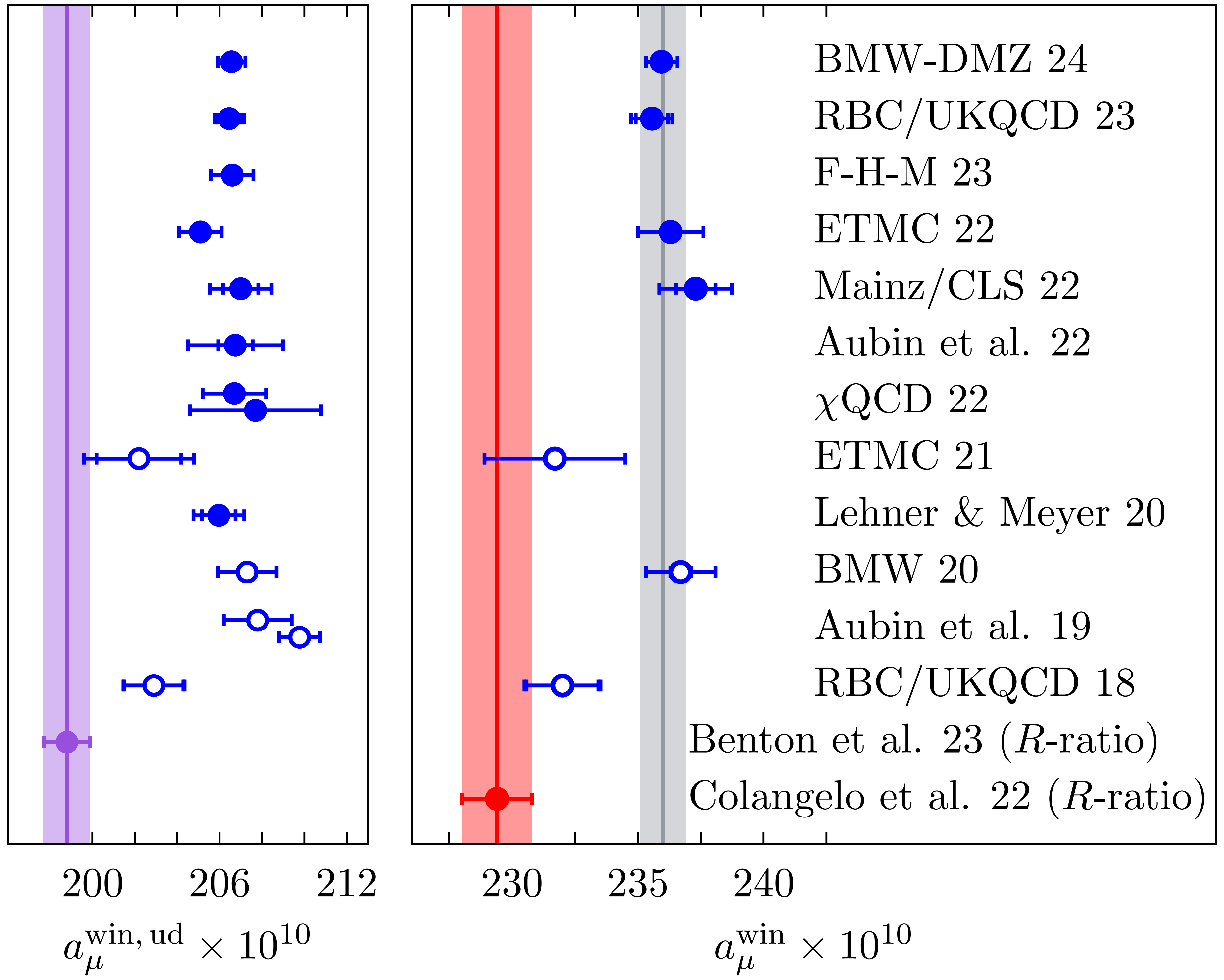
Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks



$$a_\mu^{\text{win,ud}} = (207.0 \pm 0.8 \pm 1.2) \cdot 10^{-10}$$

[Cè et al., Phys Rev D106 (2022) 114502]

Window observable: Lattice QCD vs. R-ratio



Left: dominant light-quark contribution to $a_\mu^{\rm win}$

Right: including sub-leading contributions

- Dominant light-quark contribution confirmed for wide range of discretisation with sub-percent precision
- Significant tension with results based on the R-ratio*

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

Lattice average: $a_\mu^{\rm win} = (236.0 \pm 0.9) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

- Tension of 4.0σ in the window observable evaluated from e^+e^- data* and four lattice calculations

$$a_\mu^{\rm win}\Big|_{\langle{\rm lat}\rangle} - a_\mu^{\rm win}\Big|_{e^+e^-} = (6.60 \pm 1.66) \cdot 10^{-10} \quad [4.0\,\sigma]$$

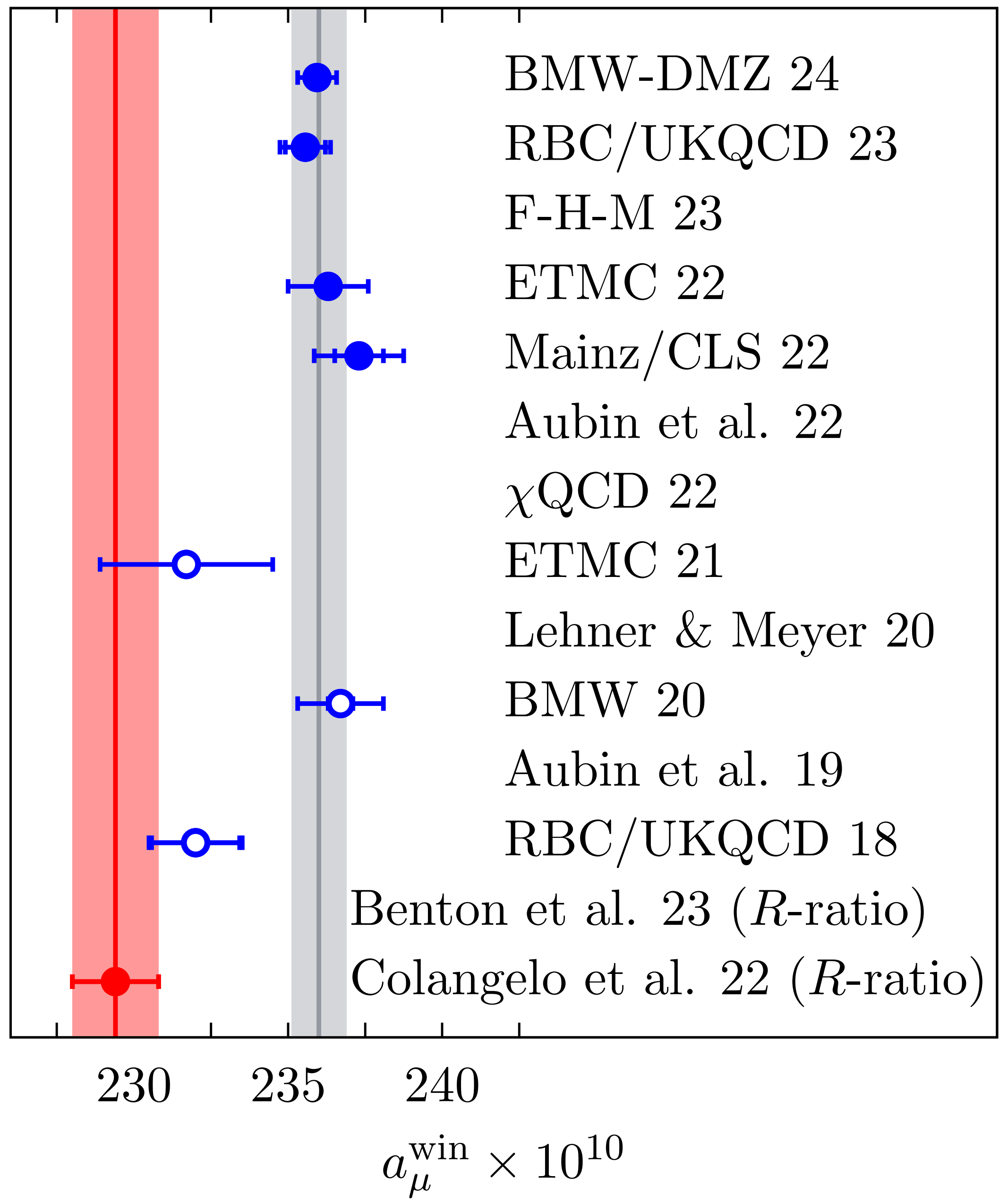
*excluding the CMD-3 result

What can we learn from $a_\mu^{\rm win}$?

Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k \sum_{\vec{x}} \left\langle j_k^{\rm em}(\vec{x},t)\, j_k^{\rm em}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R(s)^{\rm lat}\, s\, {\rm e}^{-\sqrt{s}t}$$

$$a_\mu^{\rm win}\big|_{\rm lat} > a_\mu^{\rm win}\big|_{e^+e^-} \quad \Rightarrow \quad R(s)^{\rm lat} > R(s)^{e^+e^-}$$ in some interval of $\sqrt{s}$



Interval $600 \leq \sqrt{s} \leq 900\,\mathrm{MeV}$ contributes the same fraction to $a_\mu^{\rm hvp}$ and $a_\mu^{\rm win}$

$\sqrt{s}$ interval	$a_\mu^{\rm hvp}$	$(a_\mu^{\rm hvp})^{\rm SD}$	$(a_\mu^{\rm hvp})^{\rm ID}$	$(a_\mu^{\rm hvp})^{\rm LD}$	$\bar{\Pi}(1\ {\rm GeV}^2)$
Below 0.6 GeV	15.5	1.5	5.5	23.5	8.2
0.6 to 0.9 GeV	58.3	23.1	54.9	65.4	52.6
Above 0.9 GeV	26.2	75.4	39.6	11.1	39.2
Total	100.0	100.0	100.0	100.0	100.0

[Cè et al., Phys Rev D106 (2022) 114502]

What can we learn from a_μ^{win} ?

- Phenomenological model for R-ratio predicts [Mainz/CLS, Cè et al., Phys Rev D 106 (2022) 114502]

$$\sqrt{s} = 600 - 900\,\text{MeV}: \qquad \frac{R(s)^{\text{lat}}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\text{hvp}})^{\text{lat}}}{(a_\mu^{\text{hvp}})^{e^+e^-}} \approx \frac{(a_\mu^{\text{win}})^{\text{lat}}}{(a_\mu^{\text{win}})^{e^+e^-}} = 1 + 0.6\epsilon$$

- Lattice average vs. R-ratio: $(a_\mu^{\text{win}})^{\text{lat}}/(a_\mu^{\text{win}})^{e^+e^-} = 1.029(7)$

  $\Rightarrow$ $R(s)^{\text{lat}}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,\text{MeV}$

- If confirmed, it would imply that BMW-2020 might be too low….

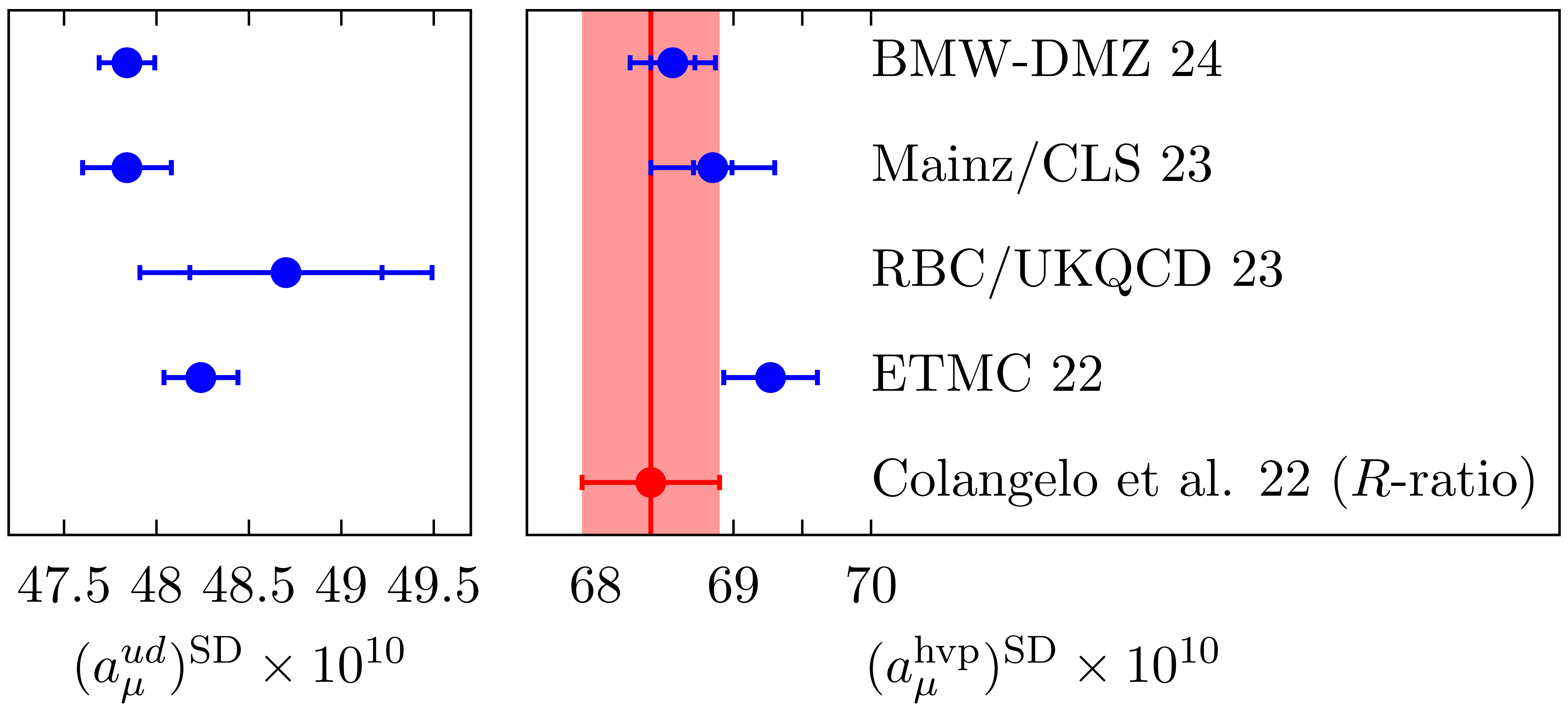
Similar conclusions

- Dispersive treatment of pion form factor [Colangelo, Hoferichter, Stoffer, PLB 814 (2021) 136073]
- "Energy-smeared" R-ratio from lattice data [ETMC, Alexandrou et al., PRL 130 (2023) 241901]

More windows….

Short-distance window

- Finite-volume correction negligible
- Uncertainty dominated by discretisation effects
- 5% enhancement of $R(s)^{\rm lat}$ for $\sqrt{s} = 0.6 - 0.9\,{\rm GeV}$ increases $(a_\mu^{\rm win})^{\rm SD}$ by $+1 \times 10^{-10}$
- Expectation confirmed by lattice calculations



Long-distance window

- New result by RBC/UKQCD for light-quark contribution: $(a_\mu^{\rm ud,\,conn})^{\rm LD} = (411.4 \pm 4.3 \pm 2.3) \cdot 10^{-10}$
  [Ch. Lehner @ Lattice 2024]

One-sided long-distance window

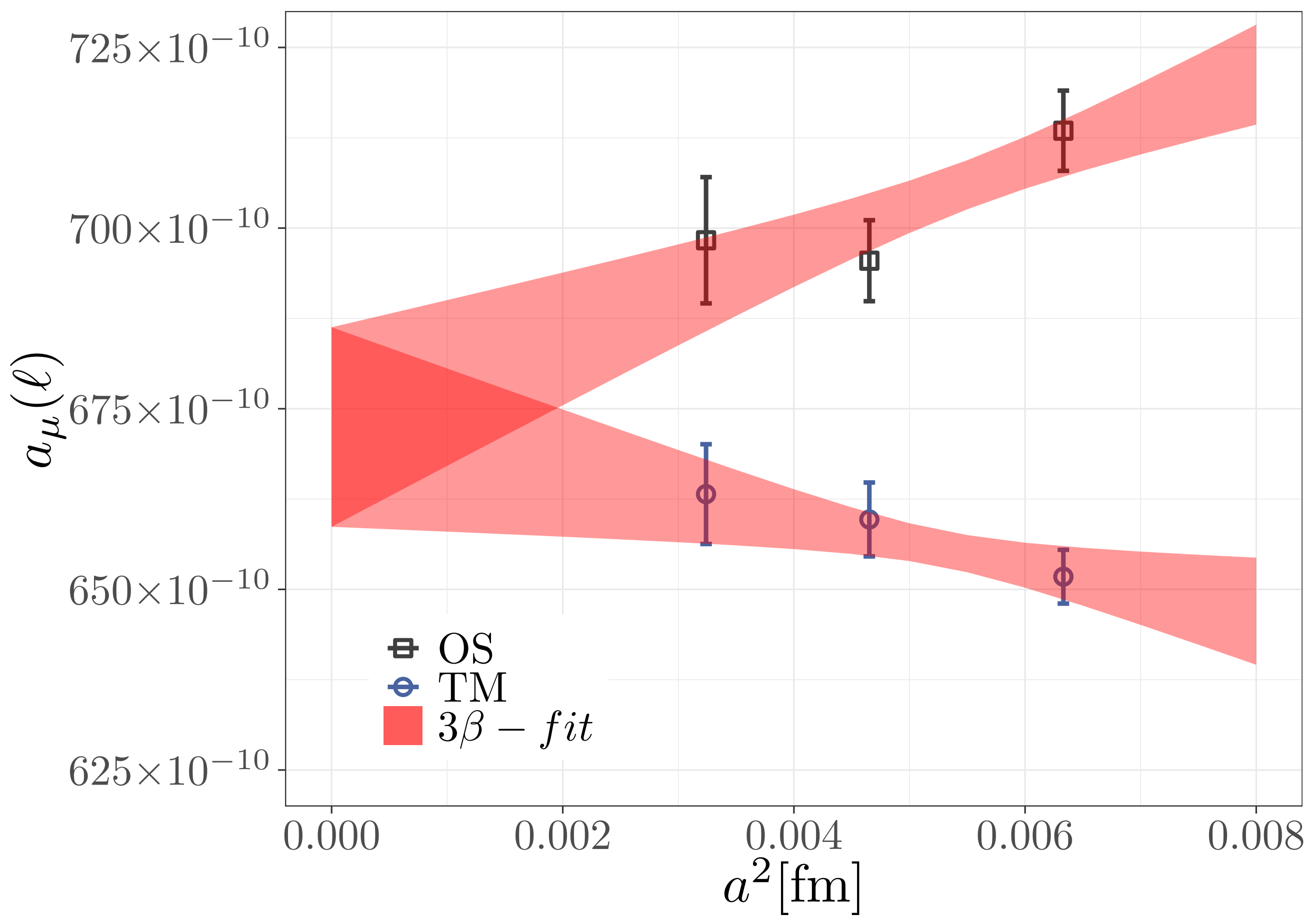
- BMW-DMZ compute window from $t = 0 - 2.8\,{\rm fm}$: $(a_\mu^{\rm hvp})^{00\to28} = (686.4 \pm 1.9 \pm 2.3) \cdot 10^{-10}$
- Combine with data-driven method for $t = 2.8\,{\rm fm} \to \infty$

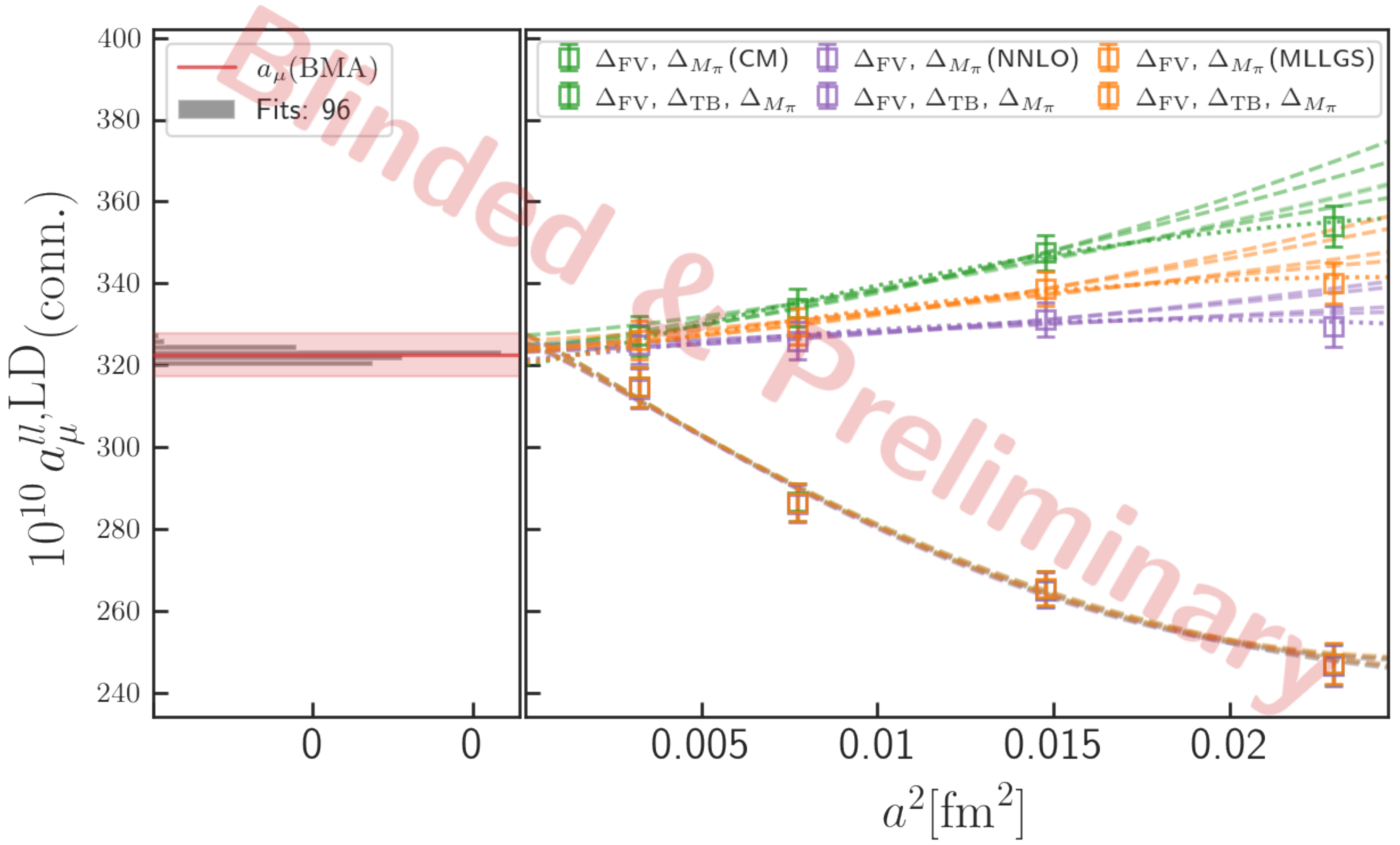
[Boccaletti et al., arXiv:2407.10913]

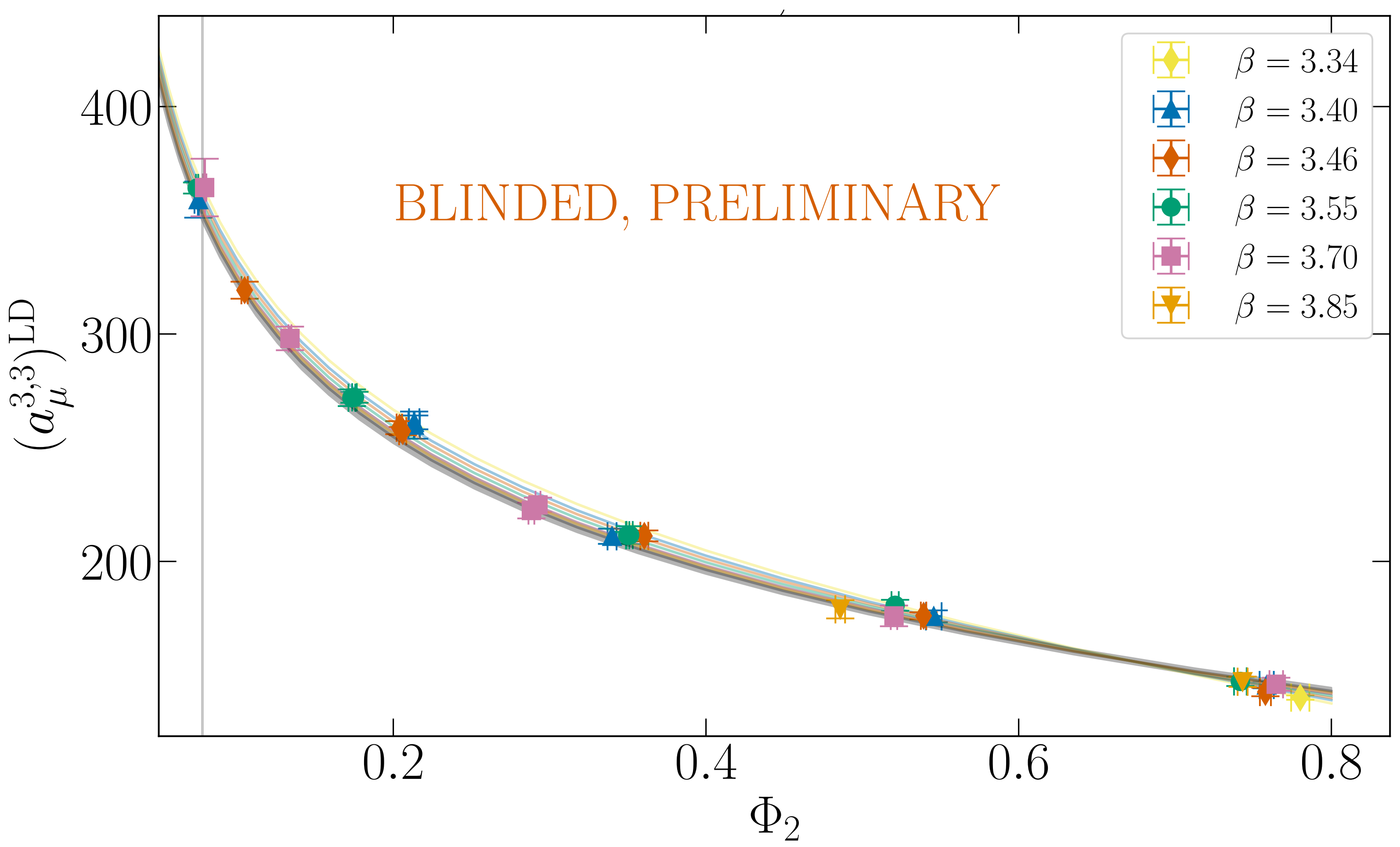
Ongoing calculations

Several collaborations prepare for publication of results with $\lesssim 1\,\%$ by end of 2024

- Deadline for inclusion in White Paper Mark II: 15 Nov 2024
- Blinding now routinely applied







ETMC

- twisted-mass Wilson fermions
- four lattice spacings

[Garofalo @ Lattice 2024]

Fermilab-HPQCD-MILC

- staggered fermions
- five lattice spacings

[Lynch @ Lattice 2024]

Mainz/CLS

- $\mathrm{O}(a)$ improved Wilson fermions
- six lattice spacings

[Kuberski @ Lattice 2024]

Ongoing calculations

Several collaborations prepare for publication of results with $\lesssim 1\,\%$ by end of 2024

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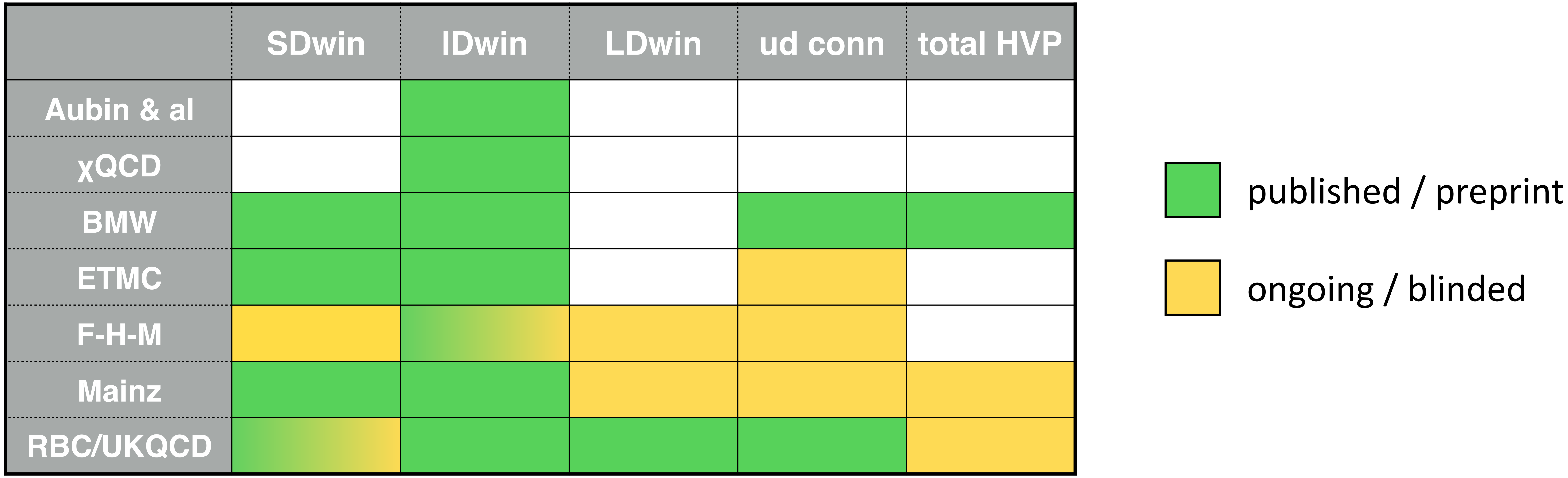
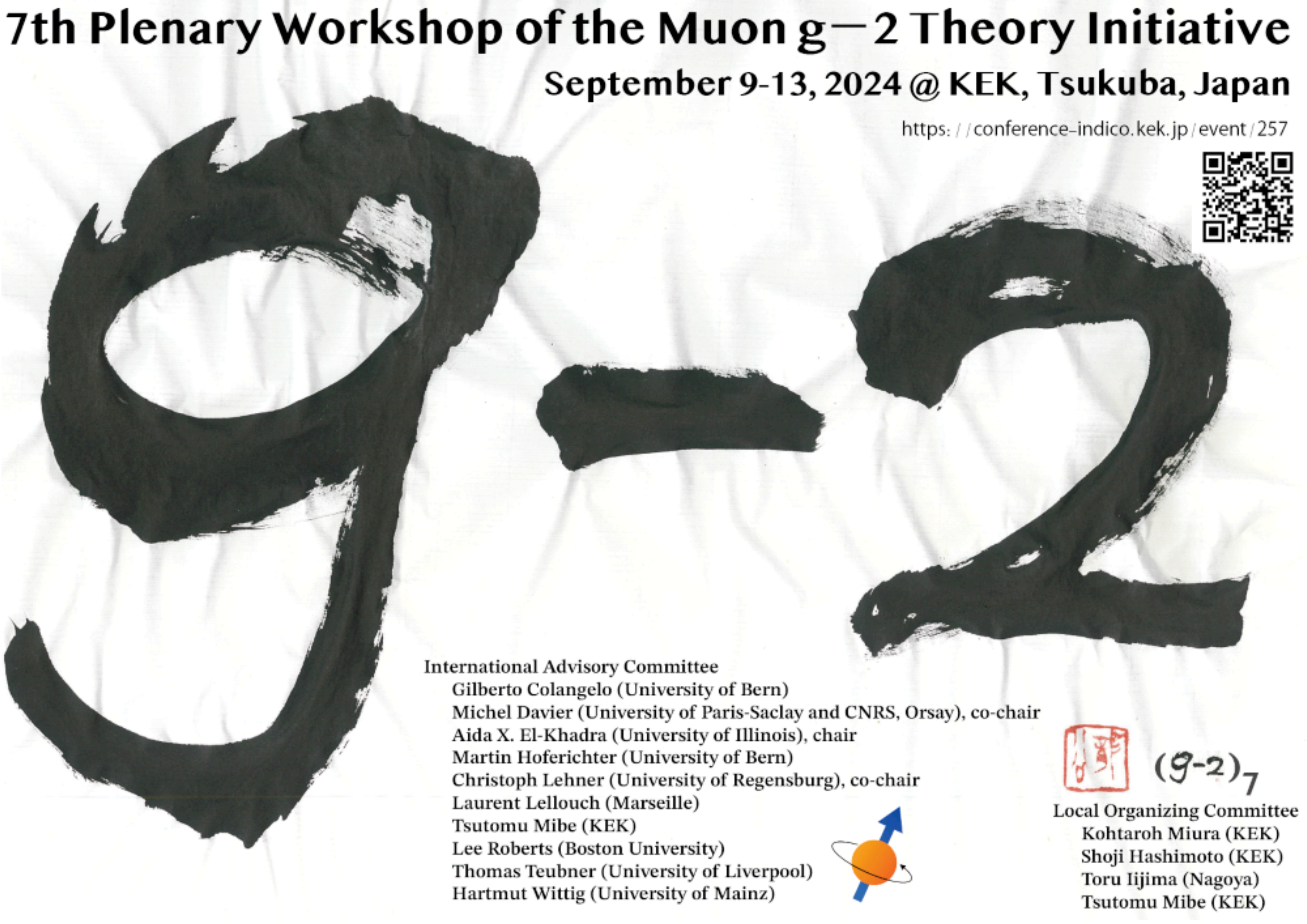
“Landscape” of lattice calculations of the HVP contribution:




7th Plenary Workshop of the Muon g−2 Theory Initiative
September 9-13, 2024 @ KEK, Tsukuba, Japan
https://conference-indico.kek.jp/event/257
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Summary and outlook

“Agreement is not a useful scientific concept”

Guido Martinelli, ca. 1994

Summary and outlook

No straightforward interpretation of the Fermilab E989 experiment

Discrepant determinations of the HVP contribution:

- Tensions between lattice QCD and the spectral function determined from e^+e^- data*
- Tension in $\pi^+\pi^-$channel between BaBar vs. KLOE and CMD-3 vs. all other results

Lattice results for ID window are remarkably consistent

More lattice results to come for total HVP contribution with $\lesssim 1\,\%$ precision

Experimental measurement of the HVP contribution by MUonE experiment

Second White Paper out by the time E989 release their final result (early 2025)

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*pre-2023